Cool But Not (necessarily) Supercool: RS Phase Transition

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Outline

- Brief review RS and phase transition
 - Strong constraint on curvature (N in dual theory)
 - Supercooling in perturbative scenarios
 - Important for viability of RS
 - Important due to potential GW implications
 - Ideas to weaken constraint
 - Potential implications for gravity wave signal
- Recent work with Mishra:
 - I: Add self-interactions to GW field
 - Original scenario assumed only a mass term
 - II: Mimic KKLT/warped compactification
 - Phase transition driven by "shrinking circle"
 - Strong back-reaction on IR geometry
 - Black brane phase very different at low temperature
 - Potential implications for gravity wave signautre
- Conclude

Review: Phase Transition Deconfined to Confined High Temperature BB to RS

 Two solutions to EE at finite temperature with neg 5d cc

$$\ell_{\text{AdS}} = 1$$

$$r = 0$$

$$r = r_{\text{ir}}$$

$$\begin{split} \ell_{\text{AdS}} &= 1 \\ \textbf{BBB} \quad ds^2 &= -\bar{e}^{2r} dt^2 \left(1 - e^{4(r-r_h)} \right) + \bar{e}^{2r} d\vec{x}^2 + \frac{dr^2}{1 - e^{4(r-r_h)}} & r = 0 \quad r = r_h \\ 0 &\leq r \leq r_h \\ \text{An approximate solution, exact in the} \\ \text{Imit of UV brane sent to boundary.} & \text{Temperature:} \quad T = \frac{1}{\pi} e^{-r_h} & \text{UV} & \text{Horizon} \end{split}$$

Deconfined

w/o Stabilization, BB is Thermodynamically Preferred

Creminelli, Nicolis, Rattazzi

$$F_{BB} - F_{RS} = -2\pi^4 M_5^3 T^4$$



 $\Delta F < 0$ at all temperatures.

Without additional ingredients, black brane always preferred Would be no phase transition

With Stabilization

- Without stabilization phase transition never happens
 - Potential for radion is flate (in flat space)
- With stabilization, extra contribution from stabilizing potential
 - Calculate in 4d or 5d?
 - All calculations so far do a 4d calculation
- For perturbative consistency require a "light" radion
- Assumption is we can neglect KK modes
 - Turns out rate always suppressed with this assumption
 - Can still be phenomenologically viable
 - But constrained and supercooled
 - Can be strong constraint

(In our second model we inch toward a strongly coupled iR where KK modes should play a role)

With Stabilizing Scalar

 χ : a 5D scalar with brane localized and bulk potential

$$S_{\chi} = \int d^5 x \sqrt{g} \left(-\frac{1}{2} \left(\partial \chi \right)^2 - V_B(\chi) \right) - \sum_i \int d^4 x \sqrt{g_i} V_i(\chi)$$

 $\Delta F = 0$ at $T = T_c$

 $T > T_c$: BB geometry (deconfined phase) favored.

 $T < T_c$: RS geometry (confined phase) favored.

 $F_{RS} \approx V(\varphi_{\min}) + \mathcal{O}(T^4)$, $V(\varphi_{\min}) < 0$ $F_{BB} - F_{RS} = -2\pi^4 M_5^3 T^4 - V(\varphi_{\min}) \equiv 2\pi^4 M_5^3 (T_c^4 - T^4)$





Simple Examples

$$\begin{split} V_B(\chi) &= 2\epsilon\chi^2 & \blacktriangle & V_B(\chi) = 2\epsilon\chi^2 & \blacksquare \\ \chi_{\rm UV} &= v_{\rm uv}, \chi_{\rm IR} = v_{\rm IR}, & \chi_{\rm UV} = v_{\rm uv}, \chi'_{\rm IR} = -\alpha \\ V(\varphi) &\sim \varphi^4 \left(1 - \left(\frac{\varphi}{\varphi_{\rm min}}\right)^\epsilon\right)^2 - \epsilon\varphi^4 & V(\varphi) \sim \varphi^4 \left(1 - \frac{1}{1 + \epsilon/4} \left(\frac{\varphi}{\varphi_{\rm min}}\right)^\epsilon\right) \\ T_c &\sim \epsilon^{3/8}\varphi_{\rm min} & T_c \sim \epsilon^{1/4}\varphi_{\rm min} \end{split}$$

- •For validity of 4d EFT (neglect KK) modes ϵ must be small
- •The critical temperature is below the minimum value of $\boldsymbol{\Phi}$

Dynamics Phase Transition

- First order!
- Can lead to visible GW signal



Assume bubble action dominated by radion



Phase transition completes when $\Gamma>H^4$, $H^2\sim
ho_{
m vac}/M_{
m pl}^2$, $ho_{
m vac}\sim 2\pi^4 M_5^3 T_c^4$

Examples

$$\begin{split} V(\varphi) &\sim \varphi^4 \left(1 - \left(\frac{\varphi}{\varphi_{\min}}\right)^{\epsilon} \right)^2 - \epsilon \varphi^4 \qquad S_3/T \approx 0.13 \frac{N^2}{\epsilon^{9/8} (v_{ir}/N)^{3/2}} \frac{T_c/T}{(1 - (T/T_c)^4)^2} \\ V(\varphi) &\sim \varphi^4 \left(1 - \frac{1}{1 + \epsilon/4} \left(\frac{\varphi}{\varphi_{\min}}\right)^{\epsilon} \right) \qquad S_3/T \approx 8 \frac{N^2}{(\epsilon\lambda)^{3/4}} \frac{T_c/T}{(1 - (T/T_c)^4)^2} \end{split}$$

General structure:
$$\Gamma \sim \exp\left(-\frac{N^2}{\delta}f(T/T_c)\right)$$

- Small ϵ makes big bubble size
 - Associated with light radion, small EFT breaking
- Large N makes overall action too big
 - Associated with viable phenomenology, justifiable geometric interpretaiton
- f of order unity associated with thin wall approximation

Rate Generally Too Small

- Might not complete at all
- Or might complete only at very low temperature
- Either way, without modification puts bound on N and viability of RS
- (Note: small rate generally leads to big GW signal)
- How to avoid?
- Bigger ε (most solutions so far)
 - Leads to viable theories but still strong bound
- Smaller N
 - Smaller N in IR than UV; still have to accommodate phenomenology
- Use thick wall ? (dictated by theory)
- Are there solutions within "radion EFT"
- Are there solutions with perturbative reliability?

How generic are these results? Are there motivated alternatives that improve situation?

- Increase δ (in previous models some power of ϵ)
 - IR contribution dominates instanton action so only need larger running in IR
 - Can get hierarchy due to small CFT breaking with small $\boldsymbol{\delta}$
 - But smaller bubble action due to bigger effective δ in IR
- One example by Csaki, Geller, Heller-Algazi, Ismail
 - Relevant Dilaton Stabilization
- Another by Agashe, Du, Ekterachian, Kumar, Sundrum
 - Transition between two CFTs
- More truly 5d calculation Gustafson, Hite, Hubisz, Sambasivam, Umuth-Jockey
- Our example (w/Mishra) similar in spirit to latter but more generic
 - We simply include an interaction term
 - Just cubic for simplicity but sufficiently general to illustrate point
 - Small contribution in UV when field small
 - Increases due to RG flor in IR

End result can be Large IR CFT breaking

Our Model I: Add Self-interactions in Bulk

$$V_B(\chi) = 2\epsilon_2 \chi^2 + \frac{4}{3}\epsilon_3 \chi^3$$
$$V_{\rm uv} = \beta \left(\chi - v_{\rm uv}\right)^2, \beta \to \infty$$

Choose $\epsilon_2 < 0, \epsilon_3 < 0$

to grow the deformation in the IR

 $V_{
m ir}(\chi)=2lpha_{
m ir}\chi$

$$S = \int d^4x \sqrt{g} \left(-12M_5^3 \left(\partial \varphi \right)^2 - V(\varphi) \right) , \quad \varphi = e^{-r_{\rm ir}}$$

$$V(\varphi) = 24M_5^3 \kappa^4 \varphi^4 \left(1 + \frac{a_2}{24M_5^3 \kappa^4} \frac{\lambda \varphi^{\epsilon_2}}{1 - \lambda \varphi^{\epsilon_2}} - \frac{a_3}{24M_5^3 \kappa^4} \log(1 - \lambda \varphi^{\epsilon_2}) \right)$$

$$\lambda = \frac{v_{\rm uv} \epsilon_{32}}{1 + v_{\rm uv} \epsilon_{32}}, \quad \epsilon_{32} = \frac{\epsilon_3}{\epsilon_2}, \quad a_2 = -\frac{1}{32} \epsilon_2 \alpha_{\rm ir}^2 - \frac{\epsilon_2}{\epsilon_3} \alpha_{\rm ir} + 2\alpha_{\rm ir}, \quad a_3 = \frac{1}{2} \frac{\epsilon_2}{\epsilon_3} \alpha_{\rm ir}$$

In Original Coordinate (approx) Solution

$$egin{aligned} V_B(\chi) &= 2\epsilon_2\chi^2 + rac{4}{3}\epsilon_3\chi^3 \ V_{
m uv} &= eta\left(\chi - v_{
m uv}
ight)^2, eta
ightarrow \infty \end{aligned}$$

 $V_{
m ir}(\chi)=2lpha_{
m ir}\chi$

Approximate solutions

 $ert \epsilon_2 ert \ll 1, v_{
m uv} \ll 1, r_{
m ir} \gg 1, ert \epsilon_2 ert r_{
m ir} \lesssim 1, ert \epsilon_3 ert r_{
m ir} \lesssim 1$ $r_{
m ir} \rightarrow r_h$ (for the BB solution)

Choose $\epsilon_2 < 0, \epsilon_3 < 0$

to grow the deformation in the IR

$$\begin{split} \chi_{\rm RS}(r) &= -\frac{\alpha_{\rm ir}}{4} e^{4(r-r_{\rm ir})} + \frac{v_{\rm uv}e^{-\epsilon_2 r}}{1+v_{\rm uv}\epsilon_3 \left(\frac{1-e^{-\epsilon_2 r}}{\epsilon_2}\right)} , \quad 0 \le r \le r_{\rm ir} \\ \chi_{\rm BB}(r) &= \frac{v_{\rm uv}e^{-\epsilon_2 r}}{1+v_{\rm uv}\epsilon_3 \left(\frac{1-e^{-\epsilon_2 r}}{\epsilon_2}\right)} , \qquad 0 \le r \le r_h \end{split}$$



Radion Potential

$$V(\varphi) = 24M_5^3 \kappa^4 \varphi^4 \left(1 + \frac{a_2}{24M_5^3 \kappa^4} \frac{\lambda \varphi^{\epsilon_2}}{1 - \lambda \varphi^{\epsilon_2}} - \frac{a_3}{24M_5^3 \kappa^4} \log(1 - \lambda \varphi^{\epsilon_2}) \right)$$

$$\begin{split} V(\varphi) &= \varphi^4 \left(b_0 + b_1 \lambda \varphi^{\epsilon_2} + b_2 \lambda^2 \varphi^{2\epsilon_2} + b_3 \lambda^3 \varphi^{3\epsilon_2} + \cdots \right) \\ b_0 &= 24 M_5^3 \kappa^4 \\ b_1 &= -v_{\rm uv} \left(\frac{1}{2} \alpha_{\rm ir} - 2\alpha_{\rm ir} \frac{\epsilon_3}{\epsilon_2} + \frac{1}{32} \alpha_{\rm ir}^2 \epsilon_3 \right) \left(1 + \frac{v_{\rm uv} \epsilon_3}{\epsilon_2} \right)^{-1} , \\ b_2 &= -v_{\rm uv} \left(\frac{v_{\rm uv} \epsilon_3}{\epsilon_2} \right) \left(\frac{3}{4} \alpha_{\rm ir} - 2\alpha_{\rm ir} \frac{\epsilon_3}{\epsilon_2} + \frac{1}{32} \alpha_{\rm ir}^2 \epsilon_3 \right) \left(1 + \frac{v_{\rm uv} \epsilon_3}{\epsilon_2} \right)^{-2} , \\ b_3 &= -v_{\rm uv} \left(\frac{v_{\rm uv} \epsilon_3}{\epsilon_2} \right)^2 \left(\frac{3}{4} \alpha_{\rm ir} - 2\alpha_{\rm ir} \frac{\epsilon_3}{\epsilon_2} + \frac{1}{32} \alpha_{\rm ir}^2 \epsilon_3 \right) \left(1 + \frac{v_{\rm uv} \epsilon_3}{\epsilon_2} \right)^{-2} . \end{split}$$

 $\epsilon_3=0$, only b_0, b_1 are non-zero.

• When $\lambda \sim \epsilon$, V $\sim \epsilon$

 $\epsilon_3 \neq 0$: many terms.

Potential can be ~1 when Φ is large Can't go all the way to nonperturbative But lesson is clear

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In more detail



- Radion potential is deeper
 - here minimum shifts slightly but can also fix minimum to see same effect
- Point is this is more realistic model of strong IR breaking
- Different terms in radion potential can balance
- Not as suppressed: mimics truly strong CFT breaking in IR

Resulting Reduction in Bounce Action And Less Restrictive Bound on N



Important Consequence for GW

- Chief lesson is that less supercooling, less strongly first order
- Implies weaker GW signal



New (and more radically different) Model

Designed to mimic warped compactification

What Drives IR Brane?



- KKLT has a warped dimension
- Also five compact dimensions
- Warp factor essentially due to shrinking S₂ due to decreasing flux as we move to IR (dual)
- At some point it caps off
- Effectively reducing N (of SU(N))
 - Strong backreaction allows for nonperturbative regime so that no longer N² suppressed

Buchel wrote a "bestiary" of black hole phases

He found (even w/o a GW field) that phase transition will occur His model effectively 5d RS-like theory with 7 additional scalars We asked if we can get essence of result with a simplified model



Figure 17: Phase diagram in the canonical ensemble at $\mu/\Lambda = 0$: the reduced free energy density $\hat{\mathcal{F}}$, see (6.1), versus the reduced temperature T/Λ for different states in the theory. Vertical dashed lines indicate critical temperatures T_c (black) for the confinement-deconfinement phase transition, $T_{\chi SB}$ (red) for the onset of the spontaneous chiral symmetry breaking, and T_u (brown) for the bifurcation point of the \mathcal{T}^s_{decon} states with positive/negative specific heat.

Model II

Critically We Will Include Backreaction

$$\begin{split} S &= S_{\rm GR} + S_{\phi} + S_{\rm bdy} ,\\ S_{\rm GR} &= 2M_5^3 \int \mathrm{d}^5 x \sqrt{-g} \Big(\left(1 - \phi/\phi_c\right)^n \, R - 2 \left(1 - \phi/\phi_c\right)^m \, \Lambda \Big) \\ S_{\phi} &= 2M_5^3 \int \mathrm{d}^5 x \sqrt{-g} \Big(-a(\partial\phi)^2 - v(\phi) \Big) \, . \qquad \ell_{\rm AdS} = 1 \end{split}$$

$$v(\phi) = 2\epsilon\phi^2, \epsilon < 0$$

- Number of "colors" changes as ψ grows in IR
- Leading to small coefficient of 5d R in IR
- Note that Φ starts small so only after growing does coefficient have significant impact
- Can find less supercooling
- Significant change in IR to deconfined phase

Einstein Frame Action

After Weyl rescaling

$$S_{\phi} = 2M_5^3 \int \mathrm{d}^5 x \sqrt{-g} \left(-\frac{1}{2} g^{MN} G(\phi) \,\partial_M \phi \,\partial_N \phi - V(\phi) \right)$$

$$\begin{split} G(\phi) &= 2a \left(1 - \frac{\phi}{\phi_c}\right)^{-n} + \frac{8n^2}{3\phi_c^2} \left(1 - \frac{\phi}{\phi_c}\right)^{-2},\\ V(\phi) &= 2\epsilon\phi^2 \left(1 - \frac{\phi}{\phi_c}\right)^{-\frac{5n}{3}} + 2\Lambda \left(1 - \frac{\phi}{\phi_c}\right)^{-\frac{5n}{3} + m}. \end{split}$$

Choices to simplify the calculations

m=5n/3: only cosmological constant survives for $\epsilon=0$ n=2, a=1: simpler kinetic term

Canonically normalized field

$$\begin{split} S_{\sigma} &= 2M_5^3 \int d^5 x \sqrt{-g} \left(-\frac{1}{2} (\partial \sigma)^2 - V(\sigma) \right) \ , \ \frac{\sigma}{\sigma_c} &= -\log \left(1 - \frac{\phi}{\phi_c} \right) \ , \ \sigma_c = \left(\frac{32}{3\phi_c^2} + 2 \right)^{1/2} \phi_c \\ \phi &= \phi_{\rm uv} \ll 1 \Rightarrow \sigma \to 0 \ , \phi \to \phi_{\rm c} \Rightarrow \sigma \to \infty \end{split}$$

Scalar Potential

$$S_{\sigma} = 2M_5^3 \int d^5x \sqrt{-g} \left(-\frac{1}{2} (\partial \sigma)^2 - V(\sigma) \right)$$

$$V(\sigma) = 2\Lambda + 2\,\widetilde{\epsilon}\,\sigma_c^2\,e^{\frac{10}{3}\frac{\sigma}{\sigma_c}}\left(1 - e^{-\sigma/\sigma_c}\right)^2$$

$$\widetilde{\epsilon} = \epsilon (\phi_c / \sigma_c)^2$$

$$V(\sigma) =_{\sigma \ll \sigma_c} 2\Lambda + 2\tilde{\epsilon}\sigma^2 + \frac{14}{3}\frac{\tilde{\epsilon}}{\sigma_c}\sigma^3 + \frac{101}{18}\frac{\tilde{\epsilon}}{\sigma_c^2}\sigma^4 + \cdots$$
$$V(\sigma) =_{\sigma \gg \sigma_c} 2\tilde{\epsilon}\sigma_c^2 e^{\frac{10}{3}\frac{\sigma}{\sigma_c}}$$

BB Metric

Without back reaction:
$$ds^2 = \rho^2 \left(1 - \frac{\rho_h^4}{\rho^4}\right) \mathrm{d}t_E^2 + \frac{\mathrm{d}\rho^2}{\rho^2 \left(1 - \frac{\rho_h^4}{\rho^4}\right)} + \rho^2 \mathrm{d}x^2, \ \rho_h \le \rho \le \rho_{\mathrm{uv}}$$

Metric ansatz:
$$ds^2 = a^2(\rho)\rho^2 \left(1 - \frac{\rho_h^4}{\rho^4}\right) dt_E^2 + \frac{b^2(\rho)d\rho^2}{\rho^2 \left(1 - \frac{\rho_h^4}{\rho^4}\right)} + c^2(\rho)\rho^2 dx^2$$
 Define ρ such that $c(\rho) = 1$

Define
$$\xi \equiv \frac{\rho^4 - \rho_h^4}{\rho_{uv}^4 - \rho_h^4}$$
 UV brane: $\xi = 1$
Horizon: $\xi = 0$ $\alpha = \frac{\rho_h^4}{\rho_{uv}^4 - \rho_h^4} \approx \rho_h^4 / \rho_{uv}^4 \ll 1$

$$ds^{2} = a^{2}(\xi) \, \xi \, dt_{E}^{2} + \frac{b^{2}(\xi)}{\xi} \, d\xi^{2} + \sqrt{\xi + \alpha} \, dx^{2}$$

 $\begin{array}{rcl} a(\xi) & = & \frac{1}{(\xi + \alpha)^{1/4}} \,, \\ b(\xi) & = & \frac{1}{4 \, (\xi + \alpha)^{1/2}} \,. \end{array} \end{array}$

Solve for $a(\xi), b(\xi), \phi(\xi)$.

Need Boundary Conditions

- Challenge is UV values specified
- IR requires smoothness of metric
 - Do IR expansion
 - Relate derivative to values
- Can then find UV values that yield consistent solutions $\xi = 0$ $\xi = 1$

$$\xi = 0 \qquad \xi = 1$$
Horizon
$$\begin{vmatrix} \xi = \xi_{ir} \ll 1 \\ 0 \\ \xi = \xi_{ir} \ll 1 \\ 0 \\ \xi = \xi_{ir} \ll 1$$

$$\downarrow 0 \\ 0 \\ \xi = \xi_{ir} \ll 1$$

$$\downarrow 0 \\ 0 \\ \xi = \xi_{ir} \ll 1$$

$$\downarrow 0 \\ 0 \\ \xi = \xi_{ir} \ll 1$$

Solving

 $\xi = 1$ UV brane



$$a_1(a_0, \phi_0) = \frac{a_0}{32\alpha} \left(\frac{3}{G(\phi_0)} \left(\frac{V'(\phi_0)}{V(\phi_0)} \right)^2 - 8 \right)$$

$$\phi_1(a_0, \phi_0) = -\frac{3}{4\alpha} \frac{1}{G(\phi_0)} \frac{V'(\phi_0)}{V(\phi_0)}$$

Fix: ϕ_{uv}, a_{uv} Choose: ϕ'_{uv}, a'_{uv} Solve: $1 \ge \xi \ge \xi_{ir} \ll 1$ Calculate: $\phi_{ir}, \phi'_{ir}, a_{ir}, a'_{ir}$ Check: $a'_{ir} = a_1(\phi_{ir}, a_{ir})$, $\phi'_{ir} = \phi_1(\phi_{ir}, a_{ir})$

Solutions

$$\phi_{\rm uv}=1, \xi_{\rm ir}/\alpha=10^{-2}.$$

- A: $\epsilon = -1/50, \ \phi_c = 3/2$. B: $\epsilon = -1/60, \ \phi_c = 3/2$.
- ${\bf C} {:} \qquad \epsilon = -1/100, \; \phi_c = 3/2 \; .$
- **D:** $\epsilon = -1/60, \ \phi_c = 2$.



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- a, b smaller when backreaction
 Φ growth larger for larger ε smaller Φ_c
- Saturates at α
- Smaller α yields bigger growth in Φ

Can Now Evaluate Thermodynamic Quantities



• Everything in terms of calculable finite quantities

Big Finding: Minimum Temperature! When large enough backreaction



- Only when large back-reaction
 - Potential grows comparable to 5d cc
- This has been seen in RS, global AdS, etc

Entropy not Single Valued



• Same temperature but different entropies

Free Energy Also Not Single-Valued



• This is important for phase structure

Focus on Strongly Back-Reacted Cases



Back-Reacted Phase Diagram



- RS is preferred stable phase at low temperature
- There might be a spinoidal phase transition in GW signal (or some other sign of instability)

Consequences and Generality?

- This was a specific model but reason to think can be generic
 - Featured in examples
 - Extra contribution from scalar field can significantly modify metric when no longer in perturbative regime
- Important Consequences

Not necessarily first order phase transition Not necessarily supercooling

• Can be other GW signals

RS might be less constrained Really a 5d analysis necessary

Conclusions

• RS Cosmology subtle

- Seems to depend fairly strongly on model

Can have GW signatures and supercooling

But connected

Models with less supercooling associated with less strong GW signal

- We explored one perturbative model where constraints weaked
- And one ultimately nonperturbative model where it will be very interesting to see GW consequences Story old, but not over!