Neutrino masses and Mixing driven by Randomness

Sudhir K Vempati

Centre for High Energy Physics,
Indian Institute of Science
Bangalore , India
vempati@iisc.ac.in

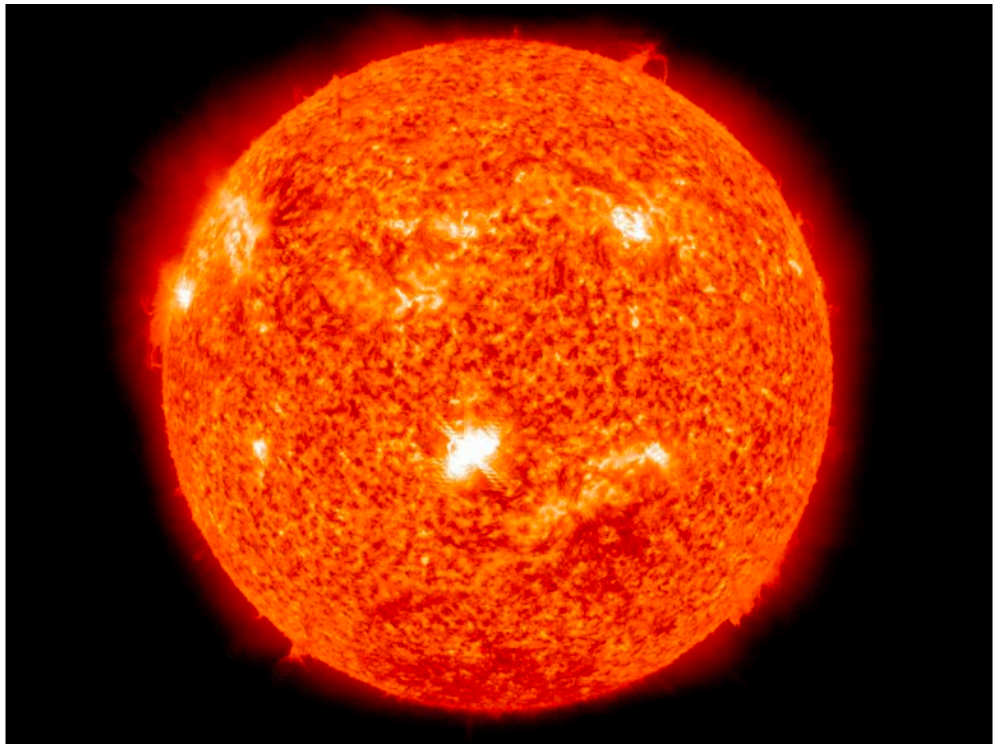
With Aadarsh Singh (2401.XXXX, 2401.XXXX)
And Aadarsh Singh , Alejandro Ibarra (2401.XXXX)

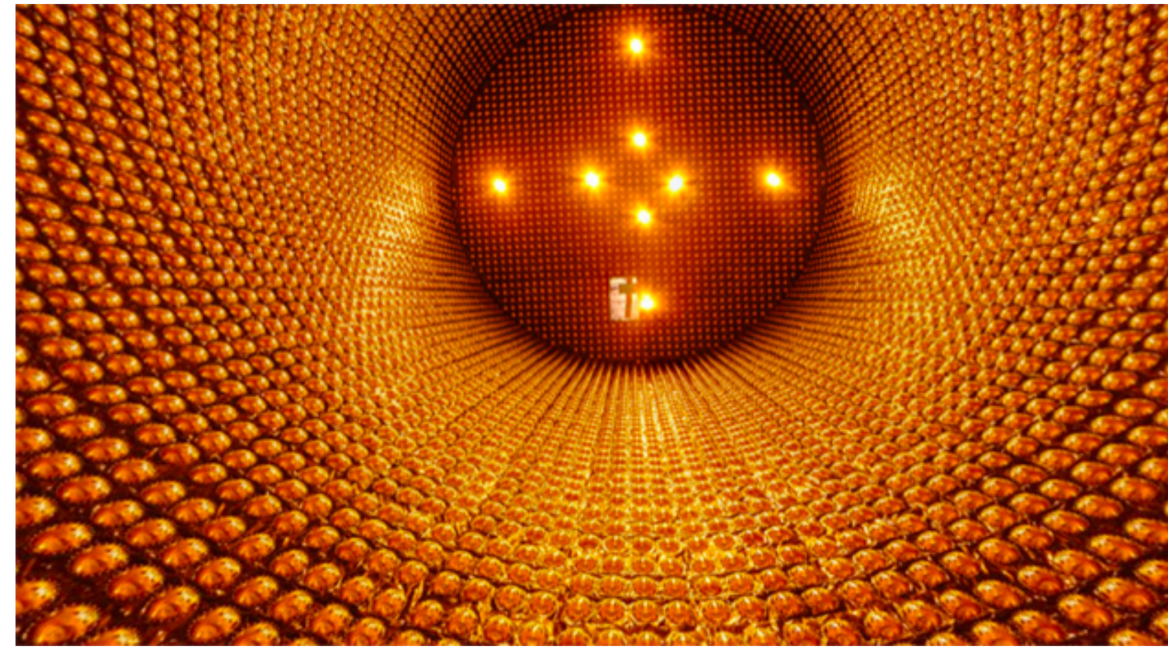
with Alejandro Ibarra and Ashwani Kushwaha
arXiv:1711.02070 (Phys. Lett. B)

center for high energy physics

BSM@50 ICISE Vietnam

IISc





$$m_{\nu 2} \sim \sqrt{\Delta m^2_{\odot}} \sim 0.008 \text{ eV}$$

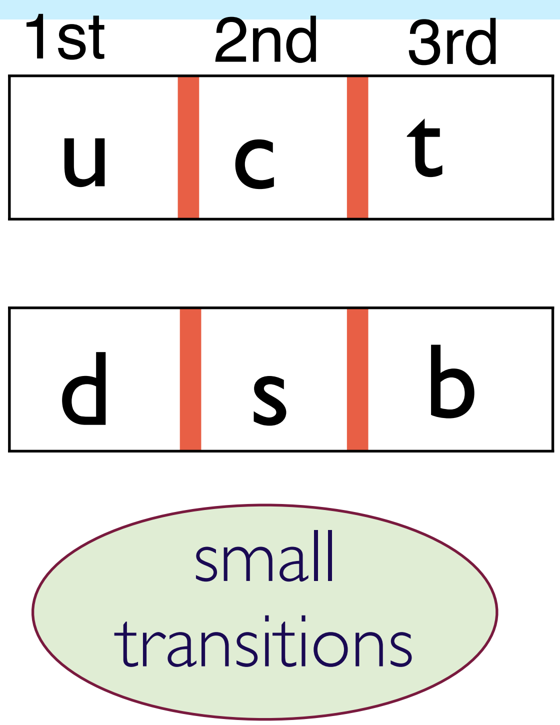
$$m_{\nu_3} \sim \sqrt{\Delta m^2_{\textbf{ATM}}} \sim 0.05 \textbf{ eV}$$

Neutrinos Oscillate and this can be explained by tiny Sub eV masses

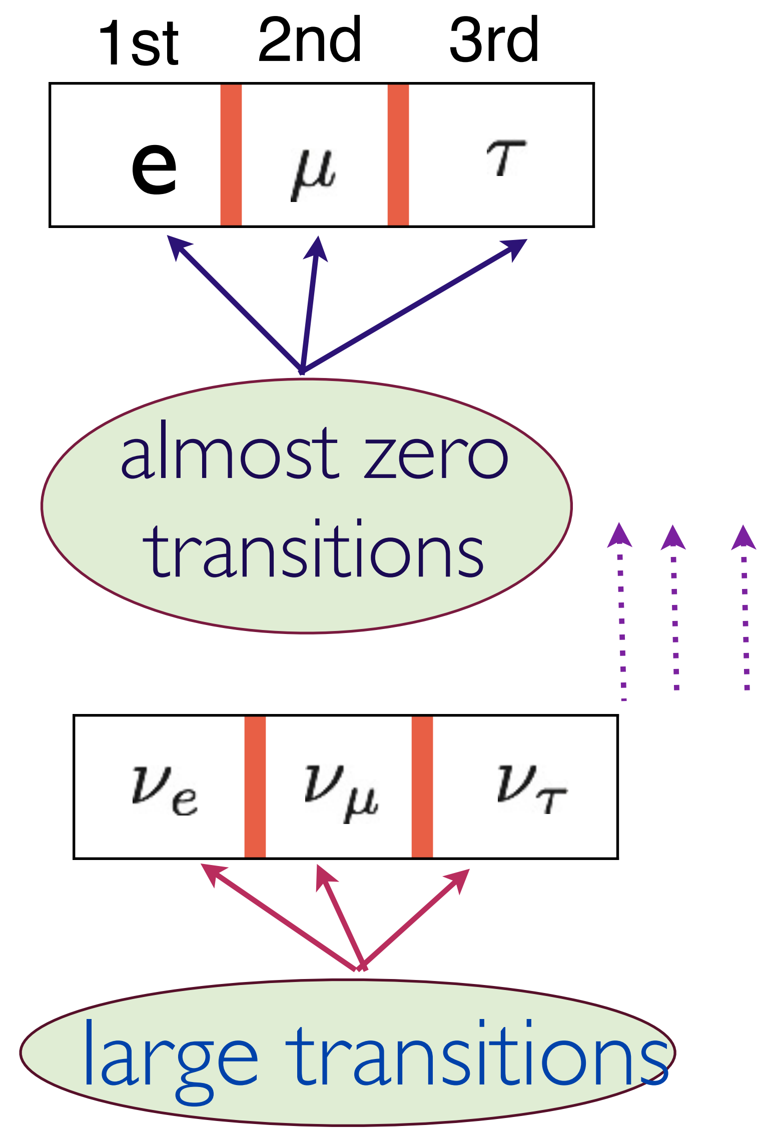
(assuming normal hierarchy)

Neutrino Mixing angles

Inter-generational transitions are small for quarks and almost zero for charged leptons



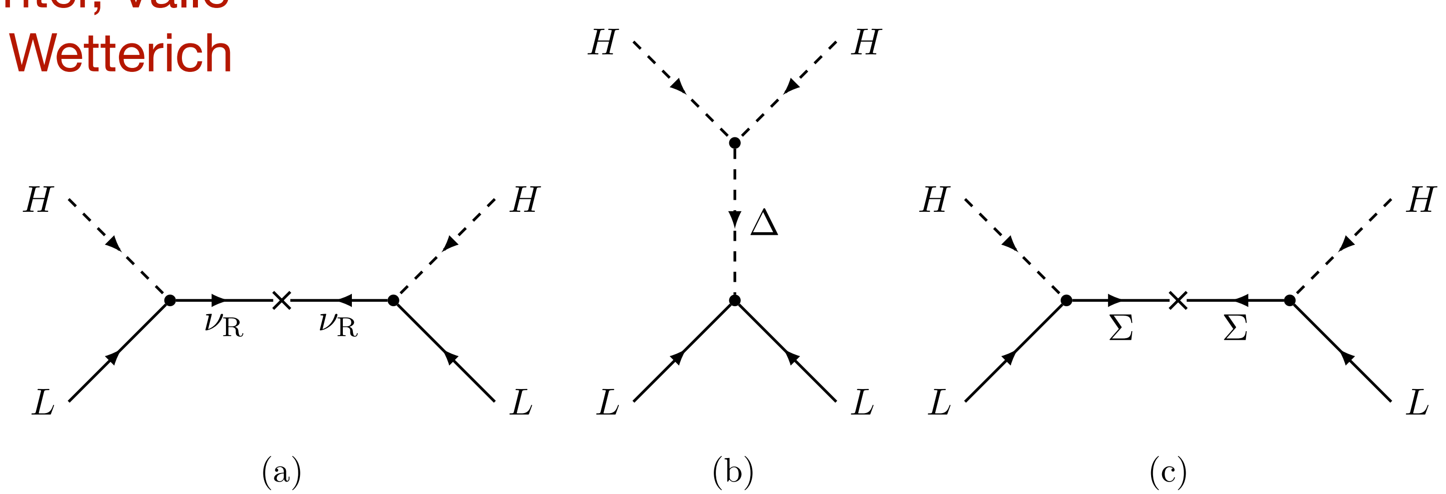
Information of large mixing can pass from the neutrino sector to other sectors giving severe constraints on the model parameter space



- Typically a large mass/small vev is required to generate the small masses
- in seesaw like mechanisms
- Can fit naturally in GUTs

Minkowski
Senjanovic, Mohapatra,
GellMann, Ramond, Slansky
Yanagida, Schechter, Valle
Lazarides, Shafi, Wetterich

S. F. King, 2003
A. De Gouvea, 2016
Davidson et.al, 2008
Cai, et.al, 2018

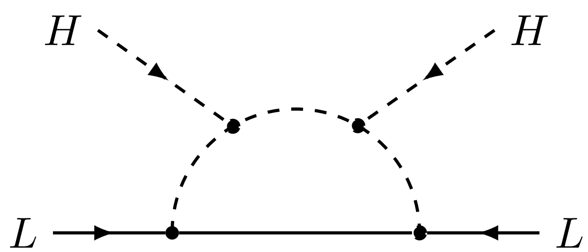


Type I **Type II** **Type III**

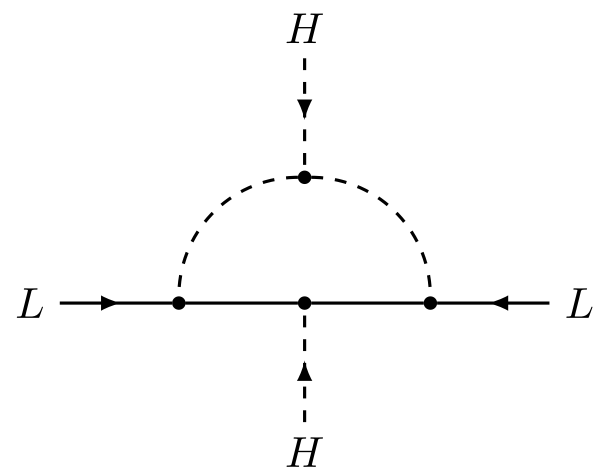
Radiative Mechanisms

E. Ma,
Babu
Zee

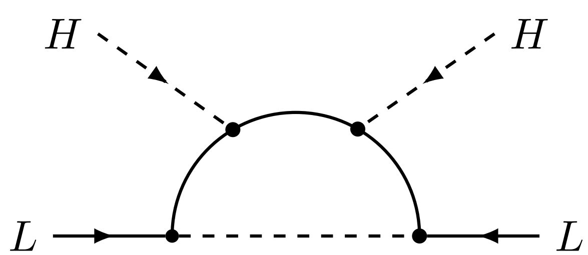
Babu Leung, 2001
Cai et.al 2017



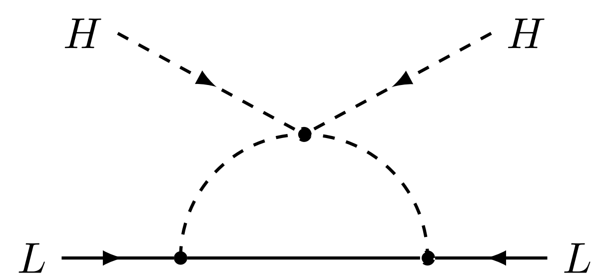
(a) T1-i



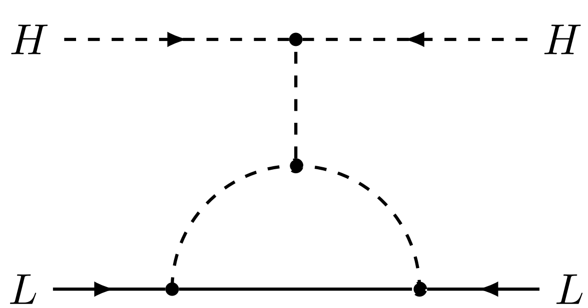
(b) T1-ii



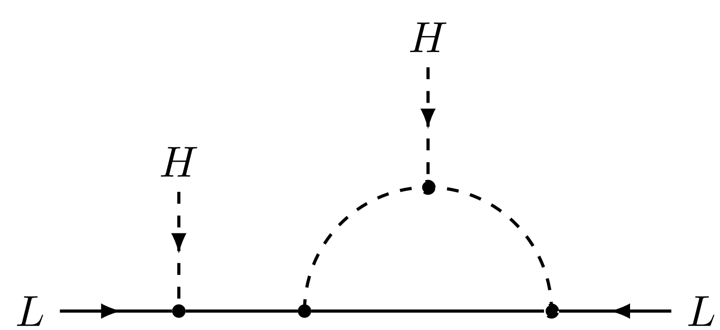
(c) T1-iii



(d) T3



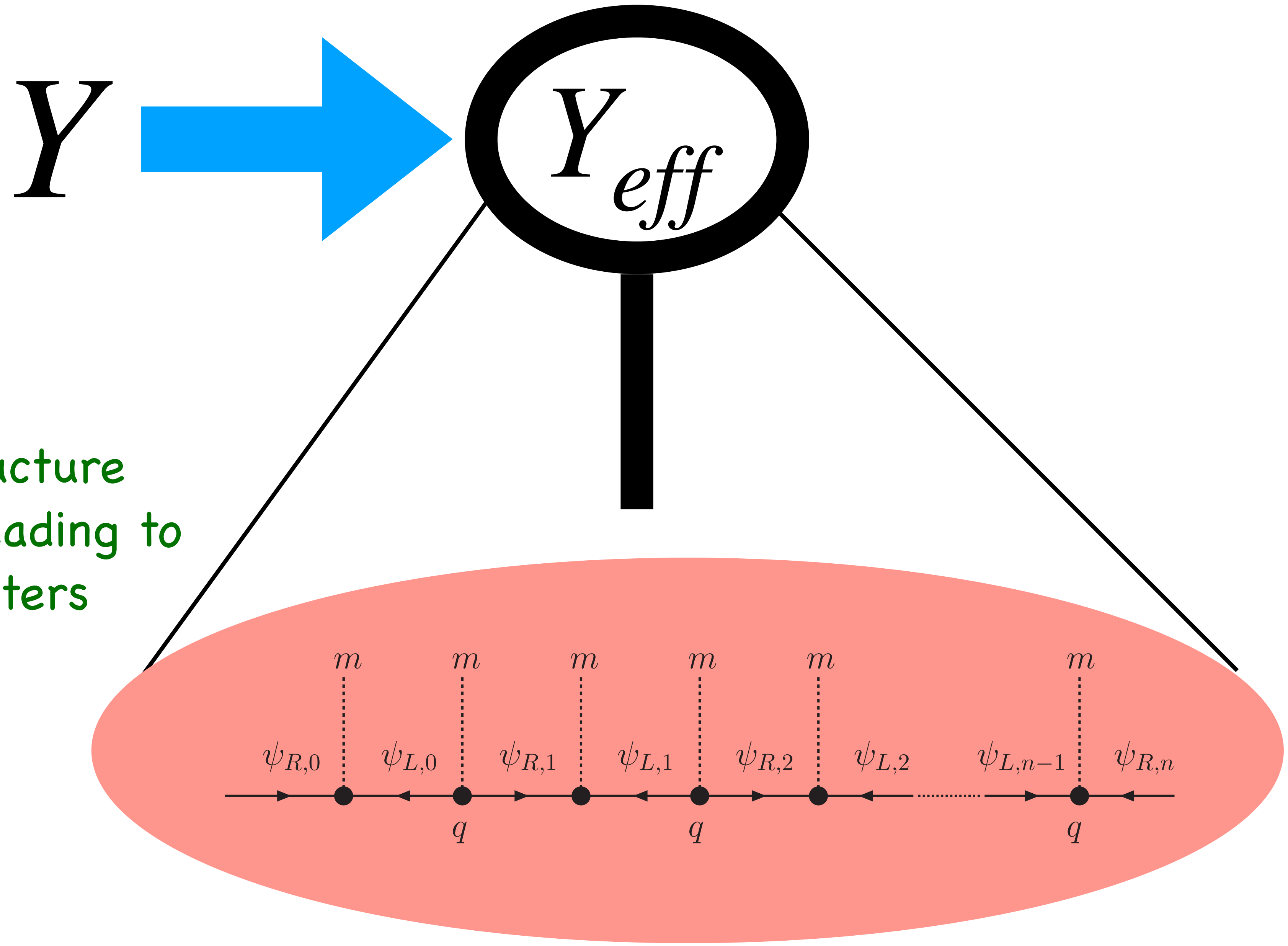
(e) T4-2-i



(f) T4-3-i

Consider Dirac Masses

$$\mathcal{L}_{SM} + Y \bar{\nu}_L \nu_R \tilde{H}$$



A deeper heavier structure
With O(1) parameters, leading to
hierarchial parameters

BSM@50, Jan 2024

Clockwork For Dirac Neutrinos

Choi and Im, JHEP 2016
Kaplan and Rattazzi, 2016

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \mathcal{L}_{\rm Clockwork} + \mathcal{L}_{\rm int} \ ,$$

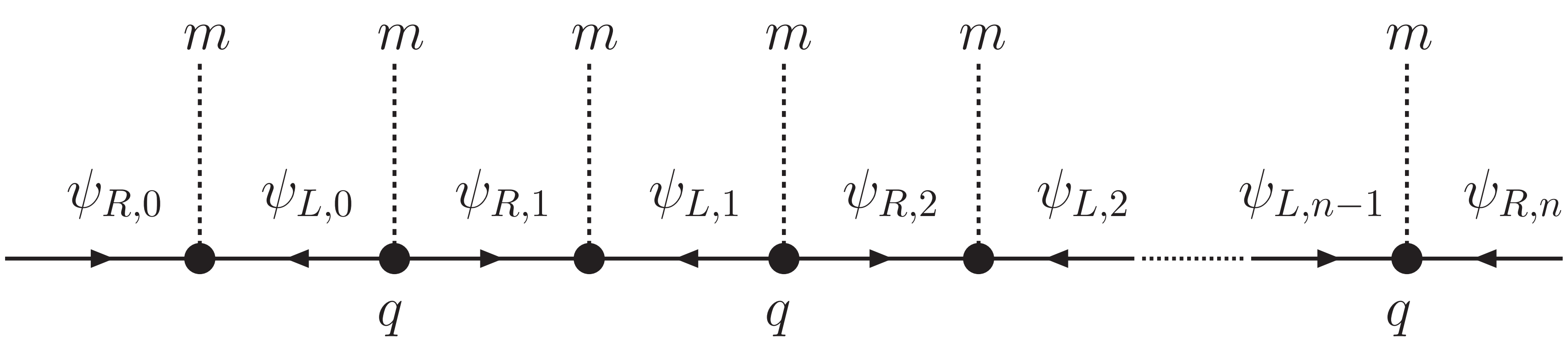
Giudice and McCoullough,2016

The clockwork sector contains (0,1,...n-1) left handed chiral fields and (0,1,....n) right handed chiral fields.

$$H_{ij}^{CW} = m\delta_{ij} + qm \ \delta_{i+1,j}$$

$$\mathcal{L}_{\rm int} = -Y\widetilde{H}\overline{L}_L\psi_{Rn} \ ,$$

We begin with one generation and the generalise to N generations.

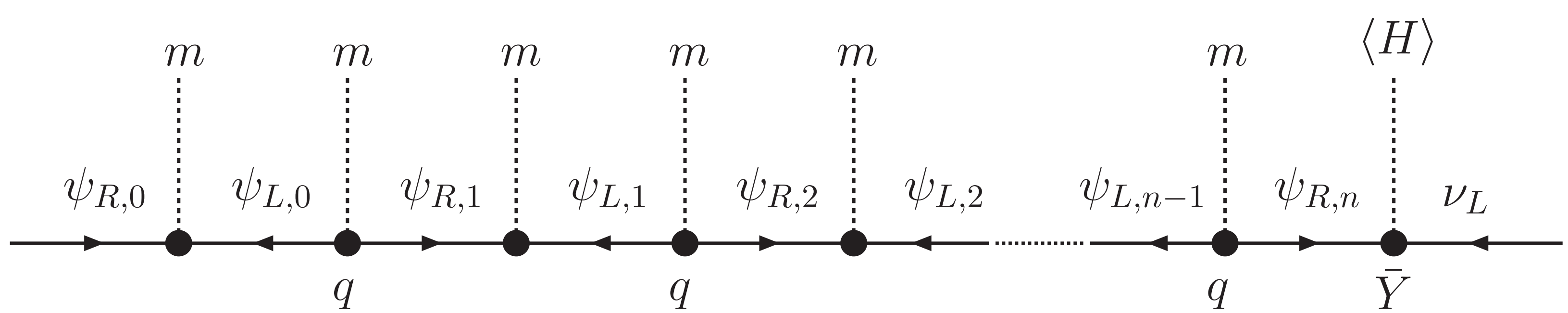


$$\mathcal{L} = \mathcal{L}_{\text{Kin}} - m \sum_{j=0}^{n-1} \left(\overline{\psi}_{L,j}\psi_{R,j} - q\,\overline{\psi}_{L,j}\psi_{R,j+1} + H.c\right) \equiv \mathcal{L}_{\text{Kin}} - \left(\overline{\psi}_L M_\psi \psi_R + H.c\right)$$

$$M_\psi = m \begin{pmatrix} 1 & -q & 0 & \cdots & & 0 \\ 0 & 1 & -q & \cdots & & 0 \\ 0 & 0 & 1 & \cdots & & 0 \\ \vdots & \vdots & \vdots & \ddots & & \vdots \\ & & & & -q & 0 \\ 0 & 0 & 0 & \cdots & 1 & -q \end{pmatrix}$$

one zero mode, and n Dirac fermions

After EW symmetry breaking from the interaction term



$$m_\nu \approx vY_0$$

$$m_\nu^D = \begin{array}{c} \\ \nu_L \\ N_{L1} \\ N_{L2} \\ \vdots \\ N_{Ln} \end{array} \begin{array}{c} \begin{array}{ccccc} N_{R0} & N_{R1} & N_{R2} & \cdots & N_{Rn} \end{array} \\ \begin{pmatrix} vY_0 & vY_1 & vY_2 & \cdots & vY_n \\ 0 & M_1 & 0 & \cdots & 0 \\ 0 & 0 & M_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & M_n \end{pmatrix} \end{array} .$$

$$Y_0 \equiv Y(u_R)_n = \frac{Y}{q^n}\sqrt{\frac{q^2-1}{q^2-q^{-2n}}} \,,$$

$$Y_k \equiv Y(U_R)_{nk} = Y\sqrt{\frac{2}{(n+1)\lambda_k}}\left[q \sin\frac{nk\pi}{n+1}\right] \,, \qquad k = 1, ..., n \,.$$

Kushwaha, Ibarra and Vempati,2017

a kind of multi-degenerate-seesaw mechanism for Dirac neutrinos, where large n reduces the neutrino mass

At least two clockworks for two mass scales.

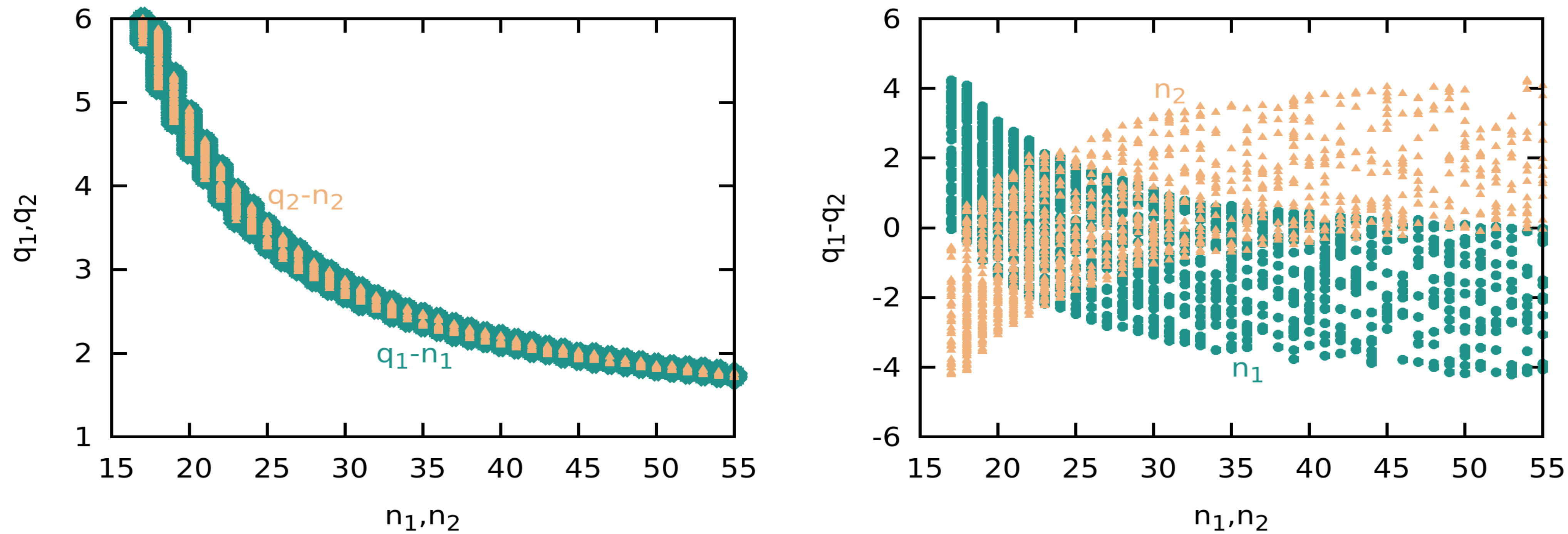


Figure 2: Values of q_1 and q_2 (left panel) and difference between them (right panel), as a function of n_1 and n_2, compatible with the measured values of the neutrino mass splittings and mixing angles within 1σ, for a scenario with two clockwork generations.

Results with three clockworks similar

Kushwaha, Ibarra and Vempati,2017

Anderson localisation in particle physics

Craig Sutherland
2017

Using randomness in couplings to generate exponential hierarchies.
Applications to neutrino masses

Sources of randomness :

(I) stringy landscapes — Balasubramaniam et.al

(II) dark sectors — Dienes, kumar et.al

"Anderson localisation" in 4D

$$S = \sum_{j=1}^{N} \int d^4x \{ \bar{\psi} \left(i\gamma^\mu D_\mu \right) \psi + \left(\overline{L_j} \Phi_{j,j+1} R_{j+1} + \overline{L_{j+1}} \Phi_{j+1,j} R_j \right)$$

$$+ \overline{L_j} M R_j + h.c. \}$$

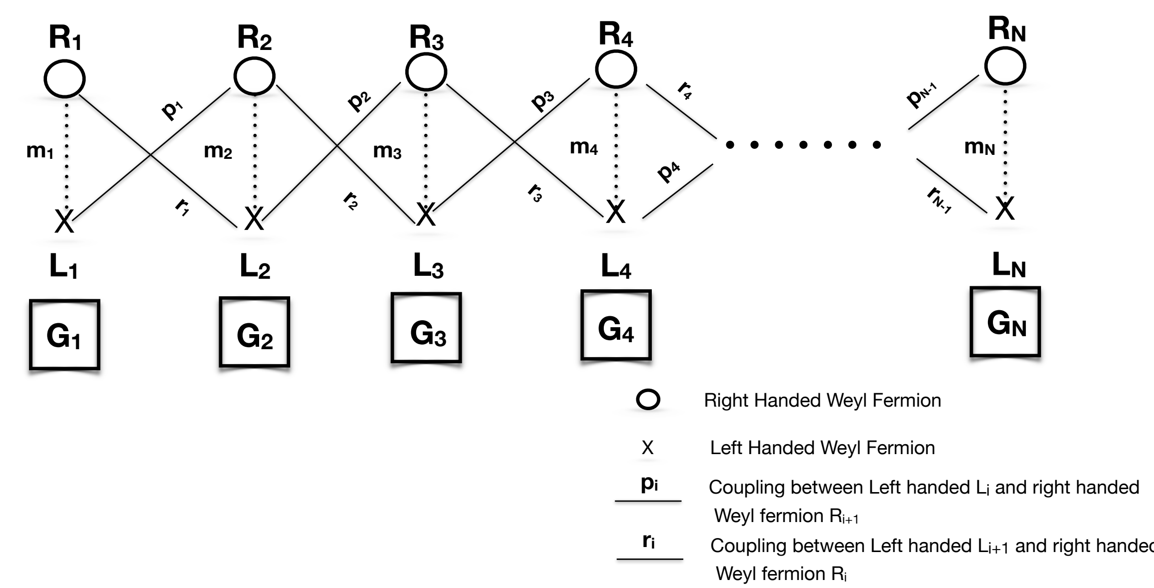
$$\mathcal{L}_{NP} = \mathcal{L}_{kin} - \sum_{i,j=1}^{n} \overline{L_i} \mathcal{H}_{i,j} R_j + h.c.$$

$$\mathcal{H}_{i,j} = \epsilon_i \delta_{i,j} - t_i (\delta_{i+1,j} + K \delta_{i,j+1})$$

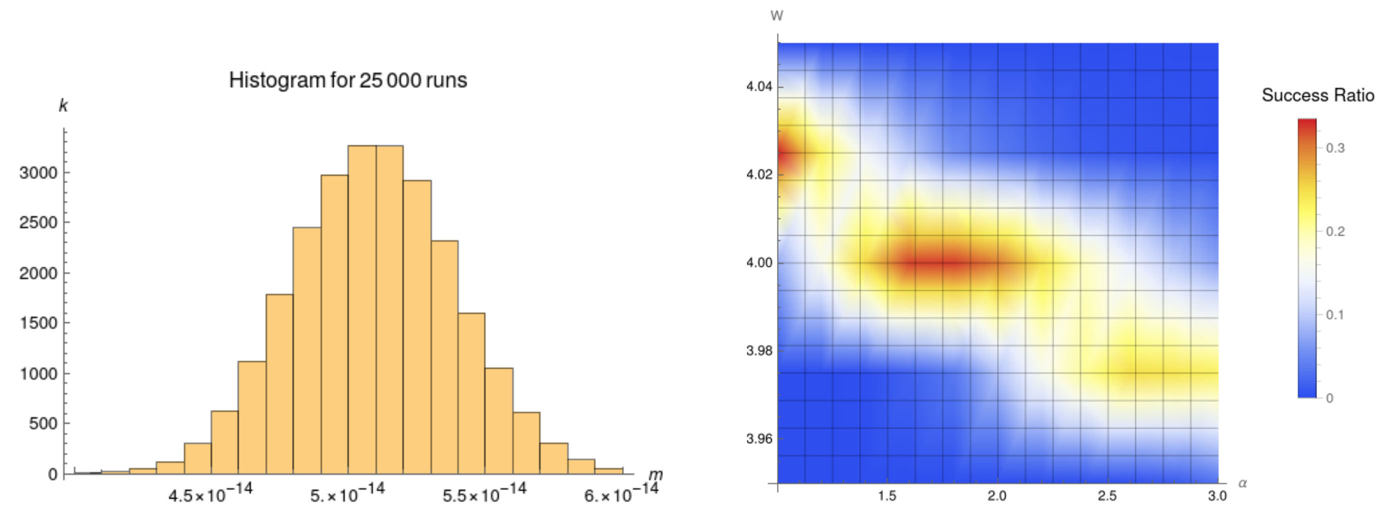
$$M_{mass} = \begin{bmatrix} 0 & M_A \\ M_A & 0 \end{bmatrix}$$

$$M_A = \begin{bmatrix} \epsilon_1 & -t & 0 & \dots & 0 \\ -t & \epsilon_2 & -t & \dots & 0 \\ 0 & -t & \epsilon_3 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & -t & \epsilon_N \end{bmatrix}$$

Deconstruction Model



Sutherland and Craig, 2017



Plot 3 - Histogram for mass distribution of hierarchical mass produced by lattice with 2% randomness in ϵ_i for 25000 runs [Left]. Heat density plot for success ratio for values of W (TeV) and α (% randomness in ϵ_i) [Right].

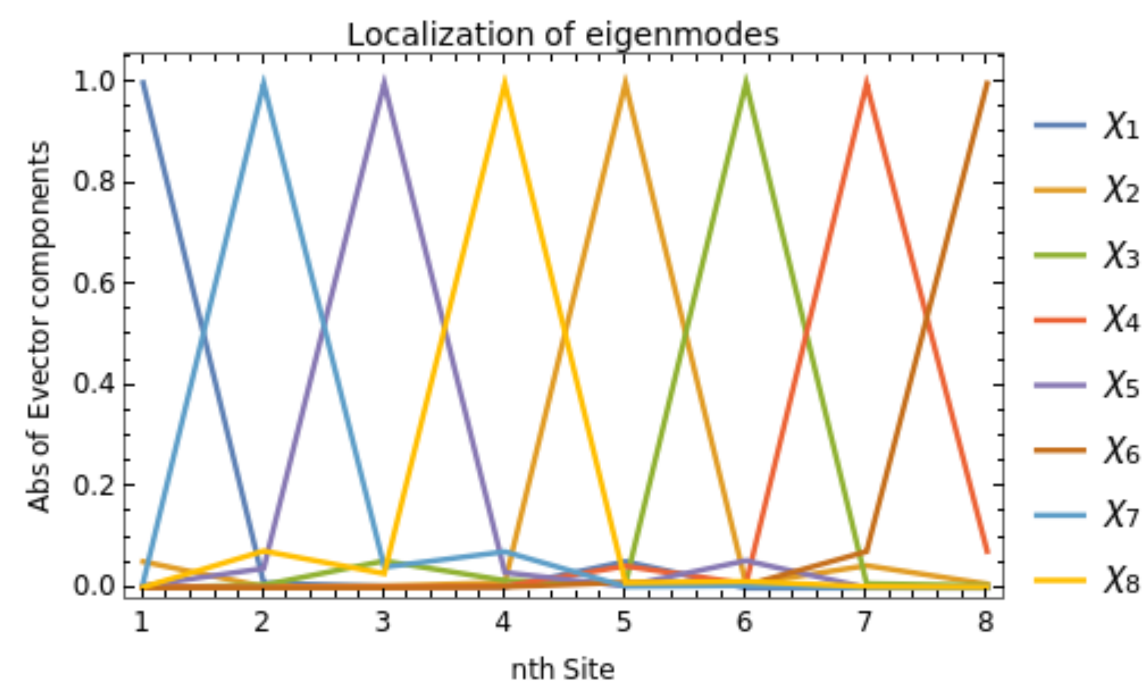
$$\epsilon_i \in [0, W]$$

For t=1, W = 3, N = 30

Strong localisation limit

$W \gg t$

$$L\left(m_i^2, t, W\right) \sim \left(\ln \frac{W}{2t} - 1\right)^{-1}$$



(1) Generalised Clockwork

$$L_{CW} = L_{kin} - \sum_{i=1}^{n} \bar{\psi}_{L_i} H_{ij} \psi_{R_j} + H.C$$

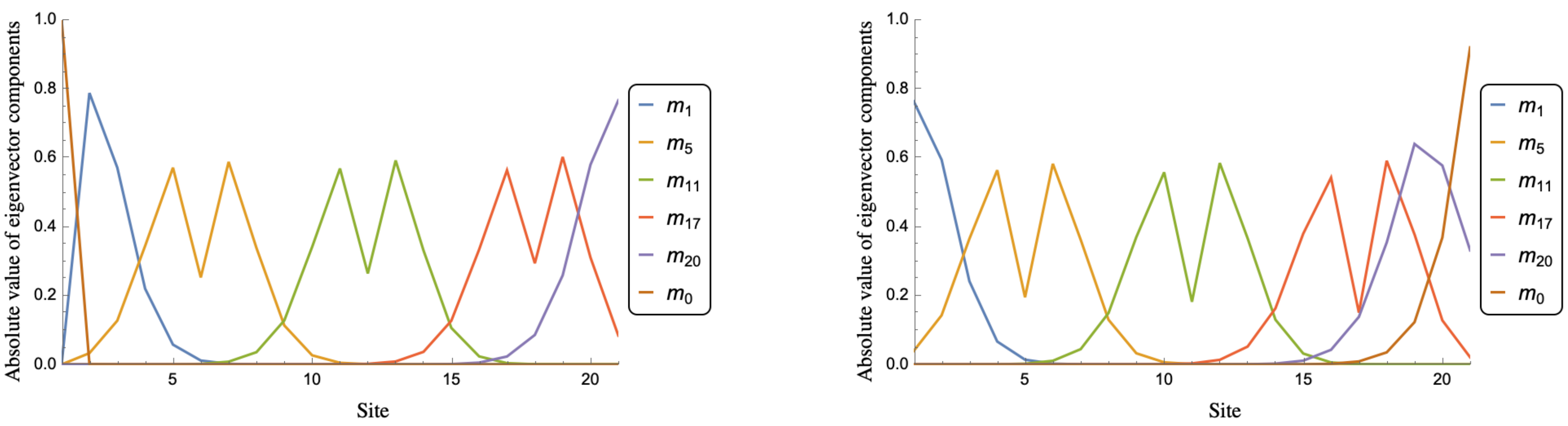
$$H_{ij} = m_i\ \delta_{ij} + q_i m_i\ \delta_{i+1,j}$$

Zero Mode !

Localisation possible for regions of parameters (no large hierarchies)

Tiny Dirac neutrino masses !

Hong, Kurup, Perelstein



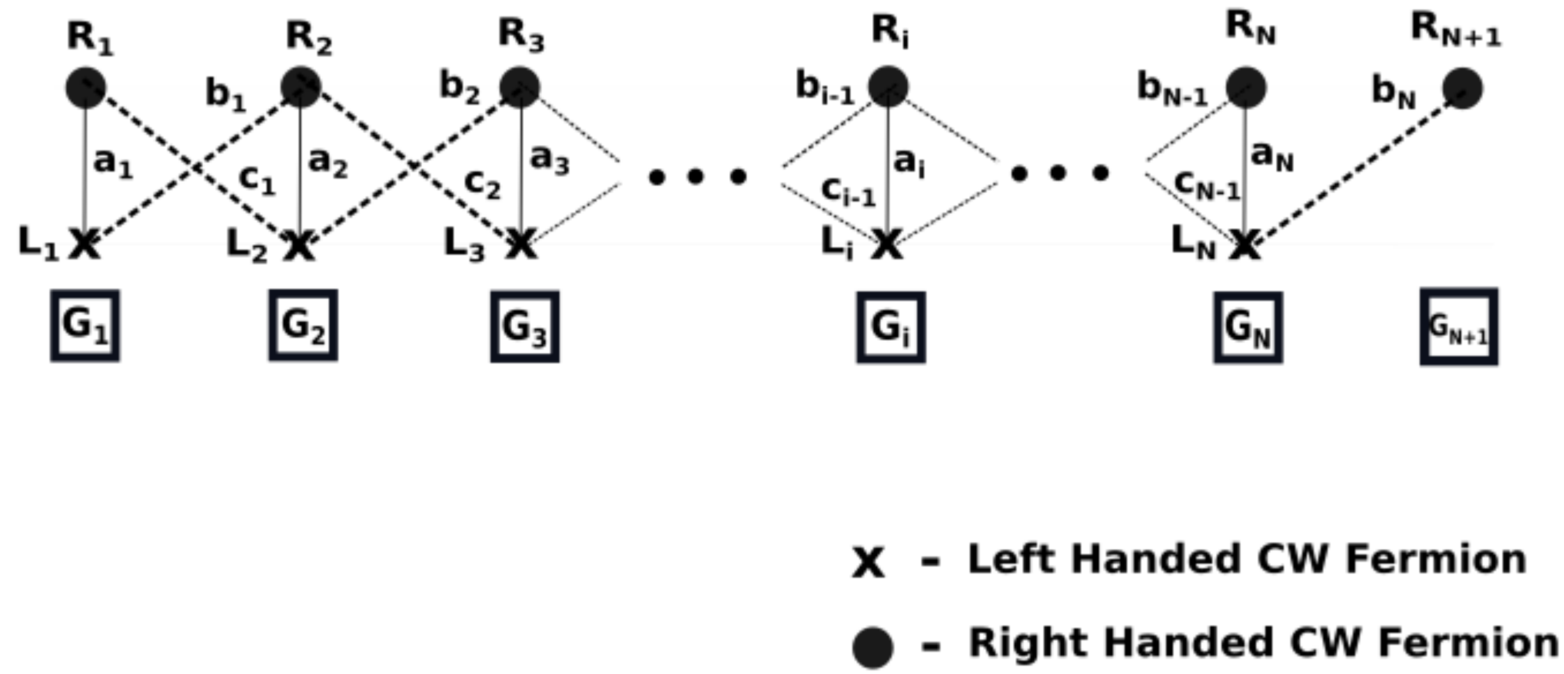
Plot 1(B) - Left plot shows the absolute value of left-handed mass eigenvectors in terms of CW fields and the right plot for right-handed mass eigenbasis with y = 0.1.

(2) Two Sided Clockwork

$$L_{CW} = L_{kin} - \sum_{i=1}^{n} \bar{\psi}_{L_i} H_{ij} \psi_{R_j} + H.C \qquad H_{ij} = m(\delta_{ij} + q_i \delta_{i+1,j} + q' \delta_{i,j+1})$$

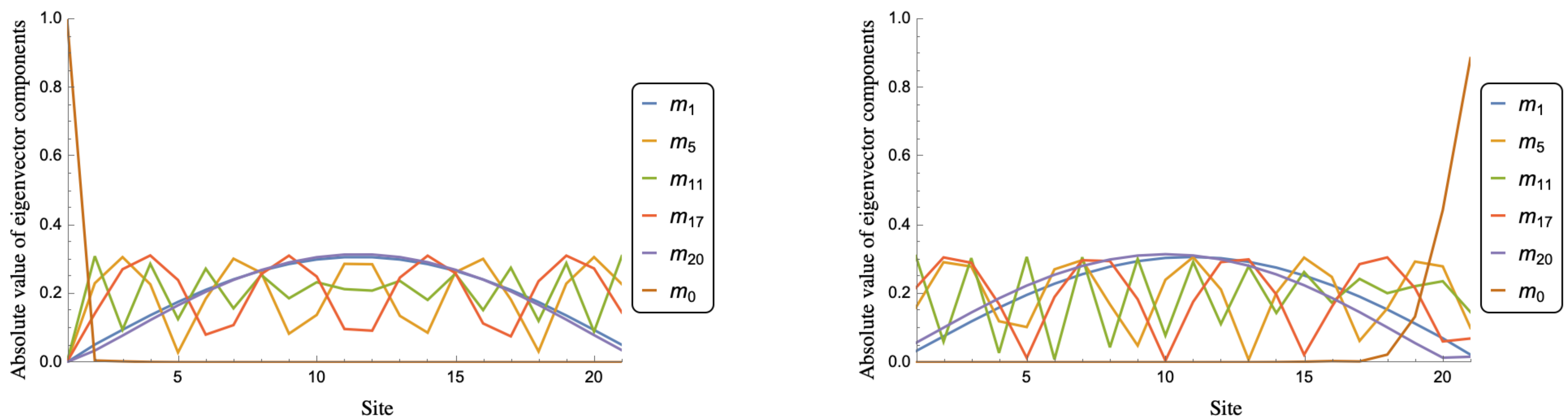
Tiny Dirac neutrino masses !

Deconstruction Model
Linear Moose



Zero Mode !

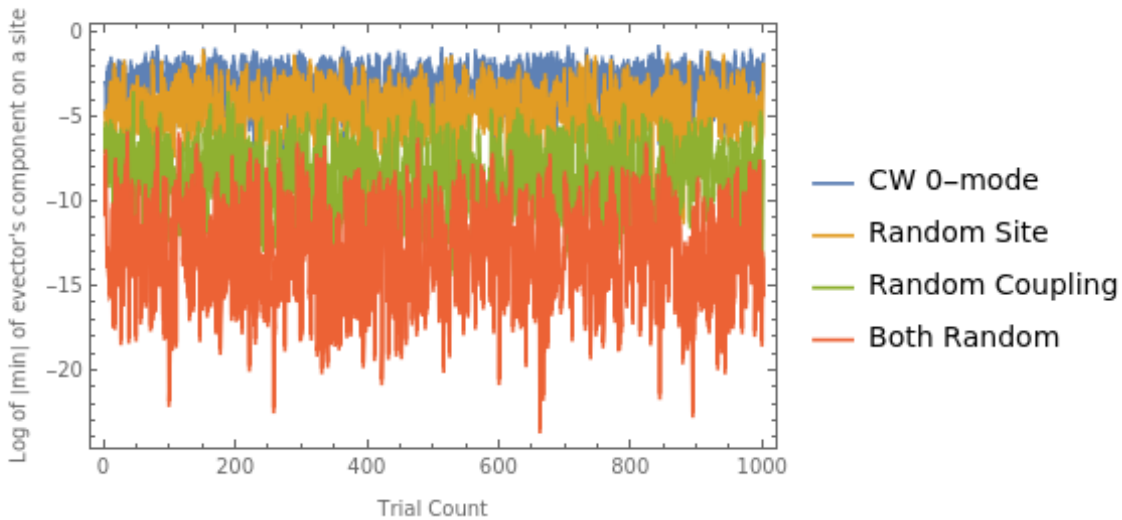
Localisation possible for regions of parameters (no large hierarchies)



Plot 2(B) - Left plot shows the absolute value of left-handed mass eigenvectors in terms of CW fields and the right plot for right-handed mass eigenbasis with y = 0.1.

Extremely efficient localisation with randomness/disorder

Gero Gresdroff



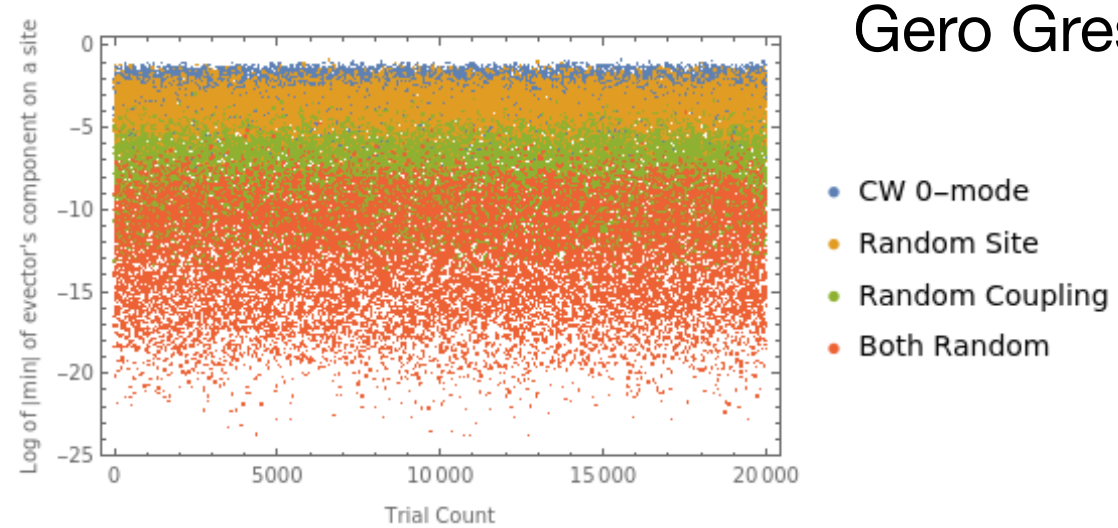
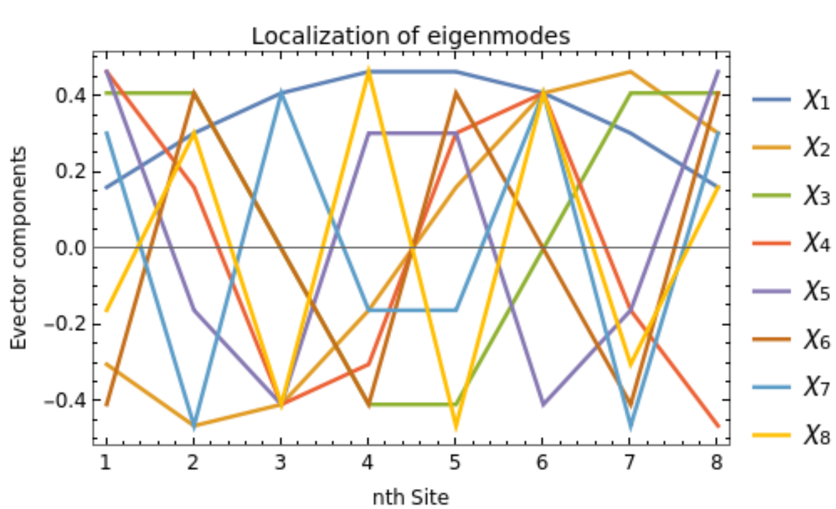
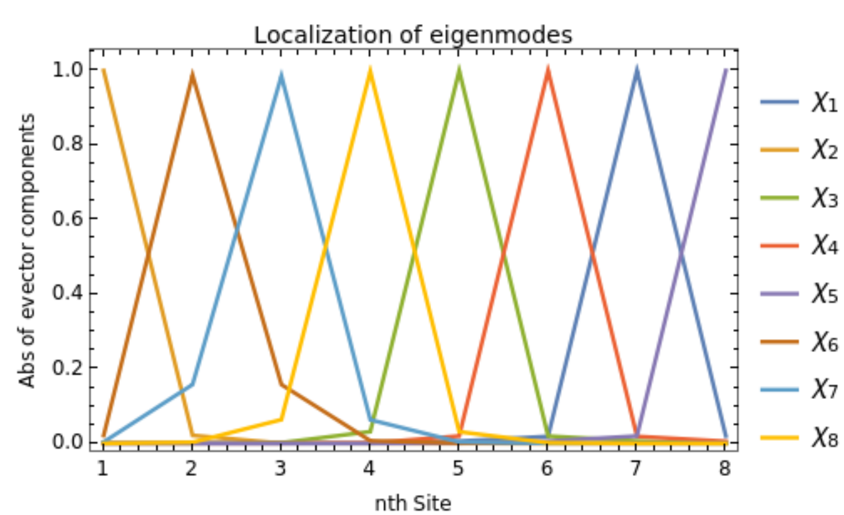


Fig.6 - Figure shows the Log of minimum component 0-mode of CW and lightest mode of disorder models achieved with n = 10 sites.





All modes localised

Fig.2 - Mass modes of Local lattice with uniform sites ϵ_i = W & $t_i = t$ (left) and random sites $t_i = t$ & $\epsilon_i \in$ [2W, -2W] (right) for W = 4 and t = 1/4 with N = 8 sites..

Singh and vempati, 2023

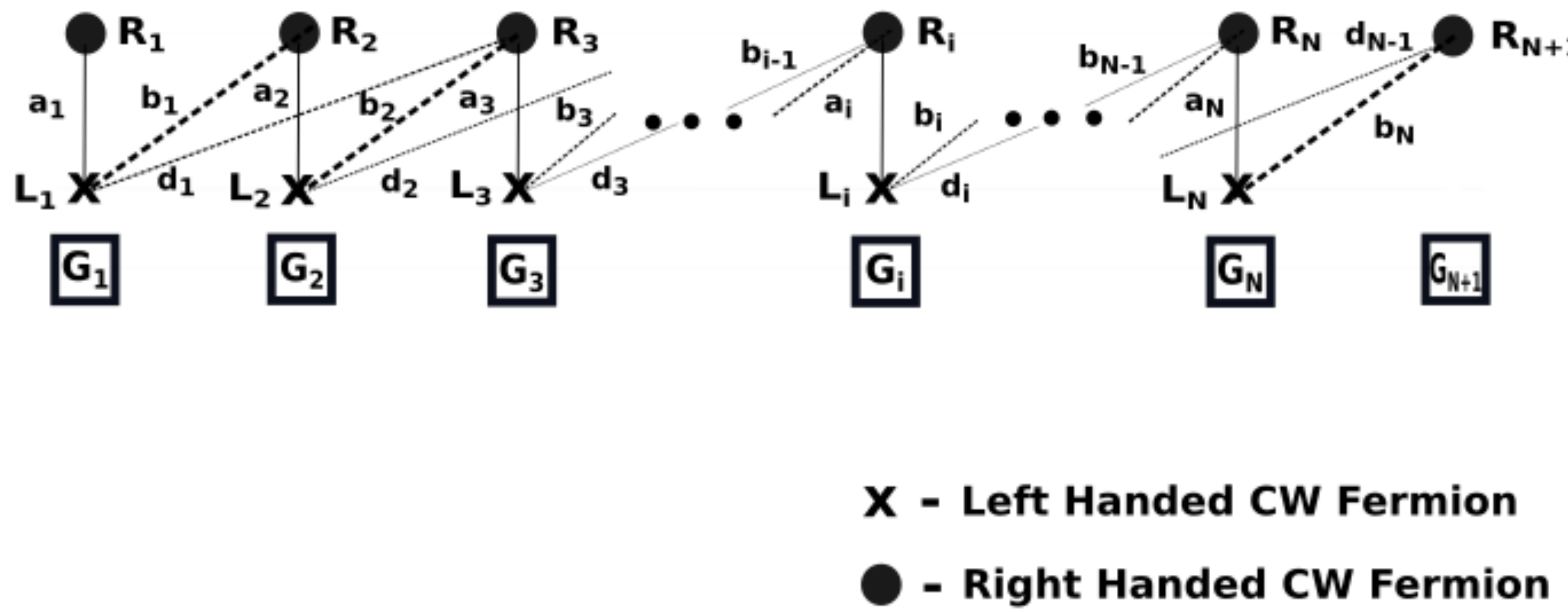
(3) Non-local Interactions

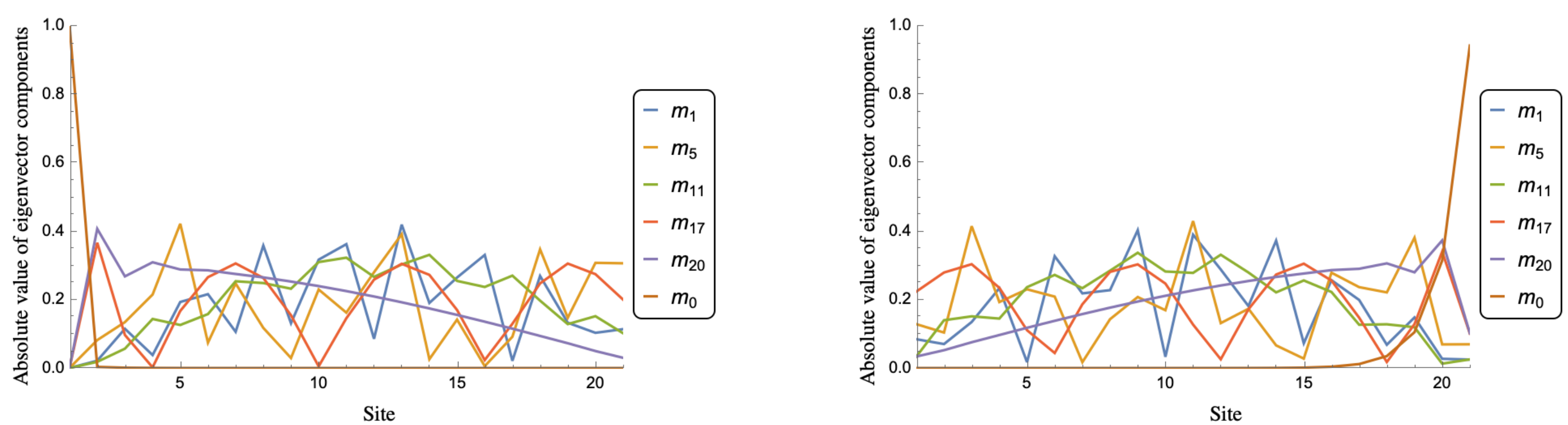
$$\mathcal{H}_{i,j} = a_i \delta_{i,j} + b_i \delta_{i+1,j} + d_i \delta_{i+2,j}$$

Tiny Dirac neutrino masses !

Zero Mode !

Localisation possible for regions of parameters (no large hierarchies)

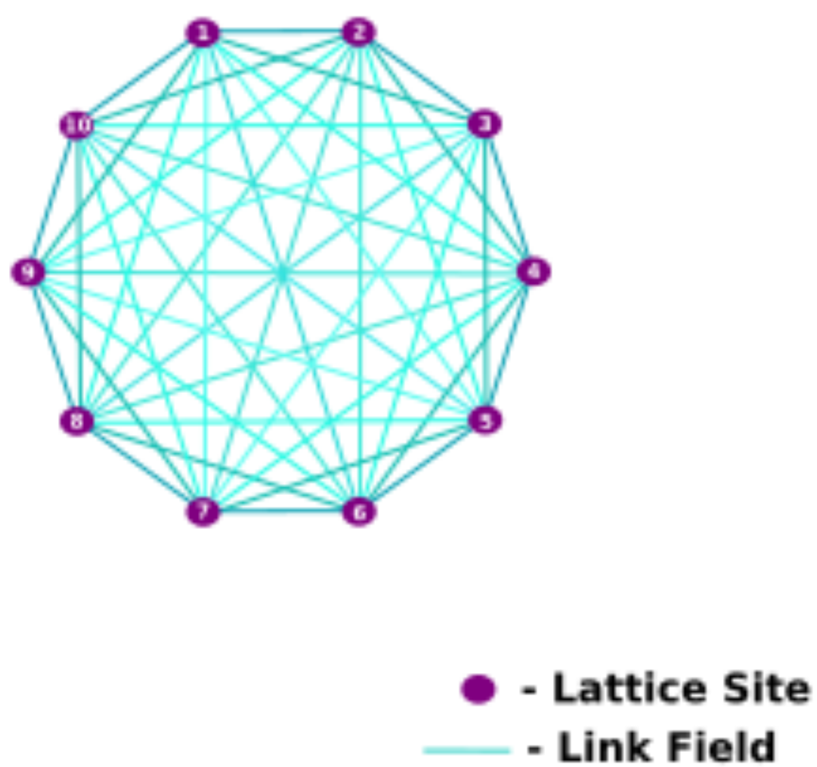




Plot 3(B) - Left plot shows the absolute value of left-handed mass eigenvectors in terms of CW fields and the right plot for right-handed mass eigenbasis with y = 0.1.

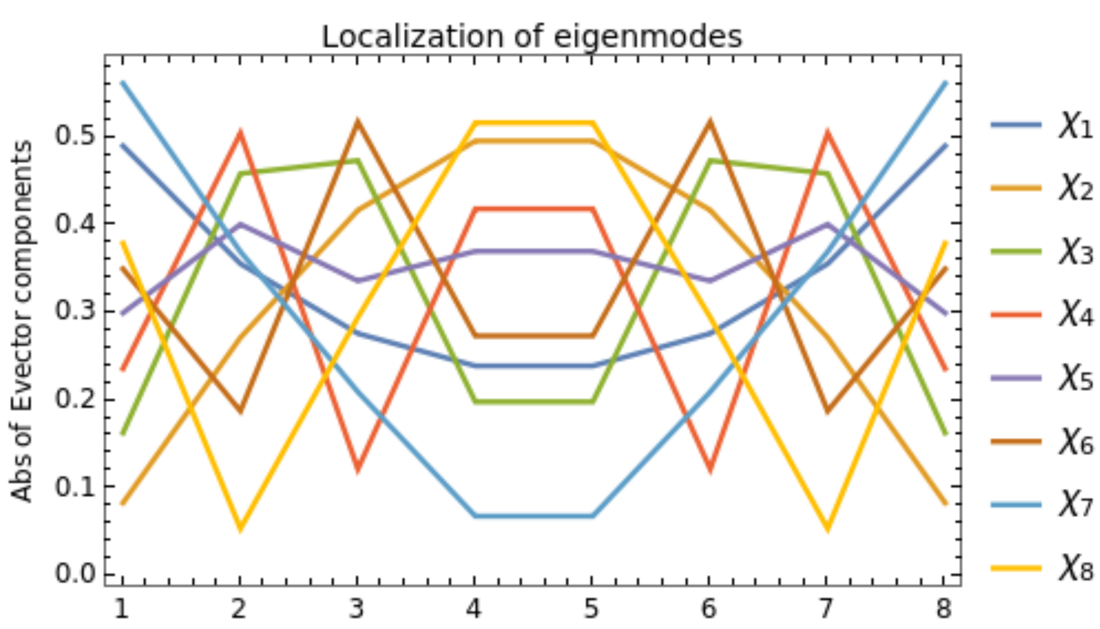
Singh and vempati, 2024

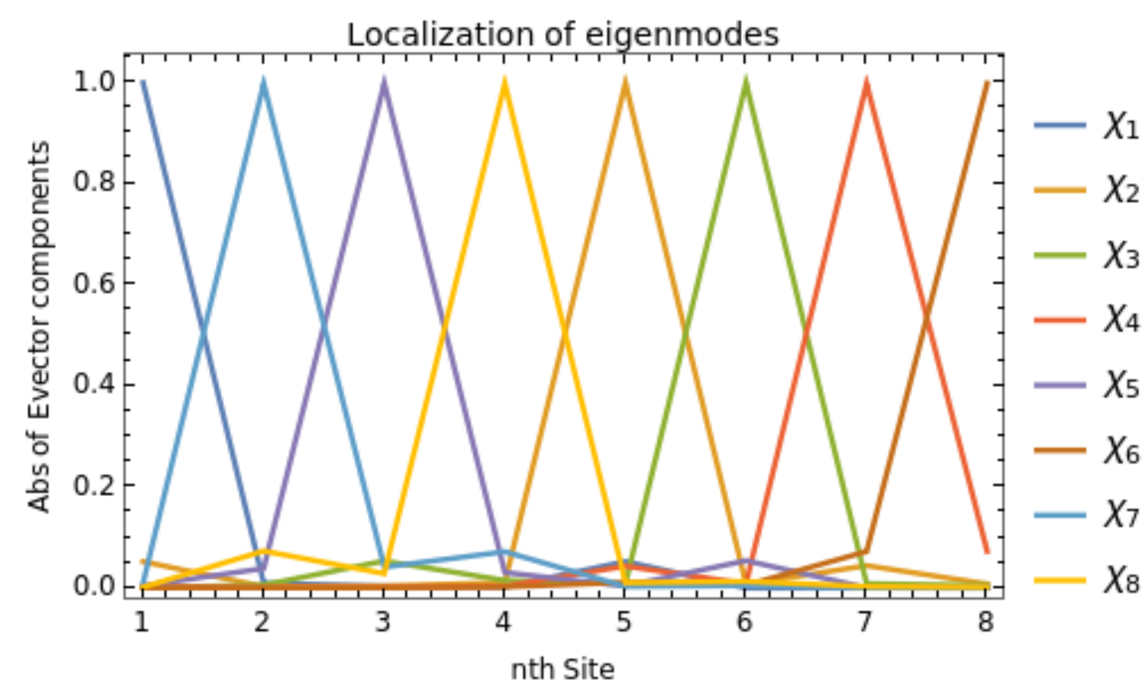
(3a) Completely Non-local



$$\mathcal{L}_{long-range} = L_{Kin} - \sum_{i,j=1}^{N} \overline{L_i}\epsilon_{i,j}R_j - \sum_{i,j=1}^{N} \overline{L_i}\frac{g}{b^{|i-j|}}\left(1-\delta_{i,j}\right)R_j + h.c.$$

$$M_{long-range} = \begin{bmatrix} \epsilon_1 & \frac{g}{b} & \frac{g}{b^2} & \cdots & \frac{g}{b^{N-1}} \\ \frac{g}{b} & \epsilon_2 & \frac{g}{b} & \cdots & \frac{g}{b^{N-2}} \\ \frac{g}{b^2} & \frac{g}{b} & \epsilon_3 & \cdots & \frac{g}{b^{N-3}} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \frac{g}{b^{N-1}} & \cdots & \cdots & \frac{g}{b} & \epsilon_N \end{bmatrix}$$



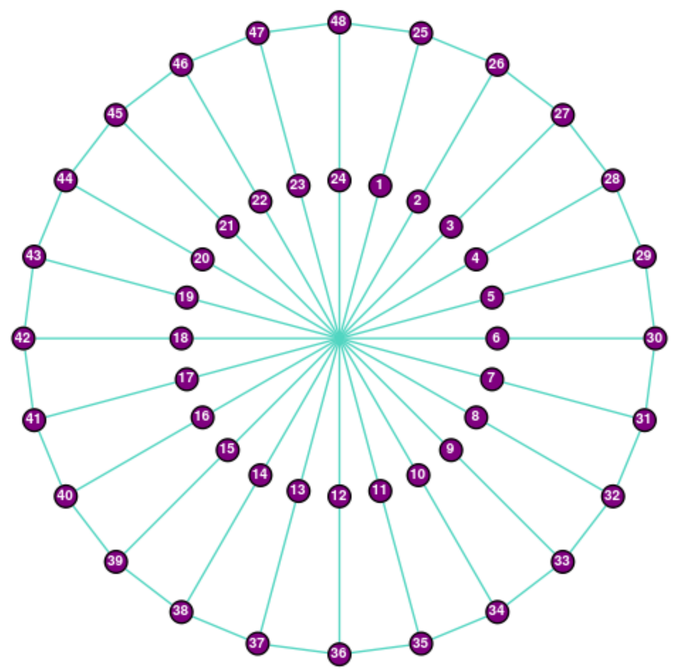


$\epsilon_i = 2\mathrm{W},\ \mathrm{g} = 1,\ \mathrm{N} = 8$ b=0.7, W = 4

Ji Ji fan et.al

Singh and Vempati, 2401.XXXX

Pietersen Graph



$$L_{Petersen} = L_{Kin} - \sum_{i,j=1}^{N} \overline{L_i}\epsilon_{i,j}R_j - \sum_{i,j=1}^{N/4} \overline{L_i}\frac{g}{b^{|i-j|}}\left(\delta_{i,j+N/4}+\delta_{i+N/4,j}\right)R_j$$
$$-\sum_{i,j=1}^{N/2} \overline{L_i}\frac{g}{b^{|i-j|}}\left(\delta_{i,j+N/2}+\delta_{i+N/2,j}\right)R_j - \sum_{i,j=N/2+1}^{N} \overline{L_i}\frac{g}{b^{|i-j|}}\left(\delta_{i,j+1}\right)R_j$$
$$-\sum_{i,j=N/2+1}^{N} \overline{L_i}\frac{g}{b^{|i-j|}}\left(\delta_{i+1,j}\right)R_j + h.c.$$

Partially non local

$$M_{Petersen} = \begin{bmatrix} \epsilon_1 & 0 & \frac{g}{b^2} & 0 & \frac{g}{b^4} & 0 & 0 & 0 \\ 0 & \epsilon_2 & 0 & \frac{g}{b^2} & 0 & \frac{g}{b^4} & 0 & 0 \\ \frac{g}{b^2} & 0 & \epsilon_3 & 0 & 0 & 0 & \frac{g}{b^4} & 0 \\ 0 & \frac{g}{b^2} & 0 & \epsilon_4 & 0 & 0 & 0 & \frac{g}{b^4} \\ \frac{g}{b^4} & 0 & 0 & 0 & \epsilon_5 & \frac{g}{b} & 0 & \frac{g}{b^3} \\ 0 & \frac{g}{b^4} & 0 & 0 & \frac{g}{b} & \epsilon_6 & \frac{g}{b} & 0 \\ 0 & 0 & \frac{g}{b^4} & 0 & 0 & \frac{g}{b} & \epsilon_7 & \frac{g}{b} \\ 0 & 0 & 0 & \frac{g}{b^4} & \frac{g}{b^3} & 0 & \frac{g}{b} & \epsilon_8 \end{bmatrix}$$

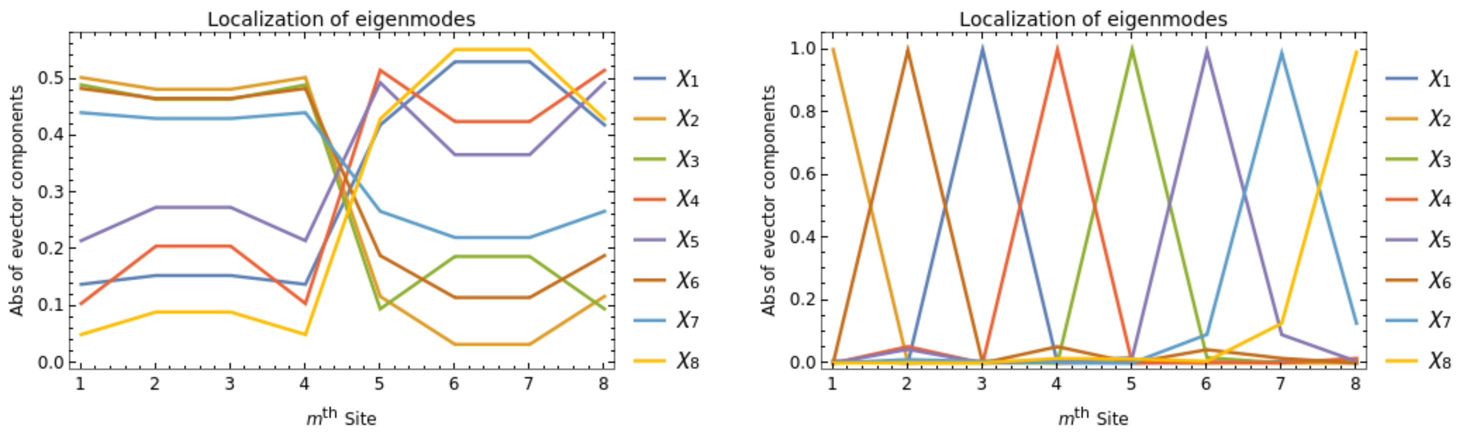


Fig.6 - Mass modes of Petersen graph with uniform sites (left) and random sites(right) for N = 8, W = 5, g = 1/4 and b = 1.4.

Singh and Vempati, 2401.XXXX

Strong Localisation Limit :

$$\epsilon \gg t$$

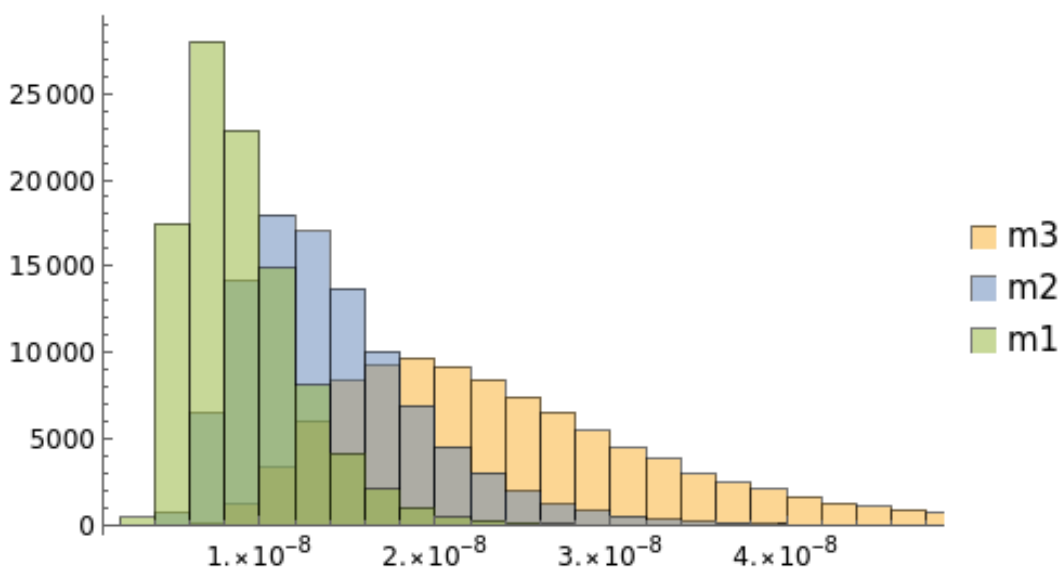
$$\mathcal{H}_{i,j} = \epsilon_i \delta_{i,j} - t_i(\delta_{i+1,j} + K\delta_{i,j+1})$$

-independent of geometry of the Chain

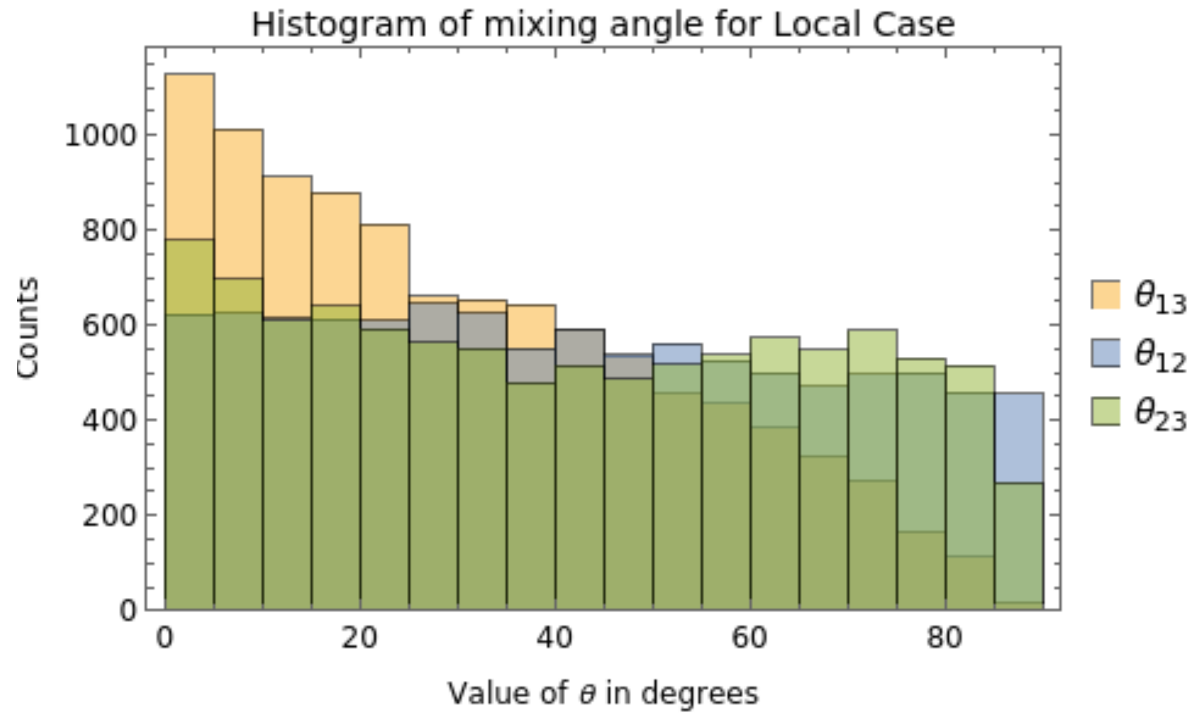
- Some universal features for neutrino masses and mixing.

Dirac Case

$$\mathcal{L}_{int.} = Y_1 \bar{\nu}_L H R_1 + Y_2 \bar{\nu}_R H L_n + h.c.$$



O(1) eV neutrino masses (Demonstration)

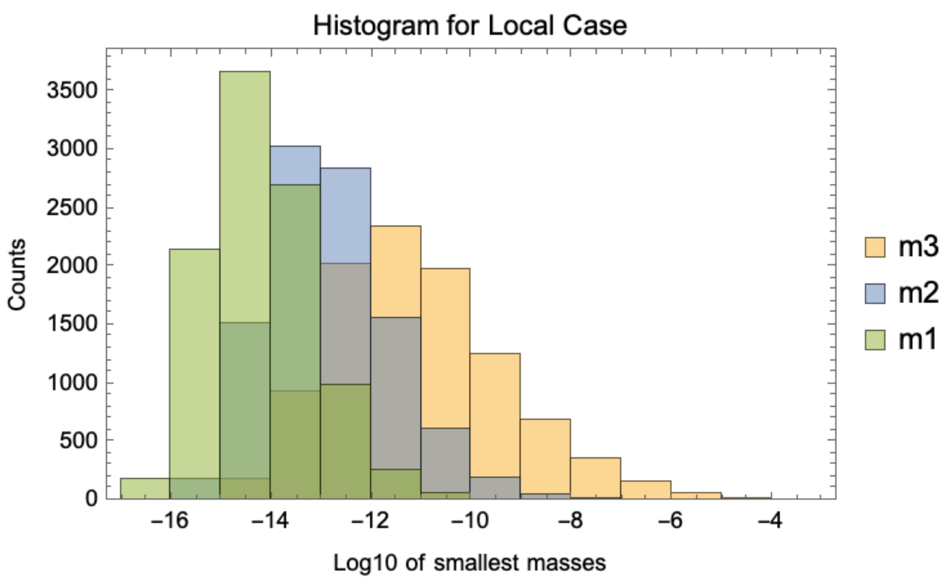


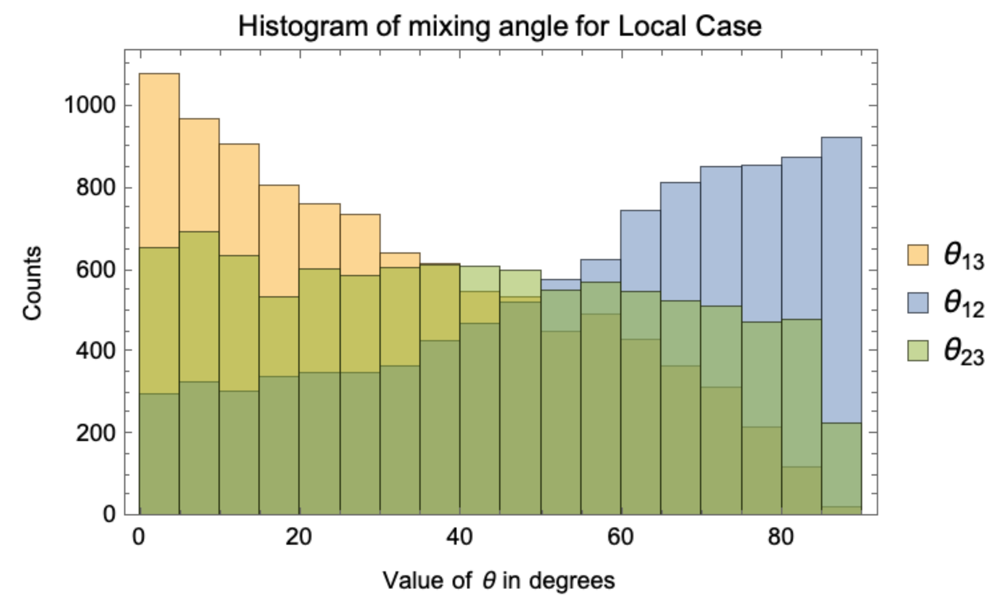
Mixing angles are anarchical.

BSM@50, Jan 2024

Majorana Case

$$\mathcal{L}_{NP} = L_{kin} - t\bar{L}_1\Psi - \sum_{i,j=1}^{n} \overline{L_i}\mathcal{H}_{i,j}R_j - W\Psi\Psi + h.c.$$





Hierachial neutrino masses with suppression but anarchical mixing angles.

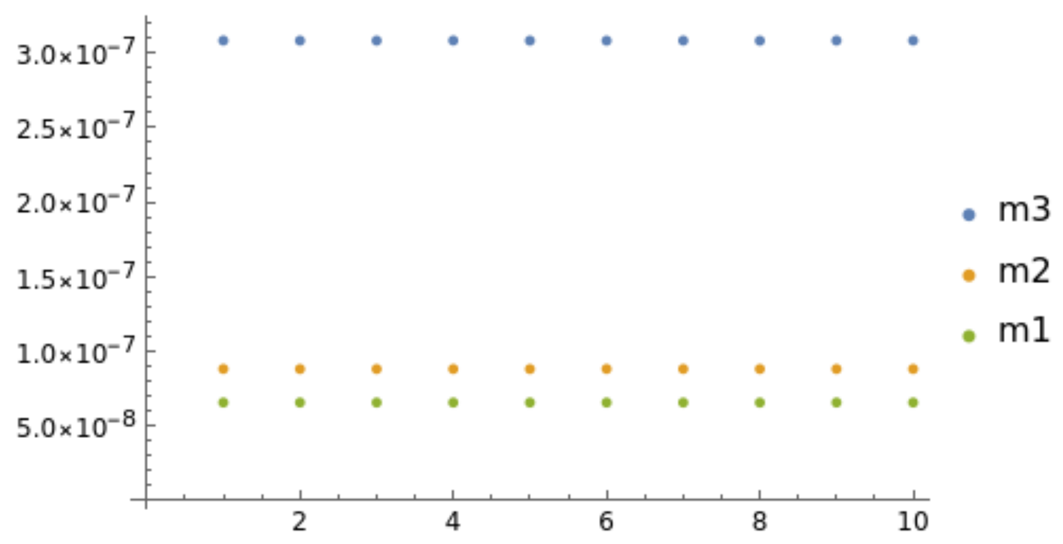
Hierarchial neutrino masses with anarchic mixing angles is a feature of the strong localisation regime independent of the type of geometry, couplings (non-local, partially local etc.)

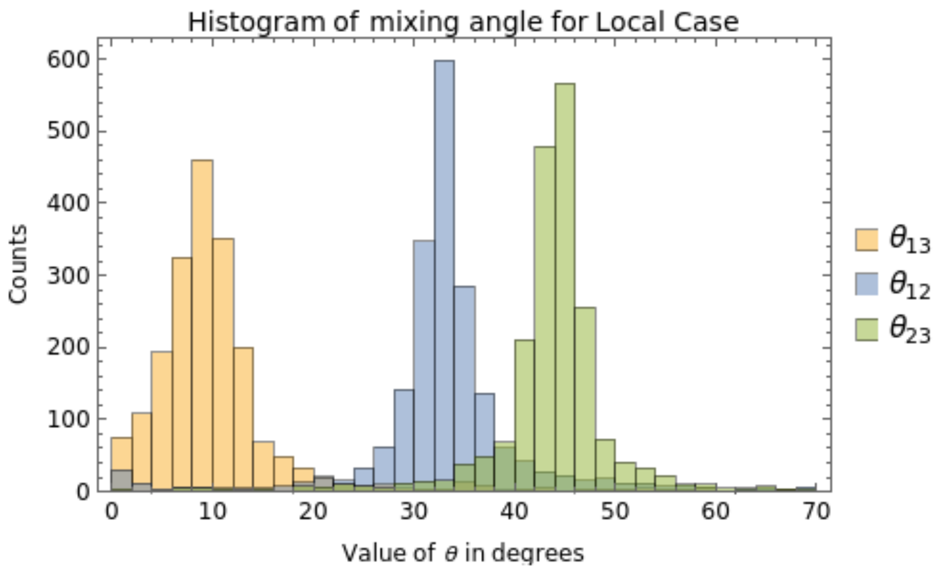
In the case of strong disorder in couplings (t) parameter, $t \gg \epsilon$, geometry does play a mild role, but mixing angles are mostly anarchic, except one !.

Role of Geometry : Weak Disorder

Dirac Scenario : Local Lattice (only nearest neighbour)

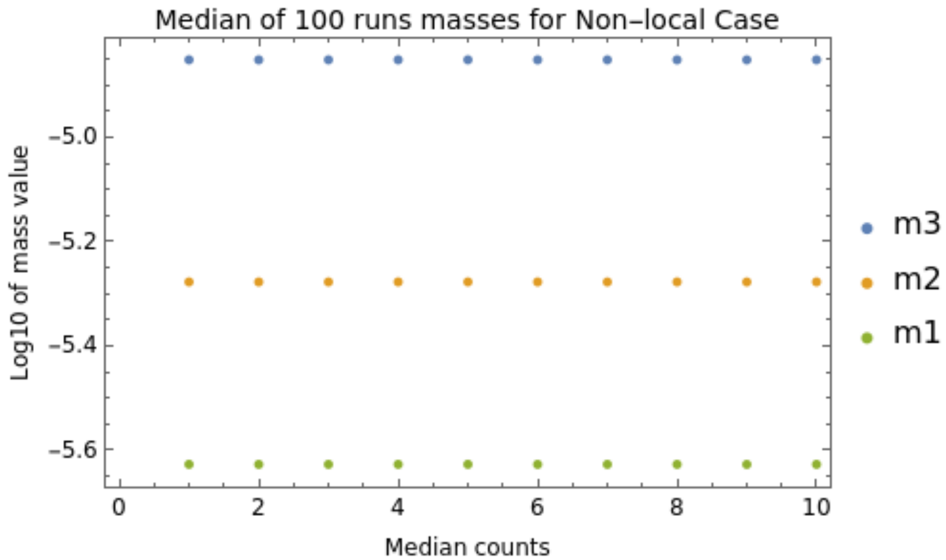
$$\epsilon \lesssim t$$

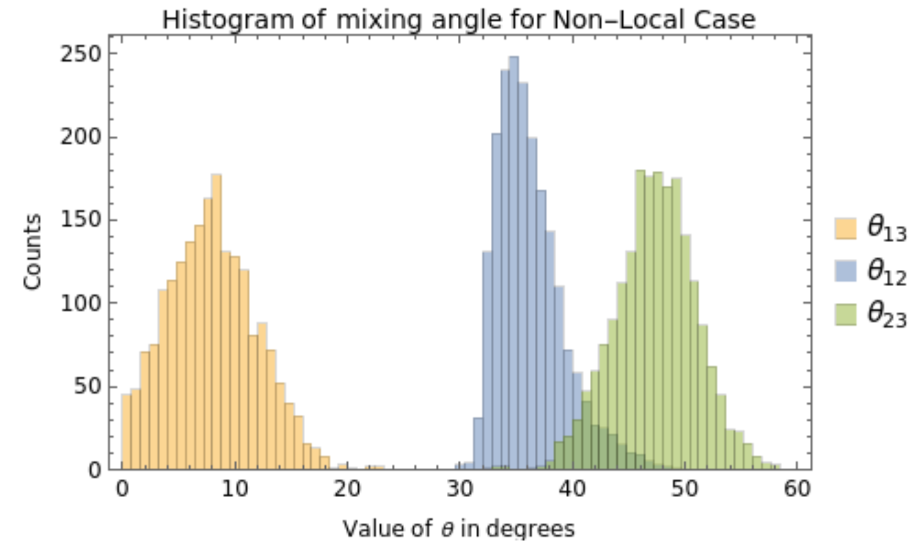




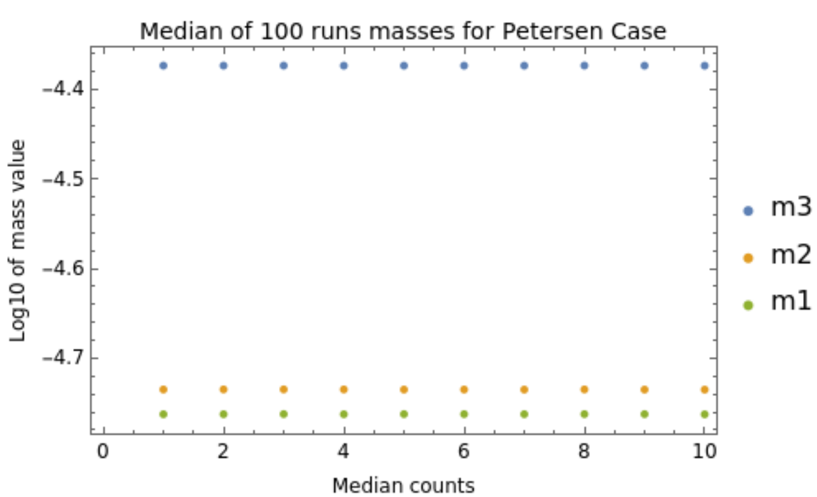
Mixing angles are “localised”.

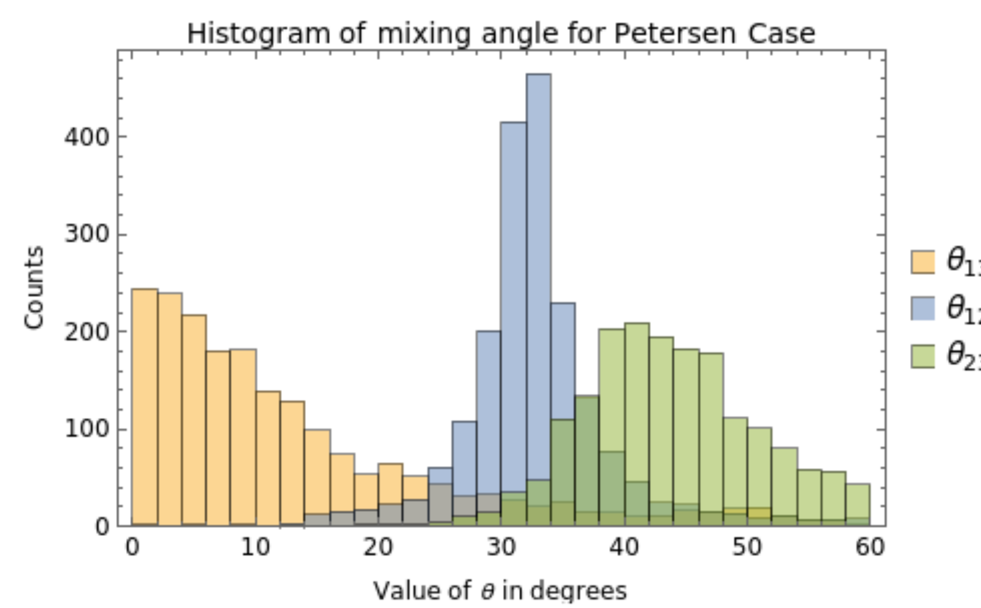
Fully non-local



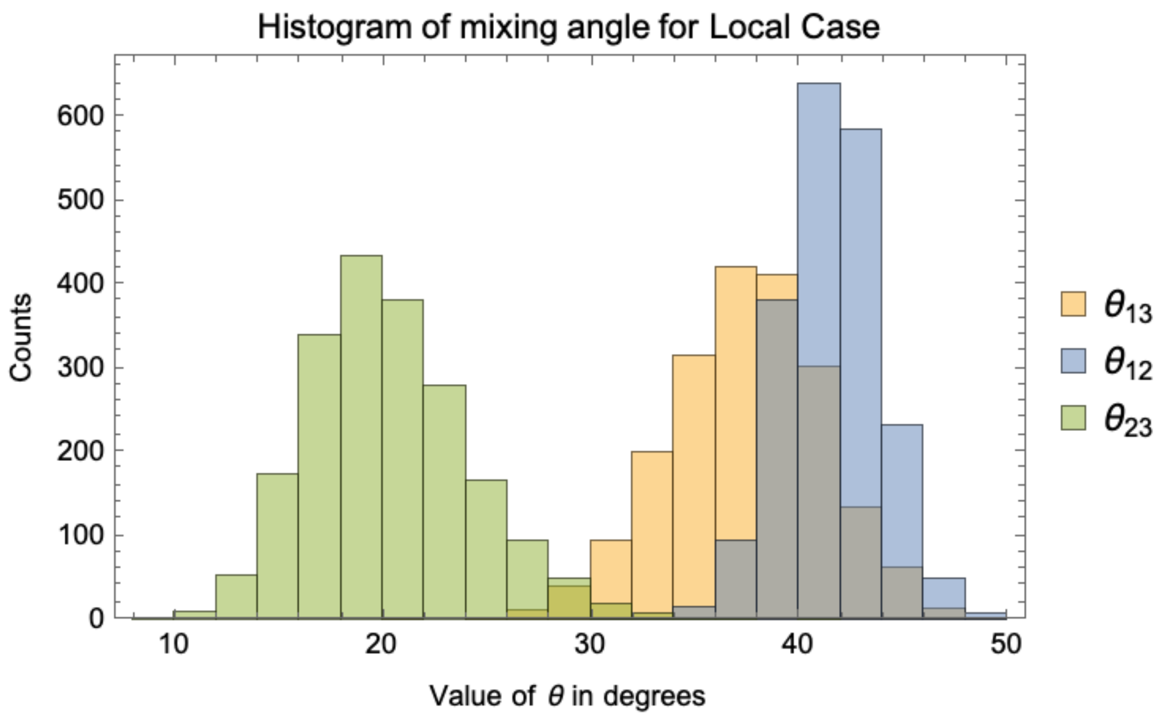


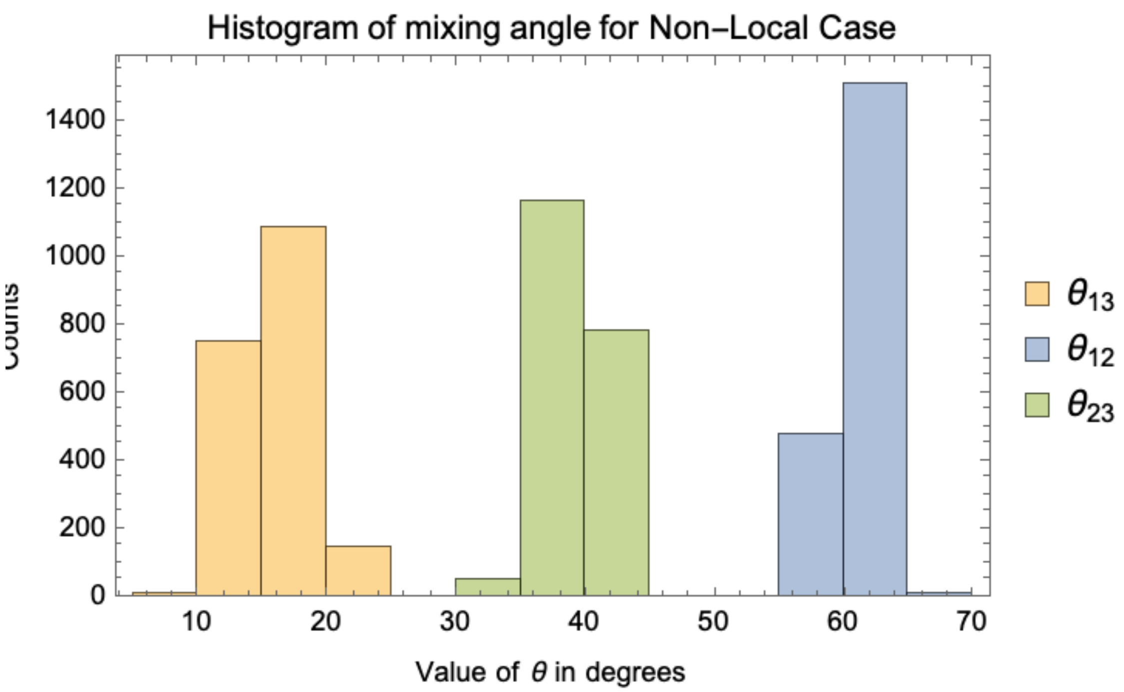
Partially non-local



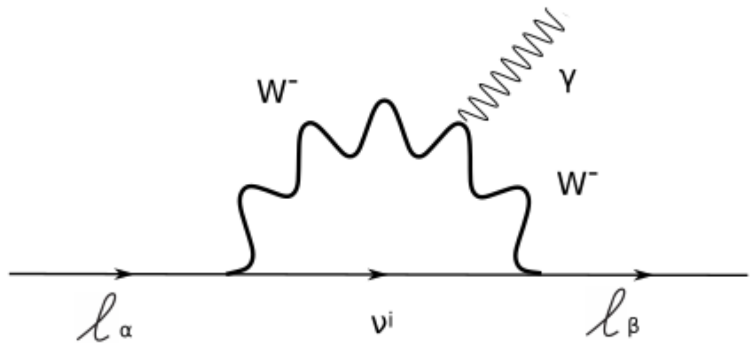


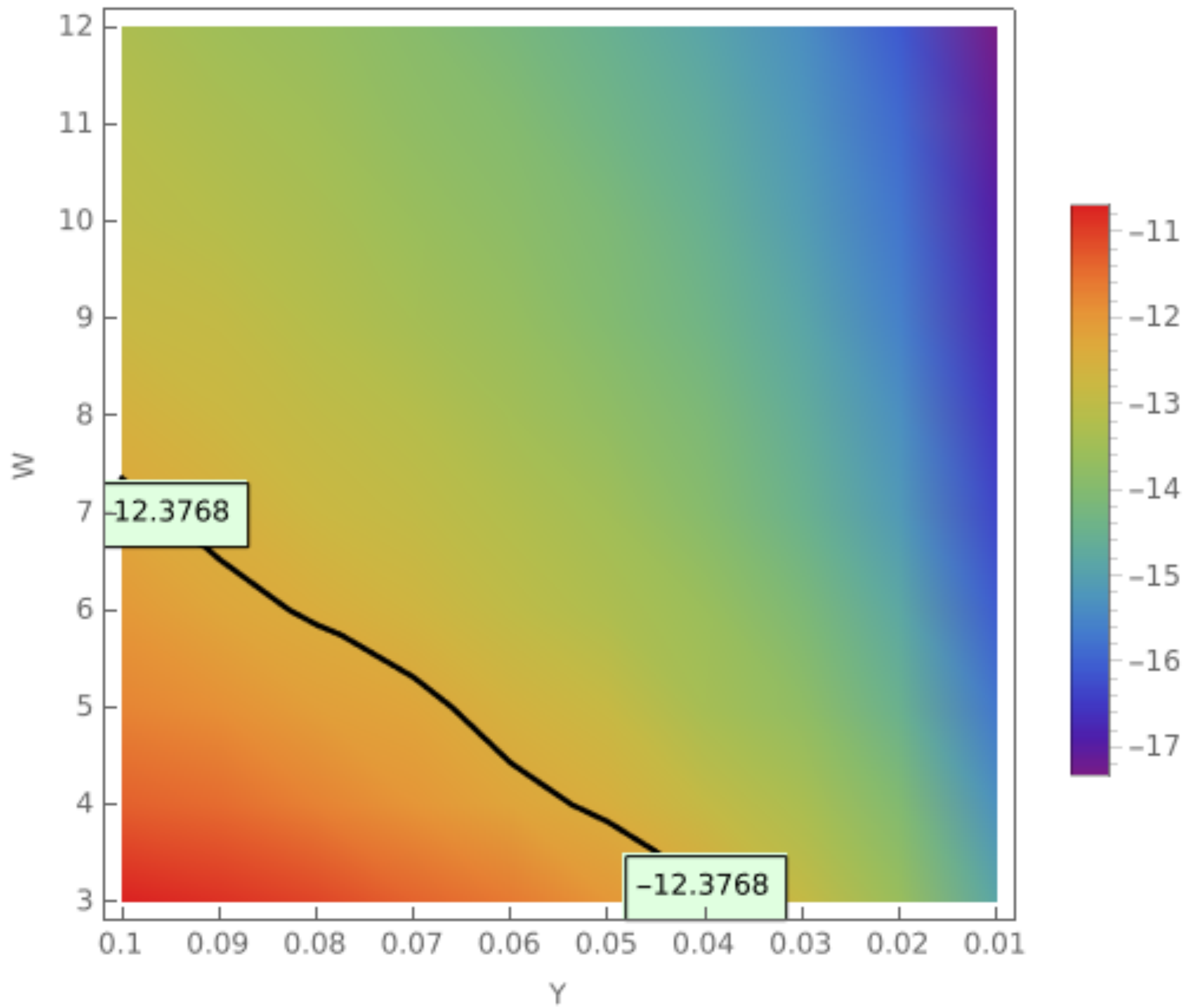
For the Majorana case, we get similar " localisation"





Phenomenology





Constraints become weaker for non-local and partially local case.

Outlook

Randomness in couplings can lead to
exponentially hierarchal couplings.

In the regime of strong coupling, the geometry of the mass chains
does not matter significantly. They predict hierarchal neutrino masses
and anarchical mixing angles for both Dirac or Majorana scenarios.

In the weak coupling regime, geometry does play a role and
can be chosen carefully to ``localise” the mixing angles.

Experimental signatures become weaker for
non-local /partially non-local cases compared to local case.

Majorana Case

The gears have large couplings as before.

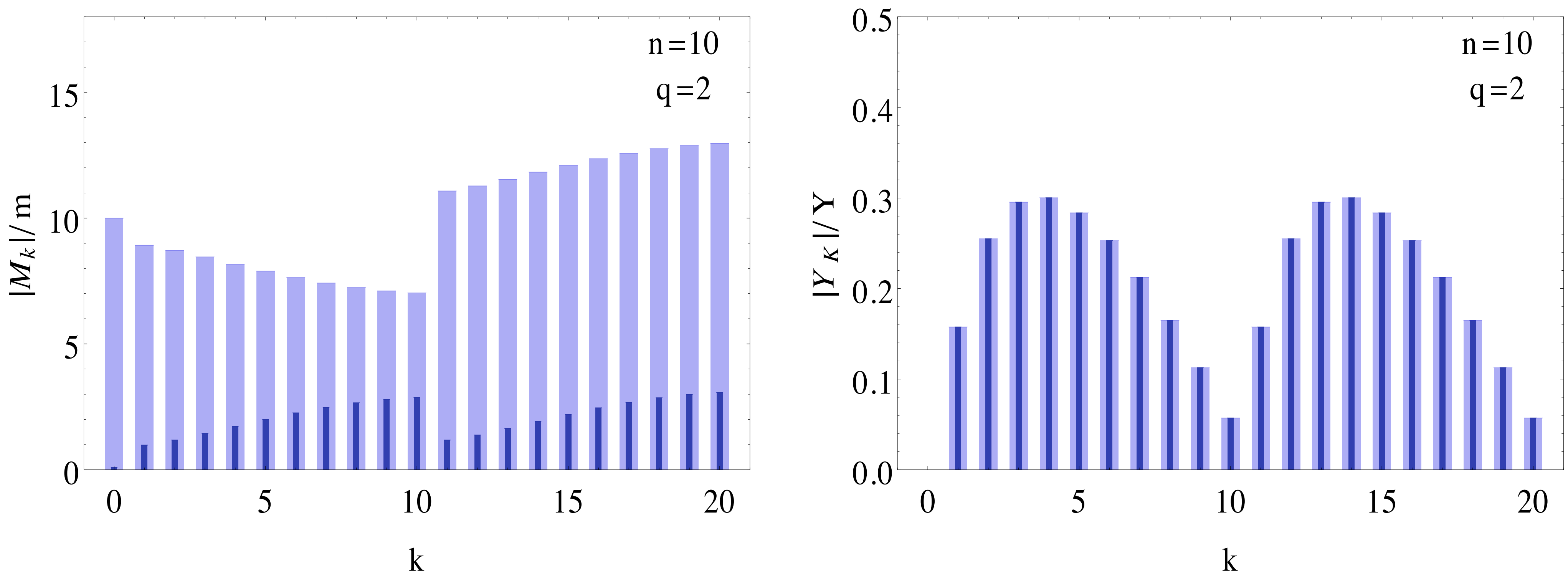


Figure 3: Majorana masses (left panel) and Yukawa couplings (right panel) of the singlet fermions of the clockwork sector, normalized respectively to m and Y, for the specific case $n = 10$, $q = 2$ and $\widetilde{q} = 0.1$ (dark blue) or $\widetilde{q} = 10$ (light blue).

Generalisation with Majorana Masses for the New Fermions

$$\mathcal{L}_{\text{Clockwork}} = \mathcal{L}_{\text{kin}} - \sum_{i=0}^{n-1} \left(m_i \overline{\psi}_{Li} \psi_{Ri} - m'_i \overline{\psi}_{Li} \psi_{Ri+1} + \text{h.c.} \right) - \sum_{i=0}^{n-1} \frac{1}{2} M_{Li} \overline{\psi^c_{Li}} \psi_{Li} - \sum_{i=0}^{n} \frac{1}{2} M_{Ri} \overline{\psi^c_{Ri}} \psi_{Ri} \ ,$$

$m_i = m$, $m'_i = mq$ $M_{Ri} = M_{Li} = m\widetilde{q}$ for all i.

related works:
Hambye et. al
Park et. al

$$\mathcal{M} = m \begin{pmatrix} \widetilde{q} & 0 & \cdots & 0 & 1 & -q & \cdots & 0 \\ 0 & \widetilde{q} & \cdots & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \widetilde{q} & 0 & 0 & 0 & -q \\ 1 & 0 & \cdots & 0 & \widetilde{q} & 0 & \cdots & 0 \\ -q & 1 & \cdots & 0 & 0 & \widetilde{q} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & -q & 0 & 0 & 0 & \widetilde{q} \end{pmatrix} ,$$

$$\begin{aligned} M_0 &= m\widetilde{q} \ , \\ M_k &= m\widetilde{q} - m\sqrt{\lambda_k} \ , \qquad k = 1, \ldots, n \ , \\ M_{n+k} &= m\widetilde{q} + m\sqrt{\lambda_k} \ , \qquad k = 1, \ldots, n \ , \end{aligned}$$

No Zero mode !!

$$\lambda_k \equiv q^2 + 1 - 2q \cos \frac{k\pi}{n+1} \ .$$

can be diagonalised the matrix

$$\mathcal{U} = \begin{pmatrix} \vec{0} & \frac{1}{\sqrt{2}} U_L & -\frac{1}{\sqrt{2}} U_L \\ \vec{u}_R & \frac{1}{\sqrt{2}} U_R & \frac{1}{\sqrt{2}} U_R \end{pmatrix} .$$

$$\vec{0}_j = 0 , \quad j = 1, ..., n ,$$

$$(u_R)_j = \frac{1}{q^j} \sqrt{\frac{q^2 - 1}{q^2 - q^{-2n}}} , \quad j = 1, ..., n ,$$

$$(U_L)_{jk} = \sqrt{\frac{2}{n+1}} \sin \frac{jk\pi}{n+1} , \qquad j, k = 1, ..., n ,$$

$$(U_R)_{jk} = \sqrt{\frac{2}{(n+1)\lambda_k}} \left[q \sin \frac{jk\pi}{n+1} - \sin \frac{(j+1)k\pi}{n+1} \right] , \quad j = 0, .., n, \quad k = 1, ..., n ,$$

under the universality assumption, the presence of the Majorana masses does not change the mixing matrices !!.

The purely majorana mass mode has same features as the zero mode

Generalisation with Majorana Masses for the New Fermions

$\tilde{q} \ll q$

$q \ll \tilde{q}$

pseudo-Dirac Masses

Phenomenology
unexplored

Perhaps ICECUBE

Normal Seesaw like scenario

$$m_\nu \approx \sum_k \frac{Y_k^2 v^2}{M_k} \, .$$

Neutrino mass limits push
the gear masses to GUT scale.

Sterile neutrino phenomenology needs to be explored

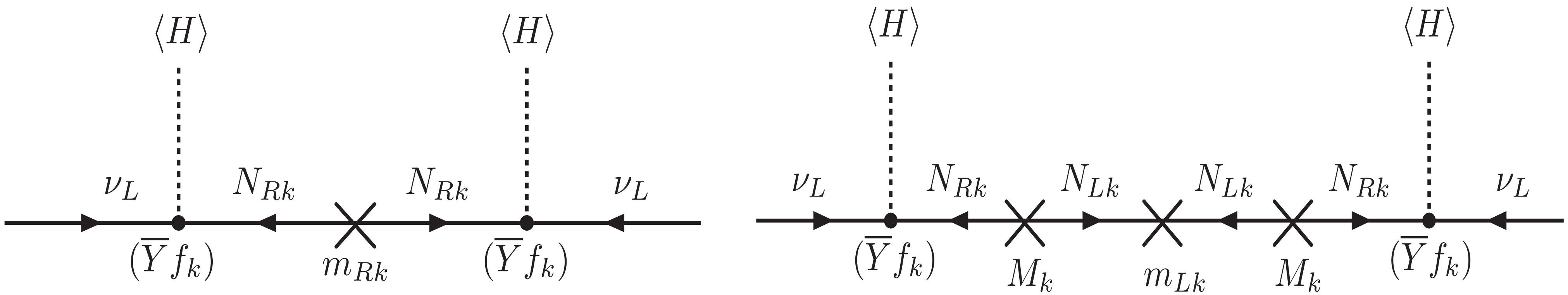


Figure 9: Neutrino Mass at tree level in Majorana Case.

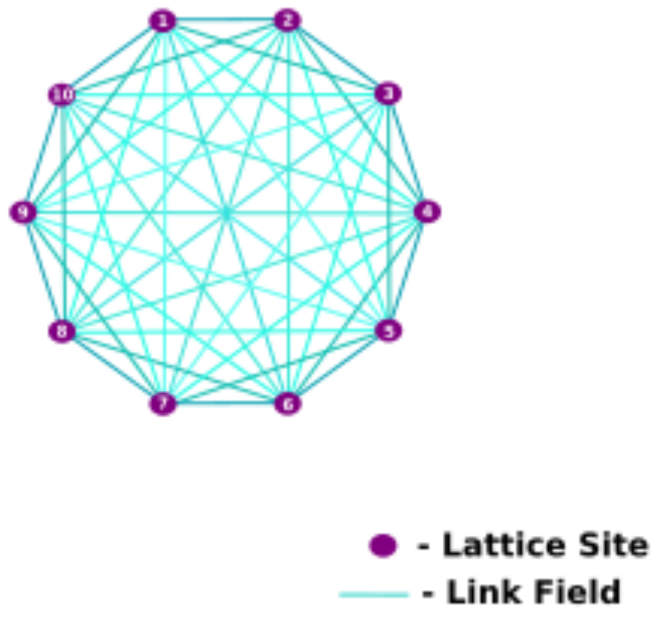
Gear masses are pushed to the GUT scale as they give large corrections to the neutrino masses.

In this case, no signals at the weak scale due to "gears", the new fermions.

Back Up

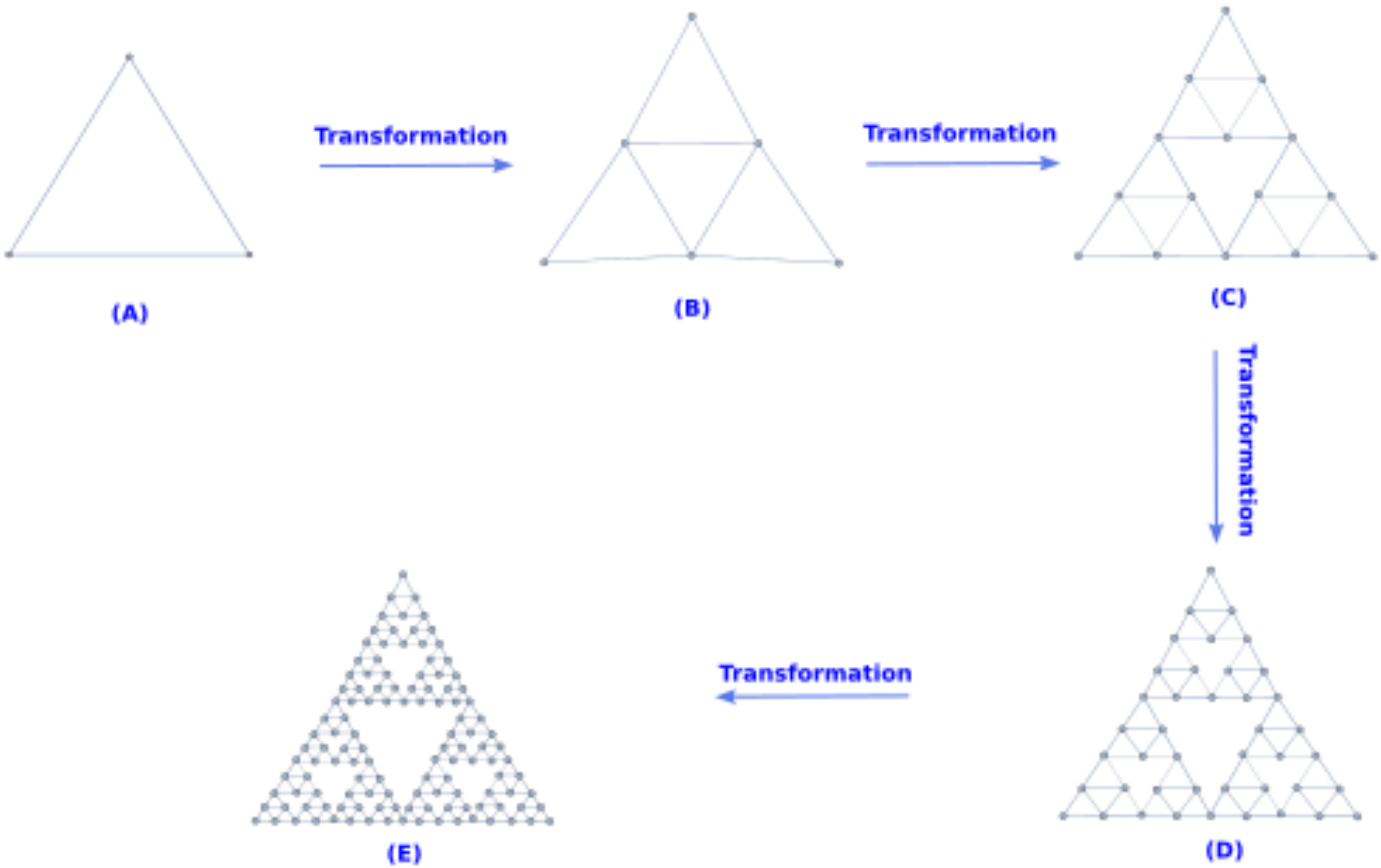
$$M_{fermion} = \begin{bmatrix} 0 & v_1^1 & v_1^2 & v_1^3 & \ldots & v_1^n \\ v_n^1 & \lambda_1 & 0 & 0 & \ldots & 0 \\ v_n^2 & 0 & \lambda_2 & 0 & \ldots & 0 \\ v_n^3 & 0 & 0 & \lambda_3 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ v_n^n & 0 & 0 & 0 & \ldots & \lambda_n \end{bmatrix}$$

$$m_0 \approx \sum_{i=1}^{n} \frac{v_1^i v_n^i}{\lambda_i} \propto \sum_{i=1}^{n} v^2 \frac{e^{-\frac{n}{L_n}}}{\lambda_i}$$



Non Local and Two Dimensional Graphs

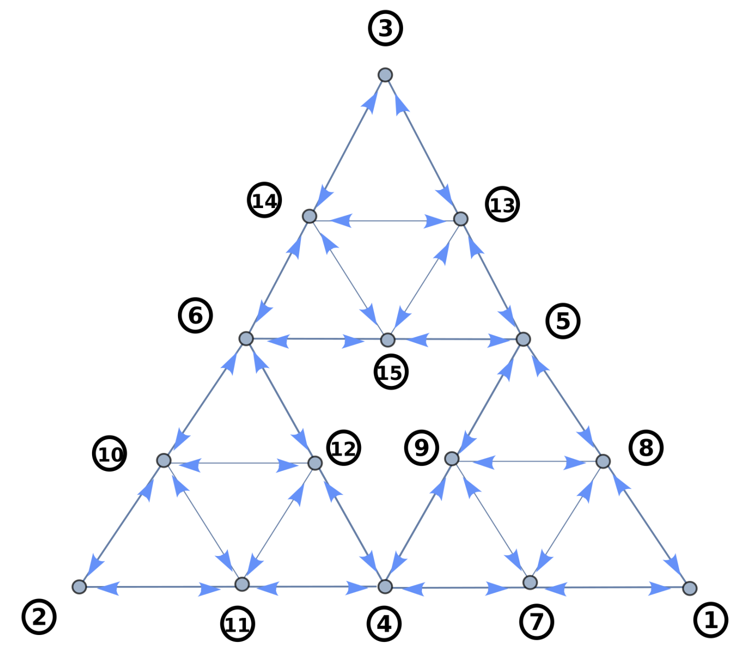
Sierpinski Fractal Graphs

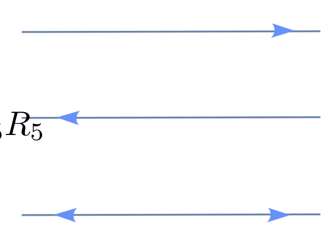


Example with 15 vertices :

- three zero modes !

-localisation of the zero modes



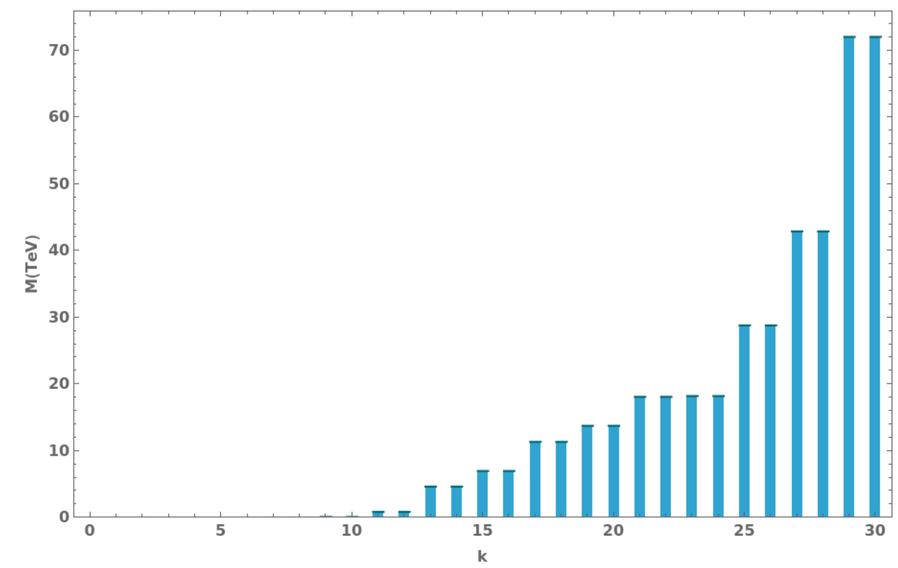


- Link Field Between L_i and R_j
- Link Field Between L_j and R_i
- Link Field Between L_i and R_j & L_j and R_i

$$
\begin{aligned}
\mathcal{L}_{NP} =& \mathcal{L}_{kin} - \sum_{i,j=1}^{15} m_i \overline{L_i} \delta_{i,j} R_j + m\Big(\overline{L_1} q_{1,7} R_7 + \overline{L_1} q_{1,8} R_8 + \overline{L_7} q_{7,4} R_4 + \overline{L_7} q_{7,9} R_9 + \overline{L_7} q_{7,8} R_8 + \overline{L_8} q_{8,5} R_5 \\
&+ \overline{L_8} q_{8,9} R_9 + \overline{L_4} q_{4,9} R_9 + \overline{L_4} q_{4,11} R_{11} + \overline{L_4} q_{4,12} R_{12} + \overline{L_9} q_{9,5} R_5 + \overline{L_5} q_{5,13} R_{13} + \overline{L_5} q_{5,15} R_{15} + \\
&\overline{L_2} q_{2,10} R_{10} + \overline{L_2} q_{2,11} R_{11} + \overline{L_{10}} q_{10,6} R_6 + \overline{L_{10}} q_{10,12} R_{12} + \overline{L_{10}} q_{10,11} R_{11} + \overline{L_{11}} q_{11,12} R_{12} + \overline{L_6} q_{6,12} R_{12} \\
&+ \overline{L_6} q_{6,14} R_{14} + \overline{L_6} q_{6,15} R_{15} + \overline{L_3} q_{3,13} R_{13} + \overline{L_3} q_{3,14} R_{14} + \overline{L_3} q_{3,15} R_{15} + \overline{L_{13}} q_{13,14} R_{14} + \overline{L_{14}} q_{14,15} R_{15} \Big) \\
&+ m \overline{L_i} q_{i \leftrightarrow j} R_j + h.c.
\end{aligned}
$$

One graph for all the three neutrinos !!

$$M_{Fractal} = \begin{pmatrix}
2m & mf & mf^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{m}{f} & 2m & mf & mf^2 & mf^3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{m}{f^2} & \frac{m}{f} & 2m & 0 & mf^2 & mf^3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{m}{f^2} & 0 & 2m & mf & 0 & mf^3 & mf^4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{m}{f^3} & \frac{m}{f^2} & \frac{m}{f} & 2m & mf & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{m}{f^3} & 0 & \frac{m}{f} & 2m & 0 & 0 & mf^3 & mf^4 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{m}{f^3} & 0 & 0 & 2m & mf & 0 & 0 & mf^4 & mf^5 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{m}{f^4} & 0 & 0 & \frac{m}{f} & 2m & 0 & 0 & 0 & mf^4 & mf^5 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{m}{f^3} & 0 & 0 & 2m & mf & 0 & 0 & 0 & mf^5 & mf^6 \\
0 & 0 & 0 & 0 & 0 & \frac{m}{f^4} & 0 & 0 & \frac{m}{f} & 2m & 0 & 0 & mf^3 & mf^4 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{m}{f^4} & 0 & 0 & 0 & 2m & mf & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{m}{f^5} & \frac{m}{f^4} & 0 & 0 & \frac{m}{f} & 2m & mf & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{m}{f^5} & 0 & \frac{m}{f^3} & 0 & \frac{m}{f} & 2m & mf & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{m}{f^5} & \frac{m}{f^4} & 0 & 0 & \frac{m}{f} & 2m & mf \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{m}{f^6} & 0 & 0 & 0 & 0 & \frac{m}{f} & 2m
\end{pmatrix}$$



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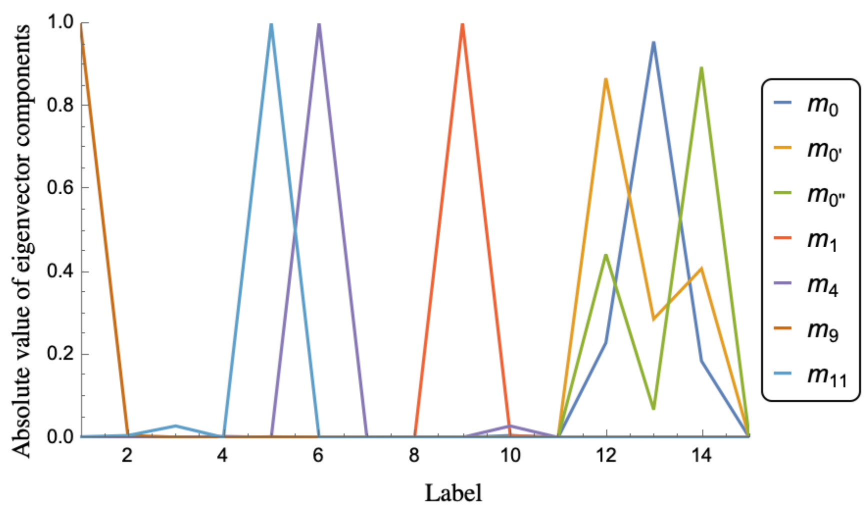
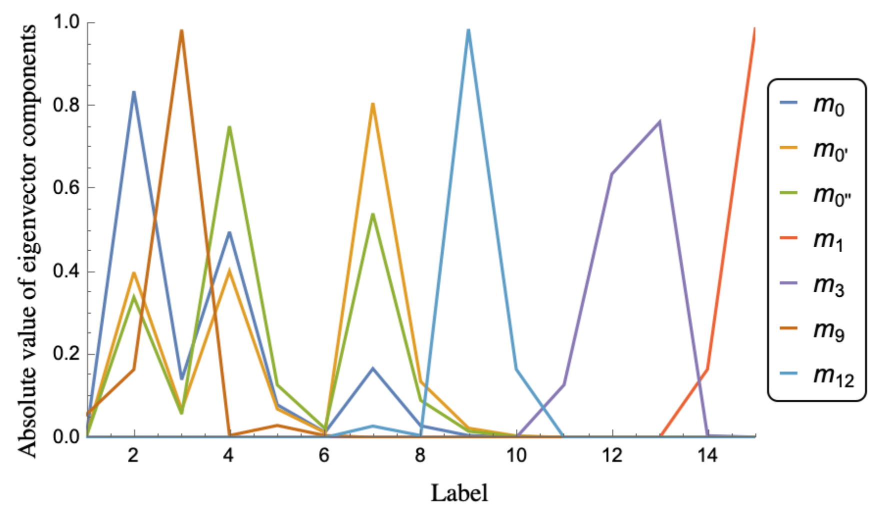




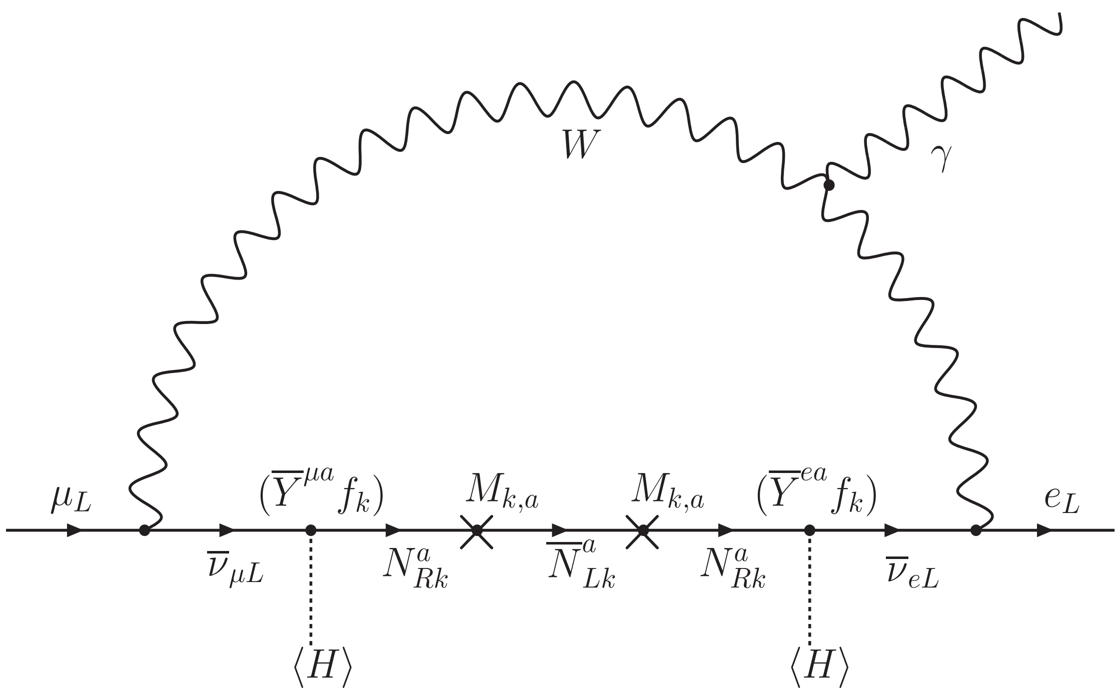
Plot 2(B) - Left plot shows the absolute value of components of left-handed mass eigenvectors and the right plot for the right-handed mass eigenvector.

Singh and vempati, 2023

Phenomenology

Putting in the full Standard Model (leptonic sector)

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{Clockwork}} + \mathcal{L}_{\text{int}} \ ,$$



Exchange of clockwork gears leads to lepton flavour violation.

Consider for example a rare process, which has not yet Been discovered…similar to rare flavour violating processes in the Hadronic sector.

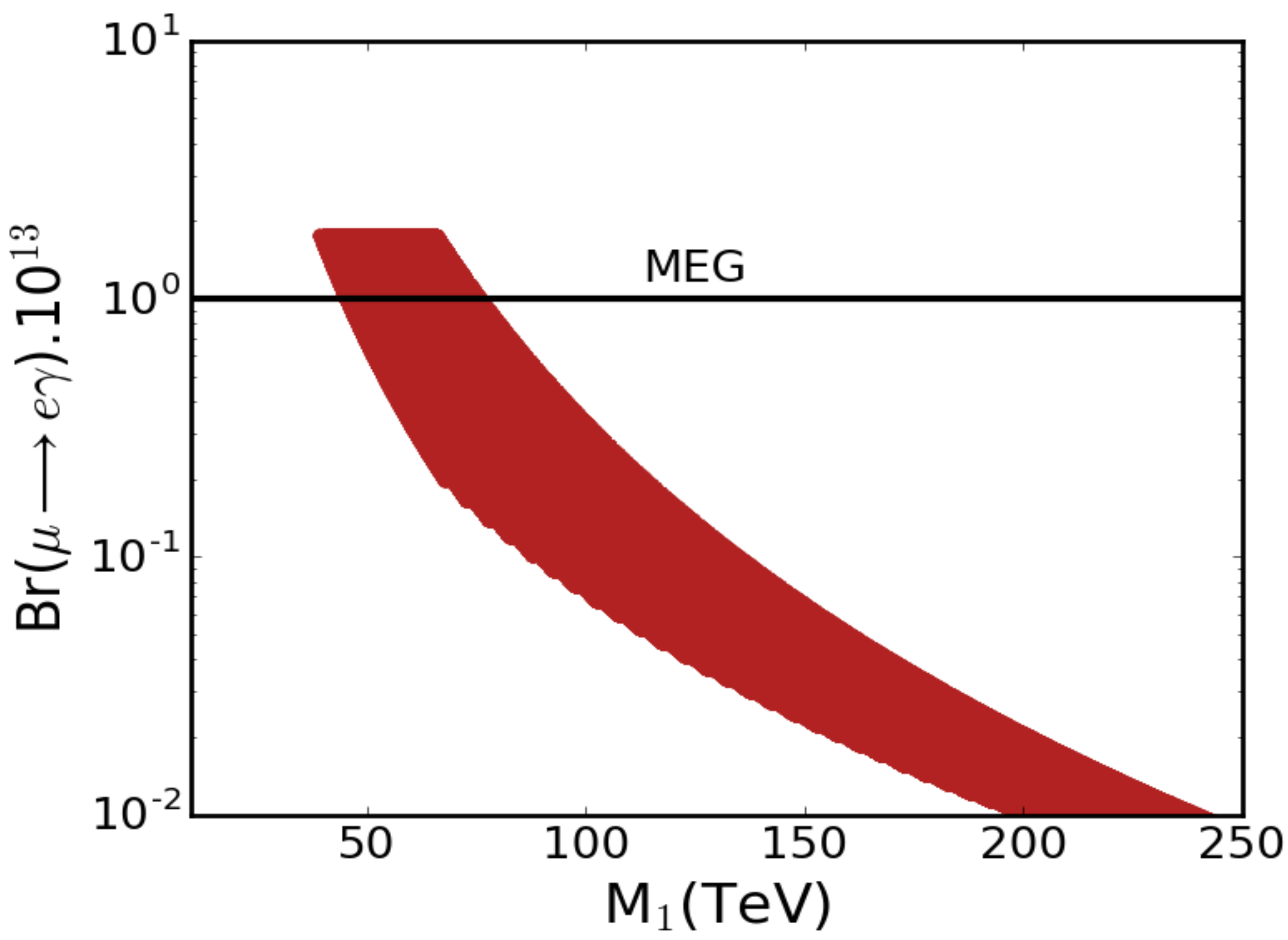
$$\mu \to e + \gamma$$

But there are strong limits on it

$$B(\mu \to e\gamma) \simeq \frac{3\alpha_{\text{em}} v^4}{8\pi} \left| \sum_{\alpha=1}^{N} \sum_{k=1}^{n_\alpha} \frac{Y_k^{e\alpha} Y_k^{\mu\alpha}}{M_k^{\alpha\,2}} F(x_k^\alpha) \right|^2 ,$$

$$F(x) \equiv \frac{1}{6(1-x)^4}(10 - 43x + 78x^2 - 49x^3 + 4x^4 - 18x^3 \log x) \ ,$$

Lepton Flavour Violation

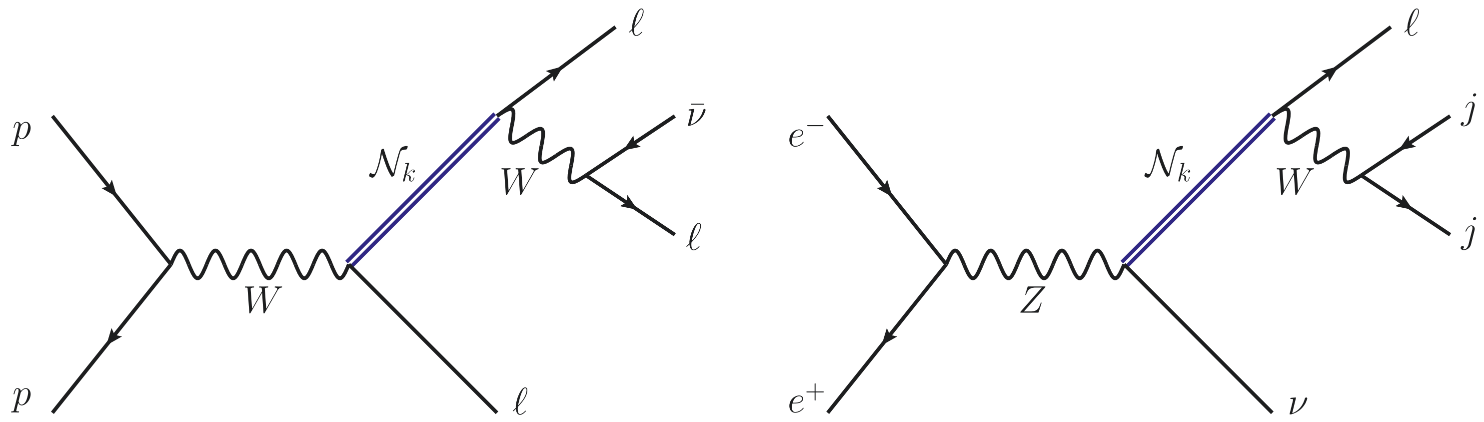


Present limit of around 40 TeV !!

Fig. 6 - These Feynman diagrams show the 1-loop contribution of fermions in Higgs mass radiative corrections. The Left diagram shows it for the same fermions in the loop with y_{ii} coupling and the right diagram shows it for different fermions in the loop with y_{ij} coupling strength.

Higgs corrections !!

LFV at colliders



A lot of things still left to be explored.

Conclusions

We presented localisation in models which are "finite" not equavilent to extra dimensions and they provide interesting phenomena.