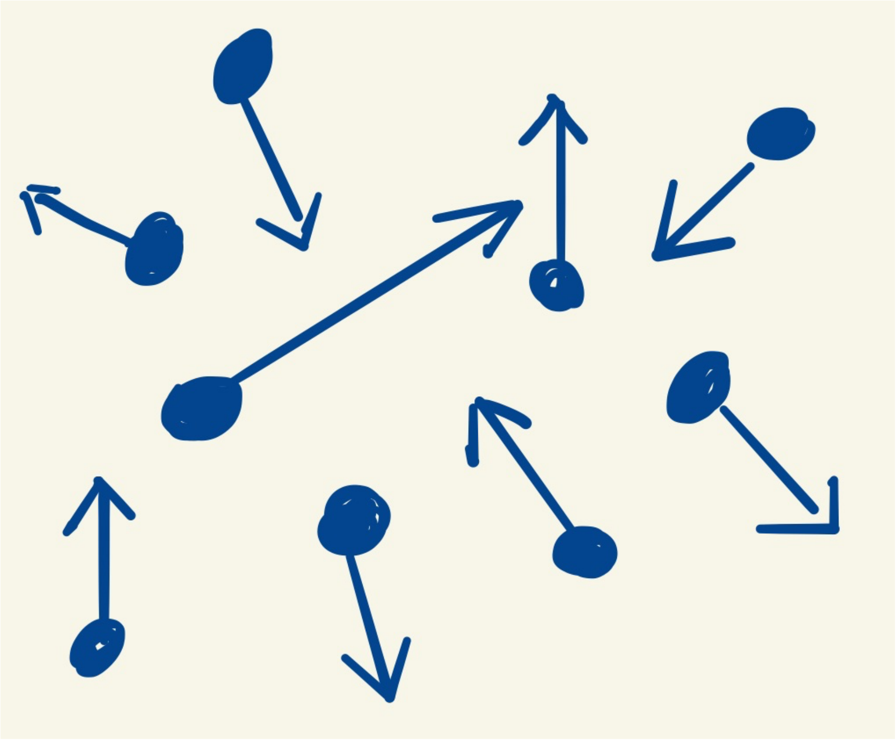
Leonardo Senatore (ETH)

On the
Effective Field Theory of
Large Scale Structure

What is a fluid?



wikipedia: credit
National Oceanic and Atmospheric Administration/
Department of Commerce

$$\partial_t \rho_\ell + \partial_i \left(\rho_\ell v^i_\ell\right) = 0$$

$$\partial_t v^i_\ell + v^j_\ell \partial_j v^i_\ell + \frac{1}{\rho_\ell}\partial_i p_\ell = \text{viscous terms}$$

– From short to long

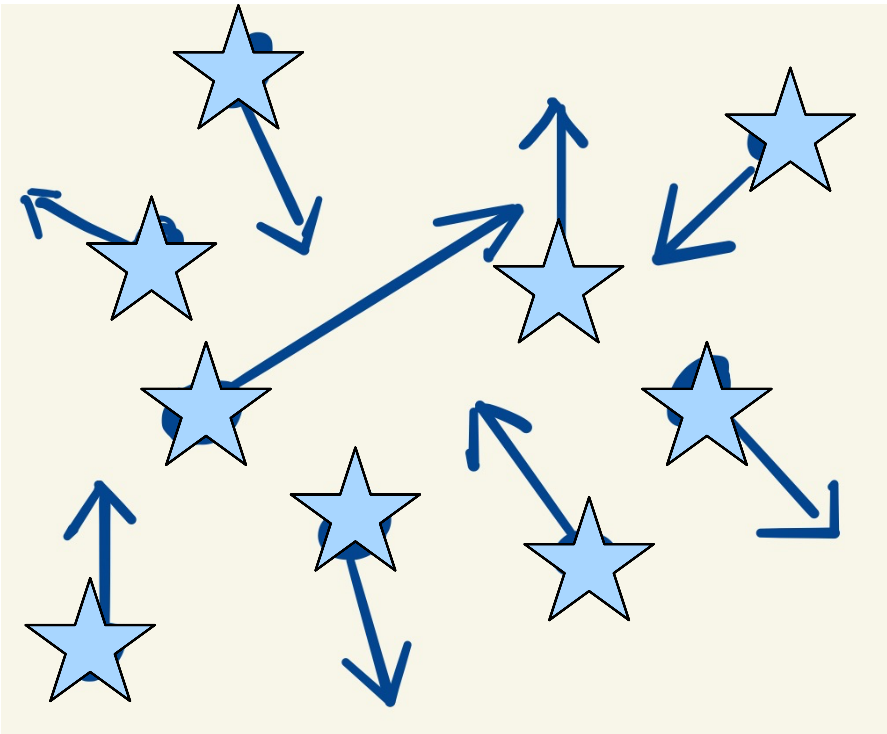
– The resulting equations are simpler

– Description arbitrarily accurate

  – construction can be made without knowing the nature of the particles.

– short distance physics appears as a non trivial stress tensor for the long-distance fluid

Do the same for matter in our Universe



credit NASA

with Baumann, Nicolis and Zaldarriaga **JCAP 2012**
with Carrasco and Hertzberg **JHEP 2012**

$$\nabla^2\Phi_\ell = H^2\left(\delta\rho_\ell/\rho\right)$$
$$\partial_t\rho_\ell + H\rho_\ell + \partial_i\left(\rho_\ell v_\ell^i\right) = 0$$
$$\partial_t v_\ell^i + v_\ell^j\partial_j v_\ell^i + \partial_i\Phi_\ell = \partial_j\tau^{ij}$$

– From short to long

– The resulting equations are simpler

– Description arbitrarily accurate

  – construction can be made without knowing the nature of the particles.

– short distance physics appears as a non trivial stress tensor for the long-distance fluid

$$\tau_{ij} \sim \delta_{ij}\,\rho_{\rm short}\,\left(v_{\rm short}^2 + \Phi_{\rm short}\right)$$

Dealing with the Effective Stress Tensor

- For long distances: expectation value over short modes (integrate them out)

$$\langle \tau_{ij}(\vec{x},t)\rangle_{\text{long fixed}} = f_{\text{very complicated}}\left(\{H, \Omega_m, \dots, m_{\text{dm}}, \dots, \rho_\ell(x)\}_{\text{past light cone}}\right)$$

At *long* wavelengths $\Downarrow$ Taylor Expansion

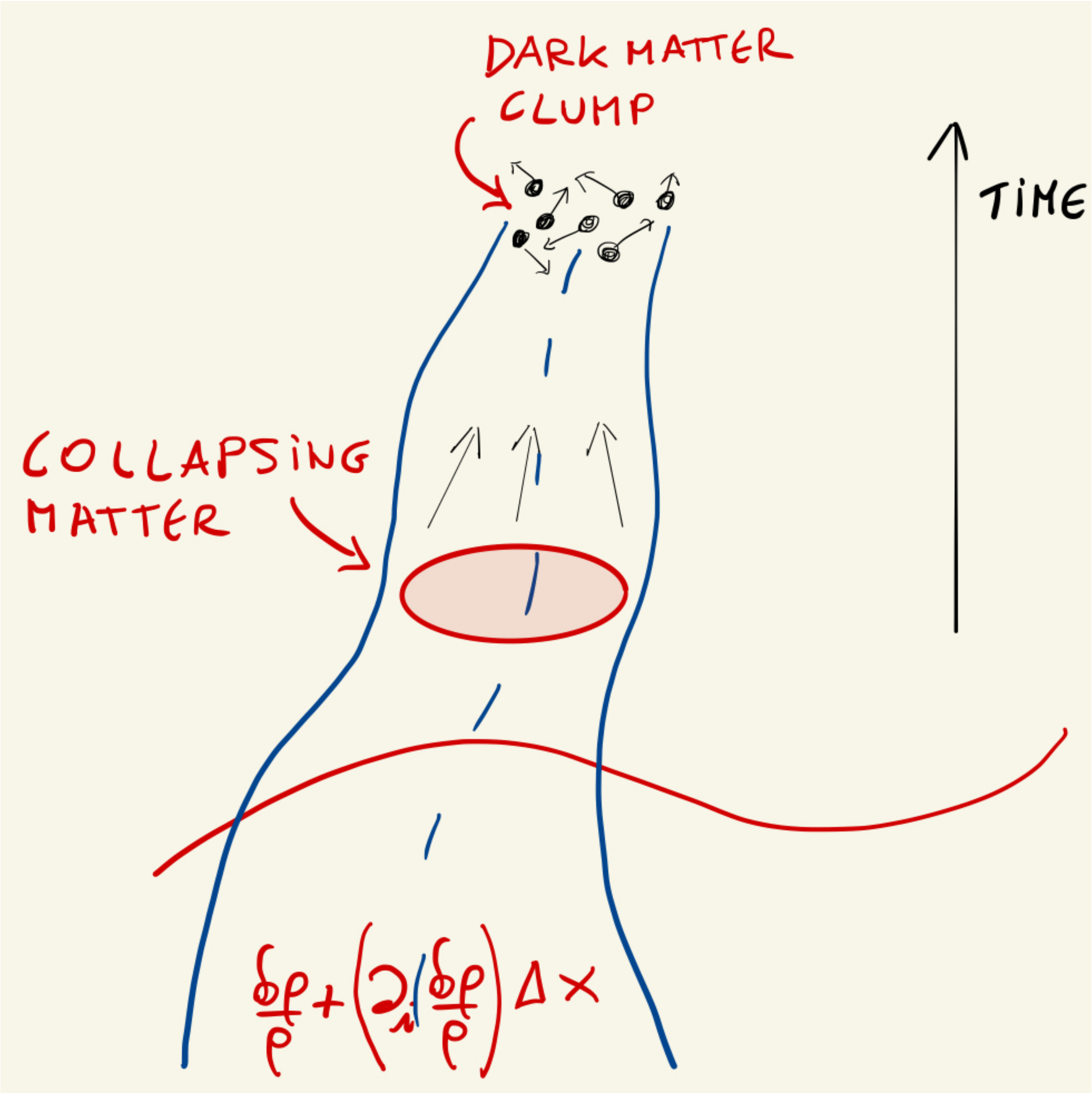
$$\langle \tau_{ij}(\vec{x},t)\rangle_{\text{long fixed}} = \int^t dt' \left[c(t,t')\,\frac{\delta\rho_\ell}{\rho}(\vec{x}_{\text{fl}},t') + \mathcal{O}\left((\delta\rho_\ell/\rho)^2\right)\right]$$

- Equations with only long-modes

$$\partial_t v^i_\ell + v^j_\ell \partial_j v^i_\ell + \partial_i \Phi_\ell = \partial_j \tau^{ij}$$

$$\tau_{ij} \sim \delta\rho_\ell/\rho + \dots$$

every term allowed by symmetries



- each term contributes as factor of

$$\frac{\delta\rho_l}{\rho} \sim \frac{k}{k_{\text{NL}}} \ll 1$$

Perturbation Theory within the EFT

- In the EFT we can solve iteratively $\delta_\ell, v_\ell, \Phi_\ell \ll 1$, where $\delta_\ell = \frac{\delta\rho_\ell}{\rho}$

$$\nabla^2\Phi_\ell = H^2\left(\delta\rho_\ell/\rho\right)$$
$$\partial_t\rho_\ell + H\rho_\ell + \partial_i\left(\rho_\ell v_\ell^i\right) = 0$$
$$\partial_t v_\ell^i + v_\ell^j\partial_j v_\ell^i + \partial_i\Phi_\ell = \partial_j\tau^{ij}$$

$$\tau_{ij} \sim \delta\rho_\ell/\rho + \ldots$$

- Two scales:

$$k\,[\text{Mean Free Path Scale}] \sim k\left[\left(\frac{\delta\rho}{\rho}\right) \sim 1\right] \sim k_{\rm NL}$$

Perturbation Theory within the EFT

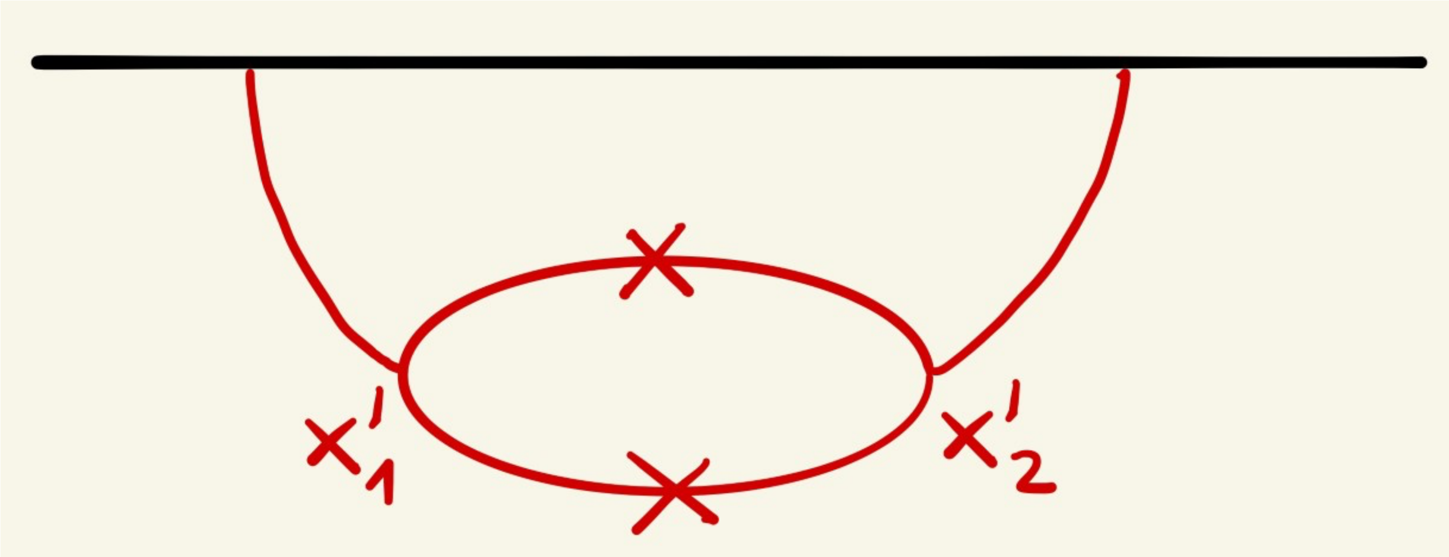
- Solve iteratively some non-linear eq. $\delta_\ell = \delta_\ell^{(1)} + \delta_\ell^{(2)} + \ldots \ll 1$
- Second order:

$$\partial^2 \delta_\ell^{(2)} = \left(\delta_\ell^{(1)}\right)^2 \quad \Rightarrow \quad \delta_\ell^{(2)}(x) = \int d^4x' \ \mathrm{Greens}(x, x') \ \left(\delta_\ell^{(1)}(x')\right)^2$$

- Compute observable:

$$\langle \delta_\ell(x_1)\delta_\ell(x_2)\rangle \supset \langle \delta_\ell^{(2)}(x_1)\delta_\ell^{(2)}(x_2)\rangle \sim \int d^4x_1' d^4x_2' \ (\mathrm{Green's})^2 \ \langle \delta_\ell^{(1)}(x_1')^2 \delta_\ell^{(1)}(x_2')^2\rangle$$

- We obtain Feynman diagrams



- Sensitive to short distance

$$x_2' \to x_1'$$

  - Need to add counterterms from $\tau_{ij} \supset c_s^2 \delta_\ell$ to correct

- Loops and renormalization applied to galaxies

…. lots of work ….

Galaxy Statistics

Senatore **1406**
with Lewandowsky *et al* **1512**
with Perko *et al*. **1610**

Galaxies in the EFTofLSS

Senatore **1406**

- On galaxies, a long history before us, summarized by McDonald, Roy **2010** .
  - Senatore **1406** provided first complete parametrization.

- Nature of Galaxies is very complicated

$$n_{\rm gal}(x) = f_{\rm very\ complicated}\left(\{H, \Omega_m, \ldots, m_e, g_{ew}, \ldots, \rho(x)\}_{\rm past\ light\ cone}\right)$$

Galaxies in the EFTofLSS

Senatore **1406**

$$n_{\rm gal}(x) = f_{\text{very complicated}}\Big(\{H,\Omega_m,\ldots,m_e,g_{ew},\ldots,\rho(x)\}_{\text{past light cone}}\Big)$$

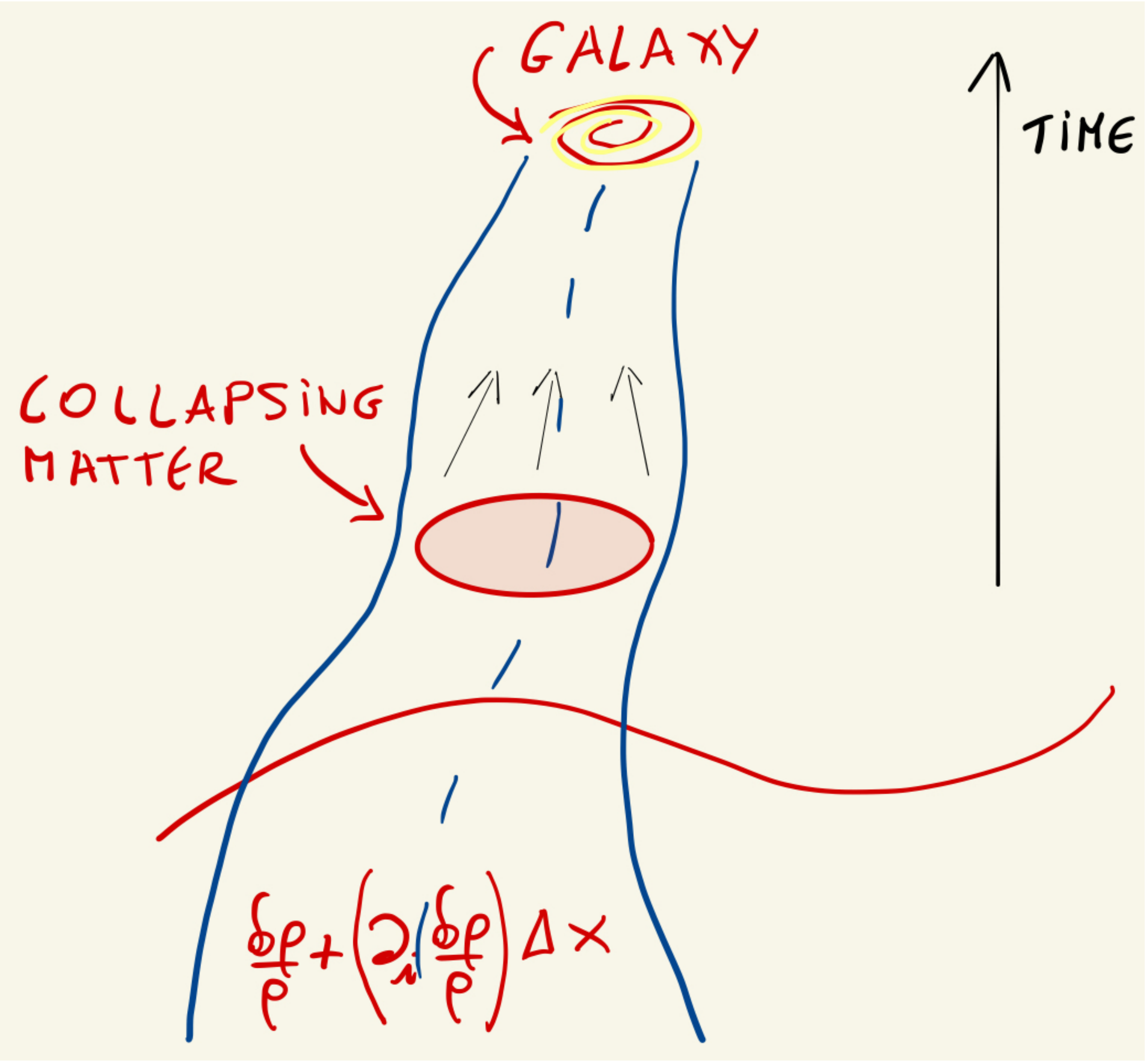
At *long* wavelengths $\Downarrow$ Taylor Expansion

$$\left(\frac{\delta n}{n}\right)_{{\rm gal},\ell}(x) \sim \int^t dt' \left[c(t,t')\,\left(\frac{\delta\rho}{\rho}\right)(\vec x_{\rm fl},t') + \ldots\right]$$

- all terms allowed by symmetries
- all physical effects included
  - e.g. assembly bias
- .

$$\left\langle \left(\frac{\delta n}{n}\right)_{{\rm gal},\ell}(x)\left(\frac{\delta n}{n}\right)_{{\rm gal},\ell}(y)\right\rangle =$$

$$= \sum_n \text{Coeff}_n \cdot \langle \text{matter correlation function}\rangle_n$$



It is familiar in dielectric E&M

- Polarizability:

$$\vec{P}(\omega) = \chi(\omega)\vec{E}(\omega) \quad \Rightarrow \quad \vec{P}(t) = \int dt' \chi(t-t')\vec{E}(t')$$

  – Here we work in time-Fourier space, and theory is practically linear.

- The EFT of Non-Relativistic binaries Goldberger and Rothstein **2004** is non-local in time

  – Here we solve perturbatively the inspiralling regime, and feed it into the long-distance theory (again time-Fourier space).

Consequences of non-locality in time

with Carrasco, Foreman, Green **1310**
Senatore **1406**

- The EFT is non-local in time $\Longrightarrow$ $\langle \tau_{ij}(\vec{x},t)\rangle_{\rm long\ fixed} \sim \int^t dt'\ K(t,t')\ \delta\rho(\vec{x}_{\rm fl},t') + \dots$
- Perturbative Structure has a decoupled structure

$$\delta\rho(x,t') = D(t')\delta\rho(\vec{x})^{(1)} + D(t')^2\delta\rho(\vec{x})^{(2)} + \dots$$

- A few coefficients for each *irrelevant* counterterm:

$$\Rightarrow \quad \langle \tau_{ij}(\vec{x},t)\rangle_{\rm long\ fixed} \sim \int^t dt'\ K(t,t')\ \left[D(t')\delta\rho(\vec{x})^{(1)} + D(t')^2\delta\rho(\vec{x})^{(2)} + \dots\right] \simeq$$

$$\simeq c_1(t)\ \delta\rho(\vec{x})^{(1)} + c_2(t)\ \delta\rho(\vec{x})^{(2)} + \dots$$

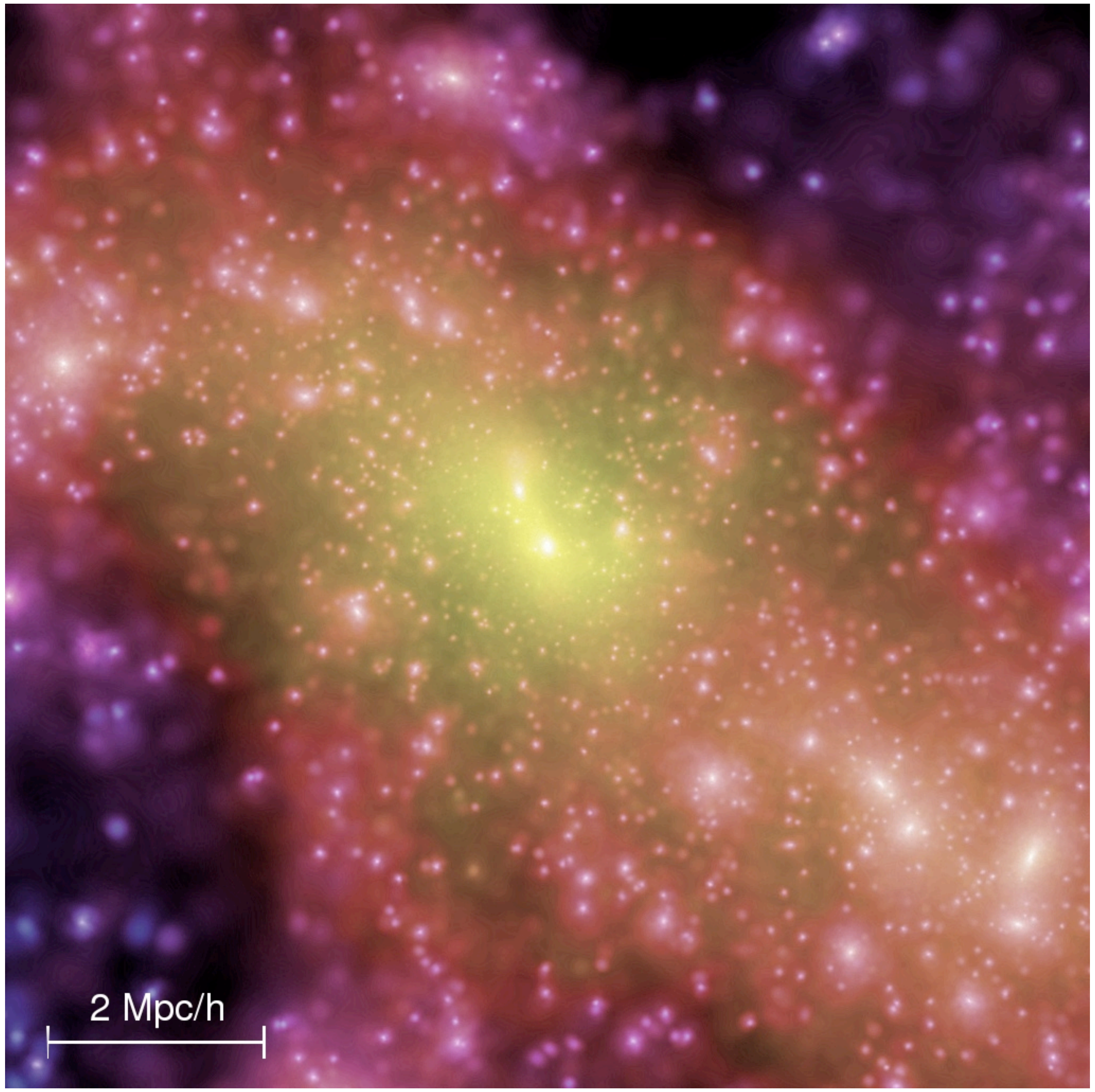
- where $c_i(t) = \int dt'\ K(t,t')\ D(t')^i$

- Difference: Time-Local QFT: $c_1(t)\left[\delta\rho(\vec{x})^{(1)} + \delta\rho(\vec{x})^{(2)} + \dots\right]$
  Non-Time-Local QFT: $c_1(t)\ \delta\rho(\vec{x})^{(1)} + c_2(t)\delta\rho(\vec{x})^{(2)} + \dots$

  - More terms, but not a disaster

Baryonic effects

- When stars explode, baryons behave differently than dark matter



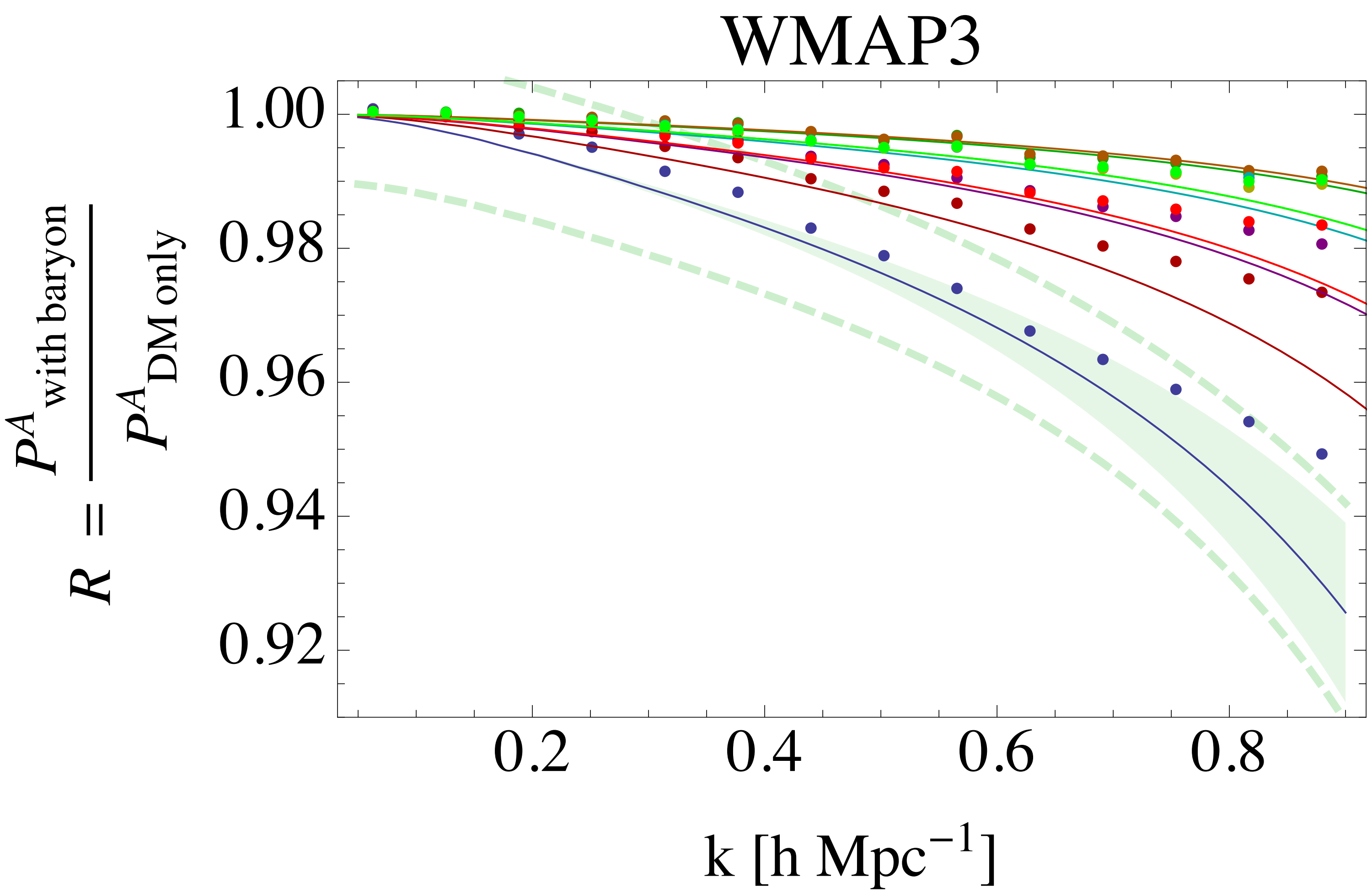
credit: Millenium Simulation, Springel *et al*. (2005)

- They cannot be reliably simulated due to large range of scales

Baryons

- Idea for EFT for dark matter:
  - Dark Matter moves $1/k_{\rm NL} \sim 10\,{\rm Mpc}$
    - $\Longrightarrow$ an effective fluid-like system with mean free path ~ $1/k_{\rm NL}$
- Baryons heat due to star formation, but move the same:
  - Universe with CDM+Baryons $\Longrightarrow$ EFTofLSS with 2 specie

$$\Delta P_b(k) \simeq c_\star^2 \left(\frac{k}{k_{\rm NL}}\right)^2 P_{11}^A(k)$$



Baryons

- EFT Equations:

$$\text{Continuity:} \quad \dot{\rho}_\sigma + 3H\rho_\sigma + a^{-1}\partial_i \pi^i_\sigma = 0 \ ,$$

$$\text{Momentum:} \quad \dot{\pi}^i_c + 4H\pi^i_c + a^{-1}\partial_j\left(\frac{\pi^i_c\pi^j_c}{\rho_c}\right) + a^{-1}\rho_c\partial_i\Phi = +a^{-1}\gamma^i - a^{-1}\partial_j\tau^{ij}_c \ ,$$

$$\dot{\pi}^i_b + 4H\pi^i_b + a^{-1}\partial_j\left(\frac{\pi^i_b\pi^j_b}{\rho_b}\right) + a^{-1}\rho_b\partial_i\Phi = -a^{-1}\gamma^i - a^{-1}\partial_j\tau^{ij}_b \ .$$

Baryons

- EFT Equations:

$$\text{Continuity:} \quad \dot{\rho}_\sigma + 3H\rho_\sigma + a^{-1}\partial_i \pi^i_\sigma = 0\ ,$$

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$$\dot{\pi}^i_b + 4H\pi^i_b + a^{-1}\partial_j\left(\frac{\pi^i_b \pi^j_b}{\rho_b}\right) + a^{-1}\rho_b \partial_i \Phi = -a^{-1}\gamma^i - a^{-1}\partial_j \tau^{ij}_b\ .$$

effective force

dynamical friction

- Counterterms:

$$\gamma^i \propto v^i_{\text{rel}}$$

no derivative: marginal operator

A marginal operator

- Dynamical friction term is indeed needed for renormalization of the theory, i.e. it is generated.

- Dynamical friction is a relevant operator: i.e. it cannot be treated perturbatively: it is an essential part of the linear *equations:*

$$a^2\delta_I^{(1)''}(a,\vec{k})+\left(2+\frac{a\mathcal{H}'(a)}{\mathcal{H}(a)}\right)a\delta_I^{(1)'}(a,\vec{k})=\int^a da_1 g(a,a_1)\, a_1\delta_I^{(1)'}(a_1,\vec{k})\ .$$

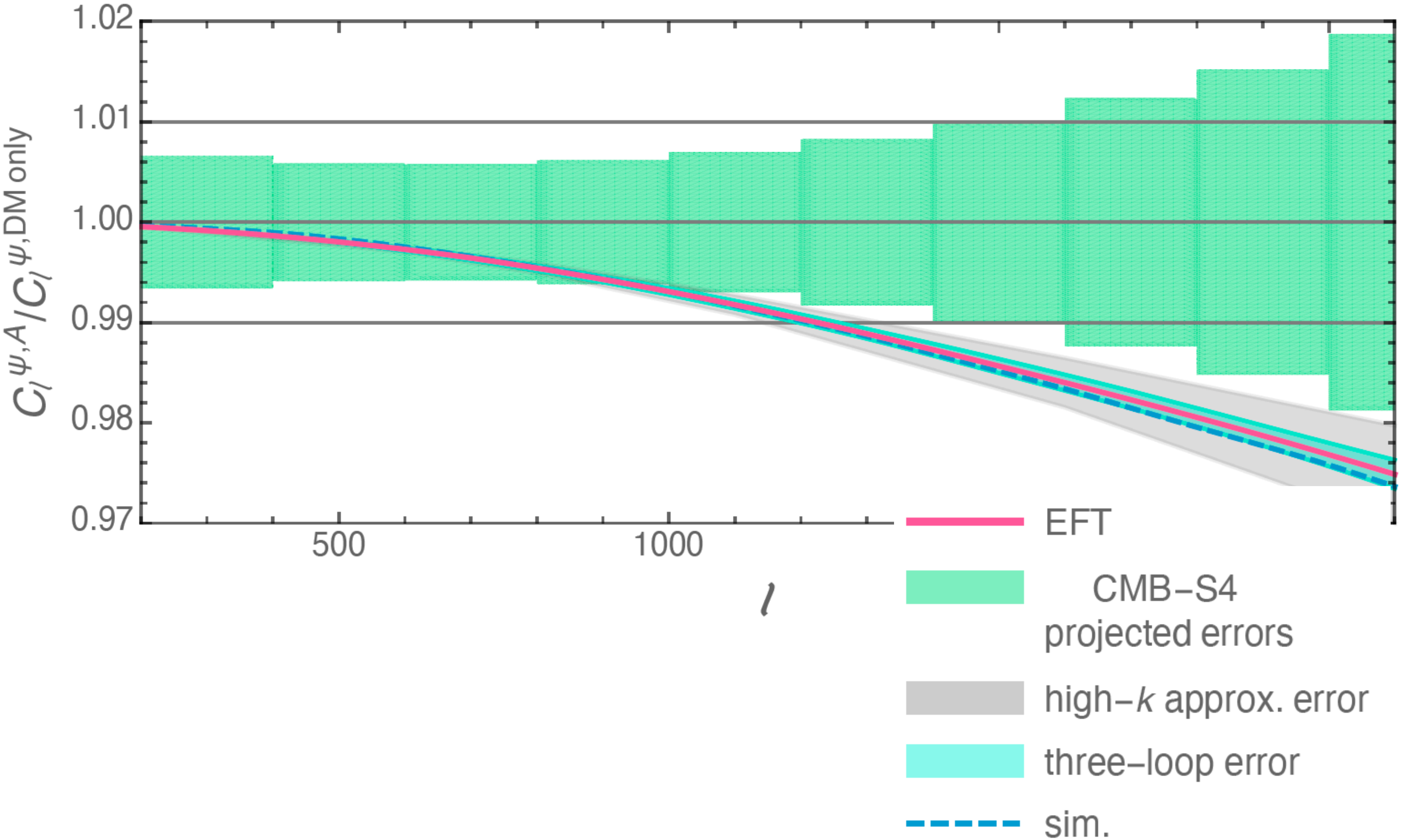
  – due to the time-translation breaking and actually even non-locality, very very very very very very hard to handle consistently.

    - we can make some guesses

- Luckily: it only affect the decaying mode of the isocurvature, which is very very very very very small by the time this effect kicks in.

Predictions for CMB Lensing

- Baryon corrections are detectable in next CMB S-4 experiments. But we can predict it:



Bispectrum at one loop

with D'Amico, Donath, Lewandowski, Zhang **2206**

Bispectrum

- The tree level bispectrum had been already used for cosmological parameter analysis in

  with Guido D'Amico, Jerome Gleyzes,
  Nickolas Kockron, Dida Markovic, Pierre Zhang, Florian Beutler, Hector Gill-Marin **1909**

  Philcox, Ivanov **2112**

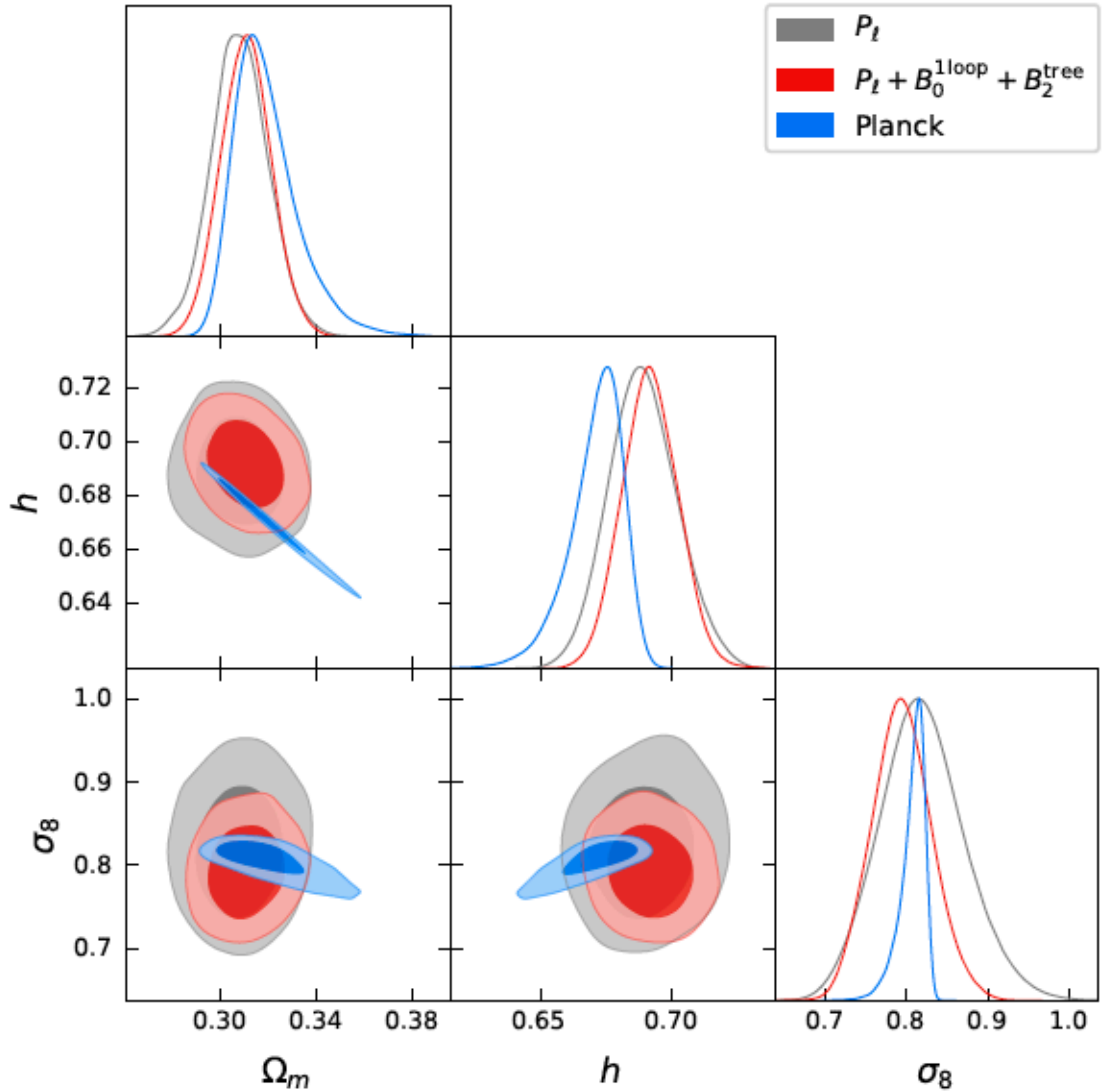
  - ~10% improvement on A_s

- Time to move to one-loop:
  - Large effort:
    - data analysis with D'Amico, Donath, Lewandowski, Zhang **2206**
    - theory model with D'Amico, Donath, Lewandowski, Zhang **2211**
    - theory integration with Anastasiou, Braganca, Zheng **2212**

Data Analysis ΛCDM

with D'Amico, Donath, Lewandowski, Zhang **2206**

- Main result: $\Lambda\mathrm{CDM}$
  - Improvements:
  - 30% on σ_8
  - 18% on h
  - 13% on Ω_m

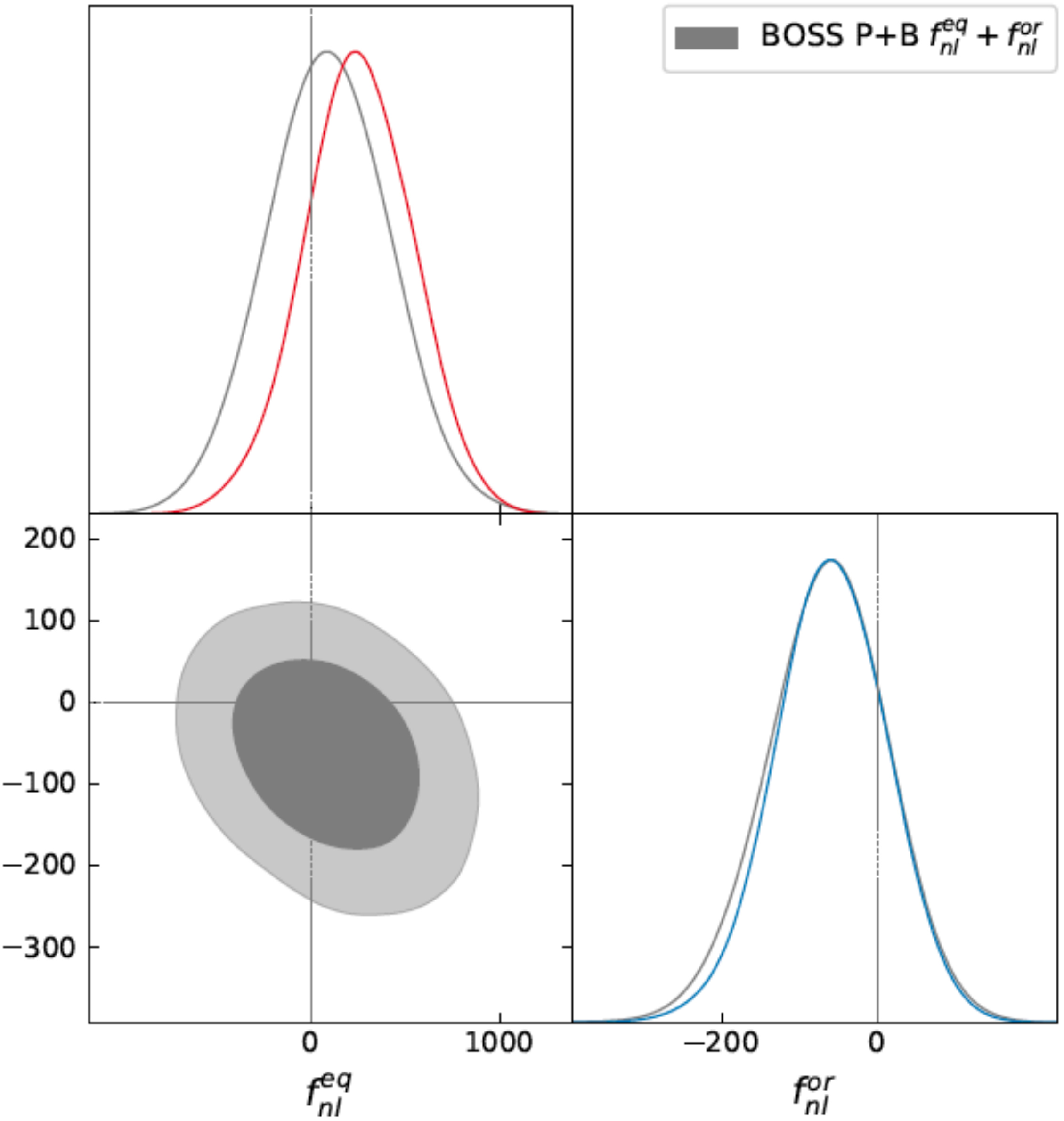


- Compatible with Planck
  - –no tensions
- Often Planck Comparable

Data Analysis Non-Gaussianities

with D'Amico, Lewandowski, Zhang **2201**

- .



	BOSS	WMAP	Planck
$f_{\rm NL}^{\rm equil.}$	245 ± 293	51 ± 136	-26 ± 47
$f_{\rm NL}^{\rm orth.}$	-60 ± 72	-245 ± 100	-38 ± 24
$f_{\rm NL}^{\rm loc.}$	7 ± 31	37.2 ± 19.9	-0.9 ± 5.1

see also contemporary Gambass, Ivanov, Simovic, Z
for only-tree-level analysis **2201**

$$S_\pi = \int d^4\sqrt{-g}\left[\frac{\dot H M_{\rm Pl}^2}{c_s^2}\left(\dot\pi^2 - c_s^2(\partial_i\pi)^2\right) + \frac{\dot H M_{\rm Pl}^2}{c_s^2}\left[\dot\pi(\partial_i\pi)^2 + \tilde c_3\,\dot\pi^3\right]\right]$$

with Cheung *et al*. **2008**

Theory Model

with D'Amico, Donath, Lewandowski, Zhang **2206**

- We add all the relevant biases (4th order) and counterterms (2nd order):

$$P_{11}^{r,h}[b_1]\ ,\quad P_{13}^{r,h}[b_1,b_3,b_8]\ ,\quad P_{22}^{r,h}[b_1,b_2,b_5]\ ,$$
$$B_{211}^{r,h}[b_1,b_2,b_5]\ ,\quad B_{321}^{r,h,(II)}[b_1,b_2,b_3,b_5,b_8]\ ,\quad B_{411}^{r,h}[b_1,\dots,b_{11}]\ ,$$
$$B_{222}^{r,h}[b_1,b_2,b_5]\ ,\quad B_{321}^{r,h,(I)}[b_1,b_2,b_3,b_5,b_6,b_8,b_{10}]\ ,$$

$$P_{13}^{r,h,ct}[b_1,c_{h,1},c_{\pi,1},c_{\pi v,1},c_{\pi v,3}]\ ,\quad P_{22}^{r,h,\epsilon}[c_1^{\rm St},c_2^{\rm St},c_3^{\rm St}]\ ,$$
$$B_{321}^{r,h,(II),ct}[b_1,b_2,b_5,c_{h,1},c_{\pi,1},c_{\pi v,1},c_{\pi v,3}]\ ,\quad B_{321}^{r,h,\epsilon,(I)}[b_1,c_1^{\rm St},c_2^{\rm St},\{c_i^{\rm St}\}_{i=4,\dots,13}]\ ,$$
$$B_{411}^{r,h,ct}[b_1,\{c_{h,i}\}_{i=1,\dots,5},c_{\pi,1},c_{\pi,5},\{c_{\pi v,j}\}_{j=1,\dots,7}]\ ,\quad B_{222}^{r,h,\epsilon}[c_1^{(222)},c_2^{(222)},c_5^{(222)}]\ .$$

- IR-resummation:
  - For the power spectrum, we use the correct and controlled IR-resummation.
  - For the bispectrum, we use an approximate method

Ivanov and Sibiryakov **2018**

Derivation of theory model

with D'Amico, Donath, Lewandowski, Zhang
2211

Derivation of theory model

with D'Amico, Donath, Lewandowski, Zhang **2211**

- Counterterms: major algebraic effort for 4th order and some theoretical subtle aspects.
- Renormalization of velocity
  - In the EFTofLSS, the velocity is a composite operator $v^i(x) = \frac{\pi^i(x)}{\rho(x)}$, so, it needs to be renormalized:

$$[v^i]_R = v^i + \mathcal{O}_v^i \ ,$$

  - Under a diffeomorphisms:

$$v^i \to v^i + \chi^i \quad \Rightarrow \quad \mathcal{O}_v^i \text{ is a scalar}$$

  - In redshift space, we have local product of velocities, which need to be renormalized but have non-trivial transformations under diff.s:

$$[v^i v^j]_R \to [v^i v^j]_R + [v^i]_R \chi^j + [v^j]_R \chi^i + \chi^i \chi^j$$

  - To achieve this, one can do: (so must include products $v^i \cdot \mathcal{O}_v^i$)

$$[v^i v^j]_R = [v^i]_R [v^j]_R + \mathcal{O}_{v^2}^{ij} \ , \quad \text{where } \ \mathcal{O}_{v^2}^{ij} \ \text{ is a scalar}$$

Derivation of theory model

with D'Amico, Donath, Lewandowski, Zhang **2211**

- Counterterms: major algebraic effort for 4th order and some theoretical subtle aspects.
- Spatially non-locally-contributing counterterm:
  - This is a normal effect, just strange-looking in the EFTofLSS context.
  - Normally, counterterms are local, but, contributing through non-local Green's functions, they contribute non-locally.

Derivation of theory model

with D'Amico, Donath, Lewandowski, Zhang **2211**

- Counterterms: major algebraic effort for 4th order and some theoretical subtle aspects.
- Spatially non-locally-contributing counterterm:
  - In the EFTofLSS, the Green's function is simple: $\dfrac{1}{\partial^2}$
  - Counterterms typically come with $\partial^2\mathcal{O}_{\rm local} \quad \Rightarrow \quad \delta_{\rm counter} \sim \dfrac{1}{\partial^2}\partial^2\mathcal{O}_{\rm local} \sim \mathcal{O}_{\rm local}$
    - result almost trivial
  - But at second order, and for velocity fields, contracted along the line of sight, derivatives do not simplify, so we get

$$\delta_{\rm counter}(\vec x) \sim \hat z^i \hat z^j \partial_i \pi^j_{(2)}(\vec x) \quad \sim \quad \hat z^i \hat z^j \frac{\partial_i\partial_j\partial_k\partial_m}{\partial^2}\mathcal{O}_{\rm local}$$

$$\sim \quad \hat z^i \hat z^j \frac{\partial_i\partial_j\partial_k\partial_m}{\partial^2}\left(\frac{\partial_k\partial_l}{H^2}\Phi(\vec x)\frac{\partial_l\partial_m}{H^2}\Phi(\vec x)\right)$$

  - This is truly non-locally contributing, truly non-trivial.

  - We check that all these terms are *needed and sufficient* for renormalization

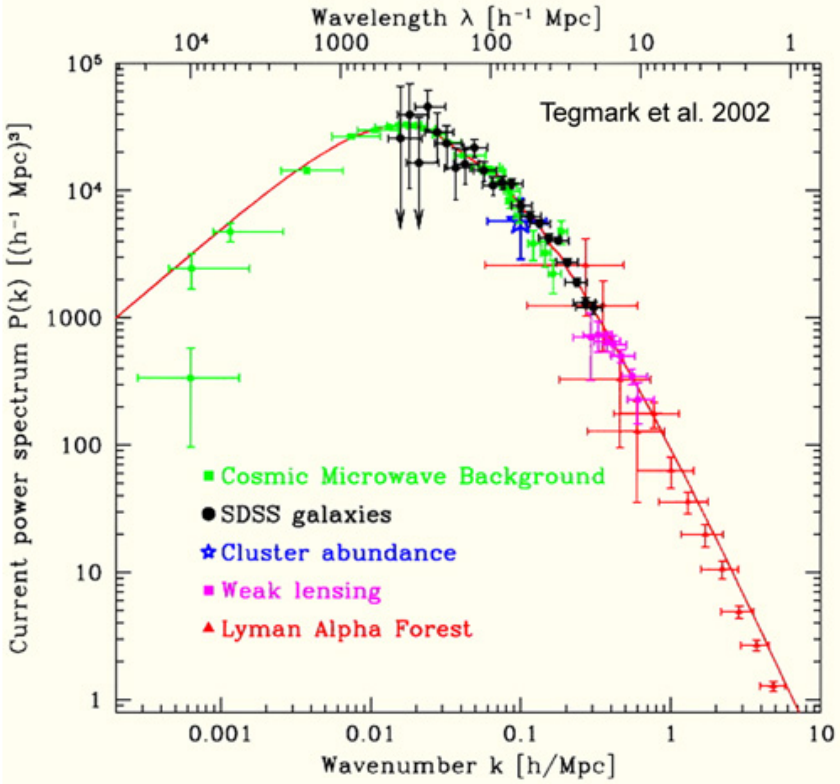
Evaluational/Computational Challenge

with Anastasiou, Braganca, Zheng **2212**

The best approach so far

Simonovic, Baldauf, Zaldarriaga,
Carrasco, Kollmeier **2018**

- Nice trick for fast evaluation of the loops integrals
- The power spectrum is a numerically computed function
- Decompose linear power spectrum



$$P_{11}(k) = \sum_n c_n \, k^{\mu + i\alpha\, n}$$

- Loop can be evaluated analytically

$$P_{1-\text{loop}}(k) = \int_{\vec{q}} K(\vec{q},\vec{k}) \; P_{11}(k-q) \; P_{11}(q) =$$

$$= \sum_{n_1,n_2} c_{n_1} c_{n_2} \left(\int_{\vec{q}} K(\vec{q},\vec{k}) \; k^{\mu+i\alpha n_1} \; k^{\mu+i\alpha n_2} \right) = \sum_{n_1,n_2} c_{n_1} c_{n_2} M_{n_1,n_2}(k)$$

–using quantum field theory techniques

– $M_{n_1 n_2}$ is cosmology independent $\Rightarrow$ so computed once

Computational Challenge

Philcox, Ivanov, Cabass, Simonovic, Zaldarriaga **2022**

- Two difficulties:

$$P_{1-\text{loop}}(k) = \int_{\vec{q}} K(\vec{q},\vec{k})\; P_{11}(k-q)\, P_{11}(q) =$$

$$= \sum_{n_1,n_2} c_{n_1} c_{n_2} \left(\int_{\vec{q}} K(\vec{q},\vec{k})\; k^{\mu+i\alpha n_1}\; k^{\mu+i\alpha n_2} \right) = \sum_{n_1,n_2} c_{n_1} c_{n_2} M_{n_1,n_2}(k)$$

  - integrals are complicated due to fractional, complex exponents
  - many functions needed, the matrix $M_{n_1 n_2 n_3}$ for bispectrum is about 50Gb, so, ~impossible to load on CPT for data analysis

- In order to ameliorate (solve) these issues, we use a different basis of functions.

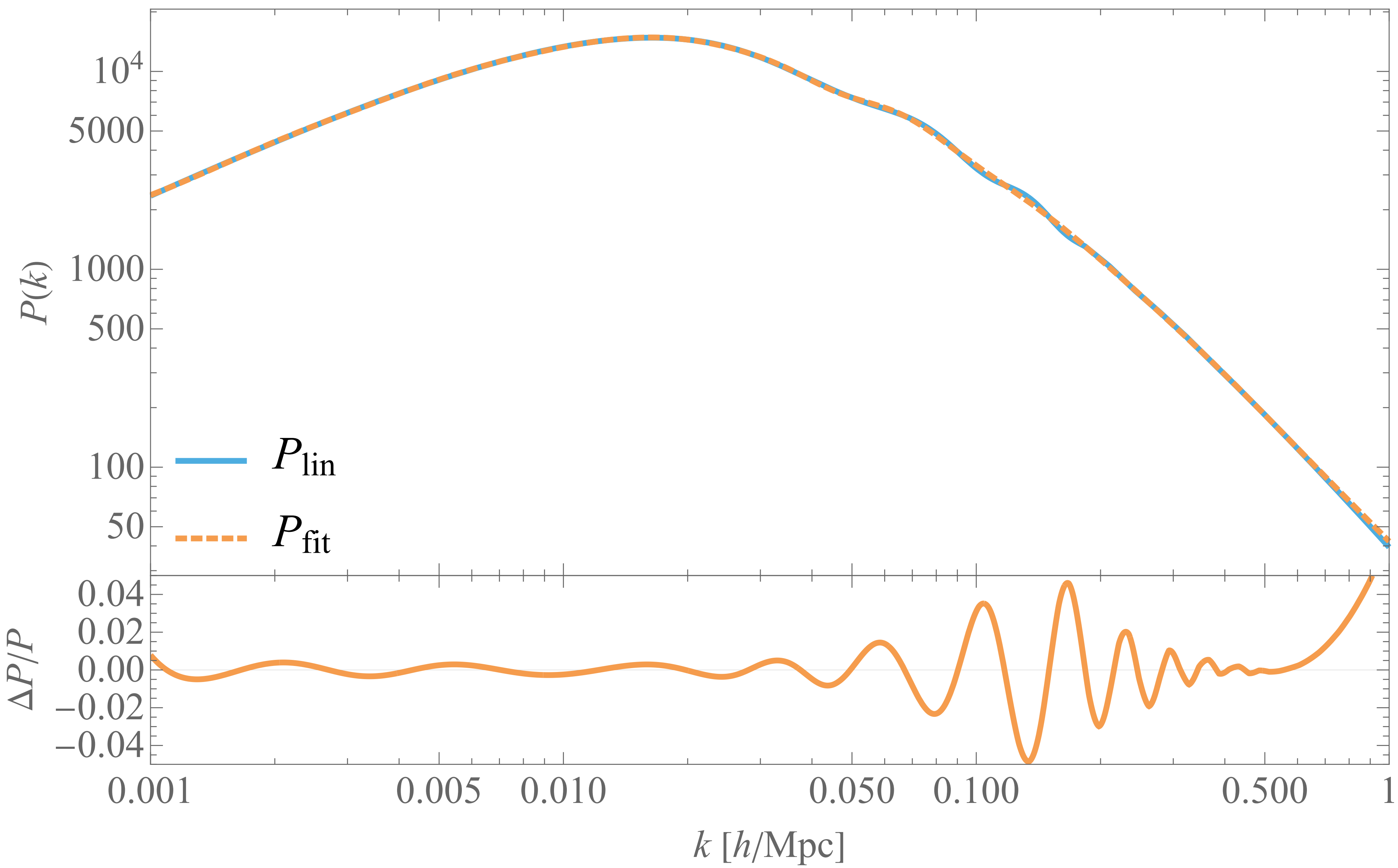
Complex-Masses Propagators

with Anastasiou, Braganca, Zheng
2212

- Use as basis:

$$f(k^2, k^2_{\rm peak}, k^2_{\rm UV}, i, j) \equiv \frac{\left(k^2/k_0^2\right)^i}{\left(1+\frac{(k^2-k^2_{\rm peak})^2}{k^4_{\rm UV}}\right)^j},$$

- With just 16 functions:



Complex-Masses Propagators

with Anastasiou, Braganca, Zheng
2212

- This basis is equivalent to massive propagators to integer powers

$$\frac{1}{\left(1+\frac{(k^2-k_{\rm peak}^2)^2}{k_{\rm UV}^4}\right)^j} = \frac{k_{\rm UV}^{4j}}{\left(k^2-k_{\rm peak}^2-i\,k_{\rm UV}^2\right)^j\left(k^2-k_{\rm peak}^2+i\,k_{\rm UV}^2\right)^j},$$

$$\frac{k_{\rm UV}^2}{\left(k^2-k_{\rm peak}^2-i\,k_{\rm UV}^2\right)\left(k^2-k_{\rm peak}^2+i\,k_{\rm UV}^2\right)} = -\frac{i/2}{k^2-k_{\rm peak}^2-i\,k_{\rm UV}^2}+\frac{i/2}{k^2-k_{\rm peak}^2+i\,k_{\rm UV}^2}$$

- So, each basis function:

$$f(k^2,k_{\rm peak}^2,k_{\rm UV}^2,i,j) = \sum_{n=1}^{j} k_{\rm UV}^{2(n-i)}\,k^{2i}\left(\frac{\kappa_n}{(k^2+M)^n}+\frac{\kappa_n^*}{(k^2+M^*)^n}\right)$$

Complex-Masses Propagators

with Anastasiou, Braganca, Zheng
2212

- This basis is equivalent to massive propagators to integer powers

$$\frac{1}{\left(1+\frac{(k^2-k^2_{\rm peak})^2}{k^4_{\rm UV}}\right)^j} = \frac{k^{4j}_{\rm UV}}{\left(k^2-k^2_{\rm peak}-i\,k^2_{\rm UV}\right)^j\left(k^2-k^2_{\rm peak}+i\,k^2_{\rm UV}\right)^j},$$

$$\frac{k^2_{\rm UV}}{\left(k^2-k^2_{\rm peak}-i\,k^2_{\rm UV}\right)\left(k^2-k^2_{\rm peak}+i\,k^2_{\rm UV}\right)} = -\frac{i/2}{k^2-k^2_{\rm peak}-i\,k^2_{\rm UV}}+\frac{i/2}{k^2-k^2_{\rm peak}+i\,k^2_{\rm UV}}$$

Complex-Mass propagator

- So, each basis function:

$$f(k^2,k^2_{\rm peak},k^2_{\rm UV},i,j) = \sum_{n=1}^{j} k^{2(n-i)}_{\rm UV}\,k^{2i}\left(\frac{\kappa_n}{(k^2+M)^n}+\frac{\kappa_n^*}{(k^2+M^*)^n}\right)$$

Complex-Masses Propagators

with Anastasiou, Braganca, Zheng **2212**

- We end up with integral like this:

$$L(n_1,d_1,n_2,d_2,n_3,d_3) = \int_q \frac{(\boldsymbol{k}_1-\boldsymbol{q})^{2n_1}\boldsymbol{q}^{2n_2}(\boldsymbol{k}_2+\boldsymbol{q})^{2n_3}}{((\boldsymbol{k}_1-\boldsymbol{q})^2+M_1)^{d_1}(\boldsymbol{q}^2+M_2)^{d_2}((\boldsymbol{k}_2+\boldsymbol{q})^2+M_3)^{d_3}}$$

- with integer exponents.
- First we manipulate the numerator to reduce to:

$$T(d_1,d_2,d_3) = \int_q \frac{1}{((\boldsymbol{k}_1-\boldsymbol{q})^2+M_1)^{d_1}(\boldsymbol{q}^2+M_2)^{d_2}((\boldsymbol{k}_2+\boldsymbol{q})^2+M_3)^{d_3}},$$

- Then, by integration by parts, we find (i.e. QCD teaches us how to) recursion relations

$$\int_q \frac{\partial}{\partial q_\mu}\cdot\left(q_\mu t(d_1,d_2,d_3)\right) = 0$$

$$\Rightarrow \quad (3-d_{1223})\hat{0} + d_1 k_{1s}\widehat{1^+} + d_3(k_{2s})\widehat{3^+} + 2M_2 d_2\widehat{2^+} - d_1\widehat{1^+}\widehat{2^-} - d_3\widehat{2^-}\widehat{3^+} = 0$$

- relating same integrals with raised or lowered the exponents (easy terminate due to integer exponents).

Complex-Masses Propagators

with Anastasiou, Braganca, Zheng
2212

- We end up to three master integrals:

- Tadpole:

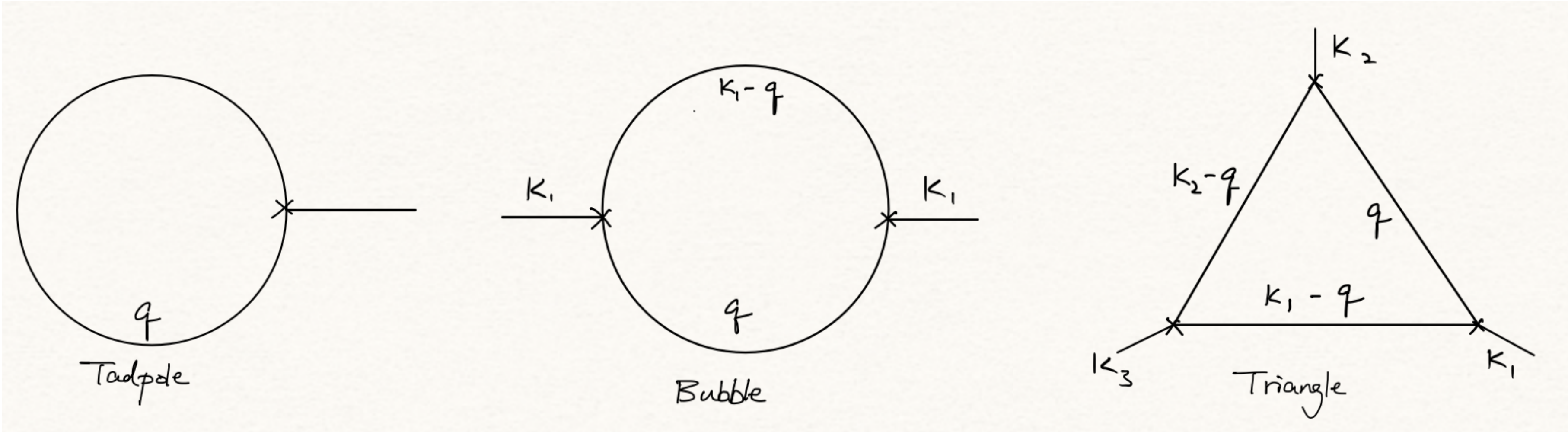
$$\mathrm{Tad}(M_j, n, d) = \int \frac{d^3\boldsymbol{q}}{\pi^{3/2}} \frac{(\boldsymbol{p}_i^2)^n}{(\boldsymbol{p}_i^2 + M_j)^d}$$

- Bubble:

$$B_{\mathrm{master}}(k^2, M_1, M_2) = \int \frac{d^3\boldsymbol{q}}{\pi^{3/2}} \frac{1}{(q^2 + M_1)(|\boldsymbol{k} - \boldsymbol{q}|^2 + M_2)}$$

- Triangle:

$$T_{\mathrm{master}}(k_1^2, k_2^2, k_3^2, M_1, M_2, M_3) = \int \frac{d^3\boldsymbol{q}}{\pi^{3/2}} \frac{1}{(q^2 + M_1)(|\boldsymbol{k}_1 - \boldsymbol{q}|^2 + M_2)(|\boldsymbol{k}_2 + \boldsymbol{q}|^2 + M_3)},$$



Complex-Masses Propagators

with Anastasiou, Braganca, Zheng
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- The master integrals are evaluated with Feynman parameters, but with great care of branch cut crossing, which happens because of complex masses.

- Bubble Master:

$$B_{\text{master}}(k^2, M_1, M_2) = \frac{\sqrt{\pi}}{k} i[\log\left(A(1, m_1, m_2)\right) - \log\left(A(0, m_1, m_2)\right) \\ - 2\pi i H(\operatorname{Im} A(1, m_1, m_2)) H(-\operatorname{Im} A(0, m_1, m_2))] \,,$$

$$A(0, m_1, m_2) = 2\sqrt{m_2} + i(m_1 - m_2 + 1) \,,$$
$$A(1, m_1, m_2) = 2\sqrt{m_1} + i(m_1 - m_2 - 1) \,,$$
$$m_1 \;=\; M_1/k^2 \;\text{ and }\; m_2 \;=\; M_2/k^2$$

- Triangle Master:

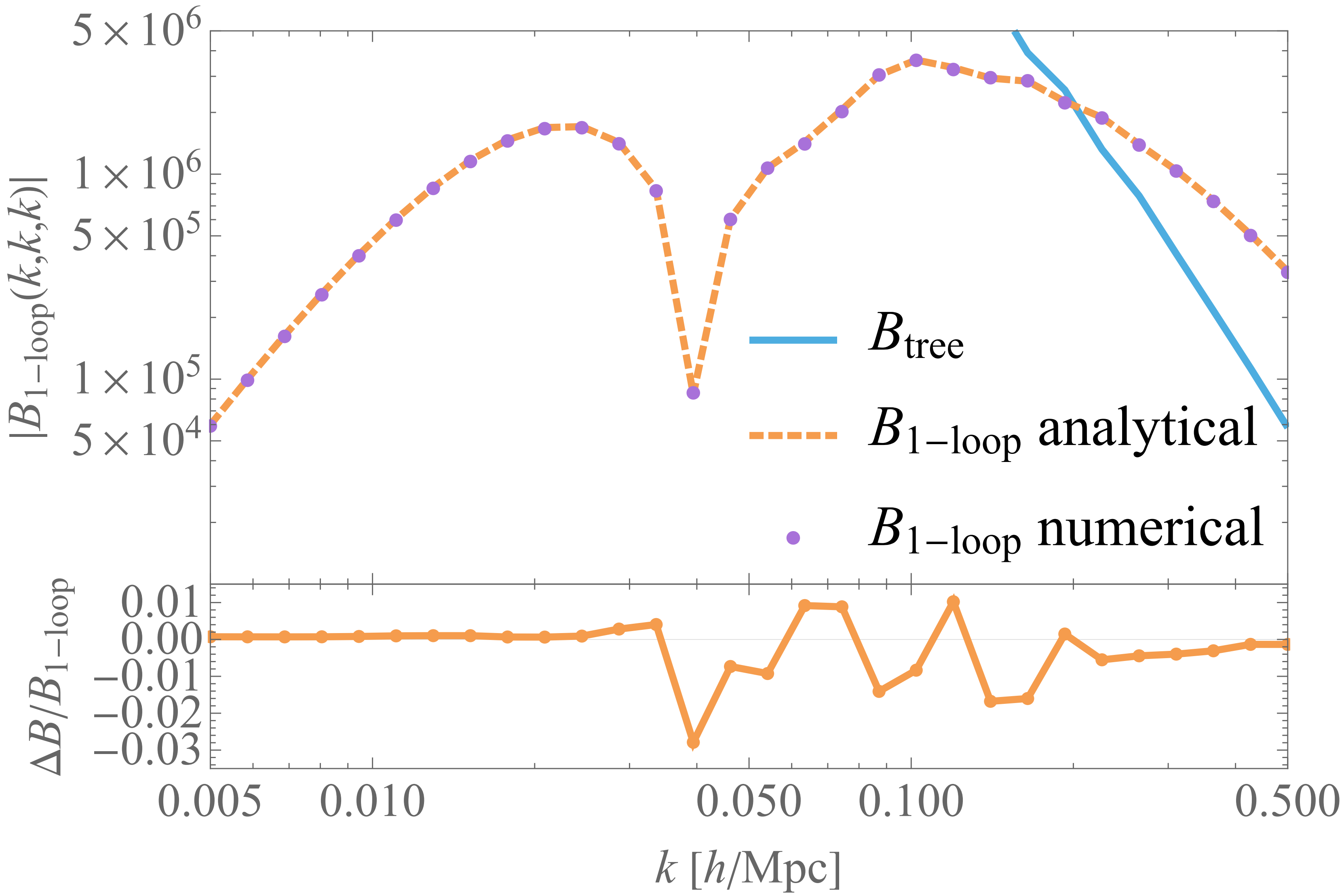
$$F_{\text{int}}(R_2, z_+, z_-, x_0) = s(z_+, -z_-) \frac{\sqrt{\pi}}{\sqrt{|R_2|}} \left. \frac{\arctan\left(\frac{\sqrt{z_+ - x}\sqrt{x_0 - z_-}}{\sqrt{x_0 - z_+}\sqrt{x - z_-}}\right)}{\sqrt{x_0 - z_+}\sqrt{x_0 - z_-}} \right|_{x=0}^{x=1} .$$

- Very simple expressions with simple rule for branch cut crossing.

Result of Evaluation

with Anastasiou, Braganca, Zheng
2212

- All automatically coded up.
- For BOSS analysis, evaluation of matrix is 2.5CPU hours and 800 Mb storage, very fast matrix contractions.
- Accuracy with 16 functions:

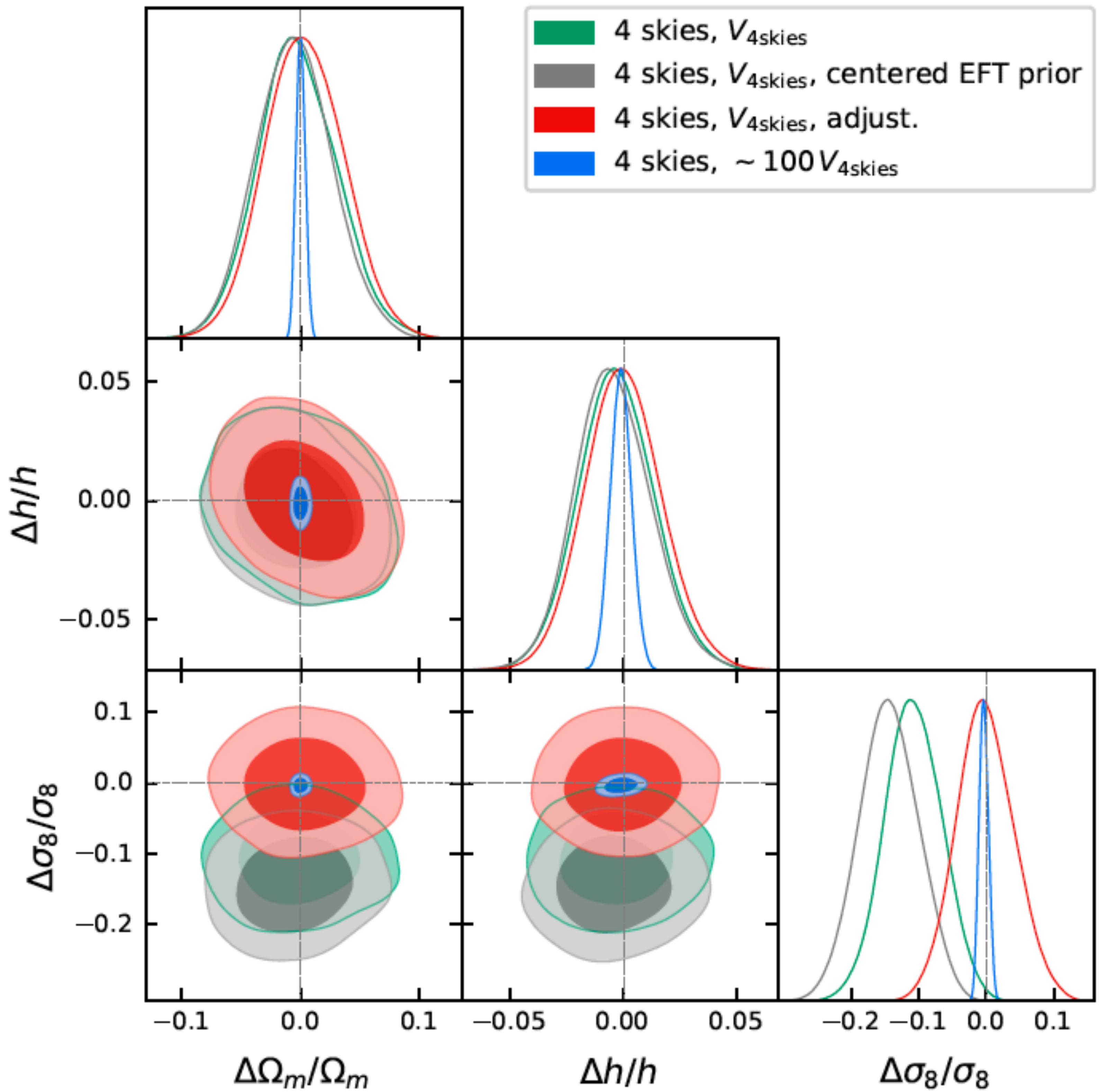


Back to data-analysis:
Pipeline Validation

with D'Amico, Donath, Lewandowski, Zhang **2206**

Measuring and fixing phase space

- We consider synthetic data, i.e. data made out of the model, and analyze them:
  - Green: biased.
- Why?
  - Priors centered on zero?
    - Grey: biased
  - Bug in pipeline?
    - Test by reducing covar.
    - Red: non-biased
- It must be phase space projection
- But the grey line offers
  - an honest measurement of it.

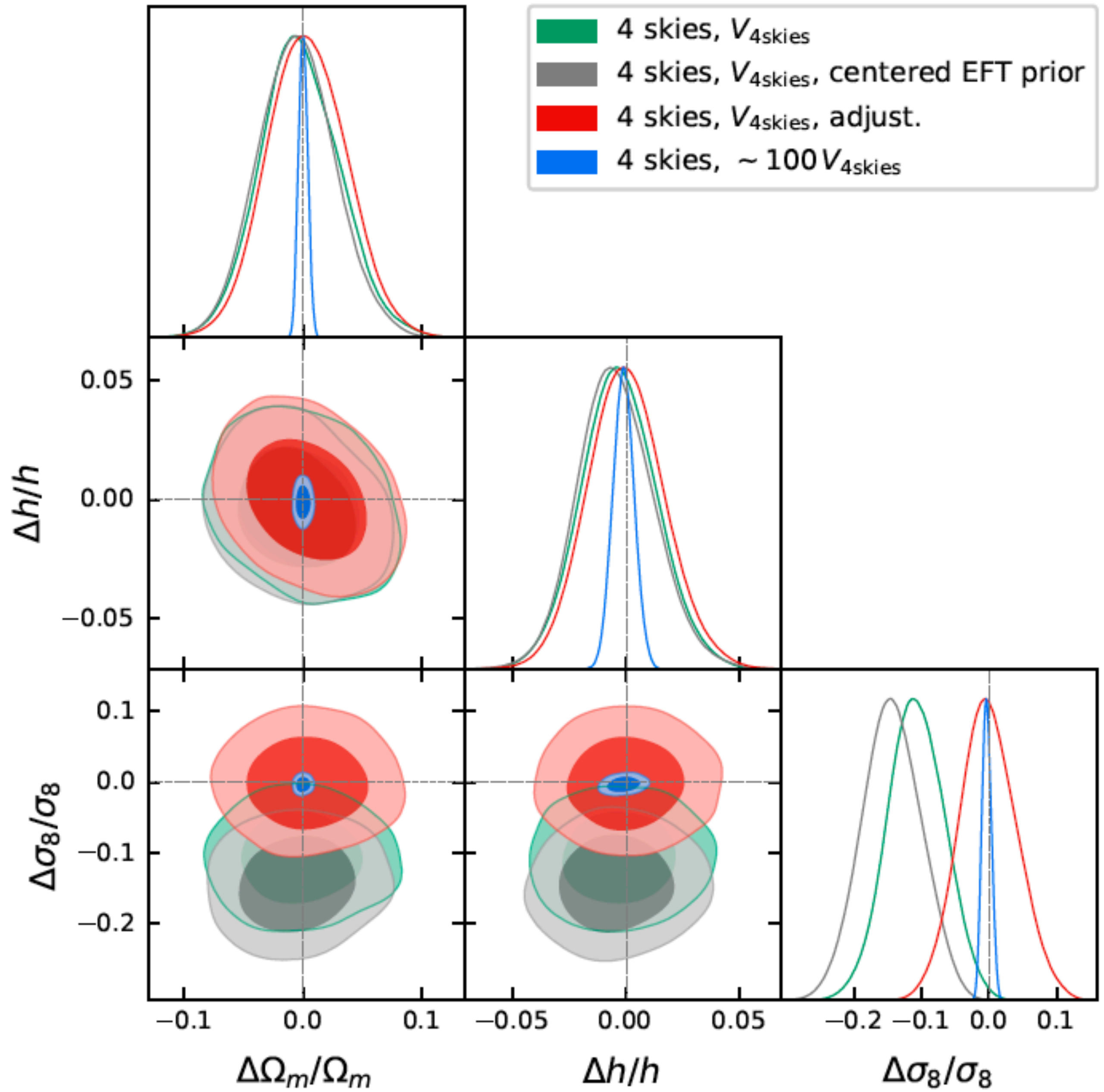


Measuring and fixing phase space

- We add:

$$\ln \mathcal{P}_{\rm pr}^{\rm ph.\ sp.\ 4sky} = -48\left(\frac{b_1}{2}\right) + 32\left(\frac{\Omega_m}{0.31}\right) + 48\left(\frac{h}{0.68}\right) ,$$

$\sigma_{\rm proj}/\sigma_{\rm stat}$	Ω_m	h	σ_8	ω_{cdm}
1 sky, $\sim 100\, V_{\rm 1sky}$	-0.1	-0.14	-0.21	-0.2
1 sky, $V_{\rm 1sky}$, adjust.	0.13	0.06	0.04	0.15
4 skies, $V_{\rm 4skies}$, adjust.	0.1	0.	-0.05	0.07



- no more proj. effect.

Scale cut from NNLO

with D'Amico, Donath, Lewandowski, Zhang **2206**

- We can estimate the $k_{\rm max}$ without the use of simulations, by adding NNLO terms, and seeing when they make a difference on the posteriors.

$$P_{\rm NNLO}(k,\mu) = \frac{1}{4} c_{r,4} b_1^2 \mu^4 \frac{k^4}{k_{\rm NL,R}^4} P_{11}(k) + \frac{1}{4} c_{r,6} b_1 \mu^6 \frac{k^4}{k_{\rm NL,R}^4} P_{11}(k)\,,$$

$$B_{\rm NNLO}(k_1,k_2,k_3,\mu,\phi) = 2 c_{\rm NNLO,1} K_2^{r,h}(\vec{k}_1,\vec{k}_2;\hat{z}) K_1^{r,h}(\vec{k}_2;\hat{z}) f \mu_1^2 \frac{k_1^4}{k_{\rm NL,R}^4} P_{11}(k_1) P_{11}(k_2)$$
$$+ c_{\rm NNLO,2} K_1^{r,h}(\vec{k}_1;\hat{z}) K_1^{r,h}(\vec{k}_2;\hat{z}) P_{11}(k_1) P_{11}(k_2) f \mu_3 k_3 \frac{(k_1^2+k_2^2)}{4 k_1^2 k_2^2 k_{\rm NL,R}^4} \Big[-2\vec{k}_1\cdot\vec{k}_2 (k_1^3\mu_1 + k_2^3\mu_2)$$
$$+ 2 f \mu_1\mu_2\mu_3 k_1 k_2 k_3 (k_1^2+k_2^2) \Big] + \text{perm.}\,, \quad (4$$

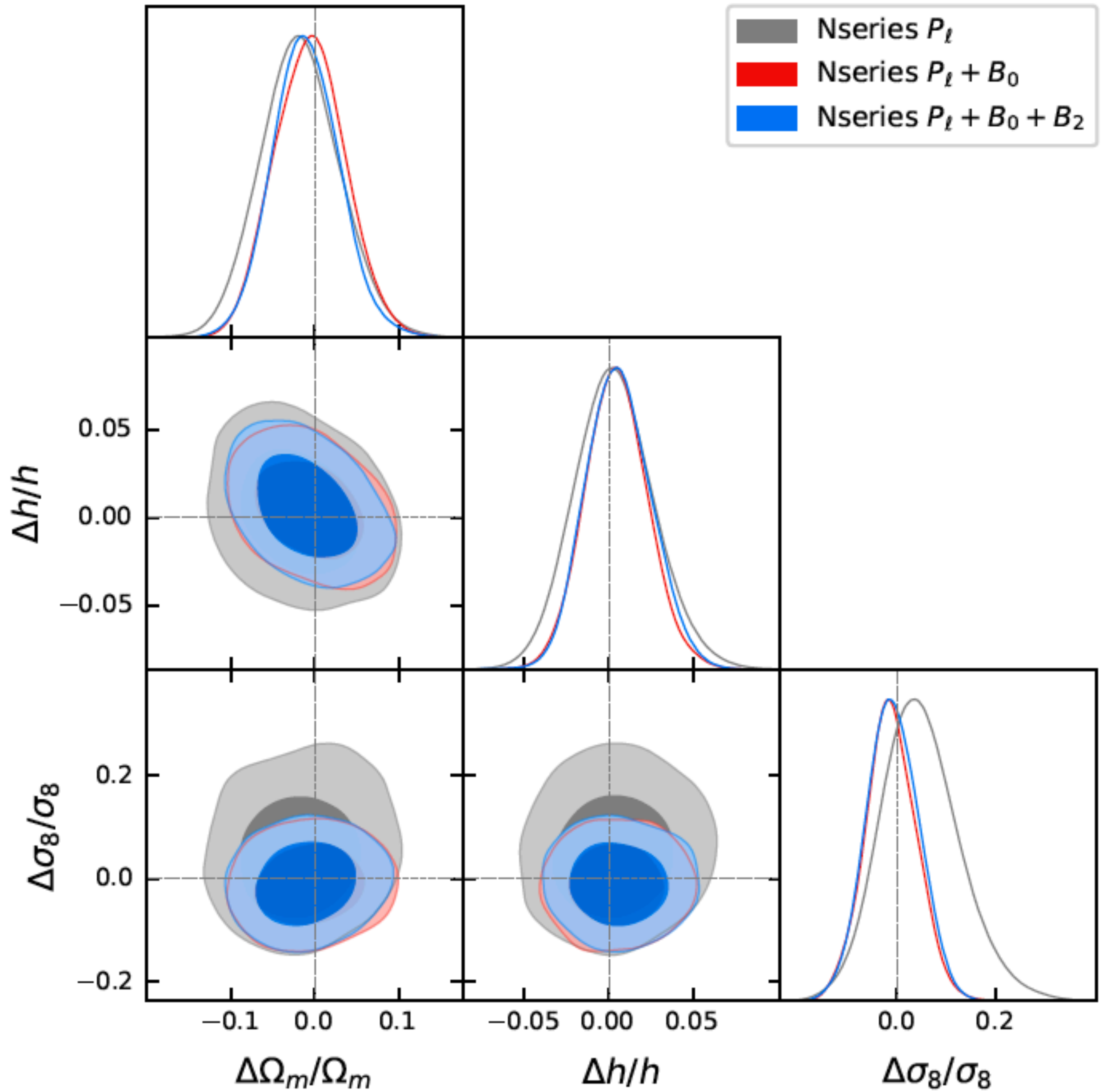
- For our $k_{\rm max}$, we find the following shifts, which are ok:

$\Delta_{\rm shift}/\sigma_{\rm stat}$	Ω_m	h	σ_8	ω_{cdm}	$\ln(10^{10}A_s)$	S_8
$P_\ell + B_0$: base - w/ NNLO	-0.03	-0.09	-0.03	-0.1	0.05	-0.04

Scale-cut from simulations

with D'Amico, Donath, Lewandowski, Zhang **2206**

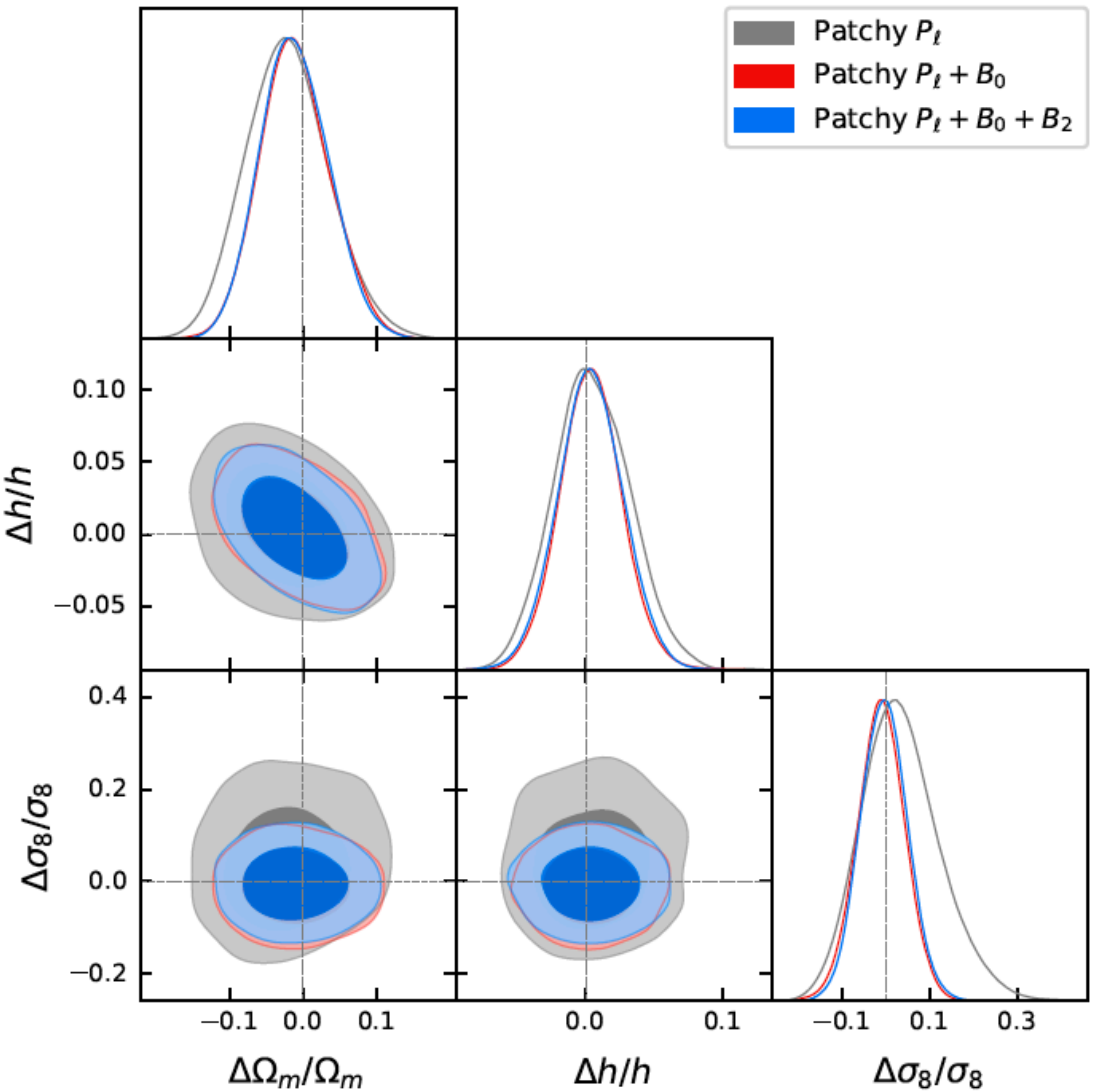
- N-series
  - Volume ~80 BOSS
  - safely within $\sigma_{\rm data}/3$
- After phase-space correction



Scale-cut from simulations

with D'Amico, Donath, Lewandowski, Zhang **2206**

- Patchy:
  - Volume ~2000 BOSS
  - safely within $\sigma_{\rm data}/3$
- After phase-space correction

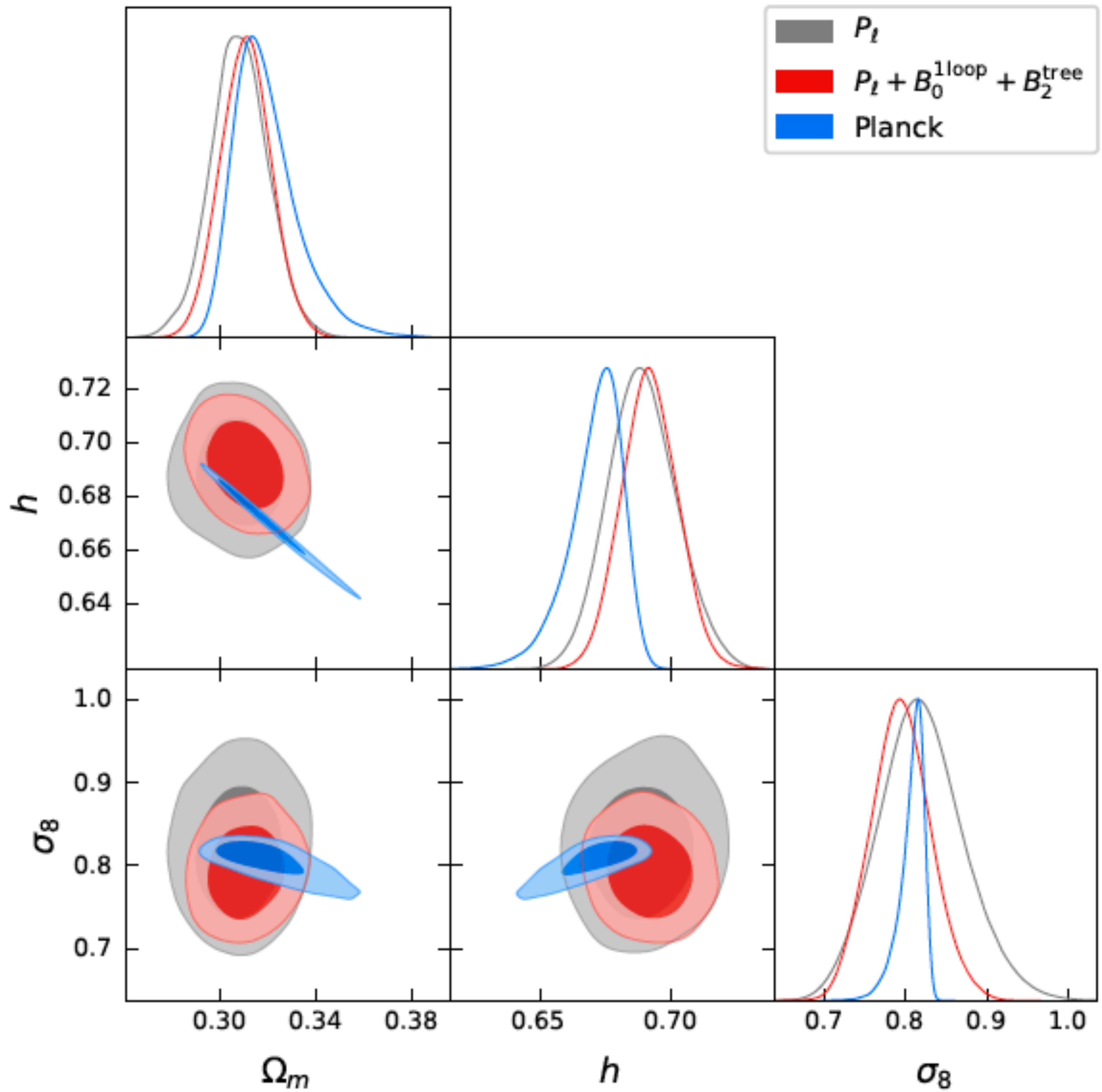


BOSS data

Data Analysis ΛCDM

with D'Amico, Donath, Lewandowski, Zhang **2206**

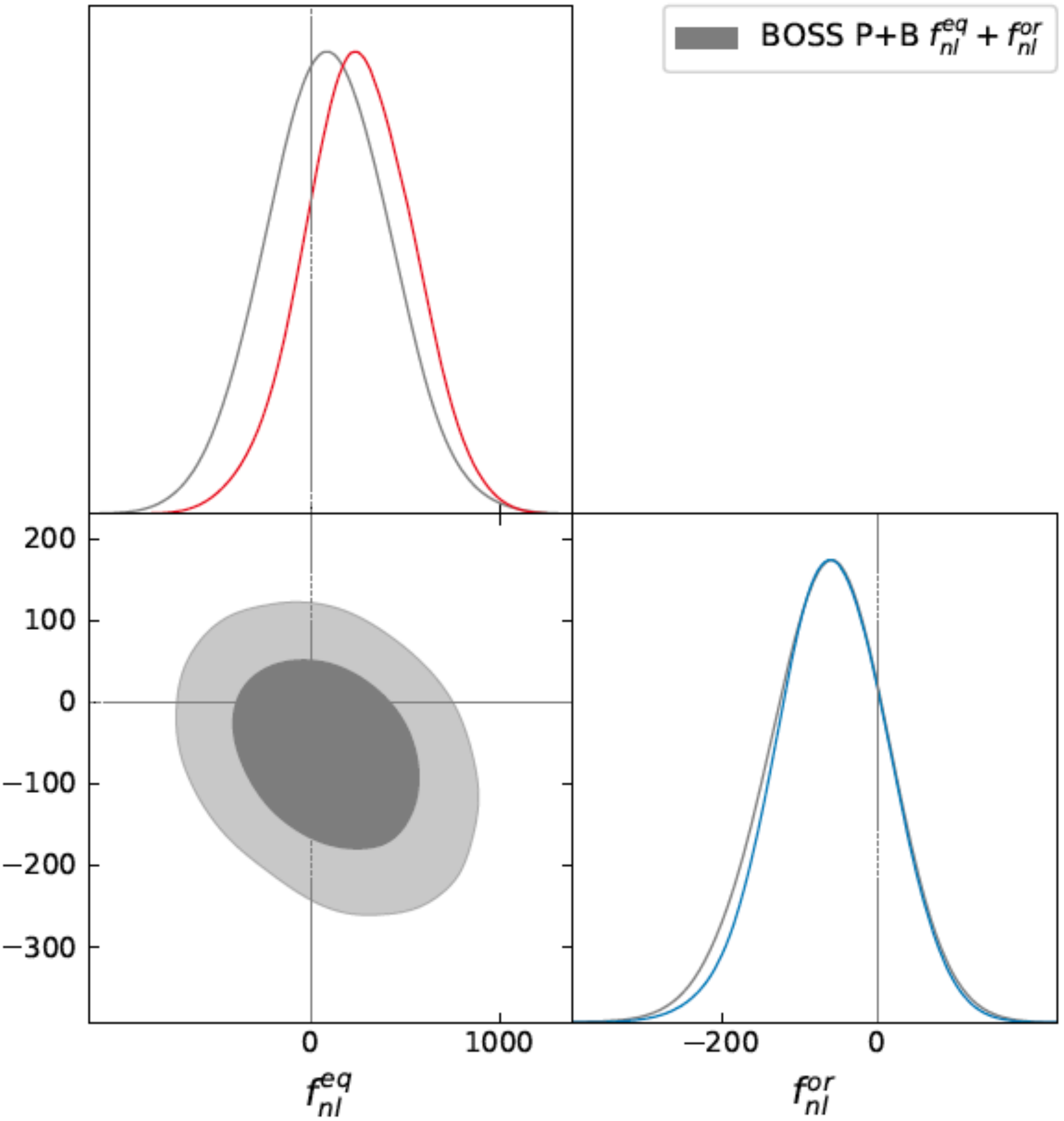
- Main result: $\Lambda\mathrm{CDM}$
  - Improvements:
  - 30% on σ_8
  - 18% on h
  - 13% on Ω_m
- Compatible with Planck
  - – no tensions
- Remarkable consistency
  - – of observables



Data Analysis Non-Gaussianities

with D'Amico, Lewandowski, Zhang **2201**

- .



	BOSS	WMAP	Planck
$f_{\rm NL}^{\rm equil.}$	245 ± 293	51 ± 136	-26 ± 47
$f_{\rm NL}^{\rm orth.}$	-60 ± 72	-245 ± 100	-38 ± 24
$f_{\rm NL}^{\rm loc.}$	7 ± 31	37.2 ± 19.9	-0.9 ± 5.1

$$S_\pi = \int d^4\sqrt{-g}\left[\frac{\dot H M_{\rm Pl}^2}{c_s^2}\left(\dot\pi^2 - c_s^2(\partial_i\pi)^2\right) + \frac{\dot H M_{\rm Pl}^2}{c_s^2}\left[\dot\pi(\partial_i\pi)^2 + \tilde c_3\,\dot\pi^3\right]\right]$$

with Cheung *et al*. **2008**

Direct Measurement of formation time of galaxies

with Donath and Lewandowski **2307**

Galaxies in the EFTofLSS

Senatore **1406**
Mirbabayi et al. **1412**

$$n_{\rm gal}(x) = f_{\text{very complicated}}\left(\{H, \Omega_m, \ldots, m_e, g_{ew}, \ldots, \rho(x)\}_{\text{past light cone}}\right)$$

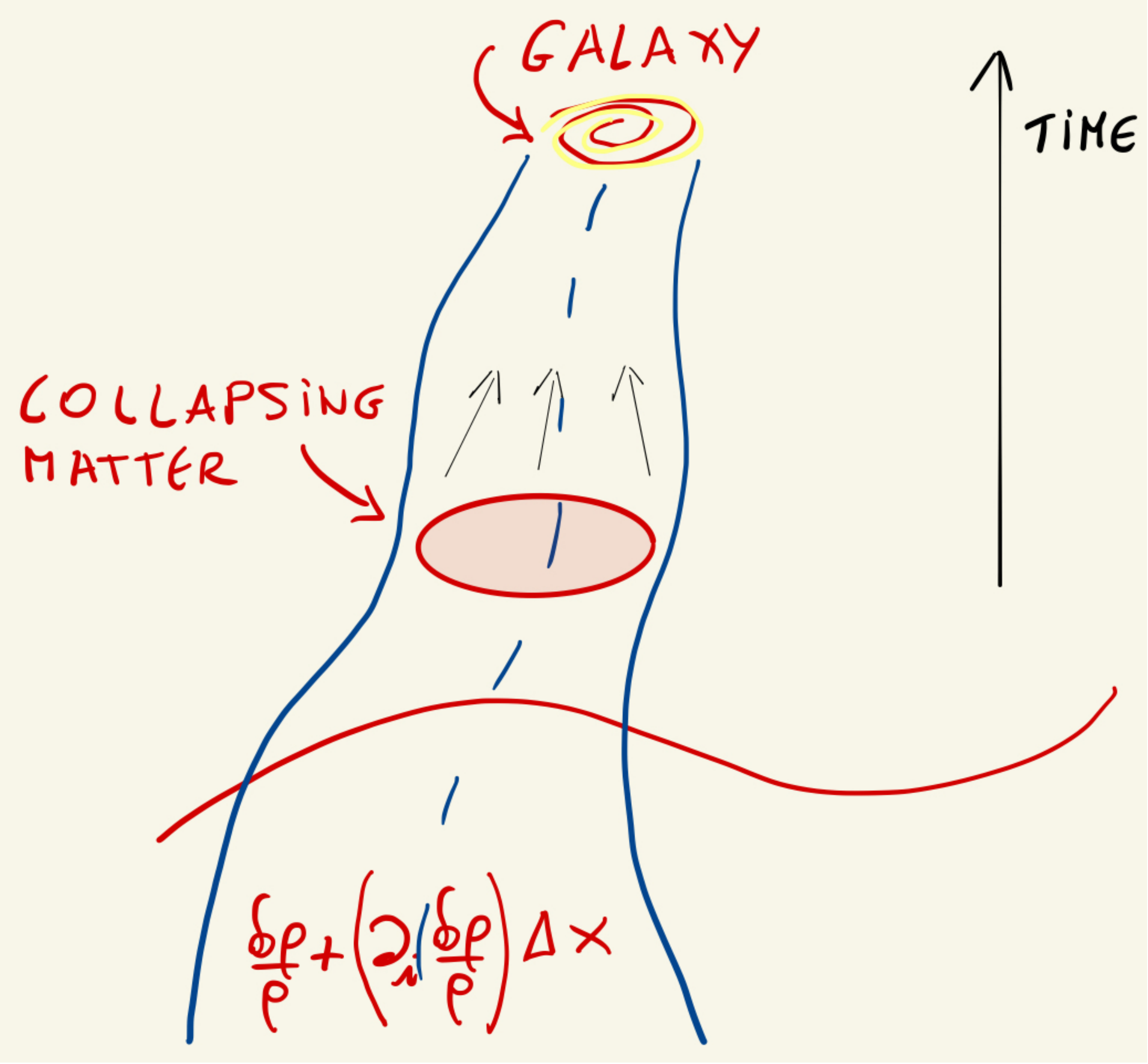
At *long* wavelengths $\Downarrow$ Taylor Expansion

$$\left(\frac{\delta n}{n}\right)_{{\rm gal},\ell}(x) \sim \int^t dt' \left[c(t,t')\ \left(\frac{\delta\rho}{\rho}\right)(\vec{x}_{\rm fl}, t') + \ldots\right]$$

- all terms allowed by symmetries
- all physical effects included
  - –e.g. assembly bias
- .

$$\left\langle \left(\frac{\delta n}{n}\right)_{{\rm gal},\ell}(x) \left(\frac{\delta n}{n}\right)_{{\rm gal},\ell}(y) \right\rangle =$$

$$= \sum_n \text{Coeff}_n \cdot \langle \text{matter correlation function} \rangle_n$$



Consequences of non-locality in time

- This means that one *does not* get the same terms as in the local-in-time expansion

- If we could measure one of these terms, we could *measure* that Galaxies take an Hubble time to form. We have never measured this: we take pictures of different galaxies at different stages of their evolution. But we have never *seen* a galaxy form in an Hubble time.
  - This would be the first direct evidence that the universe lasted an Hubble time.

- So, detecting a non-local-in-time bias would allow us to measure that, and from the size, the formation time. Unfortunately, so far, not yet.

Consequences of non-locality in time

- Mathematics again:
  - non-local in time:

$$\delta_g^{(n)}(\vec{x},t) = \sum_{\mathcal{O}_m} \int^t dt' H(t') c_{\mathcal{O}_m}(t,t') \times [\mathcal{O}_m(\vec{x}_{\rm fl}(\vec{x},t,t'),t')]^{(n)} \ ,$$

$$\mathcal{O}_{m=3} \supset \delta^2\theta, \delta^3, \ldots$$

  - local in time:

$$\Rightarrow \quad \delta_{g,\rm loc}^{(n)}(\vec{x},t) = \sum_{\mathcal{O}_m} c_{\mathcal{O}_m}(t)\, \mathcal{O}_m^{(n)}(\vec{x},t) \ ,$$

  - more non local in time:

$$[\mathcal{O}_m(\vec{x}_{\rm fl}(\vec{x},t,t'),t')]^{(n)} = \sum_{\alpha=1}^{n-m+1} \left(\frac{D(t')}{D(t)}\right)^{\alpha+m-1} \mathbb{C}_{\mathcal{O}_m,\alpha}^{(n)}(\vec{x},t)$$

$$\Rightarrow \quad \delta_g^{(n)}(\vec{x},t) = \sum_{\mathcal{O}_m} \sum_{\alpha=1}^{n-m+1} c_{\mathcal{O}_m,\alpha}(t)\, \mathbb{C}_{\mathcal{O}_m,\alpha}^{(n)}(\vec{x},t)$$

Consequences of non-locality in time

$$\delta^{(n)}_{g,\mathrm{loc}}(\vec{x},t) = \sum_{\mathcal{O}_m} c_{\mathcal{O}_m}(t)\, \mathcal{O}^{(n)}_m(\vec{x},t)\ , \qquad \delta^{(n)}_{g}(\vec{x},t) = \sum_{\mathcal{O}_m} \sum_{\alpha=1}^{n-m+1} c_{\mathcal{O}_m,\alpha}(t)\, \mathbb{C}^{(n)}_{\mathcal{O}_m,\alpha}(\vec{x},t)$$

- it turns out that up to 4th order, the two basis of operators were identical.
- but at 5th order they are not!
  - out of 29 independent operators, 3 cannot be written as local in time ones.
- $\Rightarrow$ By looking at, eg,

$$\langle \delta^{(5)}_{g_1}(\vec{x}_1)\delta^{(1)}_{g_2}(\vec{x}_2)\delta^{(1)}_{g_3}(\vec{x}_3)\delta^{(1)}_{g_4}(\vec{x}_4)\delta^{(1)}_{g_5}(\vec{x}_5)\delta^{(1)}_{g_6}(\vec{x}_6)\rangle$$

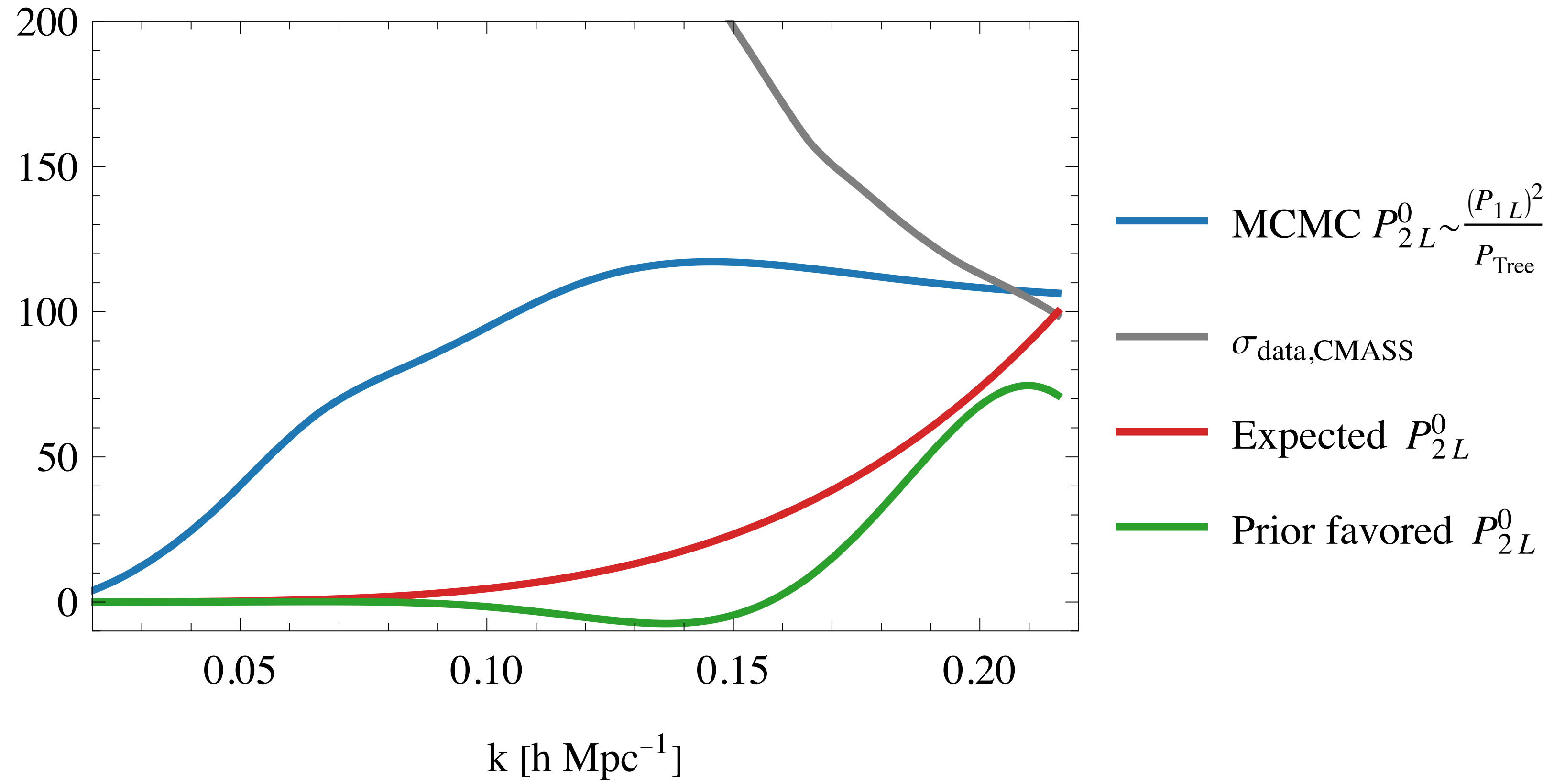
- we can detect these biases, and, from their size, determine:
  - the order of magnitude of the formation time of galaxies
  - direct evidence that the universe lasted 13 Billion years

Peeking into the next Decade

with Donath, Bracanga and Zheng **2307**

Next Decade

- After validating our technique against the MCMC's on BOSS data, we Fisher forecast for DESI and Megamapper
- Prediction of one-loop Power Spectrum and Bispectrum
- Here, and in the NG analysis, introduce a *`perturbativity prior'*: impose expected size and scaling of loop



- Also a *`galaxy formation prior'* , 0.3 in each EFT-parameter

Results: Non-Gaussianities

BOSS: $\sigma(\cdot)$	$f_{\rm NL}^{\rm loc.}$	$f_{\rm NL}^{\rm eq.}$	$f_{\rm NL}^{\rm orth.}$
$P{+}B_{\rm Tree}$	37	357	142
$P{+}B$	23	253	67
$P{+}B{+}$p.p.	17	228	62
$P{+}B{+}$p.p.+g.p.	15	163	49

DESI: $\sigma(\cdot)$	$f_{\rm NL}^{\rm loc.}$	$f_{\rm NL}^{\rm eq.}$	$f_{\rm NL}^{\rm orth.}$
$P{+}B_{\rm Tree}$	3.61	142	71.5
$P{+}B$	3.46	114	30.2
$P{+}B{+}$p.p.	3.26	91.5	27.0
$P{+}B{+}$p.p.+g.p.	3.19	77.0	21.8

MMo: $\sigma(\cdot)$	$f_{\rm NL}^{\rm loc.}$	$f_{\rm NL}^{\rm eq.}$	$f_{\rm NL}^{\rm orth.}$
$P{+}B_{\rm Tree}$	0.29	23.4	8.7
$P{+}B$	0.27	17.7	4.6
$P+B{+}$p.p.	0.26	16.0	4.2
$P{+}B{+}$p.p.+g.p.	0.26	12.6	3.4

- Just using perturbativity prior, potentially a factor of $20\,,3\,,6$ over Planck!!

Results: Curvature and Neutrinos

DESI: $\sigma(\cdot)$	h	$\ln(10^{10}A_s)$	Ω_m	n_s	Ω_k
$P{+}B$	0.004	0.035	0.002	0.011	0.013
$P+B$+p.p.	0.004	0.032	0.002	0.008	0.012
$P{+}B$+p.p.+g.p.	0.004	0.025	0.002	0.007	0.009

MMo: $\sigma(\cdot)$	h	$\ln(10^{10}A_s)$	Ω_m	n_s	Ω_k
$P{+}B$	0.002	0.0052	0.0003	0.002	0.0015
$P+B$+p.p.	0.002	0.0046	0.0003	0.002	0.0012
$P{+}B$+p.p.+g.p.	0.002	0.0044	0.0003	0.001	0.0011

- Just using perturbativity prior, potentially factor of 5 over Planck!
  - Important for the landscape of string theory.
- Neutrinos: guaranteed evidence/detection:

$$2\sigma \quad \text{DESI}, \qquad 14\sigma \quad \text{MegaMapper}$$

Where can we make better?

- Shot noise and EFT-parameters:

$\sigma(\cdot)$	h	$\ln(10^{10}A_s)$	Ω_m	n_s	$f_{\rm NL}^{\rm loc.}$	$f_{\rm NL}^{\rm eq.}$	$f_{\rm NL}^{\rm orth.}$
$P{+}B$	0.0042	0.020	0.0022	0.010	3.5	114	30
$P + B + g.p.:$	0.0042	0.018	0.0022	0.009	3.4	83	23
$P + B$: bias fixed	0.0037	0.010	0.0016	0.004	2.0	21	11
$P + B : n_b \to \infty$	0.0035	0.011	0.0009	0.005	1.7	67	17

DESI

$\sigma(\cdot)$	h	$\ln(10^{10}A_s)$	Ω_m	n_s	$f_{\rm NL}^{\rm loc.}$	$f_{\rm NL}^{\rm eq.}$	$f_{\rm NL}^{\rm orth.}$
$P{+}B$	0.0021	0.0047	0.00034	0.0017	0.27	18	4.6
$P + B + g.p.:$	0.0020	0.0045	0.00033	0.016	0.26	13	3.6
$P + B$: bias fixed	0.0016	0.0034	0.00021	0.0010	0.17	3.6	1.7
$P + B : n_b \to \infty$	0.00019	0.00045	0.000029	0.00017	0.11	5.4	1.5

MegaMapper

Summary

- After the initial, successful, application to BOSS data:
  - measurement of cosmological parameters
  - new method to measure Hubble
  - perhaps fixing tensions
- the EFTofLSS is starting to look ahead to
  - higher-order and higher-n point functions
  - enlightening what next surveys could do, and how to design them
    - an eye to BSM: primordial non-Gaussianities, neutrinos, curvature, etc..
  - learning about some astrophysics, qualitative facts on the universe

Consequences of non-locality in time

- Nice recursion relations for these operators:

- .
$$[\mathcal{O}_m(\vec{x}_{\rm fl}(\vec{x},t,t'),t')]^{(n)} = \sum_{\alpha=1}^{n-m+1} \left(\frac{D(t')}{D(t)}\right)^{\alpha+m-1} \mathbb{C}^{(n)}_{\mathcal{O}_m,\alpha}(\vec{x},t)$$

$$\Rightarrow \qquad \mathcal{O}^{(n)}_m(\vec{x},t) = \sum_{\alpha=1}^{n-m+1} \mathbb{C}^{(n)}_{\mathcal{O}_m,\alpha}(\vec{x},t) \ ,$$

equal-time completeness relation

fluid recursion

- .
$$\Rightarrow \ \mathbb{C}^{(n)}_{\mathcal{O}_m,\alpha}(\vec{x},t) = \sum_{q=m}^{n-1} \frac{1}{n-\alpha-m+1} \partial_i \mathbb{C}^{(q)}_{\mathcal{O}_m,\alpha}(\vec{x},t) \frac{\partial_i}{\partial^2}\theta(\vec{x},t)^{(n-q)},$$

- Easy higher order:

$\Rightarrow$