Axion Mass in a Composite Accidental Axion Model

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Introduction

What is **axion**?

- Axion appears in Peccei-Quinn mechanism, which solves the strong CP problem. Axion is also a dark matter candidate.

Energy scale of **Pecci-Quinn mechanism**

- Observational constrainton decay constant: $f_{\mathrm{PQ}} \gtrsim 10^9 \mathrm{GeV}$
- Even larger f_{PQ} is preferred to exlain all of the dark matter

”Axion Quality Problem”

- Higher dimensional $\mathrm{U}(1)_{\mathrm{PQ}}$-breaking operators can contribute to the effective θ-angle and θ can easily exceed the experimental upper bound.

We need some suppression mechanism for $\mathrm{U}(1)_{\mathrm{PQ}}$-breaking higher-dimensional operators. This issue is called ”axion quality problem”.

Introduction

1. Suppresions for $\mathrm{U}(1)_{\mathrm{PQ}}$-breaking operators in UV can be achieved with some gauge interactions in UV.

2. In such models, instantons not in QCD (”small instantons”) can affect the axion potential in general. [Agrawl & Howe (2018), Csáki et al (2020)].

3. Such small instanton contributions might possibly vanish, due to fermion zero modes. Case-by-case studies appear to be needed. ← My Talk

Why “fermion zero modes”?

Instanton contribution to the axion mass may vanish, if **fermion zero modes** exist.

An example of vanishing physical quantity

$$\mathcal{L} = \xi^{+} i\sigma^{\mu} D_{\mu}\xi + \eta^{\dagger} i\sigma^{\mu} D_{\mu}\eta$$

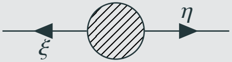
Situation: $\sigma^{\mu} D_{\mu}$ has zero modes ξ_0, η_0.

Decomposing ξ and η as

$$\xi = \lambda\xi_0 + \text{(non-zero modes)}, \quad \eta = \rho\eta_0 + \text{(non-zero modes)},$$

Path integral is

$$\int \mathrm{d}\xi^{\dagger}\mathrm{d}\xi\mathrm{d}\eta^{\dagger}\mathrm{d}\eta \ \exp[-S]\mathcal{O} \propto \int d\lambda d\rho \ \exp[-0]\mathcal{O}$$
$$= \int d\lambda d\rho \, \mathcal{O}.$$



This is vanishing, if $\mathcal{O}$ does not include ξ_0 or η_0.

Why “fermion zero modes”?

We also need to look carefully at interactions of fermions.

An example of non-vanishing physical quantity

$$L = \xi^{+} i\sigma^{\mu} D_{\mu}\xi + \eta^{\dagger} i\sigma^{\mu} D_{\mu}\eta + m(\eta\xi + \xi^{\dagger}\eta^{\dagger})$$

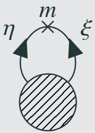
Situation: $\sigma^{\mu} D_{\mu}$ has zero modes ξ_0, η_0.

Decomposing ξ and η as,

$$\xi = \lambda\xi_0 + \text{(non-zero modes)}, \quad \eta = \rho\eta_0 + \text{(non-zero modes)},$$

Path integral is

$$\int \mathrm{d}\xi^{\dagger}\mathrm{d}\xi\mathrm{d}\eta^{\dagger}\mathrm{d}\eta \ \exp[-S]\mathcal{O} \propto \int d\lambda d\rho \ \exp[-m\rho\lambda\eta_0\xi_0]\mathcal{O} \neq 0.$$



This is non-vanishing, due to the interactions of Weyl fermions.

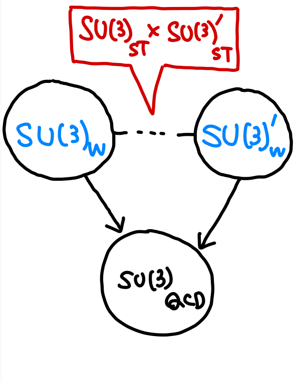
Be careful about fermion zero modes and their interactions.

Model

(A simple version of) composite accidental axion models [Redi & Sato (2016)].

- $[SU(3)_{ST}]^N \times [SU(3)_W]^N$ gauge theory with fermions. (My talk: $N = 2$)
- Dynamical scales: $\Lambda_{STRONGER} \gg \Lambda_{WEAKER}$.

Indices	A $SU(3)_{ST}$	a $SU(3)_W$	B $SU(3)'_{ST}$	b $SU(3)'_W$
ψ_A^a	$\mathbf{3}$	$\overline{\mathbf{3}}$		
ψ_A	$\mathbf{3}$	$\mathbf{1}$		
ψ_a^B		$\mathbf{3}$	$\overline{\mathbf{3}}$	
ψ^B		$\mathbf{1}$	$\overline{\mathbf{3}}$	
ψ_B^b			$\mathbf{3}$	$\overline{\mathbf{3}}$
ψ_B			$\mathbf{3}$	$\mathbf{1}$
ψ_b^A	$\overline{\mathbf{3}}$			$\mathbf{3}$
ψ^A	$\overline{\mathbf{3}}$			$\mathbf{1}$



Model

Features of the dynamics (N=2)

Energy scale

$\leftarrow \Lambda_{\mathrm{ST}}$
$\leftarrow \Lambda'_{\mathrm{ST}}$: $\mathrm{SU}(3)_{\mathrm{ST}}$ and $\mathrm{SU}(3)'_{\mathrm{ST}}$ confine.

- $\mathrm{U}(1)_{\mathrm{PQ}}$ is spontaneously broken.
- QCD appears in the diagonal subgroup of $\mathrm{SU}(3)_{\mathrm{W}} \times \mathrm{SU}(3)'_{\mathrm{W}}$

$\leftarrow \Lambda_{\mathrm{W}}$
$\leftarrow \Lambda'_{\mathrm{W}}$

Next Steps

1. Dynamics of $\mathrm{SU}(3)_{\mathrm{ST}} \times \mathrm{SU}(3)'_{\mathrm{ST}}$
2. $\mathrm{U}(1)_{\mathrm{PQ}}$ in this model

Model

Dynamics of $\mathrm{SU}(3)_{\mathrm{ST}} \times \mathrm{SU}(3)'_{\mathrm{ST}}$

Indices	A	a	B	b
	$\mathrm{SU}(3)_{\mathrm{ST}}$	$\mathrm{SU}(3)_{\mathrm{W}}$	$\mathrm{SU}(3)'_{\mathrm{ST}}$	$\mathrm{SU}(3)'_{\mathrm{W}}$
ψ^a_A	$\mathbf{3}$	$\overline{\mathbf{3}}$		
ψ_A	$\mathbf{3}$	$\mathbf{1}$		
ψ^B_a		$\mathbf{3}$	$\overline{\mathbf{3}}$	
ψ^B		$\mathbf{1}$	$\overline{\mathbf{3}}$	
ψ^b_B			$\mathbf{3}$	$\overline{\mathbf{3}}$
ψ_B			$\mathbf{3}$	$\mathbf{1}$
ψ^A_b	$\overline{\mathbf{3}}$			$\mathbf{3}$
ψ^A	$\overline{\mathbf{3}}$			$\mathbf{1}$

$$\langle \psi^a_A \psi^A_b \rangle \sim \Lambda_{\mathrm{ST}} \delta^a_b$$

$$\langle \psi_A \psi^A \rangle \sim \Lambda_{\mathrm{ST}}$$

$$\langle \psi^b_B \psi^B_a \rangle \sim \Lambda_{\mathrm{ST}} \delta^b_a$$

$$\langle \psi_B \psi^B \rangle \sim \Lambda_{\mathrm{ST}}$$

QCD appears in the diagonal subgroup of $[\mathrm{SU}(3)_{\mathrm{W}}]^2$, which is unbroken by condensations at scale Λ_{ST}.

Model

$U(1)_{PQ}$ in this model

	$U(1)_{PQ}$	$SU(3)_{ST}$	$SU(3)_{W}$	$SU(3)'_{ST}$	$SU(3)'_{W}$
ψ_A^a	1	$\mathbf{3}$	$\overline{\mathbf{3}}$		
ψ_A	-3	$\mathbf{3}$	$\mathbf{1}$		
ψ_a^B	1		$\mathbf{3}$	$\overline{\mathbf{3}}$	
ψ^B	-3		$\mathbf{1}$	$\overline{\mathbf{3}}$	
ψ_B^b	1			$\mathbf{3}$	$\overline{\mathbf{3}}$
ψ_B	-3			$\mathbf{3}$	$\mathbf{1}$
ψ_b^A	1	$\overline{\mathbf{3}}$			$\mathbf{3}$
ψ^A	-3	$\overline{\mathbf{3}}$			$\mathbf{1}$

- $U(1)_{PQ} - SU(3)_{ST} - SU(3)_{ST}$: Non-anomalous
- $U(1)_{PQ} - SU(3)_{W} - SU(3)_{W}$: Anomalous
- $U(1)_{PQ}$ is spontaneously broken by condensations at the scale Λ_{ST}.

Model

Axion quality in this model:

- $\mathrm{U}(1)_{\mathrm{PQ}}$ breaking operators are restricted by gauge symmetries.
- The lowest dimensional, $\mathrm{U}(1)_{\mathrm{PQ}}$ breaking operator (which is singlet under all the symmetries) is $\psi^a_A \psi^A_b \psi^b_B \psi^B_a$ which has a mass dimension 6.
- With larger N ($N > 2$), $\mathrm{U}(1)_{\mathrm{PQ}}$-breaking operators become more higher dimensional, and better axion quality is achieved.

Model

Axion mass in this model:

- Only a part of instantons in $\mathrm{SU}(3)_{\mathrm{W}} \times \mathrm{SU}(3)'_{\mathrm{W}}$ is in QCD, although they could potentially contribute to the axion mass.

Next Steps

We examine such small instanton contributions in a toy model.

Model

A model, effectively describing the dynamics below $\Lambda_{\rm ST}$, $\Lambda'_{\rm ST}$

- $[\mathrm{SU}(3)]^2$ gauge theory with fermions and scalars.
- Yukawa interactions: $y_1 {\Phi_1}_a^{\ b} {q_{\rm R}}^\dagger_b {\overline{Q}_{\rm L}}^{a} + y_2 {\Phi_2}_b^{\ a} {Q_{\rm R}}^\dagger_a {\overline{q}_{\rm L}}^{b} + \mathrm{h.c.}$.

	$\mathrm{SU}(3)_a$	$\mathrm{SU}(3)_b$	$\mathrm{U}(1)_{\rm PQ}$
Scalars			
Φ_1	$\mathbf{3}$	$\overline{\mathbf{3}}$	-2
Φ_2	$\overline{\mathbf{3}}$	$\mathbf{3}$	-2
Fermions			
${Q_{\rm R}}^c$	$\mathbf{3}$		1
$\overline{Q}_{\rm L}$	$\overline{\mathbf{3}}$		1
${q_{\rm R}}^c$		$\mathbf{3}$	1
$\overline{q}_{\rm L}$		$\overline{\mathbf{3}}$	1

- $\mathrm{SU}(3)_a \times \mathrm{SU}(3)_b$ corresponds to $\mathrm{SU}(3)_{\rm W} \times \mathrm{SU}(3)'_{\rm W}$.
- A QCD instanton is a pair of simultaneous instantons in $\mathrm{SU}(3)_a$ and $\mathrm{SU}(3)_b$

Model

Yukawa interactions:

$$y_1 \, {\Phi_1}_a^{\ b} \, {q_R}_b^\dagger \, \overline{Q_L}^{\ a} + y_2 \, {\Phi_2}_b^{\ a} \, {Q_R}_a^\dagger \, \overline{q_L}^{\ b} + \text{h.c.} \, .$$

- $SU(3)_a$ instanton is an example of small instantons.
- All the fermions become massive by VEV of Φ_1 and Φ_2.

Question:

Are there fermion zero modes?

Next Steps

we discuss fermion zero modes around instantons in a simpler SM-like model.

Model

SM-like $\mathrm{SU}(2)$ model, instead of axion models:

$$\mathcal{L} = -\frac{1}{4g^2}F^{\mu\nu}F_{\mu\nu} + |D_\mu H|^2 + \frac{\lambda}{4}(H^\dagger H - v^2)^2 + \begin{pmatrix} e_\mathrm{R}{}^\dagger \ell_\mathrm{L}{}^\dagger \end{pmatrix} \begin{pmatrix} yH & i\sigma^\mu \partial_\mu \\ i\bar{\sigma}^\mu D_\mu & yH^\dagger \end{pmatrix} \begin{pmatrix} \ell_\mathrm{L} \\ e_\mathrm{R} \end{pmatrix}$$

- $D = \partial - iA$, $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - \epsilon^{abc} A^a_\mu A^b_\nu$.
- $H, \ell_\mathrm{L}, e_\mathrm{R}$ are $\mathrm{SU}(2)$-doublet, doublet, singlet, respectively.

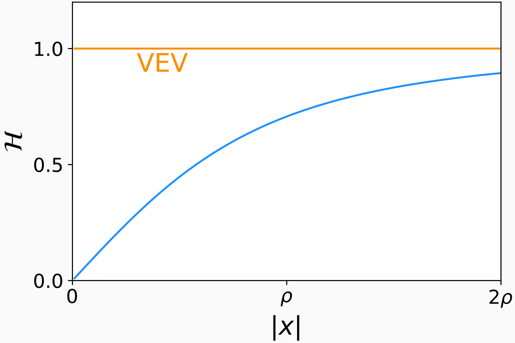
Our interests:

1. An instanton with a size sufficiently smaller than v^{-1}
2. Fermion zero modes around such an instanton

Small Instantons

An instanton with a size $\rho \ll v^{-1}$ has following profiles of H:

$$H = \begin{pmatrix} 0 & \mathcal{H} v \end{pmatrix}^T$$



Fermion zero modes

Zero mode equation for $\Psi \stackrel{\text{def}}{=} \begin{pmatrix} \ell_\mathrm{L} \\ e_\mathrm{R} \end{pmatrix}$:

$$\hat{D}\Psi \stackrel{\text{def}}{=} \begin{pmatrix} yH & i\sigma^\mu \partial_\mu \\ i\bar{\sigma}^\mu D_\mu & yH^\dagger \end{pmatrix} \begin{pmatrix} \ell_\mathrm{L} \\ e_\mathrm{R} \end{pmatrix} = 0$$

There exist fermion zero modes of $\hat{D}$. [Espinosa (1989)]

- With an anti-instanton background, Ψ has a zero mode for $\hat{D}$:

$$(\text{zero mode}) = \begin{pmatrix} \ell_\mathrm{L} \\ e_\mathrm{R} \end{pmatrix} \propto \begin{pmatrix} \mathcal{O}(1) \\ \mathcal{O}(yv) \end{pmatrix}$$

- With an instanton background, not Ψ but $\Psi^\dagger$ has a zero mode (for $\hat{D}^\dagger$) .

Fermion Zero Modes in the Axion Toy Model

$$\begin{pmatrix} q_R^\dagger & \overline{Q}_L^\dagger & \overline{q}_L^\dagger & Q_R^\dagger \end{pmatrix} \begin{pmatrix} -y_1\Phi_1 & i\sigma^\mu D_\mu^{(2)} & & \\ i\bar{\sigma}^\mu D_\mu^{(1)} & -y_1\Phi_1^\dagger & & \\ & & -y_2\Phi_2^\dagger & i\bar{\sigma}^\mu D_\mu^{(2)} \\ & & i\sigma^\mu D_\mu^{(1)} & -y_2\Phi_2 \end{pmatrix} \begin{pmatrix} \overline{Q}_L \\ q_R \\ Q_R \\ \overline{q}_L \end{pmatrix}$$

- Zero modes in $SU(3)_a$ (anti-)instanton background: $\left(Q_R^\dagger, \overline{q}_L^\dagger\right)$ & $\left(\overline{Q}_L, q_R\right)$
- Zero modes in $SU(3)_b$ (anti-)instanton background: $\left(q_R^\dagger, \overline{Q}_L^\dagger\right)$ & $\left(\overline{q}_L, Q_R\right)$

Fermion Zero Modes in the Axion Toy Model

$$\begin{pmatrix} q_R^\dagger & \overline{Q}_L^\dagger & \overline{q}_L^\dagger & Q_R^\dagger \end{pmatrix} \begin{pmatrix} -y_1\Phi_1 & i\sigma^\mu D_\mu^{(2)} & & \\ i\bar{\sigma}^\mu D_\mu^{(1)} & -y_1\Phi_1^\dagger & & \\ & & -y_2\Phi_2^\dagger & i\bar{\sigma}^\mu D_\mu^{(2)} \\ & & i\sigma^\mu D_\mu^{(1)} & -y_2\Phi_2 \end{pmatrix} \begin{pmatrix} \overline{Q}_L \\ q_R \\ Q_R \\ \overline{q}_L \end{pmatrix}$$

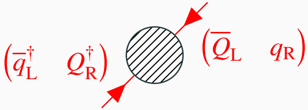
- The existence of zero modes does NOT necessarily imply the absence of small instanton contributions.

This is due to interactions between zero modes and other dynamical modes (especially Φ_1 and Φ_2) .

Fermion Zero Modes in the Axion Toy Model

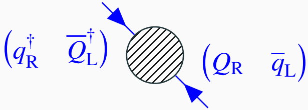
$$
\begin{pmatrix} q_R^\dagger & \overline{Q}_L^\dagger & \overline{q}_L^\dagger & Q_R^\dagger \end{pmatrix}
\begin{pmatrix}
-y_1\Phi_1 & i\sigma^\mu D_\mu^{(2)} & & \\
i\bar{\sigma}^\mu D_\mu^{(1)} & -y_1\Phi_1^\dagger & & \\
& & -y_2\Phi_2^\dagger & i\bar{\sigma}^\mu D_\mu^{(2)} \\
& & i\sigma^\mu D_\mu^{(1)} & -y_2\Phi_2
\end{pmatrix}
\begin{pmatrix} \overline{Q}_L \\ q_R \\ Q_R \\ \overline{q}_L \end{pmatrix}
$$

$SU(3)_a$ (anti-)instanton



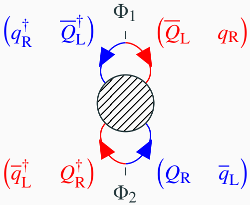
$= 0$

$SU(3)_b$ (anti-)instanton



$= 0$

QCD (anti-)instanton



Non-vanishing effects are only from QCD.

Summary

- Axion quality problem gives a restriction to UV completion of the axion model.
- Dynamics at UV, especially small instanton effects may enhance the axion mass in general.

★ I have exhibited that small instanton effects can vanish due to fermion zero modes, in a model-dependent way.

Appendix: A Bit More about Global Symnmetries

$\mathrm{U}(1)_{\mathrm{PQ}}$ in this model: $\mathrm{U}(1)_{\mathrm{PQ}} \subset \mathrm{SU}(4)_{\mathrm{Axial}} \subset [\mathrm{U}(4)]^2$ (Flavor symmetry in UV)

	$\mathrm{U}(1)_{\mathrm{PQ}}$	$\mathrm{SU}(3)_{\mathrm{ST}}$	$\mathrm{SU}(3)_{\mathrm{W}}$	$\mathrm{SU}(3)'_{\mathrm{ST}}$	$\mathrm{SU}(3)'_{\mathrm{W}}$
ψ_A^a	1	$\mathbf{3}$	$\overline{\mathbf{3}}$		
ψ_A	-3	$\mathbf{3}$	$\mathbf{1}$		
ψ_a^B	1		$\mathbf{3}$	$\overline{\mathbf{3}}$	
ψ^B	-3		$\mathbf{1}$	$\overline{\mathbf{3}}$	
ψ_B^b	1			$\mathbf{3}$	$\overline{\mathbf{3}}$
ψ_B	-3			$\mathbf{3}$	$\mathbf{1}$
ψ_b^A	1	$\overline{\mathbf{3}}$			$\mathbf{3}$
ψ^A	-3	$\overline{\mathbf{3}}$			$\mathbf{1}$

- $\mathrm{U}(1)_{\mathrm{PQ}} - \mathrm{SU}(3)_{\mathrm{ST}} - \mathrm{SU}(3)_{\mathrm{ST}}$: Non-anomalous
- $\mathrm{U}(1)_{\mathrm{PQ}} - \mathrm{SU}(3)_{\mathrm{W}} - \mathrm{SU}(3)_{\mathrm{W}}$: Anomalous
- $\mathrm{U}(1)_{\mathrm{PQ}}$ is spontaneously broken by condensations at the scale Λ_{ST}.