Weak gravitational positivity bounds and implications on Weak Gravity Conjecture

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Based on TQL, arXiv:2312.09132 [hep-th]
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Introduction

- Swampland Program [Cumrun Vafa '05]: Which low-energy EFT are consistent with non-perturbative quantum gravity (UV completed) considerations?
- Weak Gravity Conjecture (WGC) [Arkani-Hamed, et. al. '06]: roughly states that gravity should be the weakest force in Quantum Gravity (QG) theories.
- (Gravitational) Positivity Bounds: UV completion conditions put positivity constraints on EFT's (Wilson) coefficients & EFT's Cut-off scale.
- [Cheung-Remmen '14] first find possible connections between WGC on charged particles and positivity gives new charge/mass ratio $gq \gtrsim m/M_{\rm Pl}$.

Overview of positivity bounds implications' on WGC

- [Andriolo-Junghans-Noumi-Siu '18] extended the analysis to multiple $U(1)$ and motivated a stronger version of WGC called the subLattice/Tower WGC, [Chen-Huang-Noumi-Wen '18] for convex-hull type WGC for multiple $U(1)$.
- [Hamada-Noumi-Shiu '18, Bellazzini-Lewandowski-Serra '18, Arkani-Hamed et.al. '21] derived connection to BH WGC, focusing on monotonicity of the extremal curve.
- [Alberte-Rham-Jaitly-Tolley '21] incorpolated improved positivity into QED, [Aoki-TQL-Noumi-Tokuda '22] generalized to SM with magnetic WGC-type bound

$$y_e \sin\theta_{\mathcal{W}} \geq \sqrt{11/1440}\Lambda/M_{\mathrm{Pl}}.$$

- In this talk, we study $\gamma\gamma \to \gamma\gamma, HH \to HH, \gamma H \to \gamma H$ processes in the EW (Weinberg-Salam model) coupled to GR to derive possible extension of magnetic WGC: $g \gtrsim \Lambda/M_{\mathrm{Pl}}$.

Contents

UV & IR Physics

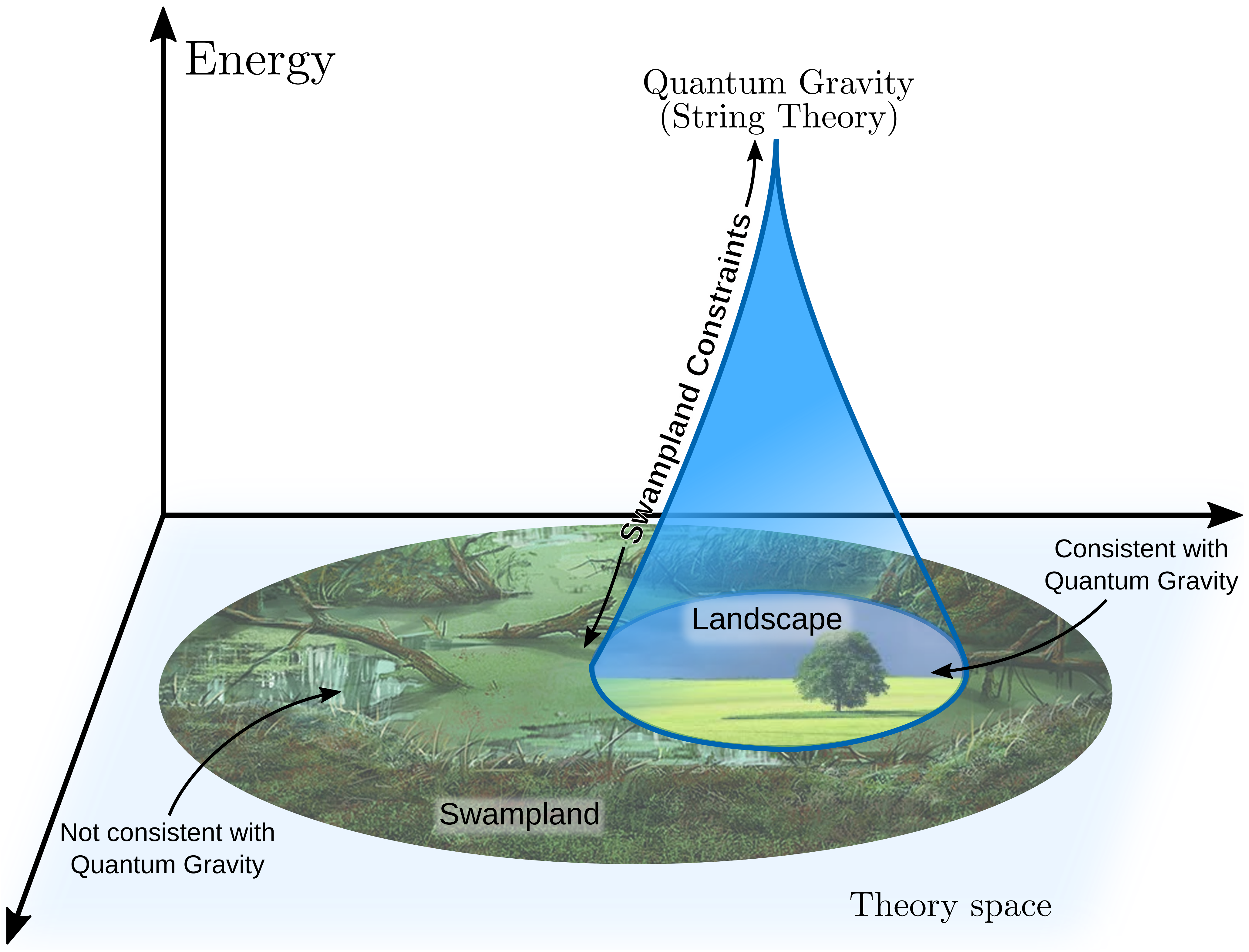


Fig. 1: The Swampland and Landscape of EFTs.
[arXiv: 2102.01111 [hep-th]](https://arxiv.org/abs/2102.01111)

Quantum Gravity

Top down ↓ ↑ Bottom up

General Principles: Dualities, Symmetries to write down EFTs' operators

UV completion conditions:
- Lorentz inv.
- Unitary
- Locality
- Causality

→ Positivity Constraints to EFTs' parameters

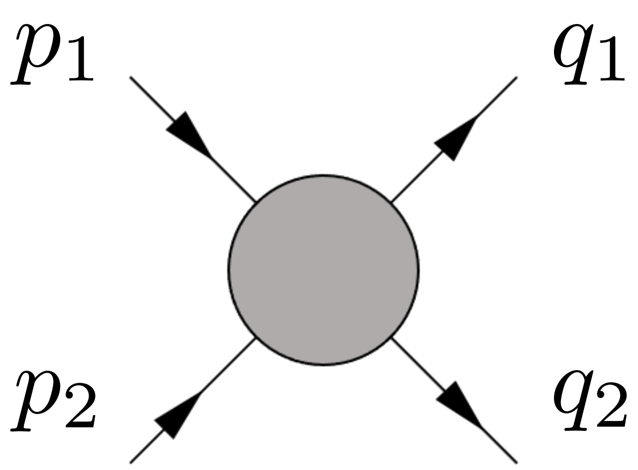
Effective Field Theories (EFTs)

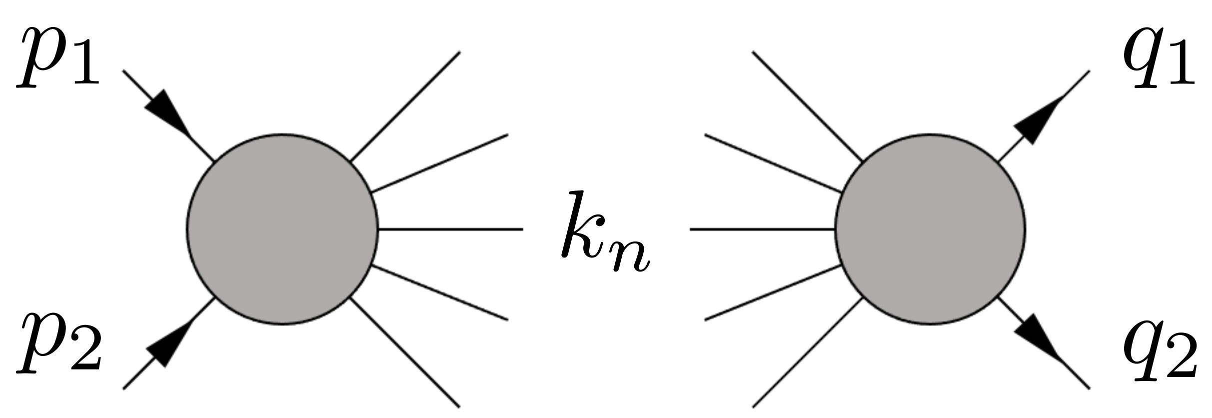
Positivity Bound: S-matrix properties (1)

Lorentz invariance

Mandelstam variables: $A(p_1, p_2, q_1, q_2) \to A(s,t)$, with Mandelstam var. $\begin{cases} s = -(p_1+p_2)^2 = E_{\rm CM}^2, \\ t = -(p_1-p_2)^2 = -\frac{s-4m^2}{2}(1-\cos\theta), \\ u = -(p_1-q_2)^2 = 4m^2 - s - t. \end{cases}$

Unitarity





Optical Theorem:

$$\begin{cases} S^\dagger S = 1 \\ S = 1 + iM \end{cases} \Rightarrow M - M^\dagger = iM^\dagger M. \quad \mathrm{Im}\ M(p_1p_2 \to q_1q_2) = \frac{1}{2}\sum_n \int \mathrm{d}\tilde{\Pi}_n M^*(p_1p_2 \to k_n) M(q_1q_2 \to k_n)$$

$$= \sqrt{s(s-4m^2)}\sigma_t > 0.$$

Locality

Froissart-Martin Bound: $\lim_{|s|\to\infty} \left|\frac{A(s,t)}{s^2}\right| = 0$ with $t \neq m^2, t < 4m^2$.

⟶ Scattering Amplitude is Polynomially bounded. Helps neglect contribution at $s \to \infty$ contours.

Causality

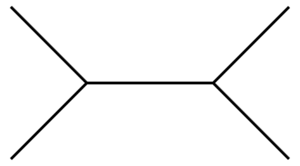
Crossing symmetry: $A(s,t)$ is invariant under $s \leftrightarrow u, t \leftrightarrow u, s \leftrightarrow t$.

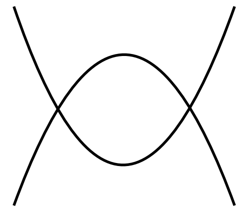
Analiticity: Analytical continuation to complex plane, integral relations and singularity conditions.

Positivity Bound: S-matrix properties (2)

Analiticity [arXiv:1605.06111 [hep-th]] [arXiv:1702.06134 [hep-th]]

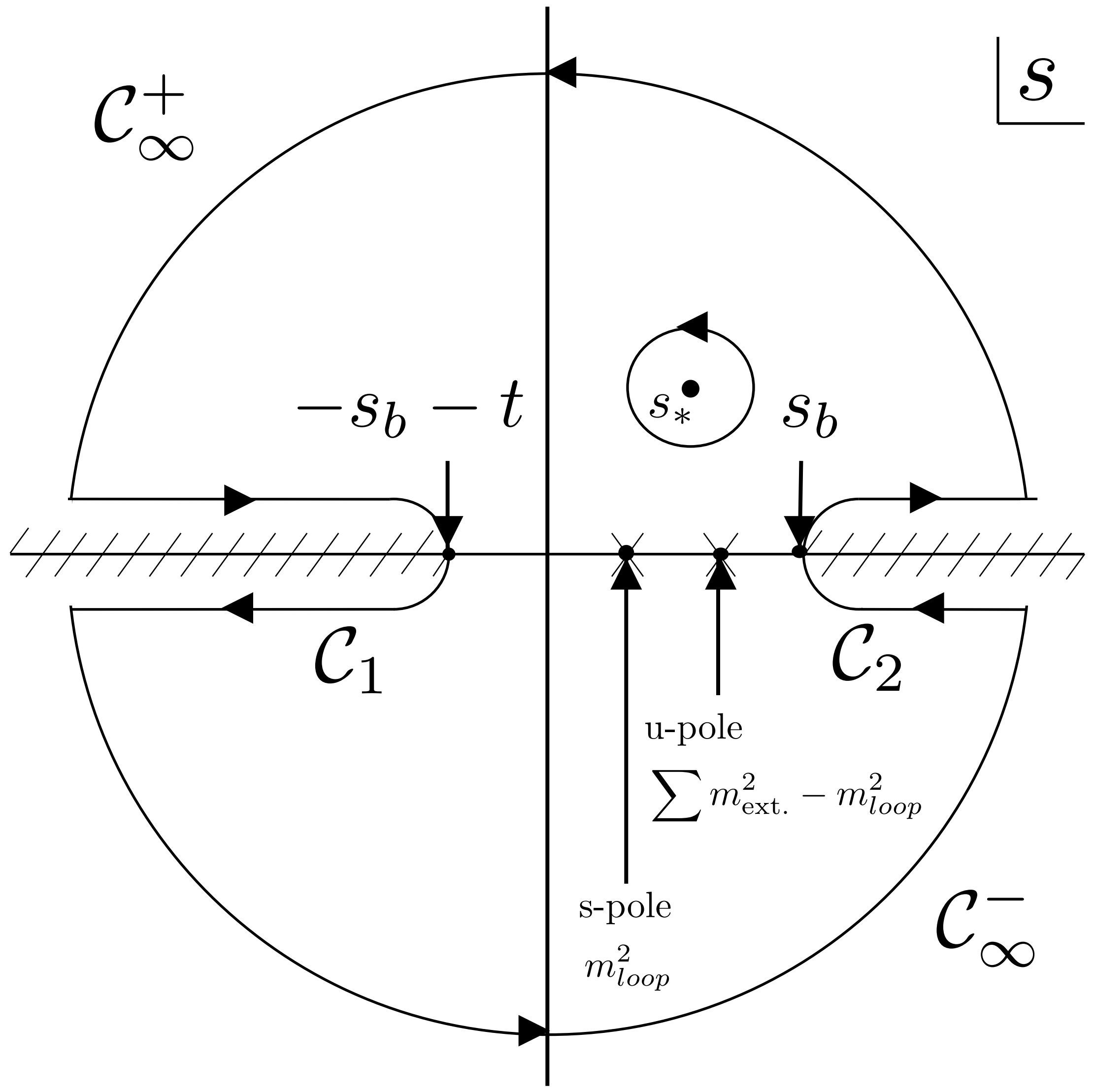
Fixed t, $u = \sum m^2_{\text{ext.}} - s - t.$

Poles from intermediary particles: 

Branch-cuts from loops: 

Dispersion relation

$$
\begin{aligned}
A(s,t) =& \frac{1}{2\pi i}\oint_C d\tilde{s}\frac{A(\tilde{s},t)}{\tilde{s}-s} \\
=& \frac{\lambda}{m^2-s} + \frac{\lambda}{m^2-u} + \int_{C\pm\infty} \mathrm{d}\tilde{s}\frac{A(\tilde{s},t)}{\tilde{s}-s} + \frac{1}{\pi}\int_{4m^2}^{\infty}\mathrm{d}\mu\left[\frac{1}{\mu-s}+\frac{1}{\mu-u}\right]\mathrm{Im}A(\mu,t)
\end{aligned}
$$



Identity

$$
\frac{1}{\mu-s} = \frac{(s-\mu_p)^2}{(\mu-\mu_p)^2}\frac{1}{\mu-s} + 2\frac{(s-\mu_p)}{(\mu-\mu_p)^2} + \frac{(\mu-s)}{(\mu-\mu_p)^2}.
$$

Twice subtracted dispersion relation

$$
\begin{aligned}
A(s,t) =& \frac{1}{2\pi i}\oint_C d\tilde{s}\frac{A(\tilde{s},t)}{\tilde{s}-s} \\
=& a(t) + \frac{\lambda}{m^2-s} + \frac{\lambda}{m^2-u} + \frac{1}{\pi}\int_{4m^2}^{\infty}\mathrm{d}\mu\left[\frac{(s-\mu_p)^2}{(\mu-\mu_p)^2(\mu-s)} + \frac{(s-\mu_p)^2}{(\mu-\mu_p)^2(\mu-s)}\right]\mathrm{Im}A(\mu,t).
\end{aligned}
$$

Positivity Bound:
Subtracted Scattering Amplitude & Improved Positivity Bound

Substracted Scattering Amplitude

$$B(s,t) = A(s,t) - \frac{\lambda}{m^2 - s} - \frac{\lambda}{m^2 - u} - \frac{\lambda}{m^2 - t}. \qquad \widetilde{B}(v,t) := B(s,t)|_{s=v+2m^2-t/2}.$$

$$B^{2N,M}(t) = \frac{1}{M!}\partial_v^{2N}\partial_t^M \widetilde{B}(v,t)\Big|_{v=0}.$$

$$B^{2N,0}(t) = \frac{(2N)!2}{\pi}\int_{4m^2}^{\infty} \mathrm{d}\mu \frac{\mathrm{Im}\, A(\mu,t)}{(\mu - 2m^2 + \frac{t}{2})^{2N+1}} > 0, \quad \text{with no t derivative (in forward limit, } t \to 0).$$

Improved positivity

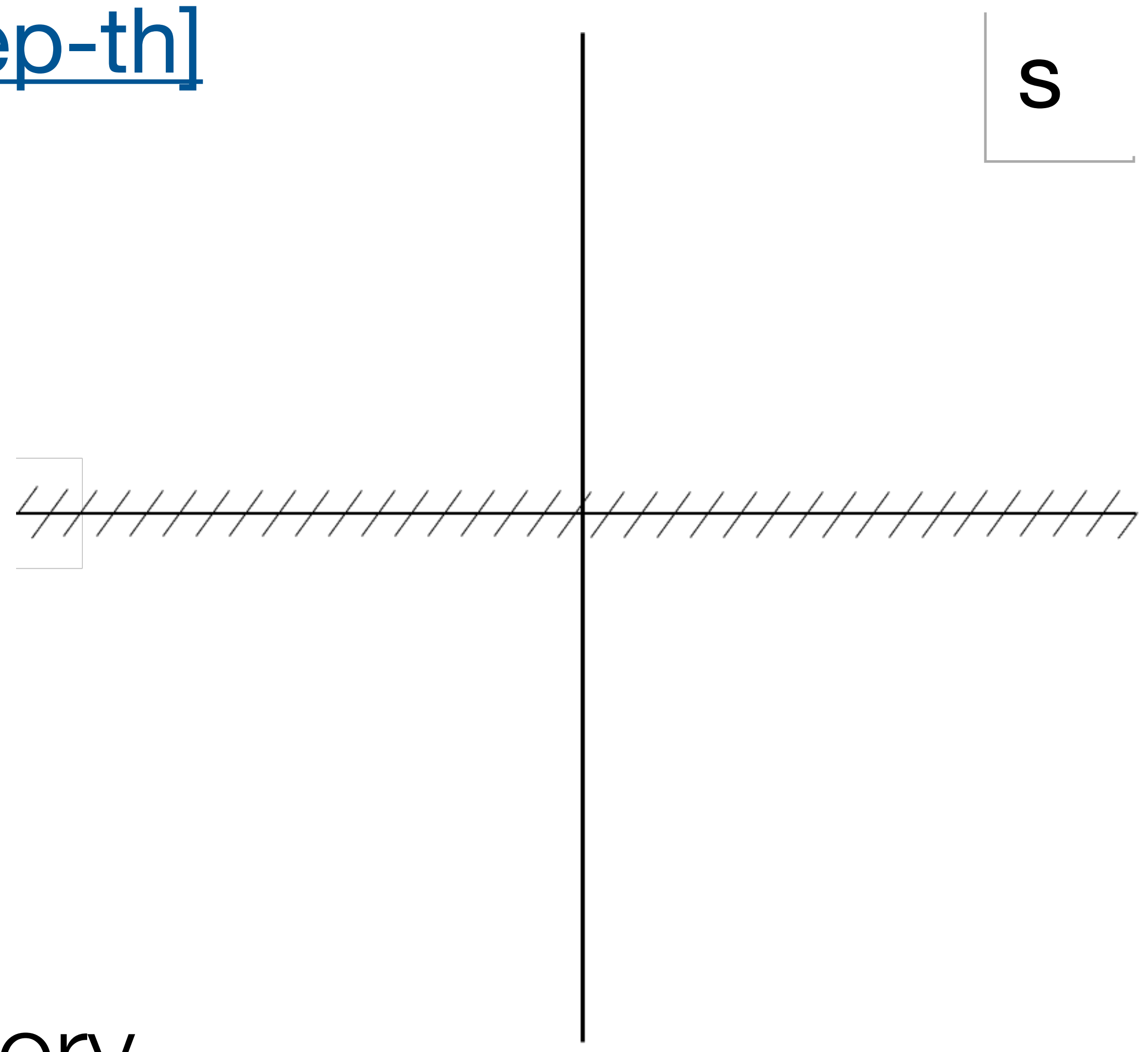
$$B^{2N,0}_{\Lambda_{EFT}}(t) := \frac{(2N)!2}{\pi}\left(\int_{4m^2}^{\infty} - \int_{4m^2}^{\Lambda_{EFT}^2}\right) \mathrm{d}\mu \frac{\mathrm{Im}\, A(\mu,t)}{(\mu - 2m^2 + \frac{t}{2})^{2N+1}} > 0$$

$$= \frac{(2N)!2}{\pi}\int_{\Lambda_{EFT}^2}^{\infty} \mathrm{d}\mu \frac{\mathrm{Im}\, A(\mu,t)}{(\mu - 2m^2 + \frac{t}{2})^{2N+1}} > 0.$$

(Subtract the known part from unknown parts to enhance the positivity).

Gravitational Positivity Bound

Adding massless Graviton causes some troubles [arXiv:2007.15009 [hep-th]]



Analytical structure

Issue: Massless loop create a branch cut along x-axis,

disconnecting the 2 half-planes.

Solution: The theory UV completed at Planck scale. In linearised theory,

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{\sqrt{32\pi}}{M_{\rm Pl}} h_{\mu\nu}.$$

→ Graviton loop is supressed at UV completion of gravity, so we can neglect it.

Gravitational Positivity Bound

Boundedness [arXiv:2007.15009 [hep-th]]

Issue: Encounter a non-gapped system: with massless particles → cannot use Froissart bound.

$$B^{(2)}(\Lambda, t) - \frac{8}{M_{\rm Pl}^2 t} = \frac{4}{\pi}\int_{\Lambda^2}^{\infty} \mathrm{d}s' \, \frac{\mathrm{Im}\,\mathcal{M}(s'+i\epsilon, t)}{(s'+(t/2))^3}\,.$$

with similar bound, $\lim_{|s|\to\infty}\left|\frac{M(s,t<0)}{s^2}\right| = 0.$

Solution: We assume the Regge behavior,

$$\mathrm{Im}\,\mathcal{M}(s,t) = f(t)\left(\frac{s}{M_s^2}\right)^{2+\alpha' t + \alpha'' t^2 + \cdots} + \cdots$$

$\alpha > 0$

with Single scaling,

$$|(\partial_t f/f)_{t=0}|, |\alpha''/\alpha'|, \alpha' \lesssim \mathcal{O}(M_s^{-2}).$$

$$\longrightarrow \quad B^{(2)}(\Lambda) := B^{(2)}(\Lambda, 0) > -\mathcal{O}(M_{\rm Pl}^{-2} M_s^{-2})\,,$$

Small amount of negativity is still allowed, R.H.S. is suppressed by not only $M_{\rm Pl}^{-2}$ but also M_s^{-2} which is small enough to provide the constraints on the SM amplitudes with gravity.

Set-up

2-by-2 processes $HH \to HH, \gamma\gamma \to \gamma\gamma, H\gamma \to H\gamma$

$$\mathcal{M}(s,t) = \sum_{i=\text{QED, Weak, Higgs}} \mathcal{M}_{\text{EW,i}}(s,t) + \mathcal{M}_{\text{GR,i}}(s,t)$$

<u>Helicity set-up</u>

$\gamma\gamma\to\gamma\gamma \quad (h_1,h_2,h_3,h_4) \in \{(\pm,\pm,\pm,\pm),(\pm,\mp,\pm,\mp)\}$

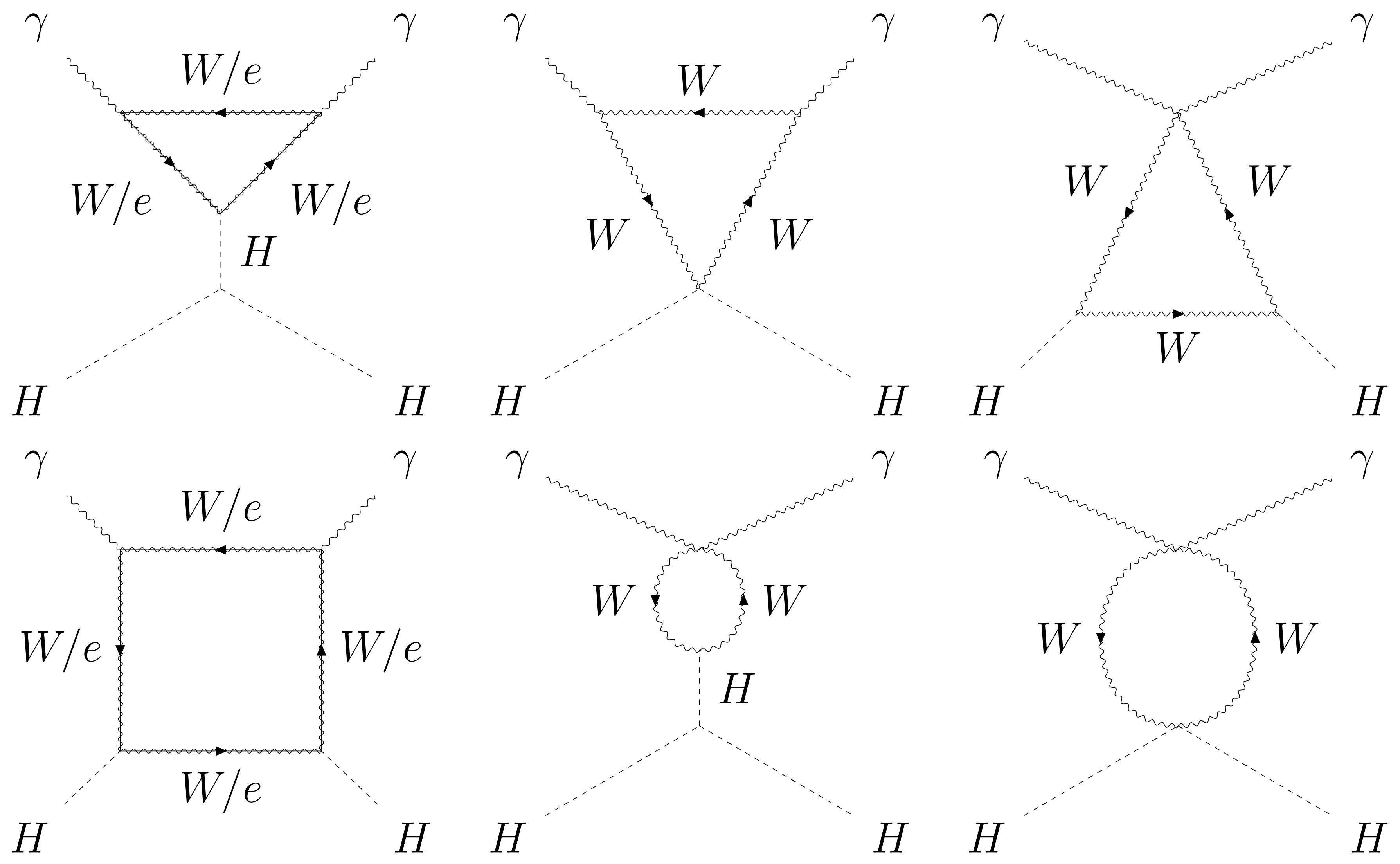
$H\gamma\to H\gamma \quad (h_1,h_3) = (+,+)$

⟶ At forward scattering limit $\mathcal{A}(s) := \mathcal{M}(s,0)$, subtracted amplitude reads,

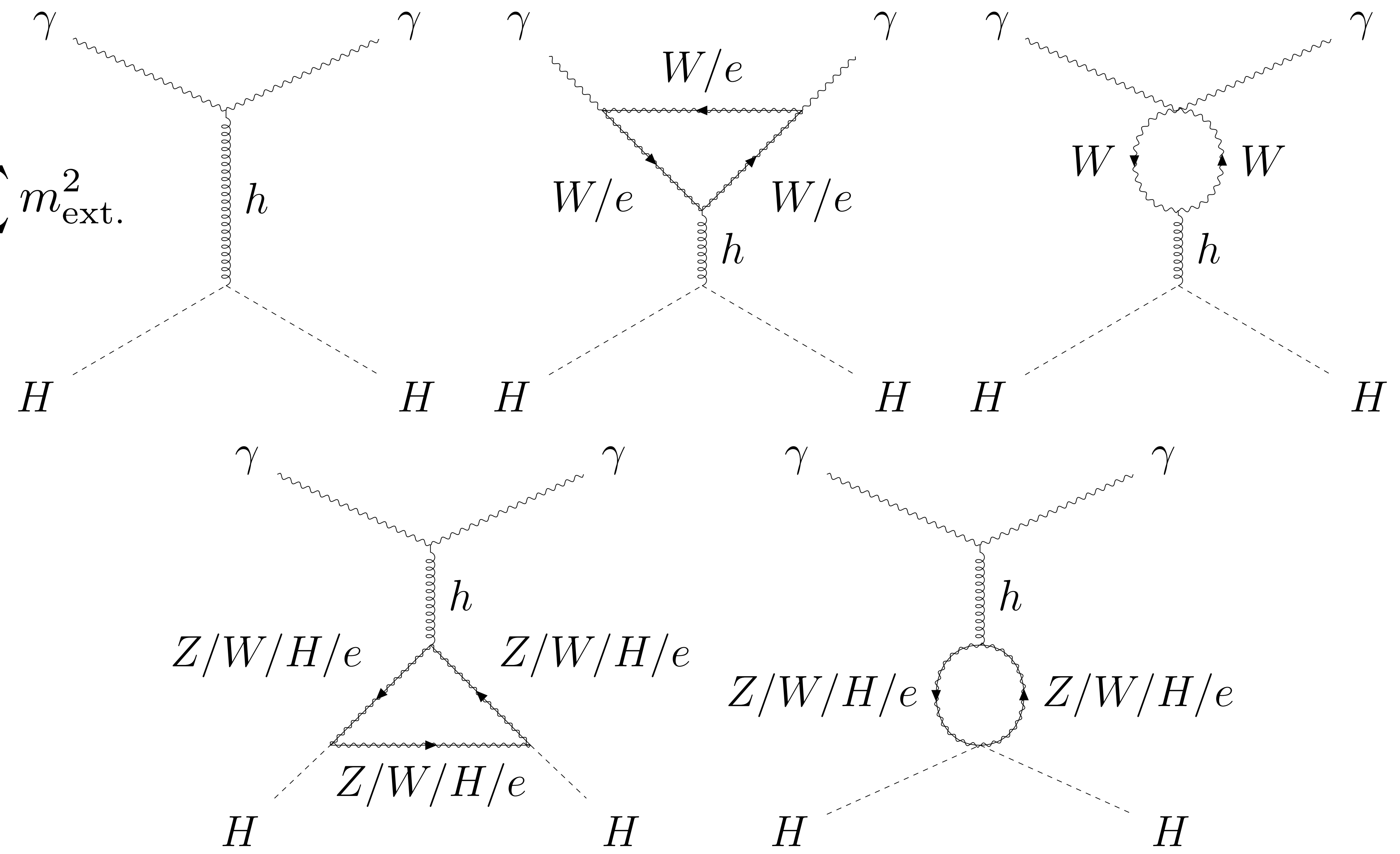
$s_* = \sum m^2_{\text{ext.}}/2 - t/2 + i\mu.$

$$B^{(2)}_{\text{EW}} = \frac{4}{\pi}\int_{\Lambda^2}^{\infty} \frac{\text{Im}\,\mathcal{A}(s)}{(s-s_*)^3}\,\text{d}s,$$

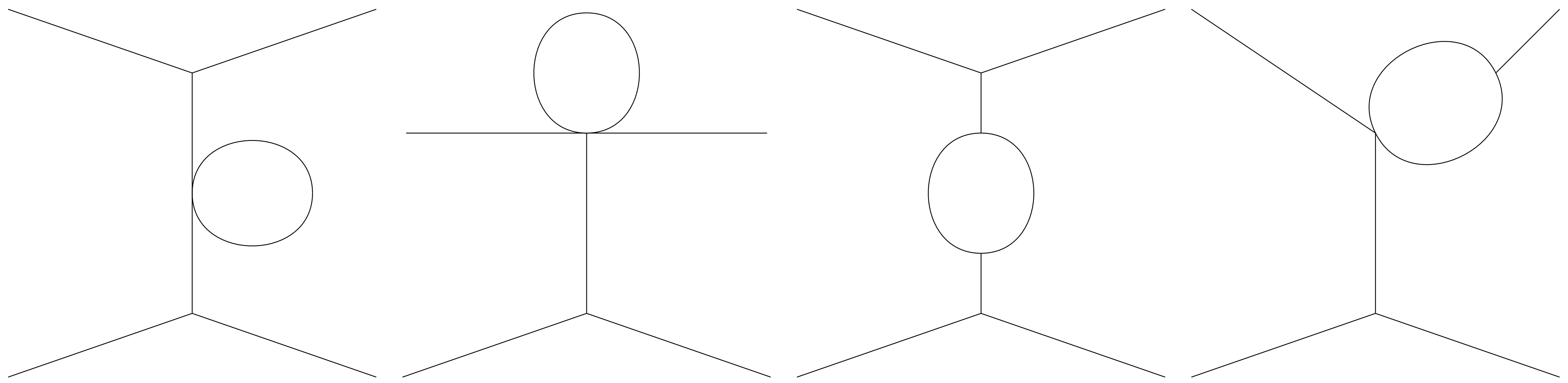
$$B^{(2)}_{\text{GR}} = \frac{d^2}{ds^2}A(s)\Big|_{s=s_*} - \frac{4}{\pi}\int_{s_b^2}^{\Lambda^2} \frac{\text{Im}\,\mathcal{A}(s)}{(s-s_*)^3}\,\text{d}s.$$



$s_b = 4m^2_{\text{loop}} - \sum m^2_{\text{ext.}}$

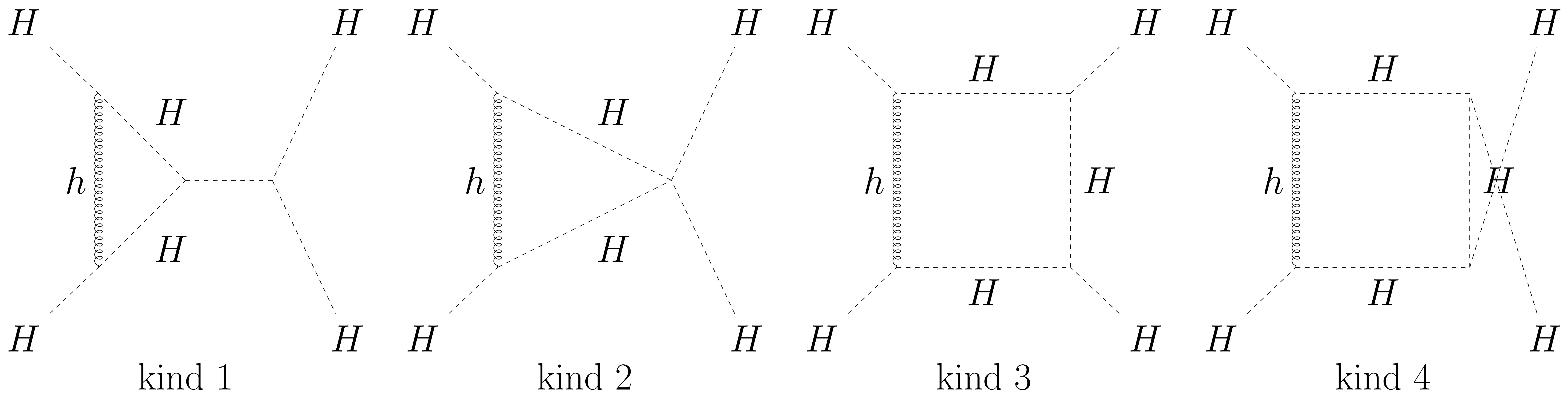


Aside: Sub-dominant contribution



Omitted topologies

- Either amplitude is less than $\mathcal{O}(s^2)$.
- Or imaginary part is suppressed $\mathcal{O}\left(\Lambda^{-2} M_{\mathrm{Pl}}^{-2}\right)$.



IR div. diagrams: [2105.01436 [hep-th]]

- Assume dressed by soft grav. cloud
- Add fictious graviton mass as a pole in graviton propagator

$$\frac{1}{M_{\mathrm{Pl}}^2} \frac{-iP^{(d)}_{\mu\nu\rho\sigma}}{q + i\epsilon} \to \frac{1}{M_{\mathrm{Pl}}^2} \frac{-iP^{(d)}_{\mu\nu\rho\sigma}}{q - m_g^2 + i\epsilon}$$

$$B^2 \sim \mathcal{O}\left(\Lambda^{-2} M_{\mathrm{Pl}}^{-2}\right).$$

→ Only t-channel diagrams with a graviton propagator contribute to $\mathcal{M}_{\mathrm{GR}}^{HH \to HH}$.

Constraint to gauge couplings at zero Higgs mass

Anomalous threshold
[Zhiboedov's Notes on the analytic S-matrix, Aoki-Huang '2023]

$HH \to HH$ $\quad g_1^2 + 3g_2^2 > \frac{125 - 8\sqrt{3}}{36} \frac{\Lambda^2}{M_{\rm Pl}^2}.$

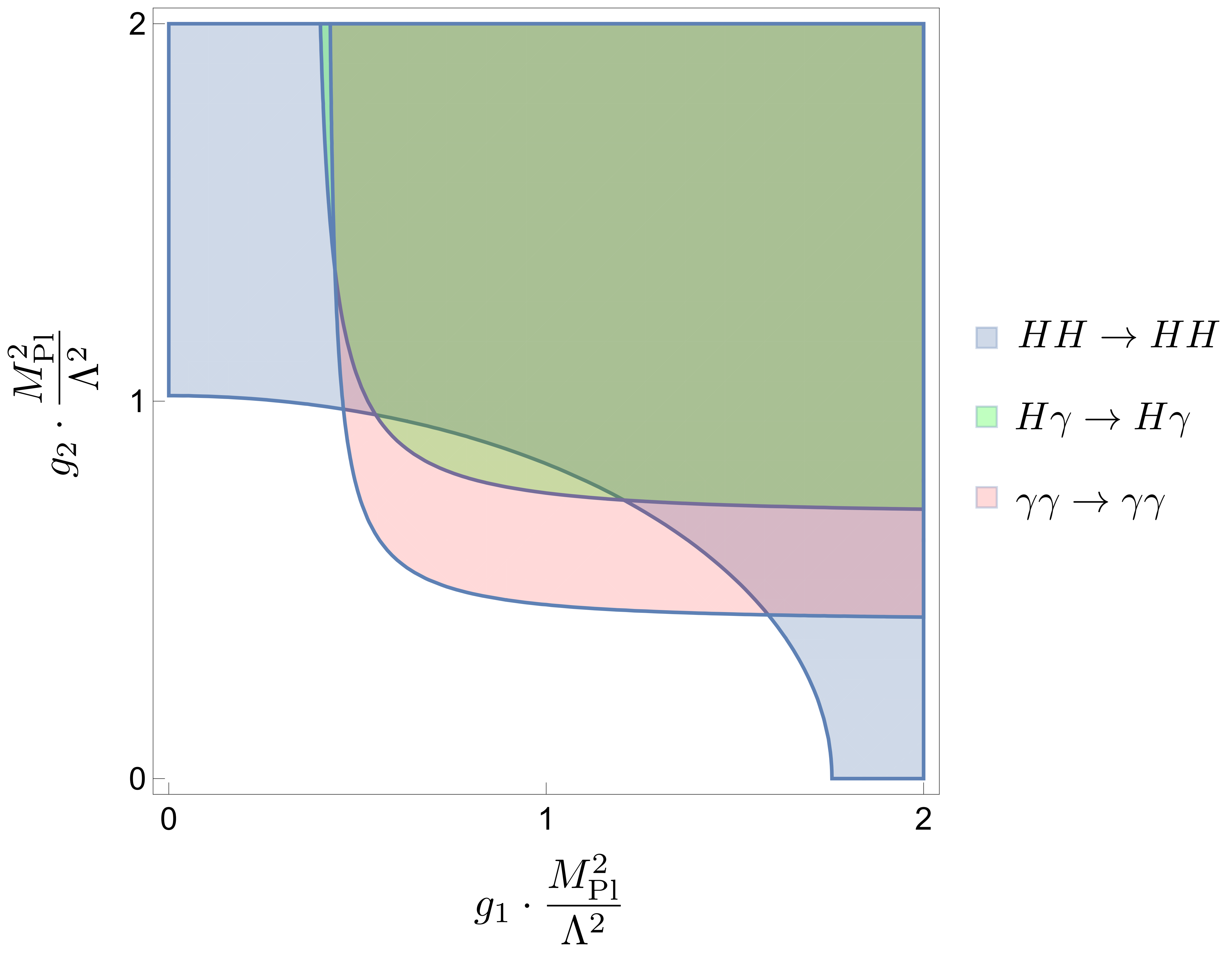
The constraint disallows both g_1 & g_2 couplings to be simultaneously small.

$H\gamma \to H\gamma$ $\quad \frac{125 - 8\sqrt{3}\pi}{576} \left(\frac{1}{g_1^2} + \frac{1}{g_2^2} \right) + \frac{7}{20} \frac{1}{g_2^2} < \frac{1}{\frac{\Lambda^2}{M_{\rm Pl}^2}}.$

$\gamma\gamma \to \gamma\gamma$ $\quad \frac{1}{g_1^2} + \frac{1}{g_2^2} < \frac{1}{\frac{11 \frac{g_2 v^2}{m_e^2} + 504}{2880} \frac{\Lambda^2}{M_{\rm Pl}^2}} \underset{m_e \gg 1}{\simeq} \frac{1}{\frac{7}{40} \frac{\Lambda^2}{M_{\rm Pl}^2}}.$

The constraints prevent either g_1 or g_2 coupling from being small on their own.

$m_W = v g_2/2, \quad m_Z = v\sqrt{g_1^2 + g_2^2}/2.$

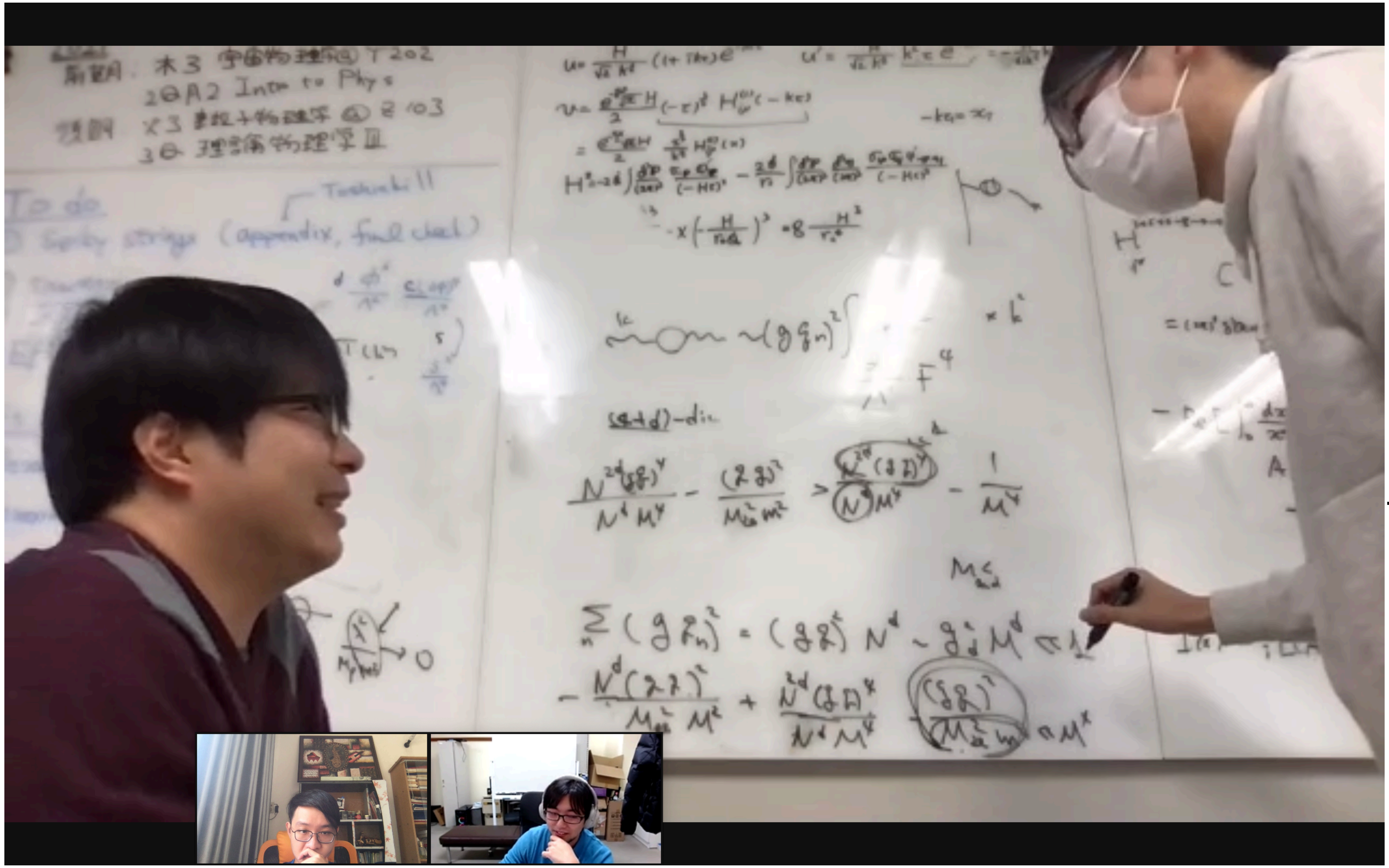


Conclusions & Outlooks

- **Conclusions:**
  - Akin to the magnetic WGC, the constraints suggest that when the coupling constants g_1, g_2 are small, the effective theory breaks down at a relatively low scale.
  - More insight to Swampland program (constraints & correlations between gauge couplings from the Electro-Weak bounds).
- **Outlooks:**
  - Conduct the analysis to determine implications from
    - massive graviton theories
    - stringy set-ups

THANK YOU FOR YOUR LISTENING!

Prof. Toshifumi Noumi (Tokyo)



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