Revisiting Metastable Cosmic String Breaking

Based on 2312.15662

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Collaboration with

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Outline

1. Metastable Cosmic String
    - Motivation.
    - Gravitational wave and NANOGrav result.

2. Breaking of Metastable String
    - Conventional approach via monopole pair production

3. Revisiting String Breaking
    - Explicit construction field configuration of breaking process.
    - Bounce solution. Numerical results.

4. Summary

Topological Object in BSM

Extension of Standard Model can have larger symmetry.

$$G_{\rm BSM} \to G_{\rm SM} = {\rm SU}(3)_C \times {\rm SU}(2)_L \times {\rm U}(1)_Y$$

Example:

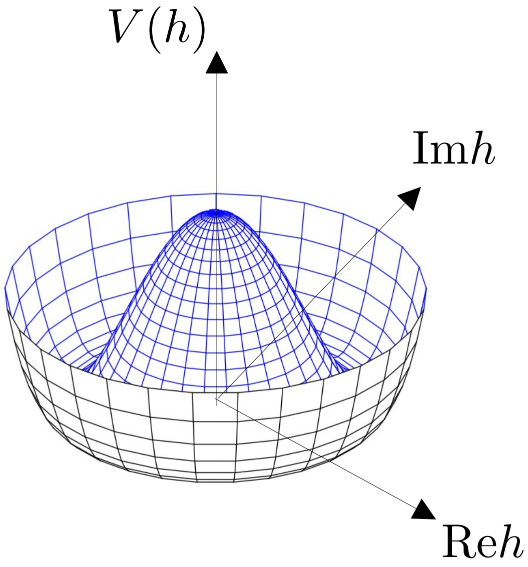
Grand Unified Theory, SU(5), SO(10)….: Monopole, cosmic string...

$U(1)_{PQ}$ extension for Axion: Domain wall, cosmic string.

Cosmic String

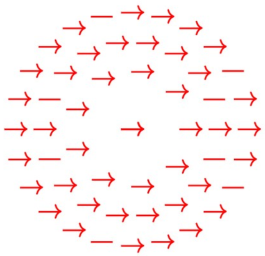
$$\mathrm{U}(1) \to \text{nothing}$$

$$\langle h \rangle \neq 0$$



SSB

Phase of h



Winding number　0

Cosmic String

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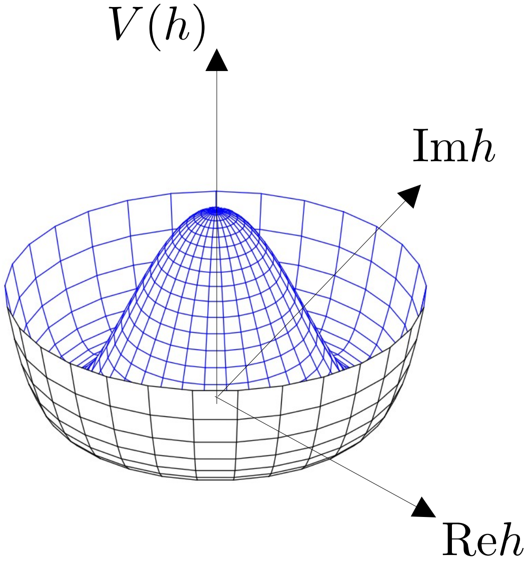

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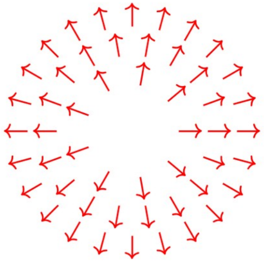

Winding number　1

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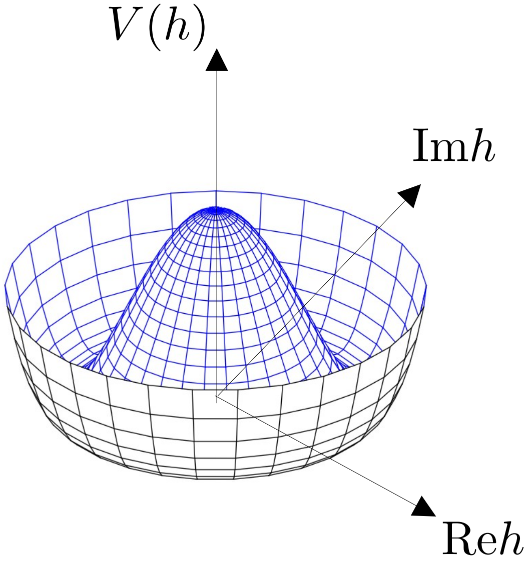
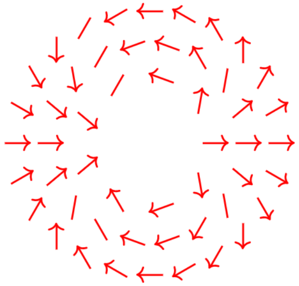


SSB

Phase of h

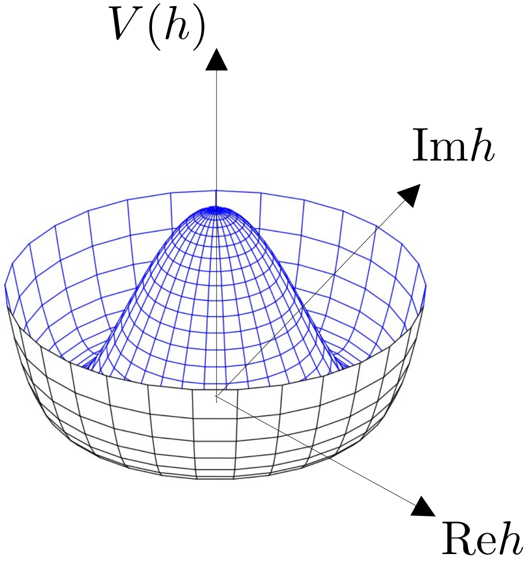


Winding number　2

Cosmic String

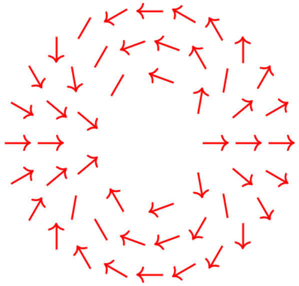
$$\mathrm{U}(1) \to \text{nothing}$$

$$\langle h \rangle \neq 0$$



SSB

Phase of h

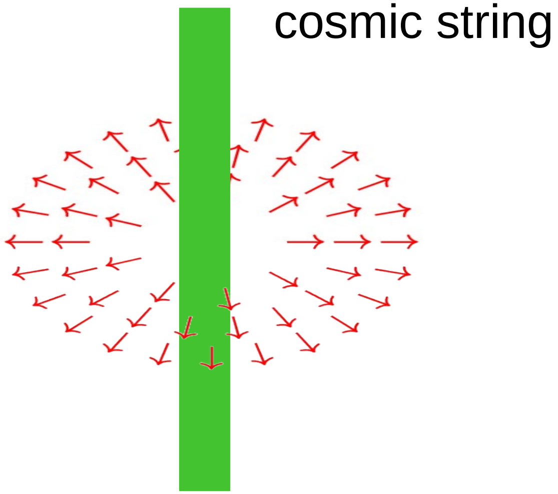


Winding number 2

$$\pi_1(\mathrm{U}(1)) = \mathbb{Z}$$

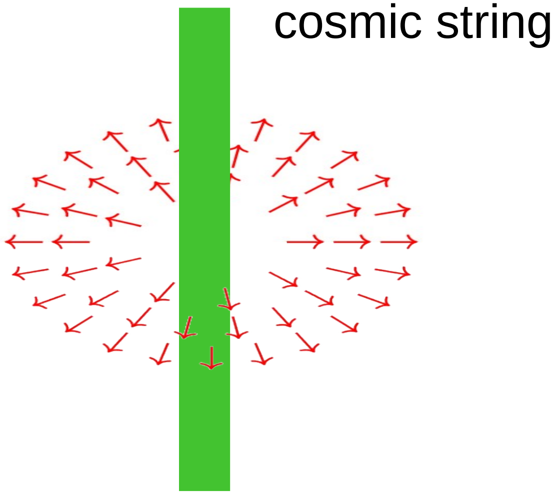
Cosmic String

$$\mathrm{U}(1) \to \text{nothing}$$



Cosmic String

$$\mathrm{U}(1)_{\mathrm{gauge}} \to \mathrm{nothing}$$



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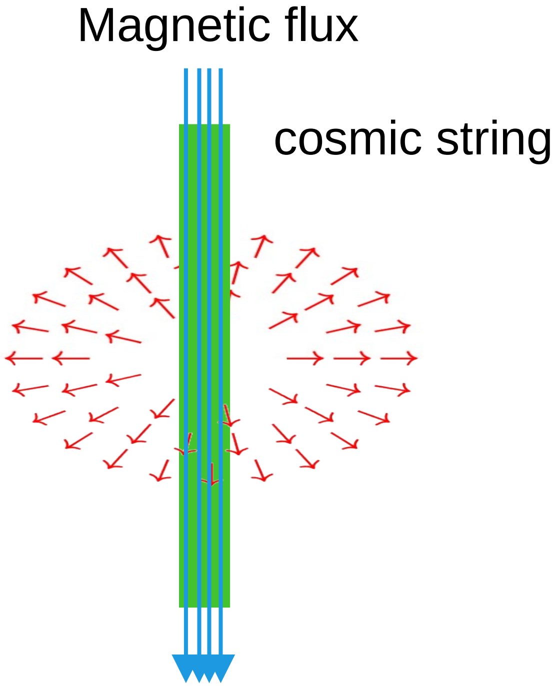

Multi-step SSB

$$\mathrm{SU}(2) \to \mathrm{nothing}$$

$$\pi_2(\mathrm{SU}(2)) = \pi_1(\mathrm{SU}(2)) = 0$$

No topological objects.

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No topological objects.

$$\mathrm{SU}(2) \underset{V}{\to} \mathrm{U}(1) \underset{v}{\to} \ \text{nothing}$$

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No topological objects.

$$\overbrace{\underbrace{\mathrm{SU}(2) \underset{V}{\rightarrow}}_{\text{Monopole}} \; \underbrace{\mathrm{U}(1) \underset{v}{\rightarrow}}_{\text{Cosmic string}}} \; \text{nothing}$$

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↓

Metastable string with $V \gg v$

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No topological objects.

$$\mathrm{SU}(2) \overbrace{\underbrace{\xrightarrow[V]{}}_{\text{Monopole}} \mathrm{U}(1) \underbrace{\xrightarrow[v]{}}_{\text{Cosmic string}}} \text{nothing}$$

Metastable string with $V \gg v$

Example:

$$\mathrm{SO}(10)_{\mathrm{GUT}} \to G_{\mathrm{SM}} \times \mathrm{U}(1)_{B-L} \to G_{\mathrm{SM}}$$

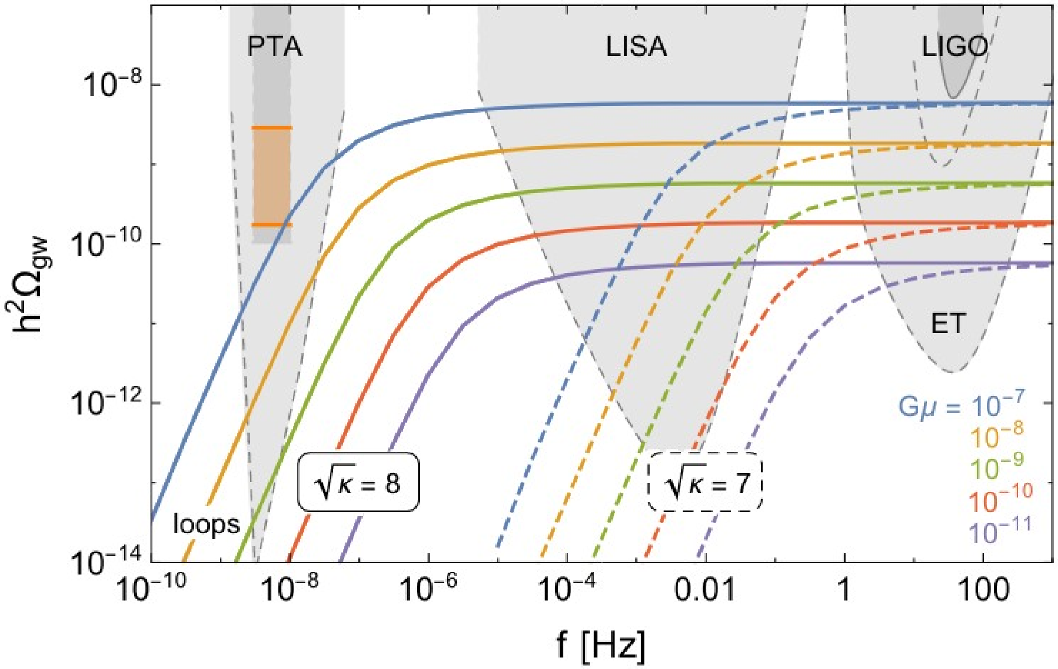
Gravitational Wave

String breaking rate per unit length

$$\Gamma = \frac{\mu}{2\pi} e^{-\pi\kappa}$$

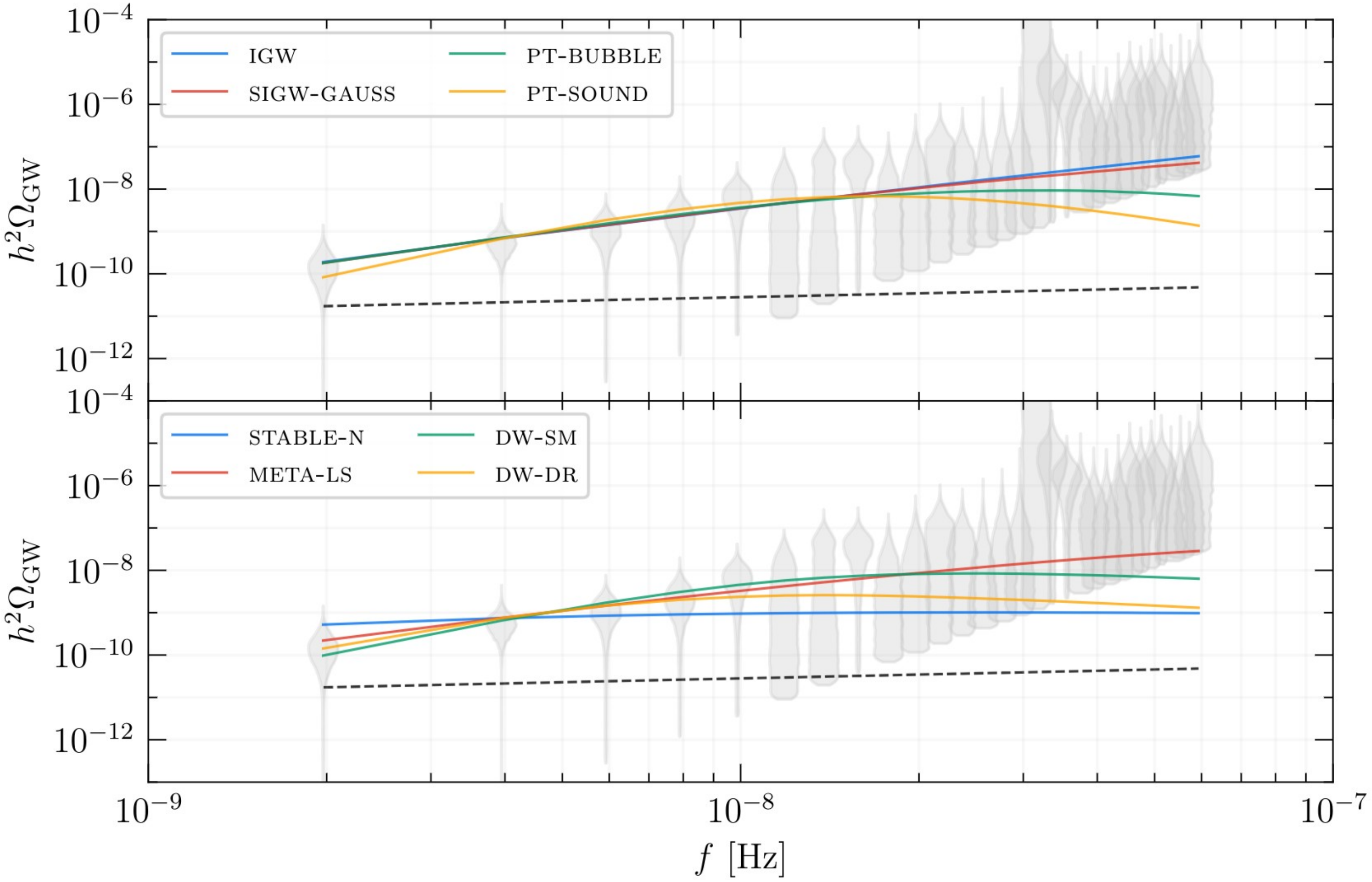
μ: String tension

[Buchmüller, Domcke, Schmitz 2107.04578]



Hint of Metastable String?

[NANOGrav, 2306.16219]



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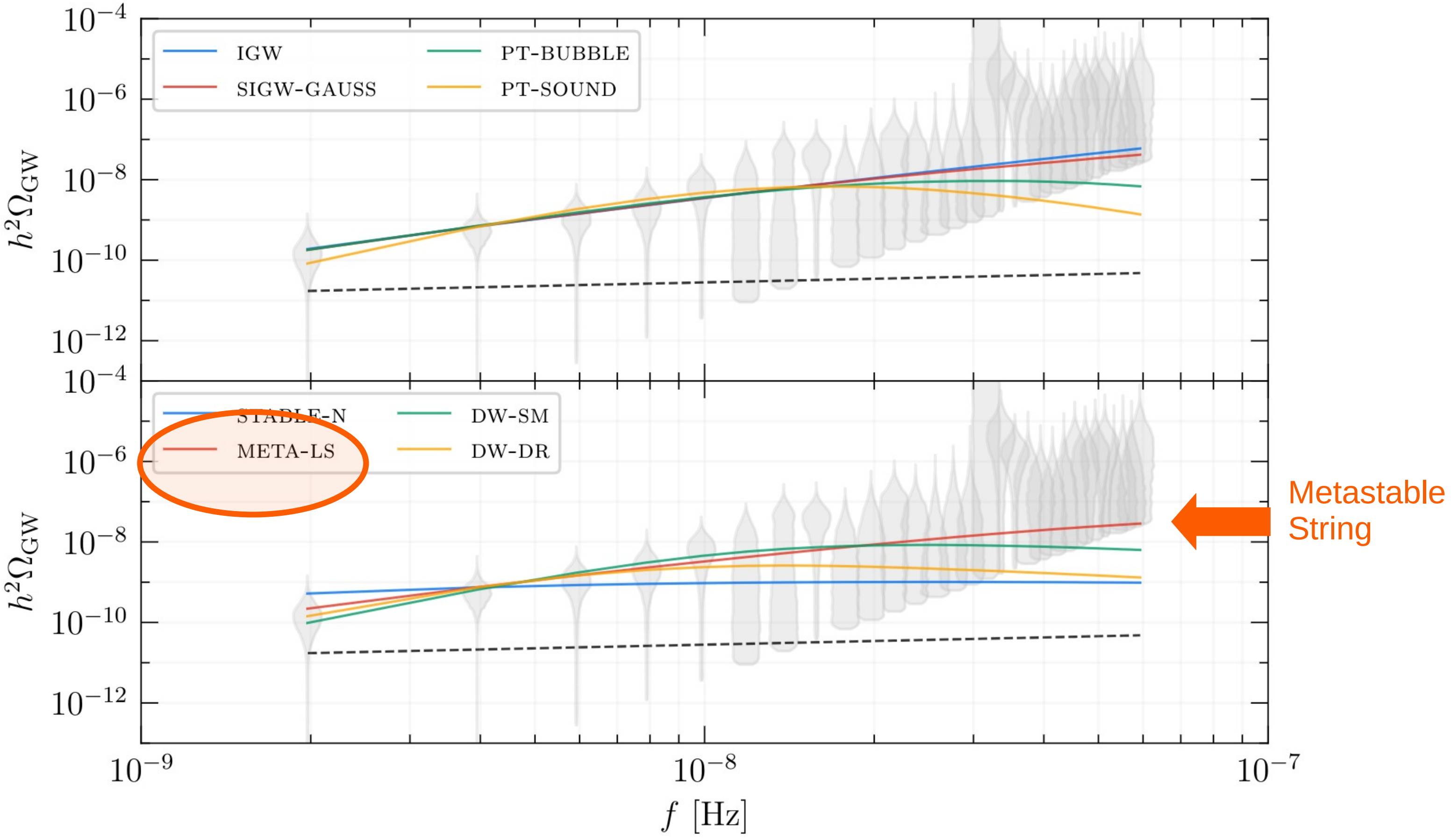

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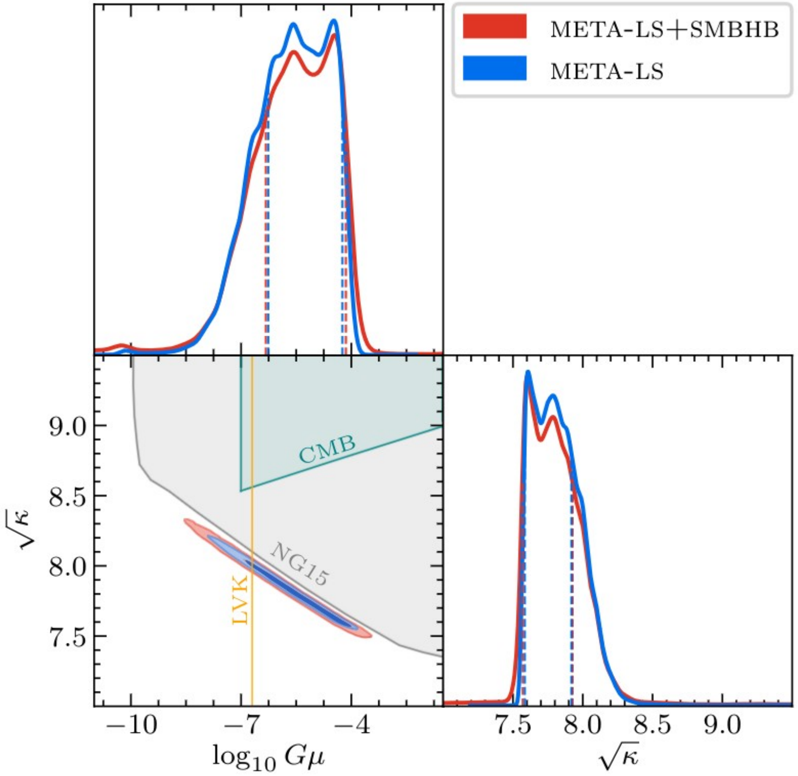

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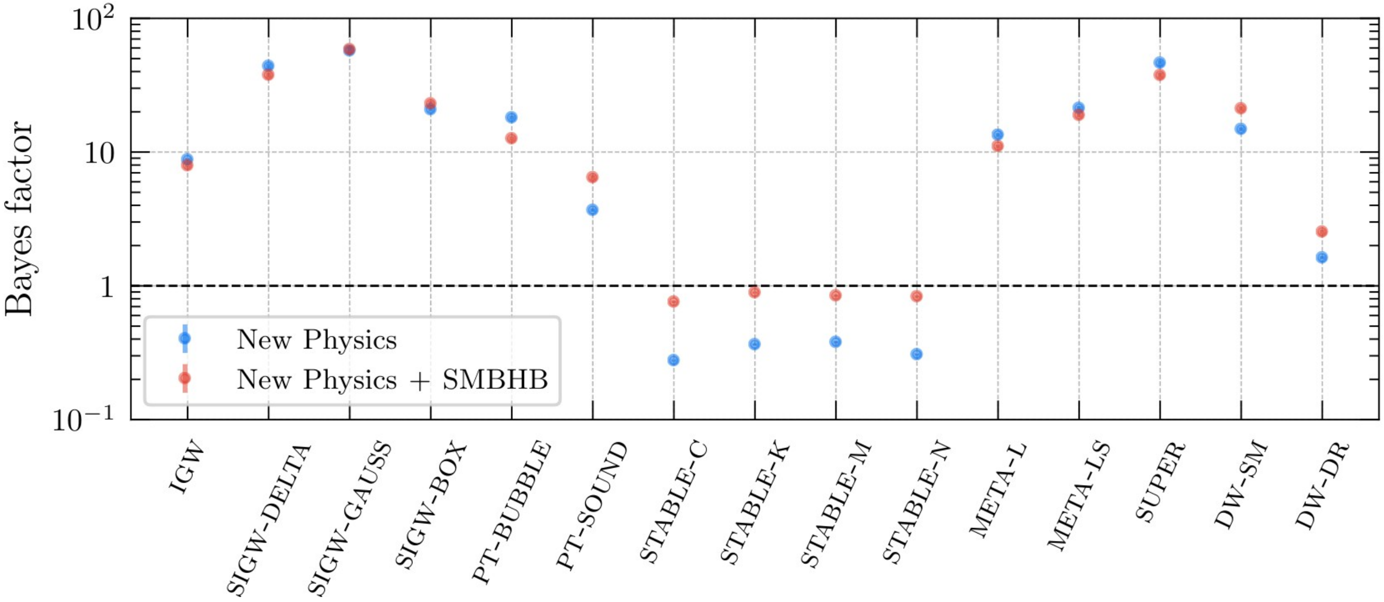

Bayes factor ~ Fitness of model for data.

Hint of Metastable String?

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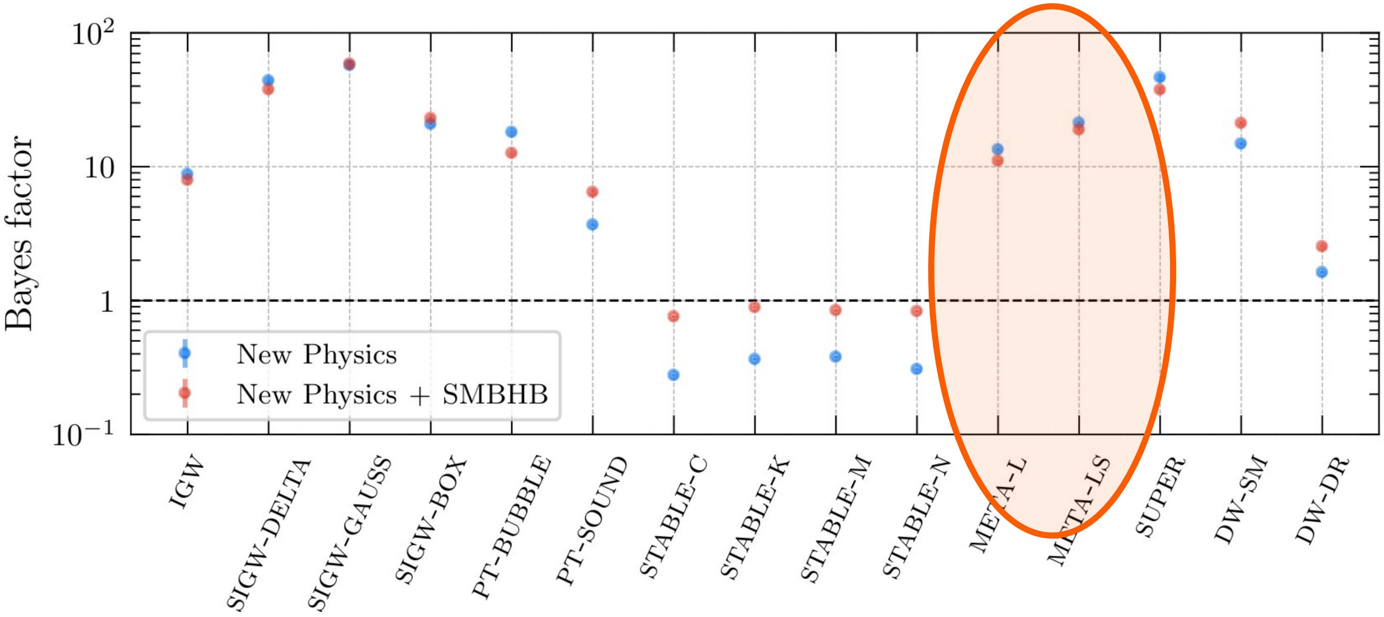
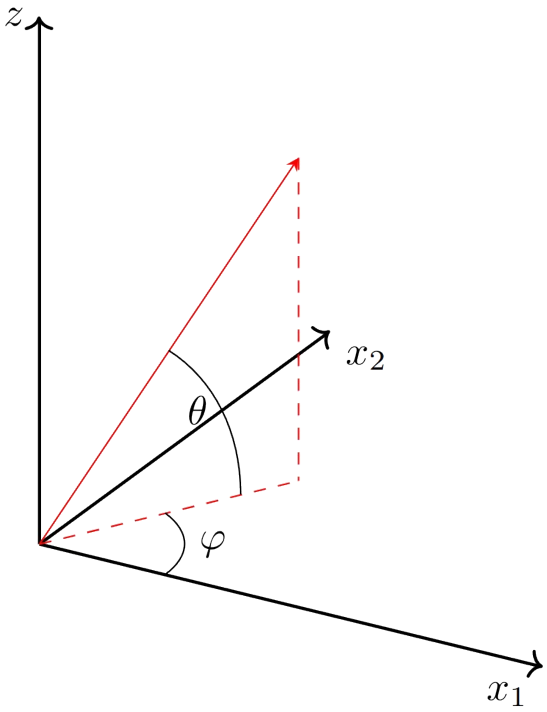


Bayes factor ~ Fitness of model for data.

Breaking of Metastable String

Coordinate

String direction



Explicit Model: SU(2) Theory

Symmetry breaking pattern:

$$\mathrm{SU}(2) \underset{V}{\rightarrow} \mathrm{U}(1) \underset{v}{\rightarrow} \ \text{nothing}$$

Higgs fields:

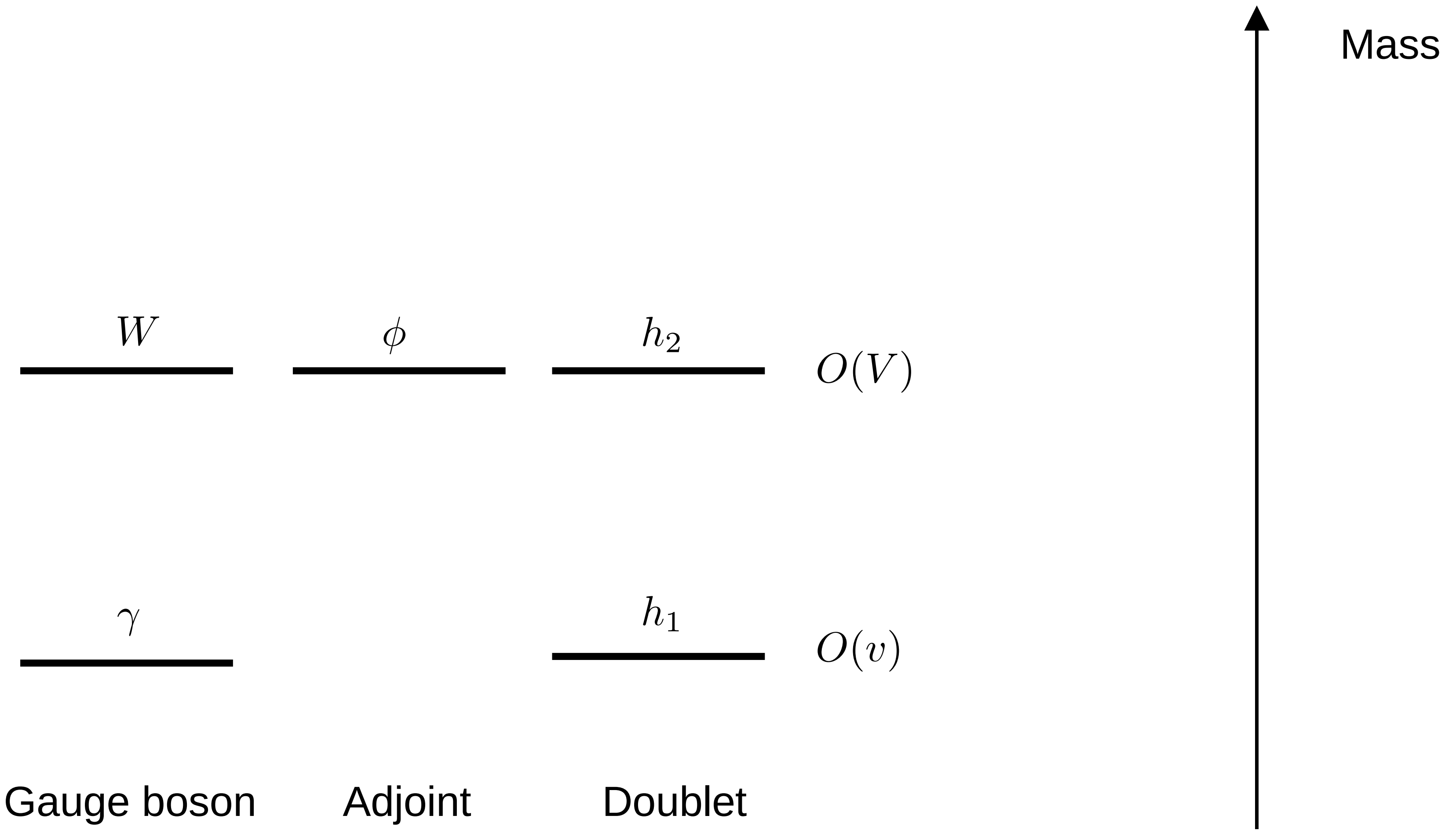
ϕ	Triplet	$\langle \phi^a \rangle = V \delta^{a3}$
h	Doublet	$\langle h \rangle = \begin{pmatrix} v \\ 0 \end{pmatrix}$

Lagrangian:

$$\mathcal{L} = -\frac{1}{2}(D_\mu \phi^a)(D^\mu \phi^a) - (D_\mu h)^\dagger (D^\mu h) - \frac{1}{4g^2} F^a_{\mu\nu} F^{a\mu\nu} \ - V_{\mathrm{Higgs}}(\phi, h)$$

$$V_{\mathrm{Higgs}}(\phi, h) := \lambda\big(|h|^2 - v^2\big)^2 + \tilde{\lambda}\big(\phi^a\phi^a - V^2\big)^2 + \gamma \left| \left(\phi - \frac{V}{2}\right) h \right|^2$$

Mass Spectrum



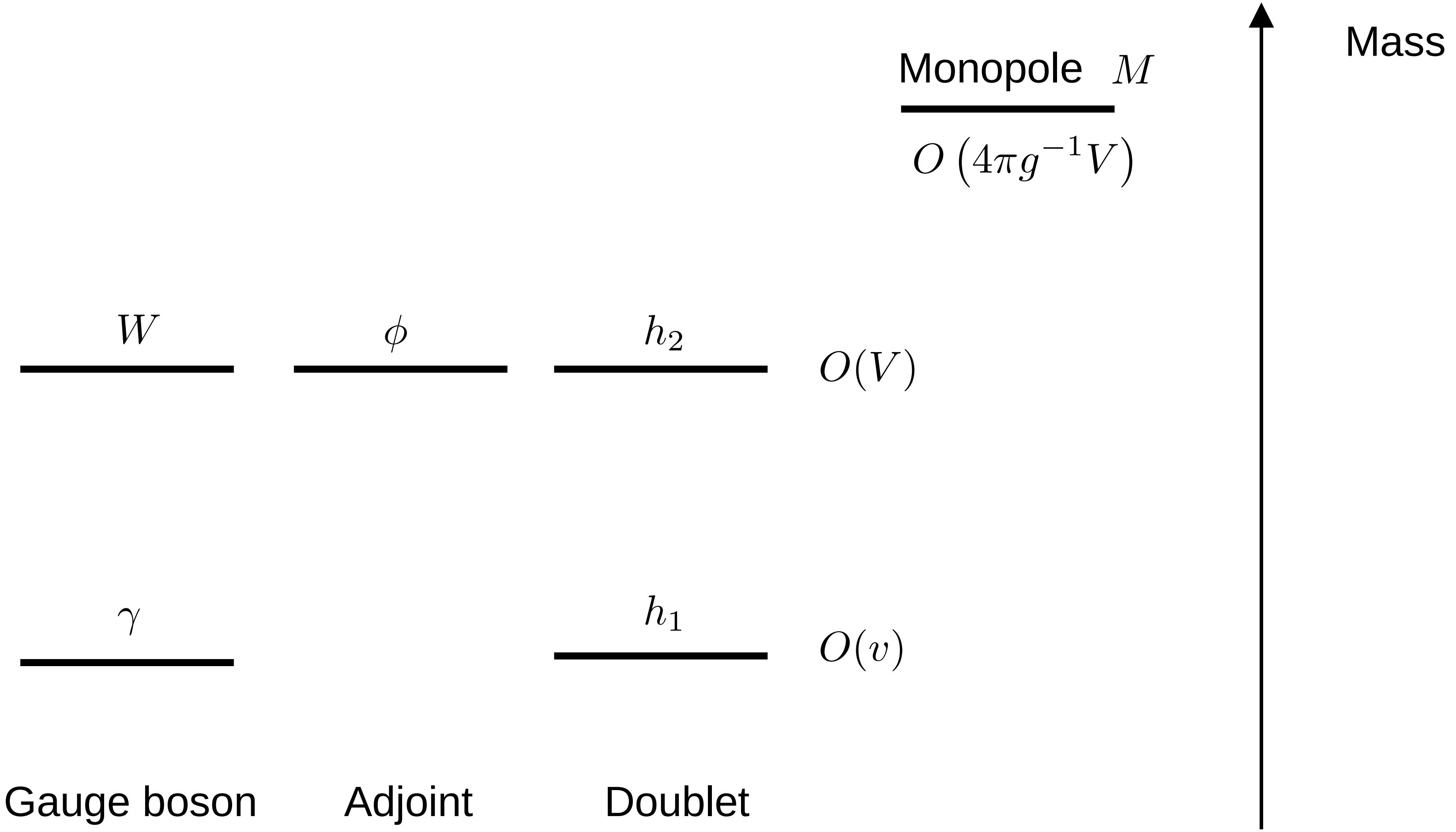
Mass

W ϕ h_2 $O(V)$

γ h_1 $O(v)$

Gauge boson Adjoint Doublet

Mass Spectrum



Mass

Monopole M

$O\left(4\pi g^{-1}V\right)$

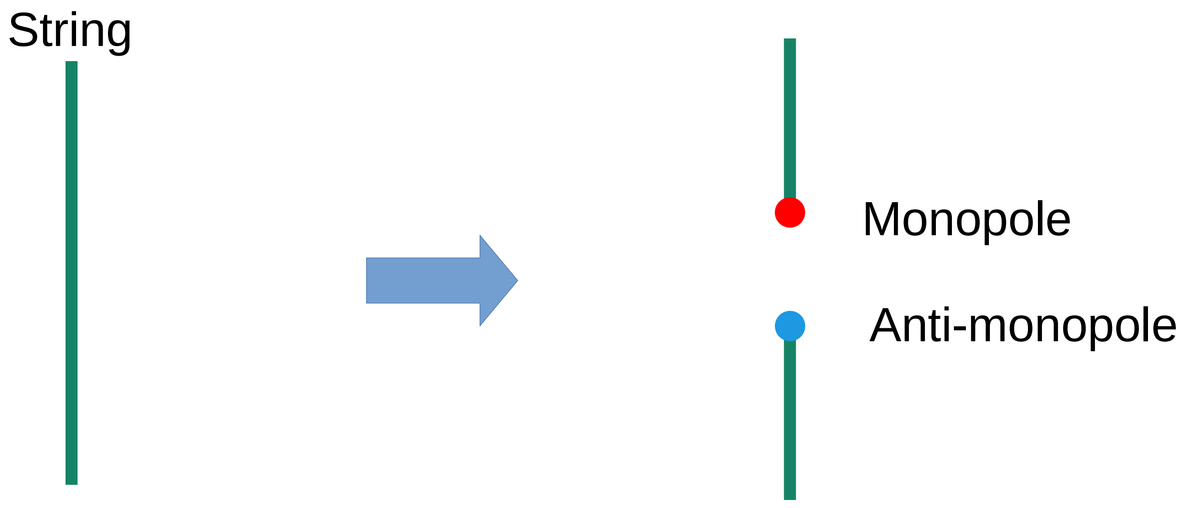
W ϕ h_2 $O(V)$

γ h_1 $O(v)$

Gauge boson Adjoint Doublet

String Breaking via Monopole

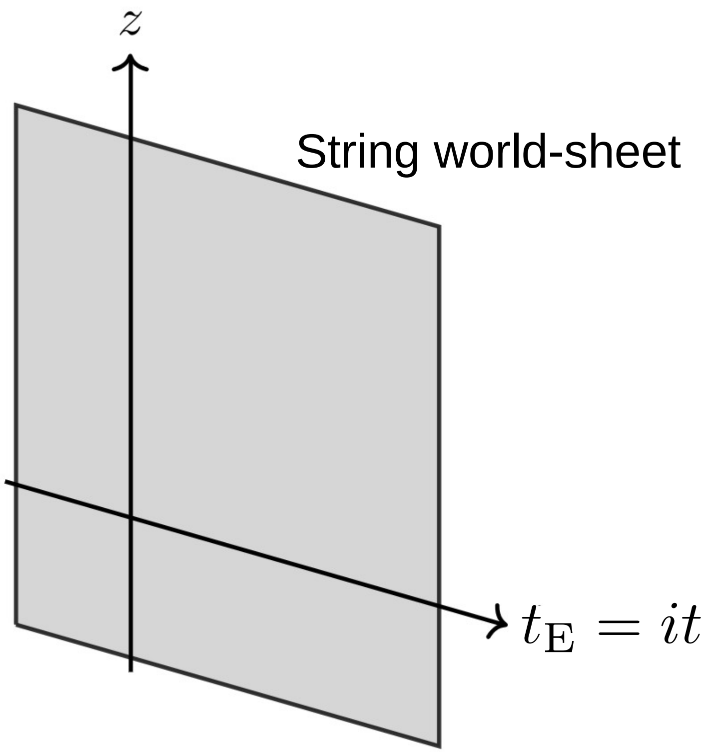
[Vilenkin, NPB 196(1982) 240]
[Preskill & Vilenkin, hep-ph/9209210]



Monopole pair production like Schwinger effect.

Conventional Approach

$V \gg v$ Infinitely thin string



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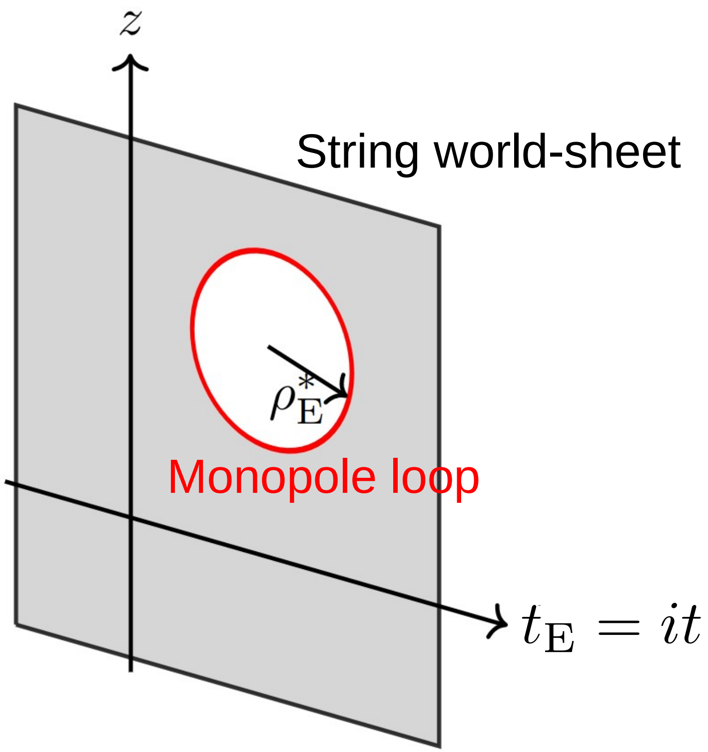

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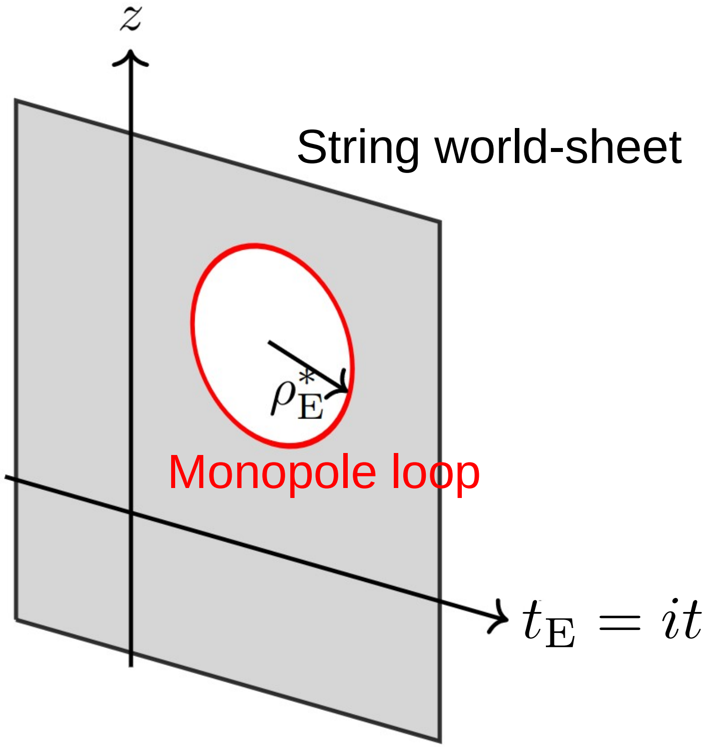

$$S_B = m_M \int_{\text{worldline}} dx - \mu \int_{\text{hole}} d^2 S$$

$$= 2\pi \rho_{\mathrm{E}}^* m_M - \pi {\rho_{\mathrm{E}}^*}^2 \mu$$

$$\Rightarrow \quad \rho_{\mathrm{E}}^* = \frac{m_M}{\mu}$$

maximization

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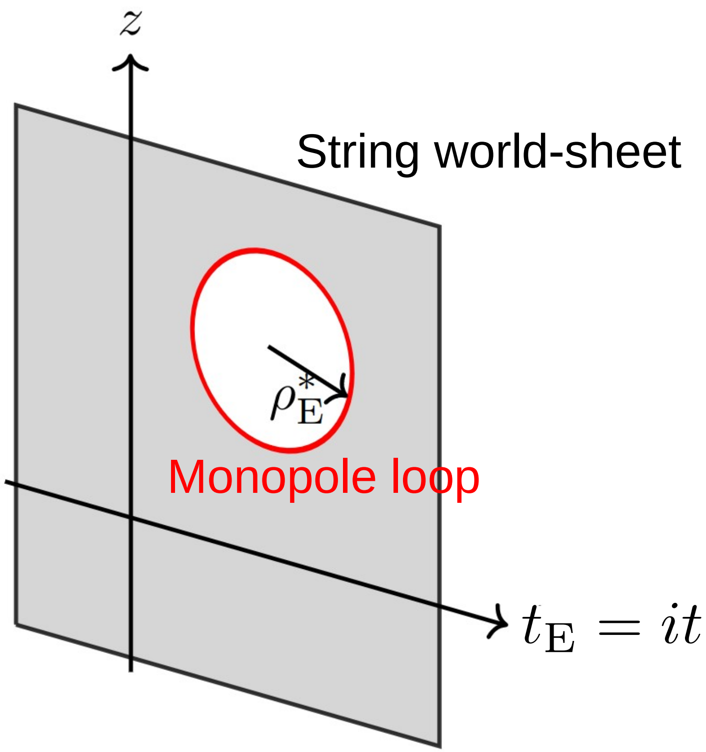

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$$= 2\pi \rho_{\rm E}^* m_M - \pi {\rho_{\rm E}^*}^2 \mu$$

➡ $$\rho_{\rm E}^* = \frac{m_M}{\mu}$$

maximization

$$S_B^{(\rm thin)} = \frac{\pi m_M^2}{\mu} =: \pi\kappa$$

$$\Gamma \propto e^{-S_B^{(\rm thin)}}$$

Nanograv Implication

NANOGrav

$$\sqrt{\kappa} = \frac{m_M}{\sqrt{\mu}} = O\left(\frac{4\pi V}{\sqrt{2\pi}gv}\right) \simeq 8 \qquad \Rightarrow \qquad V/v = O(1)$$

$$\mu \sim 2\pi v^2$$

$$M_M \sim 4\pi g^{-1} V$$

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No clear separation of SU(2) and U(1) breaking.

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Thin-string approximation is valid?

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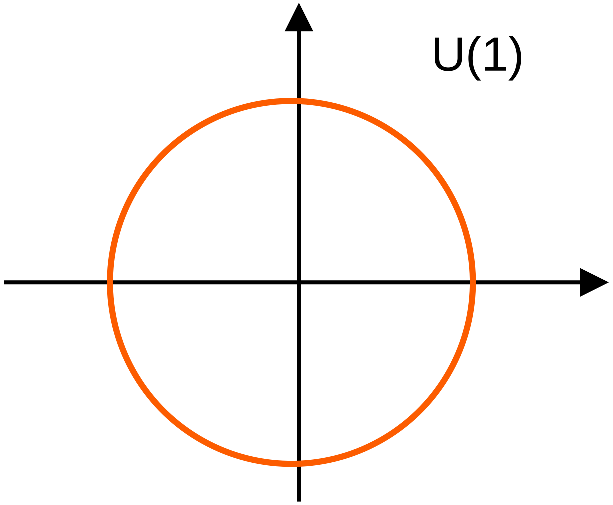
No clear separation of SU(2) and U(1) breaking.

Thin-string approximation is valid?

Any other path to break the string?

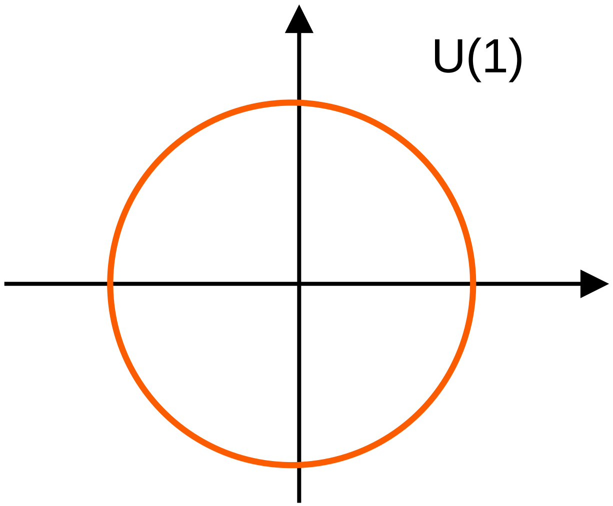
Revisiting String Breaking

U(1) in SU(2)



$$\pi_1(\mathrm{U}(1)) \neq 0$$

U(1) in SU(2)



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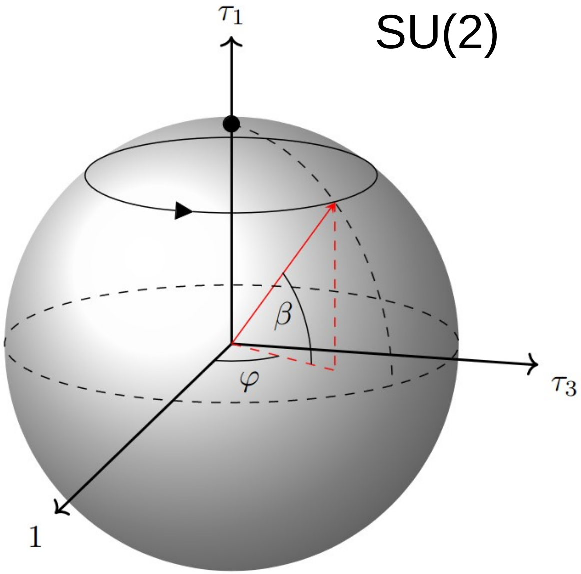
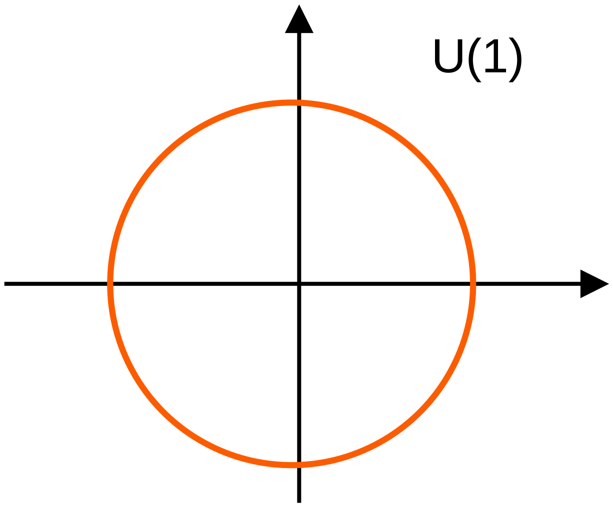


$\pi_1(\mathrm{SU}(2)) = 0$

β : Unwinding parameter.

β → π/2, no winding.

U(1) in SU(2)



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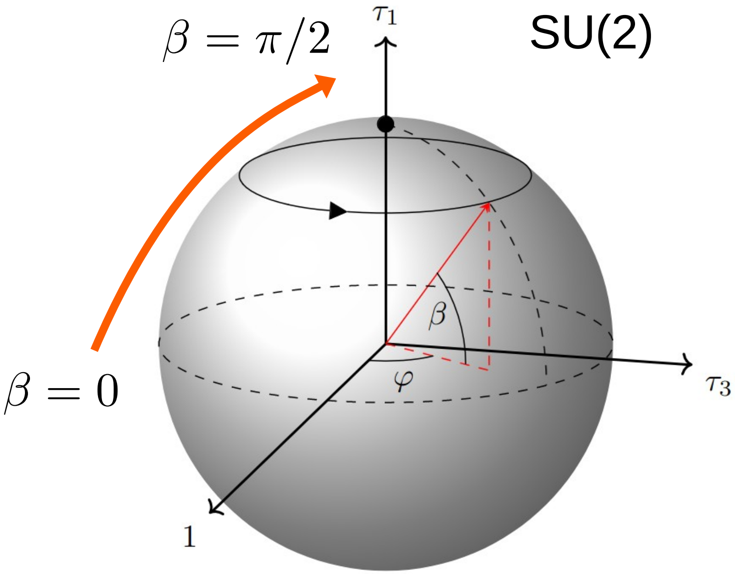

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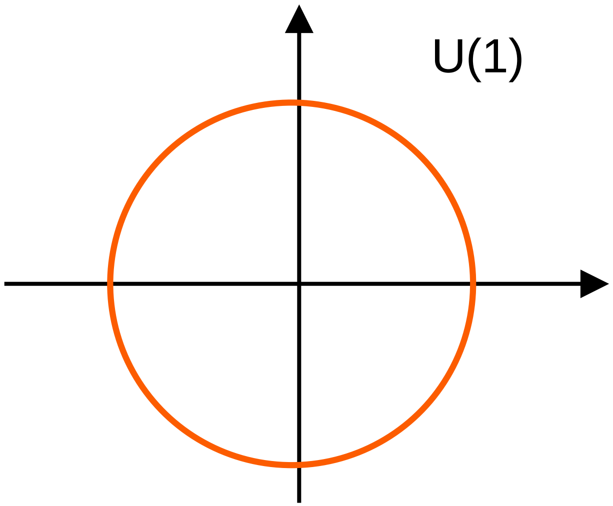

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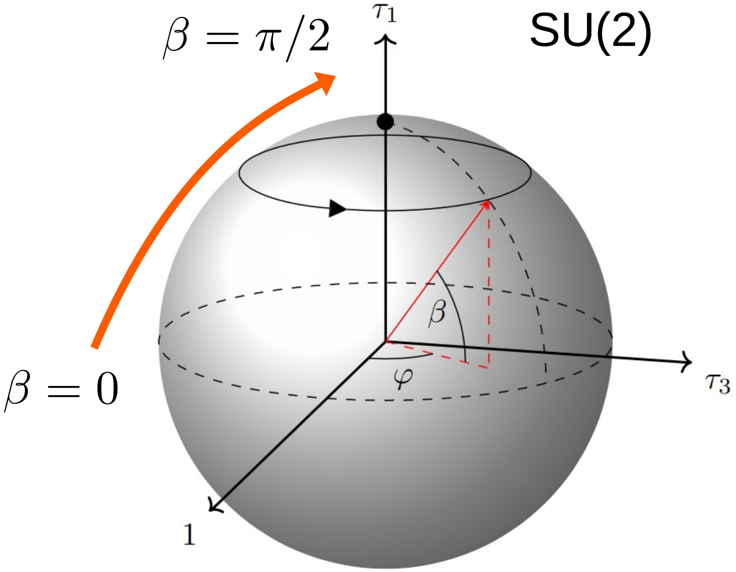

$$\pi_1(\mathrm{SU}(2)) = 0$$

β : Unwinding parameter.

β → π/2, no winding.

Using β, explicit field configuration of string breaking is possible.

Primitive Ansatz

[Shifman & Yung hep-th/0205025]

τ : Pauli matrices

$\rho = \sqrt{x_1^2 + x_2^2}$

$$\boxed{U = e^{-i\varphi\tau^3}\cos\beta + i\tau^1\sin\beta}$$

$$h(x) = U\begin{pmatrix}\xi_\beta(\rho)\\ 0\end{pmatrix}$$

$$A_\rho(x) = 0$$

$$A_\varphi(x) = iU(\partial_\varphi U^\dagger)(1 - f_\beta(\rho))$$

$$\phi(x) = VU\frac{\tau^3}{2}U^\dagger + \Delta\phi$$

$$\Delta\phi = \Phi_\beta(\rho)\left(\frac{\tau^1}{2}\sin\varphi - \frac{\tau^2}{2}\cos\varphi\right)$$

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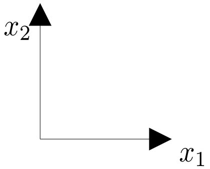
These fields yet to be determined.

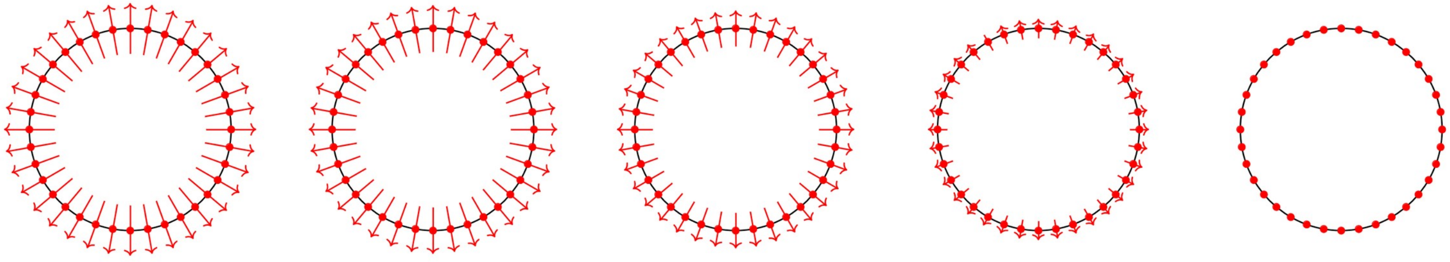
$$\phi(x) = VU\frac{\tau^3}{2}U^\dagger + \Delta\phi$$

$$\Delta\phi = \Phi_\beta(\rho)\left(\frac{\tau^1}{2}\sin\varphi - \frac{\tau^2}{2}\cos\varphi\right)$$

Field Configuration

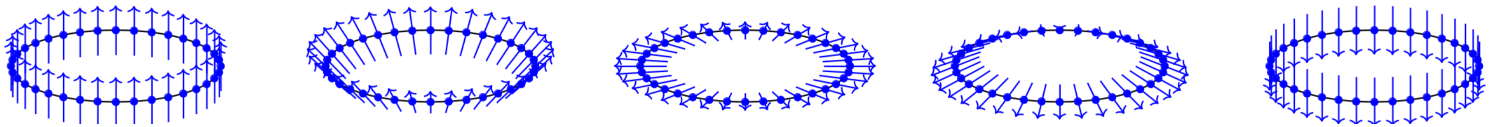
φ dependent phase of doublet h





$\beta = 0$ $\beta = \pi/8$ $\beta = \pi/4$ $\beta = 3\pi/4$ $\beta = \pi/2$

Direction of Adjoint field Φ^a



Procedure

- Promote β as collective coordinate β(t,z).

- Get effective action.

$$S_{\mathrm{Eucl}} = 2\pi \int \rho_{\mathrm{E}} d\rho_{\mathrm{E}} \left(\frac{1}{2} K_{\mathrm{eff}}(\beta)(\partial_{\rho_{\mathrm{E}}}\beta(\rho_{\mathrm{E}}))^2 + T(\beta) \right)$$

$$\rho_{\mathrm{E}} = \sqrt{t_{\mathrm{E}}^2 + z^2}$$

- Get tunneling rate with bounce solution.

Primitive Ansatz

$$h(x) = U \begin{pmatrix} \xi_\beta(\rho) \\ 0 \end{pmatrix}$$

$$A_\rho(x) = 0$$

$$A_\varphi(x) = iU(\partial_\varphi U^\dagger)(1 - f_\beta(\rho))$$

$$\phi(x) = VU\frac{\tau^3}{2}U^\dagger + \Delta\phi$$

$$\Delta\phi = \Phi_\beta(\rho)\left(\frac{\tau^1}{2}\sin\varphi - \frac{\tau^2}{2}\cos\varphi\right)$$

$$A_{t/z}(x) = -2(\partial_{t/z}\beta(t,z))\left(\frac{\tau^1}{2}\cos\varphi - \frac{\tau^2}{2}\sin\varphi\right)a_\beta(\rho)$$

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Substitute to original Lagrangian

(kinetic-term) + V_{Higgs}

$$(\partial_{\rho_{\mathrm{E}}}\beta)^2 K_{\mathrm{eff}}[\xi_\beta, f_\beta, \Phi_\beta, a_\beta] + T[\xi_\beta, f_\beta, \Phi_\beta] \qquad \rho_{\mathrm{E}} = \sqrt{t_{\mathrm{E}}^2 + z^2}$$

String Tension

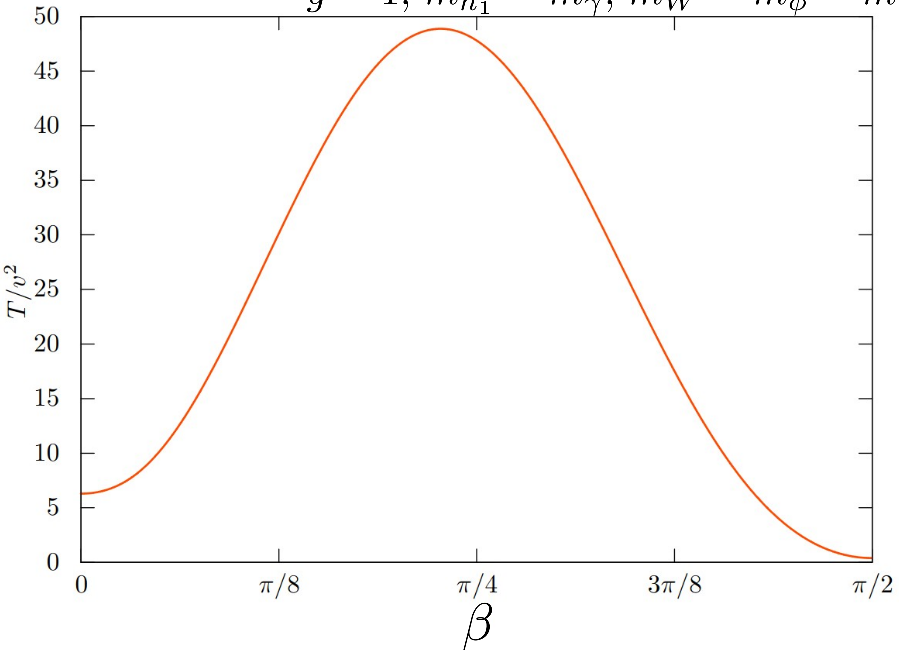
Determine f, Φ, ξ to minimize string tension for each β.

$$\rho = \sqrt{x_1^2 + x_2^2}$$

$$T(\beta) = 2\pi \int_0^\infty \rho\,\mathrm{d}\rho \left\{ \frac{2}{g^2}\frac{(\partial_\rho f_\beta)^2}{\rho^2}\cos^2\beta + (\partial_\rho \xi_\beta)^2 + \frac{f_\beta^2}{\rho^2}\xi_\beta^2\cos^2\beta \right.$$
$$+ \frac{1}{2}(\partial_\rho \Phi_\beta)^2 + \frac{1}{2\rho^2}\big[\Phi_\beta(\cos 2\beta - 2 f_\beta \cos^2\beta) + V f_\beta \sin 2\beta\big]^2$$
$$\left. + \lambda(\xi_\beta^2 - v^2) + \tilde{\lambda}\big[\Phi_\beta^2 - 2V\Phi_\beta \sin 2\beta\big]^2 + \frac{\gamma}{4}\Phi_\beta^2\xi_\beta^2 \right\} .$$

String tension

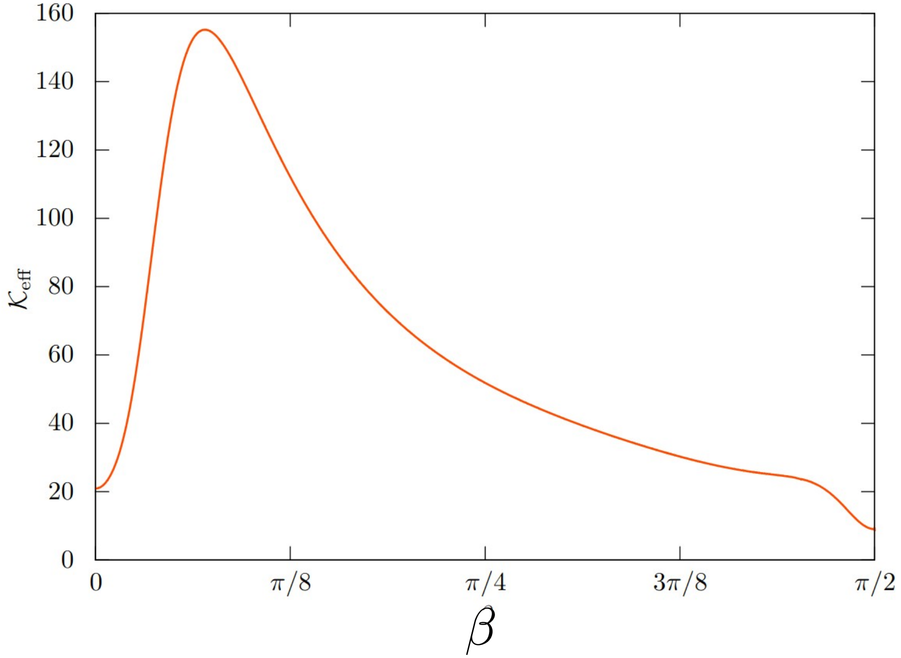
$g = 1,\, m_{h_1} = m_\gamma,\, m_W = m_\phi = m_{h_2} = 5m_\gamma$



Kinetic Term

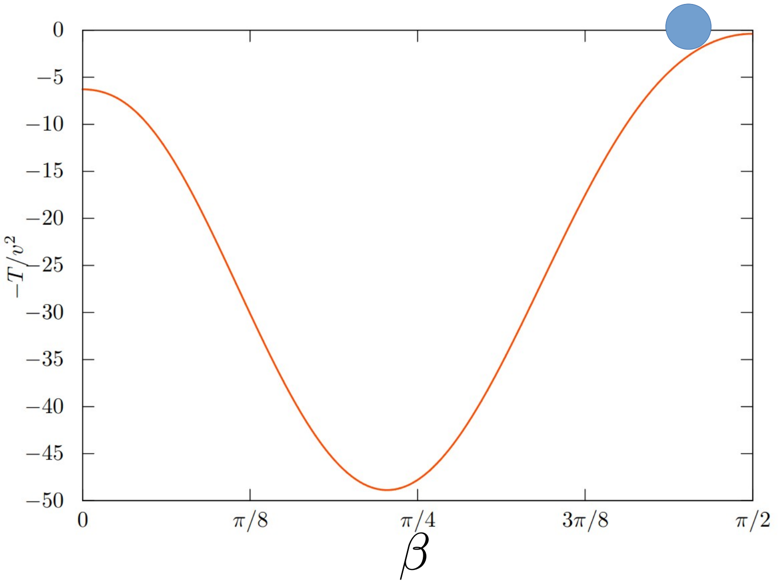
Determine a_β to minimize kinetic term for each β.

$$\begin{aligned} 8\pi\rho\Bigg[\frac{1}{g^2}(\partial_\rho a_\beta)^2 \; -a_\beta\Big\{ & -2\Phi_\beta(V\sin 2\beta-\Phi_\beta)+\frac{1}{2}V(\partial_\beta\Phi_\beta)\cos 2\beta \\ & +\frac{1}{g^2\rho^2}\left(f_\beta^2-\frac{1}{2}(\partial_\beta f_\beta)\sin 2\beta - f_\beta(1-f_\beta)\cos 2\beta\right)\Big\} \\ +a_\beta^2\Big\{\frac{1}{g^2\rho^2}(1-4f_\beta(1-f_\beta)\cos^2\beta)+V^2+\frac{1}{2}\xi_\beta^2+\Phi_\beta(-2V\sin 2\beta+\Phi_\beta)\Big\}\Bigg] & \quad \ldots \end{aligned}$$



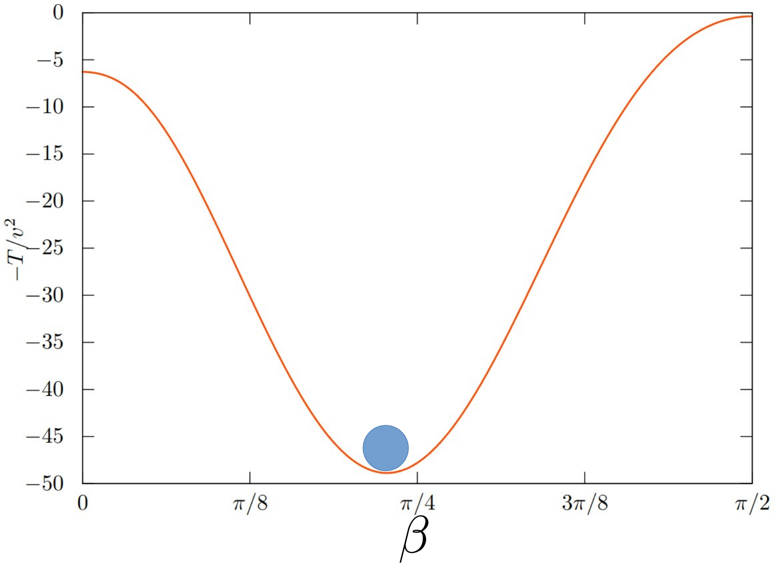
Effective Theory of β(ρE)

$$S_{\mathrm{Eucl}} = 2\pi \int \rho_{\mathrm{E}}\, \mathrm{d}\rho_{\mathrm{E}} \left[\frac{1}{2} \mathcal{K}_{\mathrm{eff}}(\beta(\rho_{\mathrm{E}}))(\partial_{\rho_{\mathrm{E}}}\beta(\rho_{\mathrm{E}}))^2 + T(\beta(\rho_{\mathrm{E}})) \right]$$



Effective Theory of β(ρ_E)

$$S_{\mathrm{Eucl}} = 2\pi \int \rho_{\mathrm{E}}\, \mathrm{d}\rho_{\mathrm{E}} \left[\frac{1}{2} \mathcal{K}_{\mathrm{eff}}(\beta(\rho_{\mathrm{E}}))(\partial_{\rho_{\mathrm{E}}}\beta(\rho_{\mathrm{E}}))^2 + T(\beta(\rho_{\mathrm{E}})) \right]$$



Effective Theory of β(ρE)

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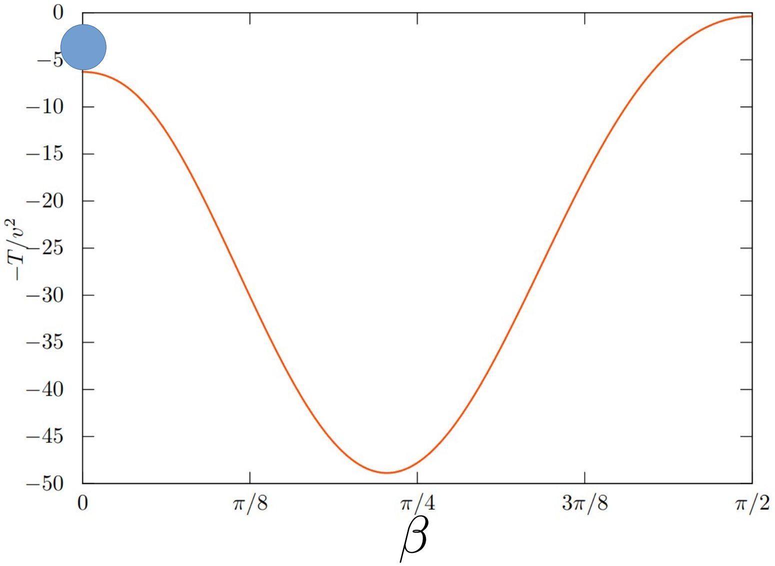
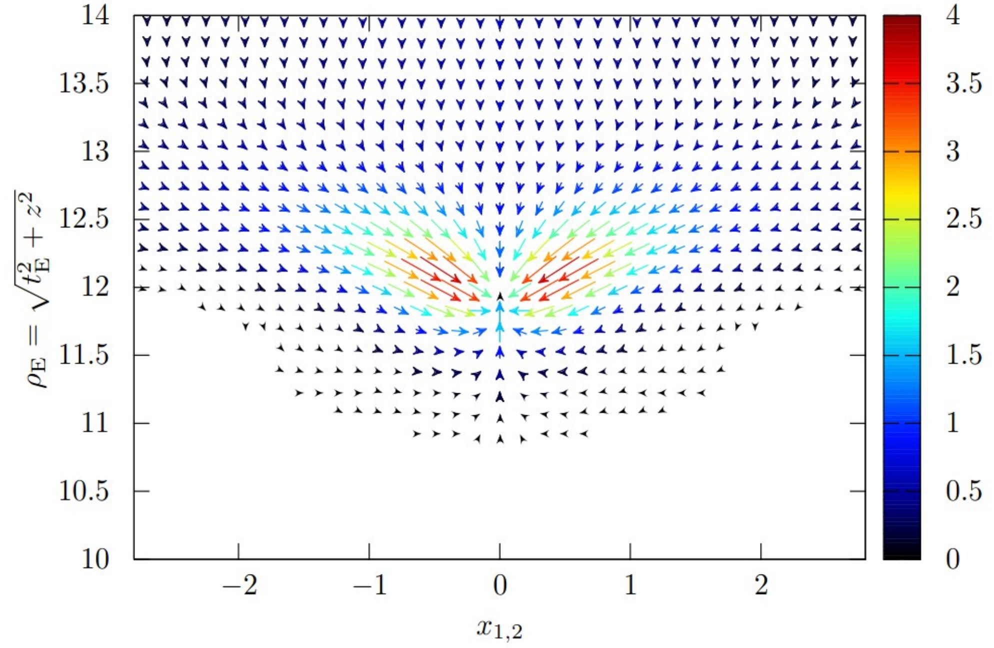


Configuration during Tunneling

β = 0: String state

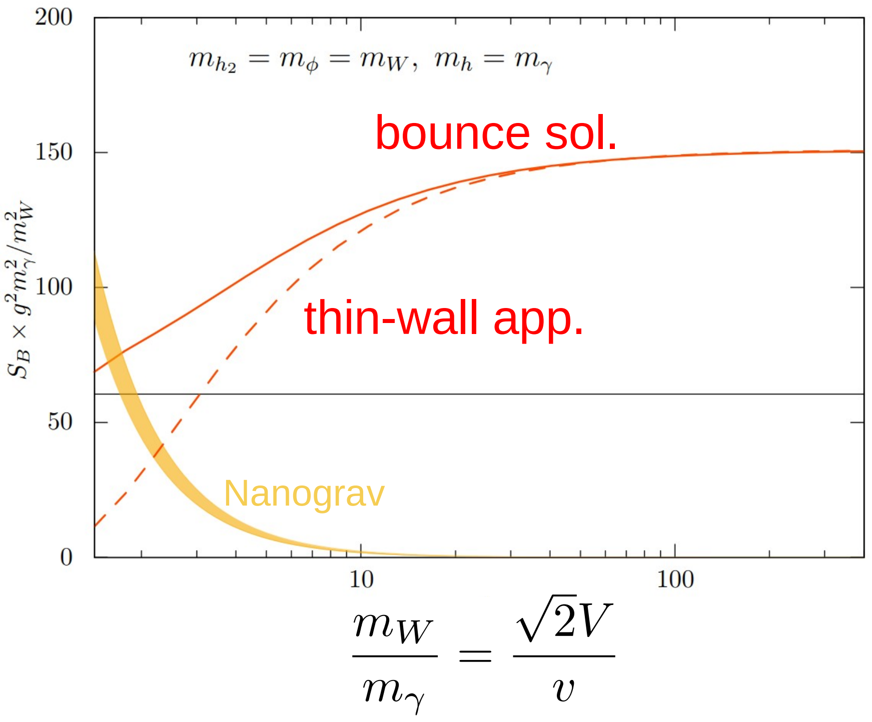
Magnetic field



β = π/2: “True” vacua

Bounce Action

Normalized S_B



Conventional estimation

$$\frac{\pi m_M^2}{\mu}$$

- The bounce solution results in larger SB.
- Thin-wall approximation fails when $V/v \lesssim 10.$

Improved Ansatz

[Shifman & Yung hep-th/0205025]

Primitive Ansatz

$$h(x) = U \begin{pmatrix} \xi_\beta(\rho) \\ 0 \end{pmatrix}$$

$$A_\rho(x) = 0$$

$$A_\varphi(x) = iU(\partial_\varphi U^\dagger)(1 - f_\beta(\rho))$$

$$\phi(x) = VU\frac{\tau^3}{2}U^\dagger + \Delta\phi$$

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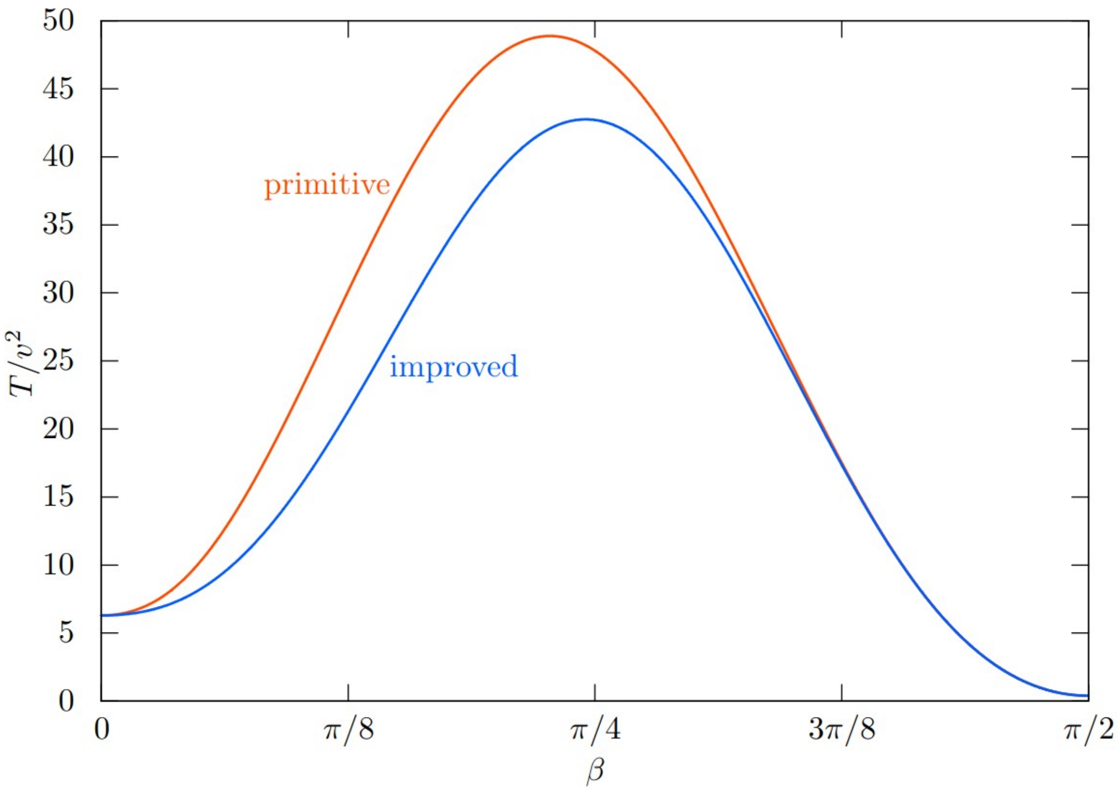
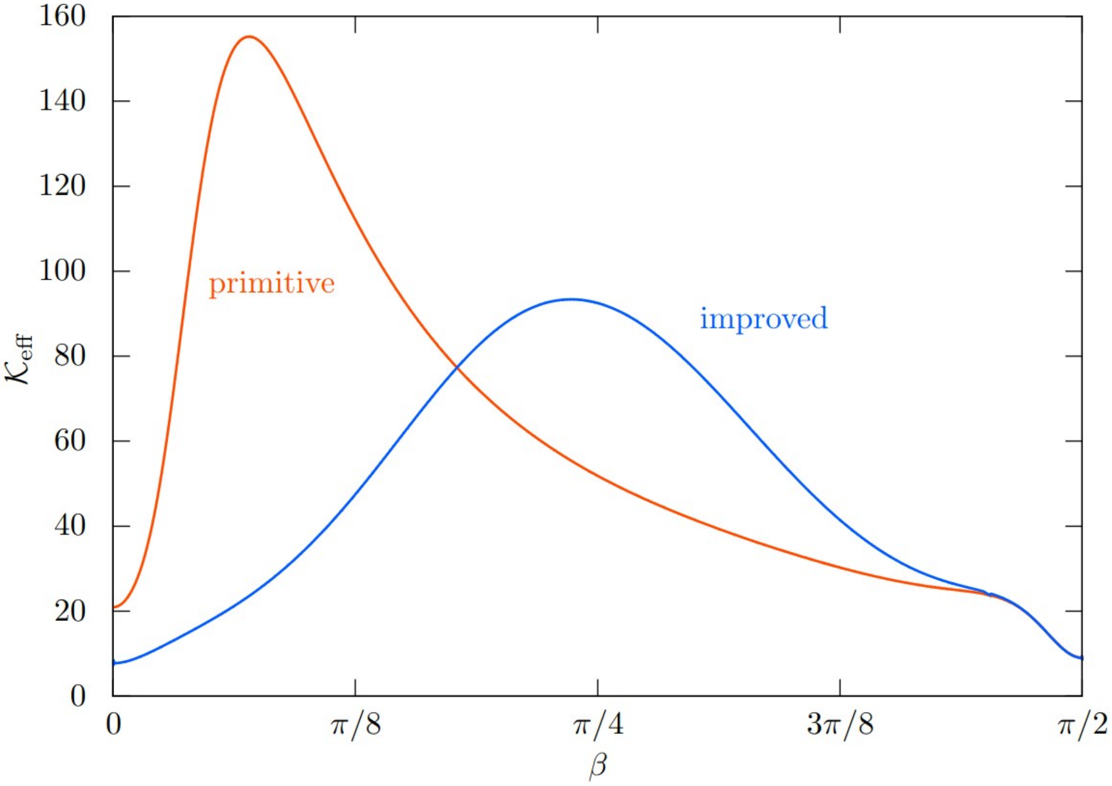
$$A \to A_W + A_\gamma$$

Separation of γ and W

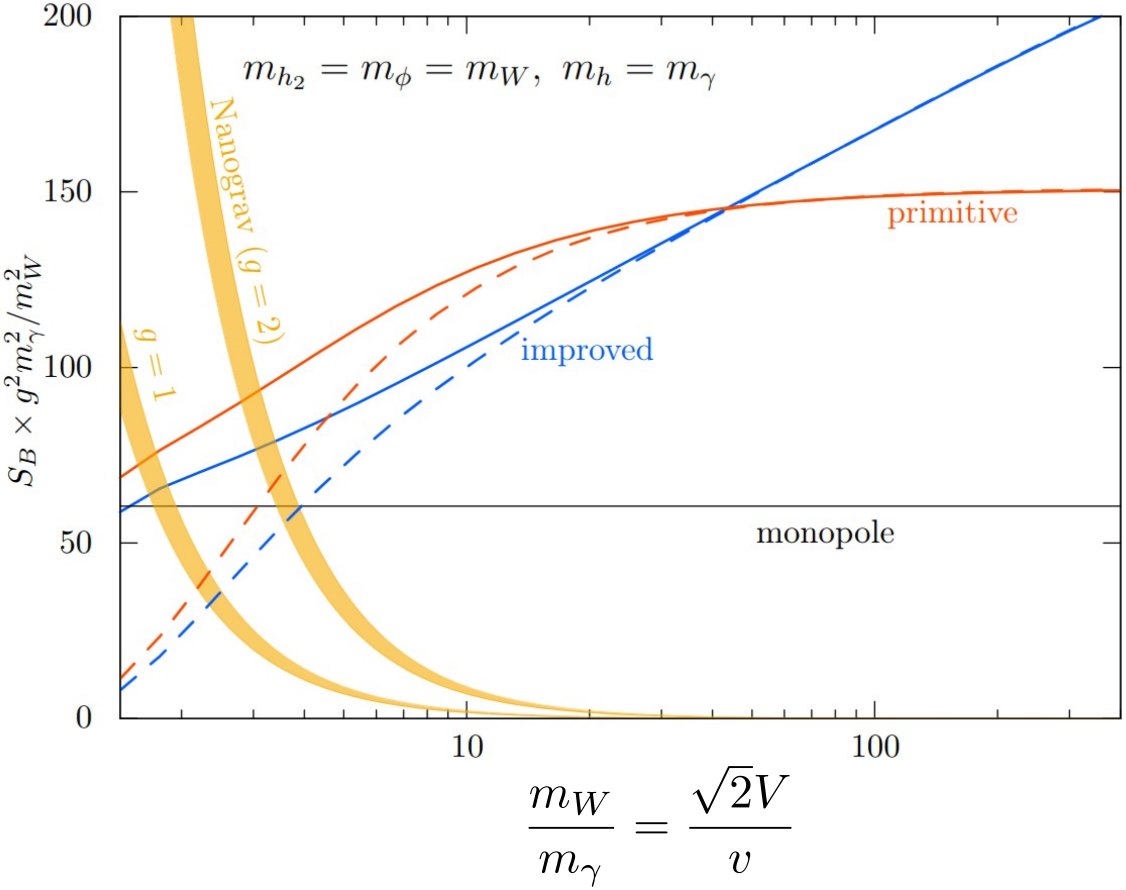
String Tension



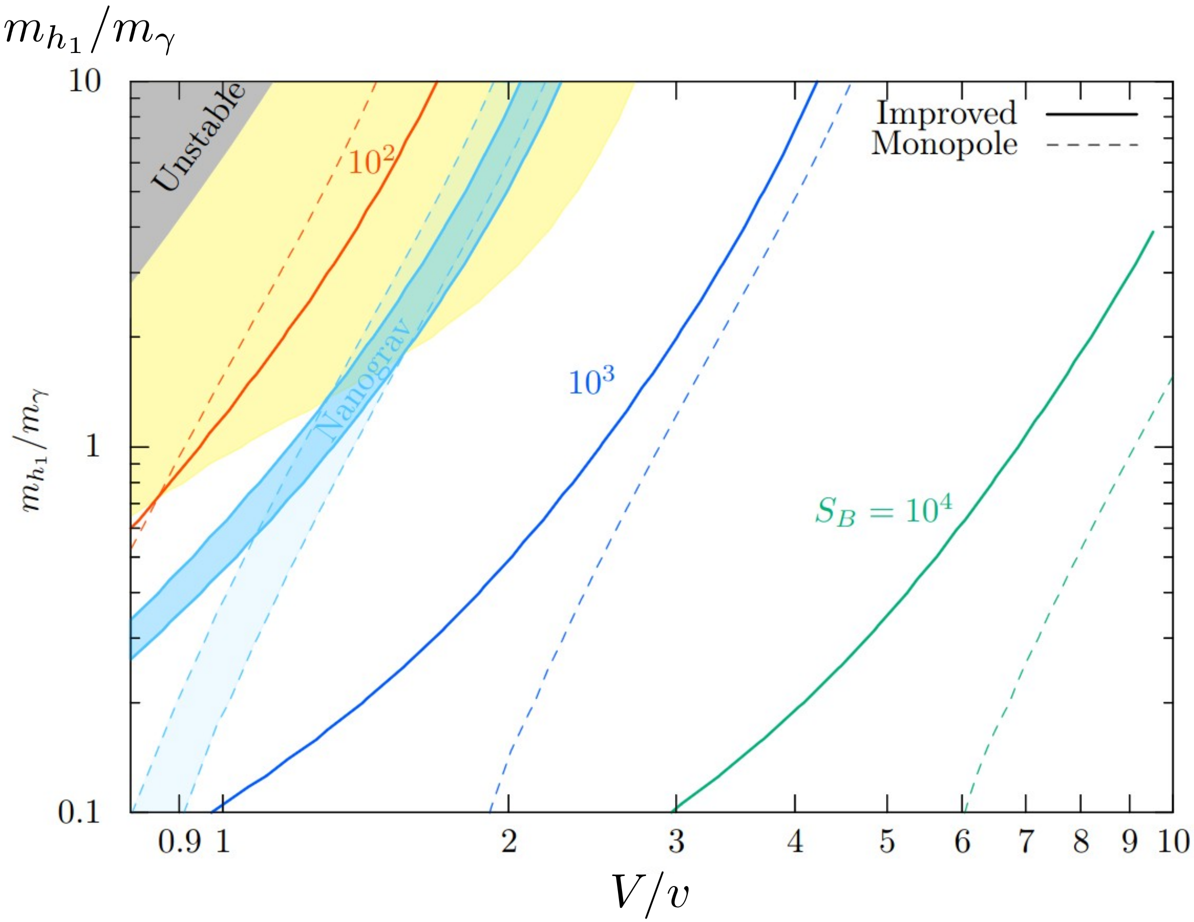
Kinetic Term



Result

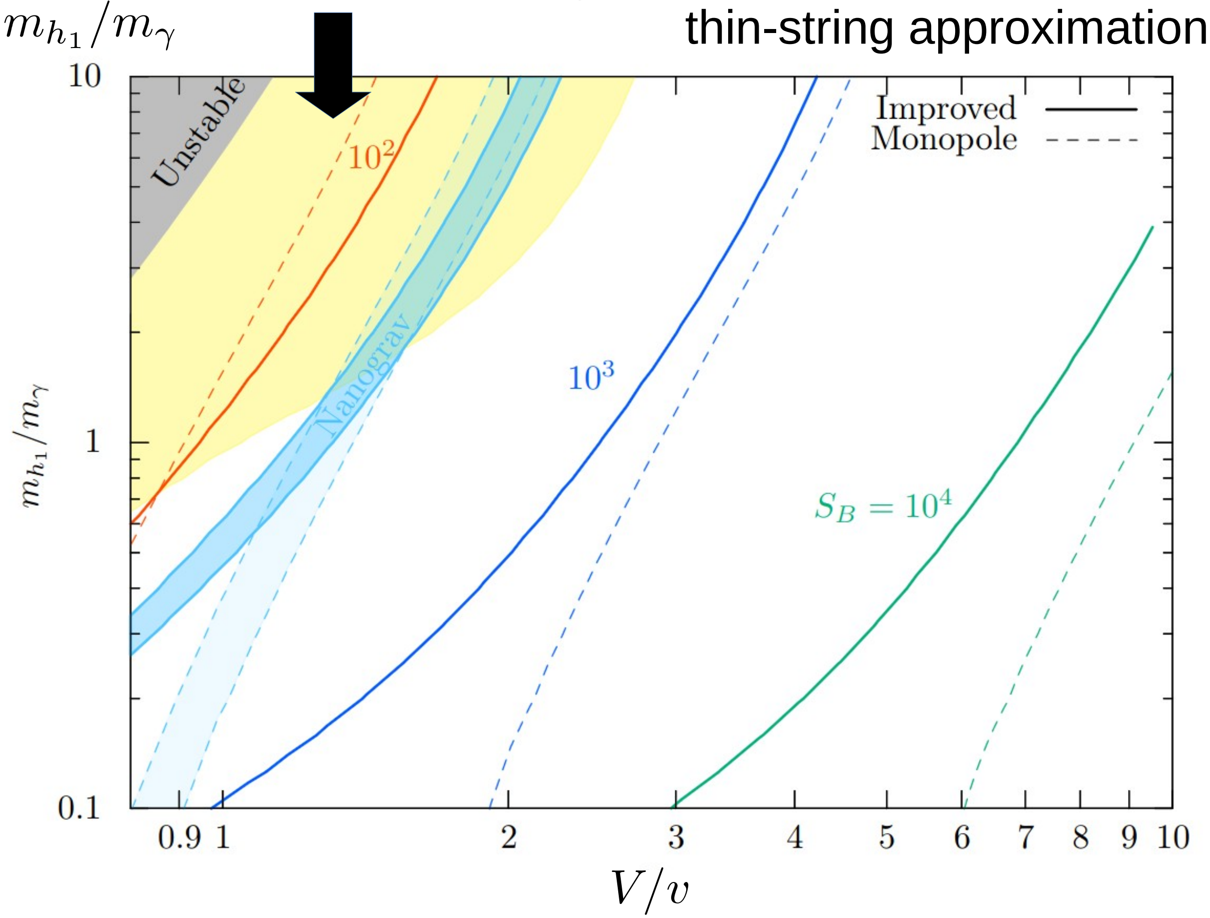

$m_{h_2} = m_\phi = m_W,\ m_h = m_\gamma$
Nanograv ($g = 2$)
$g = 1$
primitive
improved
monopole
$S_B \times g^2 m_\gamma^2 / m_W^2$
200
150
100
50
0
10
100
$\frac{m_W}{m_\gamma} = \frac{\sqrt{2}V}{v}$

Higgs Mass Effect



Higgs Mass Effect

Yellow region: Bounce solution is “better” than thin-string approximation.



Summary and Prospect

- Metastable string can appear from hierarchical symmetry breaking.
  - GW from metastable string gives good fit to NANOGrav data.

- Breaking rate of metastable string is estimated based on two Ansätze.
  - Give robust lower limit of breaking rate even with V/v = O(1).
  - Validity of conventional estimation?

- The present procedure is not best way to fastest path to string breaking.

- Application to other metastable strings, such as semi-local string.