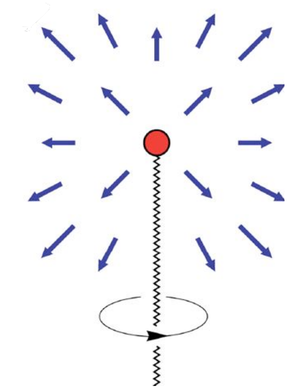
The Magnetic Monopole Unitarity Puzzle

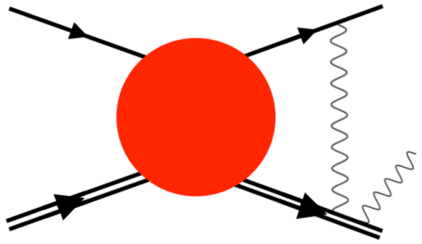
John Terning

Outline

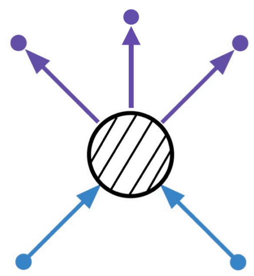
Monopole Review



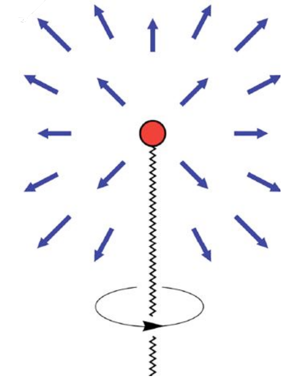
Wigner's Little Group

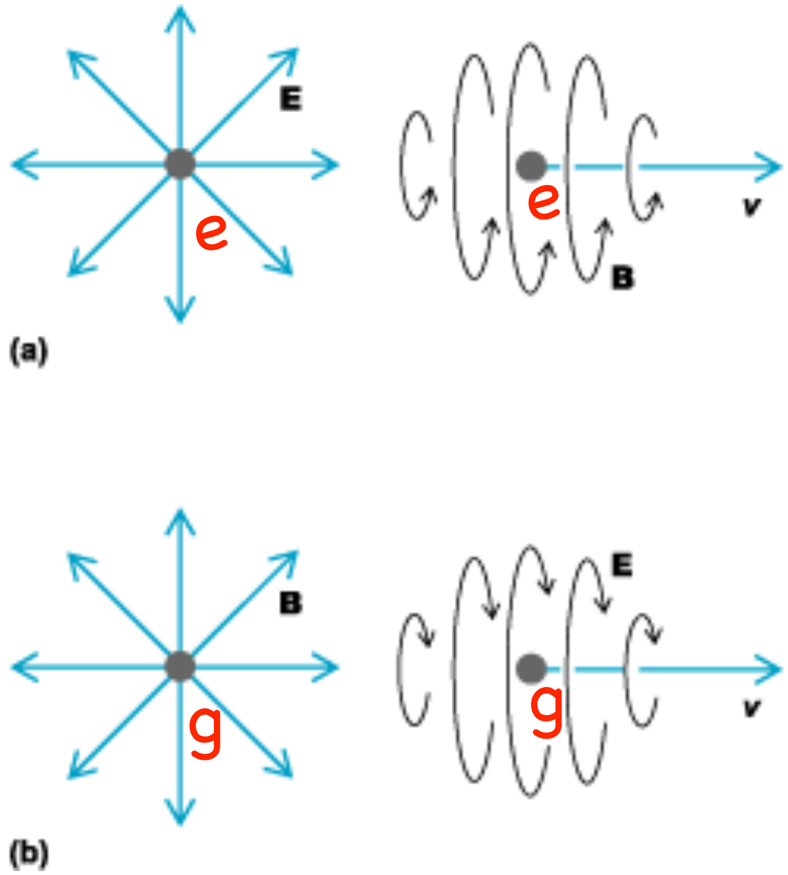


Callan's "half-particles"

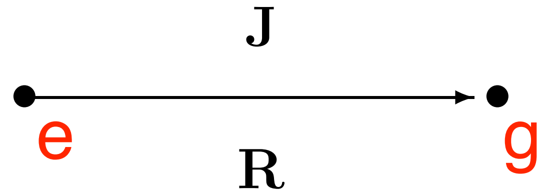


J.J. Thomson





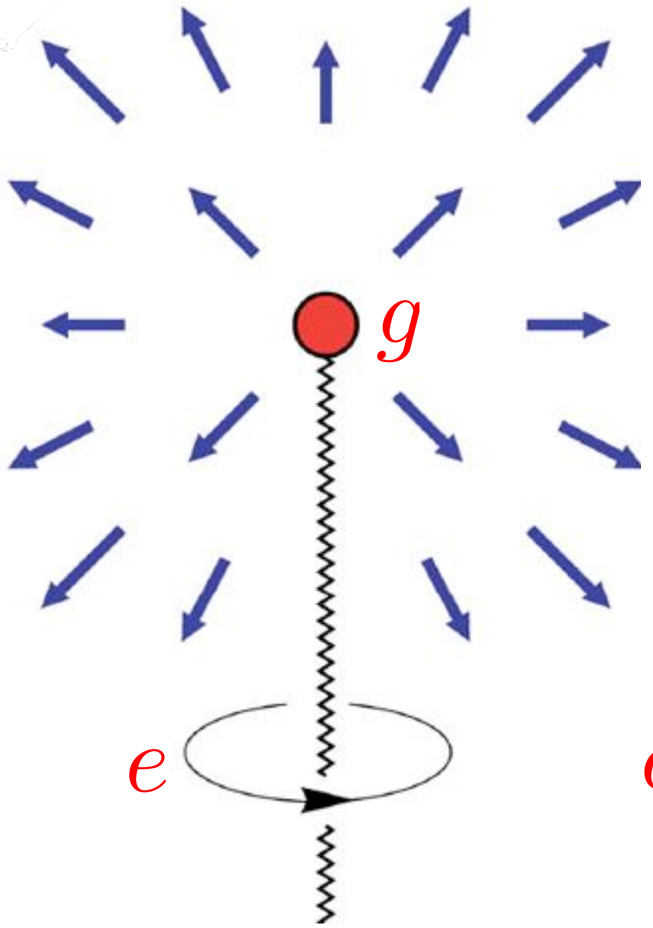
J = e g



Philos. Mag. 8 (1904) 331

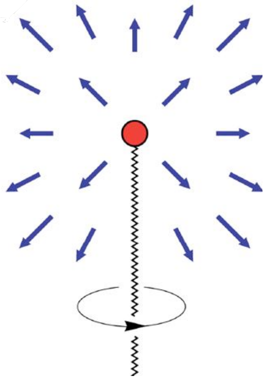
Dirac String

$$\vec{A}(\vec{r}) = \frac{g}{r} \frac{\vec{r} \times \vec{n}}{r - \vec{r} \cdot \vec{n}}$$



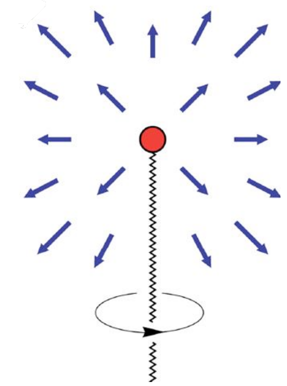
$$eg = \frac{N}{2}$$

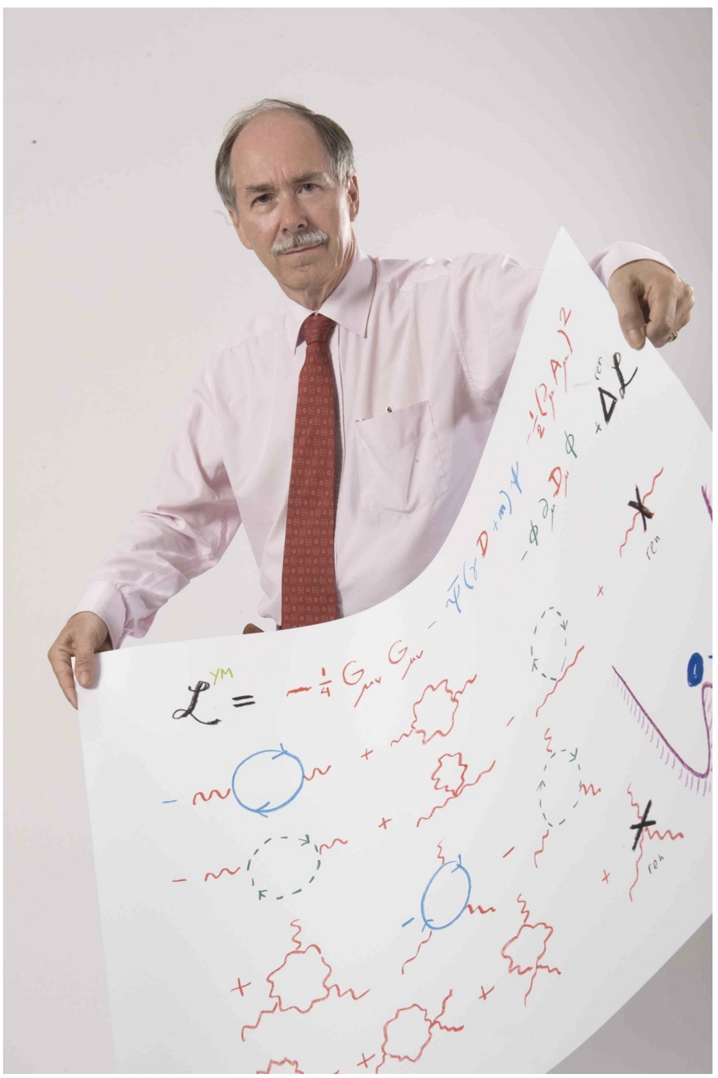
charge quantization

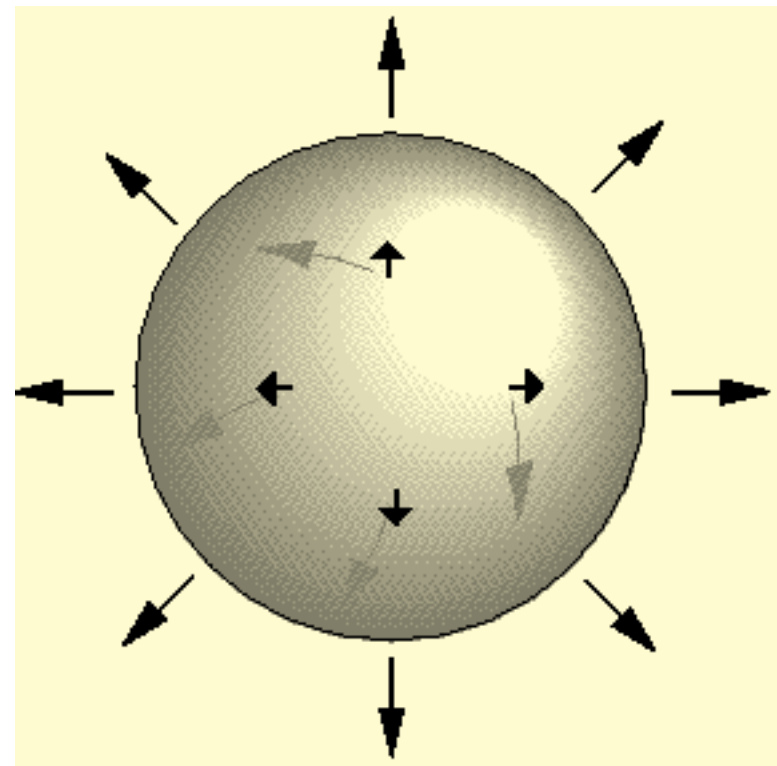


Proc. Roy. Soc. Lond. A133 (1931) 60

‘t Hooft-Polyakov



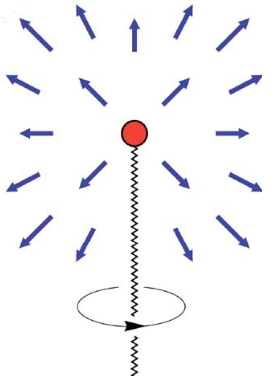


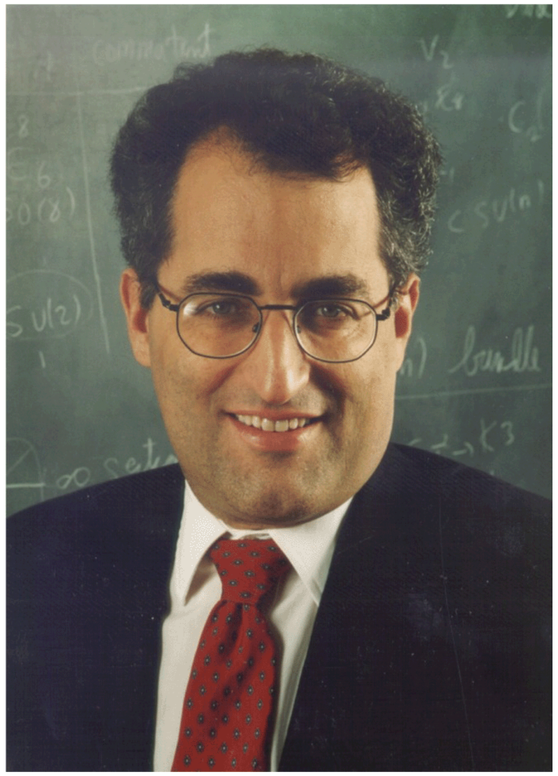


topological monopoles
every GUT predicts monopoles

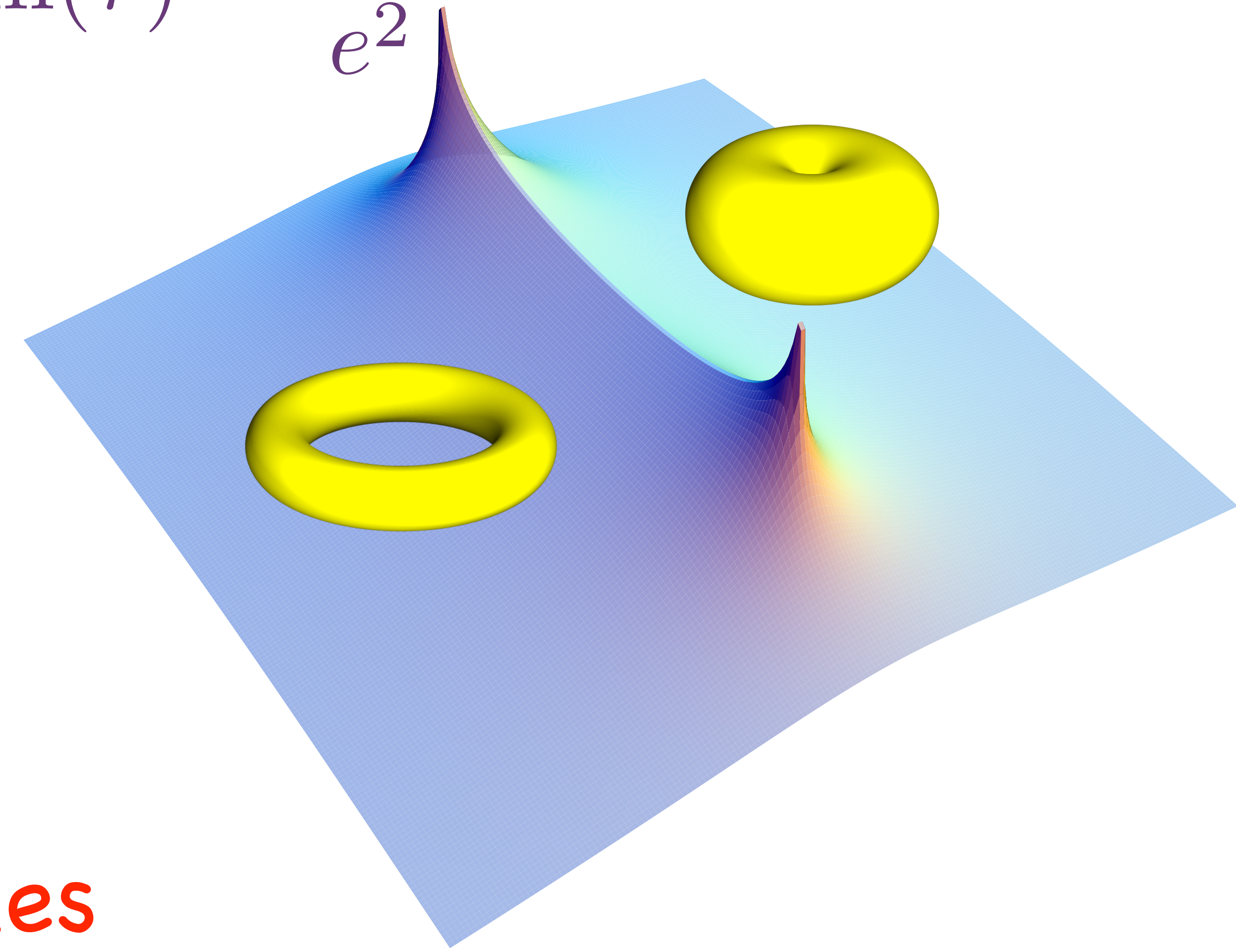
Nucl. Phys., B79 1974, 276
JETP Lett., 20 1974, 194

Seiberg-Witten





$$\mathrm{Im}(\tau) = \frac{4\pi}{e^2}$$

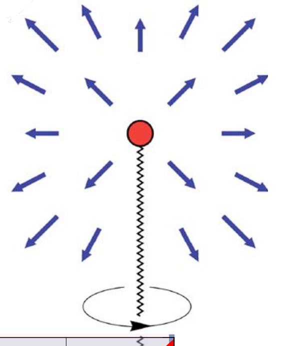


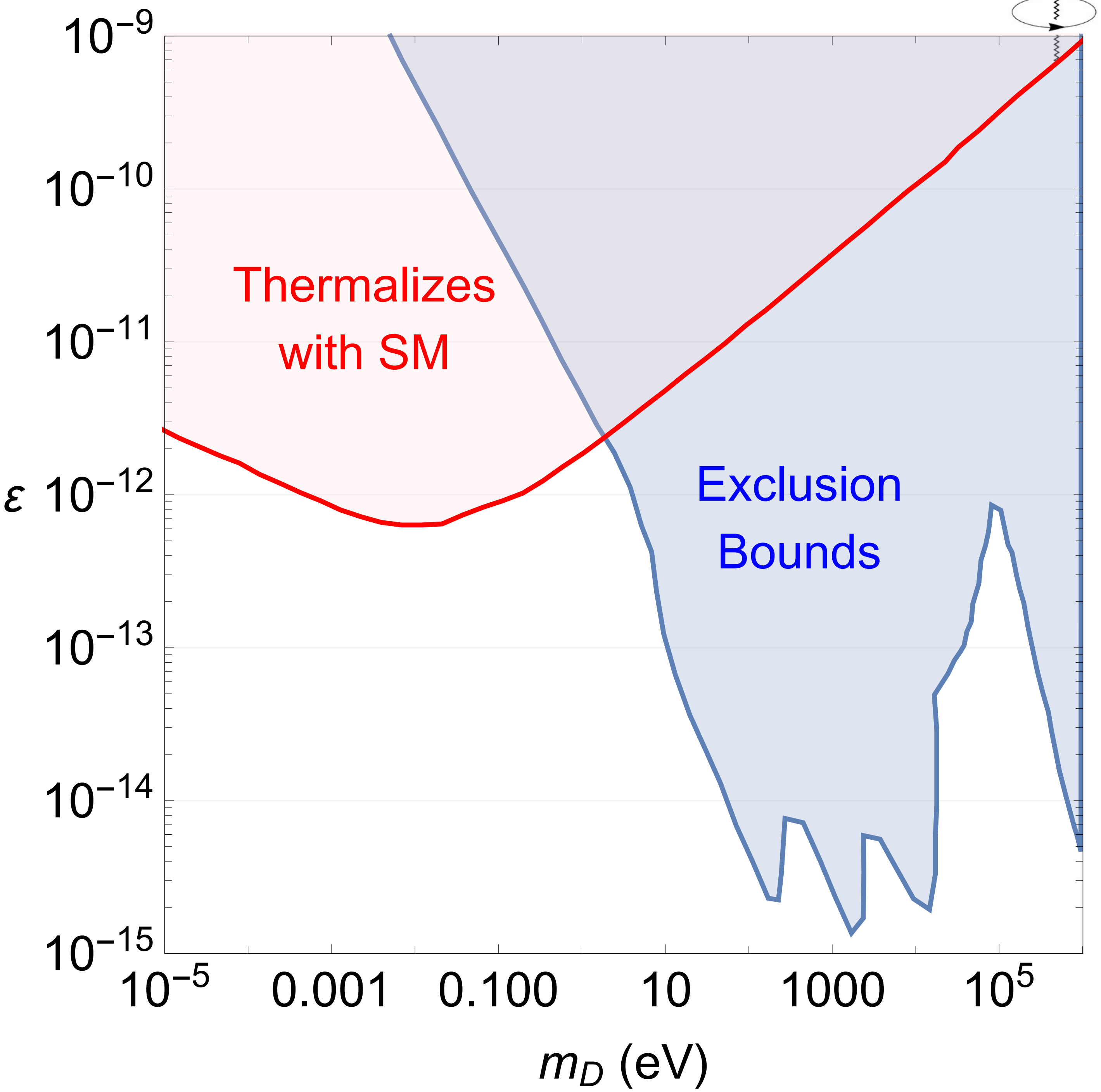
massless monopoles

size smaller than Compton wavelength

hep-th/9407087

Holdom: Dark Photons



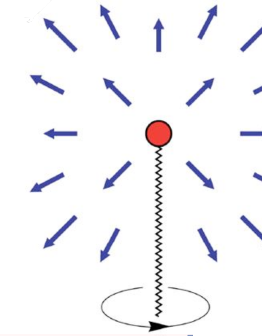


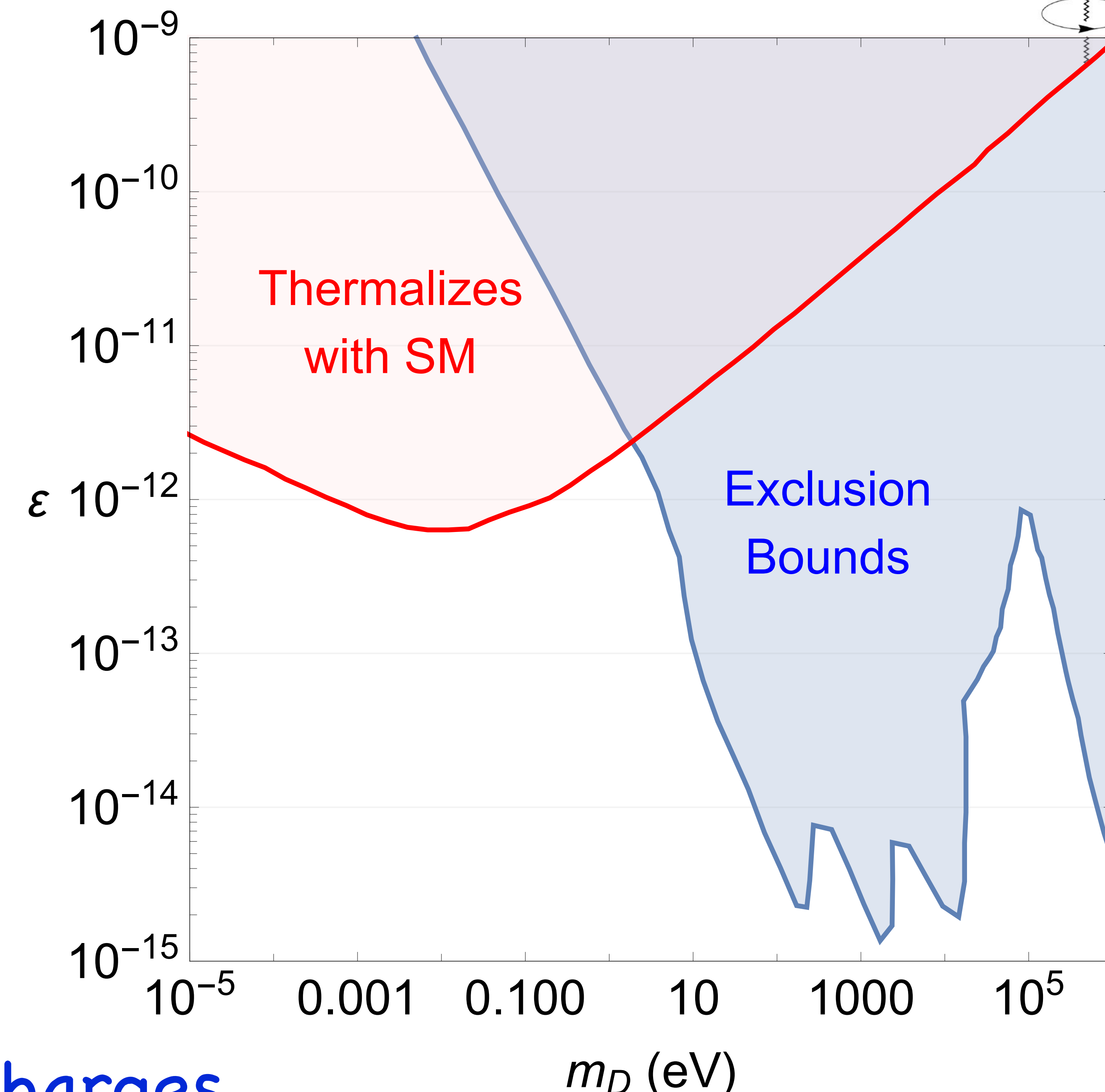
$$\mathcal{L}_{\rm mix} = \frac{\varepsilon}{2} F_{\mu\nu} F_D^{\mu\nu}$$

Holdom Phys. Lett. 166B (1986) 196

An, Pospelov, Pradler hep-ph/1304.3461

Holdom: Dark Photons





$$\mathcal{L}_{\rm mix} = \frac{\varepsilon}{2}\epsilon_{\mu\nu\alpha\beta}F^{\mu\nu}F_D^{\alpha\beta}$$

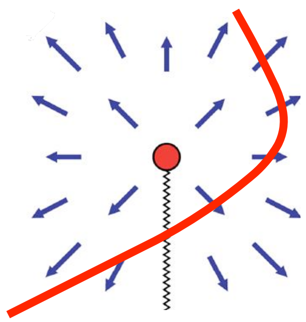
dark charges

get fractional magnetic charges

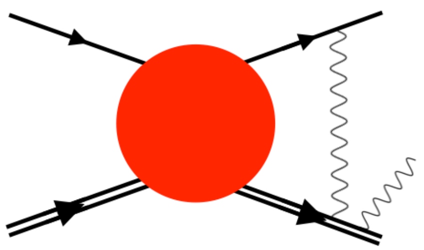
JT, Verhaaren hep-th/1808.09459, hep-ph/1906.00014

Three Problems with Monopoles

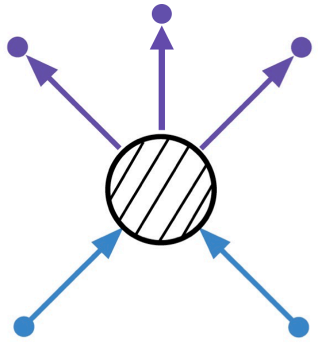
Weinberg Paradox: Lorentz violating poles



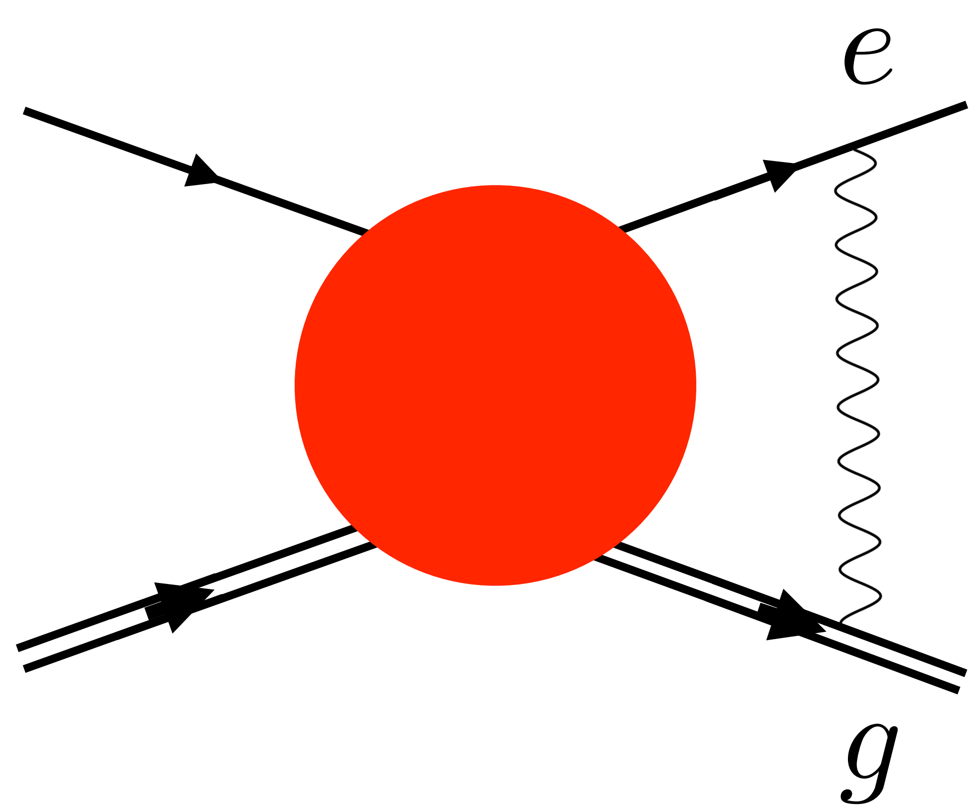
Multiparticle States are not tensor products of Wigner's 1-particle states



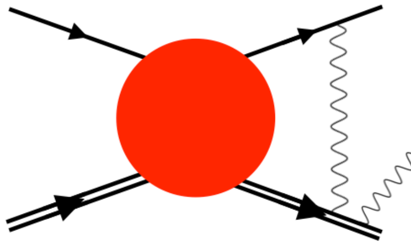
Unitarity Puzzle: Callan's "half-particles" or gauge charge violation



Are multiparticle states products of 1-particle states?



Monopole QM

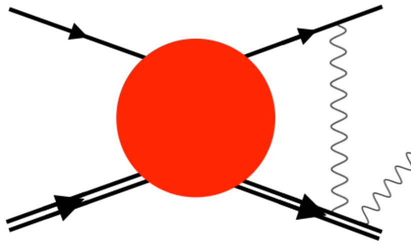


$\vec{L} = \vec{r} \times \vec{p}$ does not satisfy $[L_i, L_j] = i\epsilon_{ijk} L_k$

$\vec{L} = \vec{r} \times \vec{p} + eg\hat{r}$ does

Lipkin, Weisberger, Peshkin Annals Phys. 53 (1969) 203

Monopole QM



$\vec{L} = \vec{r} \times \vec{p}$ does not satisfy $[L_i, L_j] = i\epsilon_{ijk} L_k$

$\vec{L} = \vec{r} \times \vec{p} + eg\hat{r}$ does

Dirac quantization

$$e_1\, g_2 \;-\; e_2\, g_1 \;=\; \frac{n}{2}$$

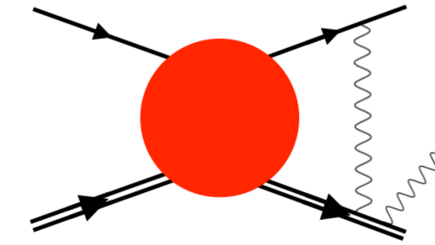
is angular momentum quantization

Lipkin, Weisberger, Peshkin Annals Phys. 53 (1969) 203

Schwinger Science 165 (1969) 757

Zwanziger Phys. Rev. 176 (1968) 1489

Wigner's Little Group

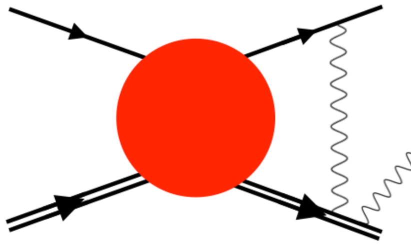


massive: labeled by spin
rest frame invariant
under 3D rotations

boosted states can transform by a
rotation that leaves the momentum fixed

Annals Math **40** (1939) 149

Wigner's Little Group



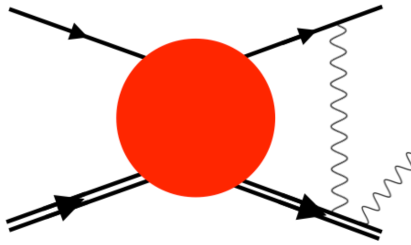
massive: labeled by spin
rest frame invariant
under 3D rotations

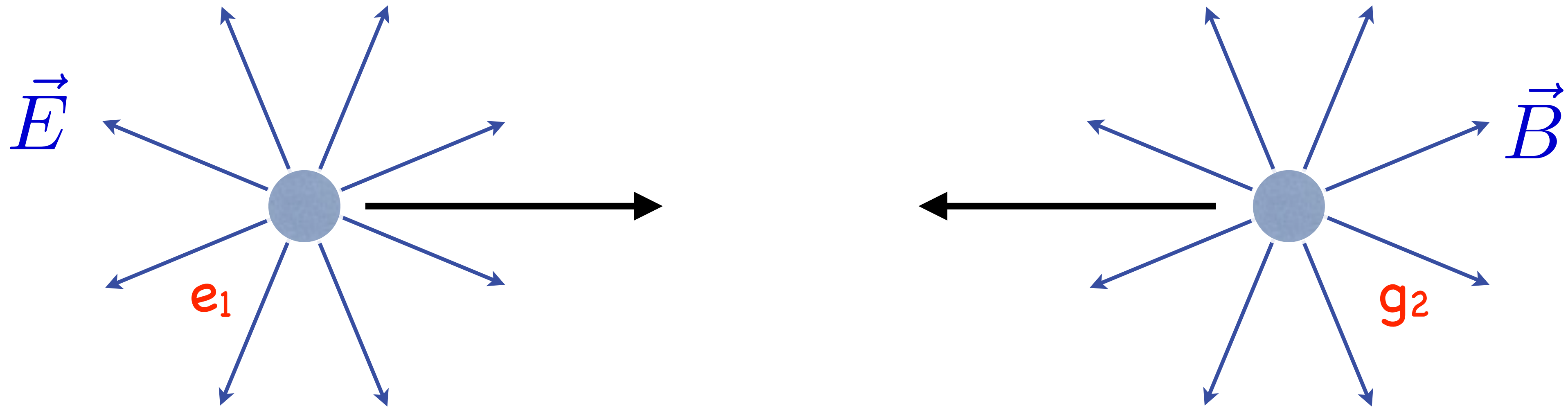
massless: labeled by helicity
momentum invariant
under rotation around
momentum axis

boosted states can transform by a
rotation that leaves the momentum fixed

Annals Math **40** (1939) 149

Wigner's Monopole Friend

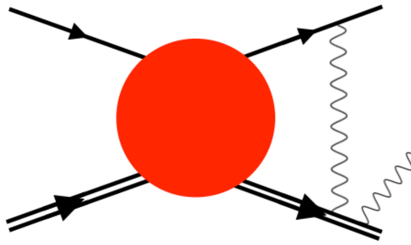


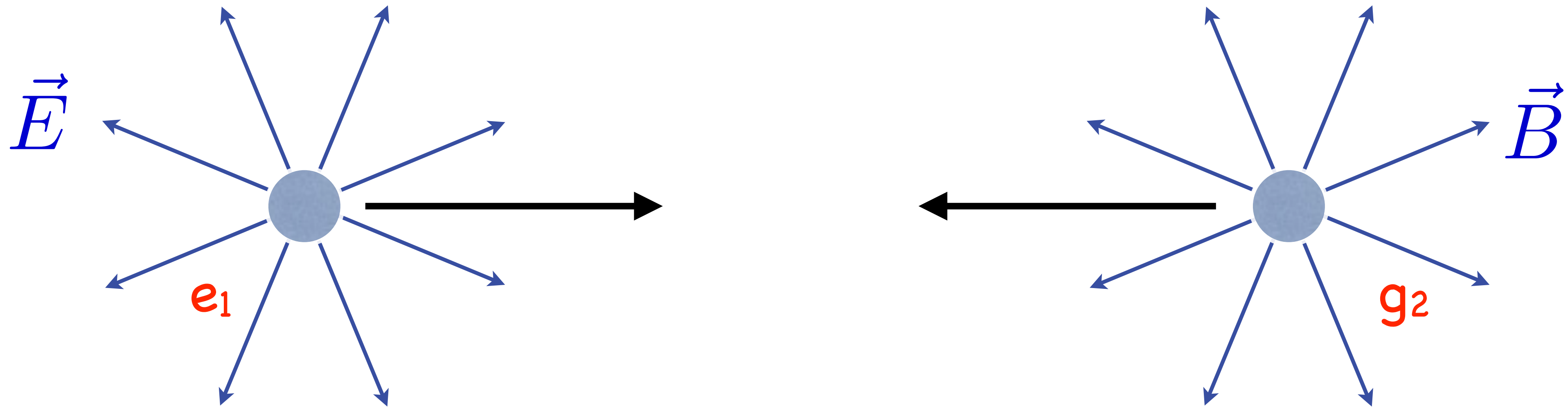


rotations about COM axis leave system invariant

Csáki, Hong, Shirman, Telem, JT hep-th/2009.14213
hep-th/2010.13794

Wigner's Monopole Friend





rotations about COM axis leave system invariant

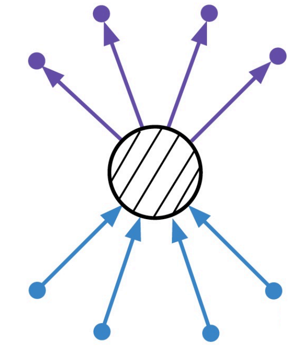
when $q_{12} = e_1\, g_2 \; - \; e_2\, g_1 \neq 0$

there is an extra phase from J in field

Pairwise Little Group phase

Csáki, Hong, Shirman, Telem, JT hep-th/2009.14213
hep-th/2010.13794

♭ Momenta



for each pairwise helicity:

$$\left(k_{ij}^{\flat\pm}\right)_\mu = p_c\,(1,0,0,\pm 1) \qquad p_c = \sqrt{\frac{(p_1\cdot p_2)^2 - m_1^2 m_2^2}{s}}$$

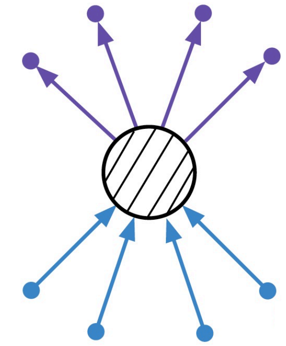
$$p_{ij}^{\flat\pm} = L_p\, k_{ij}^{\flat\pm}$$

$$p_i = \frac{1}{2p_c}\left[\left(E_i^c + p_c\right) p_{ij}^{\flat +} + \left(E_i^c - p_c\right) p_{ij}^{\flat -}\right]$$

$$p_j = \frac{1}{2p_c}\left[\left(E_j^c + p_c\right) p_{ij}^{\flat -} + \left(E_j^c - p_c\right) p_{ij}^{\flat +}\right]$$

Kosower hep-th/0406175

Pairwise Spinors



$$\left|k_{ij}^{\flat +}\right\rangle_\alpha = \sqrt{2\,p_c}\begin{pmatrix}1\\0\end{pmatrix} \quad , \quad \left|k_{ij}^{\flat -}\right\rangle_\alpha = \sqrt{2\,p_c}\begin{pmatrix}0\\1\end{pmatrix}$$

$$\left|p_{ij}^{\flat \pm}\right\rangle_\alpha = (L_p)_\alpha^{\ \beta}\left|k_{ij}^{\flat \pm}\right\rangle_\beta \quad , \quad \left[p_{ij}^{\flat \pm}\right|_{\dot\alpha} = \left[k_{ij}^{\flat \pm}\right|_{\dot\beta}\left(\tilde{L}_p\right)^{\dot\beta}_{\ \dot\alpha}$$

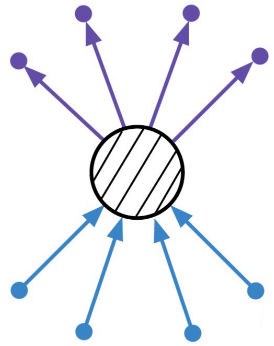
$$p_{ij}^{\flat \pm}\cdot\sigma_{\alpha\dot\alpha} = \left|p_{ij}^{\flat \pm}\right\rangle_\alpha\left[p_{ij}^{\flat \pm}\right|_{\dot\alpha}$$

$$\Lambda_\alpha^{\ \beta}\left|p_{ij}^{\flat \pm}\right\rangle_\beta = e^{\pm\frac{i}{2}\phi(p_i,p_j,\Lambda)}\left|\Lambda p_{ij}^{\flat \pm}\right\rangle_\alpha \quad , \quad \left[p_{ij}^{\flat \pm}\right|_{\dot\beta}\tilde{\Lambda}^{\dot\beta}_{\ \dot\alpha} = e^{\mp\frac{i}{2}\phi(p_i,p_j,\Lambda)}\left[\Lambda p_{ij}^{\flat \pm}\right|_{\dot\alpha}$$

spinors transforming covariantly under pairwise LG, with opposite weights

Csáki, Hong, Shirman, Telem, JT hep-th/2009.14213

Massless Limit



$$m_i \to 0$$

$$\left|p_{ij}^{\flat +}\right\rangle_\alpha \to |\,i\,\rangle_\alpha \qquad , \qquad \left[p_{ij}^{\flat +}\right|_{\dot\alpha} \to [\,i\,|_{\dot\alpha}$$

$$\left|p_{ij}^{\flat -}\right\rangle_\alpha \to \sqrt{2p_c}\,|\hat\eta_i\rangle_\alpha \qquad , \qquad \left[p_{ij}^{\flat -}\right|_{\dot\alpha} \to \sqrt{2p_c}\,[\hat\eta_i|_{\dot\alpha}$$

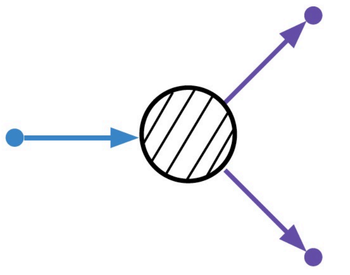
Parity flipped

$$\left[p_{ij}^{\flat +}\, i\right] = \left\langle i\, p_{ij}^{\flat +}\right\rangle = \left[\hat\eta_i\, p_{ij}^{\flat -}\right] = \left\langle p_{ij}^{\flat -}\, \hat\eta_i\right\rangle = 0$$

$$\left[p_{ij}^{\flat -}\, i\right] = \left\langle i\, p_{ij}^{\flat -}\right\rangle = \left[\hat\eta_i\, p_{ij}^{\flat +}\right] = \left\langle p_{ij}^{\flat +}\, \hat\eta_i\right\rangle = 2p_c$$

origin of mandatory helicity-flip in the lowest partial wave for charge-monopole scattering

All 3-pt EM Amplitudes



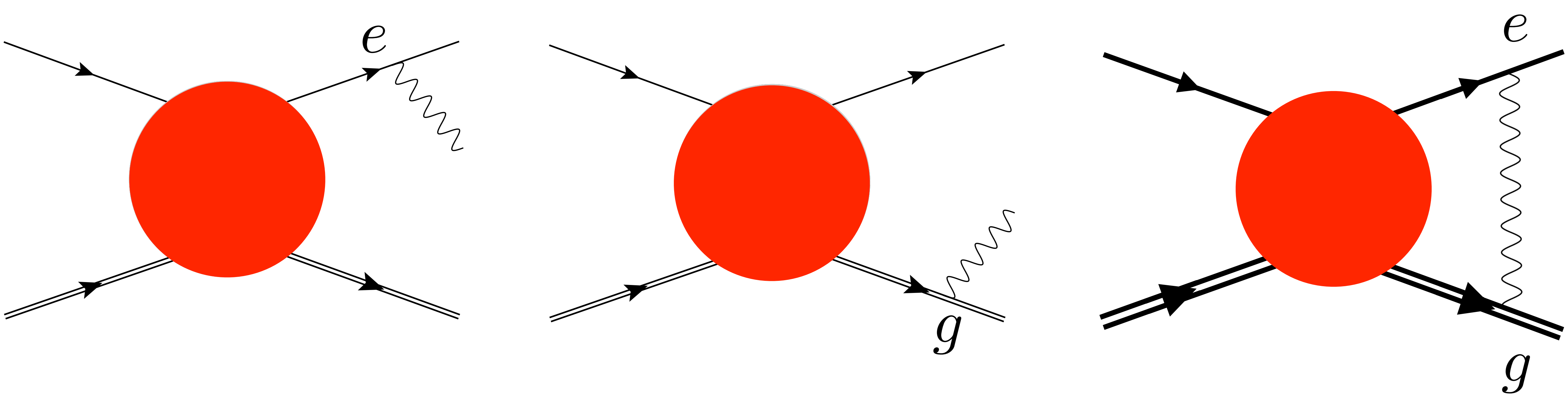
write most general Lorentz invariant expressions consistent with Little Group and Pairwise Little Group

$\text{exponents} \geq 0 \Longrightarrow$ selection rule:

the selection rules are more restrictive than the q=0 case in Arkani-Hamed at al.

Csáki, Hong, Shirman, Telem, JT hep-th/2009.14213

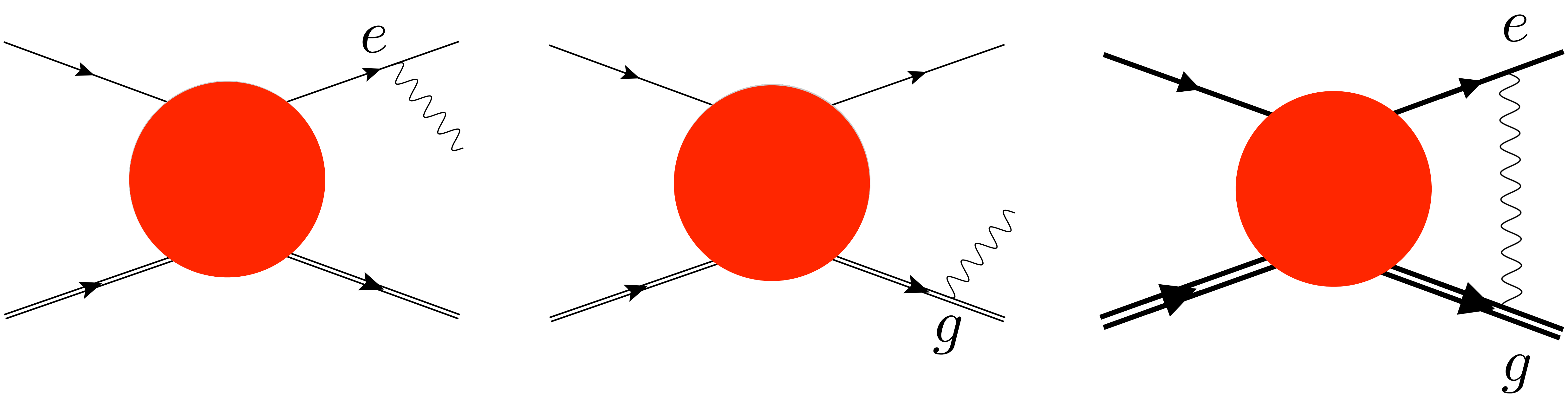
Dressed/Pairwise States



sum up all possible numbers of soft photons

|dressed state> = |particle + coherent state of photons>

Dressed/Pairwise States



sum up all possible numbers of soft photons

|dressed state> = |particle + coherent state of photons>

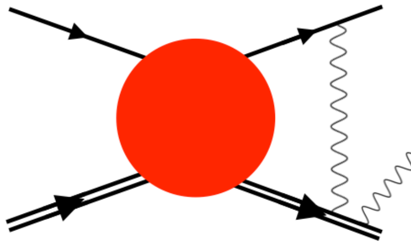
rotate photon cloud and Dirac string

Berry phase of rotating Dirac string of dressed state

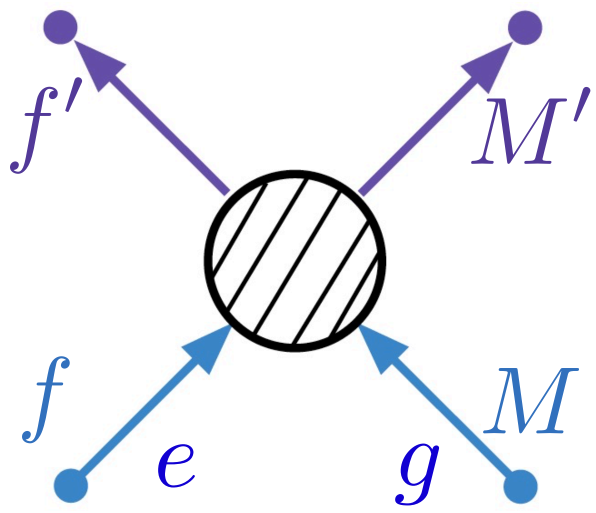
= Pairwise Little Group phase

Csáki, Dong, Telem, JT, Yankielowicz hep-th/2209.03369

Check: Lowest Partial Wave



selection rule: $J \geq |q| - \frac{1}{2}$ $\quad m \to 0$



$$J = 0 \Rightarrow \quad |\uparrow_{\text{elec.}}\rangle\,|\downarrow_{\text{field}}\rangle \quad \text{or} \quad |\downarrow_{\text{elec.}}\rangle\,|\uparrow_{\text{field}}\rangle$$

$q < 0$ only RH fermion to LH fermion

$q > 0$ only LH fermion to RH fermion

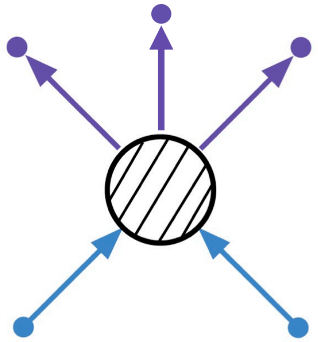
helicity flip vanishes for higher partial waves

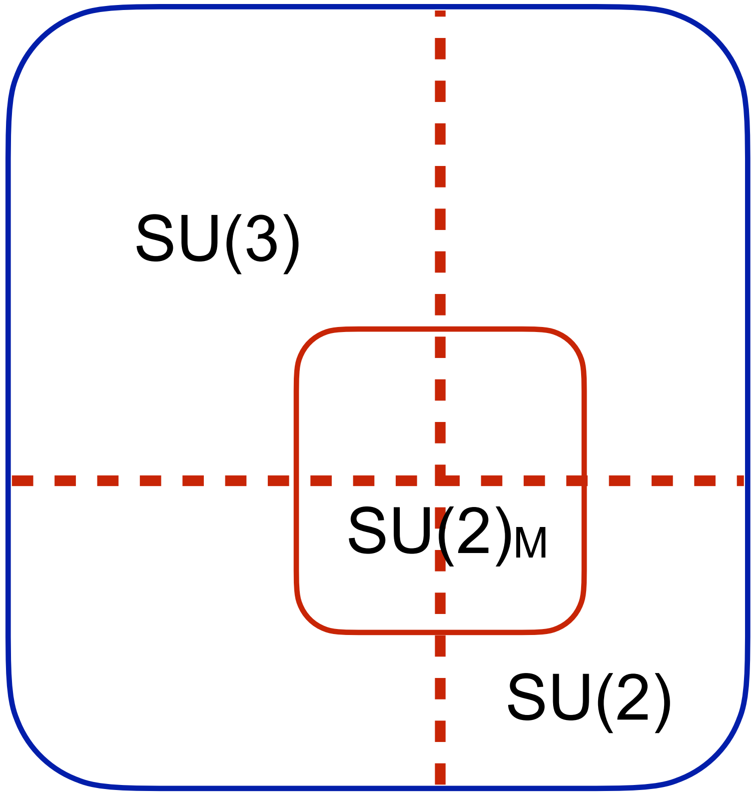
Pairwise Spinor Helicity hep-th/2009.14213

exactly reproduces

Kazama, Yang, Goldhaber Phys Rev D15 (1976) 2287

Georgi-Glashow: SU(5) GUT

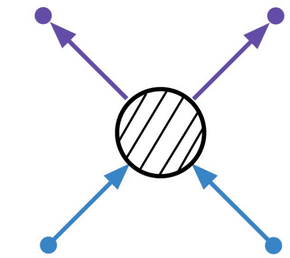




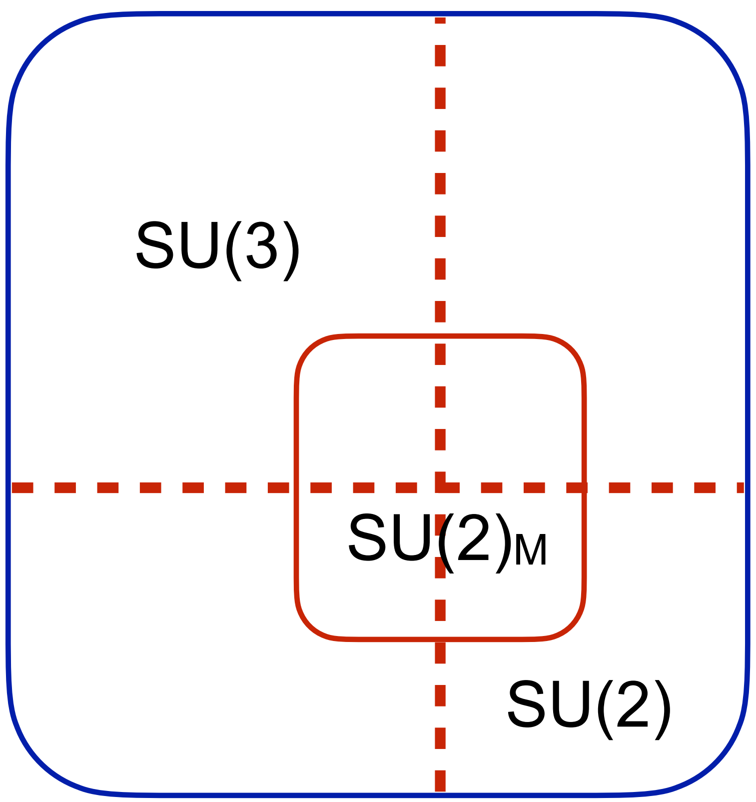
$$\bar{5} = (\bar{d}^1, \bar{d}^2, \bar{d}^3, e^-, \nu_e)$$

$$10 = \begin{pmatrix} 0 & \overline{u}^3 & -\overline{u}^2 & u^1 & d^1 \\ -\overline{u}^3 & 0 & \overline{u}^1 & u^2 & d^2 \\ \overline{u}^2 & -\overline{u}^1 & 0 & u^3 & d^3 \\ -u^1 & -u^2 & -u^3 & 0 & \overline{e} \\ -d^1 & -d^2 & -d^3 & -\overline{e} & 0 \end{pmatrix}$$

Georgi-Glashow: SU(5) GUT



$$\bar{5} = (\bar{d}^1, \bar{d}^2, \bar{d}^3, e^-, \nu_e)$$



$$10 = \begin{pmatrix} 0 & \bar{u}^3 & -\bar{u}^2 & u^1 & d^1 \\ -\bar{u}^3 & 0 & \bar{u}^1 & u^2 & d^2 \\ \bar{u}^2 & -\bar{u}^1 & 0 & u^3 & d^3 \\ -u^1 & -u^2 & -u^3 & 0 & \bar{e} \\ -d^1 & -d^2 & -d^3 & -\bar{e} & 0 \end{pmatrix}$$

monopole in $SU(2)_M$

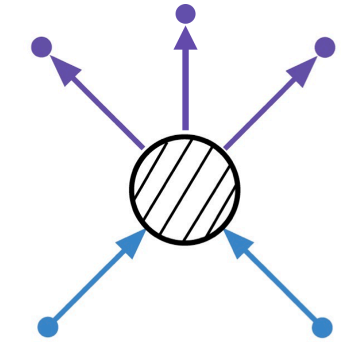
four doublets

outgoing

incoming

$$\begin{pmatrix} e \\ -\bar{d}^3 \end{pmatrix}, \quad \begin{pmatrix} \bar{u}^1 \\ u^2 \end{pmatrix}, \quad \begin{pmatrix} -\bar{u}^2 \\ u^1 \end{pmatrix}, \quad \begin{pmatrix} d^3 \\ \bar{e} \end{pmatrix}$$

Rubakov-Callan



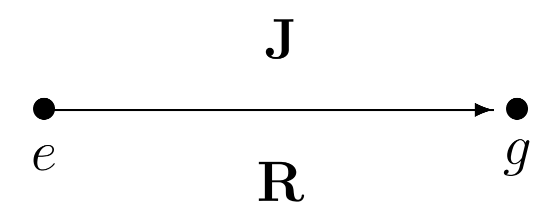
s-wave $\quad u^1 + u^2 + M \to \quad ?$

initial state

$$|\uparrow_{\mathrm{u}^1}\rangle\,|\downarrow_{\mathrm{field}}\rangle \times |\uparrow_{\mathrm{u}^1}\rangle\,|\downarrow_{\mathrm{field}}\rangle$$

$$\to \left[u^1\, p^{\flat -}_{u^1, M}\right]\left[u^2\, p^{\flat -}_{u^2, M}\right]$$

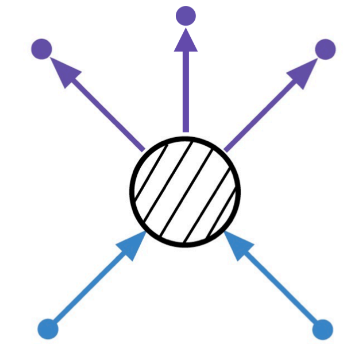
forward scattering not allowed since $|\downarrow_{\mathrm{field}}\rangle \to |\uparrow_{\mathrm{field}}\rangle$



only possibility: $|\downarrow_{e^\dagger}\rangle\,|\uparrow_{\mathrm{field}}\rangle \times |\downarrow_{\mathrm{d}^3\dagger}\rangle\,|\uparrow_{\mathrm{field}}\rangle$

$$\to \left[\bar{e}^\dagger\, p^{\flat -}_{\bar{e}^\dagger, M}\right]\left[\bar{d}^{3\dagger}\, p^{\flat -}_{\bar{d}^{3\dagger}, M}\right]$$

Callan's Puzzle



s-wave $\quad \bar{e} + M \to {\bar{u}^1}^\dagger + {\bar{u}^2}^\dagger + {\bar{d}^3}^\dagger$?

initial state $\quad \left|\uparrow_{\bar{e}}\right\rangle \left|\downarrow_{\text{field}}\right\rangle \to \left[\bar{e}\, p^{\flat -}_{\bar{e},M}\right]$

$$\begin{pmatrix} e \\ -\bar{d}^3 \end{pmatrix}, \quad \begin{pmatrix} \bar{u}^1 \\ u^2 \end{pmatrix}, \quad \begin{pmatrix} -\bar{u}^2 \\ u^1 \end{pmatrix}, \quad \begin{pmatrix} d^3 \\ \bar{e} \end{pmatrix}$$

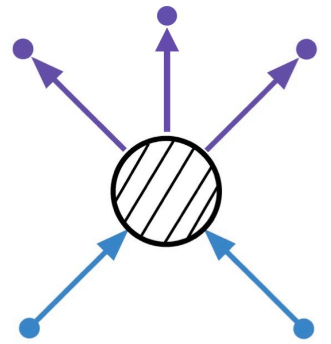
but $\quad \left|\downarrow_{\bar{\mathrm{u}}^{1\dagger}}\right\rangle \left|\uparrow_{\text{field}}\right\rangle \left|\downarrow_{\bar{\mathrm{u}}^{2\dagger}}\right\rangle \left|\uparrow_{\text{field}}\right\rangle \left|\downarrow_{\bar{\mathrm{d}}^{3\dagger}}\right\rangle \left|\downarrow_{\text{field}}\right\rangle$

has $J \neq 0$

Callan claimed only possibility: $\quad \frac{1}{2}\left(e^\dagger + {\bar{u}^1}^\dagger + {\bar{u}^2}^\dagger + d^3\right)$

fractional fermions or gauge charges conserved statistically?

Resolution of Callan's Puzzle



$$\overline{e} + M \to \overline{u}^{1\,\dagger} + \overline{u}^{2\,\dagger} + \overline{d}^{3\,\dagger}$$

incoming state $\left|\uparrow_{\overline{e}}\right\rangle \left|\downarrow_{\rm field}\right\rangle \to \left[\overline{e}\, p^{\flat -}_{\overline{e},M}\right]$

truncated 2D analysis missed

$$\left[\overline{u}^{1\,\dagger}\, p^{\flat -}_{\overline{u}^{1\,\dagger} M}\right] \left[\overline{u}^{2\,\dagger}\, p^{\flat -}_{\overline{d}^{3\,\dagger} M}\right] \left[\overline{d}^{3\,\dagger}\, p^{\flat +}_{\overline{u}^{2\,\dagger} M}\right] - (1 \leftrightarrow 2)$$

individual fermions not in J=0

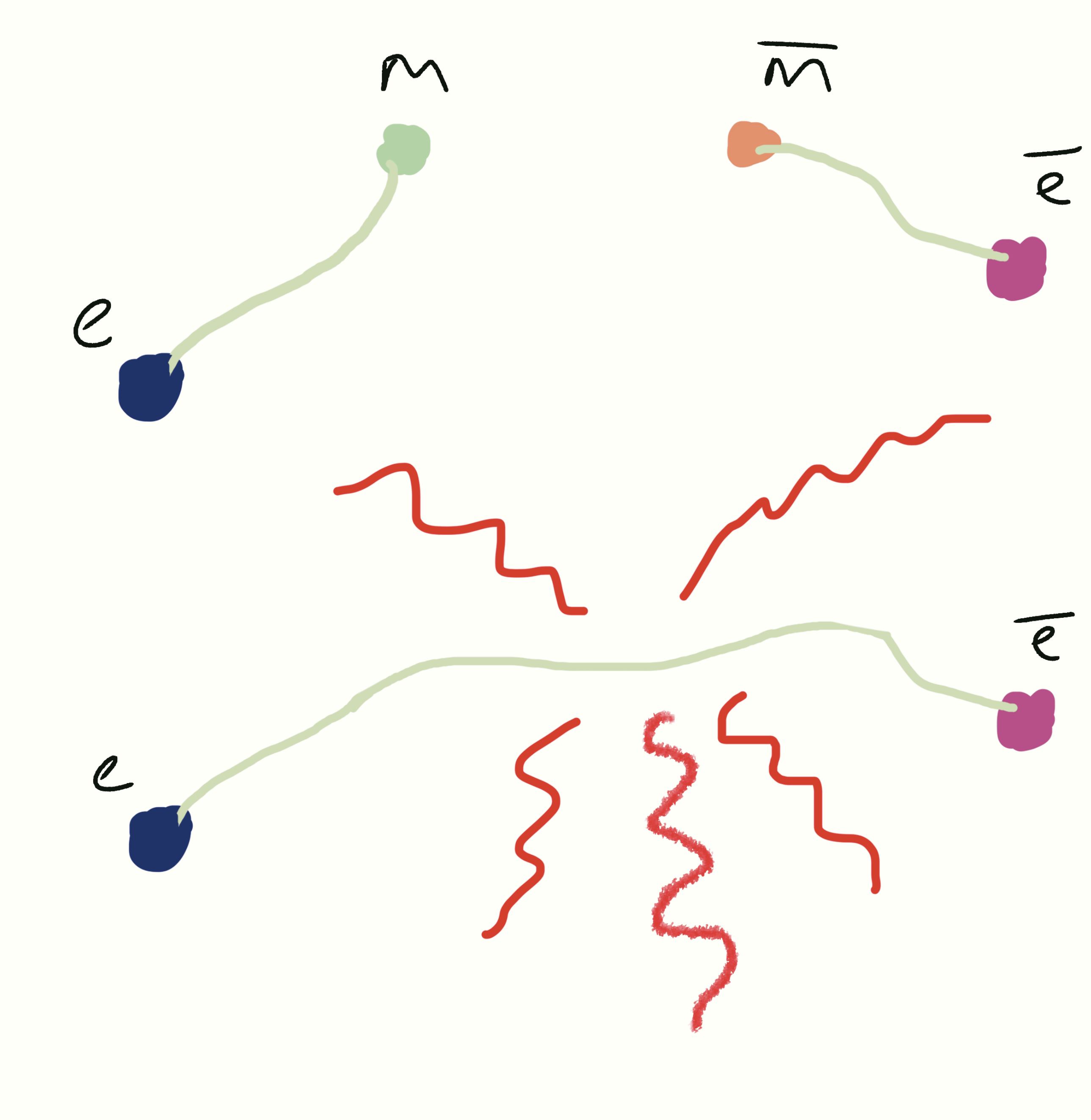
overall J=0

Csáki, Shirman, Telem, JT hep-th/2109.01145

Reinterpretation of Callan

Van Beest, Komargodski, Tong, et.al. hep-th/2306.07318

$$\frac{1}{2}\left(e^{\dagger} + \overline{u}^{1\,\dagger} + \overline{u}^{2\,\dagger} + d^{3}\right)$$



But Why Does it Work?

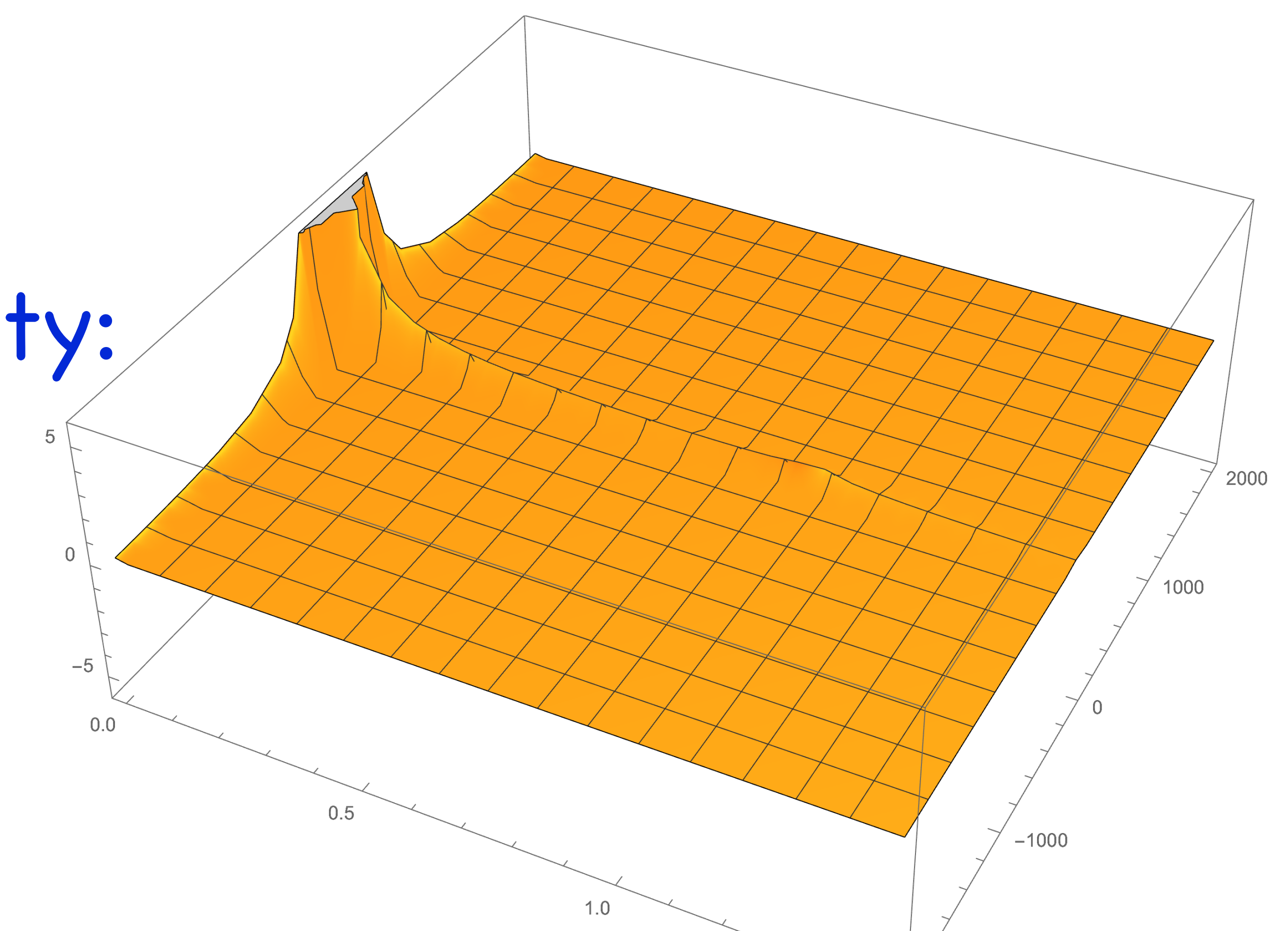
s-wave two doublet case -> 2D

Rubakov and friends showed

$$\int d^4x\, \vec{E} \cdot \vec{B} = \text{ integer}$$

Abelian Instanton

Euclidian space density:



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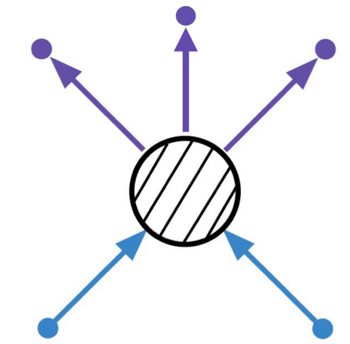
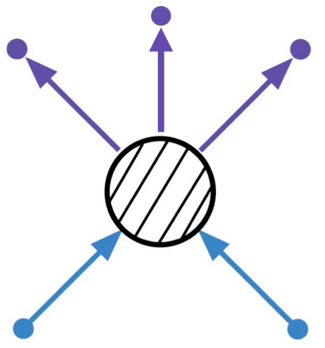


$$\int d^4x \vec{E}\cdot\vec{B} = \int r^2 dr dt d\Omega\, E_r \frac{g}{4\pi r^2} = g\int dr dt E_r = gen'$$

first Chern number

But Why Does it Work?

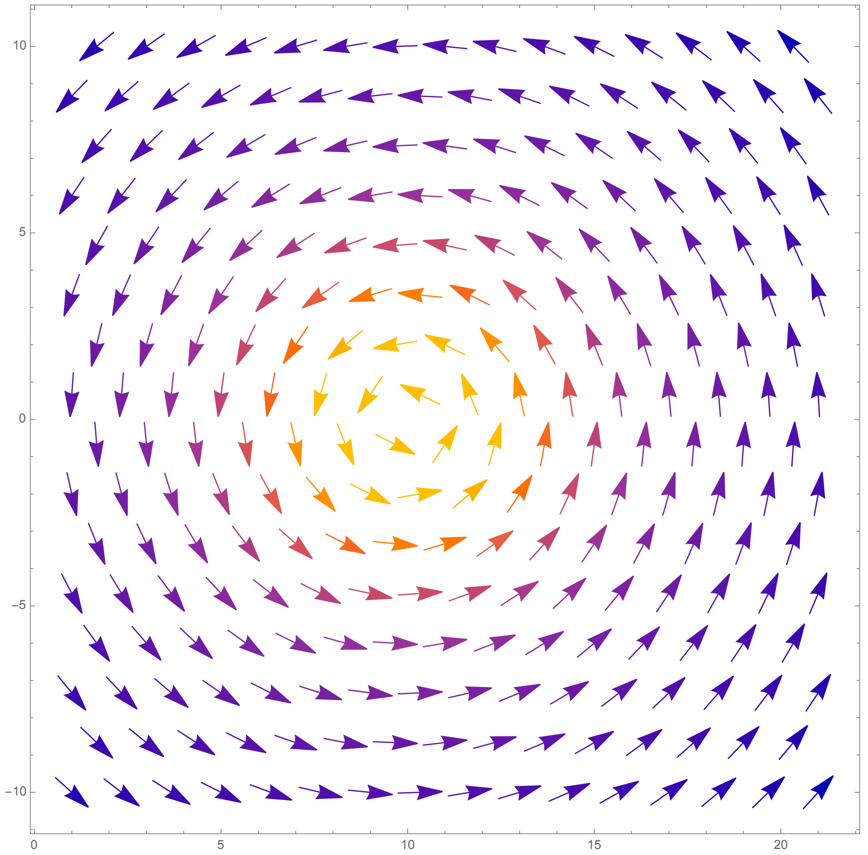


$$\int d^4x \vec{E}\cdot\vec{B} = \int r^2 dr dt d\Omega\, E_r \frac{g}{4\pi r^2} = g\int dr dt E_r = gen'$$

first Chern number

2D vortex solution:

$$A^\mu = \epsilon^{\mu\nu}\partial_\nu \frac{1}{2}\ln\left(x^2+\lambda^2\right)$$



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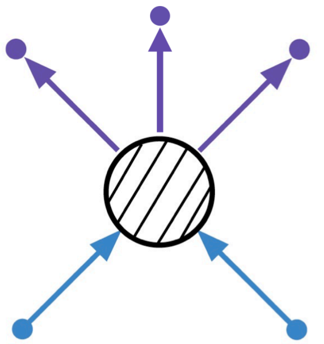

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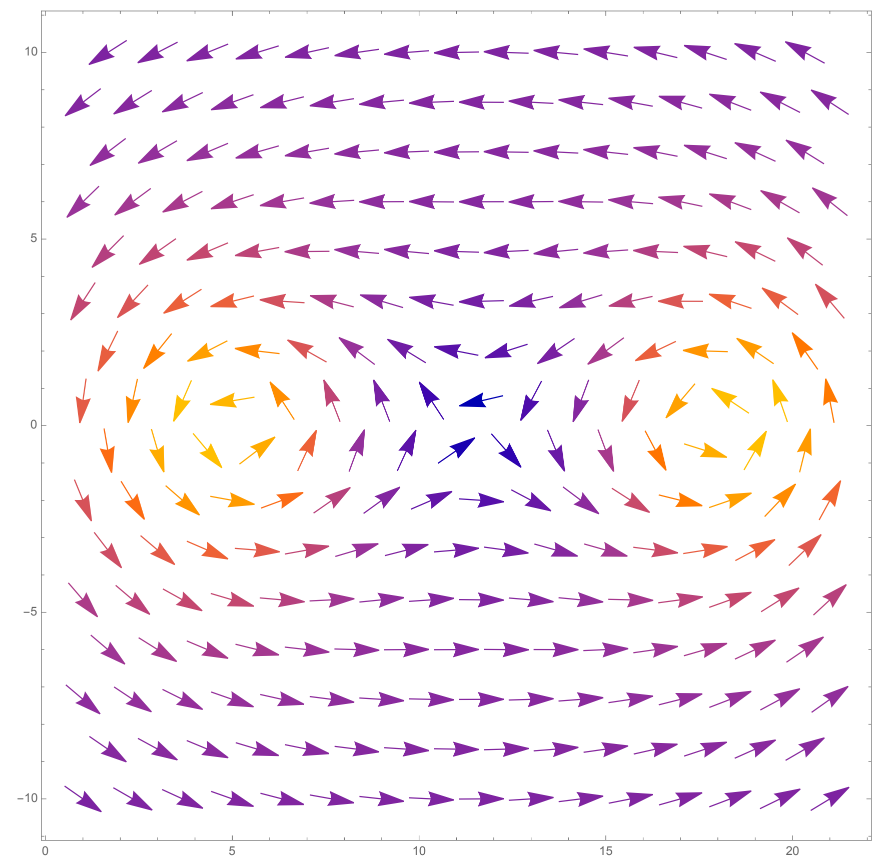

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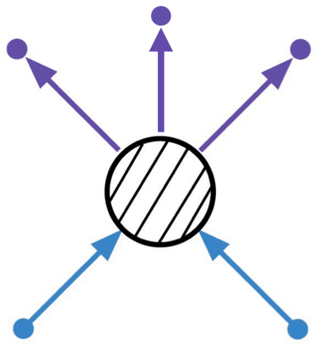

s-wave two doublet case -> 2D

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Abelian Instanton

Atiyah-Singer index theorem:

four zero modes

Conclusions

Lorentz violating terms from soft photons
exponentiate to a phase

Weinberg's Lorentz violation/gauge dependence
is hidden in a 4D topological intersection number

rotations shift phase in accordance with
Pairwise Little Group

Pairwise Little Group fixes scattering amplitudes,
without Callan's "half-particles"