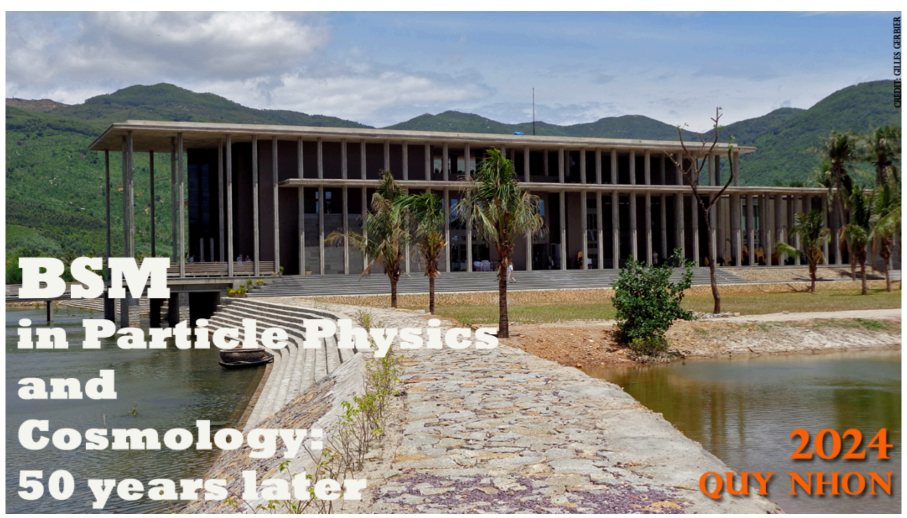
On the dynamical origin of the η' potential and the axion mass

Maximilian Ruhdorfer
Cornell University



20th Rencontres du Vietnam
January 9, 2024

based on arXiv:2307.04809 (JHEP)

with C. Csáki, R. d'Agnolo, R. Gupta, E. Kuflik *and* T. Roy

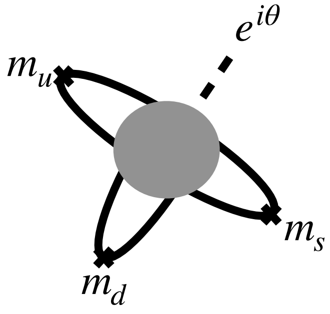
and

work in progress *with* C. Csáki *and* T. Youn

Axion and η' Mass in QCD

What generates the axion and η' mass in ordinary QCD?

typical phenomenologist: **instantons?**



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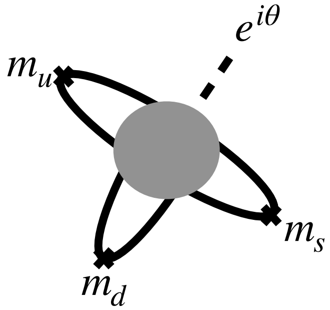

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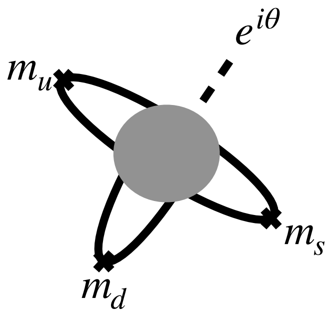

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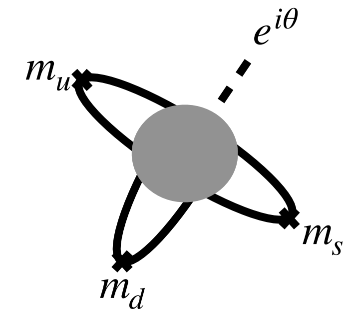

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  Vacuum energy in pure SU(N) YM

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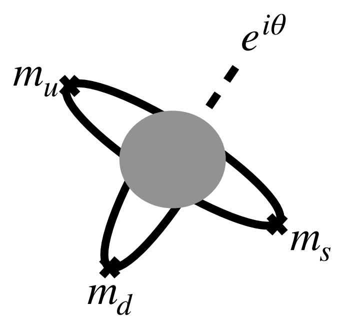

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What happens in finite N and F QCD?

η' in SUSY QCD with AMSB

- QCD is non-perturbative in IR → cannot check explicitly what is going on

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- QCD is non-perturbative in IR → cannot check explicitly what is going on

- **BUT:** can check in QCD-like theory

→ softly-broken SUSY QCD is (fully) calculable

→ SUSY QCD with AMSB has vacuum with chiral symmetry breaking for $F \leq 3N$

Murayama '21
Csaki et al '22

→ ordinary QCD would be recovered in large SUSY breaking limit

Outline

Part 1 (η': Standard lore in ordinary QCD):

- Chiral Lagrangian and η' potential
- Insights from large N QCD

Part 2 (SUSY QCD with AMSB):

- Review of SUSY QCD and AMSB
- Chiral Lagrangian in SUSY QCD with AMSB
- Spontaneous CP breaking at $\bar{\theta} = \pi$

Part 1: η' standard lore in ordinary QCD

Chiral Lagrangian

- F flavor QCD with N colors exhibits SSB at low energies

$$U(F)_L \times U(F)_R \to U(F)_d$$

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- Parameterize N^2 GBs in unitary matrix

$$U \in U(F)_A \simeq [U(1)_A \times SU(F)_A]/\mathbb{Z}_F$$

decay constants absorbed in η' and π^a

$$U = e^{i\eta'} e^{i\pi^a T^a}$$

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- $U(F)_A$ is explicitly broken by quark masses m_Q

- $U(1)_A$ additionally explicitly broken by anomaly $\partial_\mu j^\mu_A = F \frac{g^2}{8\pi^2} \mathrm{Tr} G\tilde{G}$

Chiral Lagrangian

- Lagrangian at 2-derivatives

$$\mathcal{L} = \frac{f_\pi^2}{4}\mathrm{Tr}\left[(\partial_\mu U)^\dagger \partial^\mu U\right] + \alpha\Lambda f_\pi^2 \left(\mathrm{Tr}\left[m_Q U\right] + \mathrm{h.c.}\right)$$

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- Spurion analysis for contribution from anomaly

under chiral rotation:

$$\eta' \to \eta' + 2\varphi$$

$$\frac{\theta}{8\pi^2}\mathrm{Tr}G\tilde{G} \to \frac{\theta - 2F\varphi}{8\pi^2}\mathrm{Tr}G\tilde{G}$$

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➡ Lagrangian invariant if $\theta \to \theta + 2F\varphi$

➡ invariant combination $\theta - F\eta' = \theta + i\log\det U$

η' Potential

- Simplest term

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- Potential $V_{\eta'} = -2b\Lambda^2 f_\pi^2 \cos(\theta - F\eta') \; + \mathcal{O}(m_Q)$

➡ η' mass is $O(\Lambda)$: $m_{\eta'}^2 \propto bF\Lambda^2 \gg m_\pi^2$

non-canonical kinetic term

$$\mathcal{L}_{\text{kin}} = (2F)\frac{1}{2}\partial_\mu \eta' \partial^\mu \eta'$$

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- Integrate out η'

$$\langle \eta' \rangle = \frac{\theta}{F} \quad \text{for} \quad \eta' \in \left[0, \tfrac{2\pi}{F}\right]$$

→ $U = e^{i\theta/F} e^{i\pi^a T^a}$

Large N limit

- Anomaly vanishes in $N \to \infty$ limit

$$\partial_\mu j_A^\mu \sim F \frac{g^2}{8\pi^2} \mathrm{Tr} G\tilde{G} \sim \frac{\lambda}{8\pi^2} \frac{F}{N} \mathrm{Tr} G\tilde{G} \to 0$$

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Witten '79

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- Veneziano-Witten formula for η' mass

$\theta \to \theta - F\eta'$

$$m_{\eta'}^2 = \frac{2F}{f_\pi^2} \frac{d^2 E}{d\theta^2}\bigg|_{\theta=0}^{\text{pure QCD}} \propto \frac{1}{N}$$

Witten '79
Veneziano '79

Large N η' Potential

- $m_{\eta'}^2 \propto 1/N$ suggests potential of the form

$$\mathcal{L}_{\eta'} = N\Lambda^2 f_\pi^2 (e^{-i\theta}\det U)^{1/N} + h.c.$$

inspired by SQCD

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$m_{\eta'}^2 \sim 1/N$

$\eta'^4 \sim 1/N^4$

as expected from large N

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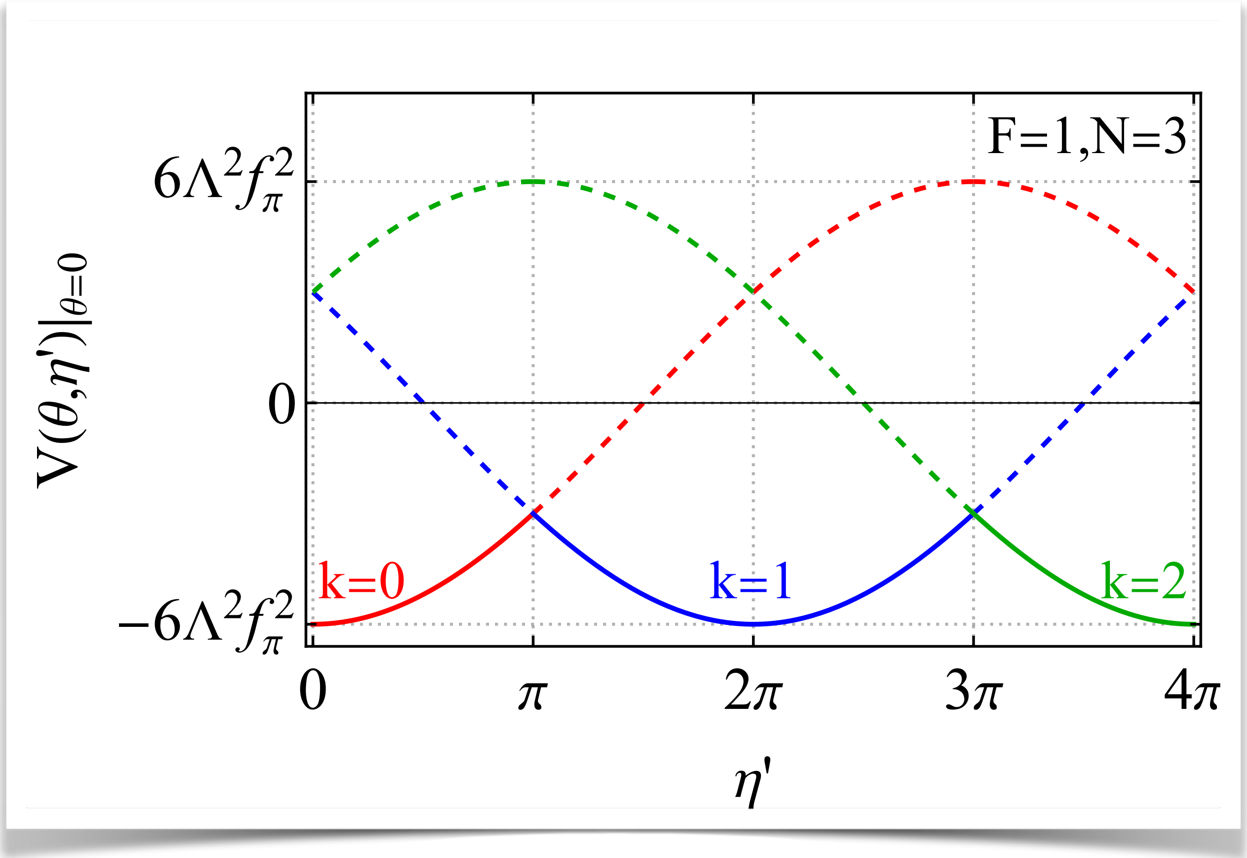

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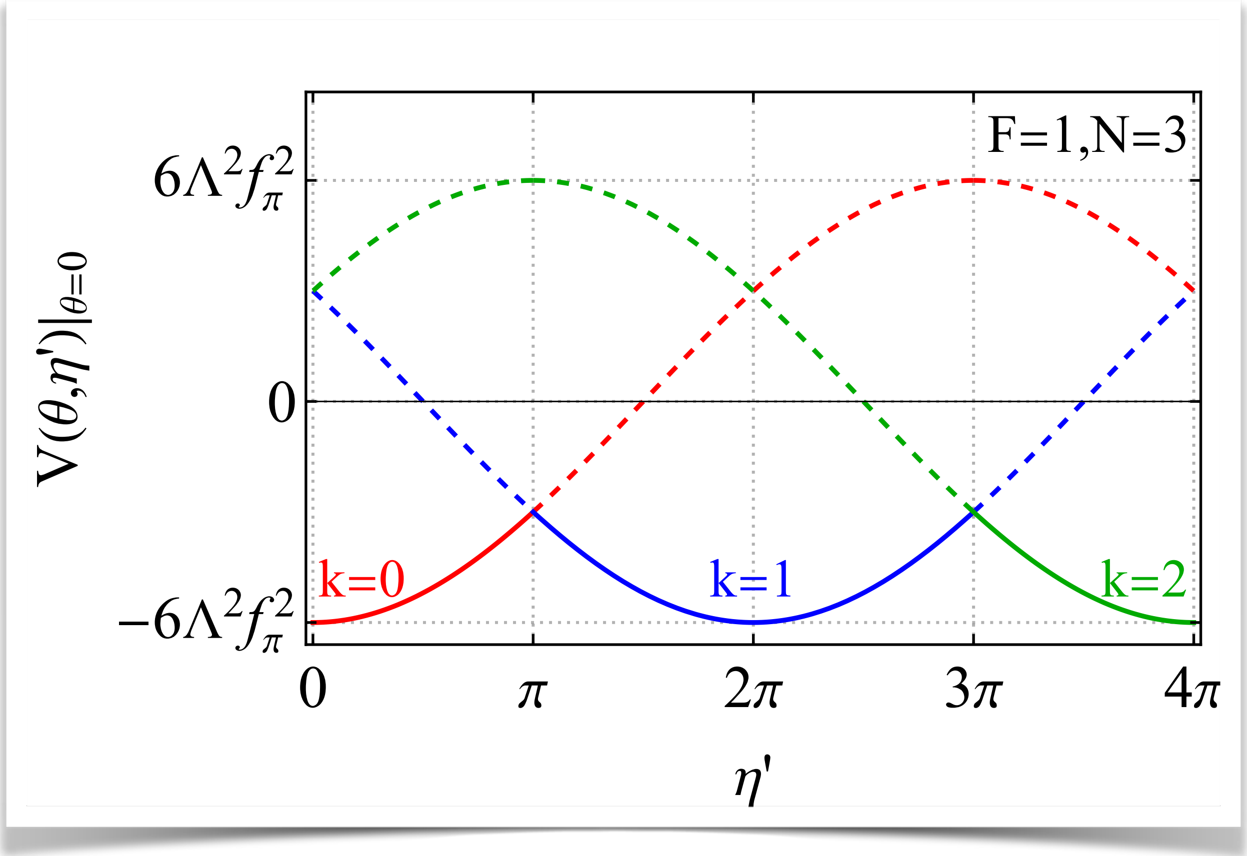

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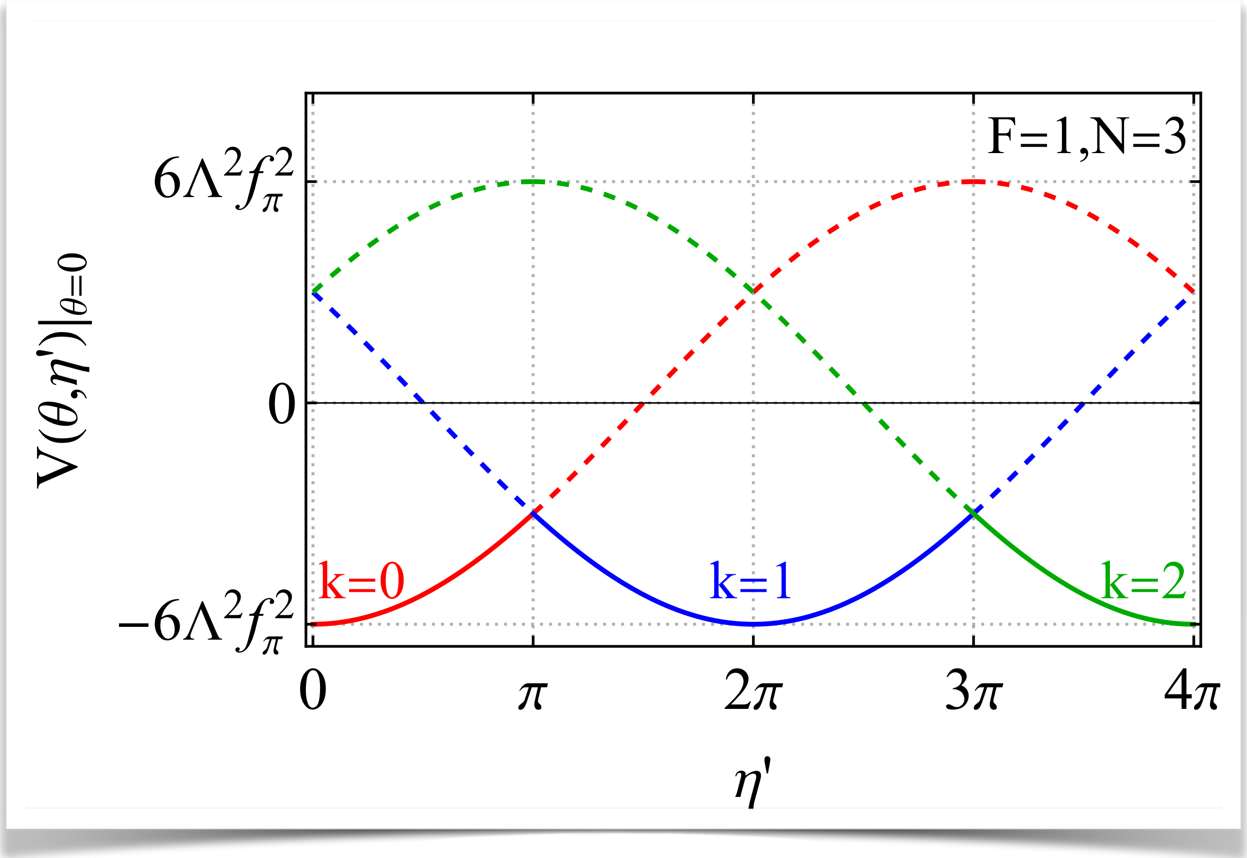

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But: branch structure is indication that it is **not** an instanton effect

Part 2:
SUSY QCD with AMSB

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SUSY QCD

Why SUSY QCD?

Non-perturbative effects are known!!

→ superpotential is uniquely fixed

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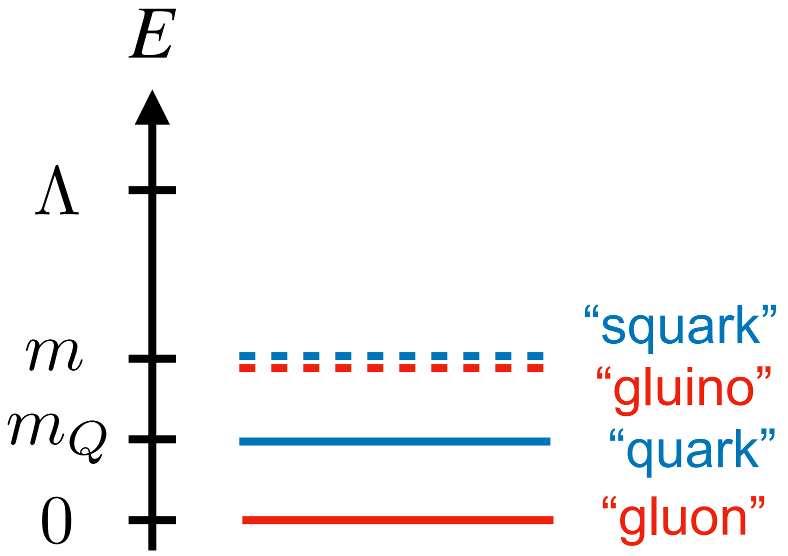
SUSY breaking $m_Q \ll m \ll \Lambda$



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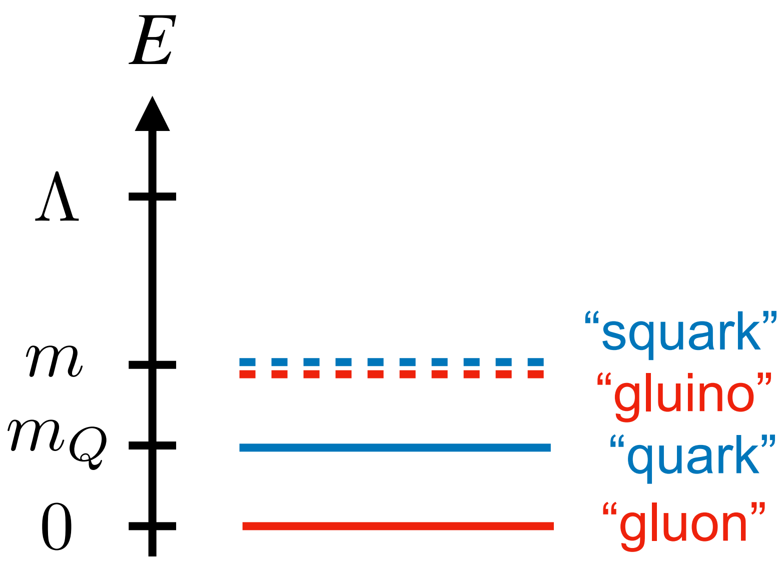

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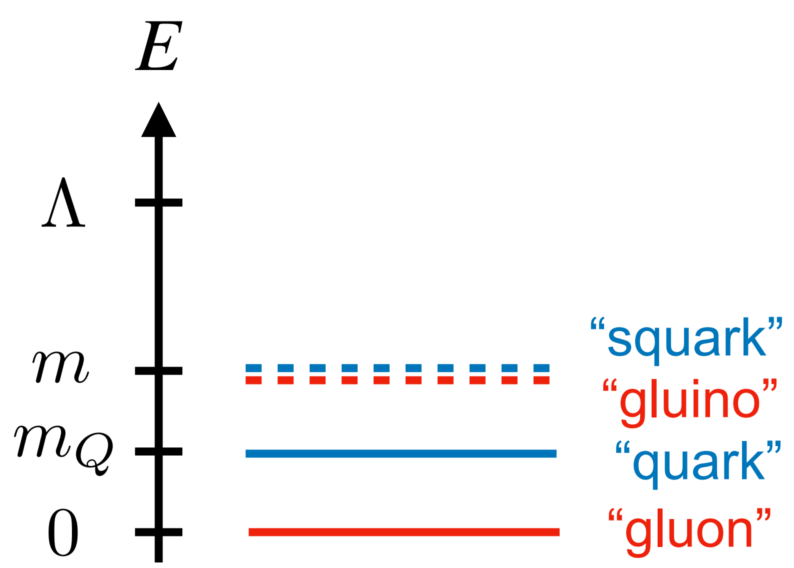

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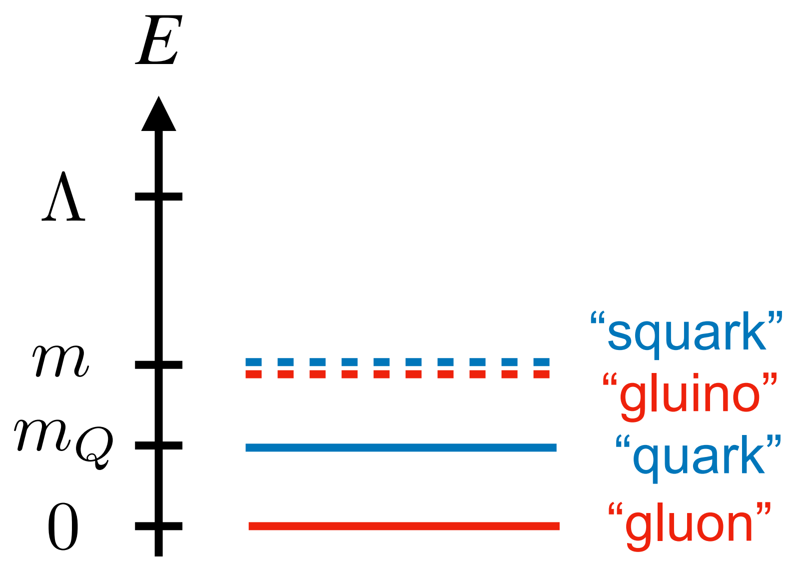

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But: can construct chiral Lagrangian from top-down

SUSY QCD

- $SU(N)$ SYM with F flavors, i.e. $Q_f, \bar{Q}_f, f = 1,\ldots,F$ in $N, \bar{N}$ representation

	$SU(N)$	$SU(F)_L$	$SU(F)_R$	$U(1)$	$U(1)_R$
Q	$\square$	$\square$	$\mathbf{1}$	1	$\frac{F-N}{F}$
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QCD-like minima $\langle M \rangle \propto \mathbb{1}$

chiral symmetry breaking
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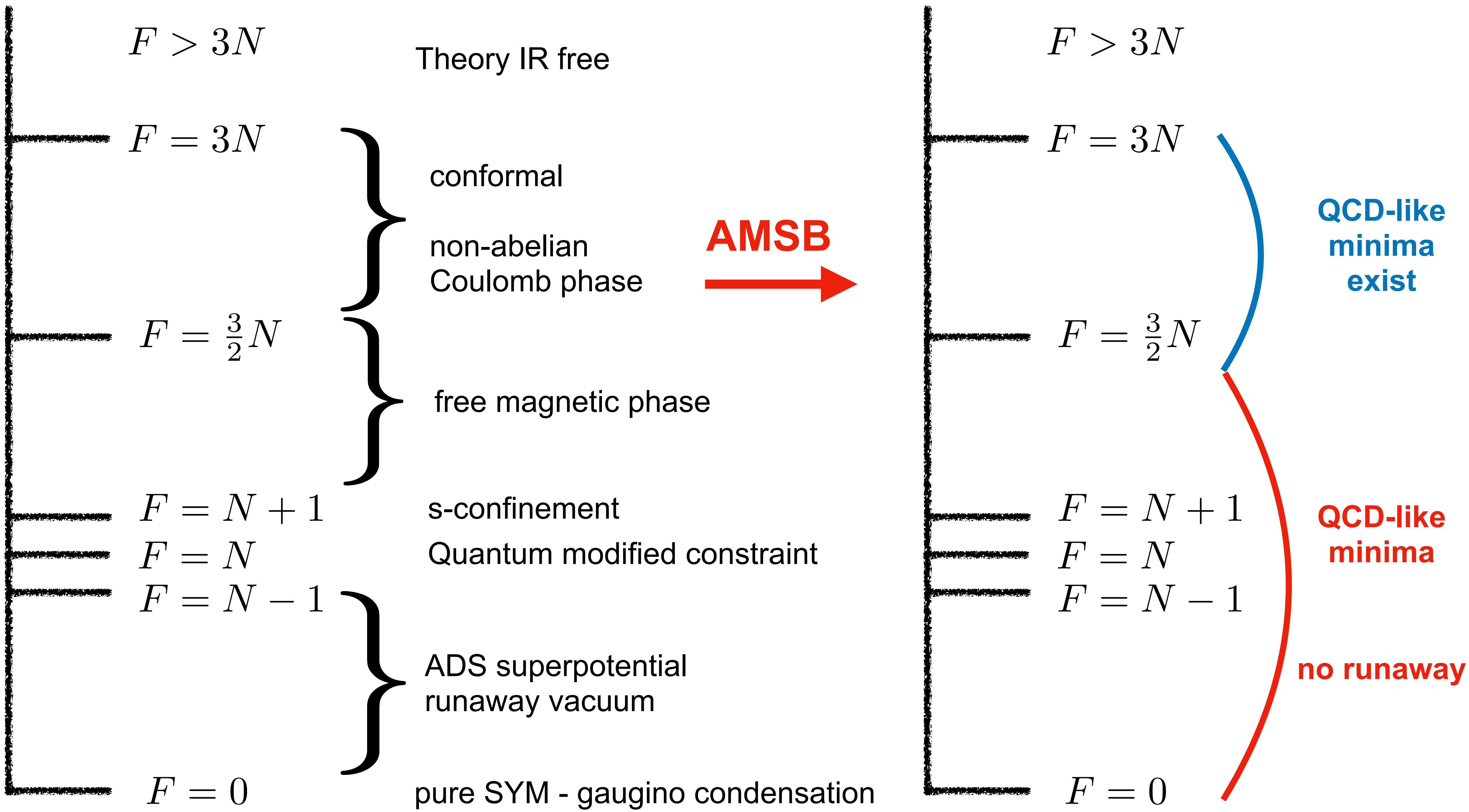
→ pure SUSY QCD has many phases!!

SUSY QCD Phase Diagram

$F > 3N$	Theory IR free
$F = 3N$ to $F = \frac{3}{2}N$	conformal non-abelian Coulomb phase
$F = \frac{3}{2}N$ to $F = N + 1$	free magnetic phase
$F = N + 1$	s-confinement
$F = N$	Quantum modified constraint
$F = N - 1$ to $F = 0$	ADS superpotential runaway vacuum
$F = 0$	pure SYM - gaugino condensation

SUSY QCD Phase Diagram



Murayama '21
Csaki et al '22

Anomaly-Mediated SUSY Breaking (AMSB)

Randall, Sundrum '98
Giudice, Luty, Murayama, Rattazzi '98
Arkani-Hamed, Rattazzi '98

- Anomaly mediation: SUSY breaking effects where scale invariance is broken

→ effects described with chiral compensator (UV insensitive)

Pomarol, Rattazzi '99

$$\mathcal{L} = \int d^4\theta \Phi^* \Phi K + \int d^2\theta \Phi^3 W + c.c. \qquad \text{with} \quad \Phi = 1 + \theta^2 m$$

- Generates tree-level scalar potential

$$V_{\text{tree}} = \partial_i W g^{ij^*} \partial_j^* W^* + m^* m \left(\partial_i K g^{ij^*} \partial_j^* K - K \right) + m \left(\partial_i W g^{ij^*} \partial_j^* K - 3W \right) + h.c.$$

Csaki et al '22

- Gluino and squark masses are loop generated

$$m_\lambda = \frac{g^2}{16\pi^2}(3N - F)m \,, \quad m_{\widetilde{Q}}^2 = \frac{g^4}{(8\pi^2)^2} 2C_i (3N - F) m^2$$

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- Similar results by Dine, Draper, Stephenson-Haskins and Xu (2016) using soft-breaking via explicit gaugino and squark masses

  → only reliable for $F < N$

Deriving the Chiral Lagrangian

Step 1: Find scalar potential

$$W(M) \qquad\qquad K(M, M^\dagger, \ldots)$$

$$V_{\text{tree}} = \partial_i W g^{ij^*} \partial_j^* W^* + m\left(\partial_i W g^{ij^*} \partial_j^* K - 3W\right) + \text{ h.c. } + m^* m\left(\partial_i K g^{ij^*} \partial_j^* K - K\right)$$

Deriving the Chiral Lagrangian

Step 1: Find scalar potential

$$W(M) \qquad\qquad K(M, M^\dagger, \ldots)$$

$$V_{\text{tree}} = \partial_i W g^{ij^*} \partial_j^* W^* + m\left(\partial_i W g^{ij^*} \partial_j^* K - 3W\right) + \text{ h.c. } + m^* m\left(\partial_i K g^{ij^*} \partial_j^* K - K\right)$$

Step 2: Minimize Scalar potential

$$\rightarrow \quad \langle M \rangle = f^2 \mathbb{1}$$

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Step 2: Minimize Scalar potential

→ $\langle M \rangle = f^2 \mathbb{1}$

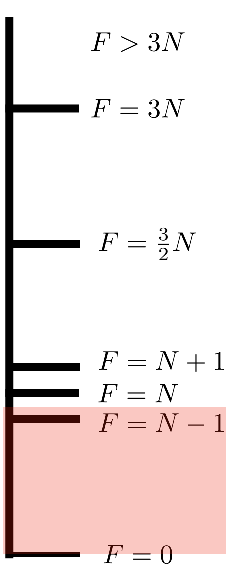
Step 3: Parameterize GBs as $M = f^2 U$

→ find chiral Lagrangian $V_{\chi PT} = V_{\rm tree}(M = f^2 U)$

F<N SUSY QCD

- IR degrees of freedom parameterized by meson matrix $M_{ff'} = \bar{Q}^a_f Q^a_{f'}$

$\langle M \rangle \propto \mathbf{1}$ breaks chiral symmetry



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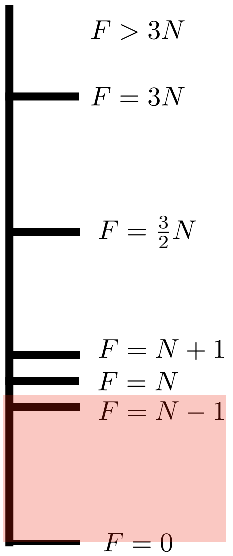

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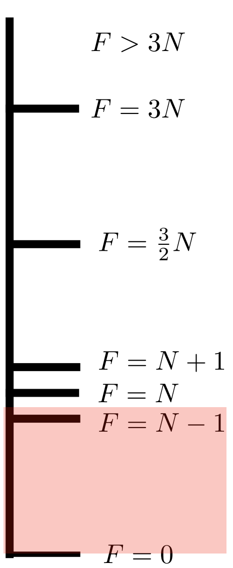
add mass term to mimick QCD

holomorphic scale $\Lambda^{b_0} = \mu^{b_0} e^{2\pi i \tau}$ with $\tau = \frac{4\pi i}{g^2} + \frac{\theta}{2\pi}$



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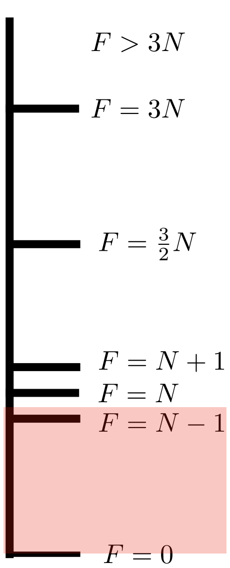
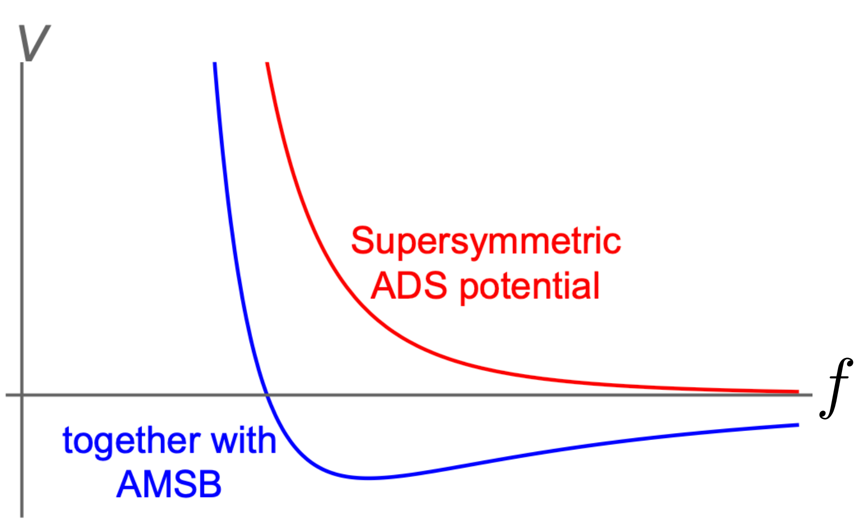
- AMSB stabilizes chiral symmetry breaking minimum





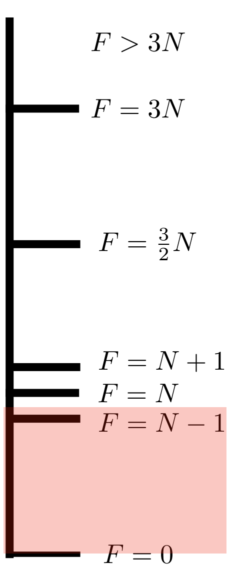
Murayama '21

with $\langle M \rangle = f^2 \mathbb{1}$

$$f = \Lambda\left(\frac{N+F}{3N-F}\frac{\Lambda}{m}\right)^{(N-F)/(2N)} + \mathcal{O}(m_Q/m)$$

F<N SUSY QCD: Chiral Lagrangian

- Parameterize GBs as fluctuations around minimum $M = |f|^2 U$



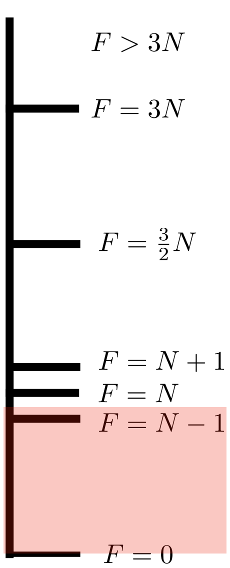
F<N SUSY QCD: Chiral Lagrangian

- Parameterize GBs as fluctuations around minimum $M = |f|^2 U$

- Chiral Lagrangian

$$V = -\,m\left[(3N-F)\left(\frac{\Lambda^{3N-F}}{|f|^{2F}}\right)^{1/(N-F)}\det(U)^{-1/(N-F)} + |f|^2\mathrm{Tr}(m_Q U)\right] + h.c.$$

$$-\,2\left(\frac{\Lambda^{3N-F}}{|f|^{2F}}\right)^{1/(N-F)}\det(U)^{-1/(N-F)}\mathrm{Tr}(m_Q^\dagger U^\dagger) + h.c\,.$$



Csaki, d'Agnolo, Gupta, Kuflik, Roy, MR

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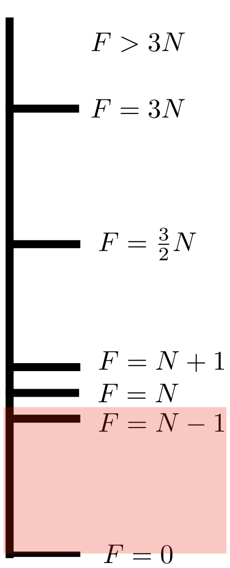
complex root of $\Lambda^{3N-F} = |\Lambda|^{3N-F} e^{i\theta}$ and $\det U = e^{iF\eta'}$

→ get branch-like structure

→ $N - F$ instead of N branches

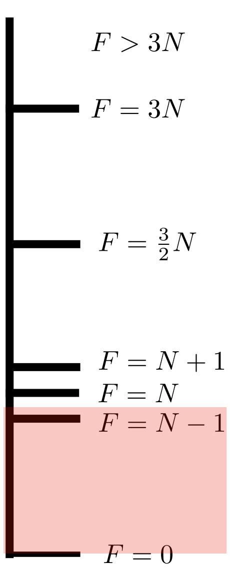
→ **not** an instanton effect but gluino condensation in unbroken $SU(N-F)$



F<N SUSY QCD: fixed F/N

- For $F/N =$ fixed the η' mass is constant

$$m_{\eta'}^2 = \frac{(x-3)^2 x}{(x+1)(x-1)^2} m^2 \,, \quad \text{with} \quad x = \frac{F}{N}$$



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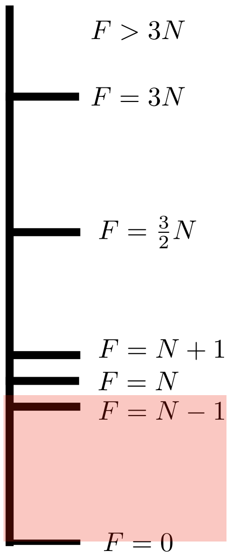
diverges for $x \to 1$, description breaks down



Maximilian Ruhdorfer (Cornell)

Csaki, d’Agnolo, Gupta, Kuflik, Roy, MR

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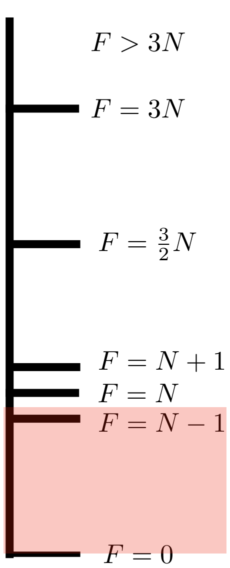
→ $m \to 0$ results in $f \to \infty$, i.e. potential becomes flat



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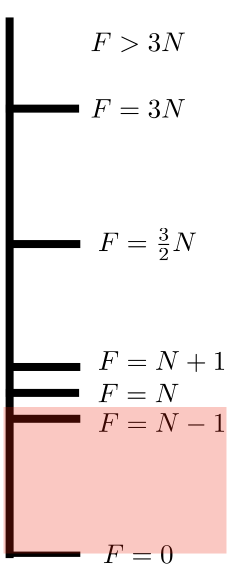
➡ non-anomalous $U(1)_R$ spontaneously broken

	$U(1)_A$	$U(1)_R$
M	2	$\frac{2(F-N)}{F}$
Λ^b	$2F$	0

η' massless as long as $U(1)_R$ not explicitly broken



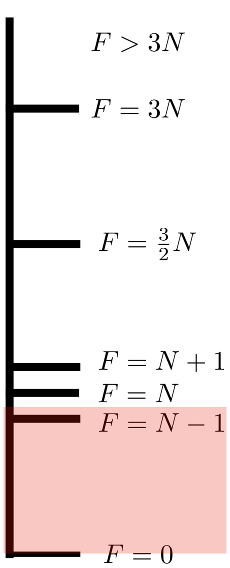
Csaki, d'Agnolo, Gupta, Kuflik, Roy, MR

F<N SUSY QCD: F=N-1

- $F = N - 1$: **no** branches and trivially 2π-periodic

$$V_k \overset{N\gg 1}{\to} -4N^{3/2}m^2|\Lambda_{\text{phys}}|^2\cos\left((N-1)\eta' - \theta\right) - 2N^{1/2}m|\Lambda_{\text{phys}}|^2\sum_{i=1}^{F} m_i \cos\left(\eta' + \theta_Q + \sum_{j=1}^{F-1} t_i^j \pi^j\right)$$
$$-4N^{1/2}m|\Lambda_{\text{phys}}|^2\sum_{i=1}^{F} m_i \cos\left(N\eta' + \theta_Q - \theta + \sum_{j=1}^{F-1} t_i^j \pi^j\right) .$$

➡ **Instanton** effect!



η' Potential SUSY QCD

- $F < N$ has $N - F$ branches
  - ➡ from gluino condensation in unbroken $SU(N-F)$
  - ➡ $F = N - 1$ has **no** branches and **is an instanton** effect
- $F = N$ and $F = N + 1$ do **not** have branches
  - ➡ consistent with an **instanton** effect
- $F > N + 1$ has $(F - N)$ branches
  - ➡ from gluino condensation in the dual $SU(F-N)$ theory

Spontaneous CP breaking at $\bar{\theta} = \pi$

- Gaiotto, Komargodski and Seiberg argued using large N and anomalies that at $\bar{\theta} = \pi$ CP might be spontaneously broken in QCD, depending on F and m_Q

see also Di Vecchia, Rossi, Veneziano, Yankielowicz '17

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- $F = 1$: there is a critical value for the quark masses $m_{Q,0} \sim \Lambda/N$ with
  - $m_Q < m_{Q,0}$ there is a unique CP conserving vacuum
  - $m_Q = m_{Q,0}$ η' is exactly massless
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We can see this explicitly in our results!

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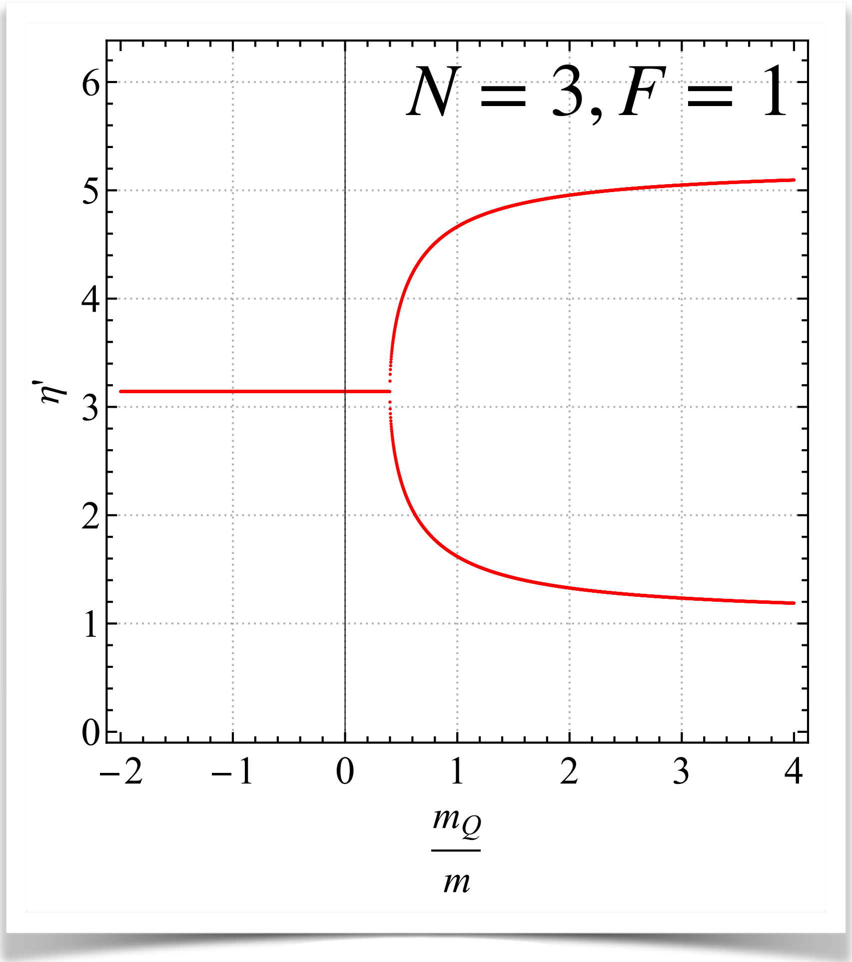
→ can be obtained analytically for $N = 3, F = 1$



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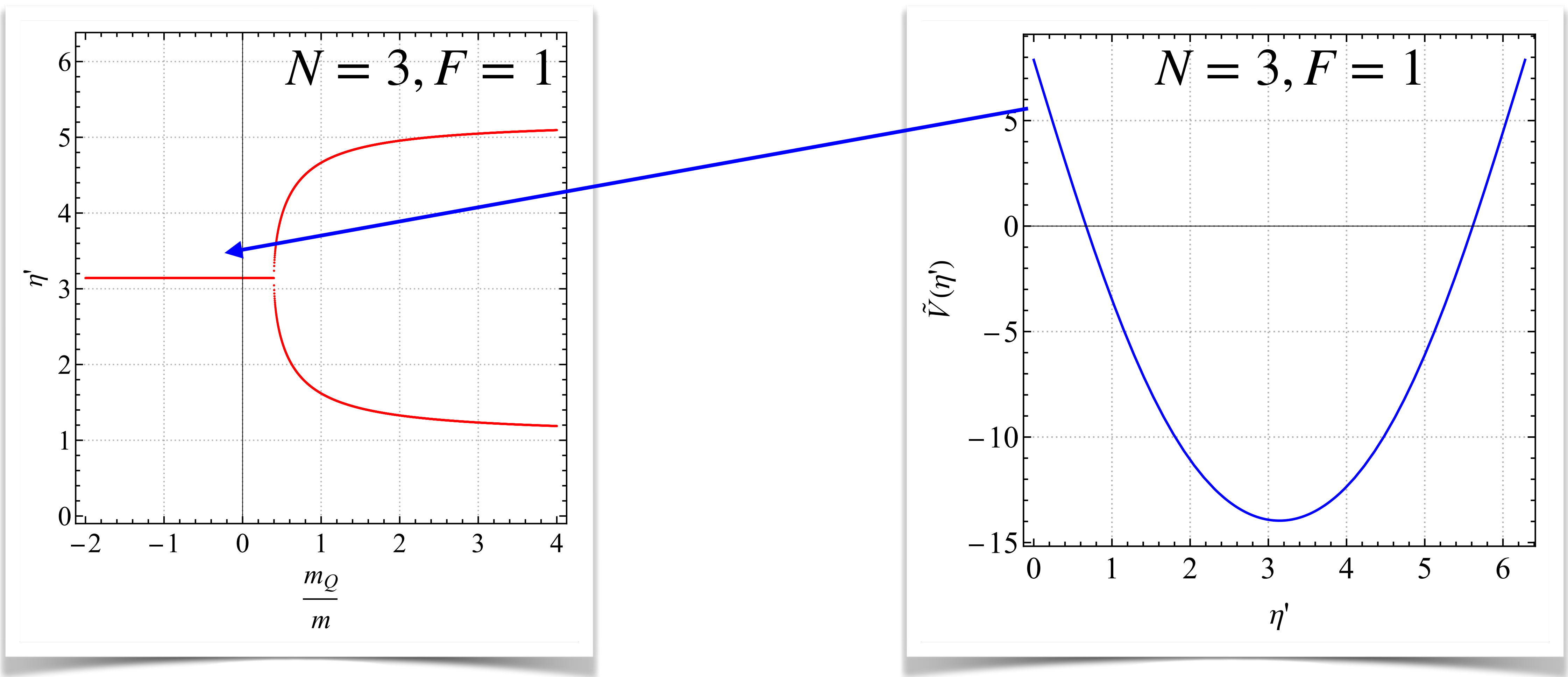

Csaki, MR, Youn in progress

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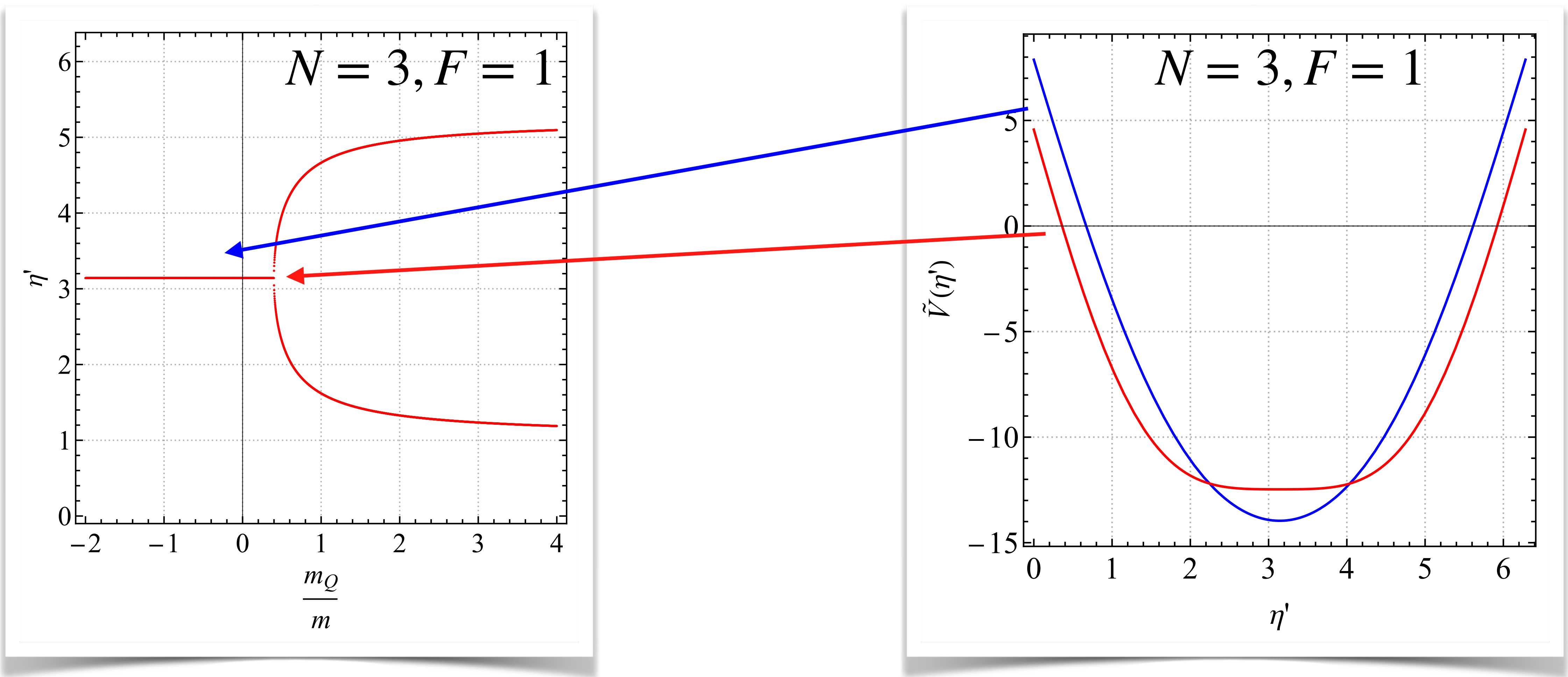

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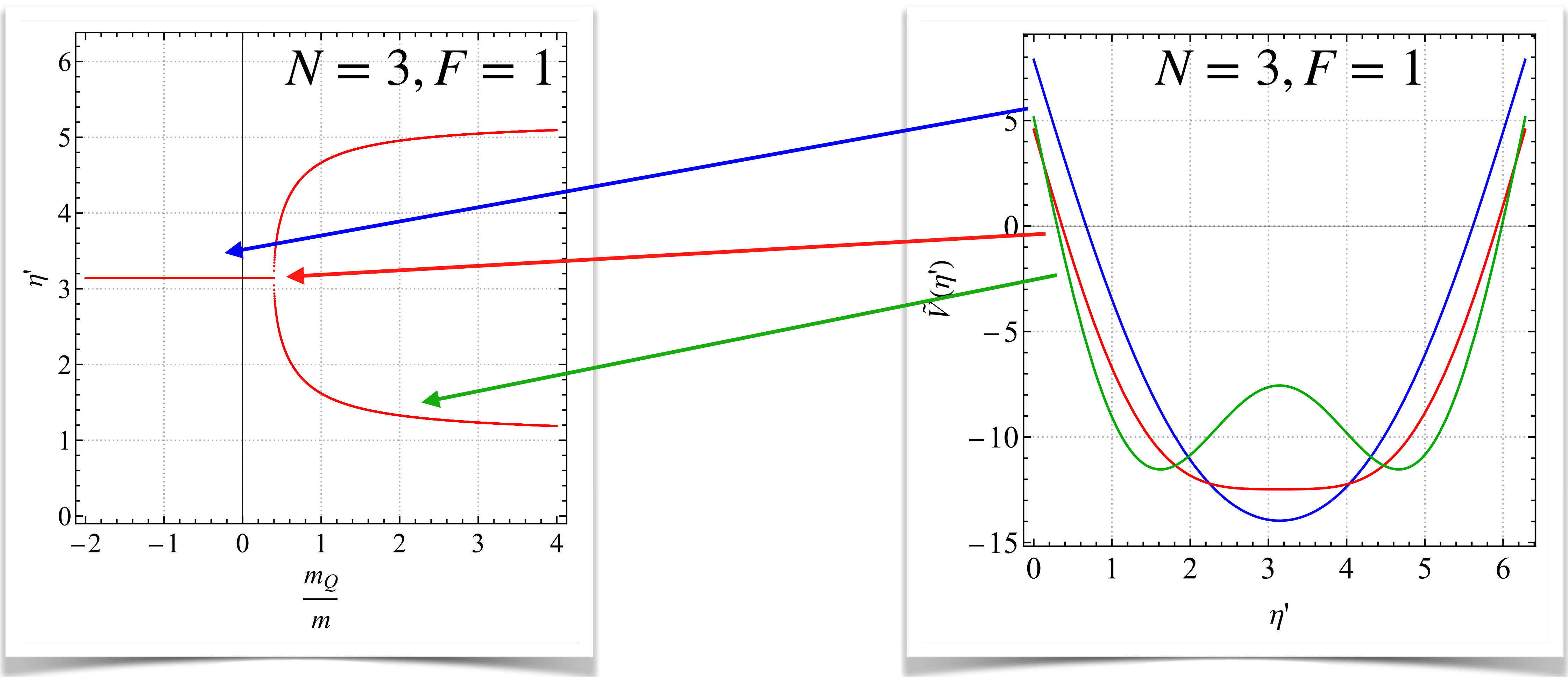

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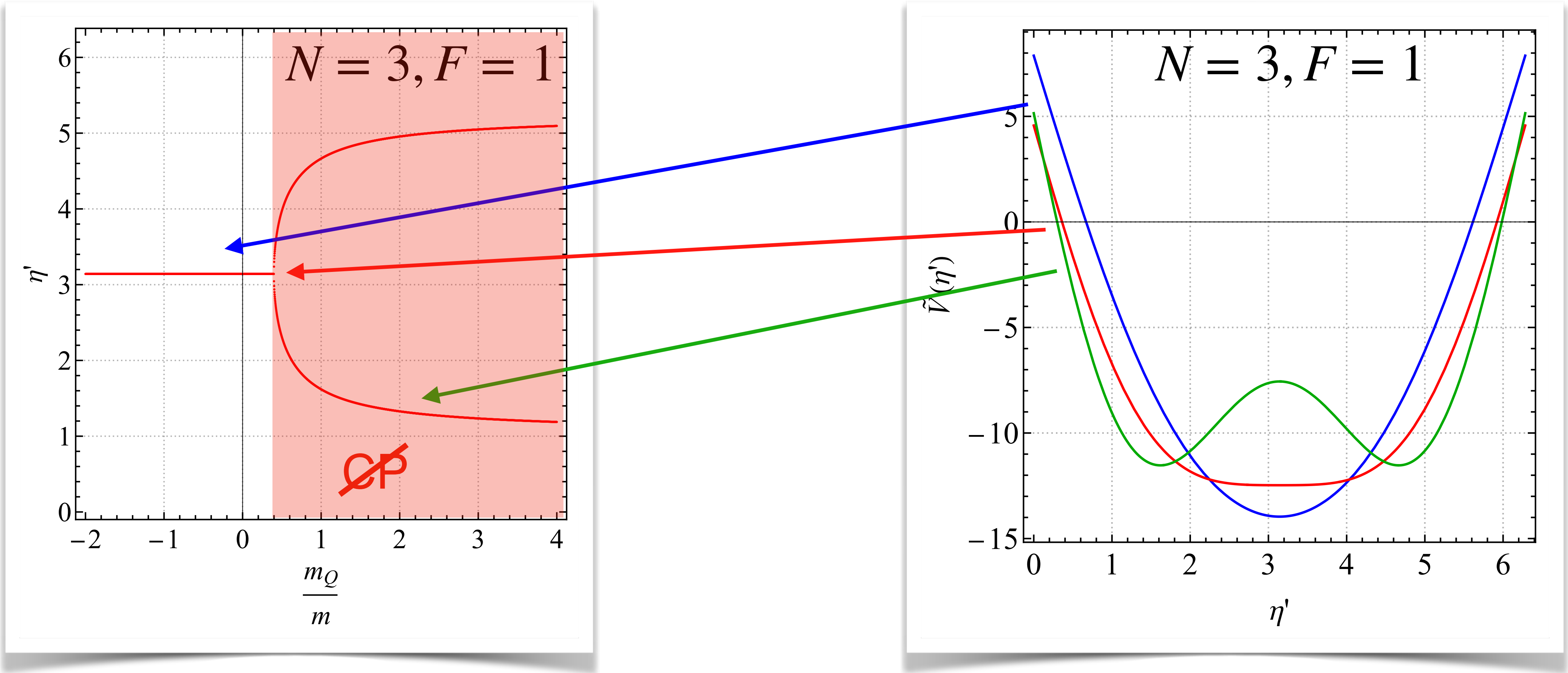

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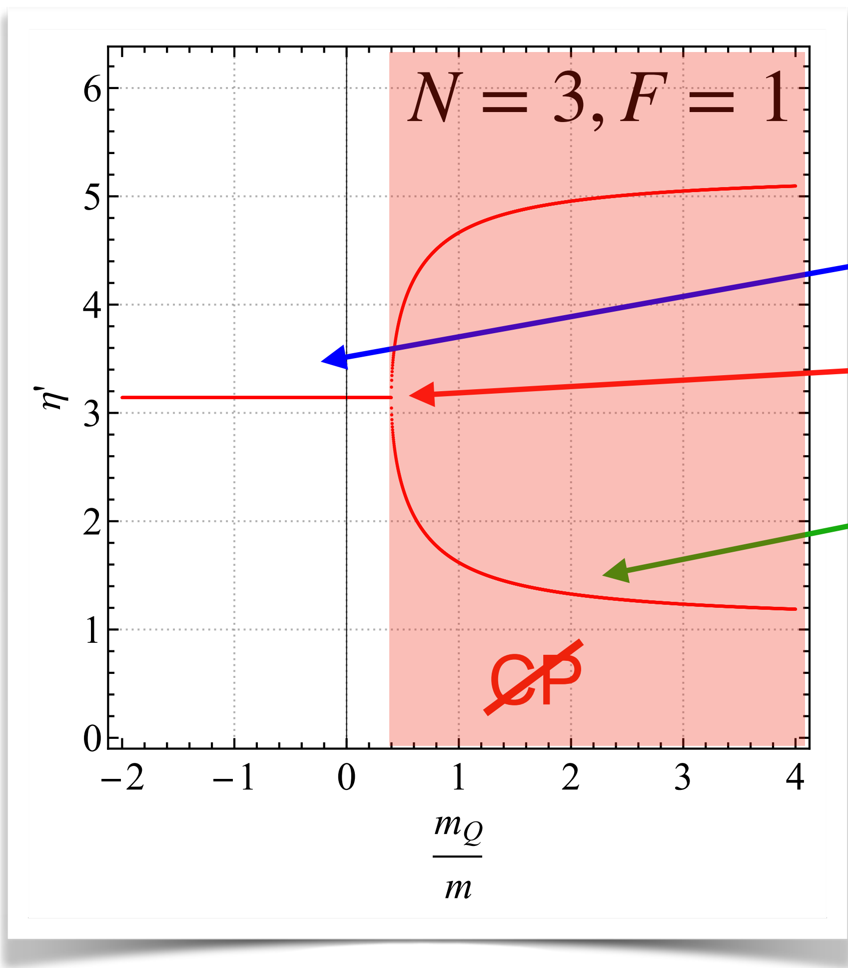
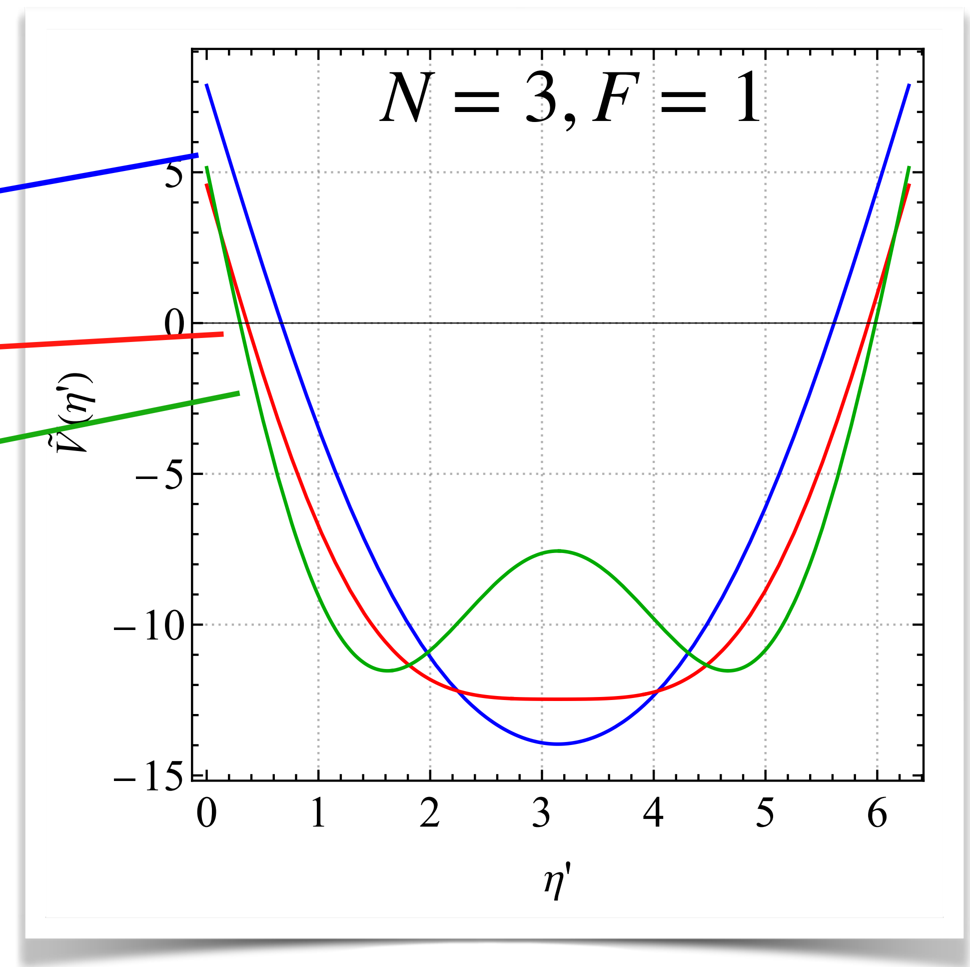
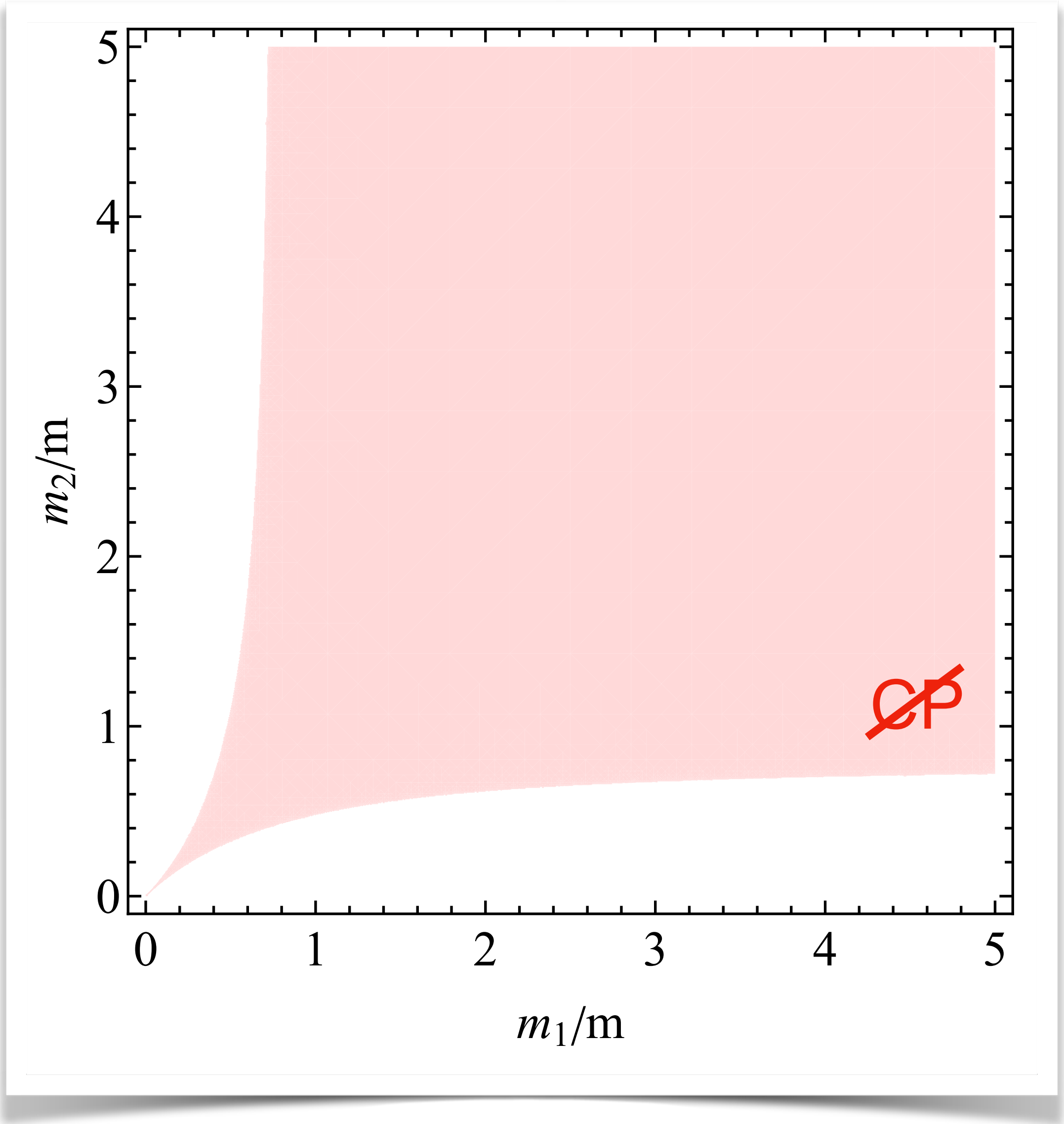




- Critical point for large N: $m_{Q,0} \xrightarrow{N \gg 1} \frac{9}{7} \frac{m}{N}$

Spontaneous CP breaking at $\bar{\theta} = \pi$

- $F > 1$: we consider $N = 3, F = 2$ as an example



Csaki, MR, Youn in progress

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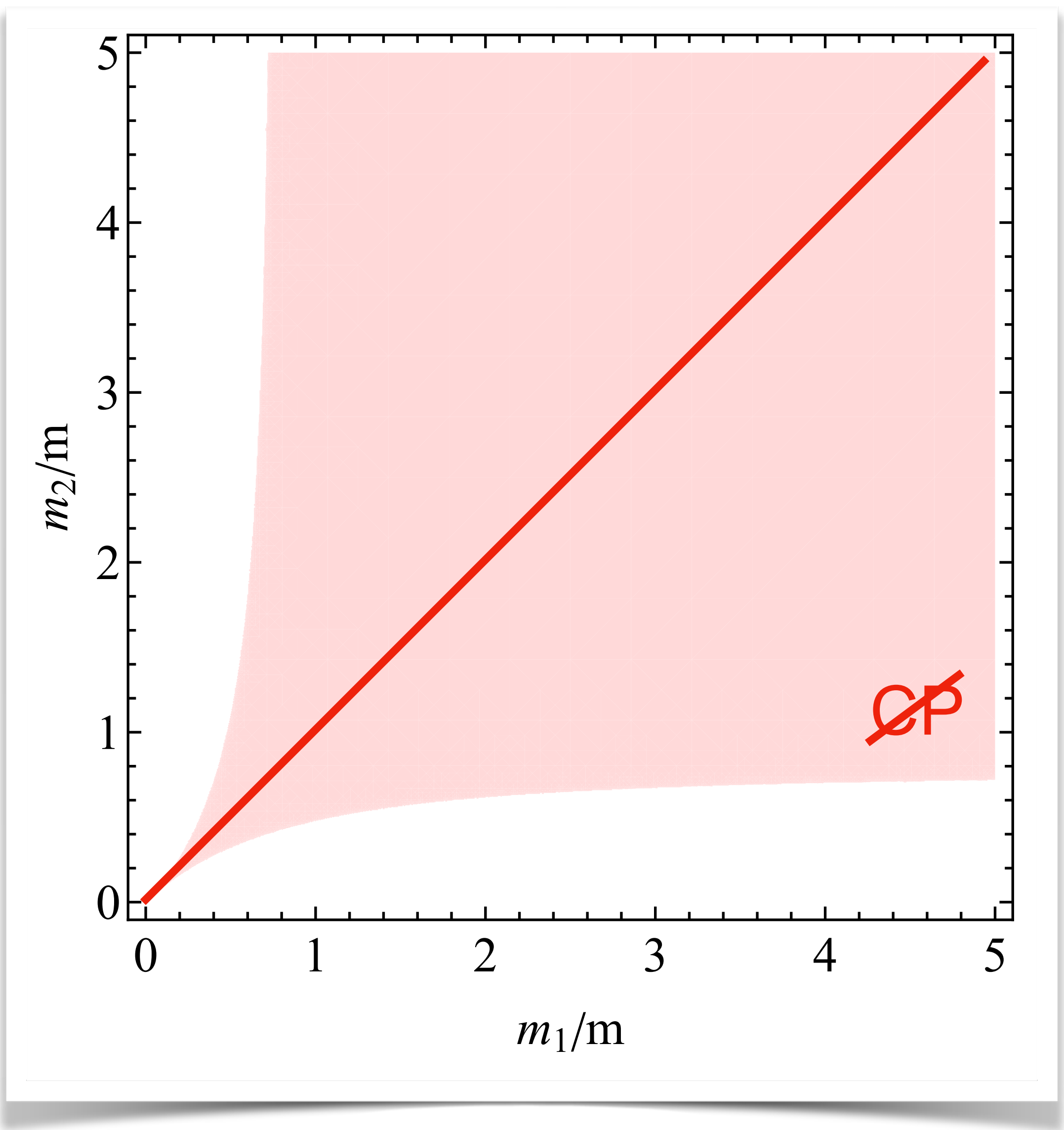

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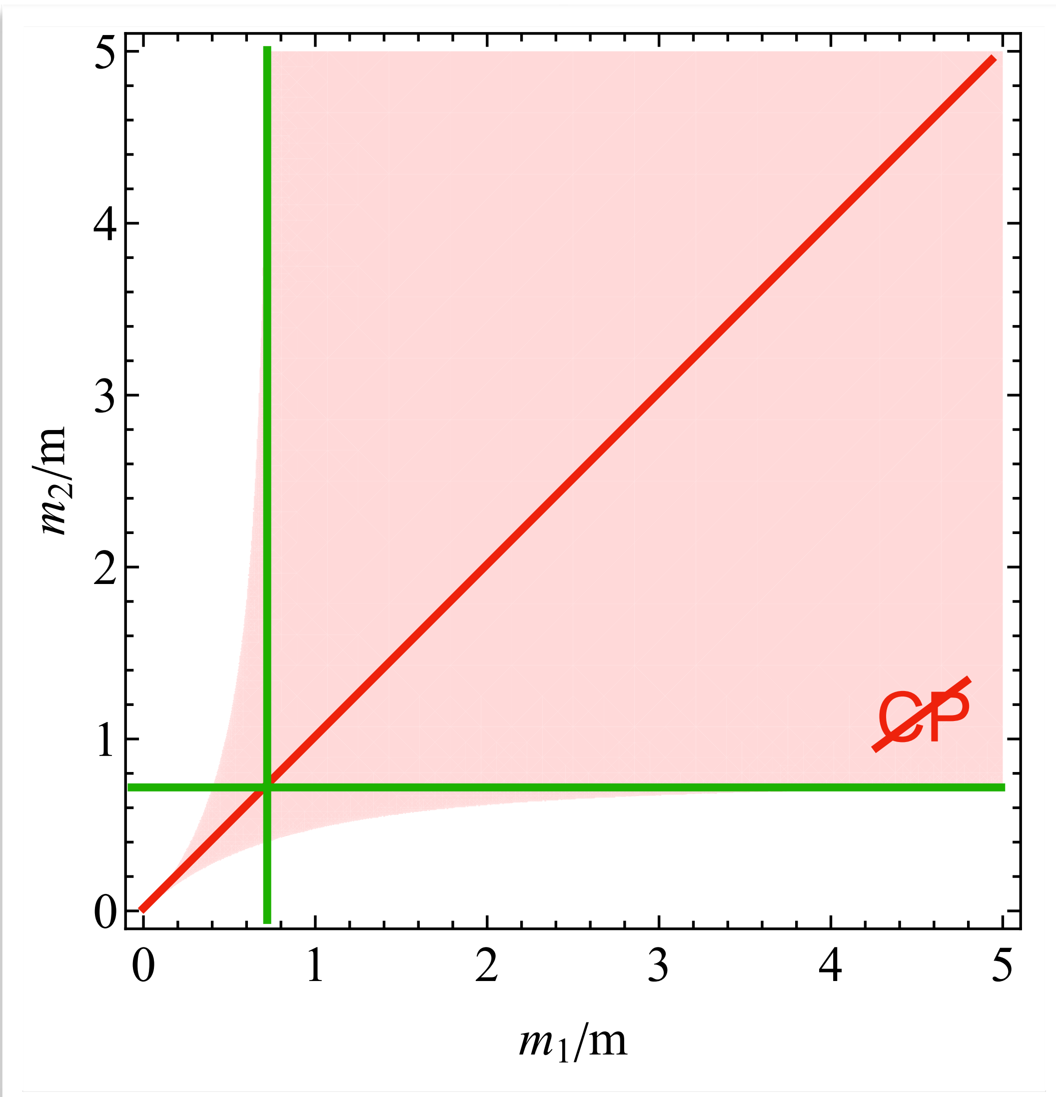

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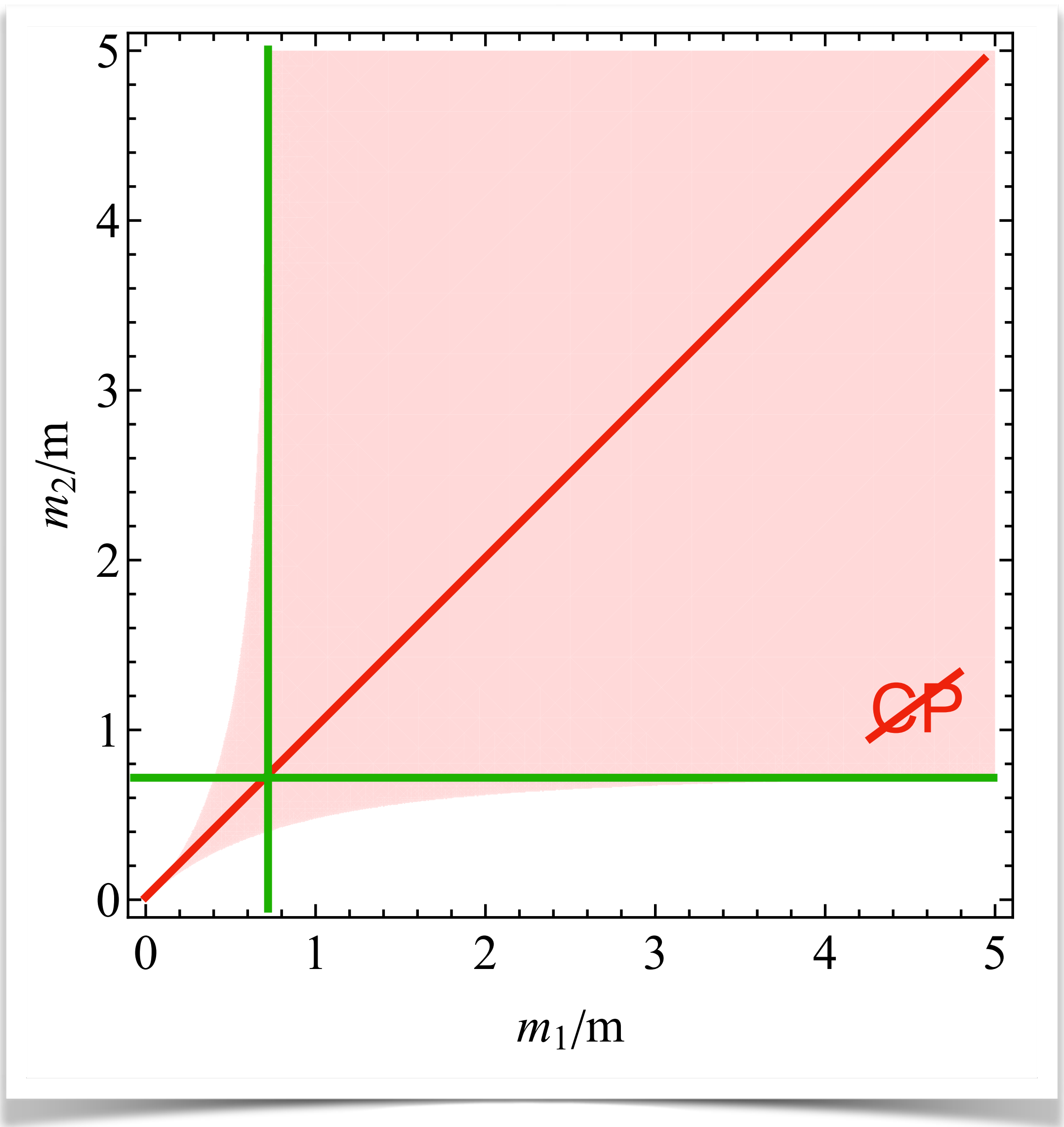

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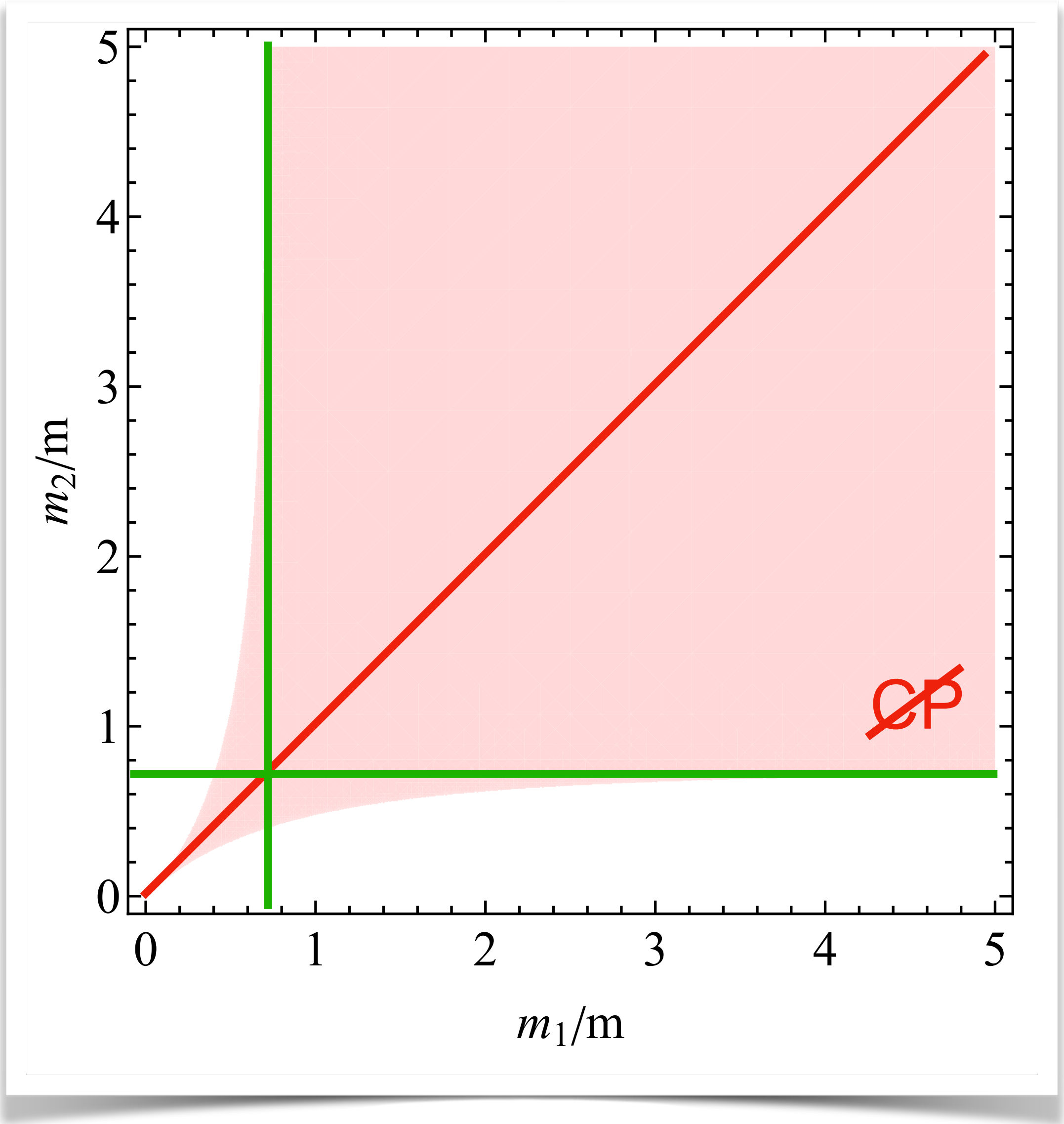

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Same structure as in large N QCD

Di Vecchia, Rossi, Veneziano, Yankielowicz ‘17

Lessons from SUSY

- η' potential has in general a branched structure with $|N-F|$ branches

  → originates from confinement dynamics and **not** instantons

- For $F = N-1, N, N+1$ we find a single branch

  → consistent with being an **instanton** effect

- η' mass vanishes for $F \ll N$ and is constant for $F/N =$ fixed

- SUSY QCD predicts spontaneous CP breaking at $\bar{\theta} = \pi$ in accordance with QCD

  Gaiotto, Komargodski and Seiberg '17
  Di Vecchia, Rossi, Veneziano, Yankielowicz '17

Which scenario is closest to ordinary QCD?

Backup

Pion Potential

$$V_\pi = -\alpha \Lambda f_\pi^2 e^{i\bar{\theta}/F} \mathrm{Tr}(m_q e^{i\pi^a T^a}) + \mathrm{h.c.}$$

- Potential for neutral GBs

quark masses

$F-1$ generators of Cartan sub-algebra

$$V_{\pi^0} = -2\alpha\Lambda f_\pi^2 \sum_{i=1}^{F} m_i \cos\left(\frac{\bar{\theta}}{F} + \sum_{j=1}^{F-1} t_i^j \pi^j\right)$$

➡ if one $m_i = 0$, can absorb $\bar{\theta}$ into pion VEVs

➡ if all $m_i \neq 0$, need extra DOF to eliminate $\bar{\theta}$: $\bar{\theta} \to \bar{\theta} + n\, a$

axion

anomaly coefficient

Axion Potential

- Chiral non-linearly realized $U(1)_{\rm PQ}$ with axion a under which

$$a \to a + \varphi \qquad\qquad \theta \to \theta - n\varphi$$

→ invariant combination $\theta + na - F\eta'$

- Full potential

$$V_k\left(\eta', a, \pi^j\right) = -2N\Lambda^2 f_\pi^2 \cos\left(\frac{\theta - F\eta' + na + 2\pi k}{N}\right) - 2\alpha\Lambda^2 f_\pi^2 \sum_{i=1}^{F} \frac{m_i}{\Lambda} \cos\left(\eta' + \theta_q + \sum_{j=1}^{F-1} t_i^j \pi^j\right)$$

axion potential is **not** generated by IR instantons!

But there can be **additional** contributions from **small** instantons!

- Integrating out η' gives standard potential

$$V_a = -2\alpha\Lambda^2 f_\pi^2 \sum_{i=1}^{F} \frac{m_i}{\Lambda} \cos\left(\frac{\bar\theta + an}{F} + \sum_{j=1}^{F-1} t_i^j \pi^j\right)$$

Axion Mass

$$V_a = -2\alpha\Lambda^2 f_\pi^2 \sum_{i=1}^{F} \frac{m_i}{\Lambda} \cos\left(\frac{\bar{\theta} + an}{F} + \sum_{j=1}^{F-1} t_i^j \pi^j\right)$$

- Easy to obtain axion mass for general F
- Integrate out pions to leading order in $\frac{f_\pi}{f_a}$ instead of diagonalizing mass matrix

➡ only need to solve linear equations

- Axion mass $m_a^2 = \alpha\Lambda n^2 \frac{f_\pi^2}{f_a^2}\left(\sum_{i=1}^{F} m_i^{-1}\right)^{-1}$

➡ in terms of pion mass $m_a^2 = \frac{n^2 F}{2(F-1)} \frac{f_\pi^2}{f_a^2} \frac{\mathrm{Tr}\, m_\pi^2}{\mathrm{Tr}\, m_q \mathrm{Tr}\, m_q^{-1}}$

➡ $F = 2:$ $m_a^2 = 2\alpha\Lambda n^2 \frac{f_\pi^2}{f_a^2} \frac{m_u m_d}{m_u + m_d} = n^2 m_\pi^2 \frac{f_\pi^2}{f_a^2} \frac{m_u m_d}{(m_u + m_d)^2}$

Maximilian Ruhdorfer (Cornell)

Csaki, d'Agnolo, Gupta, Kuflik, Roy, MR

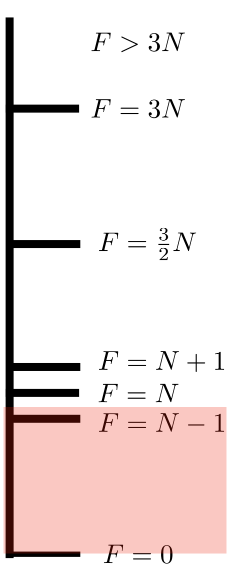
F<N SUSY QCD: Chiral Lagrangian

- Full chiral Lagrangian

$$
\begin{aligned}
V_k = & -2(3N-F)\left(\frac{N+F}{3N-F}\right)^{-F/N}\left(\frac{m}{|\Lambda|}\right)^{F/N} m|\Lambda|^3 \cos\left(\frac{F}{N-F}\eta' - \frac{\theta+2\pi k}{N-F}\right) \\
& -2\left(\frac{N+F}{3N-F}\right)^{1-F/N}\left(\frac{m}{|\Lambda|}\right)^{F/N} |\Lambda|^3 \sum_{i=1}^{F} m_i \cos\left(\eta' + \theta_Q + \sum_{j=1}^{F-1} t_i^j \pi_j\right) \\
& -4\left(\frac{N+F}{3N-F}\right)^{-F/N}\left(\frac{m}{|\Lambda|}\right)^{F/N} |\Lambda|^3 \sum_{i=1}^{F} m_i \cos\left(\frac{N}{N-F}\eta' + \theta_Q - \frac{\theta+2\pi k}{N-F} + \sum_{j=1}^{F-1} t_i^j \pi_j\right)
\end{aligned}
$$

- Has branch-like structure but with $1/(N-F)$ instead of $1/N$

→ **not** an instanton effect but gluino condensation in unbroken $SU(N-F)$



F<N SUSY QCD: Special Cases

- $F = 0$: vacuum energy as conjectured in large N pure QCD

$$V_k \xrightarrow{F=0} -6N^2 m|\Lambda_{\text{phys}}|^3 \cos\left(\frac{\theta + 2\pi k}{N}\right)$$

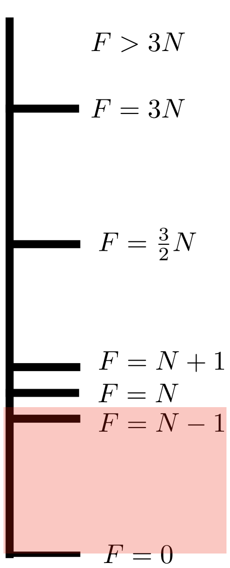
$\Lambda \sim N^{1/3}\Lambda_{\text{phys}}$

➡ **But:** this also holds for finite N

- $N \gg F$: branched η' potential

$$V_k \overset{N\gg F}{\rightarrow} -6N^2 m|\Lambda_{\text{phys}}|^3 \cos\left(\frac{F}{N}\eta' - \frac{\theta + 2\pi k}{N}\right) - \frac{14}{3}N|\Lambda_{\text{phys}}|^3 \sum_{i=1}^{F} m_i \cos\left(\eta' + \theta_Q + \sum_{j=1}^{F-1} t_i^j \pi^j\right)$$

➡ As expected η' mass goes as $m_{\eta'}^2 \propto 1/N$



What happens when both F and N are large?

Csaki, d’Agnolo, Gupta, Kuflik, Roy, MR

F=N SUSY QCD

completely antisymmetric color singlet combination of Q_f^a and $\bar{Q}_f^a$

- Larger moduli space: mesons and baryons

$$M_{ff'} = \bar{Q}_f^a Q_{f'}^a \qquad B = \epsilon^{f_1 \cdots f_N} B_{f_1 \cdots f_N}\,, \quad \bar{B} = \epsilon^{f_1 \cdots f_N} \bar{B}_{f_1 \cdots f_N}$$

- Quantum modified constraint on moduli space $\det(M) - \bar{B}B = \Lambda^{2N}$

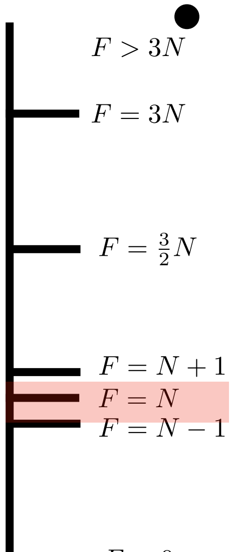
→ implemented in superpotential through Lagrange multiplier X

$$W = X\left(\frac{\det(M) - \bar{B}B}{\Lambda^{2N}} - 1\right) + \mathrm{Tr}(m_Q M)$$

$$K = \frac{\mathrm{Tr}(M^\dagger M)}{\alpha|\Lambda|^2} + \frac{X^\dagger X}{\beta|\Lambda|^4} + \frac{\bar{B}^\dagger \bar{B}}{\gamma|\Lambda|^{2N-2}} + \frac{B^\dagger B}{\delta|\Lambda|^{2N-2}}$$

- Chiral symmetry breaking vacuum exists

$$M_{ff'} = \Lambda^2 \delta_{ff'}, \quad X = -\frac{m|\Lambda|^2}{\alpha}, \quad B = \bar{B} = 0$$



Csaki, d'Agnolo, Gupta, Kuflik, Roy, MR

F=N SUSY QCD

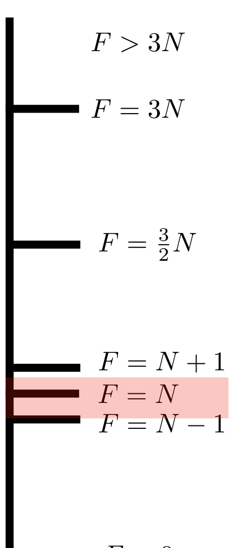
- Assume chiral symmetry breaking mesonic VEV

$$V = -2|\Lambda|^4\left[\beta + (N-2)\frac{m^2}{\alpha|\Lambda|^2}\right]\cos\left(N\eta' - \theta\right) - 4m|\Lambda|^2\sum_{i=1}^{N} m_i \cos\left(\eta' + \theta_Q + \sum_{j=1}^{N-1} t_i^j \pi^j\right)$$
$$- 2m|\Lambda|^2\sum_{i=1}^{N} m_i \cos\left((N-1)\eta' - \theta_Q - \theta - \sum_{j=1}^{N-1} t_i^j \pi^j\right)$$

No branches!
Consistent with instanton effect

$m_{\eta'} \propto \Lambda$: M **is uncharged under** $U(1)_R$
and $U(1)_A$ **broken by anomaly**

	$U(1)_A$	$U(1)_R$
M	2	$\frac{2(F-N)}{F}$
Λ^b	$2F$	0



Csaki, d'Agnolo, Gupta, Kuflik, Roy, MR

F=N+1 SUSY QCD

- Baryons in (anti-)fundamental of $SU(F)$

$$B^f = \epsilon^{f_1 \cdots f_N f} B_{f_1 \cdots f_N}$$

$$\bar{B}_f = \epsilon_{f_1 \cdots f_N f} \bar{B}^{f_1 \cdots f_N}$$

- Superpotential implements constraint on moduli space

$$W = \frac{B M \bar{B} - \det(M)}{\Lambda^{2N-1}} + \mathrm{Tr}(m_Q M)$$

Kähler:

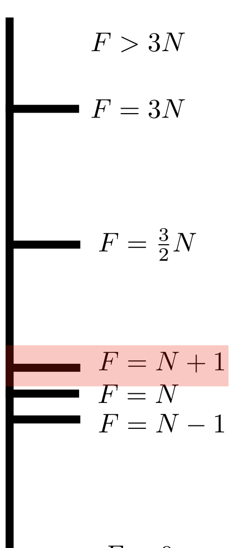
$$K = \frac{\mathrm{Tr}(M^\dagger M)}{\alpha |\Lambda|^2} + \sum_f \frac{\bar{B}_f^\dagger \bar{B}_f}{\beta |\Lambda|^{2N-2}} + \sum_f \frac{B_f^\dagger B_f}{\gamma |\Lambda|^{2N-2}}$$

- Chiral Lagrangian similar to $F = N$

$$\begin{aligned} V = & -2(N-2)\left(\frac{N-2}{N}\frac{m}{|\Lambda|}\right)^{(N+1)/(N-1)} m|\Lambda|^3 \cos\left((N+1)\eta' - \theta\right) \\ & -2\left(\frac{N-2}{N}\frac{m}{|\Lambda|}\right)^{N/(N-1)} |\Lambda|^3 \sum_{i=1}^{N+1} m_i \cos\left(N\eta' - \theta_Q - \theta - \sum_{j=1}^{N} t_i^j \pi^j\right) \\ & -4\left(\frac{N-2}{N}\frac{m}{|\Lambda|}\right)^{1/(N-1)} m|\Lambda|^2 \sum_{i=1}^{N+1} m_i \cos\left(\eta' + \theta_Q + \sum_{j=1}^{N} t_i^j \pi^j\right) . \end{aligned}$$

Again no branches!

Consistent with being an instanton effect



Csaki, d'Agnolo, Gupta, Kuflik, Roy, MR

3/2N>F>N+1 SUSY QCD

- Study weakly-coupled magnetic dual $SU(F-N)$

→ DOF: F (anti-)fundamentals $q, \bar{q}$ under $SU(F-N)$ and meson matrix M

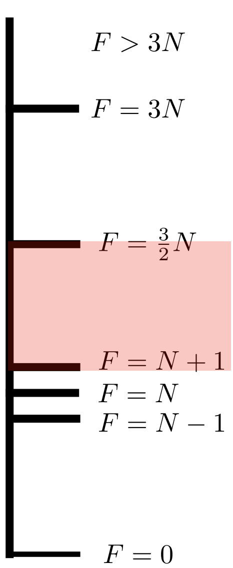
- Superpotential

$$W_d = \frac{1}{\mu} q_i M_{ij} \bar{q}_j + \mathrm{Tr}(m_Q M) \qquad \text{with} \qquad \Lambda^{3N-F} \tilde{\Lambda}^{3\tilde{N}-F} = (-1)^{F-N} \mu^F$$

mass term when M gets a VEV → integrate out dual quarks

- Pure $SU(F-N)$ SYM in IR → superpotential from gluino condensation

$$W_d^{\mathrm{eff}} = \tilde{N} \tilde{\Lambda}_{\mathrm{eff}}^3 + \mathrm{Tr}(m_Q M) = (N-F) \left(\frac{\Lambda^{3N-F}}{\det M} \right)^{1/(N-F)} + \mathrm{Tr}(m_Q M)$$



scale matching $$\left(\frac{\tilde{\Lambda}_{\mathrm{eff}}}{\det(M/\mu)^{1/F}} \right)^{3\tilde{N}} = \left(\frac{\tilde{\Lambda}}{\det(M/\mu)^{1/F}} \right)^{3\tilde{N}-F}$$

Csaki, d'Agnolo, Gupta, Kuflik, Roy, MR

3/2N>F>N+1 SUSY QCD

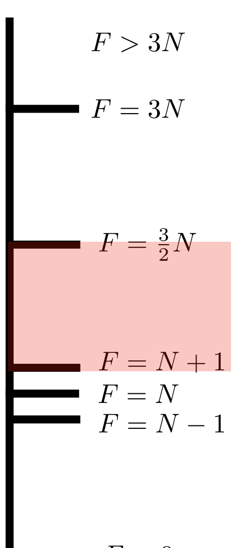
$$W_d^{\text{eff}} = \tilde{N}\tilde{\Lambda}^3_{\text{eff}} + \text{Tr}(m_Q M) = (N-F)\left(\frac{\Lambda^{3N-F}}{\det M}\right)^{1/(N-F)} + \text{Tr}(m_Q M)$$

- Same as $F<N$ with $F \leftrightarrow N$ with chiral symmetry breaking minimum

$$M_{ff'} = f^2\delta_{ij}\,, \quad \text{with} \quad f^2 = |\Lambda|^2\left(\frac{2F-3N}{N}\frac{m}{|\Lambda|}\right)^{(F-N)/(2N-F)}$$

- Chiral Lagrangian

$$\begin{aligned} V_k = &-4(3N-2F)\left(\frac{2F-3N}{N}\frac{m}{|\Lambda|}\right)^{F/(2N-F)} m|\Lambda|^3 \cos\left(\frac{F}{F-N}\eta' - \frac{\theta+2\pi k}{F-N}\right) \\ &-2F\left(\frac{2F-3N}{N}\frac{m}{|\Lambda|}\right)^{N/(2N-F)} |\Lambda|^3 \sum_{i=1}^{F} m_i \cos\left(\frac{N}{F-N}\eta' - \theta_Q - \frac{\theta+2\pi k}{F-N} - \sum_{j=1}^{F-1} t_i^j \pi_j\right) \\ &-\frac{4FN}{2F-3N}\left(\frac{2F-3N}{N}\frac{m}{|\Lambda|}\right)^{N/(2N-F)} |\Lambda|^3 \sum_{i=1}^{F} m_i \cos\left(\eta' + \theta_Q + \sum_{j=1}^{F-1} t_i^j \pi_j\right)\,, \end{aligned}$$



$F-N$ branches!

Holomorphic vs Physical Scale

- RGE invariant scale at two-loops

$$\Lambda_c = \mu \left(\frac{b_0 g_c^2(M_c)}{8\pi^2} \right)^{-b_1/(2b_0^2)} \exp\left(-\frac{8\pi^2}{b_0 g_c^2(\mu)} \right)$$

finite in large N limit since $b_0 g_c^2 \propto N g_c^2 = \text{const}$

$$b_0 = 3N - F$$

$$b_1 = 6N^2 - 2NF - 4F(N^2-1)/(2N)$$

- Holomorphic scale in superpotential (1-loop exact)

$$\Lambda = \mu e^{\frac{2\pi i \tau(\mu)}{b_0}} \qquad \text{with} \quad \tau = \frac{4\pi i}{g_h^2} + \frac{\theta}{2\pi}$$

- Relation between holomorphic and canonical coupling

wave-function renormalization

$$\mathrm{Re}(\tau) = \frac{8\pi^2}{g_c^2} + 2T(Ad)\log g_c + \sum_i T(i) \log Z_i$$

Dynkin index for adjoint and matter representations

- Relation between holomorphic and canonical scale

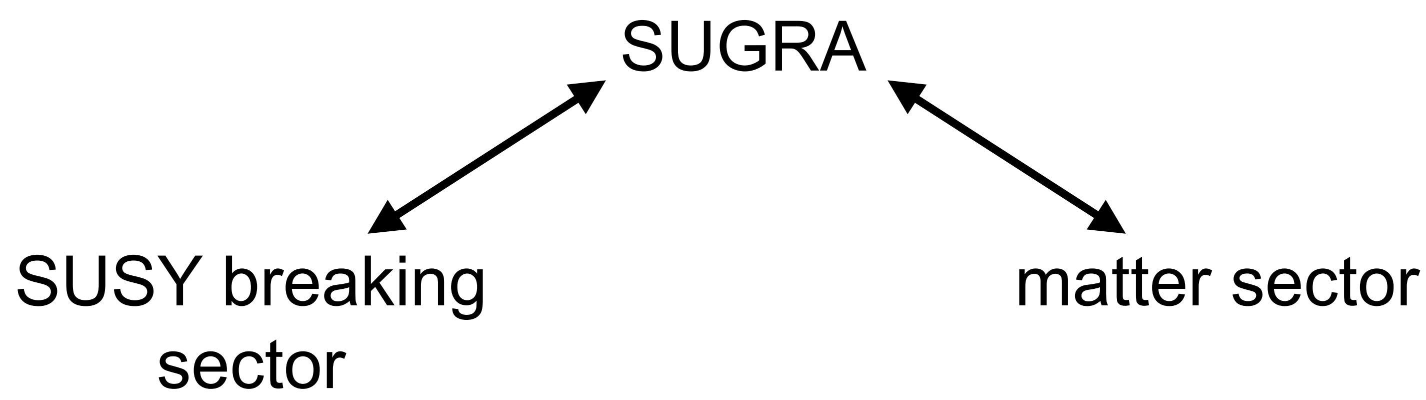
$$|\Lambda| = g_c(\mu)^{-b_1/b_0^2} \mu \exp\left(-\frac{8\pi^2}{b_0 g_c^2(\mu)} \right) = \left(\frac{b_0}{8\pi^2} \right)^{b_1/(2b_0^2)} \Lambda_c$$

for $N \gg F$:
$|\Lambda| \propto N^{1/3} \Lambda_c$

Maximilian Ruhdorfer (Cornell)

AMSB

Randall, Sundrum ’98
Giudice, Luty, Murayama, Rattazzi ’98
Arkani-Hamed, Rattazzi ‘98



- no direct interaction between SUSY breaking and matter sector
- only source of SUSY breaking is auxiliary field of SUGRA multiplet
- **all** effects in EFT are encapsulated in chiral compensator

→ UV insensitive