Fermionic Geometry for BSM

Based on 2310.02490 & work in progress
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BSM in Particle Physics and Cosmology

EFT Approach for BSM

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \sum_i c_i \mathcal{O}_i$$

- SMEFT, Higgs EFT, etc.
- Well-motivated for current status of the LHC
- Too many operators!! [84 at dim-6, 993 at dim-8]

[See Gu's talk] [Grzadkowski, Iskrzynski, Misiak, Rosiek; Murphy]

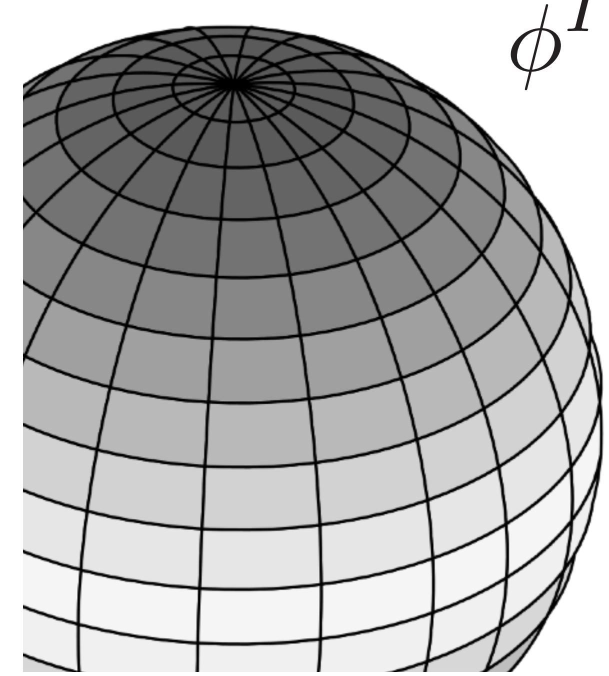
Is there a simple, physical organizing principle for the (SM)EFT?

Geometry for EFT

—Field Redefinition Invariance

[Meetz, Honerkamp, …]

$$\phi^I \to \phi'^I(\phi)$$



- Field redefinitions
  - redundancies in the operator
  - rearrange the operator basis
- Can be viewed as a coordinate transformation

$$x^\mu \to x'^\mu(x)$$

Geometry for EFT

—*scalar example*

$$\mathcal{L} = \frac{1}{2} h_{IJ}(\phi) \partial_\mu \phi^I \partial^\mu \phi^J - V(\phi)$$

$$\delta_{IJ} + c_{IJK}\phi^K + c_{IJKL}\phi^K\phi^L + \ldots$$

Incorporate all EFT operators in a two-derivative theory

But NOT all terms are physical though…

Geometry for EFT

—scalar example

$$\mathcal{L} = \frac{1}{2} h_{IJ}(\phi) \partial_\mu \phi^I \partial^\mu \phi^J - V(\phi)$$

- Field redefinition: $\phi^I \to \phi'^I(\phi)$
- Metric: $h_{IJ}(\phi) \to h'_{IJ}(\phi') = h_{AB}(\phi) \frac{\partial \phi^A}{\partial \phi'^I} \frac{\partial \phi^B}{\partial \phi'^J}$
    - Leads to the notion of *curvature*, *covariant derivatives*, etc.

Physical quantities (amplitudes, RG, etc.)

must be written in terms of geometric objects

Geometry for EFT

—scalar example

$$\mathcal{L} = \frac{1}{2} h_{IJ}(\phi) \partial_\mu \phi^I \partial^\mu \phi^J - V(\phi)$$

- Tree amplitudes

$$A_{IJKL} = \underline{R_{IJKL} s_{13}} + \underline{R_{IKJL} s_{12}} + \underline{V_{;(IJKL)}} + \ldots$$

the amplitude is arranged into geometric building blocks

Geometry for EFT

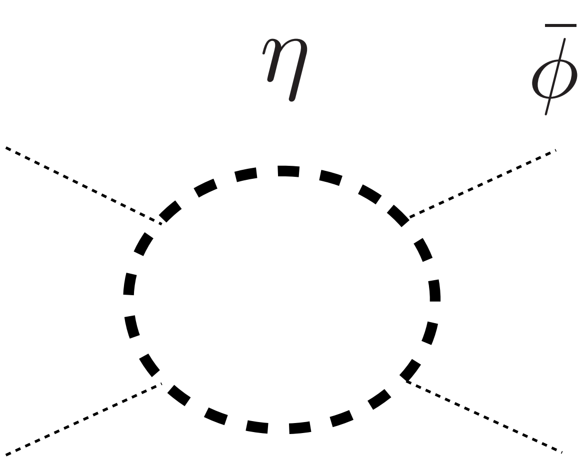
—scalar example

$$\mathcal{L} = \frac{1}{2} h_{IJ}(\phi)\partial_\mu \phi^I \partial^\mu \phi^J - V(\phi)$$

- One-loop RGEs: [t'Hooft] [See Gersdorff's talk]
  - Standard decomposition from background field method

    $\phi^I = \bar{\phi}^I + \eta^I$

  - But η^I is not covariant under field redefinition, *the geometry structure is lost.*



Geometry for EFT

—scalar example

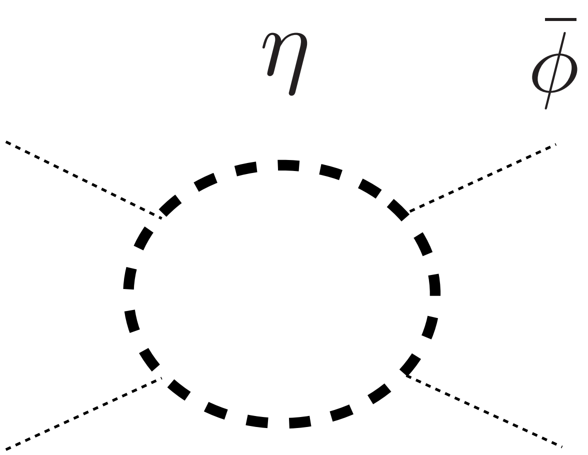
$$\mathcal{L} = \frac{1}{2} h_{IJ}(\phi) \partial_\mu \phi^I \partial^\mu \phi^J - V(\phi)$$

- One-loop RGEs: [Alonso, Jenkins, Manohar]
  - Covariant fluctuation follows from geodesic motion

$$\frac{d^2\phi^I}{d\lambda^2} + \Gamma^I_{JK}(\phi) \frac{d\phi^J}{d\lambda} \frac{d\phi^K}{d\lambda} = 0$$

$$\phi^I = \bar{\phi}^I + \eta^I - \frac{1}{2} \Gamma^I_{JK} \eta^J \eta^K + \dots$$

$$\delta^2 S = \frac{1}{2} \int d^4x \left[g_{IJ} (\mathcal{D}_\mu \eta)^I (\mathcal{D}_\mu \eta)^J - R_{IJKL} \eta^I (\mathcal{D}_\mu \phi)^J \eta^K (\mathcal{D}_\mu \phi)^L + V_{;IJ} \eta^I \eta^J \right]$$



Manifest geometry!

Geometry for EFT

—scalar example

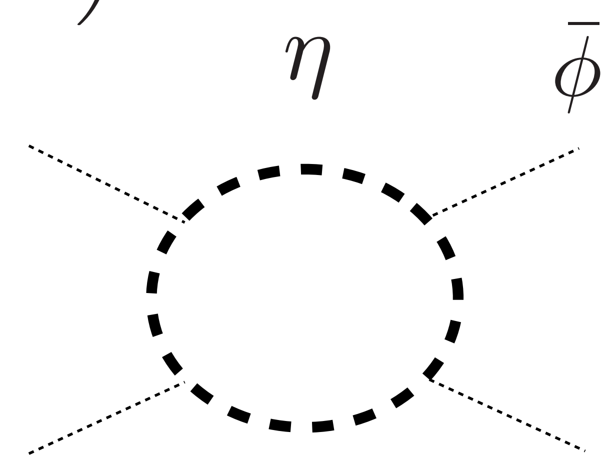
$$\mathcal{L} = \frac{1}{2} h_{IJ}(\phi) \partial_\mu \phi^I \partial^\mu \phi^J - V(\phi)$$

- One-loop RGEs: [Alonso, Jenkins, Manohar]

$$\Delta S^{1-\text{loop}} = \frac{1}{32\pi^2 \epsilon} \int d^4x \left(\frac{1}{12} \text{Tr}[Y_{\mu\nu} Y^{\mu\nu}] + \frac{1}{2} \text{Tr}[X^2] \right)$$

$$[Y_{\mu\nu}]^i{}_j \equiv [\mathscr{D}_\mu , \mathscr{D}_\nu]^i{}_j$$

$$[X]^i{}_k \equiv -R^i{}_{jkl} (D_\mu \phi)^j (D^\mu \phi)^l + g^{ij} V_{;jk}$$



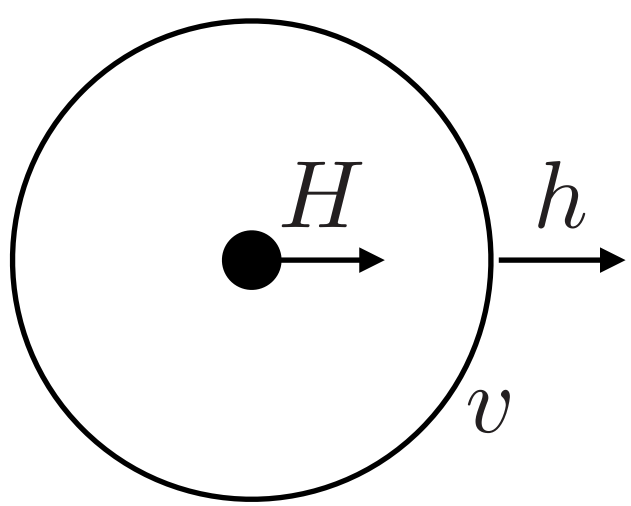
Geometric building blocks

Geometry for EFT

—scalar example

$$\mathcal{L} = \frac{1}{2} h_{IJ}(\phi) \partial_\mu \phi^I \partial^\mu \phi^J - V(\phi)$$

- SMEFT v.s. HEFT: Linear v.s. nonlinear realized EW symmetry
  [Alonso, Jenkins, Manohar; Falkowski, Rattazzi; Cohen, Craig, Lu, Sutherland;…]
    - Geometric viewpoint: O(4) invariant point in the scalar manifold



Geometry for EFT

— Challenges toward a GENERIC Geometry picture

- Generic field redefinitions
  [Cohen, Craig, Lu, Sutherland; Cheung, Helset, Parra-Martinez; Craig, Lee; Alminawi, Brivio, Davighi;…]

$$\phi \to \phi'(\phi, \partial\phi, \dots)$$

- Higher loops [Jenkins, Manohar, Naterop, Pages]
- Geometric picture for higher spin
  - Spin 1: [Helset, Jenkins, Manohar]
  - Spin 1/2: ***this talk***

    [See also Gattus, Pilaftsis, 2307.01126; Finn, Karamitsos, Pilaftsis, 2006.05831]

Fermionic Geometry for EFT

Fermionic Geometry for EFT
—*Setup*

$$\mathcal{L} = \frac{1}{2} i k_{\bar{p}r}(\phi) \left(\bar{\psi}^{\bar{p}} \gamma^{\mu} \overleftrightarrow{D}_{\mu} \psi^{r} \right) + i\omega_{\bar{p}rI}(\phi)(D_{\mu}\phi)^{I} \bar{\psi}^{\bar{p}} \gamma^{\mu} \psi^{r} - \bar{\psi}^{\bar{p}} \mathcal{M}_{\bar{p}r}(\phi) \psi^{r} + \bar{\psi}^{\bar{p}} \sigma_{\mu\nu} \mathcal{T}^{\mu\nu}_{\bar{p}r}(\phi, F) \psi^{r}$$

kinetic term ***spin connection*** ***Yukawa*** ***Dipole***

Dim-6 Examples			
trivial in Warsaw basis	$(\bar{\ell}_p \gamma^{\mu} \ell_r)\,(H^{\dagger} i \overleftrightarrow{D}_{\mu} H)$	$(H^{\dagger}H)(\bar{\ell}_p e_r H)$	$(\bar{\ell}_p \sigma^{\mu\nu} e_r H) B_{\mu\nu}$

Sufficient for all dim-6 fermonic operators* and most of dim 8.

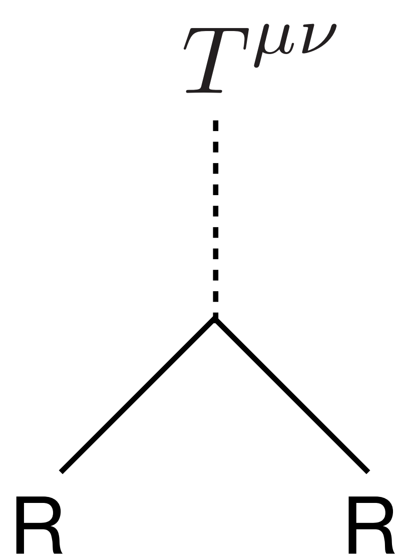
Fermionic Geometry for EFT

—*Chiral structure for dipole*

$$\mathcal{L} = \frac{1}{2} i k_{\bar{p}r}(\phi) \left(\bar{\psi}^{\bar{p}} \gamma^{\mu} \overleftrightarrow{D}_{\mu} \psi^{r} \right) + i\omega_{\bar{p}rI}(\phi)(D_{\mu}\phi)^{I} \bar{\psi}^{\bar{p}} \gamma^{\mu} \psi^{r} - \bar{\psi}^{\bar{p}} \mathcal{M}_{\bar{p}r}(\phi) \psi^{r} + \bar{\psi}^{\bar{p}} \sigma_{\mu\nu} \mathcal{T}^{\mu\nu}_{\bar{p}r}(\phi, F) \psi^{r}$$

Dipole

- The duality relation $\epsilon_{\mu\nu\rho\sigma}\sigma^{\rho\sigma} = 2i\sigma_{\mu\nu}\gamma_5$ leads to



$$\bar{\psi}\sigma_{\mu\nu}T^{\mu\nu}\psi = \bar{\psi}\sigma_{\mu\nu}T^{\mu\nu}P_R\psi$$

$$\bar{\psi}\sigma_{\mu\nu}T^{\mu\nu\dagger}\psi = \bar{\psi}\sigma_{\mu\nu}T^{\mu\nu\dagger}P_L\psi$$

for self-dual $T^{\mu\nu}$

Effective chiral structure

Fermionic Geometry for EFT

—*Setup*

$$\mathcal{L} = \frac{1}{2} i k_{\bar{p}r}(\phi) \left(\bar{\psi}^{\bar{p}} \gamma^{\mu} \overleftrightarrow{D}_{\mu} \psi^{r} \right) + i\omega_{\bar{p}rI}(\phi)(D_{\mu}\phi)^{I} \bar{\psi}^{\bar{p}} \gamma^{\mu} \psi^{r} - \bar{\psi}^{\bar{p}} \mathcal{M}_{\bar{p}r}(\phi) \psi^{r} + \bar{\psi}^{\bar{p}} \sigma_{\mu\nu} \mathcal{T}^{\mu\nu}_{\bar{p}r}(\phi, F) \psi^{r}$$

- Field redefinitions: $\psi^p \to R^p{}_r(\phi)\psi^r$

$$k_{\bar{p}r} \to \left[(R^{\dagger})^{-1} k R^{-1} \right]_{\bar{p}r} ,$$

$$\omega_{\bar{p}rI} \to \left[(R^{\dagger})^{-1} \omega_I R^{-1} \right]_{\bar{p}r} + \frac{1}{2} \left[(R^{\dagger})^{-1} k (\partial_I R^{-1}) \right]_{\bar{p}r} - \frac{1}{2} \left[(\partial_I (R^{\dagger})^{-1}) k R^{-1} \right]_{\bar{p}r}$$

Spin connections are inevitable under field redefinitions

Fermionic Geometry for EFT
— *"Supermanifold" for the SMEFT*

$$\mathcal{L} = \frac{1}{2} i k_{\bar{p}r}(\phi) \left(\bar{\psi}^{\bar{p}} \gamma^{\mu} \overleftrightarrow{D}_{\mu} \psi^{r} \right) + i \omega_{\bar{p}rI}(\phi) (D_{\mu}\phi)^{I} \bar{\psi}^{\bar{p}} \gamma^{\mu} \psi^{r} - \bar{\psi}^{\bar{p}} \mathcal{M}_{\bar{p}r}(\phi) \psi^{r} + \bar{\psi}^{\bar{p}} \sigma_{\mu\nu} \mathcal{T}^{\mu\nu}_{\bar{p}r}(\phi, F) \psi^{r}$$

- Unified geometry between scalar and fermion:

$$\begin{array}{cccc} & \phi & \psi & \bar{\psi} \end{array}$$

$$\bar{g}_{ab}(\phi,\psi) = \begin{pmatrix} h_{IJ} & -\left(\frac{1}{2}k_{\bar{s}r,I} - \omega_{\bar{s}rI}\right)\bar{\psi}^{\bar{s}} & \left(\frac{1}{2}k_{\bar{r}s,I} + \omega_{\bar{r}sI}\right)\psi^{s} \\ \left(\frac{1}{2}k_{\bar{s}p,J} - \omega_{\bar{s}pJ}\right)\bar{\psi}^{\bar{s}} & 0 & k_{\bar{r}p} \\ -\left(\frac{1}{2}k_{\bar{p}s,J} + \omega_{\bar{p}sJ}\right)\psi^{s} & -k_{\bar{p}r} & 0 \end{pmatrix} \qquad \Phi^{a} = \begin{pmatrix} \phi^{I} \\ \psi^{p} \\ \bar{\psi}^{\bar{p}} \end{pmatrix}$$

- Following the same construction as in supermanifold

Fermionic Geometry for EFT

—"Supermanifold" for the SMEFT

$$\mathcal{L} = \frac{1}{2} i k_{\bar{p}r}(\phi) \left(\bar{\psi}^{\bar{p}} \gamma^\mu \overleftrightarrow{D}_\mu \psi^r \right) + i\omega_{\bar{p}rI}(\phi)(D_\mu\phi)^I \bar{\psi}^{\bar{p}} \gamma^\mu \psi^r - \bar{\psi}^{\bar{p}} \mathcal{M}_{\bar{p}r}(\phi)\psi^r + \bar{\psi}^{\bar{p}} \sigma_{\mu\nu} \mathcal{T}^{\mu\nu}_{\bar{p}r}(\phi, F)\psi^r$$

- Unified geometry between scalar and fermion:

$$\bar{\Gamma}^p_{Is} = \bar{\Gamma}^p_{sI} = k^{p\bar{r}} \left(\frac{1}{2} k_{\bar{r}s,I} + \omega_{\bar{r}sI} \right)$$

$$\bar{\Gamma}^{\bar{p}}_{I\bar{s}} = \bar{\Gamma}^{\bar{p}}_{\bar{s}I} = \left(\frac{1}{2} k_{\bar{s}r,I} - \omega_{\bar{s}rI} \right) k^{r\bar{p}}$$

$$\bar{R}_{\bar{p}rIJ} = \omega_{\bar{p}rJ,I} - \left(\frac{1}{2} k_{\bar{p}s,I} - \omega_{\bar{p}sI} \right) k^{s\bar{t}} \left(\frac{1}{2} k_{\bar{t}r,J} + \omega_{\bar{t}rJ} \right) - (I \leftrightarrow J)$$

fermions ($\bar{p}r$) *scalars* (IJ)

Fermionic Geometry for EFT

—Tree amplitudes

- Amplitudes are composed of geometric building blocks

$$\psi^p \phi^I \to \psi^{\bar{r}} \phi^J \qquad \mathcal{A}_{pI\bar{r}J} = (\bar{u}_{\bar{r}} \not{p}_I u_p) \bar{R}_{\bar{r}pJI}$$

$$\psi^p \phi^I \to \psi^{\bar{r}} \phi^J \phi^K \qquad \mathcal{A}_{pI\bar{r}JK} = (\bar{u}_{\bar{r}} \not{p}_J u_p) \bar{\nabla}_K \bar{R}_{\bar{r}pIJ} + (\bar{u}_{\bar{r}} \not{p}_K u_p) \bar{\nabla}_J \bar{R}_{\bar{r}pIK}$$

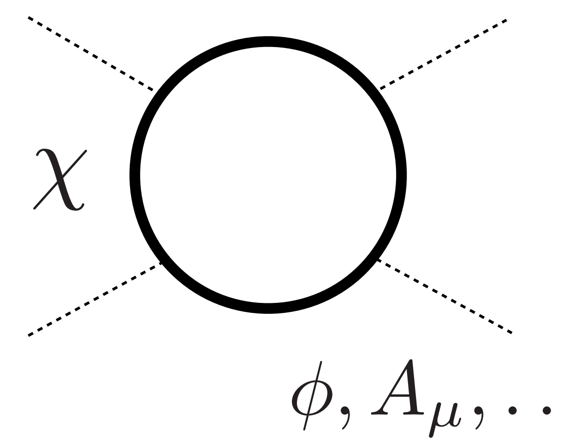
Nontrivial geometric building blocks

Fermionic Geometry for EFT

—*One-loop RGEs*

- For RGEs of bosonic operators, $\psi = \psi_0 + \chi$

$$\delta_{\bar{\chi}\chi} S = \int \mathrm{d}^4 x \left\{ \frac{1}{2} i k_{\bar{p}r} \left(\bar{\chi}^{\bar{p}} \gamma^{\mu} \overleftrightarrow{\mathcal{D}}_{\mu} \chi^{r} \right) - \bar{\chi}^{\bar{p}} \mathcal{M}_{\bar{p}r} \chi^{r} + \bar{\chi}^{\bar{p}} \sigma_{\mu\nu} \mathcal{T}^{\mu\nu}_{\bar{p}r} \chi^{r} \right\}$$

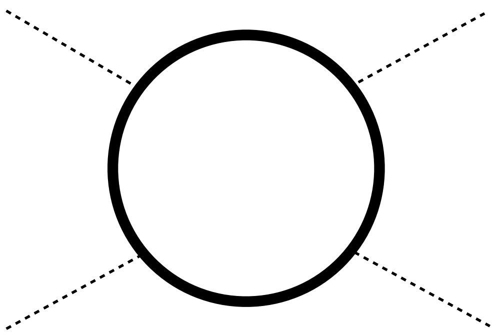


Fermionic Geometry for EFT

—*One-loop RGEs*

- For RGEs of bosonic operators, $\psi = \psi_0 + \chi$

$$\Delta S = \frac{1}{32\pi^2\epsilon}\int \mathrm{d}^4x \left\{ \frac{1}{3}\mathrm{Tr}\left[\mathcal{Y}_{\mu\nu}\mathcal{Y}^{\mu\nu}\right] + \mathrm{Tr}\left[(\mathcal{D}_\mu\mathcal{M})(\mathcal{D}^\mu\mathcal{M}) - (\mathcal{M}\mathcal{M})^2\right] - \frac{16}{3}\mathrm{Tr}[(\mathcal{D}_\mu\mathcal{T}^{\mu\alpha})(\mathcal{D}_\nu\mathcal{T}^{\nu\alpha}) - (\mathcal{T}^{\mu\nu}\mathcal{T}^{\alpha\beta})^2] - 4i\mathrm{Tr}[\mathcal{Y}_{\mu\nu}(\mathcal{M}\mathcal{T}^{\mu\nu} + \mathcal{T}^{\mu\nu}\mathcal{M})] - 8\mathrm{Tr}(\mathcal{M}\mathcal{T}^{\mu\nu})^2 \right\},$$

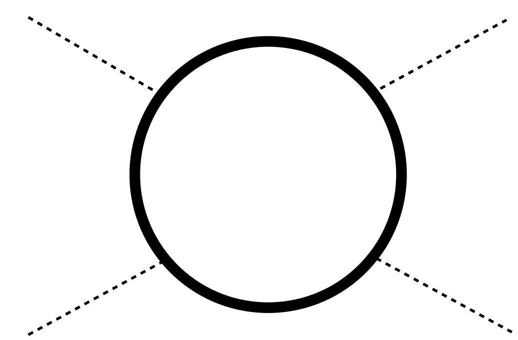


Geometric building blocks

Fermionic Geometry for EFT
—One-loop RGEs

- For RGEs of bosonic operators, $\psi = \psi_0 + \chi$

$$\Delta S = \frac{1}{32\pi^2\epsilon}\int \mathrm{d}^4x \left\{\frac{1}{3}\mathrm{Tr}\left[\mathcal{Y}_{\mu\nu}\mathcal{Y}^{\mu\nu}\right] + \mathrm{Tr}\left[(\mathcal{D}_\mu\mathcal{M})(\mathcal{D}^\mu\mathcal{M}) - (\mathcal{M}\mathcal{M})^2\right] - \frac{16}{3}\mathrm{Tr}[(\mathcal{D}_\mu\mathcal{T}^{\mu\alpha})(\mathcal{D}_\nu\mathcal{T}^{\nu\alpha}) - (\mathcal{T}^{\mu\nu}\mathcal{T}^{\alpha\beta})^2] - 4i\mathrm{Tr}[\mathcal{Y}_{\mu\nu}(\mathcal{M}\mathcal{T}^{\mu\nu} + \mathcal{T}^{\mu\nu}\mathcal{M})] - 8\mathrm{Tr}(\mathcal{M}\mathcal{T}^{\mu\nu})^2\right\},$$



$$[\mathcal{Y}_{\mu\nu}]^p{}_r = [\mathcal{D}_\mu, \mathcal{D}_\nu]^p{}_r = \bar{R}^p{}_{rIJ}(D_\mu\phi)^I(D_\nu\phi)^J + \left(\bar{\nabla}_r t^p_A\right)F^A_{\mu\nu},$$
$$(\mathcal{D}_\mu\mathcal{M})^p{}_r = k^{p\bar{t}}(\mathcal{D}_\mu\mathcal{M}_{\bar{t}r}) = k^{p\bar{t}}\left[D_\mu\mathcal{M}_{\bar{t}r} - \bar{\Gamma}^{\bar{s}}_{I\bar{t}}(D_\mu\phi)^I\mathcal{M}_{\bar{s}r} - \bar{\Gamma}^s_{Ir}(D_\mu\phi)^I\mathcal{M}_{\bar{t}s}\right],$$
$$(\mathcal{M}\mathcal{M})^p{}_r = k^{p\bar{t}}\mathcal{M}_{\bar{t}q}k^{q\bar{s}}\mathcal{M}_{\bar{s}r},$$
$$(\mathcal{D}_\mu\mathcal{T}^{\alpha\beta})^p{}_r = k^{p\bar{t}}(\mathcal{D}_\mu\mathcal{T}^{\alpha\beta}_{\bar{t}r}) = k^{p\bar{t}}\left[D_\mu\mathcal{T}^{\alpha\beta}_{\bar{t}r} - \bar{\Gamma}^{\bar{s}}_{I\bar{t}}(D_\mu\phi)^I\mathcal{T}^{\alpha\beta}_{\bar{s}r} - \bar{\Gamma}^s_{Ir}(D_\mu\phi)^I\mathcal{T}^{\alpha\beta}_{\bar{t}s}\right],$$
$$(\mathcal{T}^{\mu\nu}\mathcal{T}^{\alpha\beta})^p{}_r = k^{p\bar{t}}\mathcal{T}^{\mu\nu}_{\bar{t}q}k^{q\bar{s}}\mathcal{T}^{\alpha\beta}_{\bar{s}r}.$$

same geometric structure

Fermionic Geometry for EFT

—One-loop RGEs in the SMEFT

- We calculate the bosonic RGEs up to dim 8 using geometry.

  [Agree w/ Chala, Guedes, Ramos, Santiago; Das Bakshi, Chala, Diaz-Carmona, Guedes; Accettulli, Huber, De Angelis]

$$M_{\bar{p}r} \supset [Y_e]^\dagger_{\bar{p}r} H - {}^6C_{\underset{\bar{p}r}{leH^3}} H(H^\dagger H) - {}^8C_{\underset{\bar{p}r}{leH^5}} H(H^\dagger H)^2$$

$$T^{\mu\nu}_{\bar{p}r} \supset {}^6C_{\underset{\bar{p}r}{leBH}} H \frac{1}{2}\underbrace{\left(B^{\mu\nu} - i\tilde{B}^{\mu\nu}\right)}_{\text{chiral}} + {}^8C_{\underset{\bar{p}r}{leBH^3}} H(H^\dagger H)\frac{1}{2}\underbrace{\left(B^{\mu\nu} - i\tilde{B}^{\mu\nu}\right)}_{\text{chiral}}$$

Fermionic Geometry for EFT
—One-loop RGEs in the SMEFT

- We calculate the bosonic RGEs up to dim 8 using geometry.
  [Agree w/ Chala, Guedes, Ramos, Santiago; Das Bakshi, Chala, Diaz-Carmona, Guedes; Accettulli, Huber, De Angelis]
  - dim-6 D^2H^4 example:

$$
\begin{aligned}
{}^{6}\dot{C}_{H^4\Box} &= \frac{2}{3} g_1^2 \kappa_1 + 2 g_2^2 \kappa_2 - 2\kappa_9 - 6\kappa_{10} - 2\kappa_{11} , \\
{}^{6}\dot{C}_{H^4D^2} &= \frac{8}{3} g_1^2 \kappa_1 - 8\kappa_9 + 4\kappa_{11} \, .
\end{aligned}
$$

↑ ***Geometry groups terms together***

$$
\begin{aligned}
\kappa_1 &= \left[y_e \, {}^{6}C_{e^2 \underset{tt}{H^2} D} + 2 y_\ell \, {}^{6}C^{(1)}_{\ell^2 \underset{tt}{H^2} D} + N_c y_u \, {}^{6}C_{u^2 \underset{tt}{H^2} D} + N_c y_d \, {}^{6}C_{d^2 \underset{tt}{H^2} D} + 2 N_c y_q \, {}^{6}C^{(1)}_{q^2 \underset{tt}{H^2} D} \right] \\
\kappa_9 &= \mathrm{Tr} \left[-Y_e Y_e^{\dagger \, 6} C_{e^2 H^2 D} + Y_e^\dagger Y_e \, {}^{6}C^{(1)}_{\ell^2 H^2 D} - N_c Y_d Y_d^{\dagger \, 6} C_{d^2 H^2 D} + N_c Y_d^\dagger Y_d \, {}^{6}C^{(1)}_{q^2 H^2 D} \right. \\
&\qquad \left. + N_c Y_u Y_u^{\dagger \, 6} C_{u^2 H^2 D} - N_c Y_u^\dagger Y_u \, {}^{6}C^{(1)}_{q^2 H^2 D} \right] , \\
\kappa_{11} &= \mathrm{Tr} \left[-N_c Y_d Y_u^{\dagger \, 6} C_{udH^2D} - N_c Y_u Y_d^{\dagger \, 6} C^\dagger_{udH^2D} \right]
\end{aligned}
$$

Conclusion & Outlook

- ***Geometry*** emerges in EFT from ***field redefinition invariance***
- We generalize this picture to fermions
- Organize physical quantities, amplitudes and RGEs, into simple building blocks
- New results on fermionic RGEs
- Four fermion operators, mixed type of loops, SUSY structures, … for future work

Thank you! Cảm ơn