Axion Domain Walls, Small Instantons and Non-Invertible Symmetry Breaking

Sungwoo Hong
KAIST

(Based on 2309.05636 with Clay Cordova, Liantao Wang)

20th Rencontres du Vietnam:
BSM in particle physics and cosmology - 50 years later

Axion Domain Wall Problem

Axion Domain Wall Problem

I. Axion-QCD Theory

$$S \supset \frac{1}{2}\int \partial_\mu a\, \partial^\mu a + \frac{iK}{8\pi^2}\int \frac{a}{f_a}\, Tr(G \wedge G)$$

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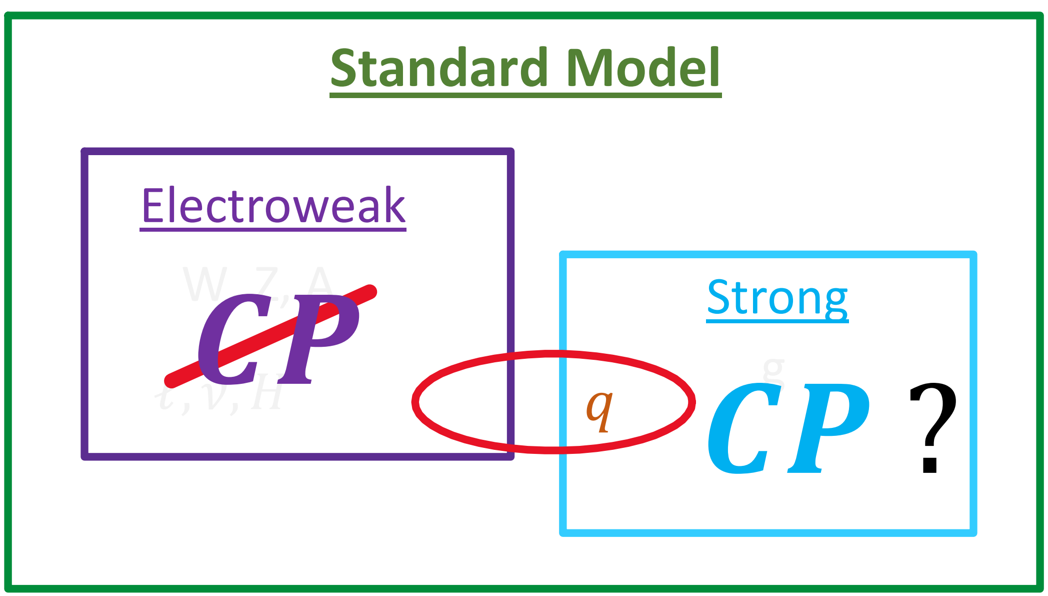
(i) Leading Theoretical Proposal to Solve Strong CP Problem



$$S_{QCD} \supset \frac{i\bar{\theta}}{8\pi^2}\int Tr(G \wedge G)$$

Neutron Electric Dipole Moment

$$d_n \sim 3 \times 10^{16}\, \bar{\theta} \;\; \rightarrow \;\; \bar{\theta} < 10^{-10}$$

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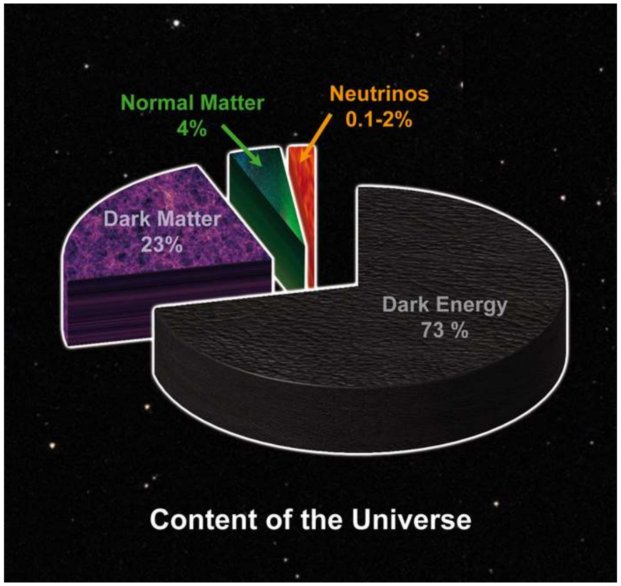
(i) Leading Theoretical Proposal to Solve Strong CP Problem
(ii) Well-motivated candidate for Dark Matter



Credit: HAP / A. Chantelauze

Axion Domain Wall Problem

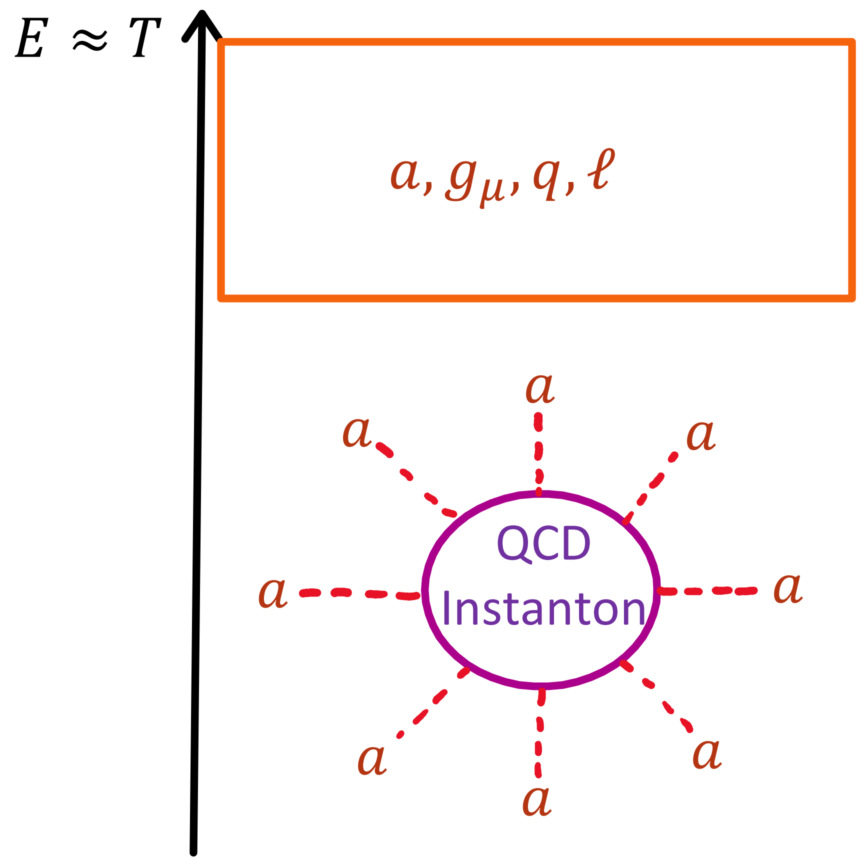
II. Axion Domain Wall Problem

$$S \supset \frac{1}{2}\int \partial_\mu a\, \partial^\mu a + \frac{iK}{8\pi^2}\int \frac{a}{f_a}\, Tr(G \wedge G)$$

cf. $S_{QCD} \supset \frac{i\bar{\theta}}{8\pi^2}\int Tr(G \wedge G)$

$U(1)_{PQ} \to Z_K :$ K-degenerate vacua

$$a \sim a + 2\pi f_a, \qquad a \to a + \frac{2\pi f_a}{K}$$



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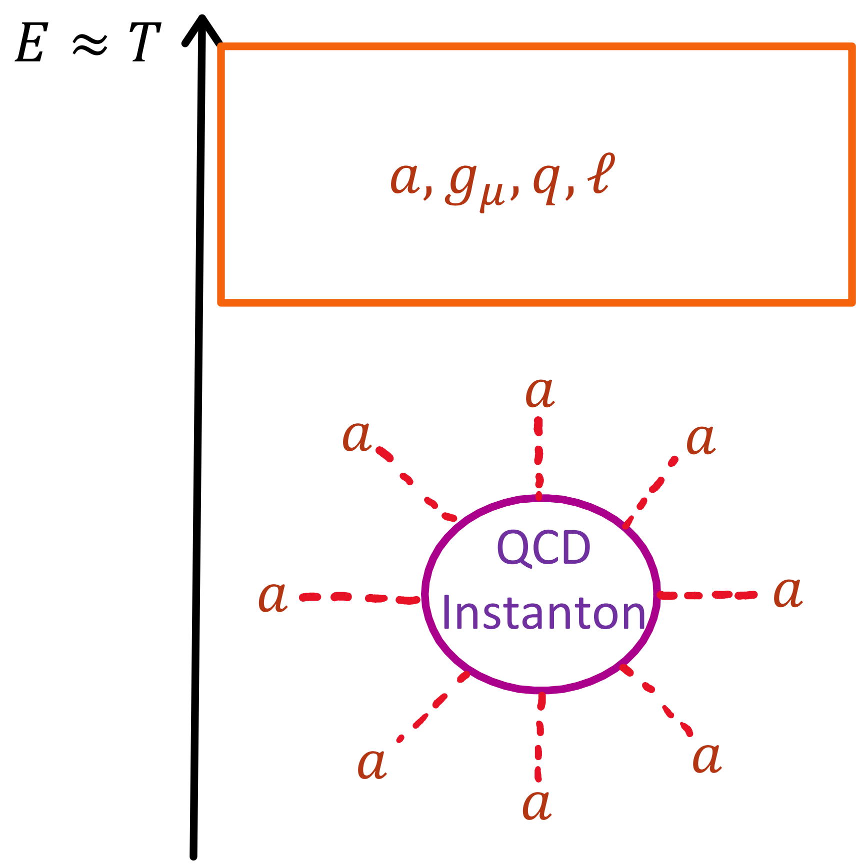
$$e^{-\frac{8\pi^2}{g^2(\mu)}} \ll 1$$



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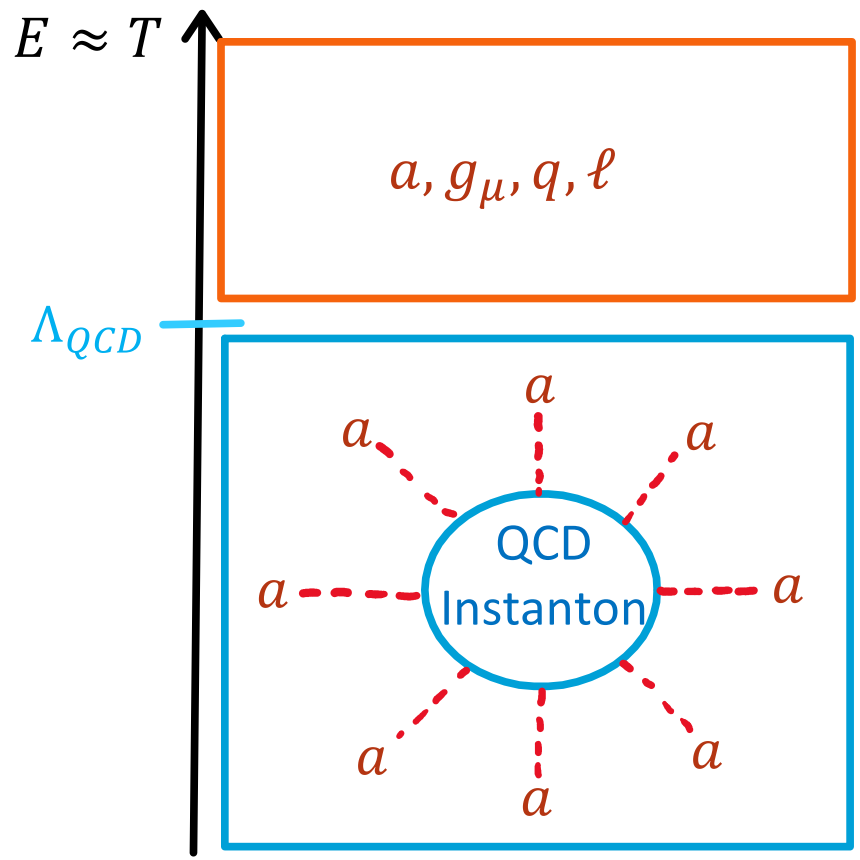
$a \sim a + 2\pi f_a,\qquad a \to a + \frac{2\pi f_a}{K}$

$$e^{-\frac{8\pi^2}{g^2(\Lambda_{QCD})}} \sim 1$$



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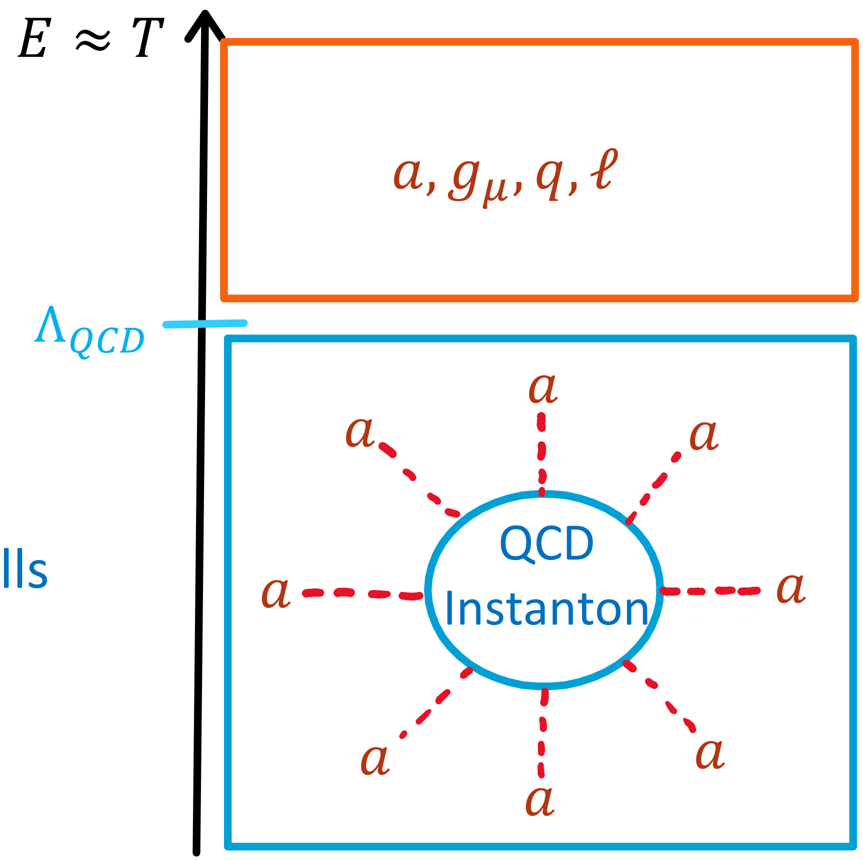
$U(1)_{PQ} \to Z_K$: K-degenerate vacua

$$V(a) = m_u \Lambda_{QCD}^3\left(1-\cos\frac{Ka}{f_a}\right)$$

Z_K Spontaneously Broken : K Domain Walls



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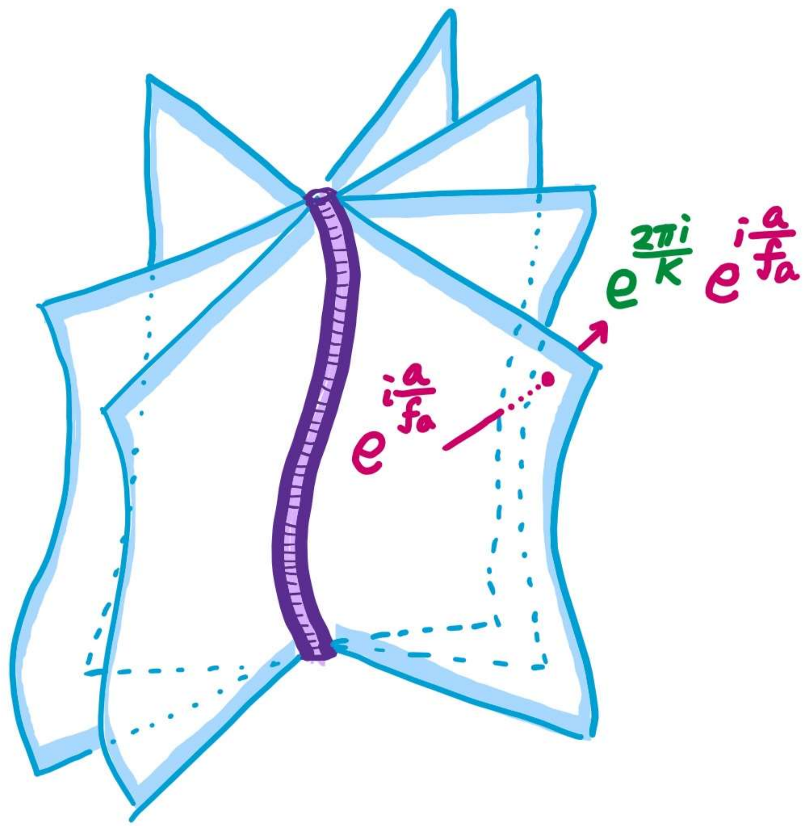

Axion Domain Wall Problem

II. Axion Domain Wall Problem

The presence of long-lived axion-Domain-Walls is inconsistent with cosmological observations

- (Dark) matter energy density: $\rho_{\rm DM} \propto R^{-3} \Rightarrow \quad R(t) \propto t^{2/3}$ **(Matter-Dominated)**
- Domain Wall energy density: $\rho_{\rm DW} \propto R^{-1} \quad \Rightarrow \quad R(t) \propto t^{2}$ **(DW-Dominated)**

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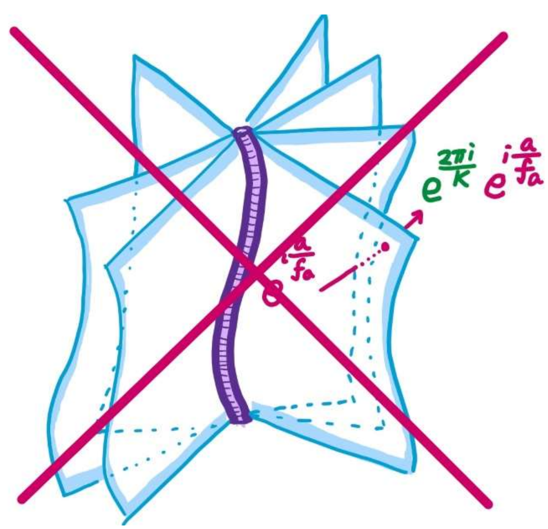


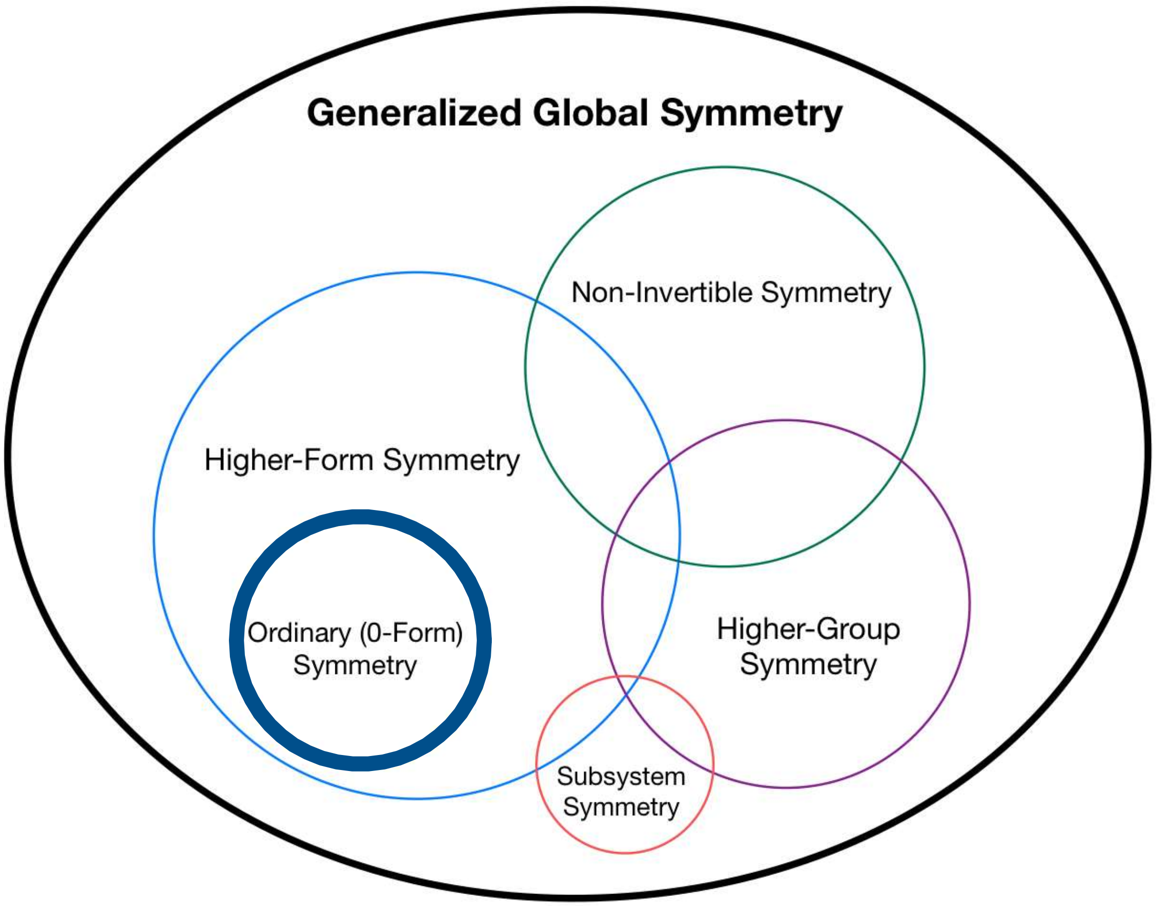
Axion Domain Wall Problem

II. Axion Domain Wall Problem

Success of axion theory hinges on first solving domain wall problem!

Axion Domain Wall Problem

III. Generalized Global Symmetries



Non-Invertible
Axion Domain Wall Problem

Non-invertible Axion Domain Wall Problem

I. Non-invertible Symmetries of Axion

Consider axion coupled to $G_g = SU(N)/Z_N$

$S \supset \frac{iK}{8\pi^2}\int \frac{a}{f_a} Tr(G \wedge G)$ $\qquad$ G = field strength of G_g

1. If $G_g = SU(N)$

   Full axion shift symmetry: $U(1)_{PQ} \rightarrow Z_K\text{(invertible)}$

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Full axion shift symmetry: $U(1)_{PQ} \rightarrow Z_K (\text{invertible})$

$a \sim a + 2\pi f_a$, under $a \rightarrow a + \frac{2\pi f_a}{z}$ with $z \in Z$ (to be determined)

$$S \rightarrow S + \frac{i2\pi K}{z}\int \frac{Tr(G \wedge G)}{8\pi^2}$$

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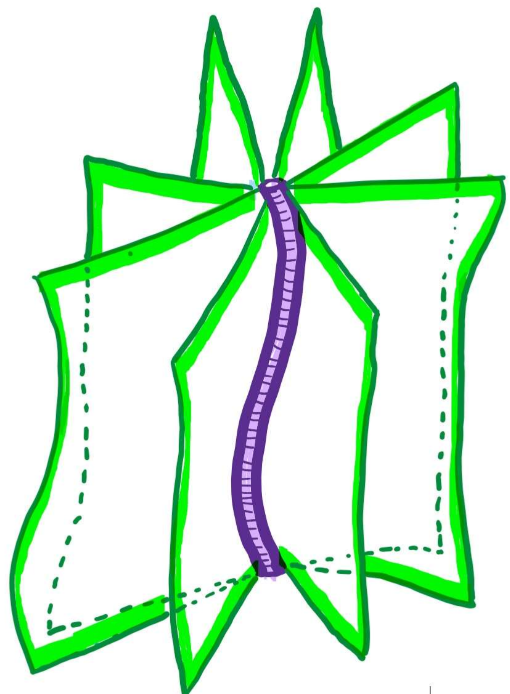

1. If $G_g = SU(N)$

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   K degenerate vacua $\rightarrow$ K invertible Domain Walls

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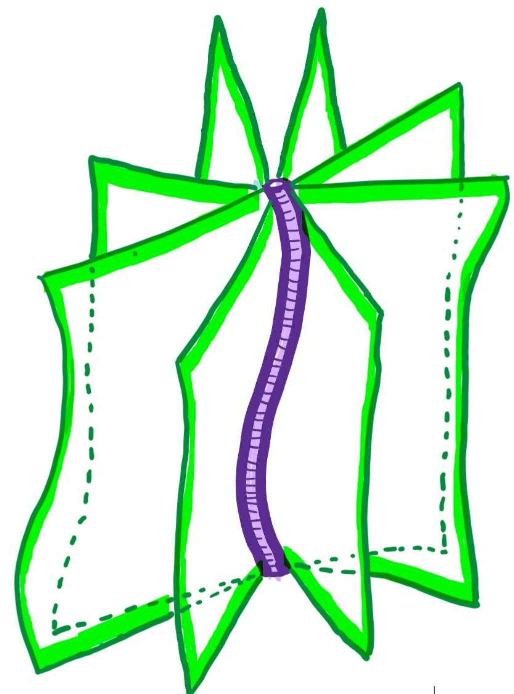

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2. If $G_g = SU(N)/Z_N$

   Expected symmetry: $Z_{K/N}(\text{invertible})$

Non-invertible Axion Domain Wall Problem

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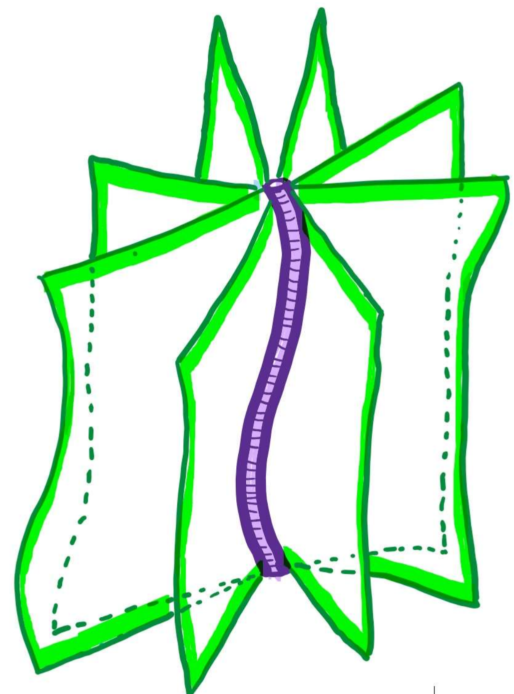

1. If $G_g = SU(N)$

   Full axion shift symmetry: $U(1)_{PQ} \rightarrow Z_K(\text{invertible})$

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2. If $G_g = SU(N)/Z_N$

   Enlarged symmetry: $Z_{K/N}(\text{invertible}) \subset Z_K(\text{non invertible})$

Non-invertible Axion Domain Wall Problem

I. Non-invertible Symmetries of Axion

$S \supset \frac{iK}{8\pi^2}\int \frac{a}{f_a} Tr(G \wedge G)$ G = field strength of G_g

We still fix: $a \sim a + 2\pi f_a$, under $a \to a + \frac{2\pi f_a}{z}$ with $z \in Z$ (to be determined)

$\Rightarrow \delta S = \frac{2\pi iK}{z}\int \frac{Tr(G\wedge G)}{8\pi^2} = \frac{2\pi iK}{z} n + \frac{2\pi iK}{z}\frac{N-1}{N}\int \frac{w\wedge w}{2} = 2\pi i Z$

$w \in H^2(M_4, Z_N)$: $\oint_{\Sigma_2} w = Z/NZ = Z_N$

2. If $G_g = SU(N)/Z_N$

Enlarged symmetry: $Z_{K/N}$(invertible) $\subset Z_K$(non invertible)

Non-invertible Axion Domain Wall Problem

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$$w \in H^2(M_4, Z_N): \quad \oint_{\Sigma_2} w = Z/NZ = Z_N$$

$$\Rightarrow n: \; U(1)_{PQ} \to Z_K \,, \qquad \frac{N-1}{N}\int \frac{w \wedge w}{2} : \; U(1)_{PQ} \to Z_{K/N}$$

$$\Rightarrow Z^{I}_{K/N} \subset Z^{NI}_{K}$$

2. If $G_g = SU(N)/Z_N$

Enlarged symmetry: $Z_{K/N}$(invertible) $\subset$ Z_K(non invertible)

Non-Invertible Symmetry with Fractional Instanton

NIS construction possible for non-abelian gauge theory with 1-form magnetic center symmetry

e.g. $G = SU(N)$

electric 1-form: Z_N

magnetic 1-form: none

Non-Invertible Symmetry with Fractional Instanton

NIS construction possible for non-abelian gauge theory with 1-form magnetic center symmetry

e.g. $G = SU(N)/Z_L$

electric 1-form: $Z_{N/L}$

magnetic 1-form: Z_L

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NIS construction possible for non-abelian gauge theory with 1-form magnetic center symmetry

e.g. $G = SU(N)/Z_L$ electric 1-form: $Z_{N/L}$

magnetic 1-form: Z_L

$U(1)_A$ with $\alpha = \frac{2\pi}{k}$, $S \to S + \frac{2\pi A i}{k} \underbrace{\int_{M_4} \frac{G \wedge G}{8\pi^2}}_{\in Z} + \frac{2\pi A i}{k} \underbrace{\left(\frac{L-1}{L}\right) \int_{M_4} \frac{w_2 \wedge w_2}{2}}_{\in Z_L}$

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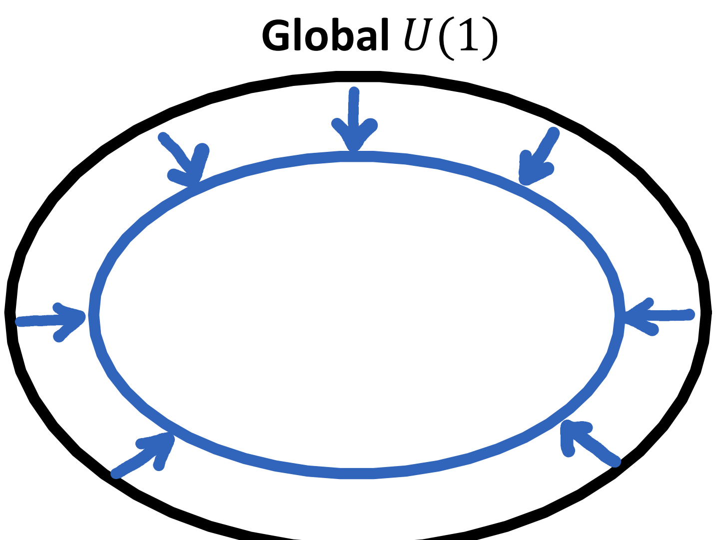

$\rightarrow$ Z Instanton

Non-Invertible Symmetry with Fractional Instanton

NIS construction possible for non-abelian gauge theory with 1-form magnetic center symmetry

e.g. $G = SU(N)/Z_L$ electric 1-form: $Z_{N/L}$

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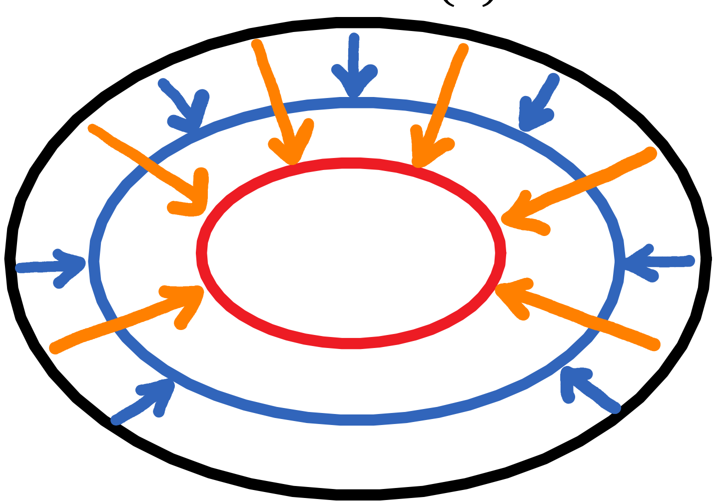
$\in Z$ $\in Z_L$



$\rightarrow Z$ Instanton

$\rightarrow Z_L$ (fractional) Instanton

Non-Invertible Symmetry with Fractional Instanton

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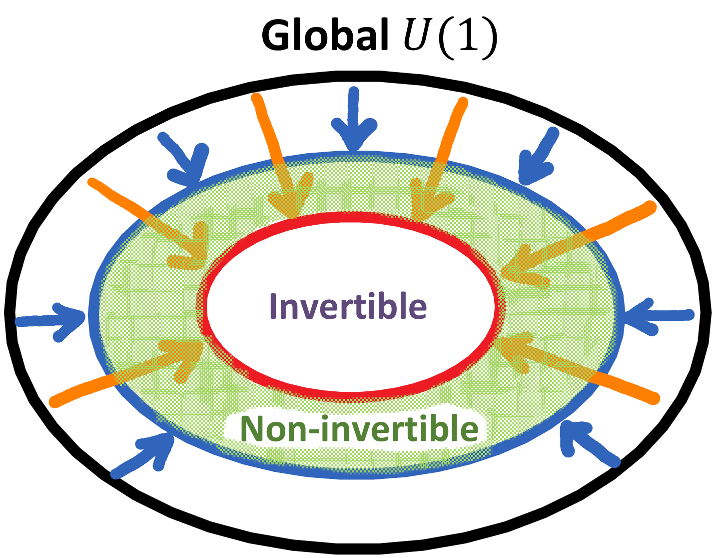

$\to$ Z Instanton

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Non-Invertible Symmetry with Fractional Instanton

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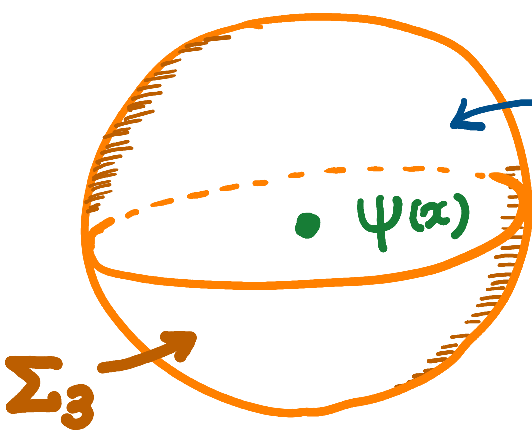
$\in Z$ $\qquad\qquad \in Z_L$



$$S_{defect} = \frac{iN}{4\pi} \int_{\Sigma_3} C \wedge dC$$

$$+ \frac{i}{2\pi} \int_{\Sigma_3} C \wedge w_2$$

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$$U\left(\frac{2\pi}{k}, \Sigma_3\right) \to D_k = U\left(\frac{2\pi}{k}, \Sigma_3\right) \times \mathcal{A}^{N,p}(w_2) \ with \ \frac{p}{N} = \frac{A}{k}$$

Non-invertible Axion Domain Wall Problem

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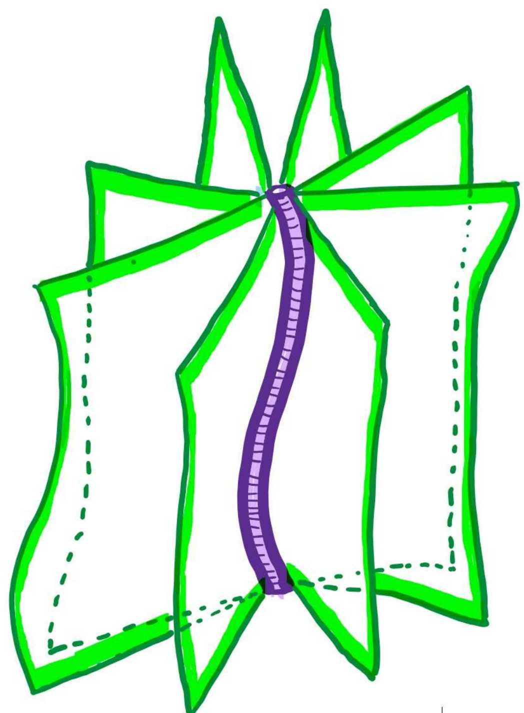

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   K degenerate vacua $\rightarrow$ K invertible Domain Walls

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   Enlarged symmetry: $Z_{K/N}(\text{invertible}) \subset Z_K(\text{non invertible})$

   Invertible $Z_{K/N}$ $\Rightarrow$ K/N degenerate vacua

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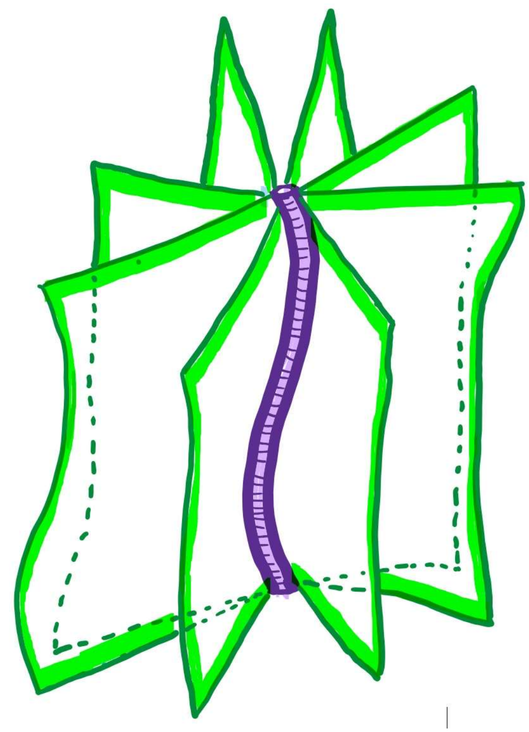

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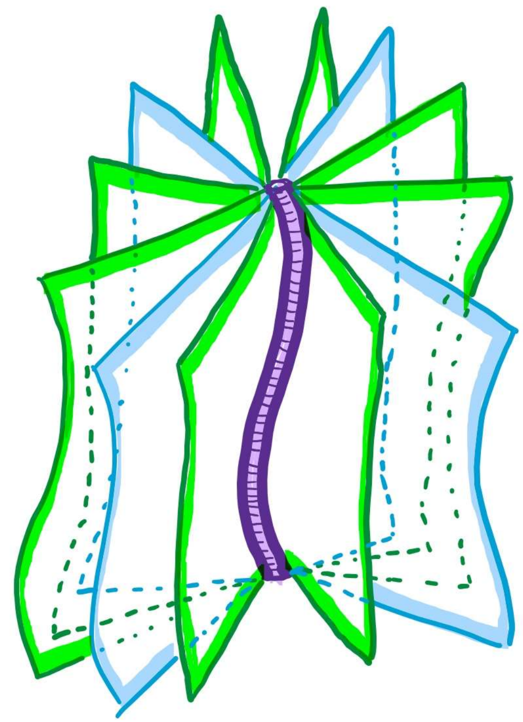

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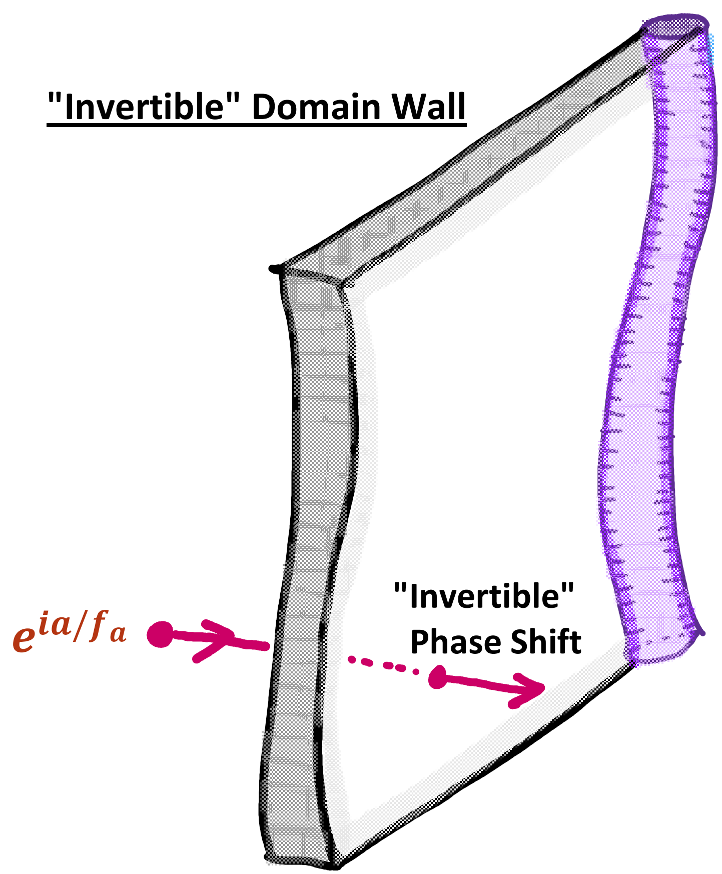
   ~~Invertible $Z_{K/N}$ $\Rightarrow$ K/N degenerate vacua~~

   K degenerate vacua $\rightarrow$ invertible + non-invertible Domain Walls

Non-invertible Axion Domain Wall Problem

Invertible DW vs Non-Invertible DW



Non-invertible Axion Domain Wall Problem

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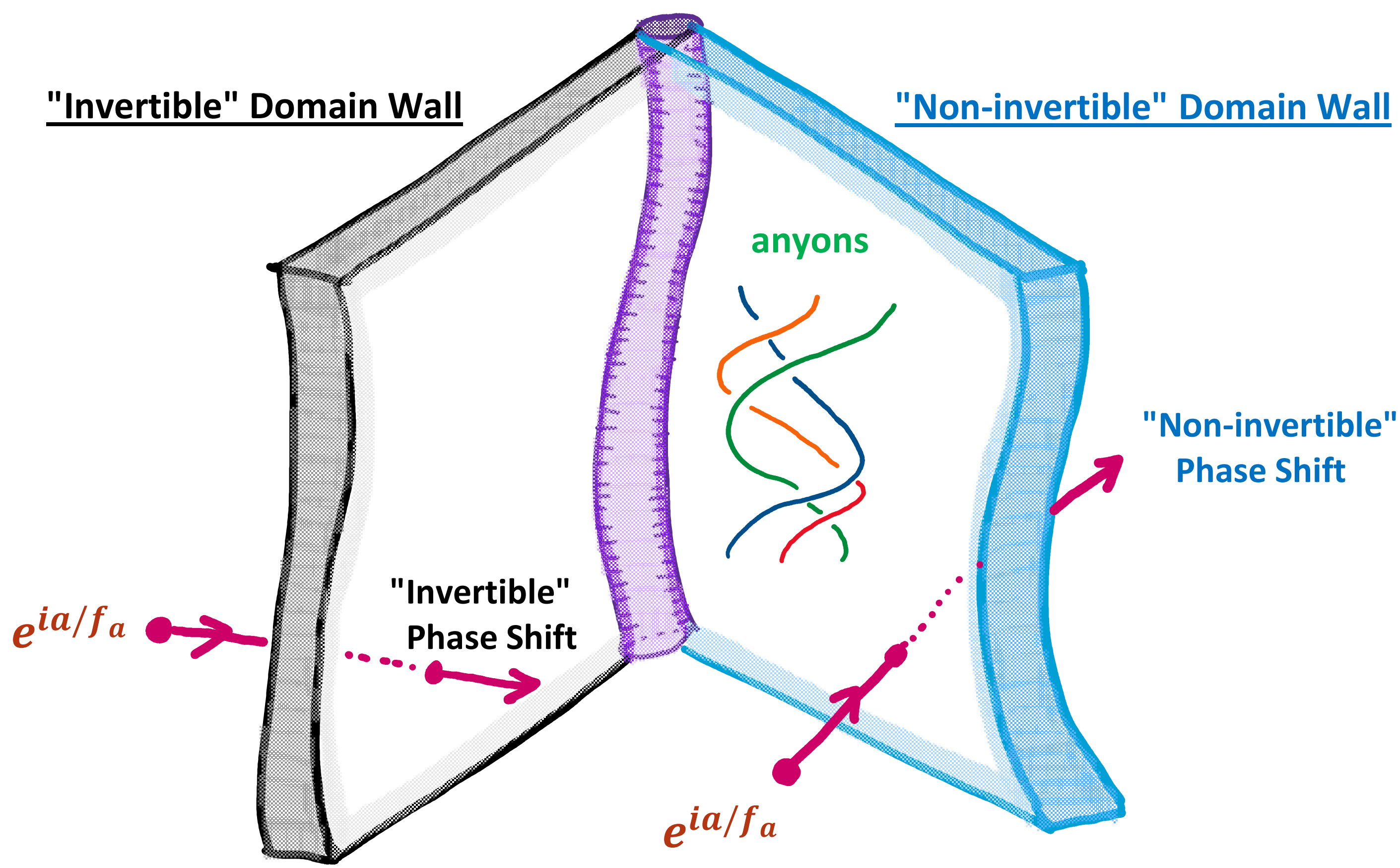

Non-invertible Axion Domain Wall Problem

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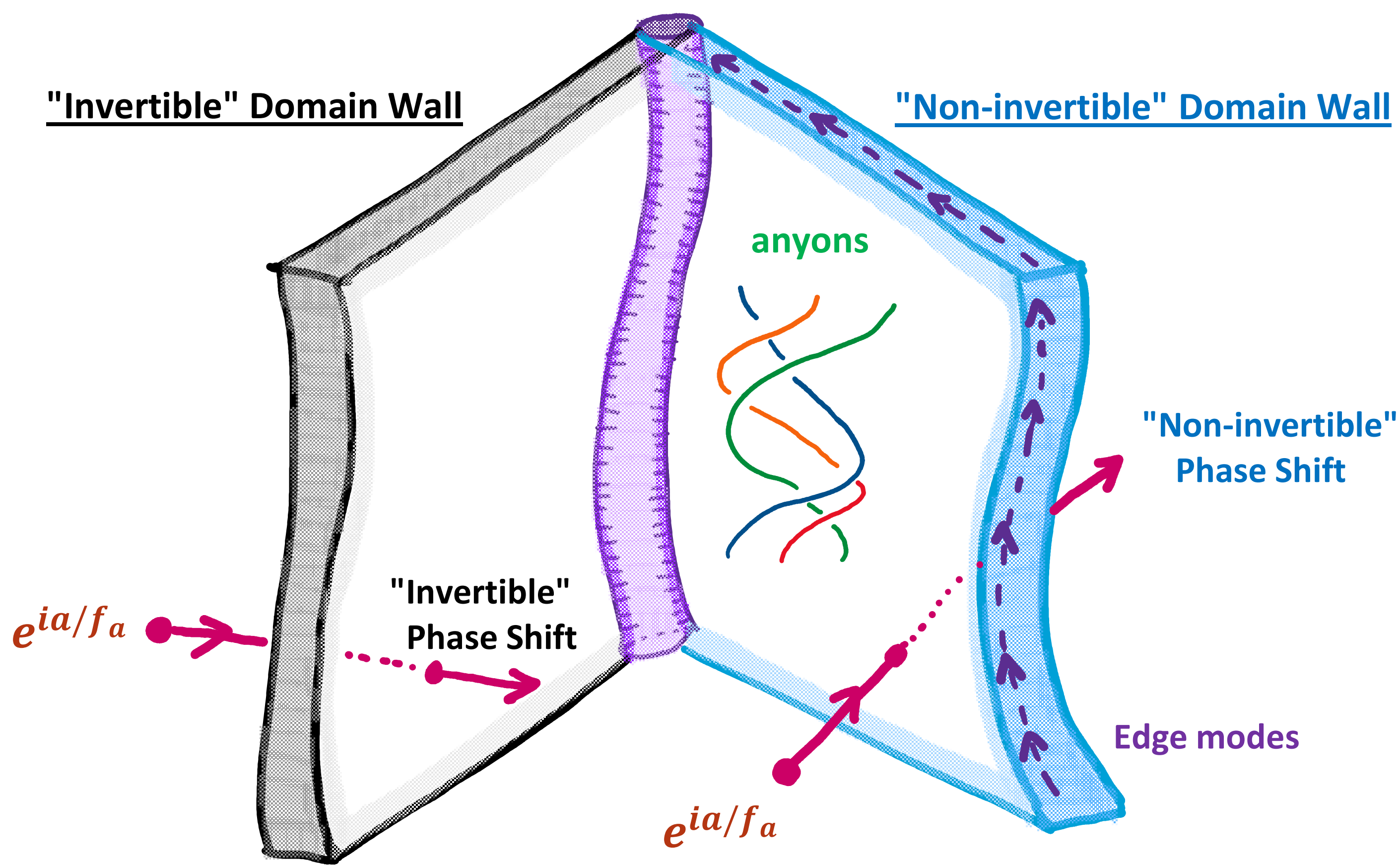

Non-invertible Axion Domain Wall Problem

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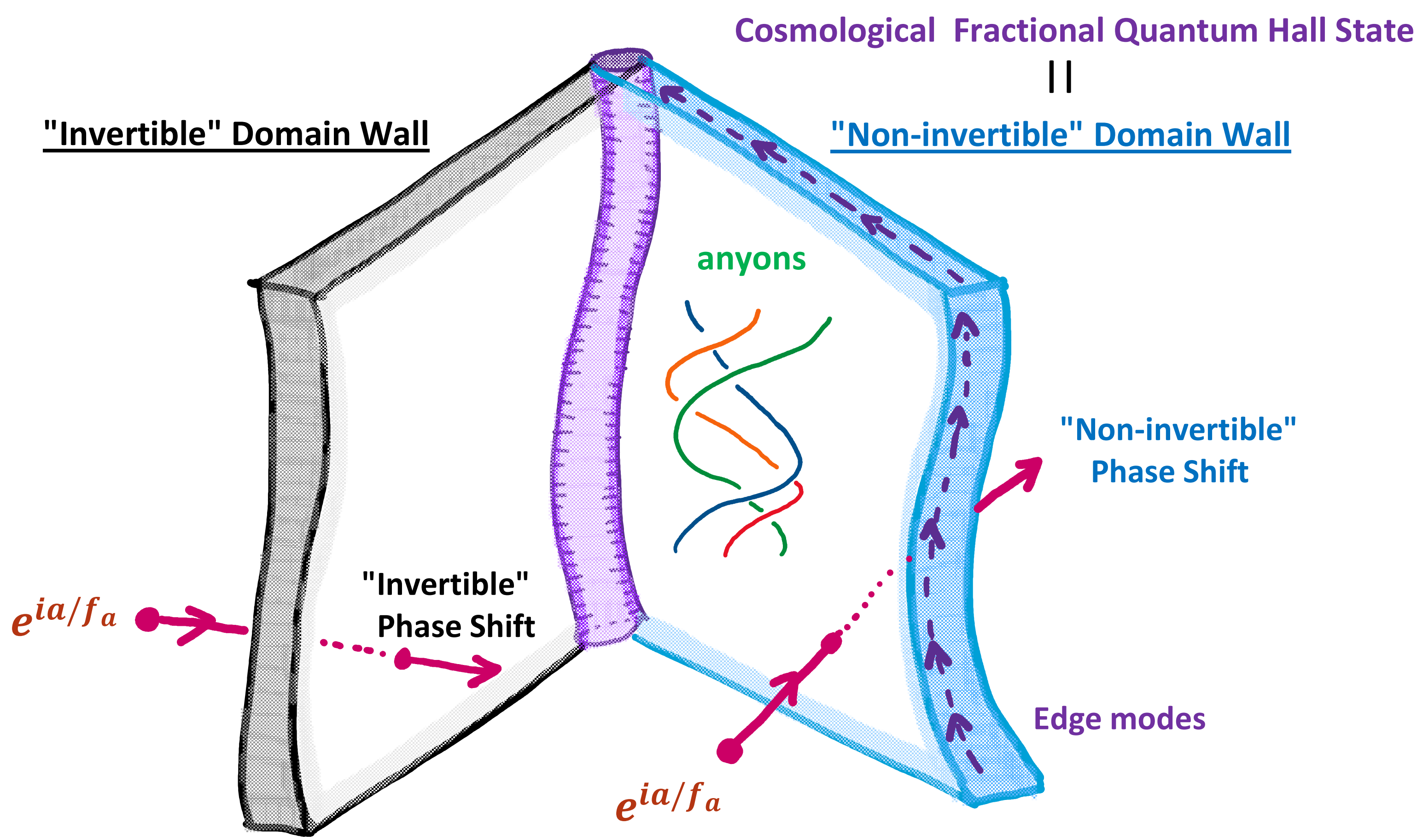

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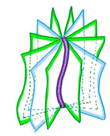
II. "Non-invertible Axion Domain Wall Problem" 

(Clay Cordova, Sungwoo Hong, Liantao Wang '23)

For given choice of the global structure of G_{SM} and anomaly coefficients, axion-SM theory can contain non-invertible-type as well as regular (invertible-type) domain wall defects. Any of these topological defects in the early universe is inconsistent with cosmological observations, and therefore should be made unstable or removed from the spectrum.

$$G_{SM} = \frac{SU(3)_C \times SU(2)_L \times U(1)_Y}{\Gamma}, \quad \Gamma = 1, Z_2, Z_3, Z_6$$

$$S \supset \frac{i\ell_3}{8\pi^2} \int \frac{a}{f_a} Tr(G \wedge G) + \frac{i\ell_2}{8\pi^2} \int \frac{a}{f_a} Tr(W \wedge W) + \frac{i\ell_1}{8\pi^2} \int \frac{a}{f_a} B \wedge B$$

Axion-Standard Model

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III. Axion-SM

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i. The entire SM matter fields are neutral under Z_6 transformation generated by

$$e^{\frac{2\pi i}{3}\lambda_8} = e^{\frac{2\pi i}{3}} I_3 \in SU(3)_C, \quad e^{\frac{2\pi i}{2}T_3} = -I_2 \in SU(2)_L, \quad e^{\frac{2\pi i}{6}Q_Y} \in U(1)_Y$$

ii. Z_6 Wilson lines are not screened $\Rightarrow$ $Z_6^{(1)}$ electric 1-form center symmetry

iii. Gauging $\Gamma = Z_p^{(1)} \subset Z_6^{(1)}$ $\Rightarrow$ $\frac{SU(3)_C \times SU(2)_L \times U(1)_Y}{\Gamma}$, $Z_{6/p}^{(1)}(e) \times Z_p^{(1)}(m)$

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$\Gamma = 1$: $\ell_{1,2,3} \in Z$
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See also: [Y. Choi, M. Forslund, H. T. Lam, S-H. Shao, 2309.03937]
[M. Reece, 2309.03939]

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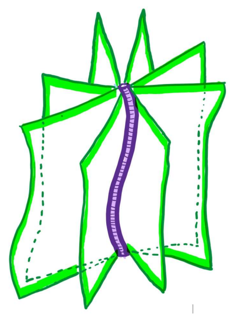
4. Non-invertible Axion Shift Symmetry: e.g. $\Gamma = Z_3$

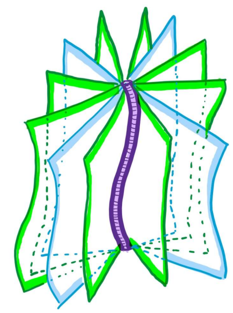
$Z_K^I \subset Z^{NI}_{\mathrm{gcd}(\ell_2,\ell_3)} \approx Z^{NI}_{\ell_3}$ where $K = \mathrm{gcd}\left(\frac{\ell_1}{3}, \ell_2, \ell_3, \frac{\ell_1+6\ell_3}{9}\right)$

Axion-Standard Model

III. Axion-SM



VS



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$Z^I_{K=1}$ where $K = \gcd\left(\frac{\ell_1}{3}, \ell_3, \frac{\ell_1 + 6\ell_3}{9}\right)$

Axion-Standard Model

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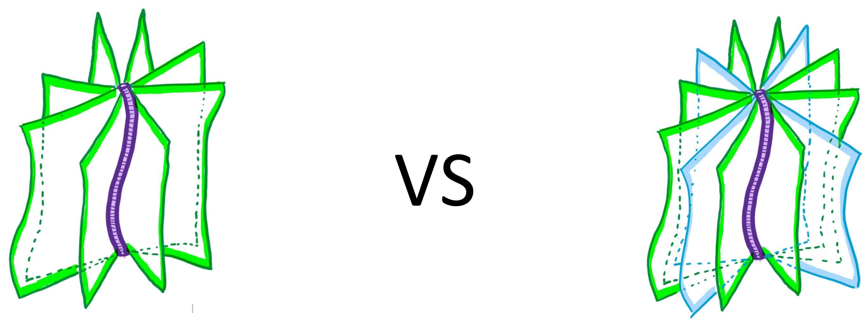

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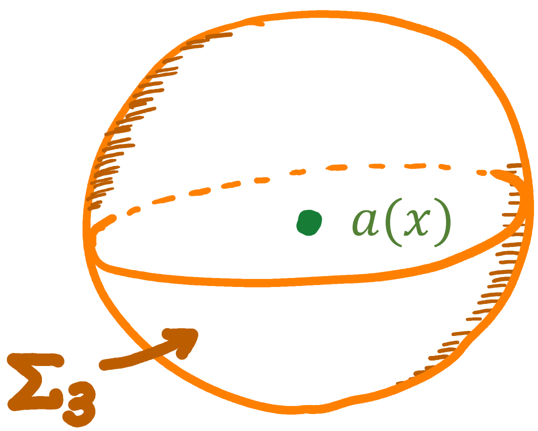
Solving DW Problem by
Non-Invertible Symmetry Breaking

Solving Axion Domain Wall Problem

I. Non-invertible Symmetry Breaking

Non-invertible axion shift symmetry = 0-form + magnetic 1-form composite



$$U(\alpha, \Sigma_3) = e^{i\alpha Q(\Sigma_3)}$$

$$Q(\Sigma_3) = \int_{\Sigma_3} d^3x\, J^0 = \int_{\Sigma_3} * J_1$$

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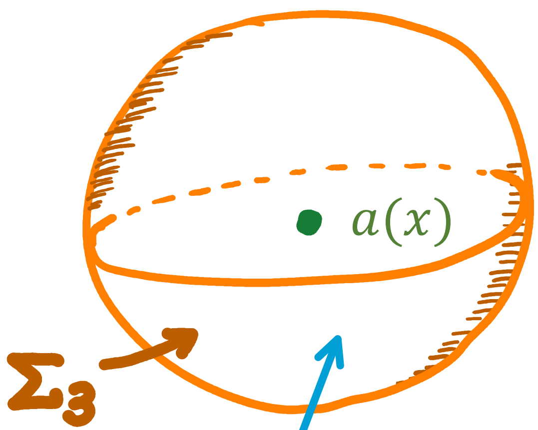

$$S_{defect} = \frac{iN}{4\pi}\int_{\Sigma_3} C \wedge dC$$

$$+ i\int_{\Sigma_3} C \wedge w$$

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$$U\left(\frac{2\pi}{k}, \Sigma_3\right) \rightarrow D_k = U\left(\frac{2\pi}{k}, \Sigma_3\right) \times \mathcal{A}^{N,p}(w)$$

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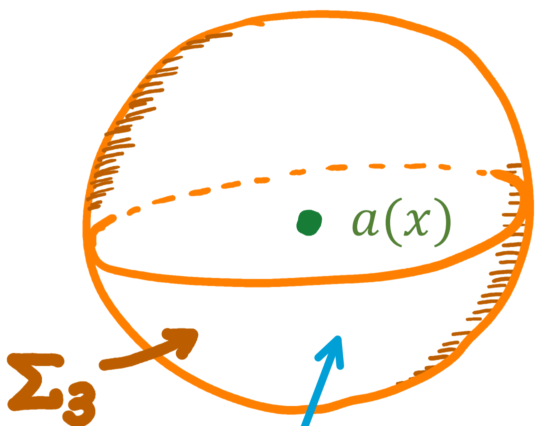

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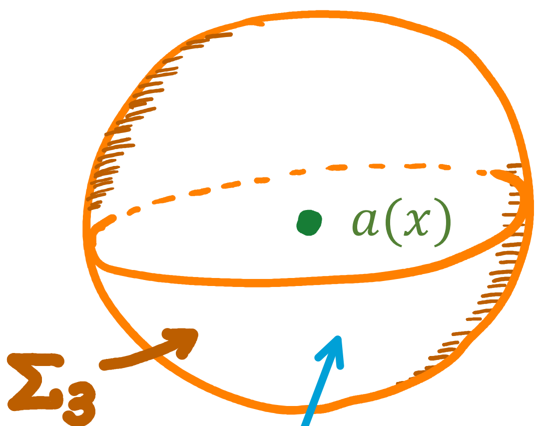

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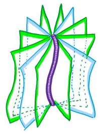
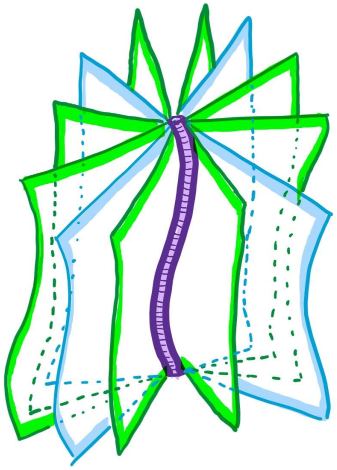
Solving Axion Domain Wall Problem

II. Solution by Non-invertible Symmetry Breaking 

(Clay Cordova, Sungwoo Hong, Liantao Wang)



Consider axion coupled to $G_g = SU(N)/Z_N$

$$S \supset \frac{iK}{8\pi^2} \int \frac{a}{f_a} Tr(G \wedge G)$$

G = field strength of G_g

Enlarged symmetry: $Z_{K/N}$(invertible) $\subset$ Z_K(non invertible)

Solving Axion Domain Wall Problem

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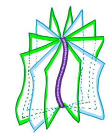

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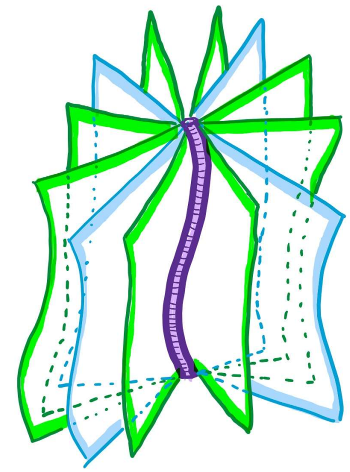

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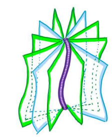
G_g (IR) Instanton effects $\Rightarrow$ Z_K^{NI} invariant $V(a)$

$$V(a) = \sum_{n=NZ} \alpha_n \cos\frac{Kn}{N}\frac{a}{f_a}\ , \qquad \alpha_n \propto e^{-\frac{8\pi^2 n}{g_{IR}^2}}$$

Solving Axion Domain Wall Problem

II. Solution by Non-invertible Symmetry Breaking 

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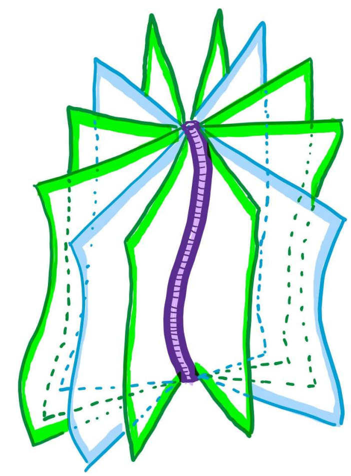

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$G_g \to G_{\text{uv}} = SU(N^2-1) \supset G_g$ with $\pi_1(G_{\text{uv}}) = 0 \Rightarrow Z_K^{NI}$ violating $\Delta V(a)$

$$\Delta V(a) = \sum_{m=0}^{N-1} \beta_m \cos\frac{Km}{N}\frac{a}{f_a}\ , \qquad \beta_m \propto e^{-\frac{8\pi^2 m}{g_{uv}^2}}$$

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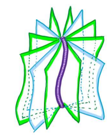
II. Solution by Non-invertible Symmetry Breaking 

(Clay Cordova, Sungwoo Hong, Liantao Wang)

$$V(a) = V(a) + \Delta V(a)$$

$$V(a) = \sum_{n=NZ} \alpha_n \cos\frac{Kn}{N}\frac{a}{f_a} \quad \text{and} \quad \Delta V(a) = \sum_{m=0}^{N-1} \beta_m \cos\frac{Km}{N}\frac{a}{f_a}$$

($\widehat{G_{IR}}$ - instantons) (G_{UV}/G_{IR} - instantons)

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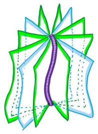
II. Solution by Non-invertible Symmetry Breaking 

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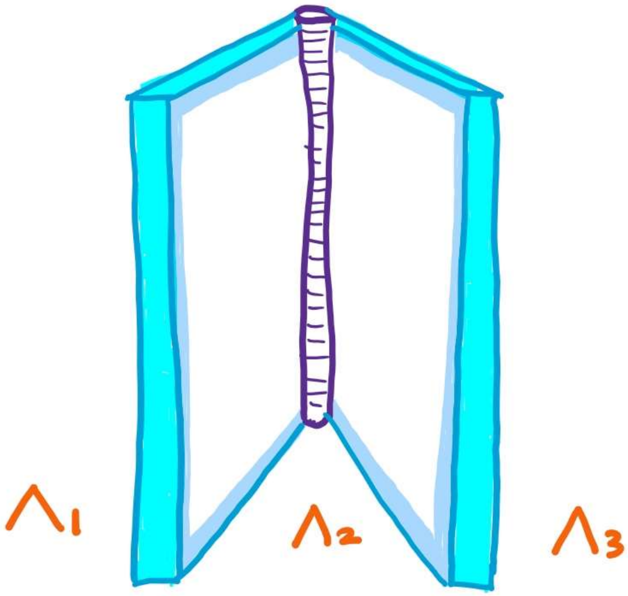
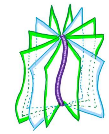


$$\Lambda_1 = \Lambda_2 = \Lambda_3$$

Solving Axion Domain Wall Problem

II. Solution by Non-invertible Symmetry Breaking 

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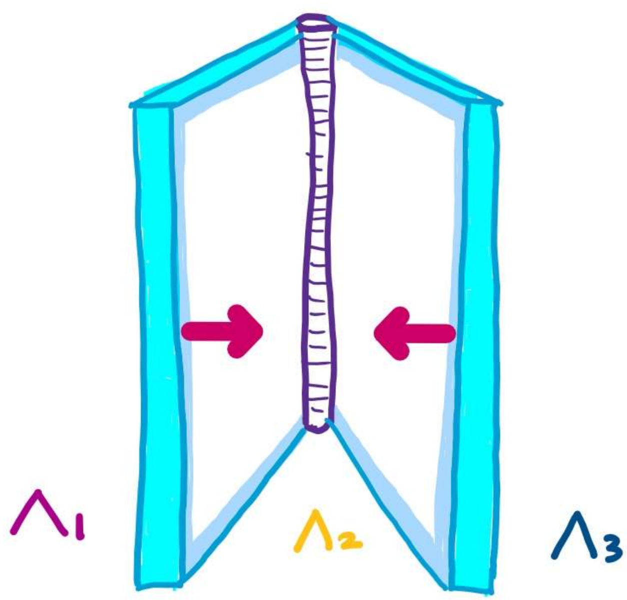
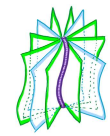
($\widehat{G_{IR}}$ - instantons) (G_{UV}/G_{IR} - instantons)



Solving Axion Domain Wall Problem

II. Solution by Non-invertible Symmetry Breaking 

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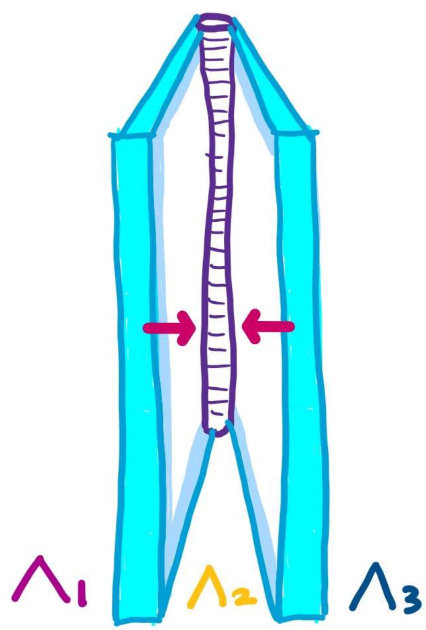
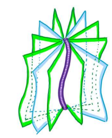


Solving Axion Domain Wall Problem

II. Solution by Non-invertible Symmetry Breaking 

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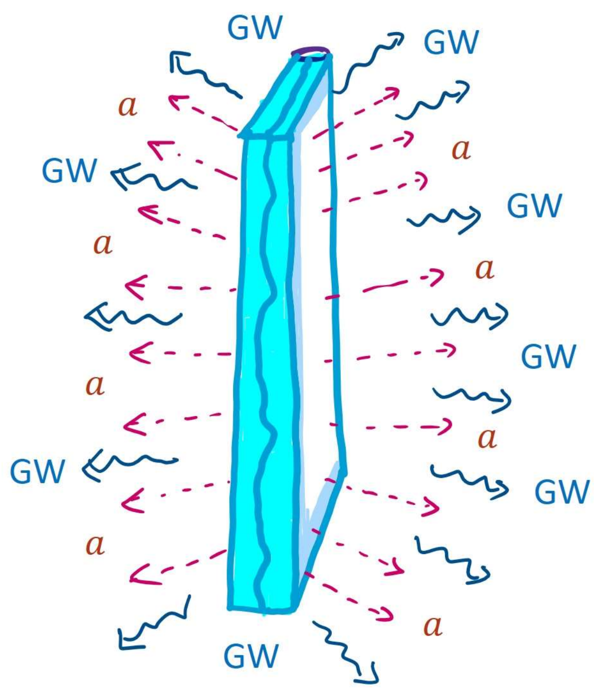


Thank You

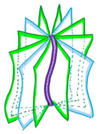
For

Your Attention!

Solving Axion Domain Wall Problem

III. Constraints on GUT Theories

(Clay Cordova, Sungwoo Hong, Liantao Wang)



1. We found that the most well-known GUTs can not lift the vacuum degeneracy

$\Gamma = \mathrm{Z}_6 : \quad SU(5)\,,\ SO(10)\,,\ E_6$

$\Gamma = Z_3 : \quad SU(4)_C \times SU(2)_L \times SU(2)_R$ [Pati-Salam]

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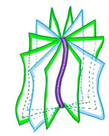
Solving Axion Domain Wall Problem

III. Constraints on GUT Theories



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index of embedding in $G_{\mathrm{UV}} \to G_{\mathrm{IR}}$: 1-IR-instanton = n-UV-instanton

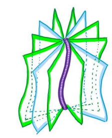
If $n \neq 1$, then $\exists\ G_{\mathrm{UV}}$-instantons not gauge-equvalent to G_{IR}-instanton

"Small Instantons"

These small instantons can break non-invertible symmetries.

Solving Axion Domain Wall Problem

III. Constraints on GUT Theories



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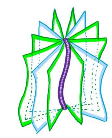
In all these cases, **"index of embedding (n)" = 1**

So, either anomaly coefficients "ℓ" in GUT should be $\boldsymbol{\ell_{\mathrm{UV}} = 1}$
or extra structures have to be supplemented to cure the DW problem

Solving Axion Domain Wall Problem



III. Constraints on GUT Theories

(Clay Cordova, Sungwoo Hong, Liantao Wang)

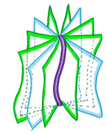
2. GUT constraints on Axion-Gauge Couplings

$$S_{SU(5)} \supset \frac{i\ell_{\mathrm{uv}}}{8\pi^2}\int \frac{a}{f_a}\, Tr(g \wedge g)$$

Solving Axion Domain Wall Problem

III. Constraints on GUT Theories

(Clay Cordova, Sungwoo Hong, Liantao Wang)



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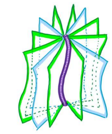
Solving Axion Domain Wall Problem

III. Constraints on GUT Theories



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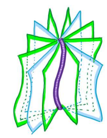
$$\boldsymbol{\ell_3 = \ell_{\rm uv},\ \ell_2 = \ell_{\rm uv},\ \ell_1 = 30\ell_{\rm uv}}$$

Solving Axion Domain Wall Problem



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This is consistent with

$\Gamma = 1$: $\ell_{1,2,3} \in Z$

$\Gamma = Z_2$: $\ell_1 \in 2Z$, $\ell_{2,3} \in Z$, and $\ell_1 + 2\ell_2 \in 4Z$

$\Gamma = Z_3$: $\ell_1 \in 3Z$, $\ell_{2,3} \in Z$, and $\ell_1 + 6\ell_3 \in 9Z$

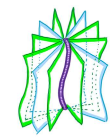
$\Gamma = Z_6$: $\ell_1 \in 6Z$, $\ell_{2,3} \in Z$, $\ell_1 + 2\ell_2 \in 4Z$, and $\ell_1 + 6\ell_3 \in 9Z$

but provides more stringent constraints.

Solving Axion Domain Wall Problem

III. Constraints on GUT Theories

(Clay Cordova, Sungwoo Hong, Liantao Wang)



2. GUT constraints on Axion-Gauge Couplings

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$SU(3)_C \times SU(3)_L \times SU(3)_R\ [Z_2]$: $\ell_3 = \ell_{\rm c},\ \ell_2 = \ell_{\rm L},\ \ell_1 = 6\ell_{\rm L} + 24\ell_R$

Thank You

For

Your Attention!

Back-up-1

(Quantization of Axion-Gauge Couplings)

Axion-Standard Model

3. Quantization of Axion-Gauge Couplings from non-trivial global form

$\Gamma = Z_3$: $\ell_1 \in 3Z$, $\ell_{2,3} \in Z$, and $\ell_1 + 6\ell_3 \in 9Z$

Axion-Standard Model

3. Quantization of Axion-Gauge Couplings from non-trivial global form

$\Gamma = Z_3 :$ $\ell_1 \in 3Z$, $\ell_{2,3} \in Z$, and $\ell_1 + 6\ell_3 \in 9Z$

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(iii) $\oint_{\Sigma_2} \frac{F}{2\pi} = \frac{1}{3} \oint_{\Sigma_2} w(A_3) \quad \Rightarrow \quad \frac{F}{2\pi} = \frac{1}{3} w(A_3) + X, \quad X \in H^2(M_4, Z)$

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(iv) Then, the axion periodicity $a \sim a + 2\pi f_a$ is respected if

$$\frac{\ell_1}{2} \int \left(\frac{w(A_3)}{3} + X\right)^2 + \ell_2 \, n_2 + \ell_3 \left(n_3 + \frac{2}{3} \int \frac{w(A_3)^2}{2}\right) \in Z$$

Axion-Standard Model

3. Quantization of Axion-Gauge Couplings from non-trivial global form

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(1) Consider first the case when $w(A_3)^2$-term vanishes

$$\frac{\ell_1}{3} \int w(A_3) \wedge X + \ell_2 n_2 + \ell_3 n_3 \in Z \;\; \Rightarrow \;\; \ell_1 \in 3Z, \; \ell_2, \ell_3 \in Z$$

Axion-Standard Model

3. Quantization of Axion-Gauge Couplings from non-trivial global form

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(2) $w(A_3)^2$: $\left(\frac{\ell_1}{9} + \frac{2\ell_3}{3}\right)\int\frac{w(A_3)^2}{2} \in Z \ \Rightarrow \ \ \ell_1 + 6\ell_3 \in 9Z$

Axion-Standard Model

III. Axion-SM

$$S \supset \frac{i\ell_3}{8\pi^2}\int \frac{a}{f_a}\, Tr(G\wedge G) + \frac{i\ell_2}{8\pi^2}\int \frac{a}{f_a}\, Tr(W\wedge W) + \frac{i\ell_1}{8\pi^2}\int \frac{a}{f_a}\, B\wedge B$$

1. G, W, B = field strength of $SU(3)_C$, $SU(2)_L, U(1)_Y$, respectively.

2. $G_{SM} = \frac{SU(3)_C \times SU(2)_L \times U(1)_Y}{\Gamma}$, $\Gamma = 1, Z_2, Z_3, Z_6$

3. Quantization of Axion-Gauge Couplings from non-trivial global form

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Back-up-2

(Non-invertible Axion Shift Symmetry)

Axion-Standard Model

4. Non-invertible Axion Shift Symmetry: e.g. $\Gamma = Z_3$

$$Z_K^I \subset Z^{NI}_{\mathrm{gcd}(\ell_2,\ell_3)} \approx Z^{NI}_{\ell_3} \quad \text{where} \quad K = \mathrm{gcd}\left(\frac{\ell_1}{3}, \ell_2, \ell_3, \frac{\ell_1+6\ell_3}{9}\right)$$

Under $a \to a + 2\pi f_a/z$ (recall: $F/2\pi = w/3 + X$)

$$\delta S = \frac{2\pi i}{z}\frac{\ell_1}{2}\int\left(\frac{w}{3}+X\right)^2 + \frac{2\pi i}{z}\,\ell_2 n_2 + \frac{2\pi i}{z}\,\ell_3\left(n_3 + \frac{2}{3}\int\frac{w^2}{2}\right) \in 2\pi i Z$$

Axion-Standard Model

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(1) For $w^2 = 0$: $\frac{\ell_1/3}{z}, \frac{\ell_2}{z}, \frac{\ell_3}{z} \in Z$

Axion-Standard Model

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Axion-Standard Model

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$Z_K^I \subset Z^{NI}_{\mathrm{gcd}(\ell_2,\ell_3)} \approx Z^{NI}_{\ell_3}$ where $K = \mathrm{gcd}\left(\frac{\ell_1}{3}, \ell_2, \ell_3, \frac{\ell_1+6\ell_3}{9}\right)$

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$\Rightarrow$ Invertible Symmetry: $Z_{\mathrm{gcd}(\ell_1/3,\ell_2,\ell_3,(\ell_1+6\ell_3)/9)}$

Axion-Standard Model

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$\Rightarrow$ Invertible Symmetry: $Z_{\gcd(\ell_1/3,\ell_2,\ell_3,(\ell_1+6\ell_3)/9)}$

(3) Regular Z-valued instanton effects: $Z_{\gcd(\ell_2,\ell_3)} \approx Z_{\ell_3}$