# Internal Supersymmetry

## and the Hierarchy Problem

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Based on work in progress with **Emanuele Gendy** & **Jessica Howard**

**Rencontres du Vietnam**

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# Predict the Higgs Mass

$\rho_{DM}$

- Free parameter in cosmological Standard Model.
- Readily explained by new phenomena over 90 decades of energy.
- No guarantee that correct explanation has measurable consequences (anthropics aside).

$m_h$

- Free parameter in particle Standard Model.
- Hard to explain without new phenomena at/below the weak scale.
- Correct explanation almost always has measurable consequences (anthropics aside).

*Predicting the Higgs mass is one of the great challenges of the era & we should approach it audaciously.*

# Possible Paths

*The goal is to predict the Higgs mass. Naturalness is a promising strategy (and the one nature has repeatedly chosen), but there are also principled frameworks explaining why it might appear to fail.*

**Naturalness**: $m_H^2 \sim m_{UV}^2$
(SUSY, global sym, discrete sym, …)

**Adjustment:** $m_H^2 \sim m_{UV}^2 + m_{IR}^2 \ll m_{UV}^2$
(relaxion, self-organization, …)

**Unnaturalness**: $m_H^2 \sim \Sigma m_{UV}^2 \ll m_{UV}^2$
(anthropics, NNaturalness, crunching, sliding…)

**Un-effectiveness**: $m_H^2 \sim m_{UVIR}^2$
(modular invariance, quantum gravity, …)

Today's talk: an audacious return to symmetries.

# Supersymmetry

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If you are wondering why you've never heard of this idea before (unless you've tried it yourself!), it's because there's a proliferation of ghosts.

In this talk I have no intention of hiding this fact or concealing any of the well-known pitfalls that ensue. My philosophy is to see how far we can get in the spirit of adventure, *a la* $E \to -E$ [Kaplan, Sundrum '05], Lee-Wick [Grinstein, O'Connell, Wise '07], or Agravity [Salvio, Strumia '14]. It's interesting enough to be well worth the effort.

I'll throw up a ⚠ DANGER sign whenever these issues begin to crop up, so you don't need to stop me to point out that the theory is pathological.

# An Aside: Lee-Wick Theory

[Grinstein, O'Connell, Wise '07, …]

Eliminate quadratic sensivitivity w/ higher derivatives: $\mathcal{L}_{\rm hd} = \frac{1}{2}\partial_\mu\hat{\phi}\partial^\mu\hat{\phi} - \frac{1}{2M^2}(\partial^2\hat{\phi})^2 - \frac{1}{2}m^2\hat{\phi}^2 - \frac{1}{3!}g\hat{\phi}^3$

Equivalently, integrate in PV ghost: $\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\partial_\mu\tilde{\phi}\partial^\mu\tilde{\phi} + \frac{1}{2}M^2\tilde{\phi}^2 - \frac{1}{2}m^2(\phi-\tilde{\phi})^2 - \frac{1}{3!}g(\phi-\tilde{\phi})^3$

Apply to hierarchy problem by introducing a Lee-Wick counterpart for every SM field.

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Apply to hierarchy problem by introducing a Lee-Wick counterpart for every SM field.

⚠ DANGER *Apparent pathologies: classical & quantum instabilities, unitarity violation.*

[Grinstein, O'Connell, Wise '07, …]: argue that finite width of LW fields, modified contour prescription imply at worst microscopic acausality.

In any case, a major who-ordered-that problem: why are there Lee-Wick partners, and *why do they share the same cutoff with their SM counterparts?*

# Supergroups: $SU(N|M)$

See e.g. [I. Bars, “Supergroups and Their Representations”, 1984]

$SU(N|M)$ algebra:
matrices of the form

$$\mathcal{H} = \begin{pmatrix} H_N & \theta \\ \theta^\dagger & H_M \end{pmatrix}$$

$N \times N$ complex Hermitian matrix

$N \times M$ complex Grassmann matrix

$M \times M$ complex Hermitian matrix

Not traceless, but supertraceless: $\mathrm{str}\,(\mathcal{H}) \equiv \mathrm{tr}(\sigma_3 \mathcal{H}) = \mathrm{tr}(H_N) - \mathrm{tr}(H_M) = 0$

$$\sigma_3 = \begin{pmatrix} \mathbb{I}_{N\times N} & 0 \\ 0 & -\mathbb{I}_{M\times M} \end{pmatrix}$$

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Conveniently organized by generators (resp. multiplying bosonic and Grassmann params)

$$T_N^a = \underbrace{\begin{pmatrix} t_N^a & 0 \\ 0 & 0 \end{pmatrix} \;,\; T_M^b = \begin{pmatrix} 0 & 0 \\ 0 & t_M^b \end{pmatrix}}_{SU(N),\, SU(M) \text{ generators}} \qquad \lambda_U \propto \underbrace{\begin{pmatrix} \mathbb{I}_N/N & 0 \\ 0 & \mathbb{I}_M/M \end{pmatrix}}_{U(1) \text{ generator}} \qquad S_i = \frac{1}{2}\begin{pmatrix} 0 & s_i \\ s_i^\dagger & 0 \end{pmatrix} \underbrace{\;,\;}_{s_i,\,(\tilde{s}_i):\ -i,\,(1) \text{ in one entry, 0 otherwise}} \tilde{S}_i = \frac{1}{2}\begin{pmatrix} 0 & \tilde{s}_i \\ \tilde{s}_i^\dagger & 0 \end{pmatrix}$$

Exponentiates to form group elements; bosonic symmetry is $SU(N) \times SU(M) \times U(1)$

# Supergroups

Generators normalized s.t. $\mathrm{str}\,(\lambda_I\lambda_J) = \frac{1}{2}g_{IJ}$

$$g_{IJ} = \left(\begin{array}{c|c}
\begin{matrix}
\underbrace{\begin{matrix}1&&\\&1&\\&&1\end{matrix}}_{SU(N)} &&&& \\
& \ddots &&& \\
&& \overbrace{\pm1}^{U(1)\,:\,\mathrm{sgn}(M-N)} && \\
&&& \underbrace{\begin{matrix}-1&&\\&-1&\\&&-1\end{matrix}}_{SU(M)} & \\
&&&& \ddots
\end{matrix}
&
\begin{matrix} 0 \\ \\ \overbrace{\qquad\qquad\qquad}^{S_i,\,\tilde{S}_i} \end{matrix}
\\ \hline
0
&
\begin{matrix}
\begin{matrix}0&i\\-i&0\end{matrix} && 0 \\
& \begin{matrix}0&i\\-i&0\end{matrix} & \\
0 && \ddots
\end{matrix}
\end{array}\right)$$

⚠ DANGER

Completeness relation: $(\lambda_I)_{ij}g^{IJ}(\lambda_J)_{kl} = \frac{1}{2}\left(\delta_{il}\delta_{jk}(-1)^{\mathrm{f}(j)\mathrm{f}(k)} - \frac{1}{N-M}\delta_{ij}\delta_{kl}\right)$

**Grading**

$$\mathrm{f}(i) = \begin{cases} 0 & \text{if } 1 \le i \le N \\ 1 & \text{if } N+1 \le i \le N+M \end{cases}$$

# Super-scalars

Scalar in the fundamental of $SU(N|M)$

$$\Phi_i = \begin{pmatrix} \phi_a \\ \psi_\alpha \end{pmatrix} \begin{array}{l} \longleftarrow \textit{N}\text{-component complex scalar} \\ \longleftarrow \textit{M}\text{-component complex ghost} \end{array}$$

(scalar w/ fermionic statistics $f(\psi_\alpha) = 1$)

This is the first place we encounter an obvious issue: we have Lorentz scalars with fermionic statistics, i.e. FP-like ghosts.

*Perhaps they are innocuous, perhaps a BRST-like symmetry comes to the rescue, perhaps they can be decoupled. Let's proceed apace with perturbation theory.*

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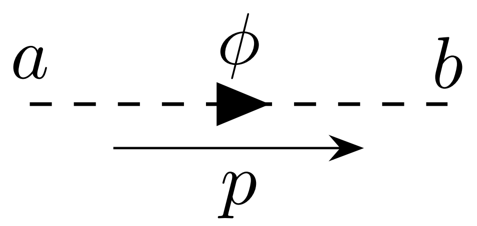
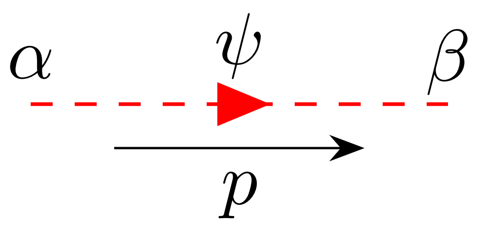
Renormalizable Lagrangian: $\mathcal{L}_\Phi = \partial_\mu \Phi_i^\dagger \partial^\mu \Phi_i - m^2 \Phi_i^\dagger \Phi_i + \lambda (\Phi_i^\dagger \Phi_i)^2$

$$\mathcal{L}_\Phi = +\partial_\mu\phi_a^\dagger\partial^\mu\phi_a + \partial_\mu\psi_\alpha^\dagger\partial^\mu\psi_\alpha - m^2\phi_a^\dagger\phi_a - m^2\psi_\alpha^\dagger\psi_\alpha + \lambda\left[(\phi_a^\dagger\phi_a)^2 + (\psi_\alpha^*\psi_\alpha)^2 + 2\phi_a^\dagger\phi_a\psi_\alpha^\dagger\psi_\alpha\right]$$



$\frac{i\delta_{ab}}{p^2-m^2}$



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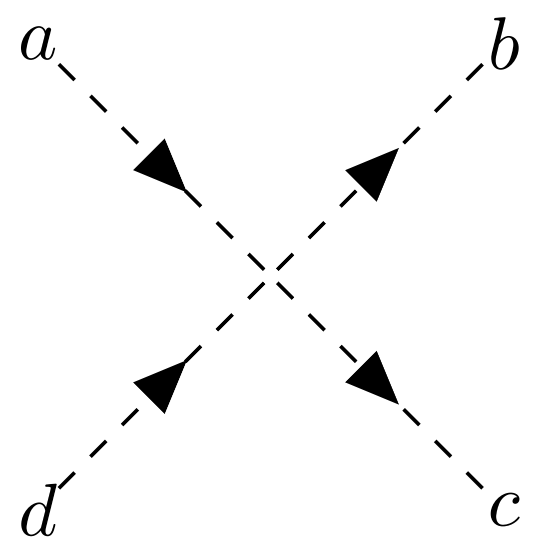
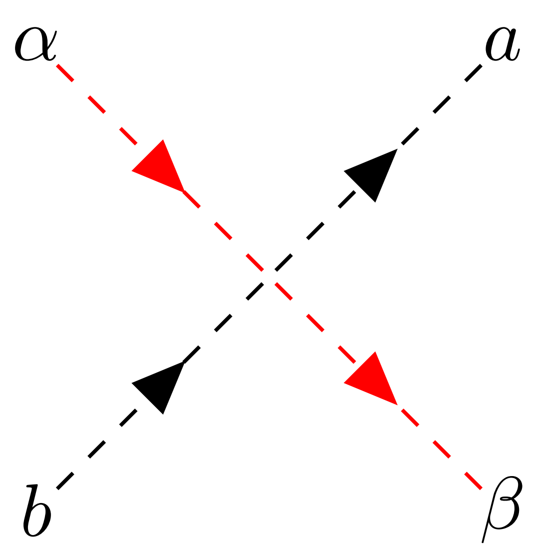


$2i\lambda(\delta_{ab}\delta_{cd} + \delta_{ac}\delta_{bd})$



$2i\lambda\delta_{\alpha\beta}\delta_{ab}$

# Super-finite!

Consider the one-loop correction to the mass of the ordinary scalars $\phi_a$:

$$\Sigma_1 = -2(N+1) \times \lambda \int \frac{d^d p}{(2\pi)^d} \frac{1}{p^2 - m^2}$$

$$\Sigma_2 = +M \times (2\lambda) \int \frac{d^d p}{(2\pi)^d} \frac{1}{p^2 - m^2}$$

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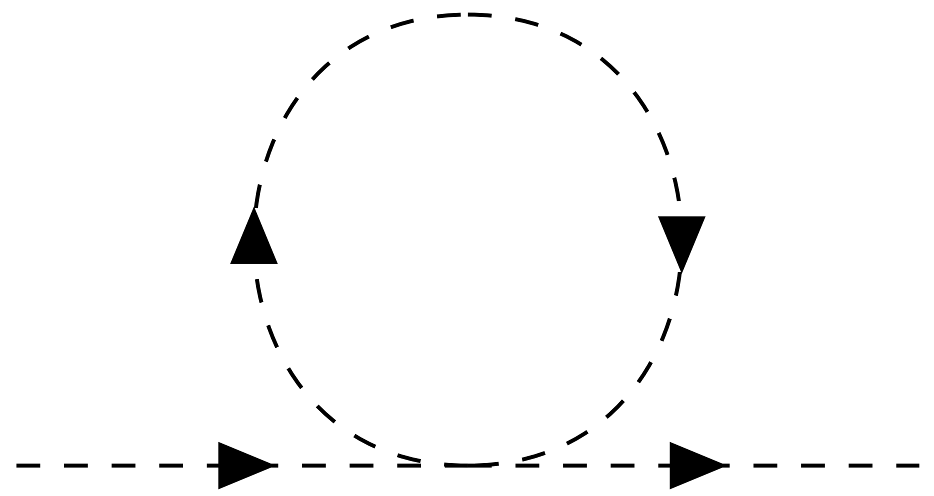

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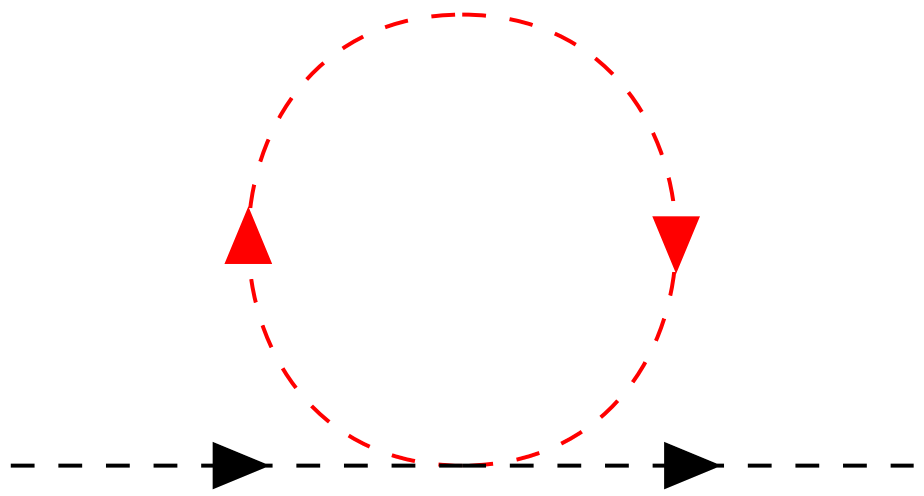

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Cancel entirely for $M = N + 1$, i.e. $SU(N|N+1)$

Soft breaking works as expected: $\Delta\mathcal{L} = -\rho^2 \psi_\alpha^\dagger \psi_\alpha \Rightarrow \delta m^2 \propto \frac{\lambda}{16\pi^2} \rho^2 \log(\Lambda^2/\rho^2)$

Amusing enough to keep going…

# Super-vectors

*First introduced in [S. Arnone, Y. A. Kubyshin, T. R. Morris and J. F. Tighe, "Gauge invariant regularization via SU(N|N)", 2001] with an eye towards systematic PV-like regularization of non-abelian gauge theories.*

Let's add $SU(N|M)$ gauge bosons.

$A_\mu$ bosonic statistics, $B_\mu$ fermionic statistics

$$\mathcal{A}_\mu = \begin{pmatrix} A^{1a}_\mu t^a_N & B^i_\mu (s_1 + \tilde{s}_i) \\ (B^\dagger_\mu)^i (s^\dagger_1 + \tilde{s}^\dagger_i) & A^{2b}_\mu t^b_M \end{pmatrix} + A^\lambda_\mu \lambda$$

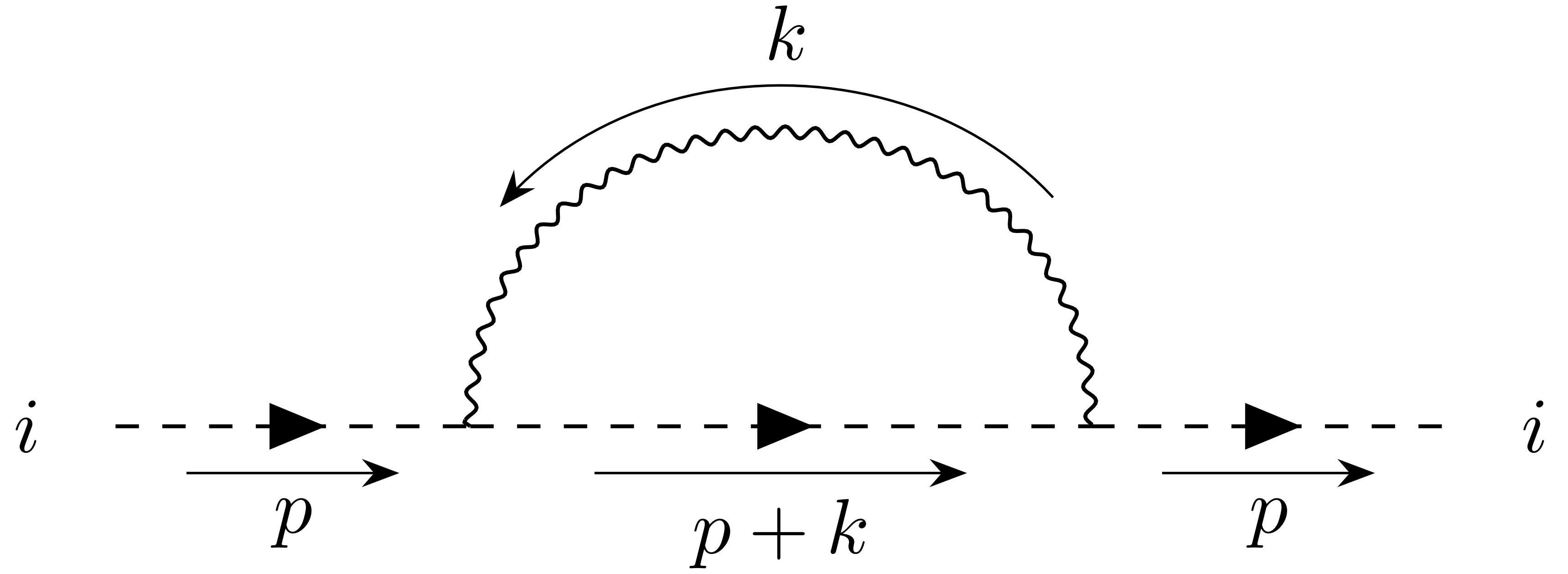
Now two pathologies: FP-like ghosts (the $B_\mu$) and PV-like ghosts (the $A^2$)

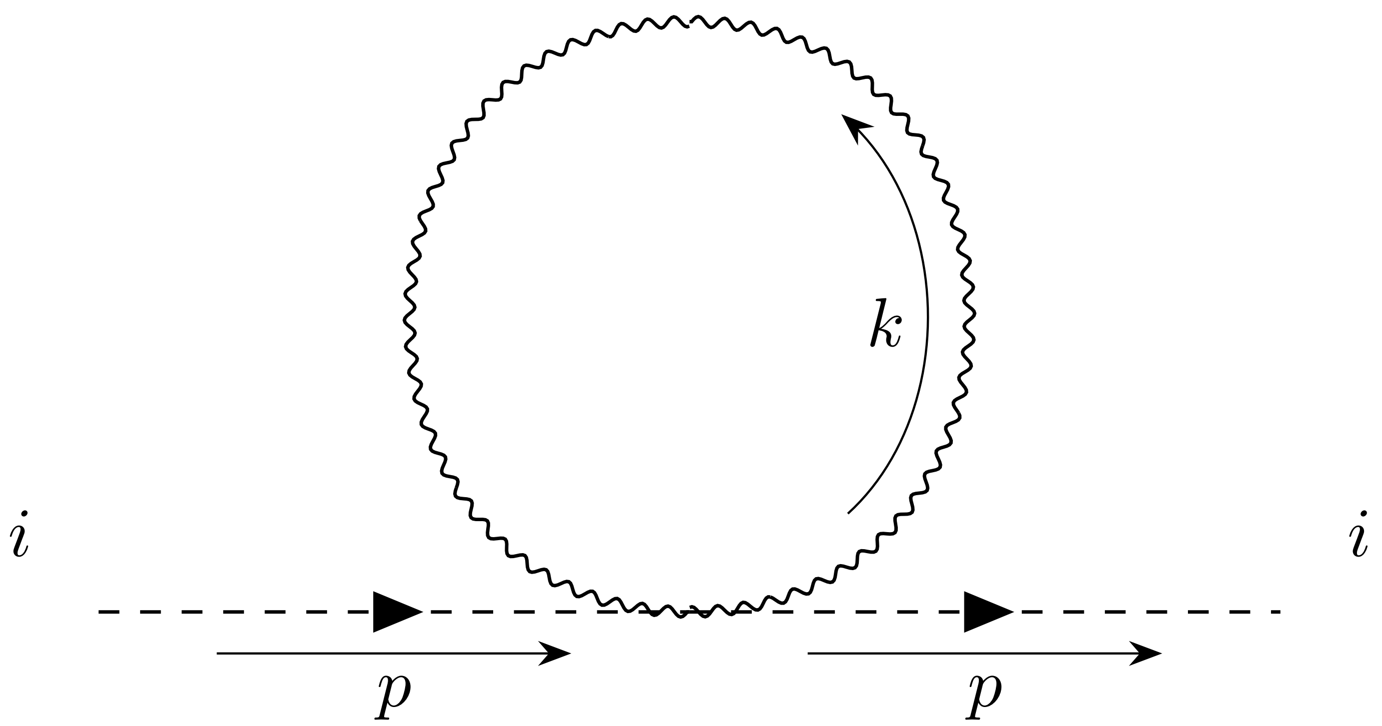
Gauge kinetic term $\mathcal{L}_{\text{gauge}} = -\frac{1}{2}\text{str}\left(\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}\right)$ involves $SU(N|M)$ metric

$$g_{IJ} = \left(\begin{array}{ccc|c} \mathbb{I}_N & & & \\ & \pm 1 & & 0 \\ & & -\mathbb{I}_M & \\ \hline & 0 & & \ddots \end{array}\right)$$

# Super-vectors

Turn the crank, compute mass renormalization of $\Phi_i$ charged under $SU(N|M)$:





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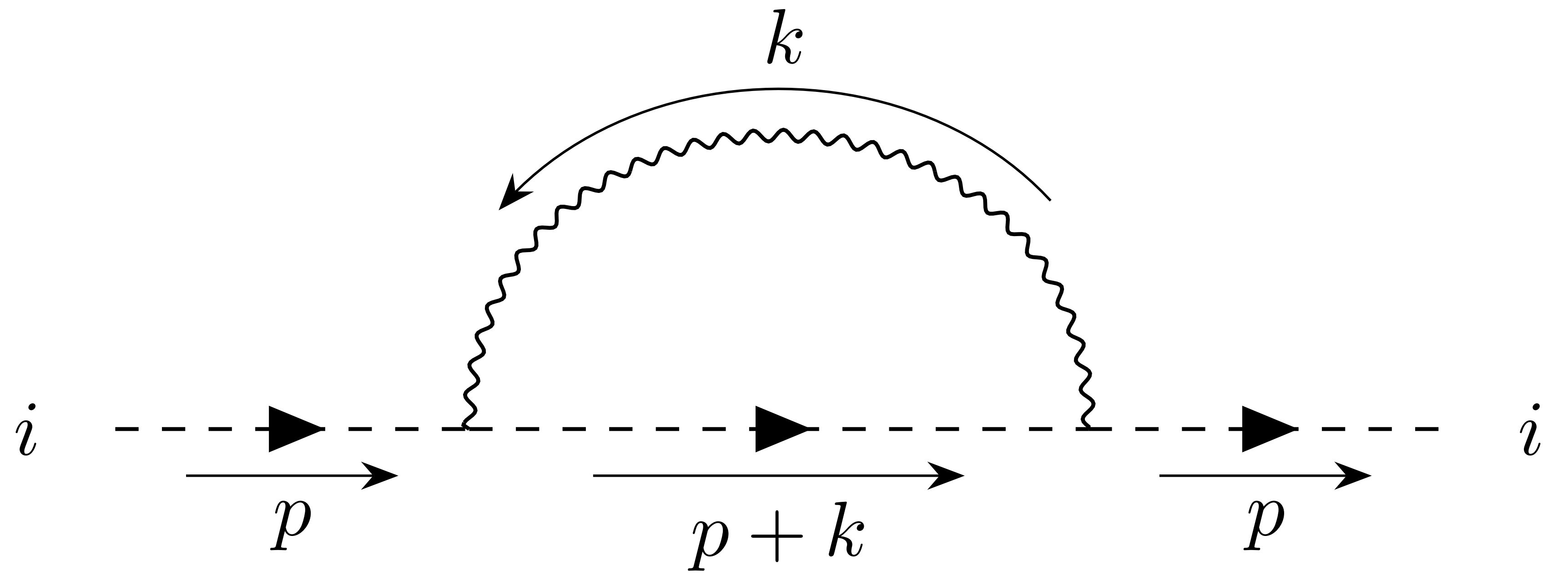

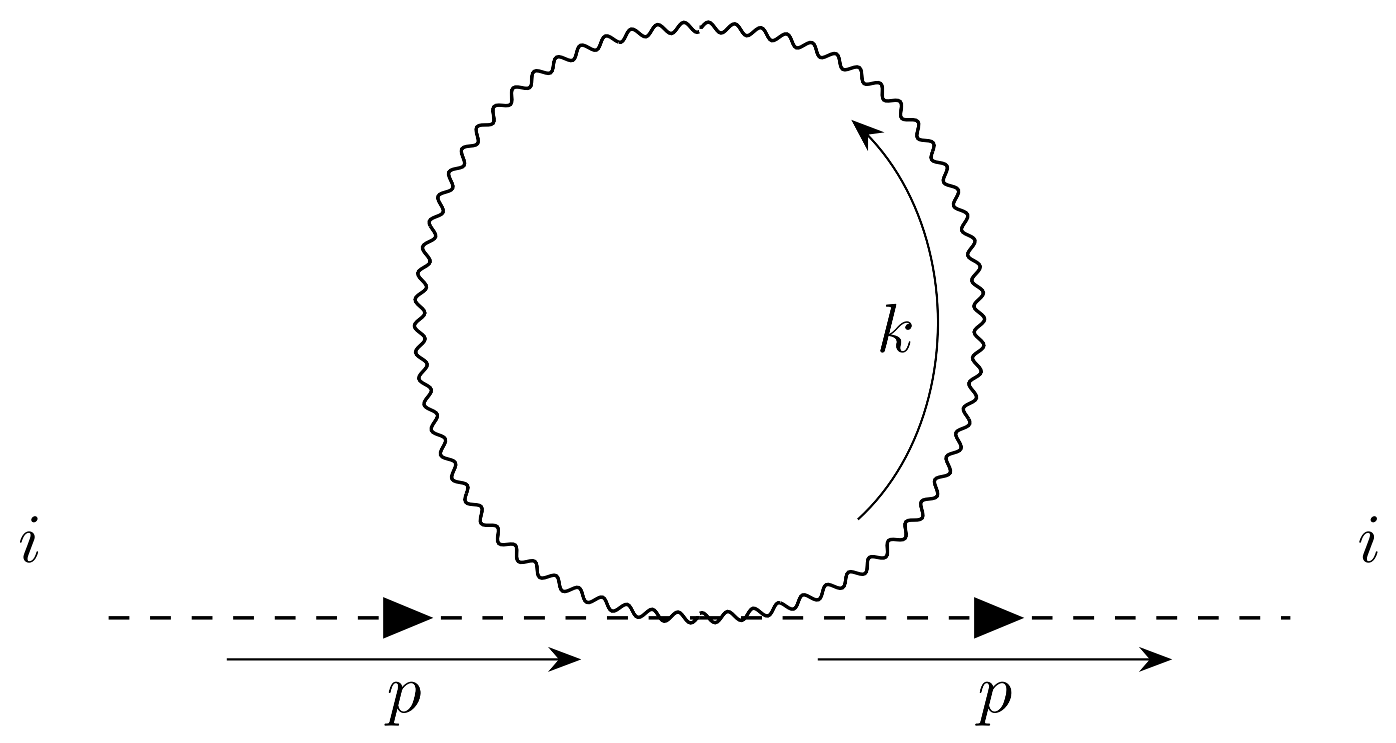

Both proportional to $$(\lambda_I)_{ik} g^{IJ} (\lambda_J)_{ki} = \frac{1}{2}\delta_{ii}\left((N-M) - \frac{1}{N-M}\right)$$ (completeness relation)

Vanish for $M = N + 1$, i.e. $SU(N|N+1)$

Amusing enough to keep going…

# Super-fermions

Can add $SU(N|M)$ fermions w/ Yukawa coupling:
simplest example is a fundamental $\xi_i$ and an adjoint $\Theta_I$

$$\mathcal{L}_{Yuk} = -y\Phi_i\bar{\xi}_j(\lambda_I)_{ij}\Theta^I$$

⚠ DANGER

Lorentz spinors with bosonic statistics.

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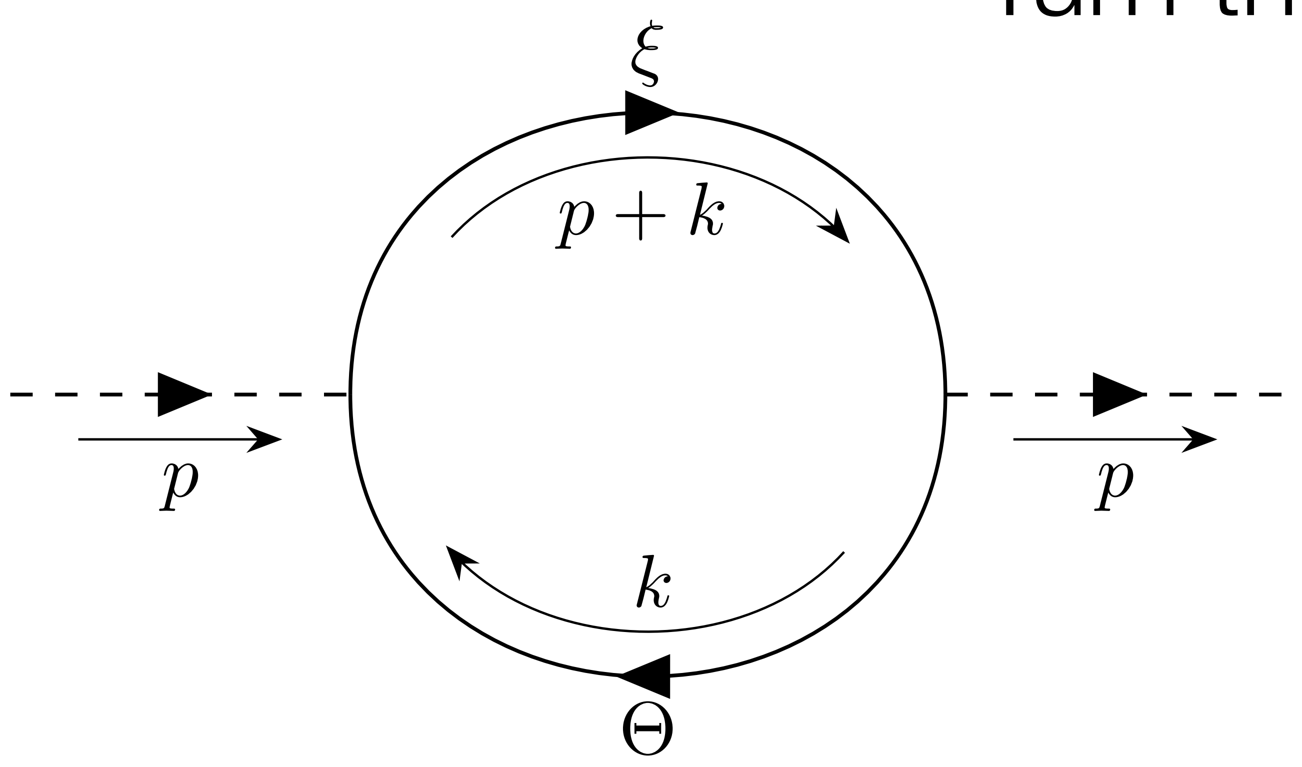

$$= (-iy)^2(i\delta_{kl})(ig^{IJ})(\lambda_I)_{ik}(\lambda_J)_{li}\int\frac{\mathrm{d}^dk}{(2\pi)^k}\frac{\mathrm{Tr}\left[(\not{p}+\not{k}+m_\xi)(\not{k}+m_\Theta)\right]}{\left[(p+k)^2-m_\xi^2+i\varepsilon\right]\left[k^2-m_\Theta^2+i\varepsilon\right]}$$

Proportional to $$(\lambda_I)_{ik}g^{IJ}(\lambda_J)_{ki} = \frac{1}{2}\delta_{ii}\left((N-M)-\frac{1}{N-M}\right)$$ (completeness relation)

Vanish for $M = N+1$, i.e. $SU(N|N+1)$

# Can this theory make sense?

We have found a very interesting theory that is UV insensitive, for reasons superficially similar to the Lee-Wick model, but now completely organized by an internal symmetry!

Both the FP-like ghosts and the PV-like ghosts are potentially fatal to the theory. Perhaps soft breaking and contour prescriptions can be used to argue away pathologies.

As a first step, we can entertain a fantasy: what if the $SU(N|N+1)$ symmetry is spontaneously broken in a way that lifts the FP-like ghosts and decouples the PV-like ghosts?

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Tempting to start w/ a scalar $\Sigma$ in the adjoint, w/ $\mathbb{Z}_2$ for simplicity, hence the most general renormalizable Lagrangian

$$\mathcal{L}_\Sigma = \mathrm{str}\left([\nabla_\mu, \Sigma]^2\right) + \mu^2 \mathrm{str}\left(\Sigma^2\right) - \lambda_1 \mathrm{str}\left(\Sigma^2\right)^2 - \frac{1}{4}\lambda_2 \mathrm{str}\left(\Sigma^4\right)$$

☹️ there are always tachyonic ghosts, no local minima.

# Spontaneous Symmetry Breaking

Second try: introduce scalar $\Sigma^i_j$ transforming as a direct product of fundamental and anti-fundamental representations, i.e. like adjoint but w/out supertracelessness.

See [S. Arnone, Y. A. Kubyshin, T. R. Morris and J. F. Tighe, "Gauge invariant regularization via SU(N|N)", 2001]

Supplement $SU(N|M)$ generators with the identity, $\lambda_I \to \lambda_{\tilde{I}} = \{\lambda_I,\, \lambda_T \equiv \frac{1}{\sqrt{2(N-M)}}\mathbb{I}\}$

Consider potential $V[\Sigma] = -\frac{1}{2}\mu^2\Sigma^{\tilde{I}}g_{\tilde{I}\tilde{J}}\Sigma^{\tilde{J}} + \frac{1}{4}\lambda_1\left(\Sigma^{\tilde{I}}g_{\tilde{I}\tilde{J}}\Sigma^{\tilde{J}}\right)^2 + \frac{1}{4}\lambda_2\Sigma^{\tilde{I}}\Sigma^{\tilde{J}}\Sigma^{\tilde{K}}\Sigma^{\tilde{L}}T_{\tilde{I}\tilde{J}\tilde{K}\tilde{L}}$

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✓ At tree level, no runaways provided $\lambda_2 < 4(M-N)\lambda_1$, persists at one loop in global limit.

✓ Fermionic scalars (FP-like ghosts) remain exactly massless in the global limit at one loop, consistent w/ Goldstone’s theorem (?!)

# Spontaneous Symmetry Breaking

$$\langle\Sigma\rangle = \rho\sigma_3 \ : SU(N|M) \to SU(N) \times SU(M) \times U(1)$$

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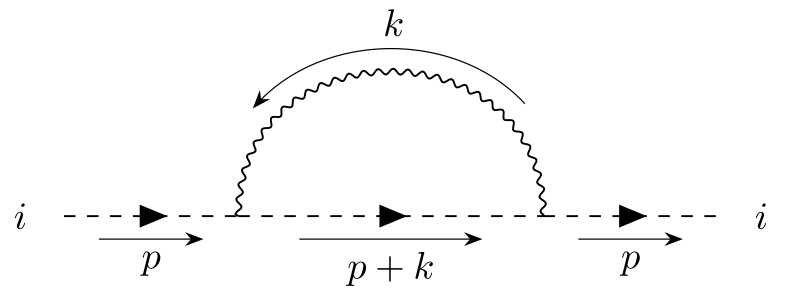
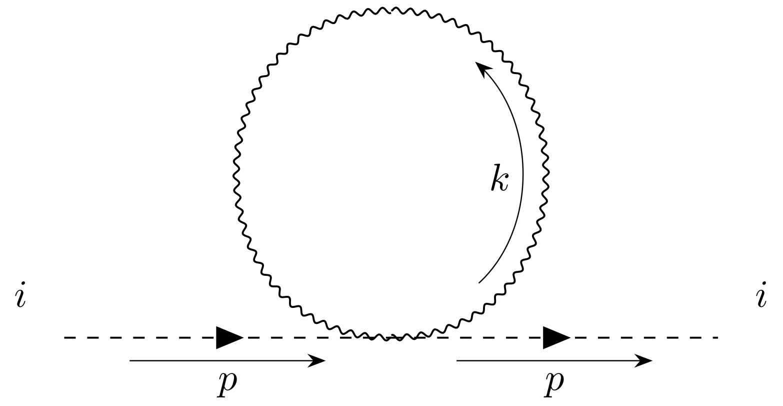


However, $m_B$ feeds into mass of scalars charged under $SU(N|N+1)$,

$$m^2_{\Phi,1-\text{loop}} \approx m^2_{\Phi,\text{tree}} + \frac{g^2}{16\pi^2}\left(\frac{N+1}{2}\right)\left(1 + 2\log(\frac{\mu^2}{m^2})\right)m_B^2$$



⇒ Tension between naturalness and fully decoupling PV-like ghosts. (Also true for soft explicit breaking.)
But a promising start for thinking about SSB, either directly or in a hidden sector.

# Internally Supersymmetric Standard Model?

*Not just a weirder version of SUSY.*

- Unlike spacetime SUSY, don’t need to embed the entire SM into a supergroup. Instead can selectively combine internal supergroups with ordinary symmetries or gauge bosonic subgroups of supergroup global symmetries.
- Even if the entire SM is embedded into a product of internal supergroups, internal SUSY is much less constraining — e.g. Higgs quartic not fixed by gauge couplings, so naturalness problems less severe (and unlike ordinary global symmetries, no tree-level Higgs coupling deviations).
- Seems worth exploring further despite dangers.

# Cảm ơn!