Gravitational waves from phase transitions during inflation

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BSM in Particle Physics and Cosmology: 50 years later

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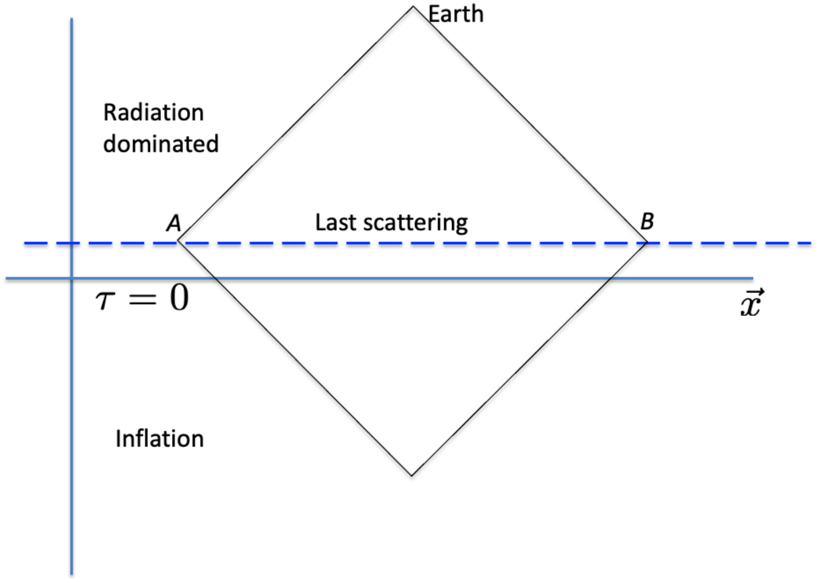
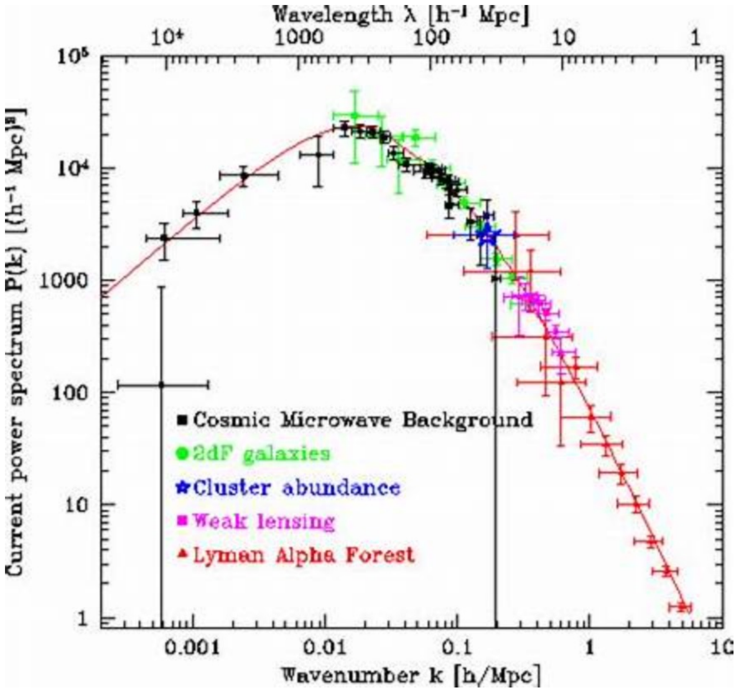
2009.12381, 2201.05171 w/ Kun-Feng Lyu, Lian-Tao Wang and Siyi Zhou
2208.14857 w/ Xi Tong and Siyi Zhou
2308.00070 w/ Boye Su, Hanwen Tai, Lian-Tao Wang, Chen Yang

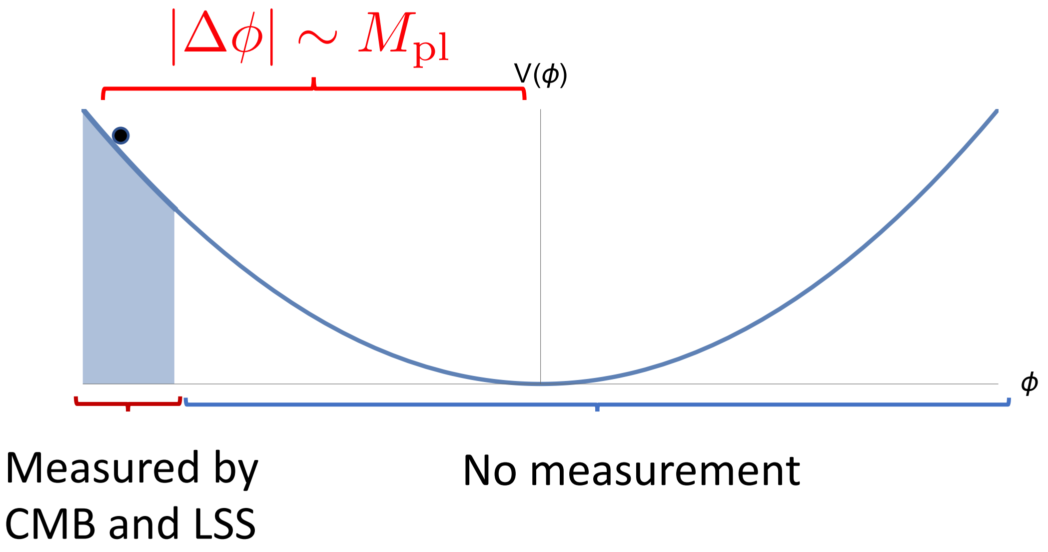
Very brief introduction of inflation





- To solve the problems, 40 to 60 e-folds is required, BUT we can only observe ten!

Slow roll models

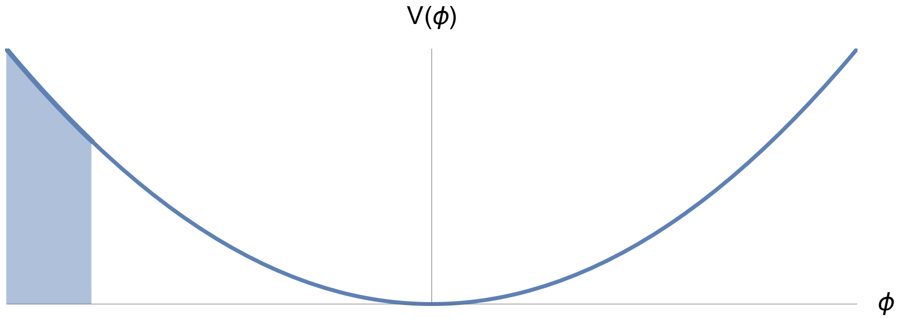


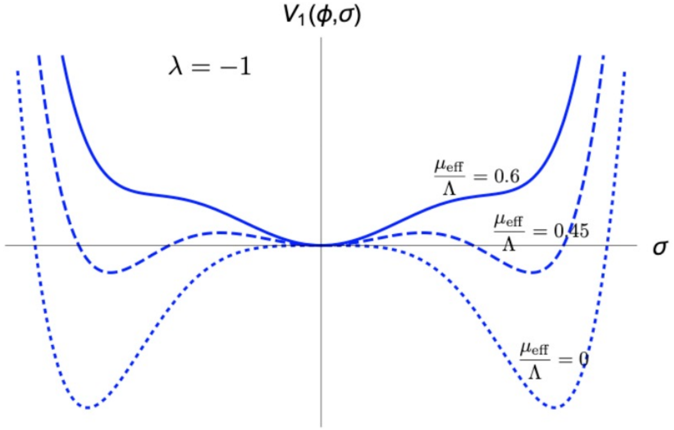
- The inflaton must couple to some spectator field.
- The masses or couplings in the spectator sector can be changed drastically due to the evolution of the inflaton field.

Induced phase transition in spectator sectors

- ϕ: inflaton field σ: spectator field

Example 1: $$V_1(\phi,\sigma) = -\frac{1}{2}(\mu^2 - c^2\phi^2)\sigma^2 + \frac{\lambda}{4}\sigma^4 + \frac{1}{8\Lambda^2}\sigma^6$$





Example 2: $$\mathcal{L}_\sigma = -(1 - \frac{c^2\phi^2}{\Lambda^2})\frac{1}{4g^2}G^a_{\mu\nu}G^{a\mu\nu}$$

Induced phase transition in spectator sectors

Evolution of inflaton

Phase transitions in spectator sector

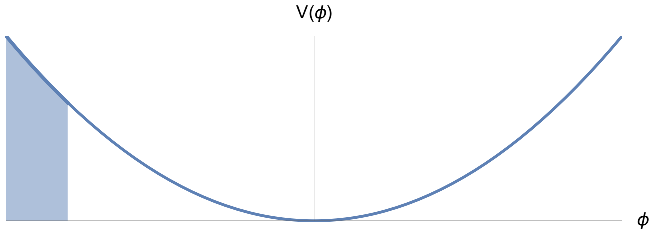
Gravitational waves

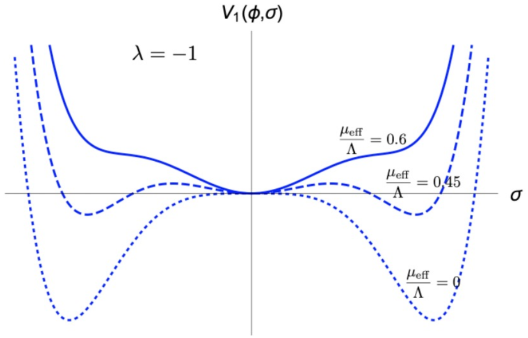
Primary GW

Curvature perturbation

Gravitational waves

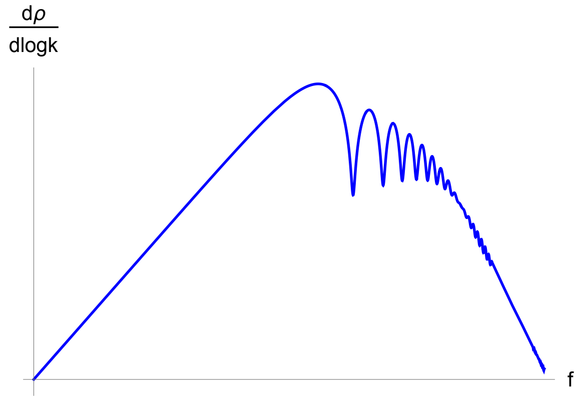
Secondary GW

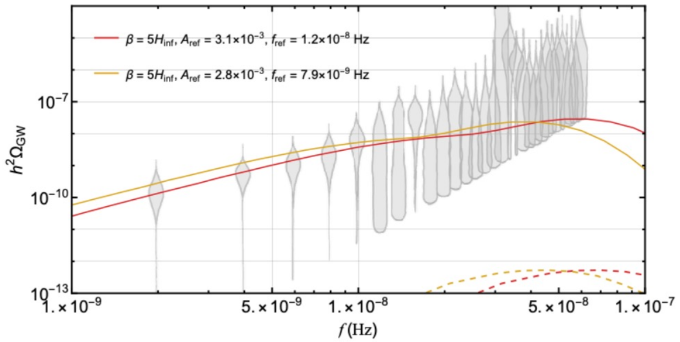


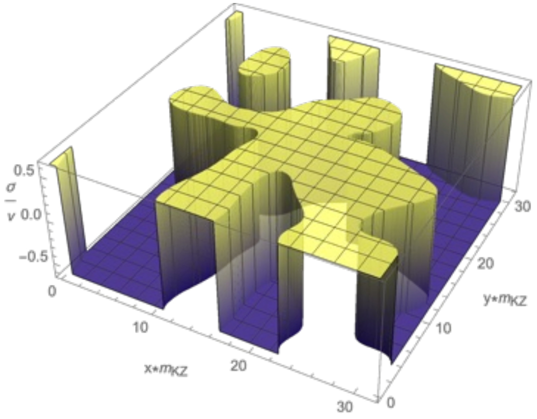


Outline

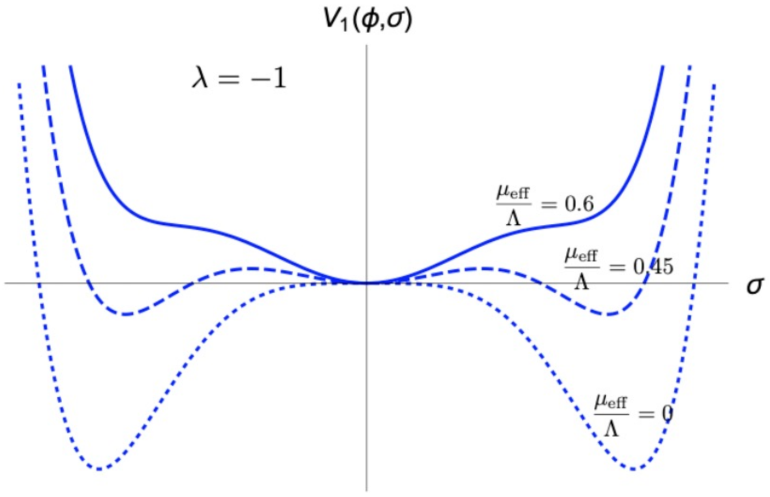
- GWs from first-order phase transitions during inflation.
  - Primary GWs
  - Curvature perturbation and secondary GWs
- GWs from second-order phase transitions (domain walls) during inflation.
- Summary and outlook



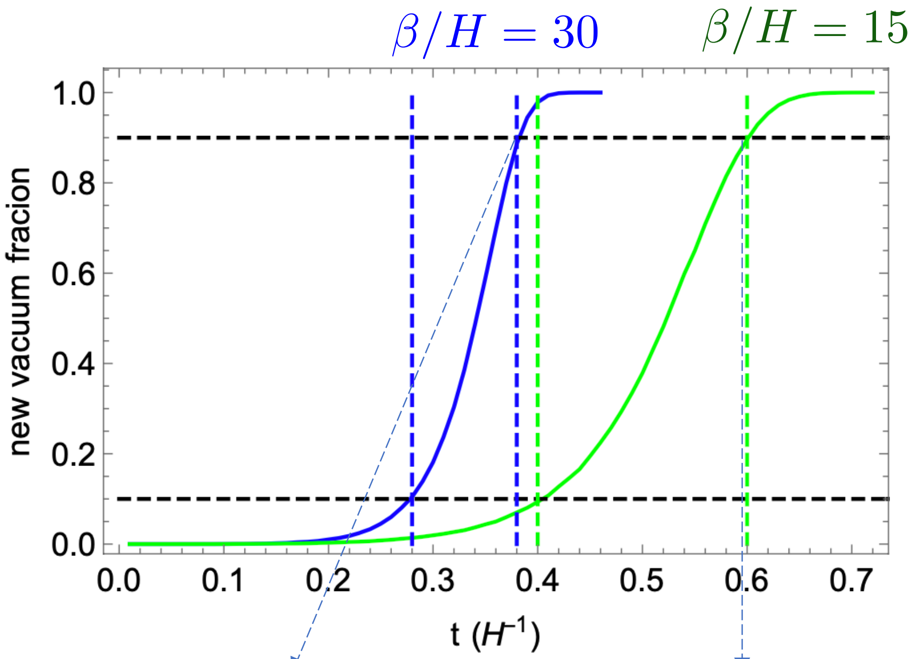


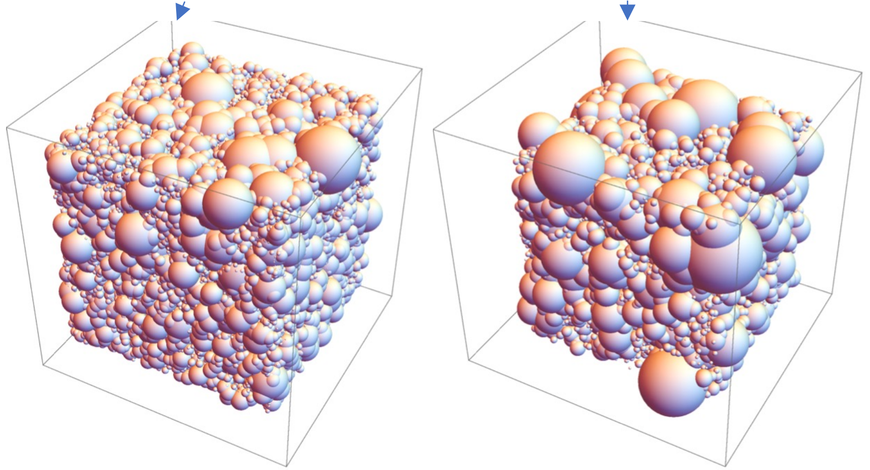


First-order phase transition during inflation



S_4 becomes smaller during





- $\beta = -\frac{dS_4}{dt}$, determines the rate of the phase transition.
- Phase transition completes if $\beta \gg H$.

How to calculate GW?

- In E&M: $\partial_\mu F^{\mu\nu} = J^\nu$
  - We solve the Green's function first.
  - We convolute the Green's function with the source.

- In GR: $G_{\mu\nu} = 8\pi G T_{\mu\nu}$
  - We solve the Green's function first. (instantaneous and local source)
  - We convolute the Green's function with the source.

GW from instantaneous and local sources (qualitative study)

- E.O.M. of GW

$$ds^2 = a^2(\tau)\left[-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j\right]$$

Traceless and transverse

$$h''_{ij} + \frac{2a'}{a}h'_{ij} - \nabla^2 h_{ij} = 16\pi^2 G_N a^2 \sigma_{ij}$$

- For an instantaneous and local source,

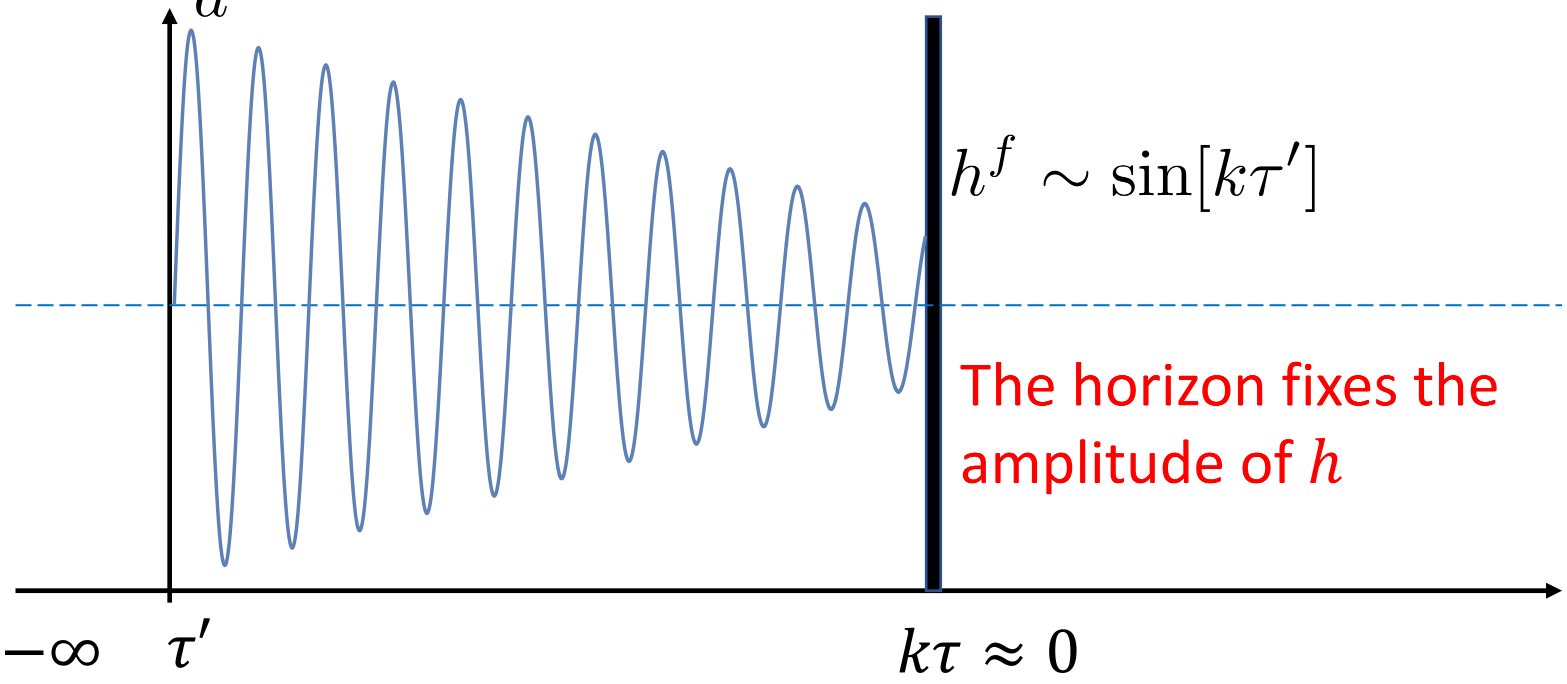
$$\sigma_{ij} \sim \delta(\mathbf{x})\delta(\tau - \tau')$$

- E.O.M. in Fourier space

$$h''(\tau, \mathbf{k}) + \frac{2a'}{a}h'(\tau, \mathbf{k}) + k^2 h(\tau, \mathbf{k}) = 16\pi G_N a^{-1} T\delta(\tau - \tau')$$

GW from instantaneous and local sources (qualitative study)

- $h''(\tau,\mathbf{k})+\frac{2a'}{a}h'(\tau,\mathbf{k})+k^2h(\tau,\mathbf{k})=16\pi G_N a^{-1}T\delta(\tau-\tau')$

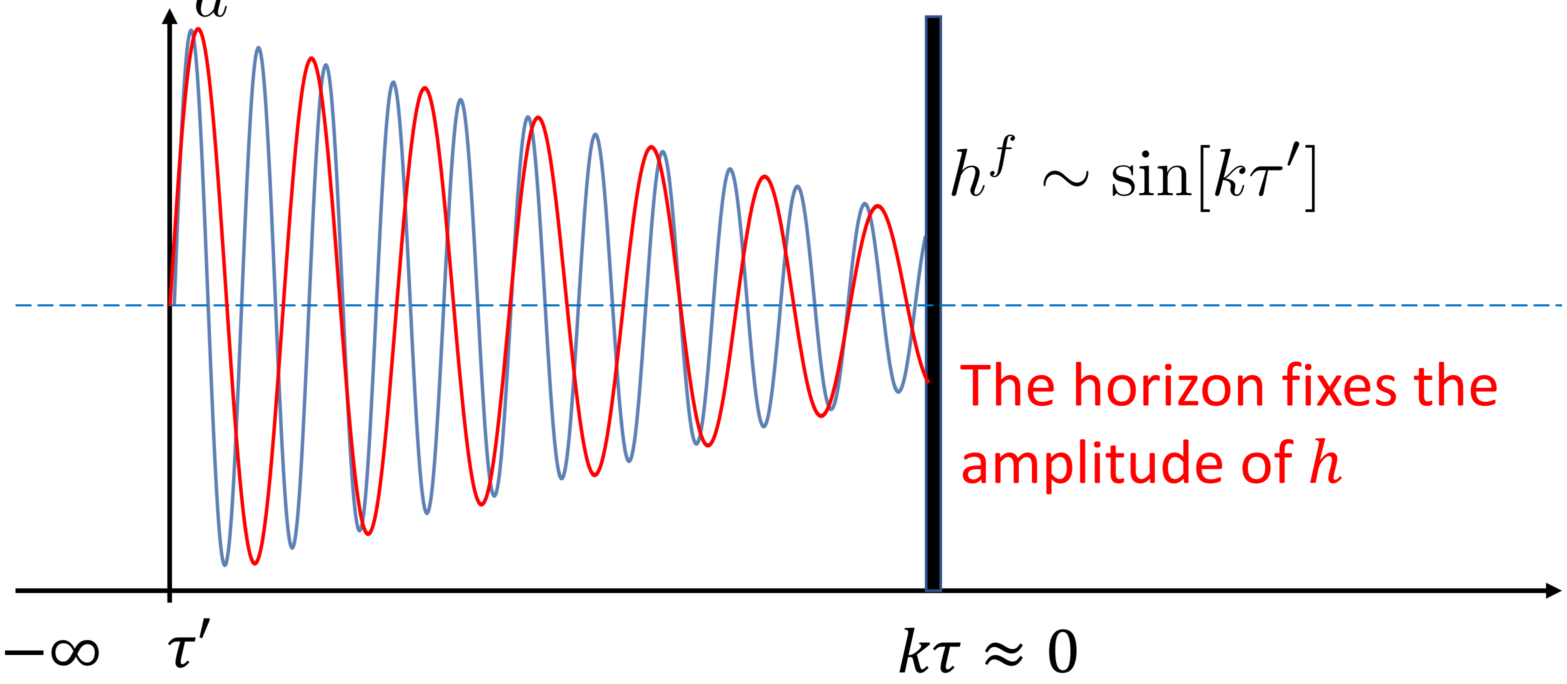


$h^f\sim\sin[k\tau']$

The horizon fixes the amplitude of h

GW from instantaneous and local sources (qualitative study)

- $h''(\tau, \mathbf{k}) + \frac{2a'}{a} h'(\tau, \mathbf{k}) + k^2 h(\tau, \mathbf{k}) = 16\pi G_N a^{-1} T \delta(\tau - \tau')$

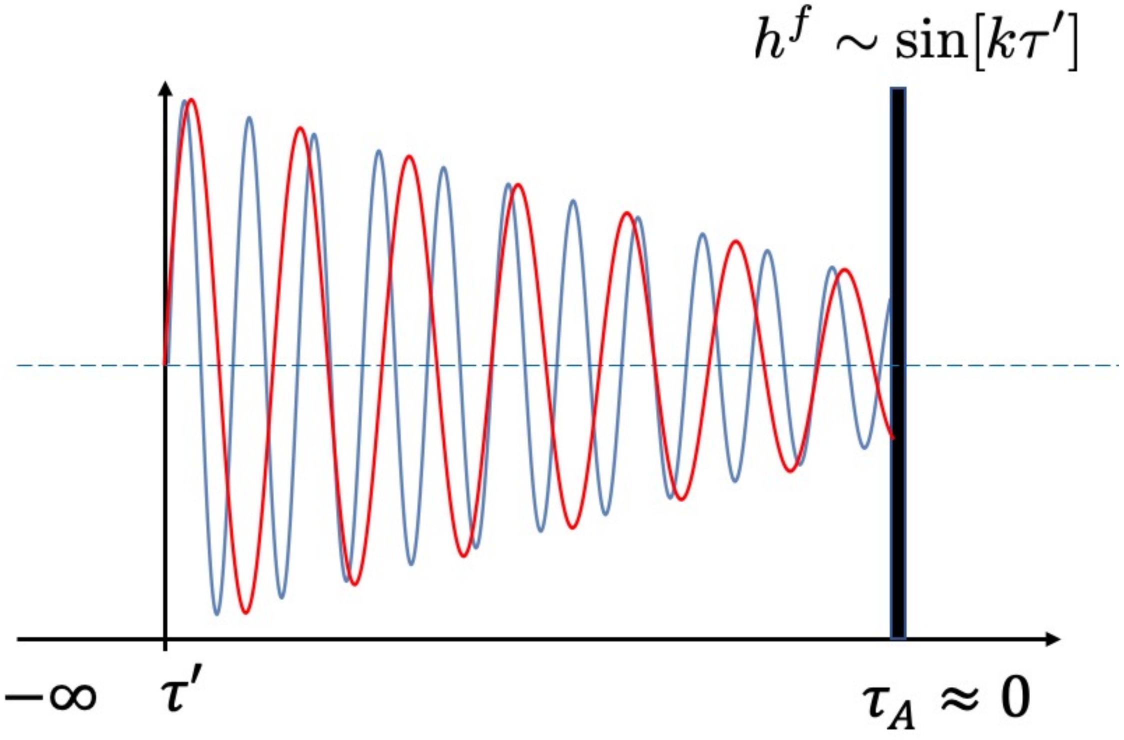


The horizon fixes the amplitude of h

GW from instantaneous and local sources (qualitative study)

- The conformal time between the source and the horizon is fixed.
- The phase of h at the source is fixed.
- The value of h^f at the horizon oscillates with k.
- h^f is the initial condition for

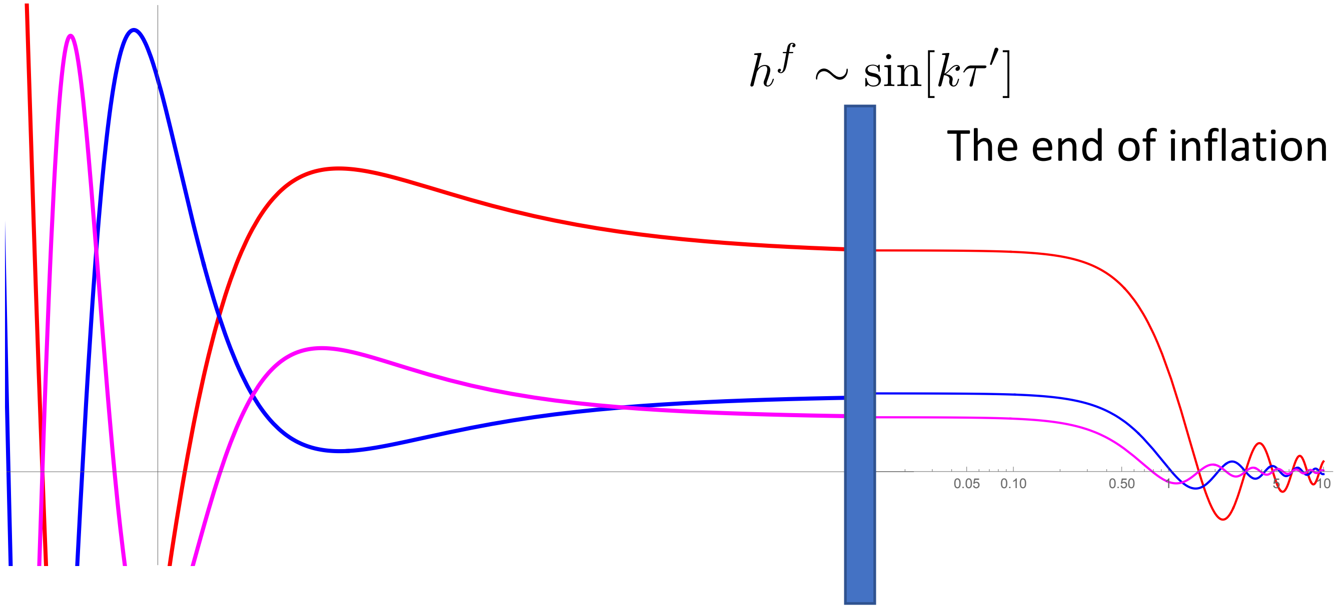
  later evolution.



$k\tau_A \approx 0$

After inflation

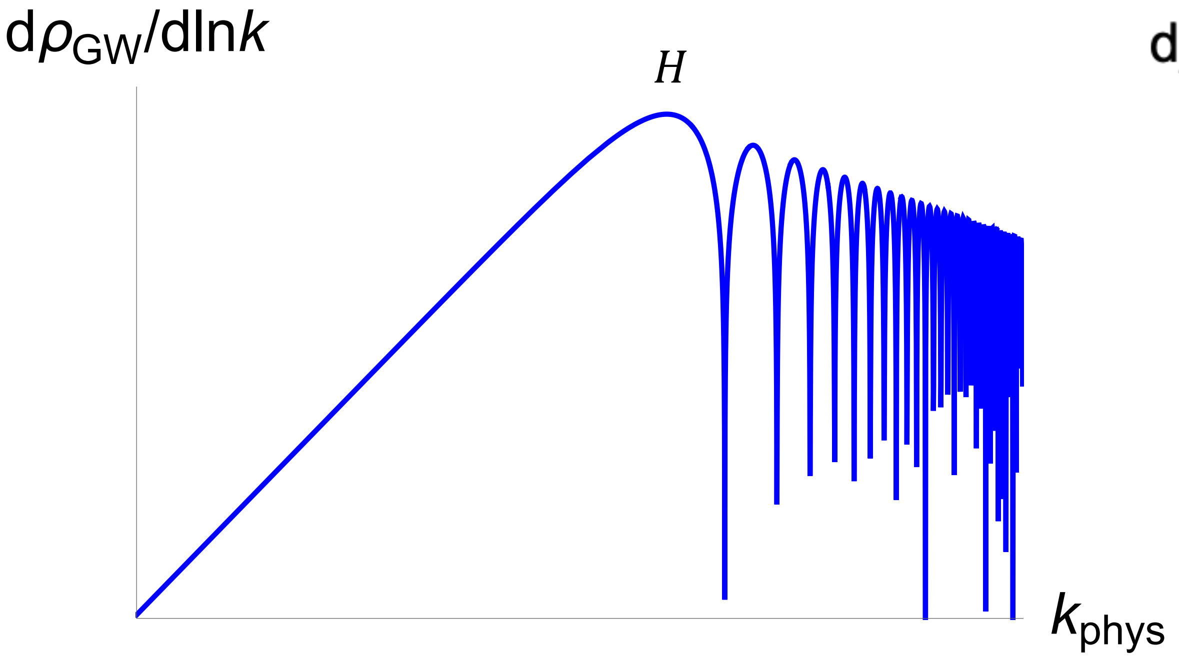
- $h^f(k)$ is the initial amplitude for the GW oscillation after inflation.
- All the modes start to oscillate with the same phase.
- Example, in RD, the oscillation is $\sin k\tau / k\tau$



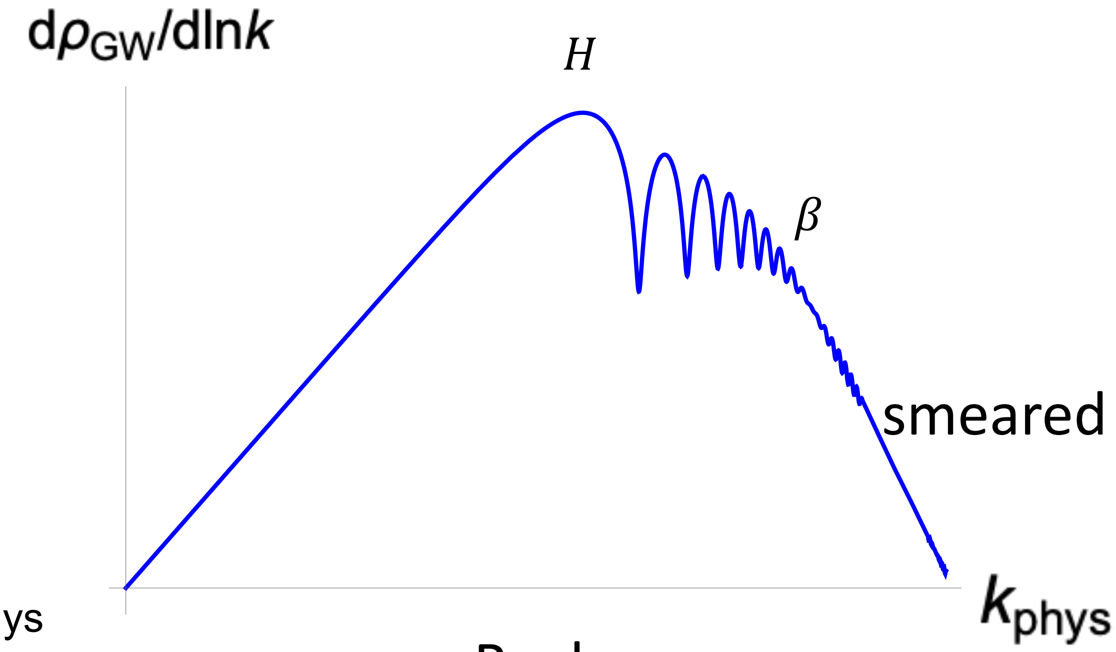
$h^f \sim \sin[k\tau']$

The end of inflation

Spectrum of GW from a real source

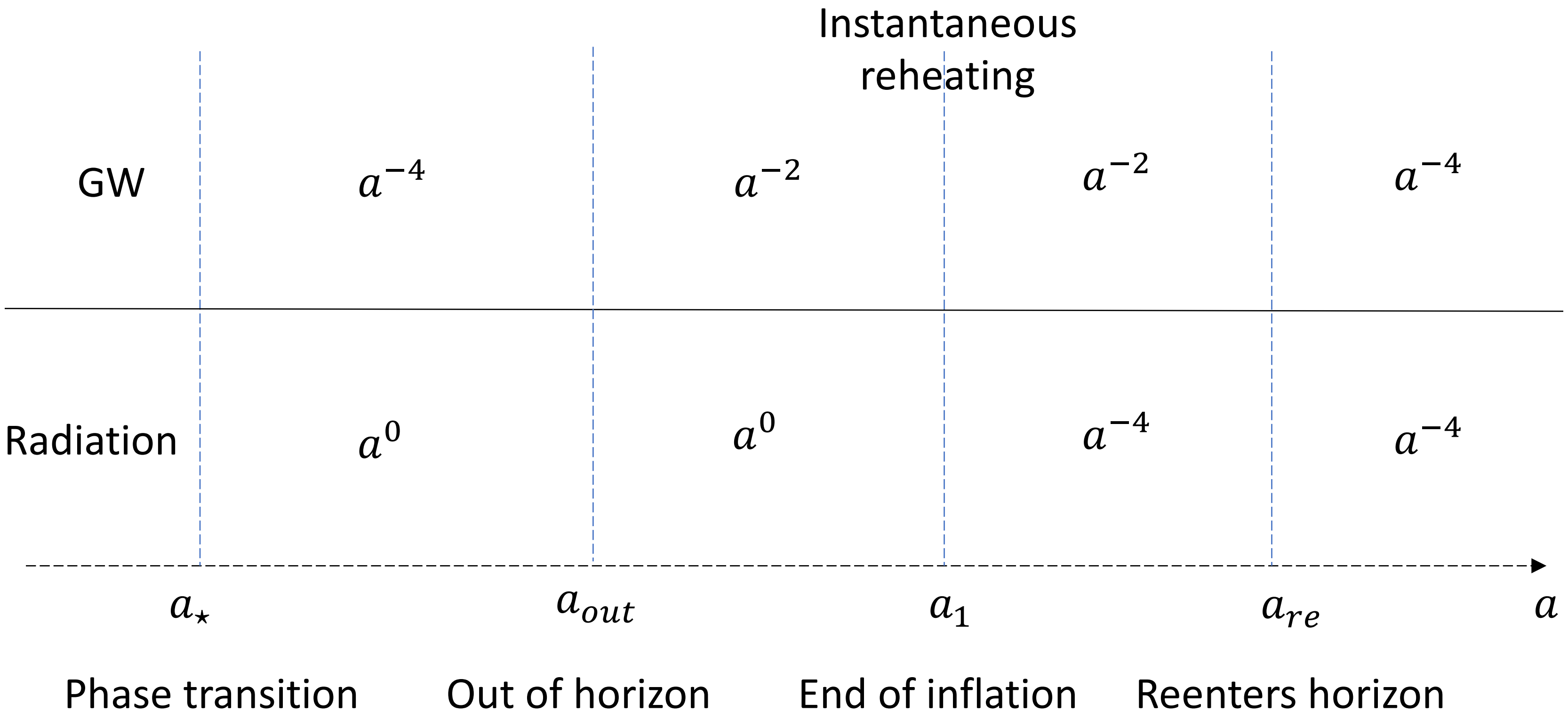


Instantaneous source



Real source

Redshifts of the GW signal



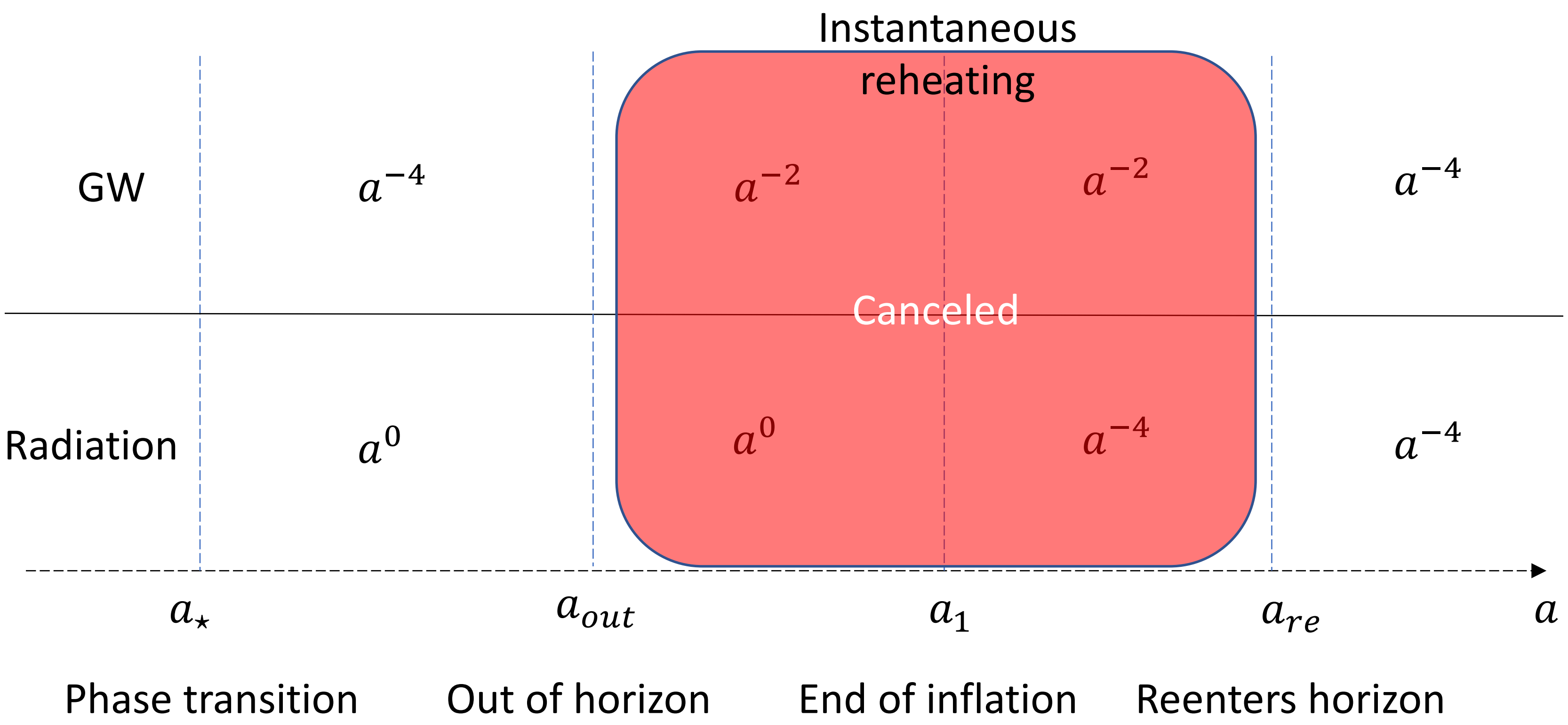
Instantaneous reheating

GW	a^{-4}	a^{-2}	a^{-2}	a^{-4}
Radiation	a^{0}	a^{0}	a^{-4}	a^{-4}

$a_\star$ a_{out} a_1 a_{re} a

Phase transition Out of horizon End of inflation Reenters horizon

Redshifts of the GW signal



Instantaneous reheating

GW	a^{-4}	a^{-2}	a^{-2}	a^{-4}
Radiation	a^{0}	a^{0}	a^{-4}	a^{-4}

Canceled

$a_\star$ a_{out} a_1 a_{re} a

Phase transition

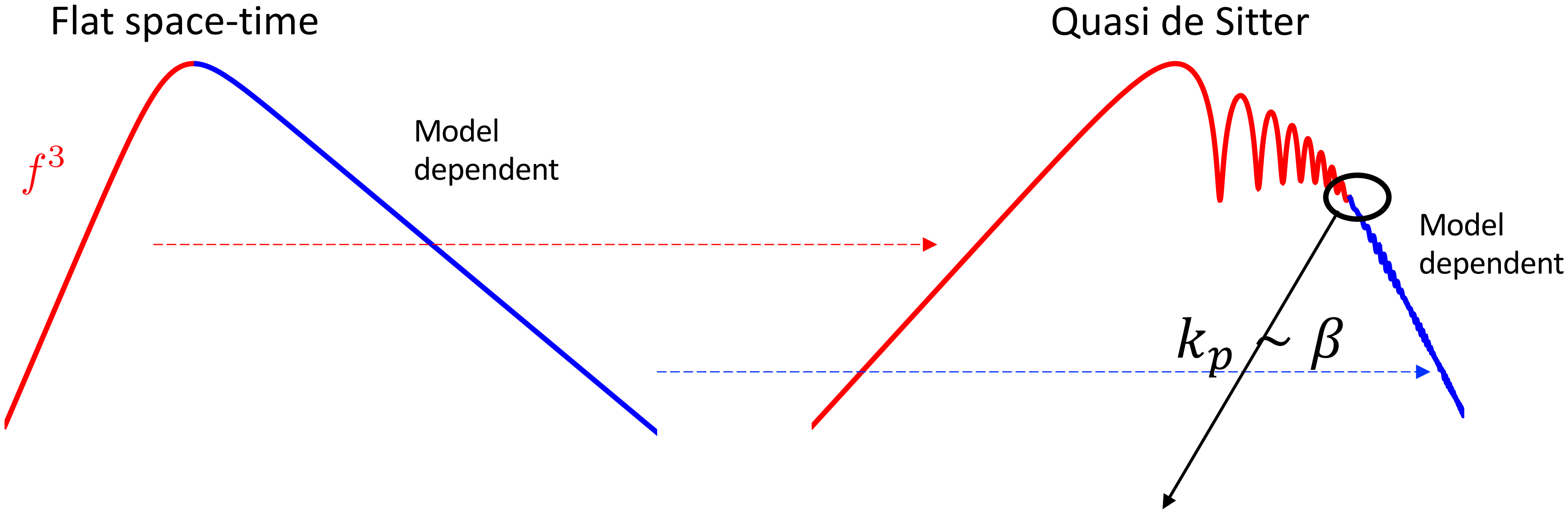
Out of horizon

End of inflation

Reenters horizon

$$\frac{\Omega_{\rm GW}}{\Omega_\gamma} \sim \left(\frac{a_\star}{a_{\rm out}}\right)^4 \sim \left(\frac{H}{\beta}\right)^4$$

Spectrum distortion by inflation



Flat space-time

Quasi de Sitter

f^3

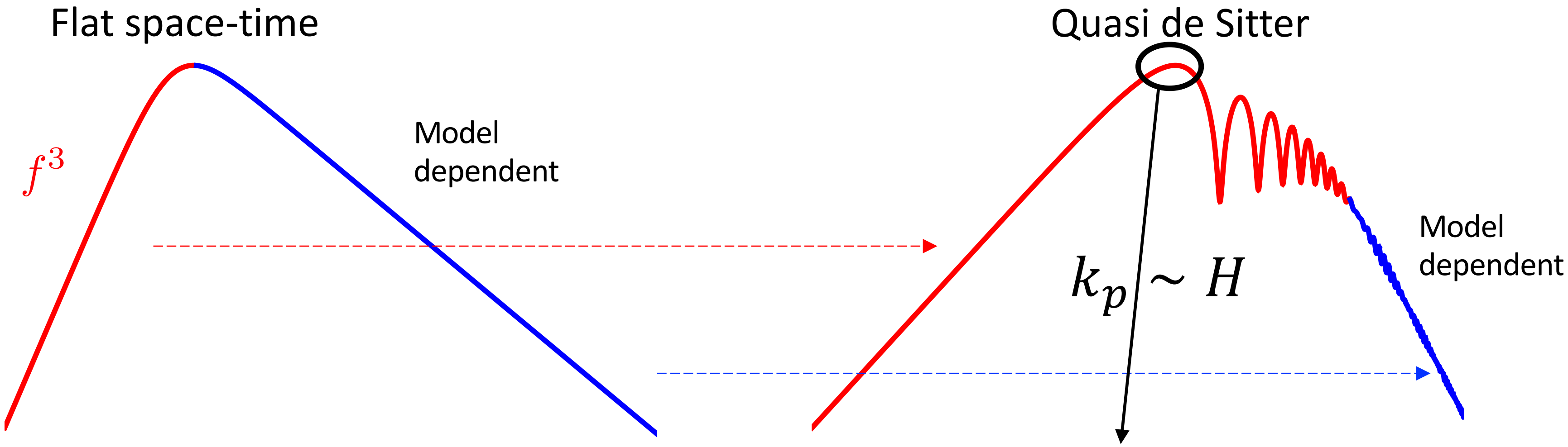
Model dependent

Model dependent

$k_p \sim \beta$

$$\Omega_{\rm GW} \approx \Omega_R \left(\frac{H_{\rm inf}}{\beta}\right)^6 \left(\frac{\Delta\rho_{\rm vac}}{\rho_{\rm inf}}\right)^2$$

Spectrum distortion by inflation



$$\Omega_{\rm GW} \approx \Omega_R \left(\frac{H_{\rm inf}}{\beta}\right)^5 \left(\frac{\Delta\rho_{\rm vac}}{\rho_{\rm inf}}\right)^2$$

$$\approx 10^{-12} \times \left(\frac{H_{\rm inf}}{0.1\beta}\right)^5 \left(\frac{\Delta\rho_{\rm vac}}{0.1\rho_{\rm inf}}\right)^2$$

$$\approx 10^{-17} \times \left(\frac{H_{\rm inf}}{0.01\beta}\right)^5 \left(\frac{\Delta\rho_{\rm vac}}{0.1\rho_{\rm inf}}\right)^2$$

First-order phase transition during inflation

- Assume quasi-dS inflation, RD re-entering, and fast reheating

$$\Omega_{\rm GW}(k_{\rm today}) = \Omega_R \underset{\substack{\downarrow \\ \text{Dilution factor}}}{\frac{H_{\rm inf}^4}{k_p^4}} \left[\frac{1}{2} + \underset{\substack{\searrow \\ \text{Smearing}}}{\mathcal{S}(k_p\beta^{-1})} \cos\left(\frac{2k_p}{H_{\rm inf}}\right)\right] \underset{\substack{\downarrow \\ \text{Suppressed by} \\ \text{the energy} \\ \text{faction}}}{\left(\frac{\Delta\rho_{\rm vac}}{\rho_{\rm inf}}\right)^2} \frac{d\rho_{\rm GW}^{\rm flat}}{\Delta\rho_{\rm vac} d\log k_p}$$

Redshift

$$\frac{f_{\rm today}}{f_\star} = \underset{\substack{\downarrow \\ e^{-N_e}}}{\frac{a(\tau_\star)}{a_1}} \left(\frac{g_{*S}^{(0)}}{g_{*S}^{(R)}}\right)^{1/3} \frac{T_{\rm CMB}}{\left[\left(\frac{30}{g_*^{(R)}\pi^2}\right)\left(\frac{3H_{\rm inf}^2}{8\pi G_N}\right)\right]^{1/4}}$$

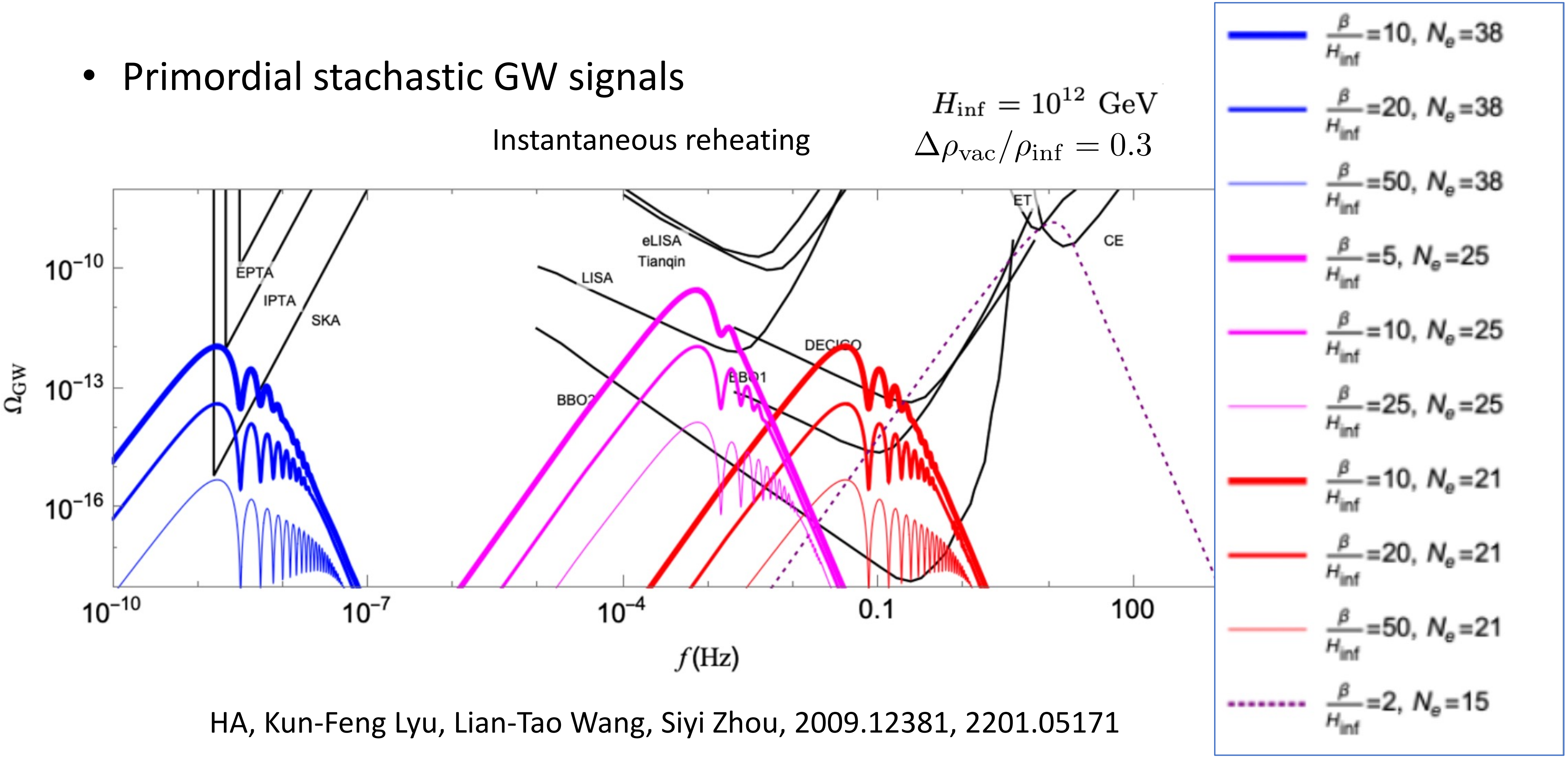
N_e: e-folds before the end of inflation

First-order phase transition during inflation

- Primordial stachastic GW signals

Instantaneous reheating

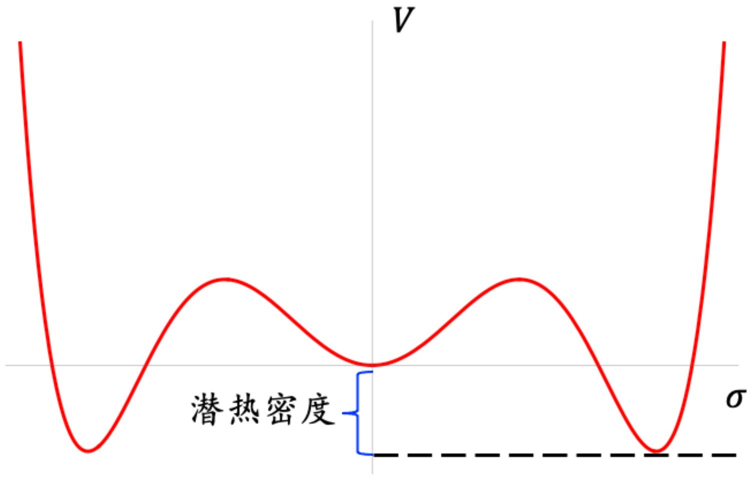
$$H_{\rm inf} = 10^{12}\ {\rm GeV}$$
$$\Delta\rho_{\rm vac}/\rho_{\rm inf} = 0.3$$

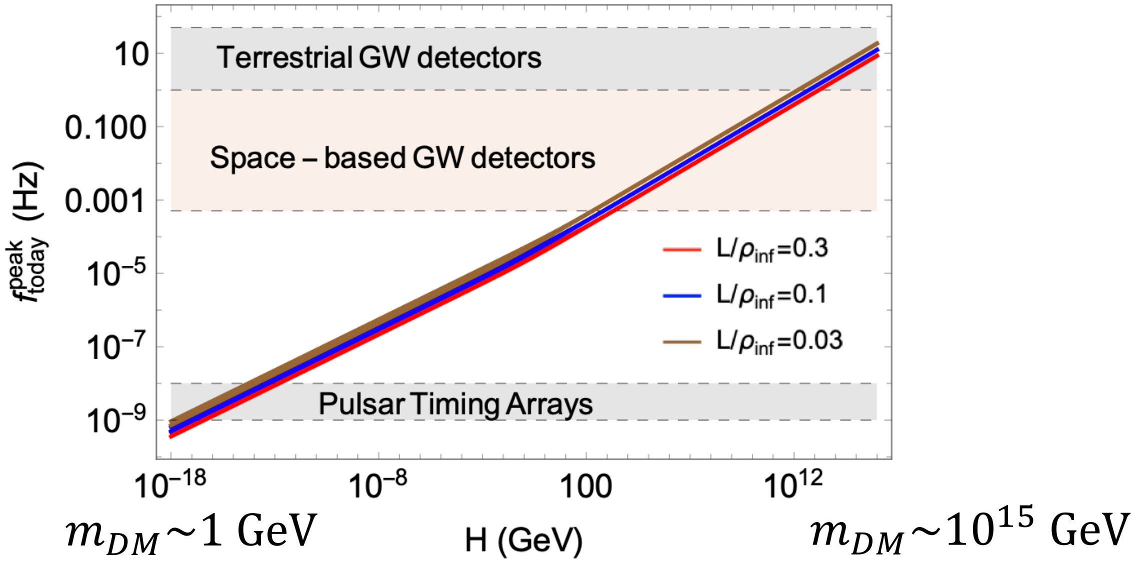


HA, Kun-Feng Lyu, Lian-Tao Wang, Siyi Zhou, 2009.12381, 2201.05171

Producing super heavy DM

- Where does the latent heat go?
- σ particles produced during bubble collision and thermalization.
- If the phase transition is symmetry-restoration, σ particles can be DM.





HA, Xi Tong, Siyi Zhou, 2208.14857

Induced phase transition in spectator sectors

Evolution of inflaton

Phase transitions in spectator sector

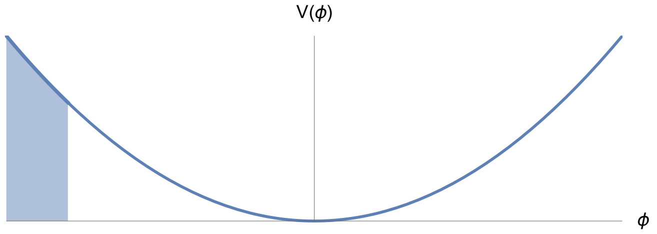
Gravitational waves

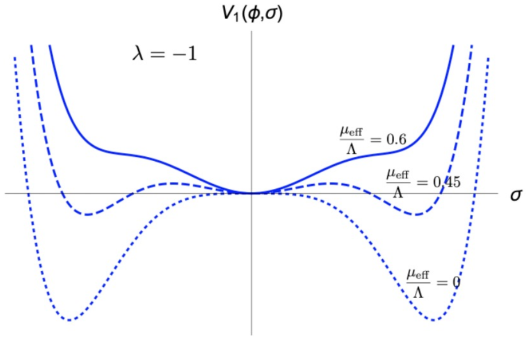
Primary GW

Curvature perturbation

Gravitational waves

Secondary GW





Induced scalar perturbation $\delta\phi$

- Interactions

$$\mathcal{L} = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - V(\phi,\sigma)$$

$$V(\phi,\sigma) = V_0(\phi) + V_1(\phi,\sigma) \xrightarrow{\phi = \phi_0 + \delta\phi} \frac{\partial V_1}{\partial\phi_0}\delta\phi$$

Source term for $\delta\phi$

$$\delta\tilde{\phi}''_{\mathbf{q}} - \frac{2}{\tau}\delta\tilde{\phi}'_{\mathbf{q}} + \left(q^2 + \frac{1}{H^2\tau^2}\frac{\partial^2 V_0}{\partial\phi_0^2}\right)\delta\tilde{\phi}_{\mathbf{q}} = \mathcal{S}_{\mathbf{q}}\ ,$$

$$\mathcal{S}_{\mathbf{q}} = -\frac{1}{H^2\tau^2}\left[\frac{\partial V_1}{\partial\phi}\right]_{\mathbf{q}} \underbrace{-\left\{\frac{2\Phi_{\mathbf{q}}}{H^2\tau^2}\left(\frac{\partial V_0}{\partial\phi_0} + \left[\frac{\partial V_1}{\partial\phi}\right]_0\right) + \frac{\dot{\phi}_0}{H\tau}\left(3\Psi'_{\mathbf{q}} + \Phi'_{\mathbf{q}}\right)\right\}}_{\text{Pure gravitational, subdominant}}$$

Induced curvature perturbation ζ

- We solve the following equations of motion numerically with a $1000\times1000\times1000$ lattice

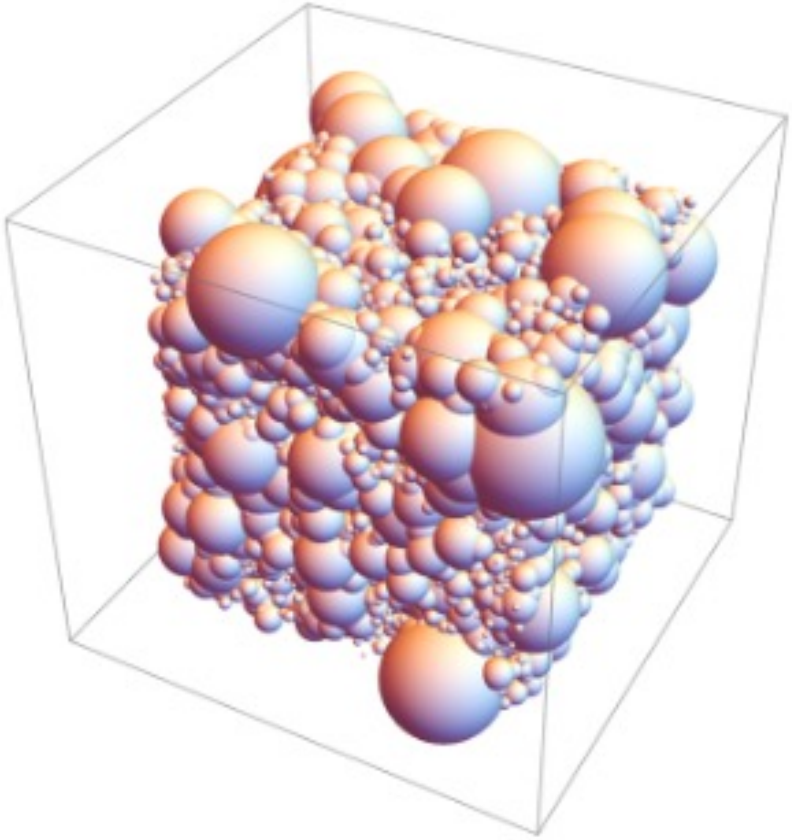
$$\delta\tilde{\phi}''_{\mathbf{q}} - \frac{2}{\tau}\delta\tilde{\phi}'_{\mathbf{q}} + \left(q^2 + \frac{1}{H^2\tau^2}\frac{\partial^2 V_0}{\partial\phi_0^2}\right)\delta\tilde{\phi}_{\mathbf{q}} = \mathcal{S}_{\mathbf{q}} \ ,$$

$$\tilde{\Psi}'_{\mathbf{q}} - \frac{\tilde{\Phi}_{\mathbf{q}}}{\tau} = -4\pi G_N\left(\frac{\dot{\phi}_0\delta\tilde{\phi}_{\mathbf{q}}}{H_{\rm inf}\tau} + \left[\frac{\partial_i}{\partial^2}(\sigma'\partial_i\sigma)\right]_{\mathbf{q}}\right)$$

$$\tilde{\pi}^S_{\mathbf{q}} = -\frac{3}{2}H_{\rm inf}^2\tau^2 q_i q_j q^{-4}\left[(\partial_i\sigma\partial_j\sigma)^{\rm TL}\right]_{\mathbf{q}}$$

- Conserved quantity after the phase transition

$$\zeta_{\mathbf{q}} = -\tilde{\Psi}_{\mathbf{q}} - \frac{H_{\rm inf}\delta\tilde{\phi}_{\mathbf{q}}}{\dot{\phi}_0}$$



Power spectrum of ζ

- After the collision of the bubbles, σ field oscillates and keeps producing ζ.
- The production of ζ lasts about H^{-1}, longer than β^{-1}.

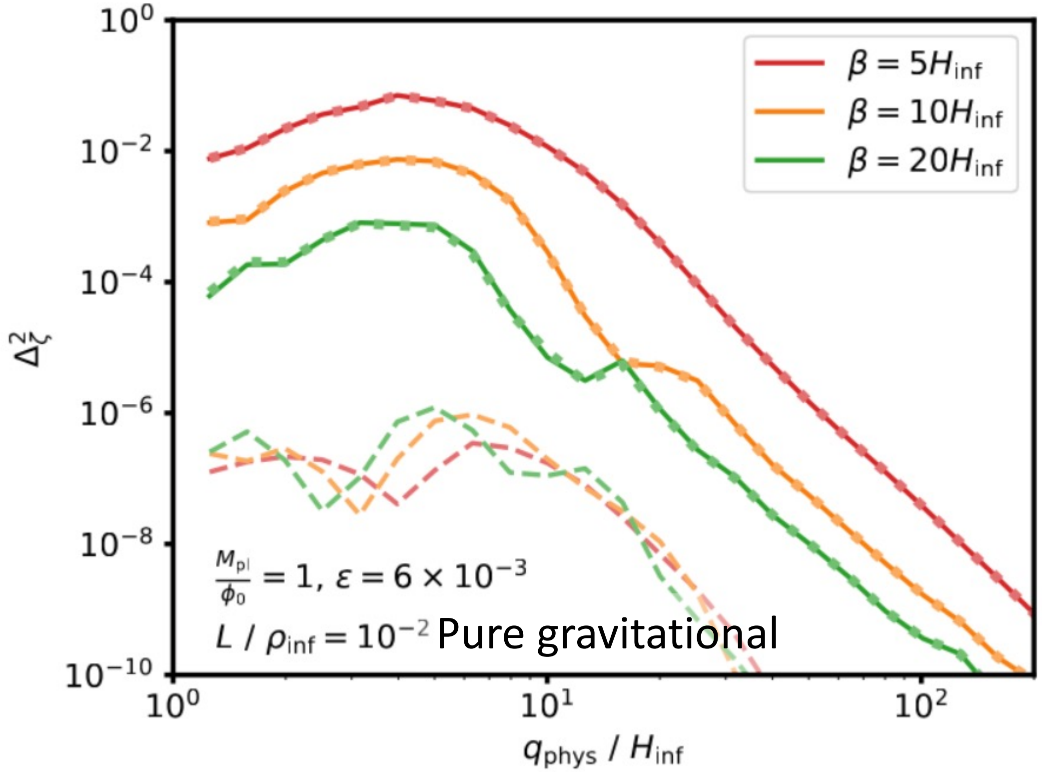
$$\zeta_{\mathbf{q}} \approx \frac{H_{\rm inf}}{\dot{\phi}_0 q^2}\int \frac{d\tau'}{\tau'}\left(\cos q\tau' - \frac{\sin q\tau'}{q\tau'}\right)\frac{c_m\phi_0[\sigma^2(\tau')]_{\mathbf{q}}}{H_{\rm inf}^2\tau'^2}$$

$$\Delta^{2\,({\rm emp})}_{\zeta}(q) = A_{\rm ref}\mathcal{F}\left(\frac{q_{\rm phys}}{H_{\rm inf}}\right)$$

$$\mathcal{F}(x) = \frac{x^3}{1+(\alpha_1 x)^4+(\alpha_2 x)^9}$$

$$A_{\rm ref} = \frac{\mathcal{A}}{\epsilon}\left(\frac{M_{\rm pl}}{\phi_0}\right)^2\left(\frac{H_{\rm inf}}{\beta}\right)^3\left(\frac{L}{\rho_{\rm inf}}\right)^2$$

$\mathcal{A} \approx 24$ $\quad L \equiv \Delta\rho$ $\quad \alpha_1 \approx 0.31, \alpha_2 \approx 0.17$



Secondary GWs

- After inflation $\zeta \to \Phi, \Psi$
- Expand the Einstein equation to second order:

$$h''_{ij} + 2\mathcal{H}h'_{ij} - \nabla^2 h_{ij} = -4\hat{\mathcal{T}}_{ij}{}^{lm}\mathcal{S}_{lm}\,,$$

$$\begin{aligned}\mathcal{S}_{ij} \equiv\ & 2\Phi\partial^i\partial_j\Phi - 2\Psi\partial^i\partial_j\Phi + 4\Psi\partial^i\partial_j\Psi + \partial^i\Phi\partial_j\Phi - \partial^i\Phi\partial_j\Psi - \partial^i\Psi\partial_j\Phi + 3\partial^i\Psi\partial_j\Psi \\ & - \frac{4}{3(1+w)\mathcal{H}^2}\partial_i\left(\Psi' + \mathcal{H}\Phi\right)\partial_j\left(\Psi' + \mathcal{H}\Phi\right) - \frac{2c_s^2}{3w\mathcal{H}^2}\left[3\mathcal{H}(\mathcal{H}\Phi - \Psi') + \nabla^2\Psi\right]\partial_i\partial_j(\Phi - \Psi)\,.\end{aligned}$$

Scalar induced GWs

Baumann, Steinhardt, Takahashi, hep-th/0703290 …

Secondary GWs

$$\Omega_{\rm GW}^{(2)}(f) = \Omega_R A_{\rm ref}^2 \mathcal{F}_2\left(\frac{q_{\rm phys}}{H_{\rm inf}}\right)$$

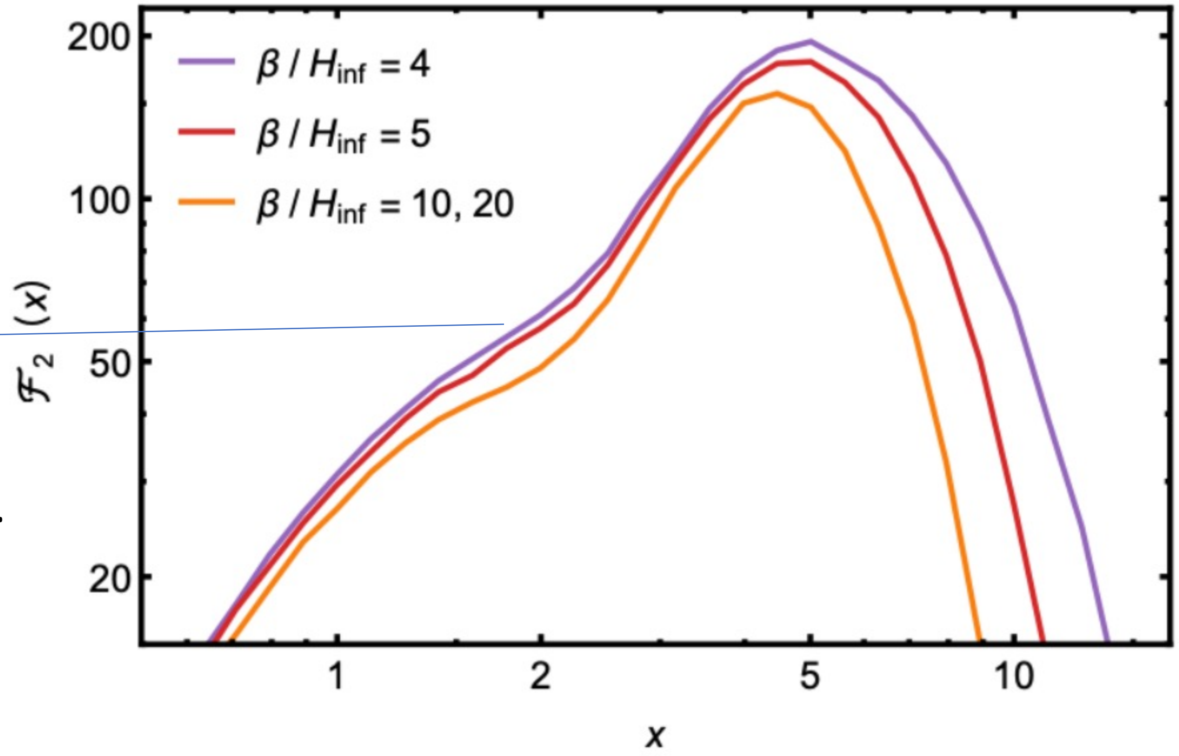
$$A_{\rm ref} = \frac{\mathcal{A}}{\epsilon}\left(\frac{M_{\rm pl}}{\phi_0}\right)^2\left(\frac{H_{\rm inf}}{\beta}\right)^3\left(\frac{L}{\rho_{\rm inf}}\right)^2$$

$$f = \frac{q}{2\pi a_0} = f_{\rm ref}\times\frac{q_{\rm phys}}{H_{\rm inf}}$$

$$f_{\rm ref} = 10^{-9}\ {\rm Hz}\times e^{40-N_e}\left(\frac{H_{\rm inf}}{10^{14}\ {\rm GeV}}\right)^{1/2}$$

$$\mathcal{F}_2^{\rm IR}(x) \approx x^3\left(\frac{6}{5}\log^2 x + \frac{16}{25}\log x + \frac{28}{125}\right)$$

$\mathcal{F}_2^{\rm max} \approx 200$



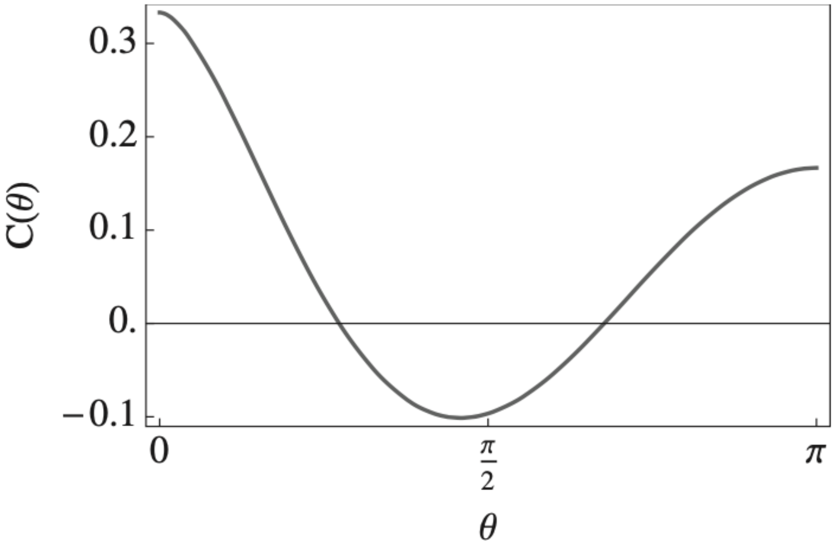
F_2 Collects information of the transfer functions.

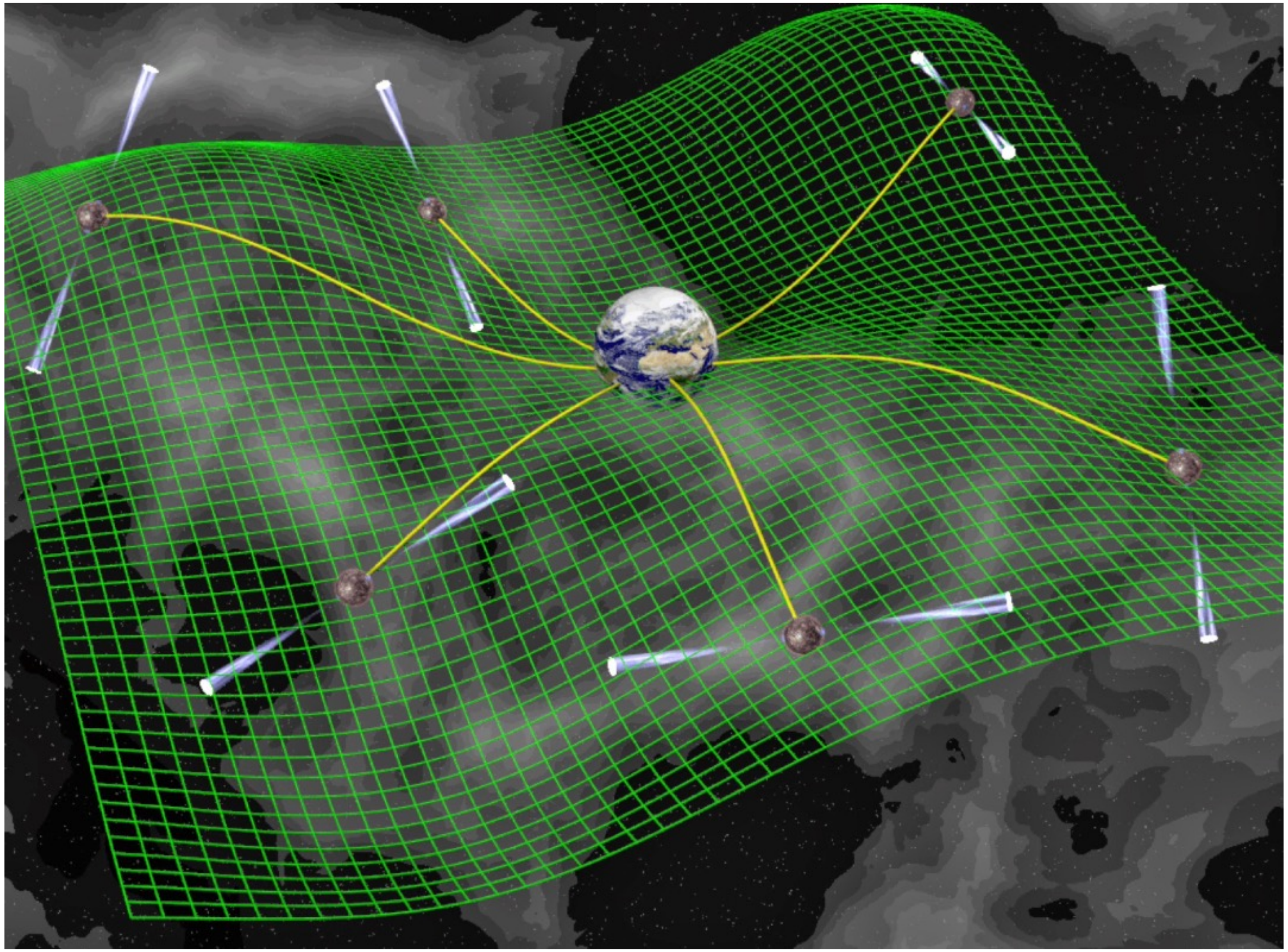
Observation from PTAs

- Hellings-Downs curve

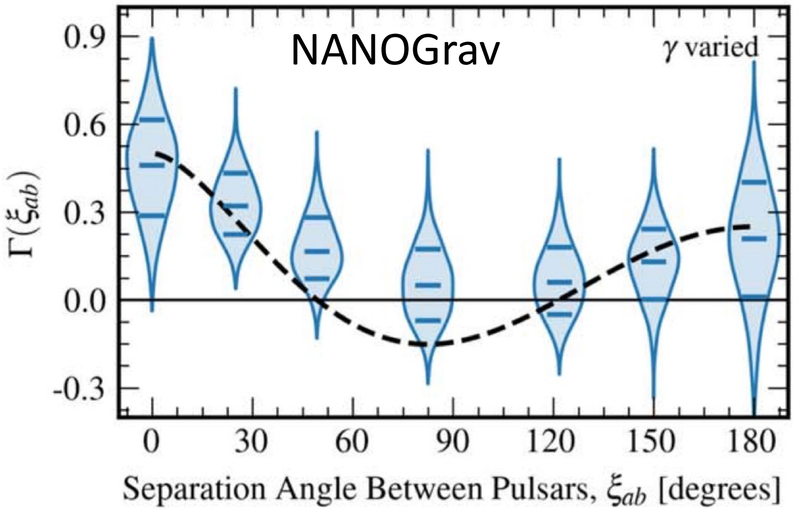
$$\langle z_a(t) z_b(t) \rangle = \underset{\substack{\downarrow \\ \text{Angular correlation}}}{C(\theta_{ab})} \int_0^\infty df\, S_h(f)$$

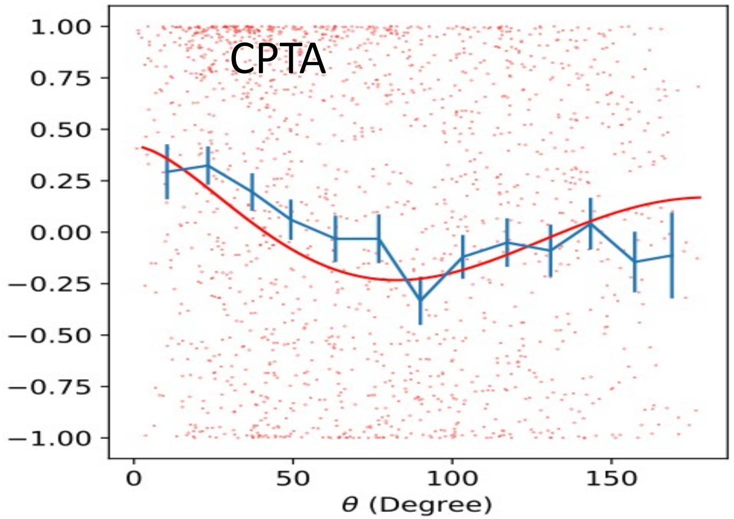
$$z_a(t) = -(\Delta\nu_a/\nu_a)(t) = \Delta T_a/T_a$$

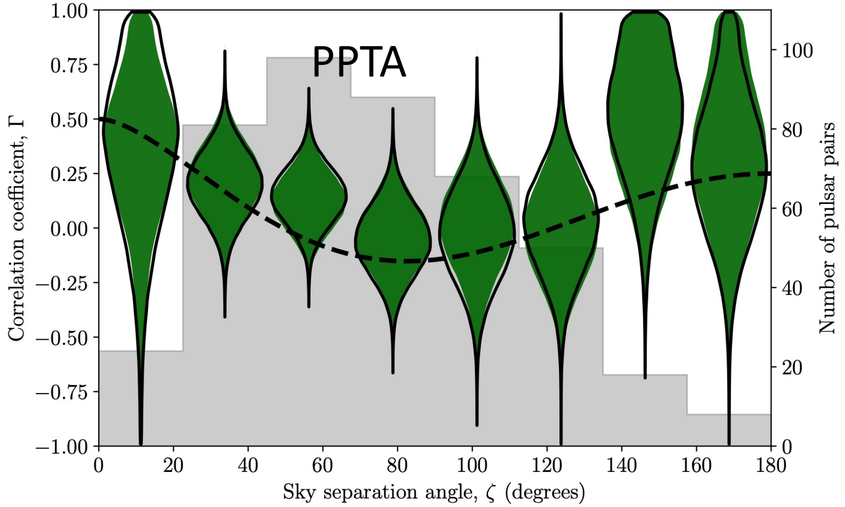


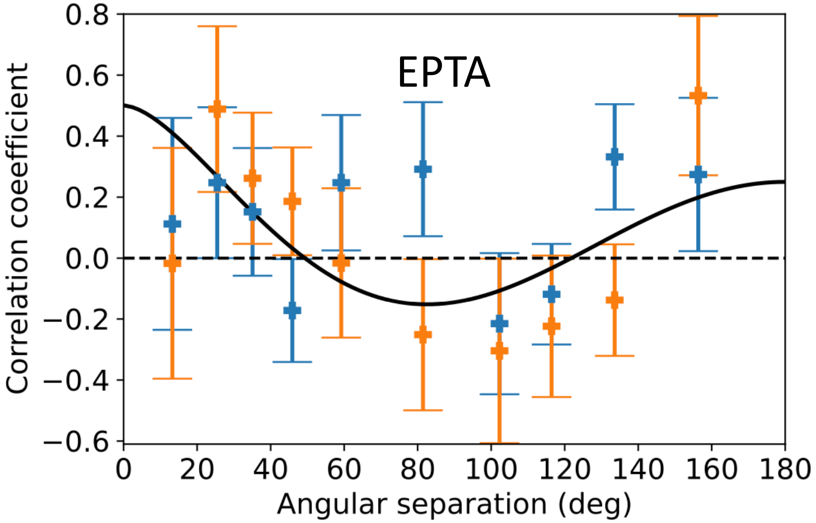


Observation from PTAs





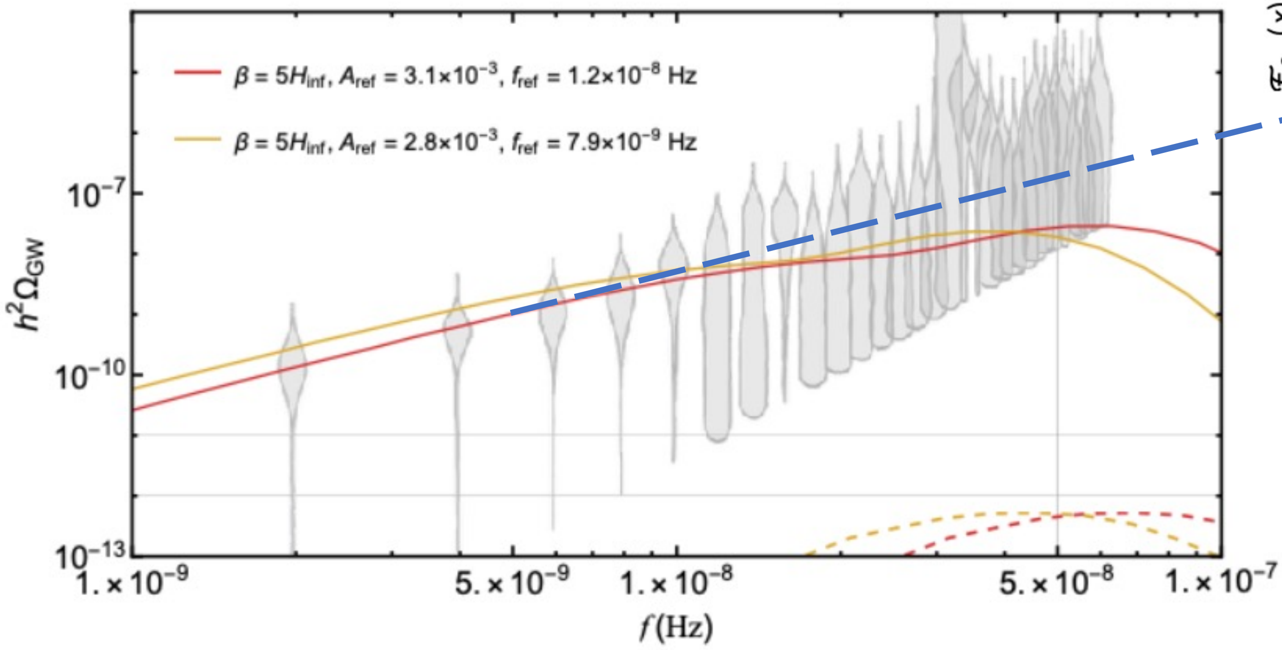


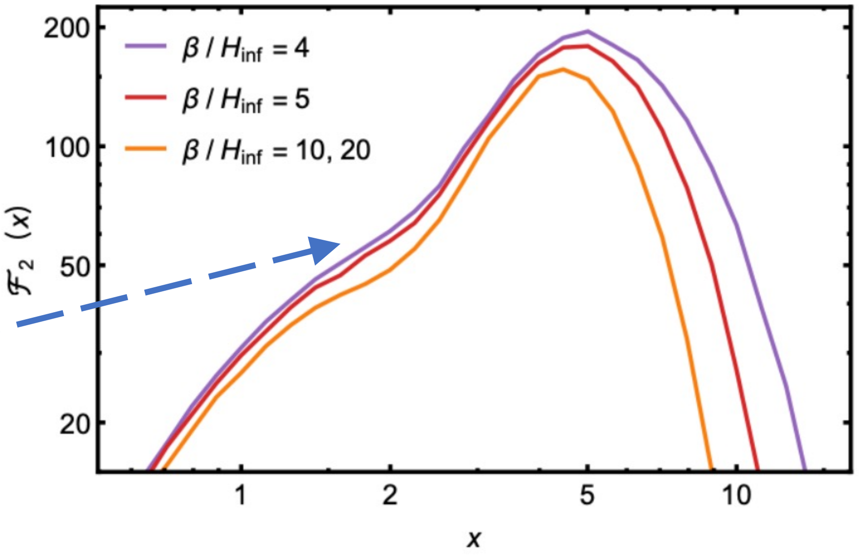


Observation from PTAs

$$\mathcal{F}_2^{\rm IR}(x) \approx x^3 \left(\frac{6}{5}\log^2 x + \frac{16}{25}\log x + \frac{28}{125}\right)$$

- The slope is around 2 in the IR region





$$\Omega_{\rm GW}^{(2)}(f) = \Omega_R A_{\rm ref}^2 \mathcal{F}_2\left(\frac{q_{\rm phys}}{H_{\rm inf}}\right)$$

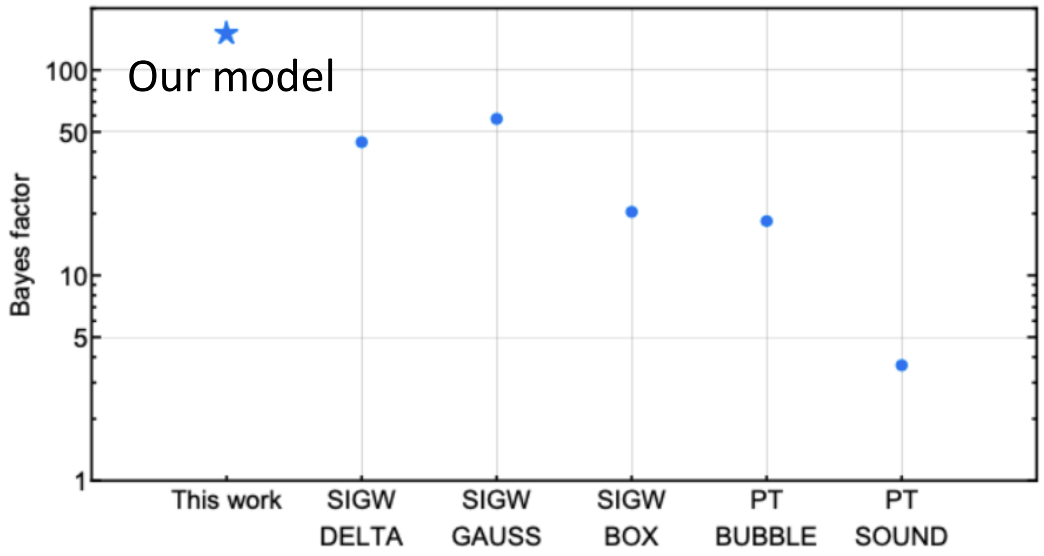
$$f = \frac{q}{2\pi a_0} = f_{\rm ref} \times \frac{q_{\rm phys}}{H_{\rm inf}}$$

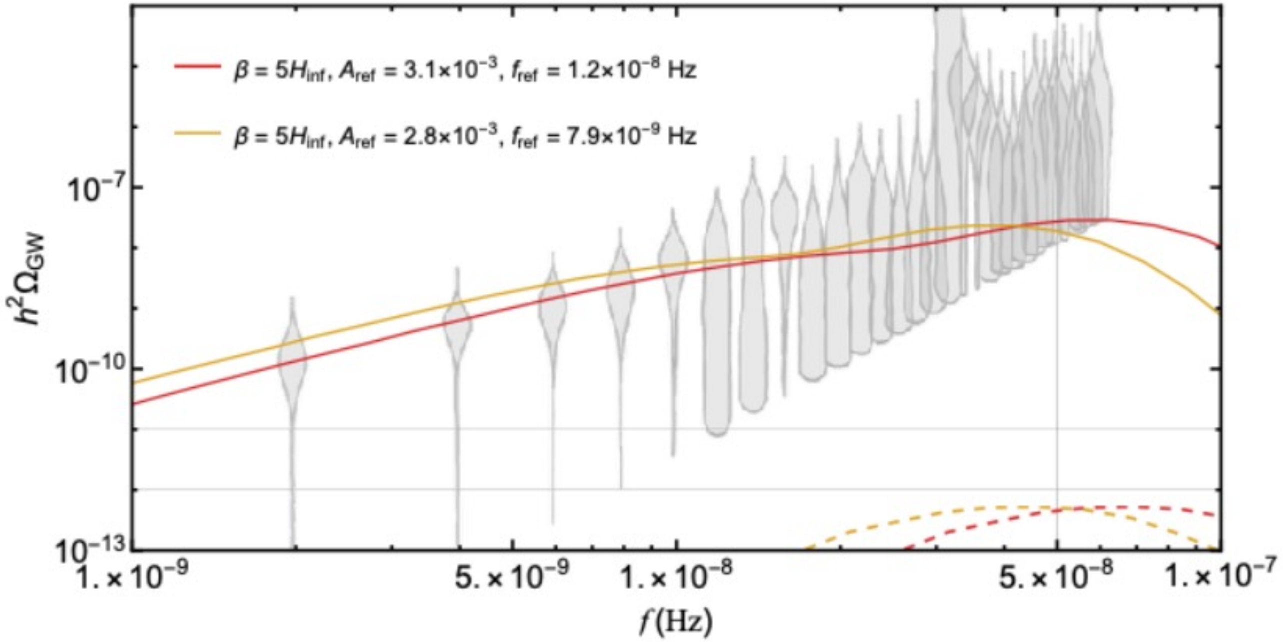
$$f_{\rm ref} = 10^{-9}\ {\rm Hz} \times e^{40-N_e}\left(\frac{H_{\rm inf}}{10^{14}\ {\rm GeV}}\right)^{1/2}$$

Observation from PTAs

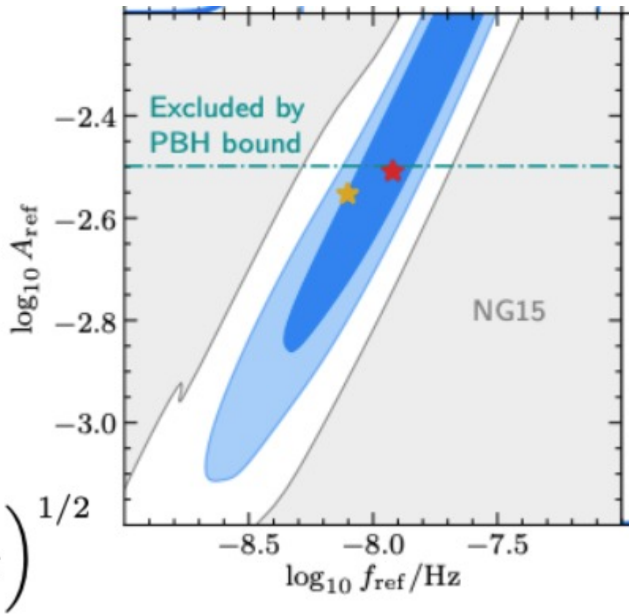
HA, Boye Su, Hanwen Tai, Lian-Tao Wang, 2308.00070

- Bayes factor against SMBHB





$$\Omega^{(2)}_{\rm GW}(f) = \Omega_R \underline{A^2_{\rm ref}} \mathcal{F}_2\left(\frac{q_{\rm phys}}{\underline{H_{\rm inf}}}\right)$$



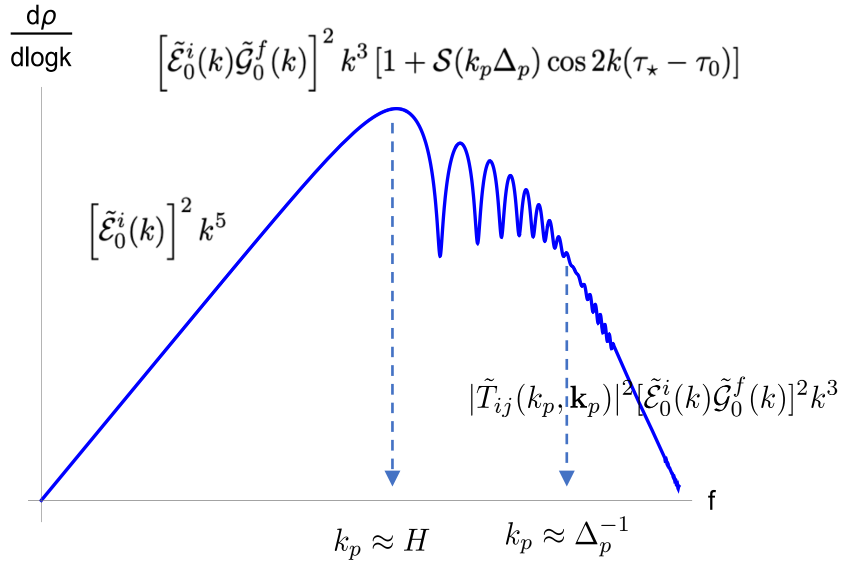
$$f = \frac{q}{2\pi a_0} = f_{\rm ref} \times \frac{q_{\rm phys}}{H_{\rm inf}}$$

$$f_{\rm ref} = 10^{-9}\ {\rm Hz} \times e^{40-N_e}\left(\frac{H_{\rm inf}}{10^{14}\ {\rm GeV}}\right)^{1/2}$$

Comparison between primary GW and secondary GW

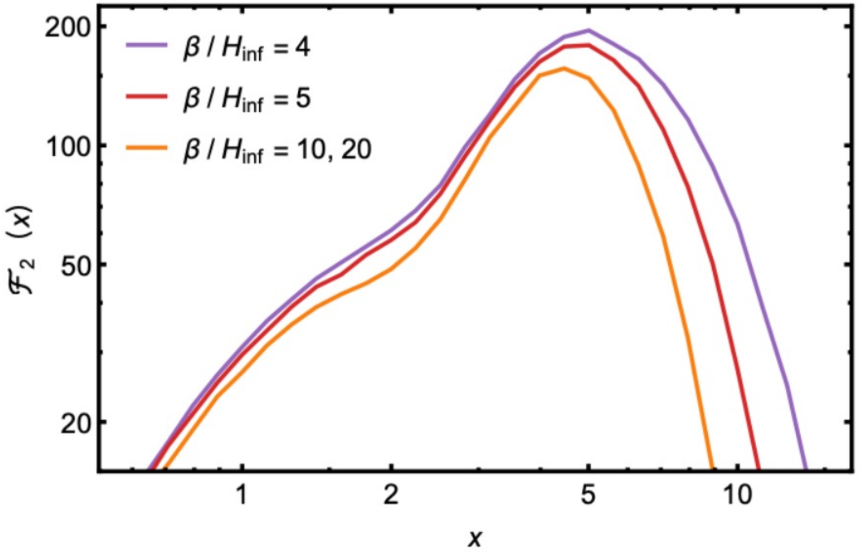
- Primary

$$\Omega_{\rm GW} \approx \Omega_R \left(\frac{H_{\rm inf}}{\beta}\right)^5 \left(\frac{\Delta\rho_{\rm vac}}{\rho_{\rm inf}}\right)^2$$

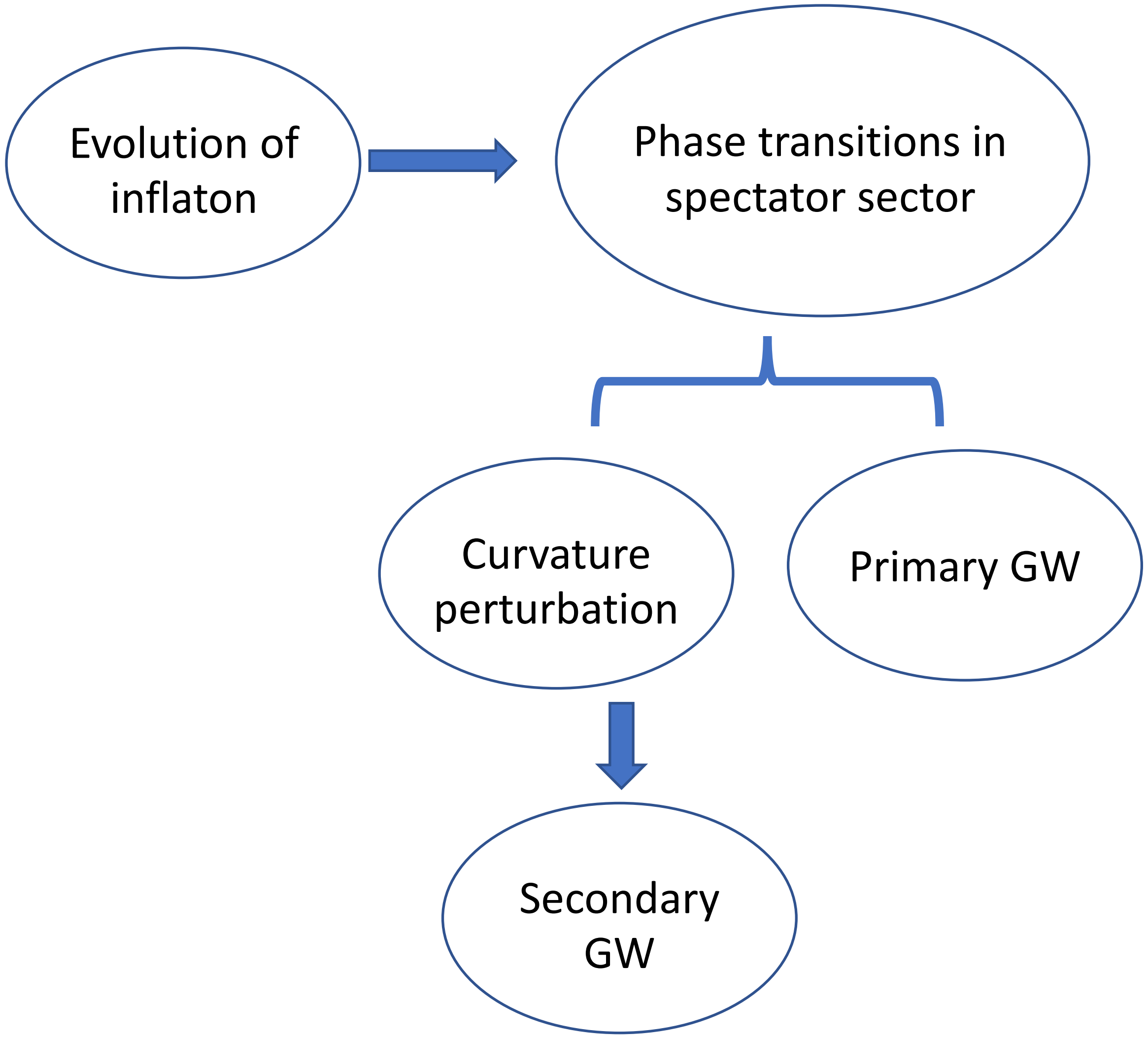


- Secondary

$$\Omega_{\rm GW} \sim \Omega_R \left(\frac{\mathcal{A}}{\epsilon}\right)^2 \left(\frac{M_{\rm pl}}{\phi_0}\right)^4 \left(\frac{H_{\rm inf}}{\beta}\right)^6 \left(\frac{L}{\rho_{\rm inf}}\right)^4$$



Summary for FOPT



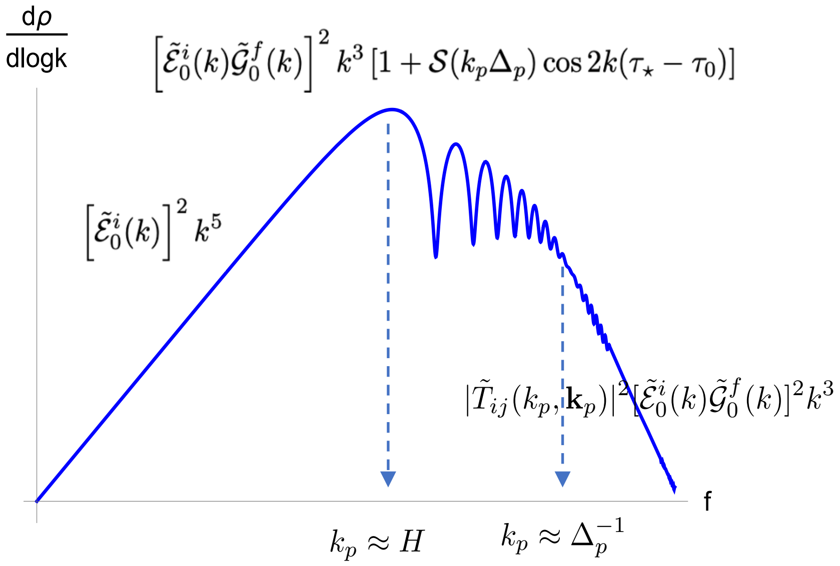
Evolution of inflaton

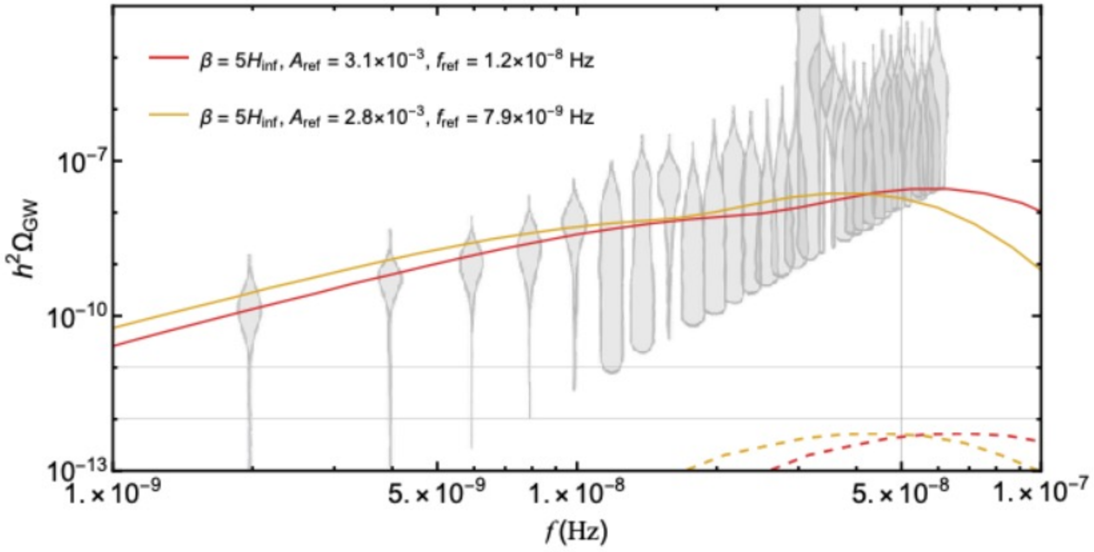
Phase transitions in spectator sector

Curvature perturbation

Primary GW

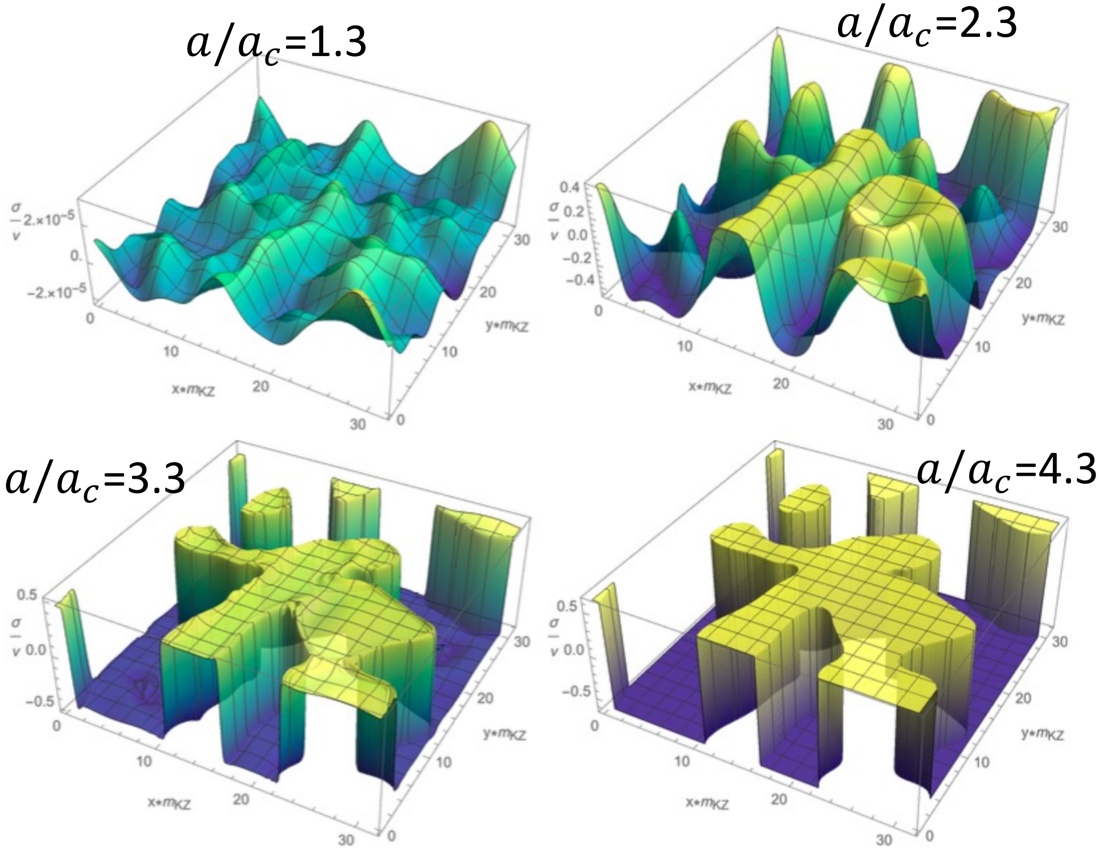
Secondary GW





Formation of domain walls

- We numerically solve the nonlinear evolution of σ field with $1000\times1000\times1000$ lattice.
- At the beginning there are fluctuations, dying out after a few e-folds.
- The configuration becomes comovingly static after a few e-folds.



a/a_c=1.3

a/a_c=2.3

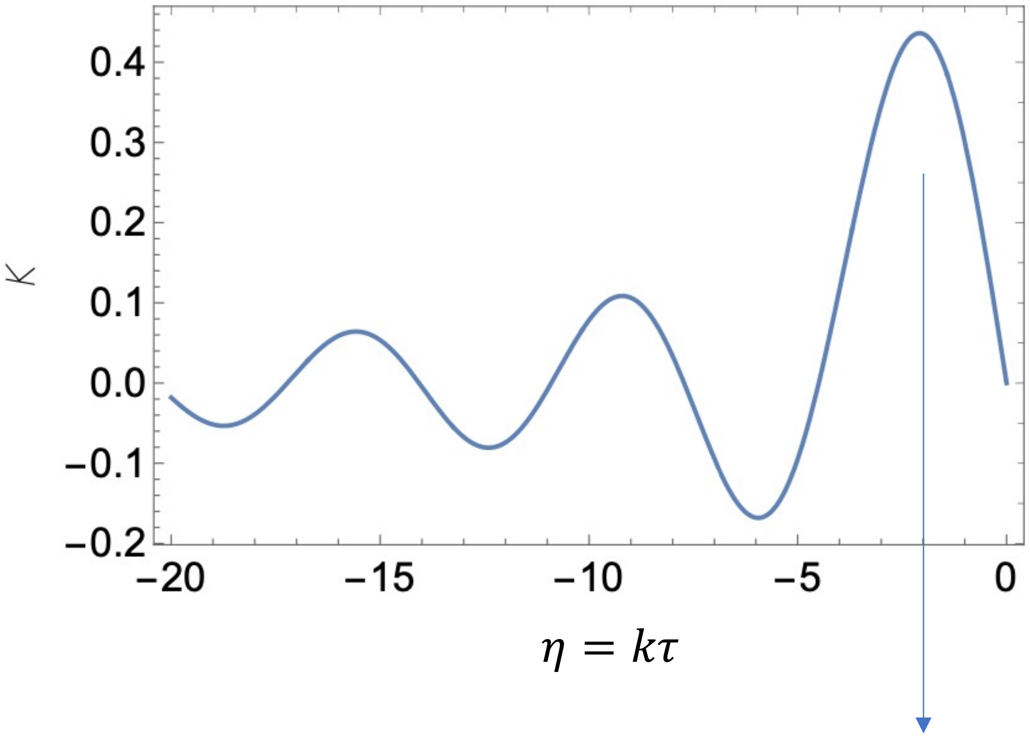
a/a_c=3.3

a/a_c=4.3

Calculation of GWs

- In Minkowski spacetime, static source cannot radiate due to energy-momentum conservation.
- During inflation, energy conservation is badly broken, so the even static source can produce GWs.

$$\tilde{h}^{f}_{ij}(\mathbf{k}) = \frac{16\pi G_N}{k} \int_{-\infty}^{0} d\tau' \mathcal{K}(k\tau') \tilde{T}^{TT}_{ij}(\tau', \mathbf{k})$$



The dominant contribution

Numerical results for GWs

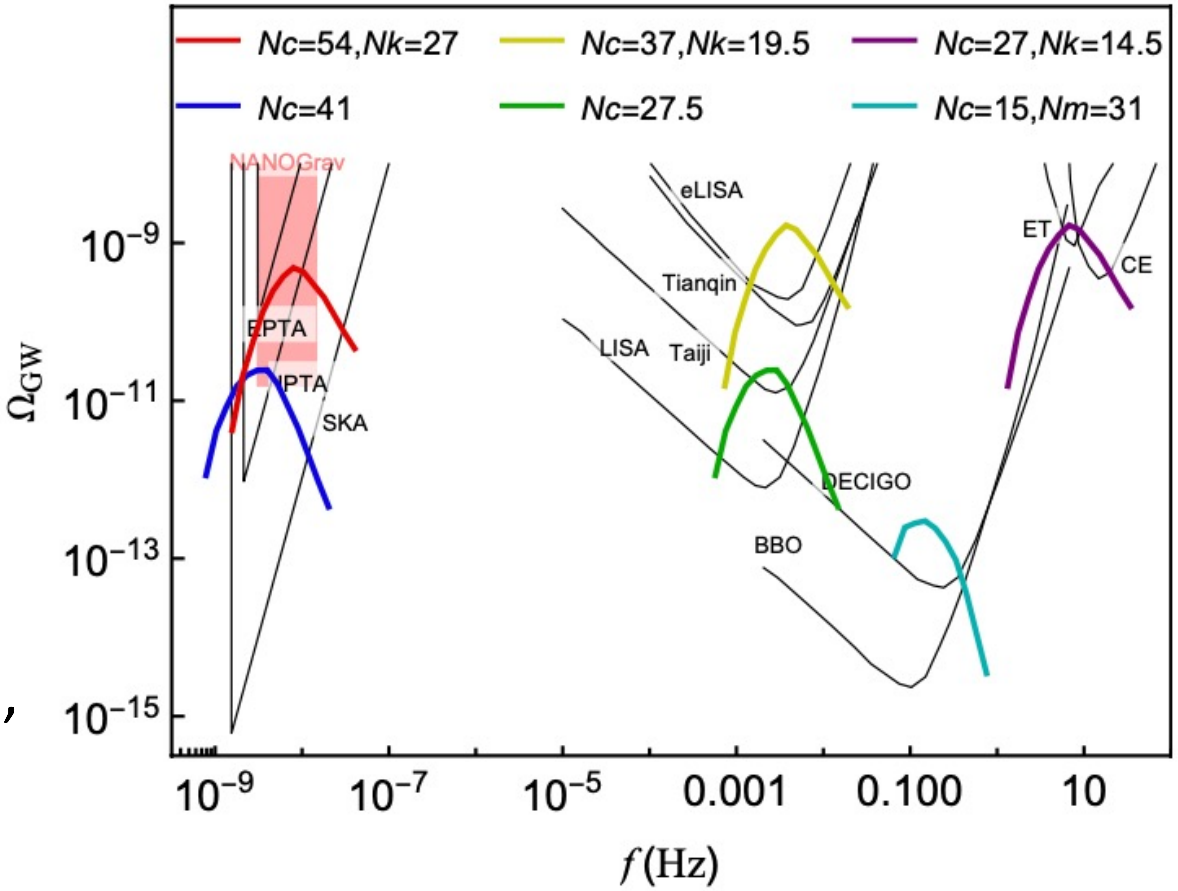
$$\Omega_{\rm GW}(f) = \Omega_R \times \rho_R^{-1} \frac{d\rho_{\rm GW}}{d\ln f}\bigg|_{\rm today}$$

$$\frac{f_{\rm today}}{f_\star} = \frac{a(\tau_\star)}{a_1}\left(\frac{g_{*S}^{(0)}}{g_{*S}^{(R)}}\right)^{1/3} \frac{T_{\rm CMB}}{\left[\left(\frac{30}{g_*^{(R)}\pi^2}\right)\left(\frac{3H_{\rm inf}^2}{8\pi G_N}\right)\right]^{1/4}}$$

The detailed shape and strength also depends on the evolution of the universe.

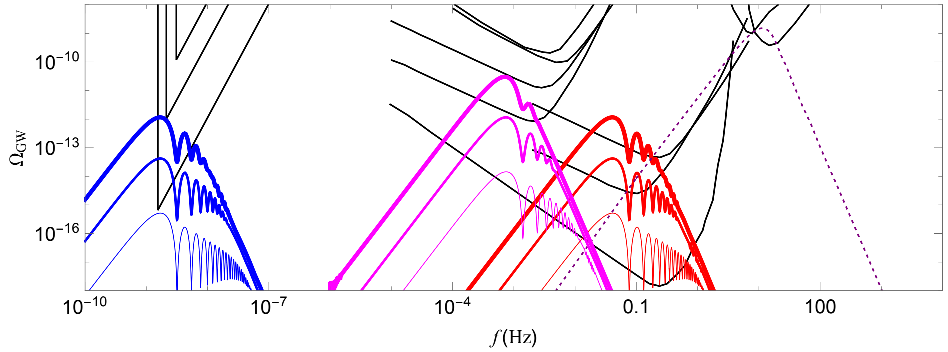
- Instantaneous reheating,
- Matter dominated intermediate stage,
- Kination dominated intermeditate stage.

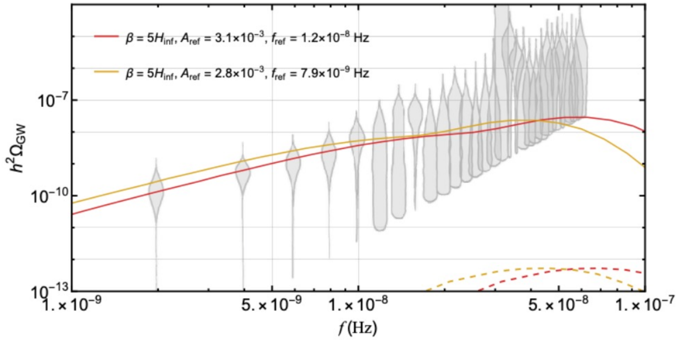
HA, Chen Yang, 2304.02361

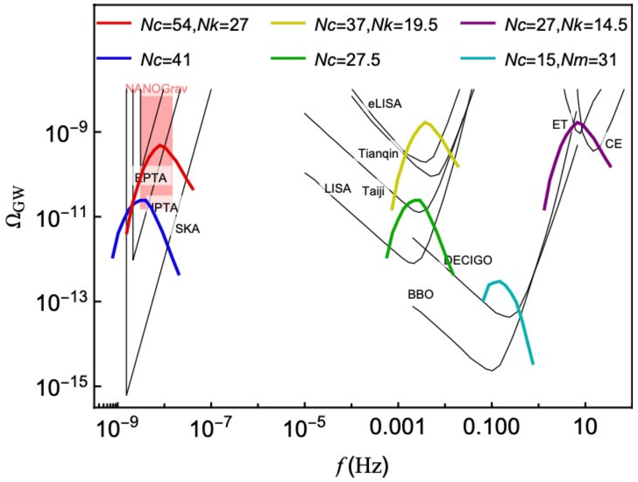


Summary

- Phase transitions can happen in a spectator sector during inflation.
- We show that there is an oscillatory feature in the GW spectrum if it is produced by first-order phase transition during inflation.
- We show that the secondary GW can be strong enough to explain the signals observed by PTAs
- Static topological defects can produce GWs during inflation.







Outlook

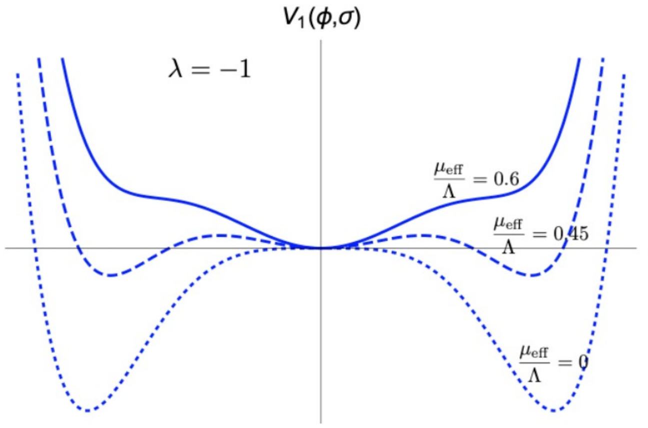
- The fate of the domain walls.
- Other topologcial defects.
- Application to high scale particle physics models.

Backups

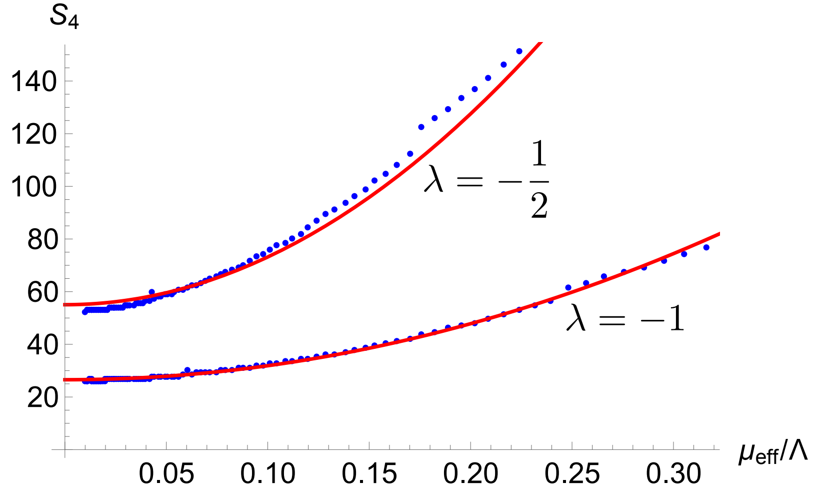
First order phase transition during inflation

- $$\beta = \left|\frac{dS_4}{dt}\right| = \frac{dS_4}{d\log\mu_{\rm eff}^2} \times \left|\frac{2\dot{\phi}}{\phi\left(1-\frac{\mu^2}{c^2\phi^2}\right)}\right| \qquad \mu_{\rm eff}^2 = -(\mu^2 - c^2\phi^2)$$

$$\Rightarrow \quad \frac{\beta}{H} = \left|\frac{dS_4}{d\log\mu_{\rm eff}^2}\right| (2\epsilon)^{1/2} \times \frac{M_{\rm pl}}{\left|\phi\left(1-\frac{\mu^2}{c^2\phi^2}\right)\right|}$$



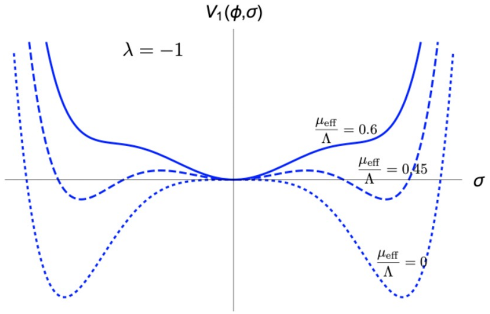
$$V_1(\phi,\sigma) = -\frac{1}{2}(\mu^2 - c^2\phi^2)\sigma^2 + \frac{\lambda}{4}\sigma^4 + \frac{1}{8\Lambda^2}\sigma^6$$



CosmoTransitions

First order phase transition during inflation

- $$\beta = \left|\frac{dS_4}{dt}\right| = \frac{dS_4}{d\log\mu_{\rm eff}^2} \times \left|\frac{2\dot\phi}{\phi\left(1-\frac{\mu^2}{c^2\phi^2}\right)}\right|$$



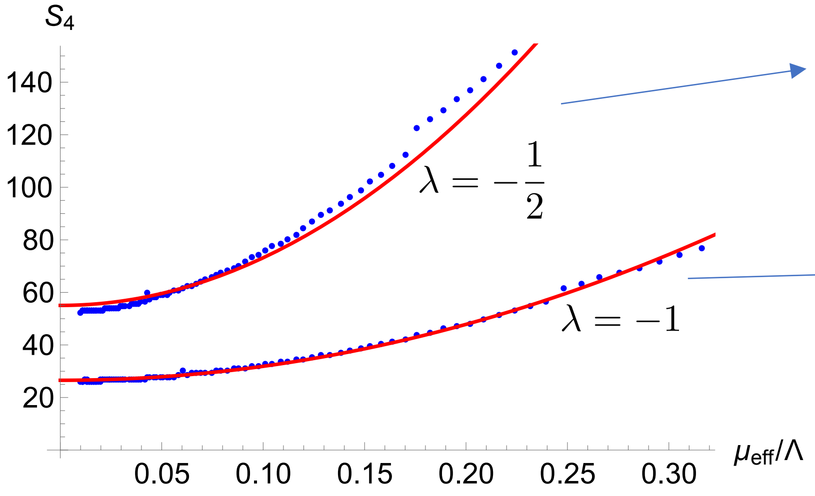
$$\Rightarrow \quad \frac{\beta}{H} = \left|\frac{dS_4}{d\log\mu_{\rm eff}^2}\right| (2\epsilon)^{1/2} \times \frac{M_{\rm pl}}{\left|\phi\,\underbrace{\left(1-\frac{\mu^2}{c^2\phi^2}\right)}_{\sim\,\mu_{\rm eff}^2/\Lambda^2}\right|}$$

$$\boxed{\int_{\phi_{\rm end}}^{\phi_{\rm PT}} \frac{d\phi}{\sqrt{2\epsilon}M_{\rm pl}} = N_{\rm e}}$$

$$\frac{\beta}{H} \sim \left|\frac{dS_4}{d\log\mu_{\rm eff}^2}\right| \times \frac{\Lambda^2}{\mu_{\rm eff}^2} \times \frac{1}{N_e}$$

First order phase transition during inflation

- $$\frac{\beta}{H} \sim \left|\frac{dS_4}{d\log\mu_{\text{eff}}^2}\right| \times \frac{\Lambda^2}{\mu_{\text{eff}}^2} \times \frac{1}{N_e}$$



$$\frac{\beta}{H} \sim \frac{3800}{N_e}$$

$$\frac{\beta}{H} \sim \frac{500}{N_e}$$

N_e: e-folds before the end of inflation

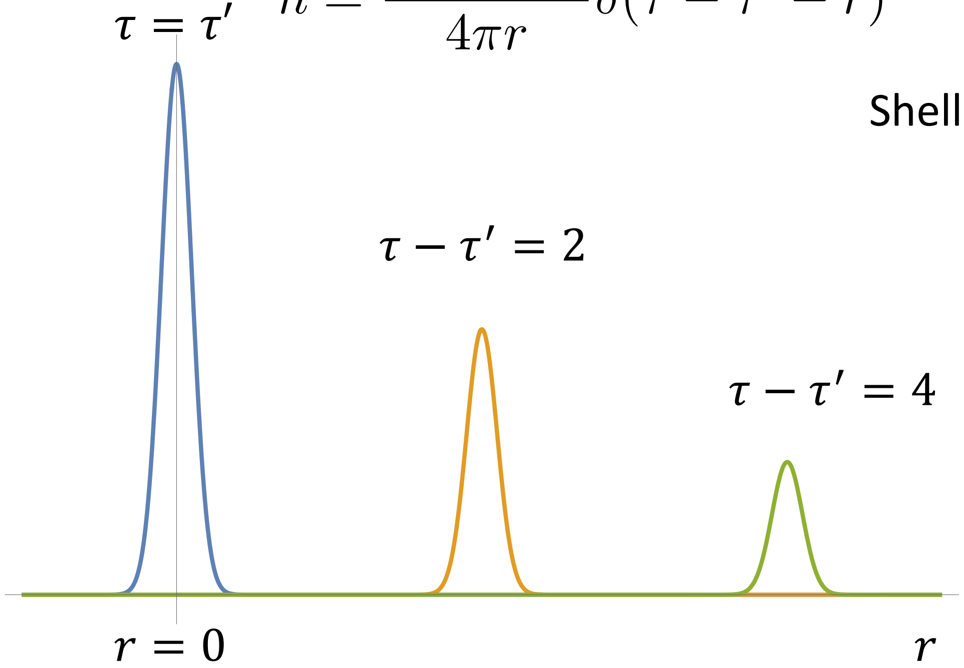
$$V_1(\phi,\sigma) = -\frac{1}{2}(\mu^2 - c^2\phi^2)\sigma^2 + \frac{\lambda}{4}\sigma^4 + \frac{1}{8\Lambda^2}\sigma^6$$

$$\frac{\beta}{H} \sim \mathcal{O}(10) - \mathcal{O}(100)$$

de Sitter inflation as an example

- What is the spatial configuration of h_{ij}?
- In Minkovski space

$$h = \frac{16\pi G_N T}{4\pi r}\delta(\tau - \tau' - r)$$



Shell with radius $|\tau - \tau'|$

de Sitter inflation as an example

- What is the spatial configuration of h_{ij}?
- In de Sitter space

$$h_{ij}(\tau,\mathbf{k}) = -16\pi G_N H T_{ij} \tau \Theta(\tau-\tau') \left[\frac{\sin k(\tau-\tau')}{k} \right.$$

$$\left. + \left(\frac{1}{k^2\tau} - \frac{1}{k^2\tau'} \right) \cos k(\tau-\tau') + \frac{1}{k^3\tau\tau'} \sin k(\tau-\tau') \right]$$

de Sitter inflation as an example

- What is the spatial configuration of h_{ij}?
- In de Sitter space

$$h_{ij}(\tau,\mathbf{k}) = -16\pi G_N H T_{ij}\tau\Theta(\tau-\tau')\left[\overset{\frac{\tau}{4\pi x}\delta(\tau-\tau'-|\mathbf{x}|)}{\overset{\nearrow}{\frac{\sin k(\tau-\tau')}{k}}}\right.$$

$$\underbrace{\left.+\left(\frac{1}{k^2\tau}-\frac{1}{k^2\tau'}\right)\cos k(\tau-\tau')+\frac{1}{k^3\tau\tau'}\sin k(\tau-\tau')\right]}_{\frac{1}{4\pi}\Theta(\tau-\tau'-|\mathbf{x}|)}$$

de Sitter inflation as an example

- What is the spatial configuration of h_{ij}?
- In de Sitter space

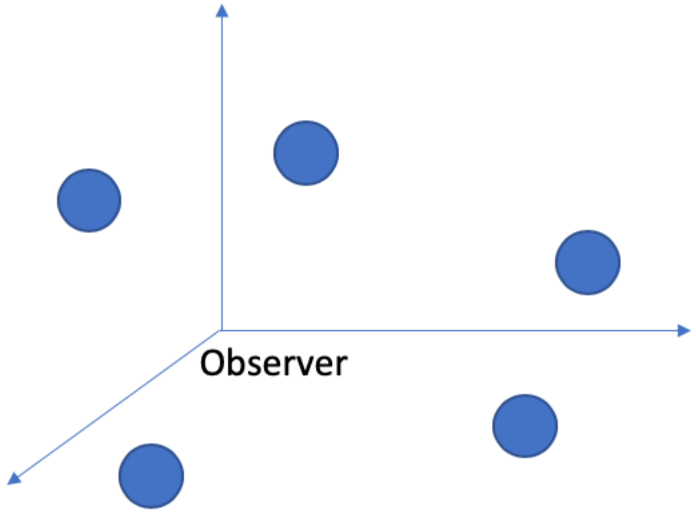
$$h(\tau, \mathbf{x}) \sim \frac{\tau}{4\pi x}\delta(\tau - \tau' - x) + \frac{1}{4\pi}\Theta(\tau - \tau' - x)$$

Similar to Minkovski	Intrinsic in de Sitter
Decreases with both x and τ	constant
Vanishes out of horizon	

de Sitter inflation as an example

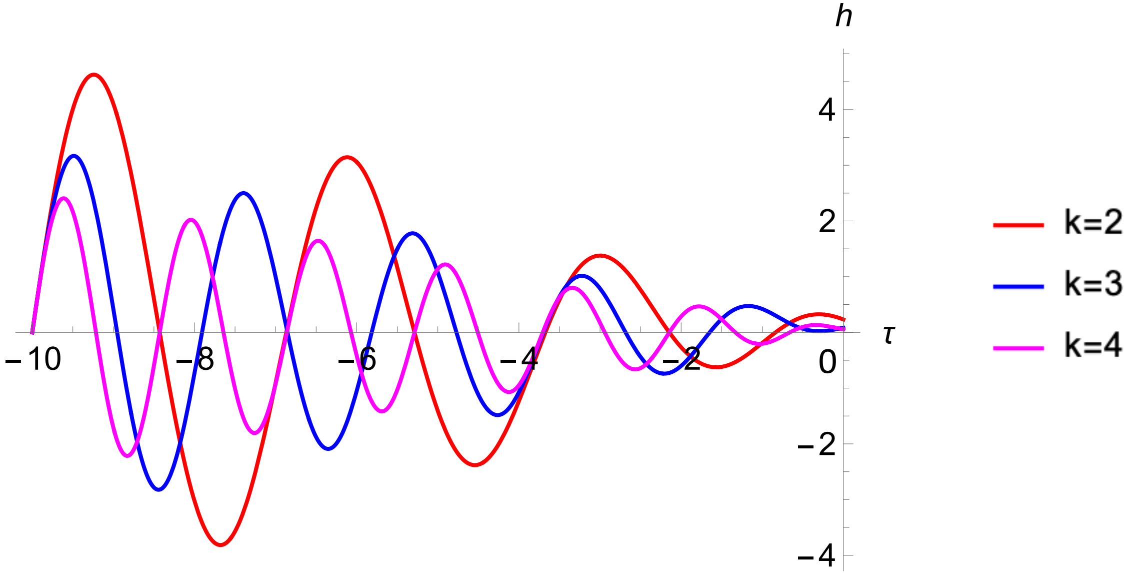
- At $\tau \to 0$ $\quad h(\tau, \mathbf{x}) \sim \frac{1}{4\pi}\Theta(|\tau'| - x)$

- A ball of GW, with radius $|\tau'|$
- h uniformally distributed inside the GW balls.
- All the balls have the same radius.



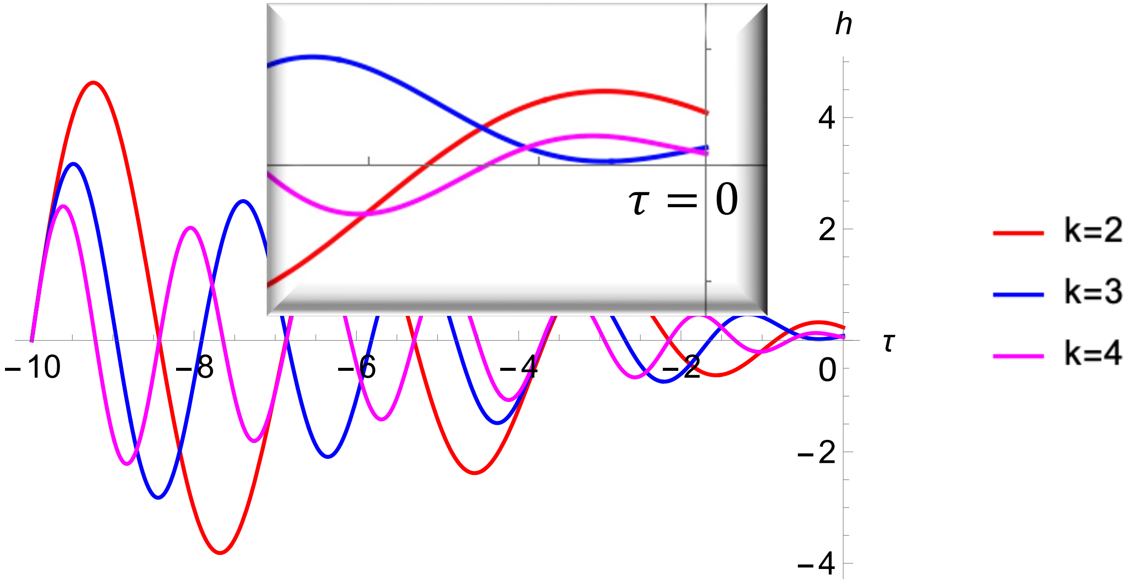
Quasi-de Sitter inflation as an example

- $$a = -\frac{1}{H\tau}$$
- $$h_{ij}(\tau, \mathbf{k}) = -\frac{16\pi G_N H T_{ij}\tau}{k}\left[\left(\frac{1}{k\tau} - \frac{1}{k\tau'}\right)\cos k(\tau - \tau') + \left(1 + \frac{1}{k^2\tau\tau'}\right)\sin k(\tau - \tau')\right]$$



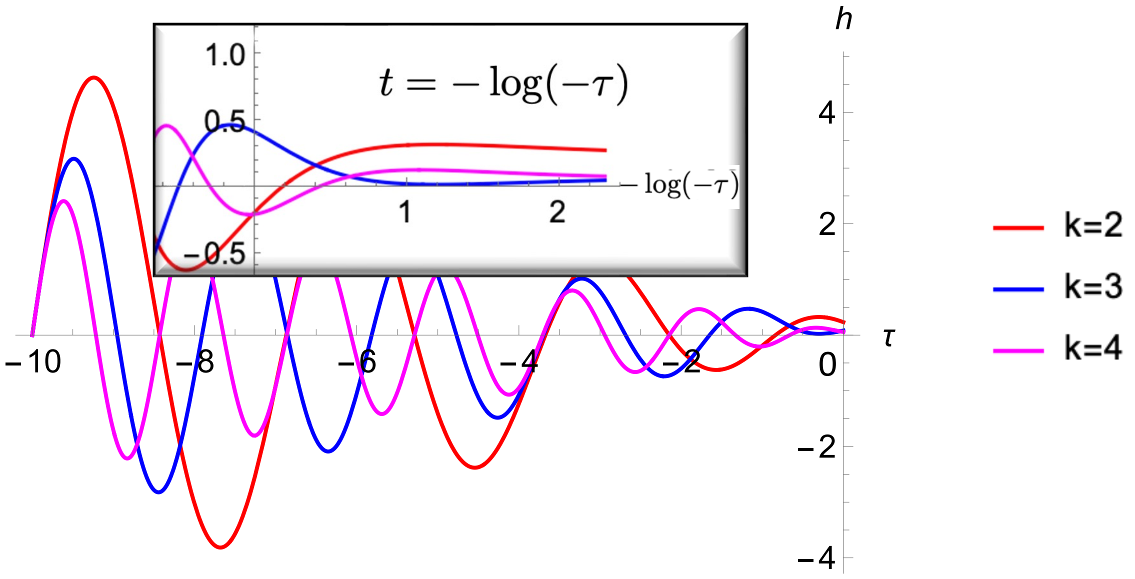
De Sitter inflation as an example

- $$a = -\frac{1}{H\tau}$$
- $$h_{ij}(\tau, \mathbf{k}) = -\frac{16\pi G_N H T_{ij}\tau}{k}\left[\left(\frac{1}{k\tau} - \frac{1}{k\tau'}\right)\cos k(\tau - \tau') + \left(1 + \frac{1}{k^2\tau\tau'}\right)\sin k(\tau - \tau')\right]$$



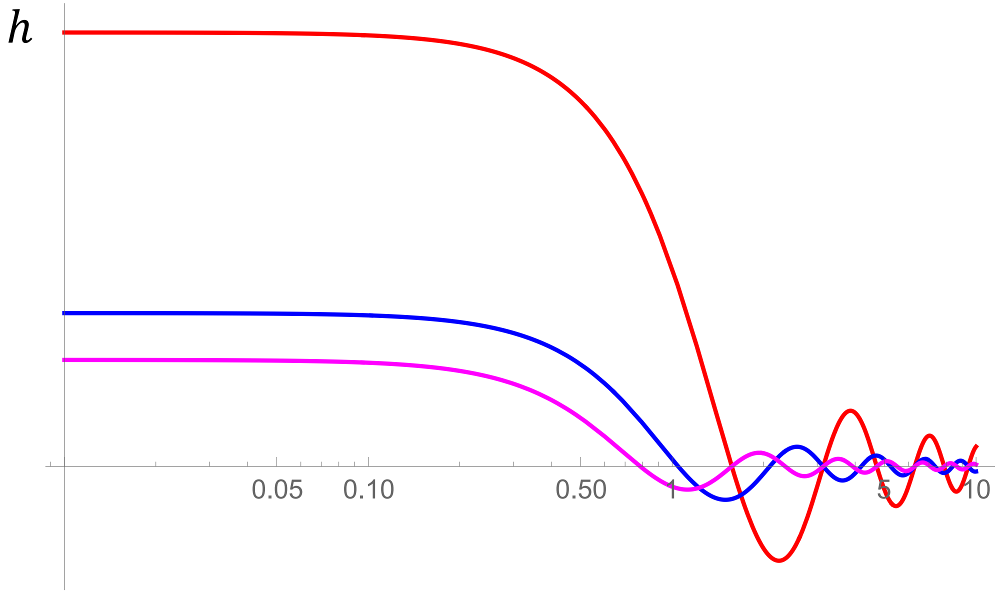
De Sitter inflation as an example

- $$a = -\frac{1}{H\tau}$$
- $$h_{ij}(\tau, \mathbf{k}) = -\frac{16\pi G_N H T_{ij}\tau}{k}\left[\left(\frac{1}{k\tau} - \frac{1}{k\tau'}\right)\cos k(\tau - \tau') + \left(1 + \frac{1}{k^2\tau\tau'}\right)\sin k(\tau - \tau')\right]$$



After inflation

- $h^f(k)$ is the initial amplitude for the GW oscillation after inflation.
- All the modes start to oscillate with the same phase.
- Example, in RD, the oscillation is $\sin k\tau / k\tau$



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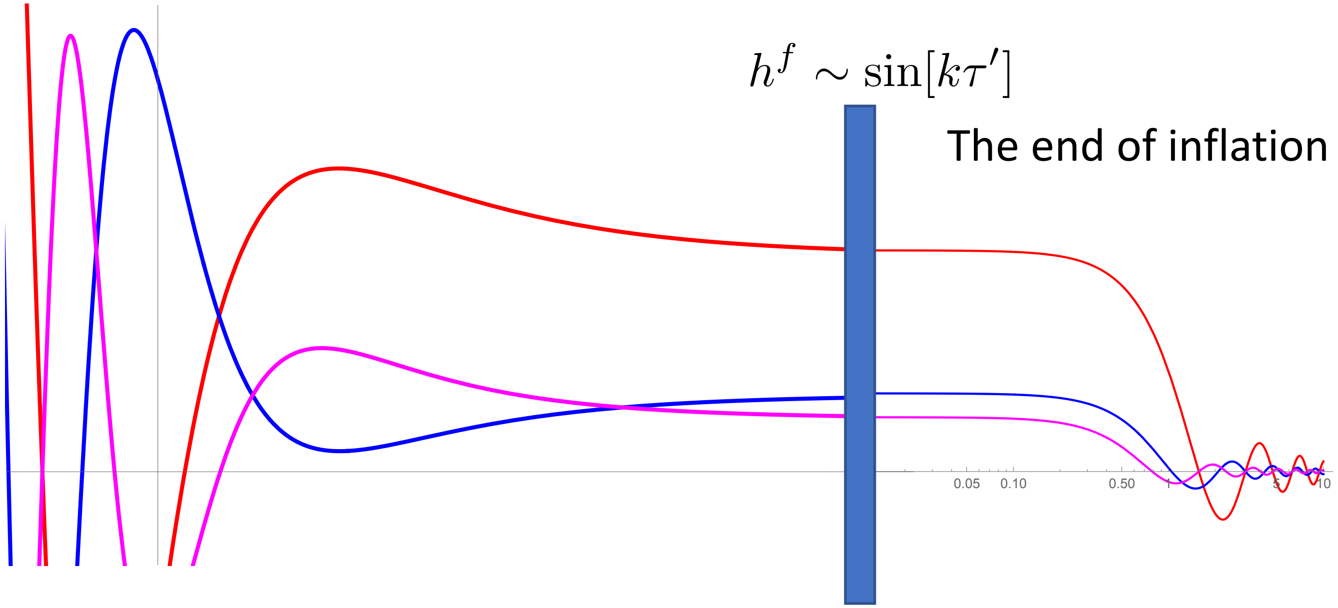
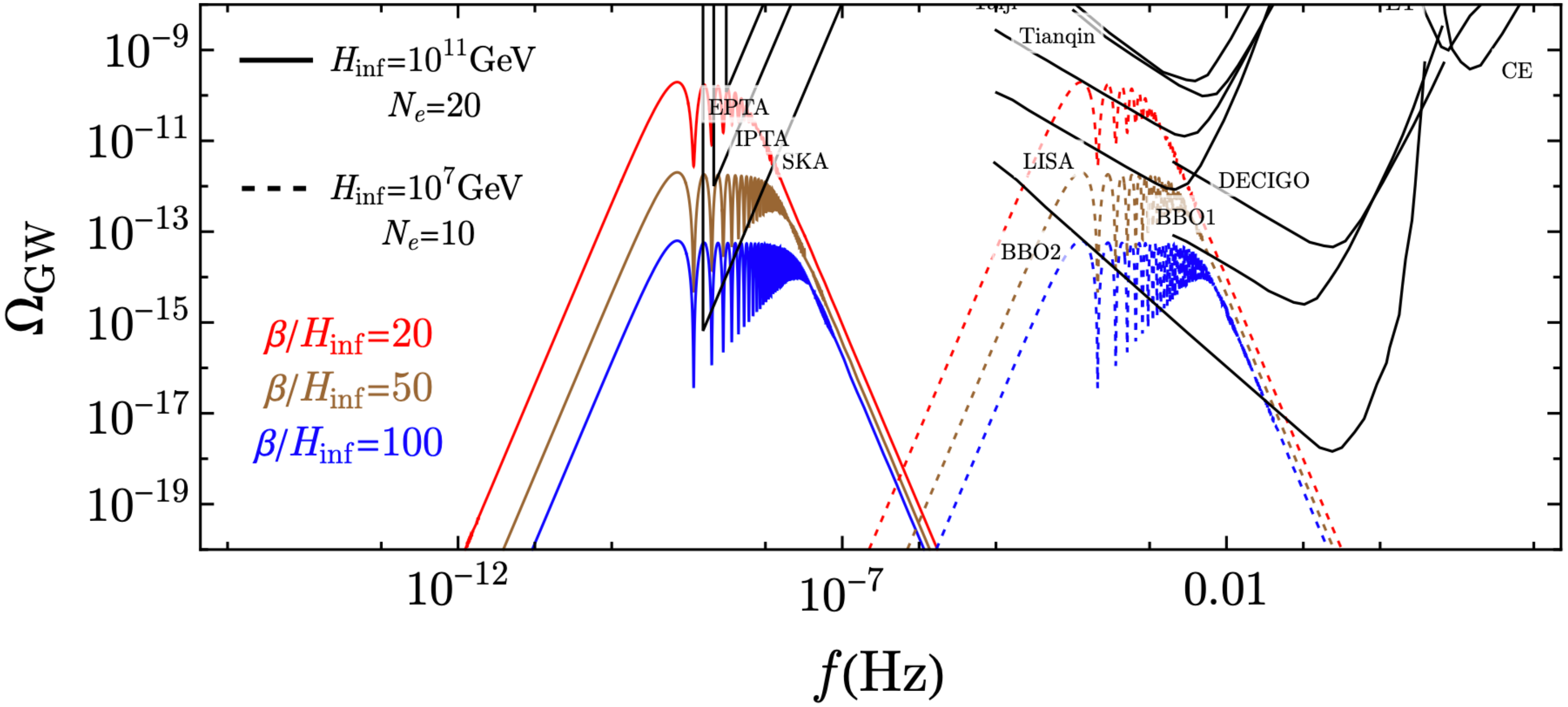


$h^f \sim \sin[k\tau']$

The end of inflation

First order phase transition during inflation

- Signal strength is also sensitive to intermediate stages


Kination domination

First order phase transition during inflation

With kination domination intermediate stage