

BBN Constraints on Ultralight Scalar Dark Matter

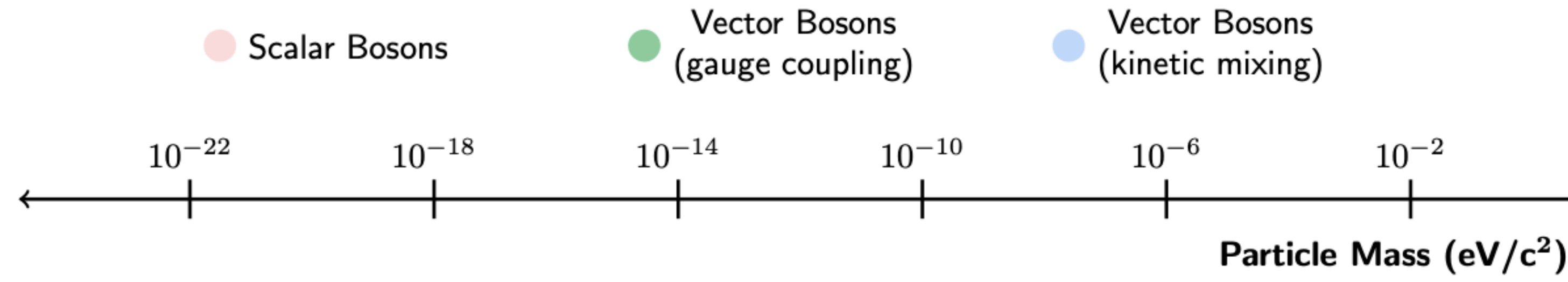
Tien-Tien Yu (University of Oregon)

based on S. Sibiryakov, P. Sørensen, TTY *JHEP* 20 (2020) 075 [arXiv: 2006.04820]
T. Bouley, P. Sørensen, TTY *JHEP* 03 (2023) 104 [arXiv: 2211.09826]

BSM in Particle Physics and Cosmology, 50 years later
January 11, 2024

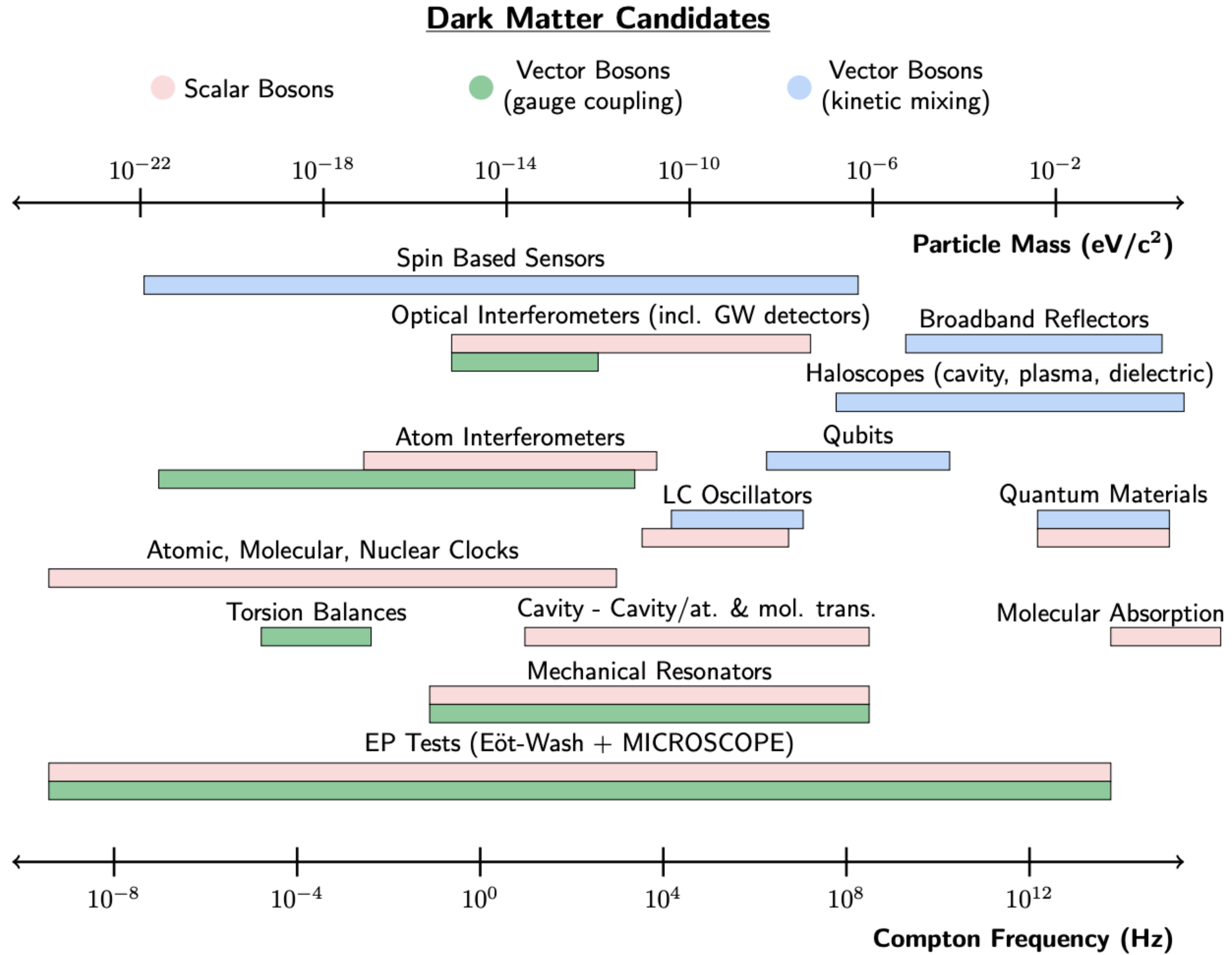
ICISE, Quy Nhon, Vietnam

Dark Matter Candidates



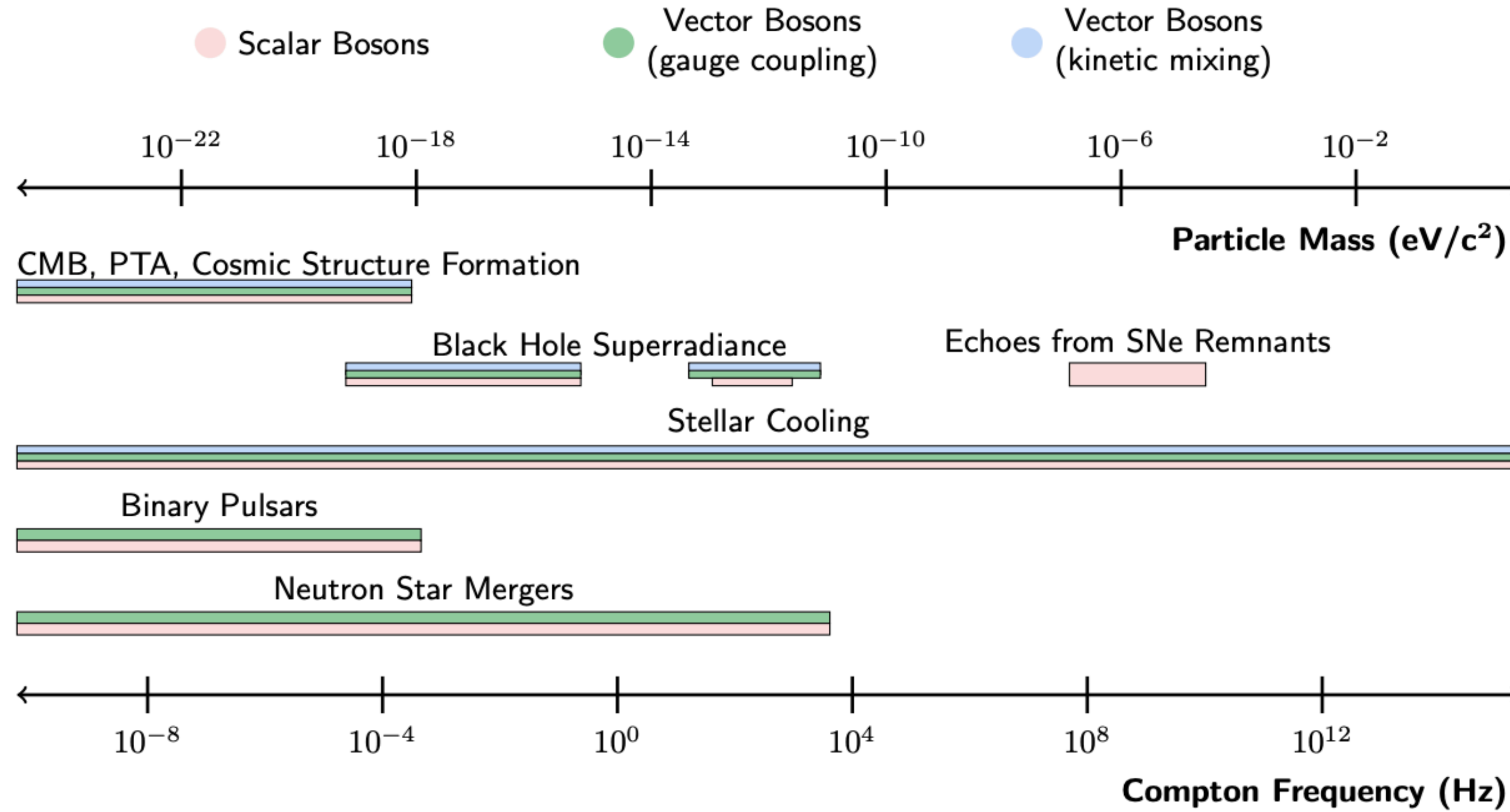
[arXiv:2203.14915]

Direct Detection



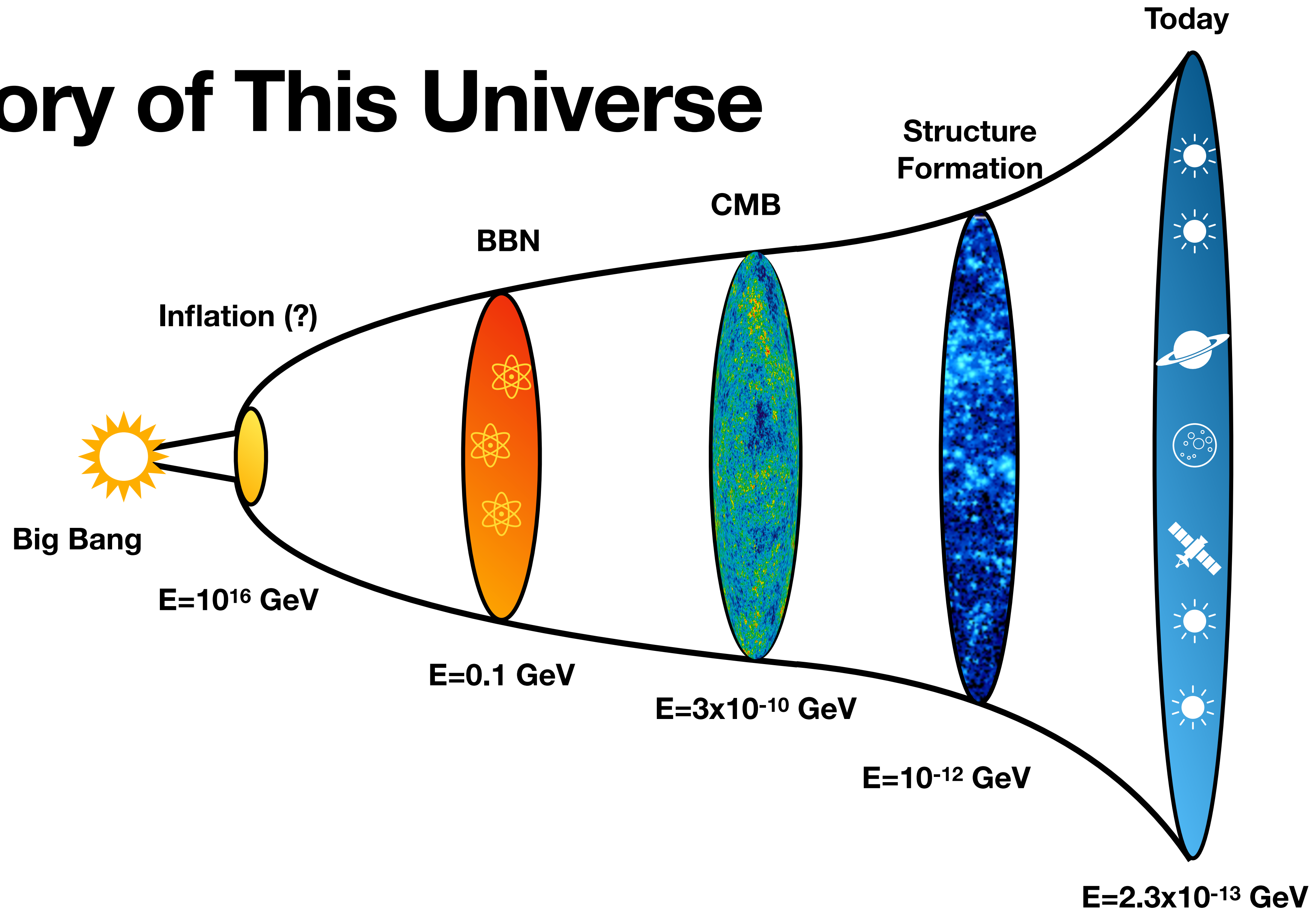
[arXiv:2203.14915]

Dark Matter Candidates

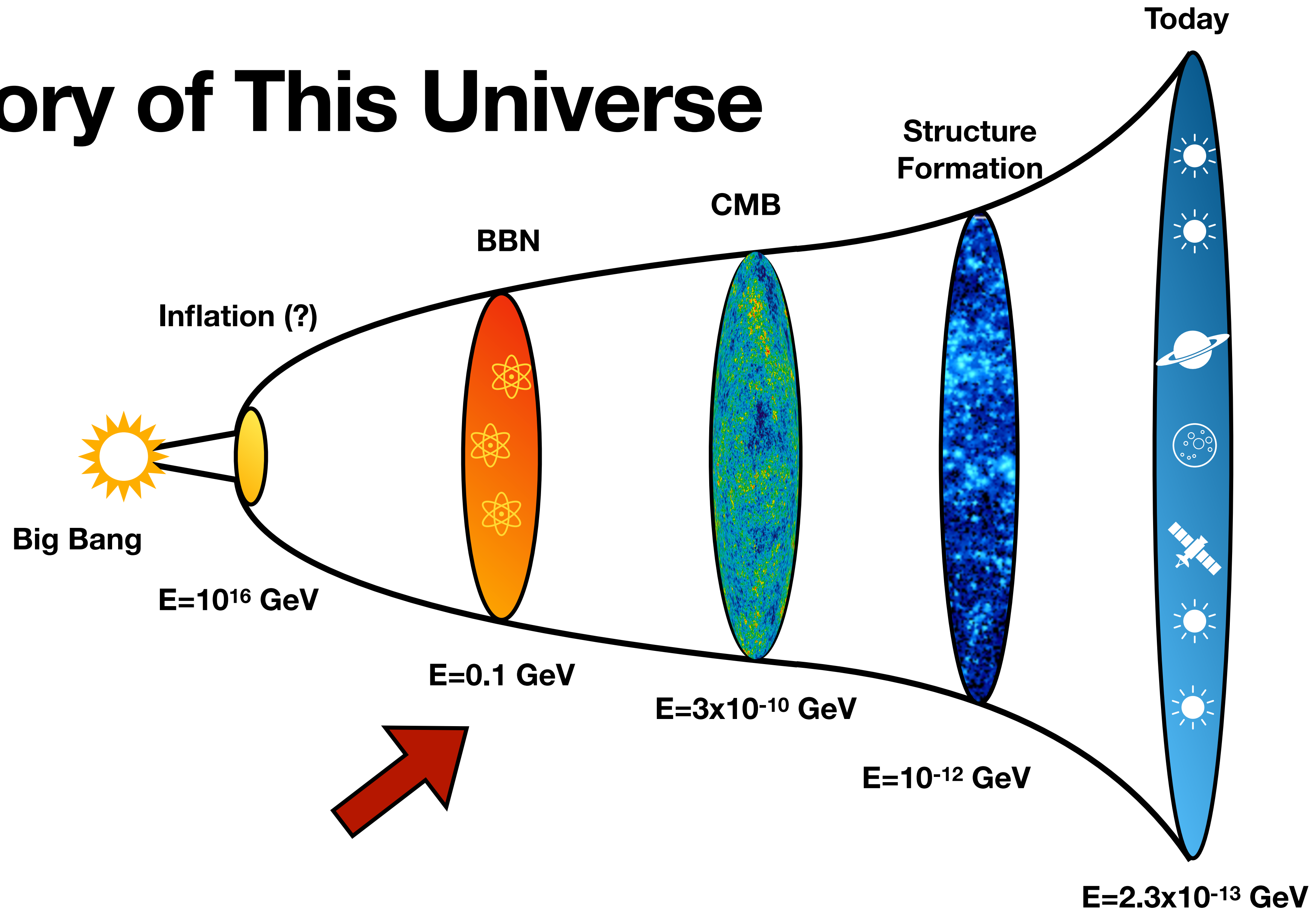


[arXiv:2203.14915]

History of This Universe

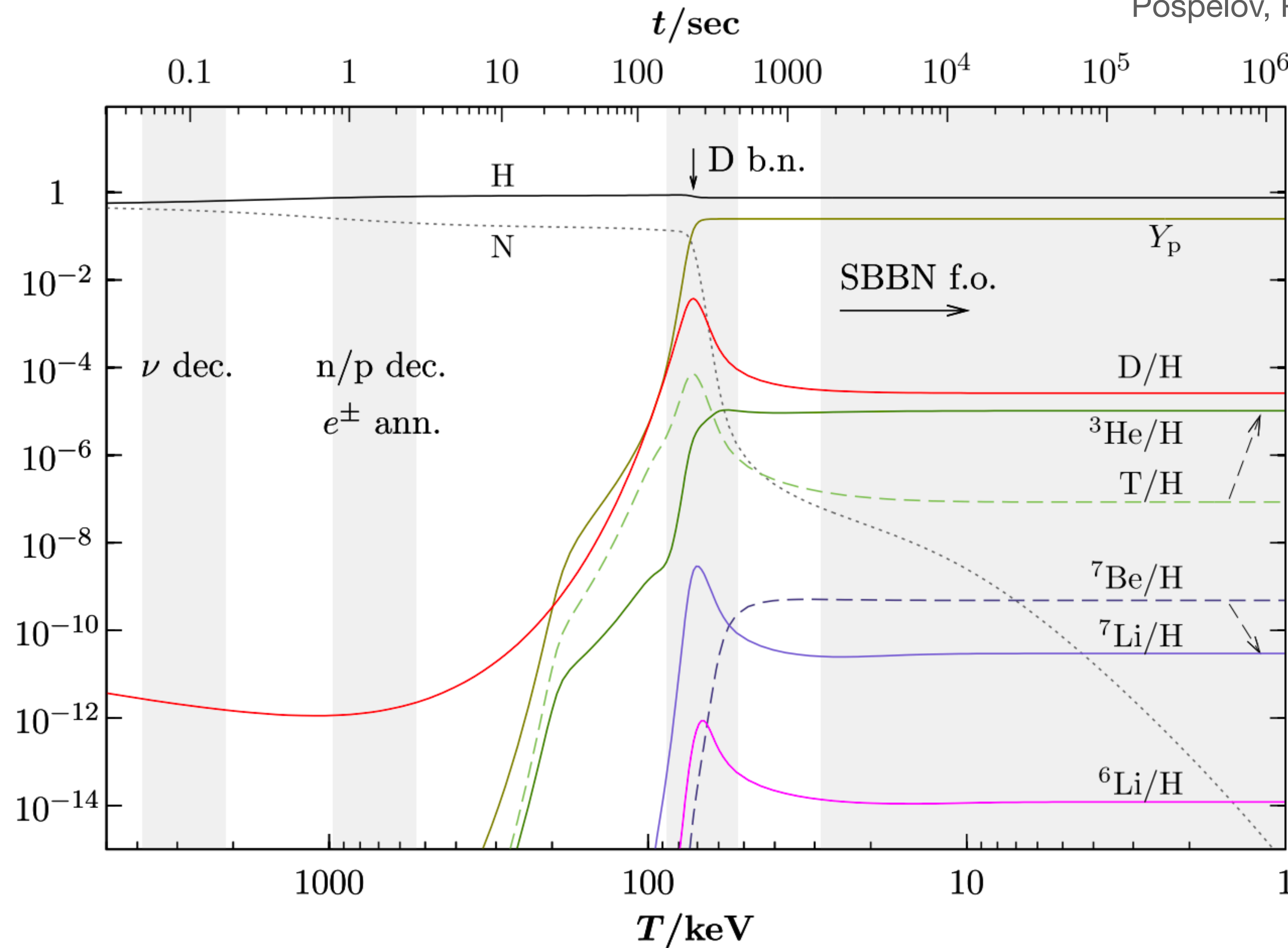


History of This Universe



standard BBN picture

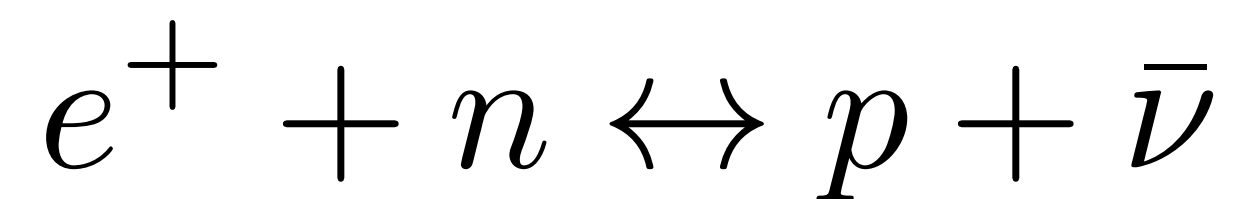
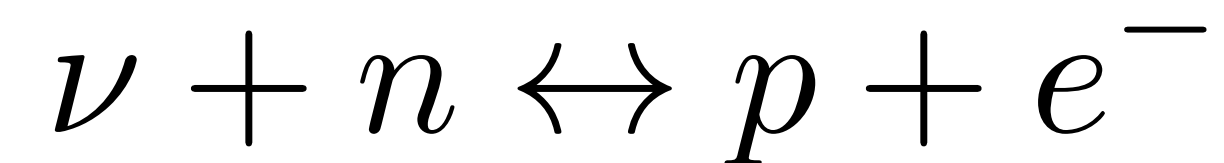
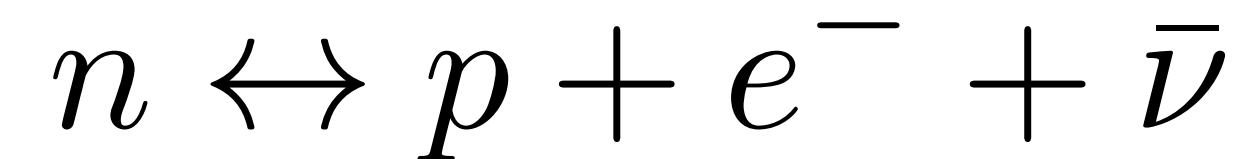
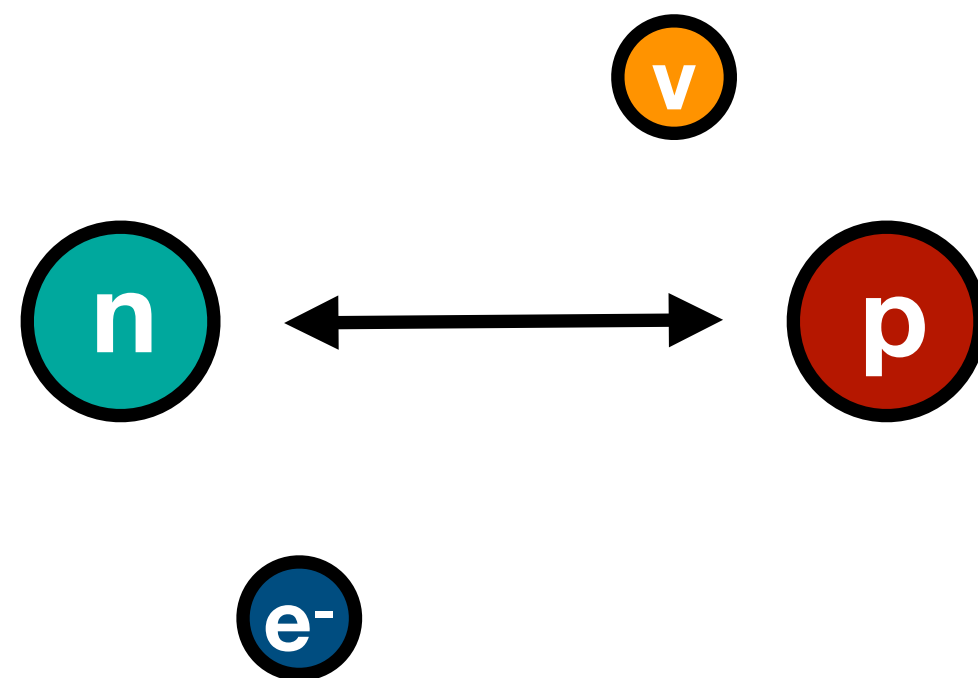
Pospelov, Pradler [arXiv:1011.1054]



big bang nucleosynthesis

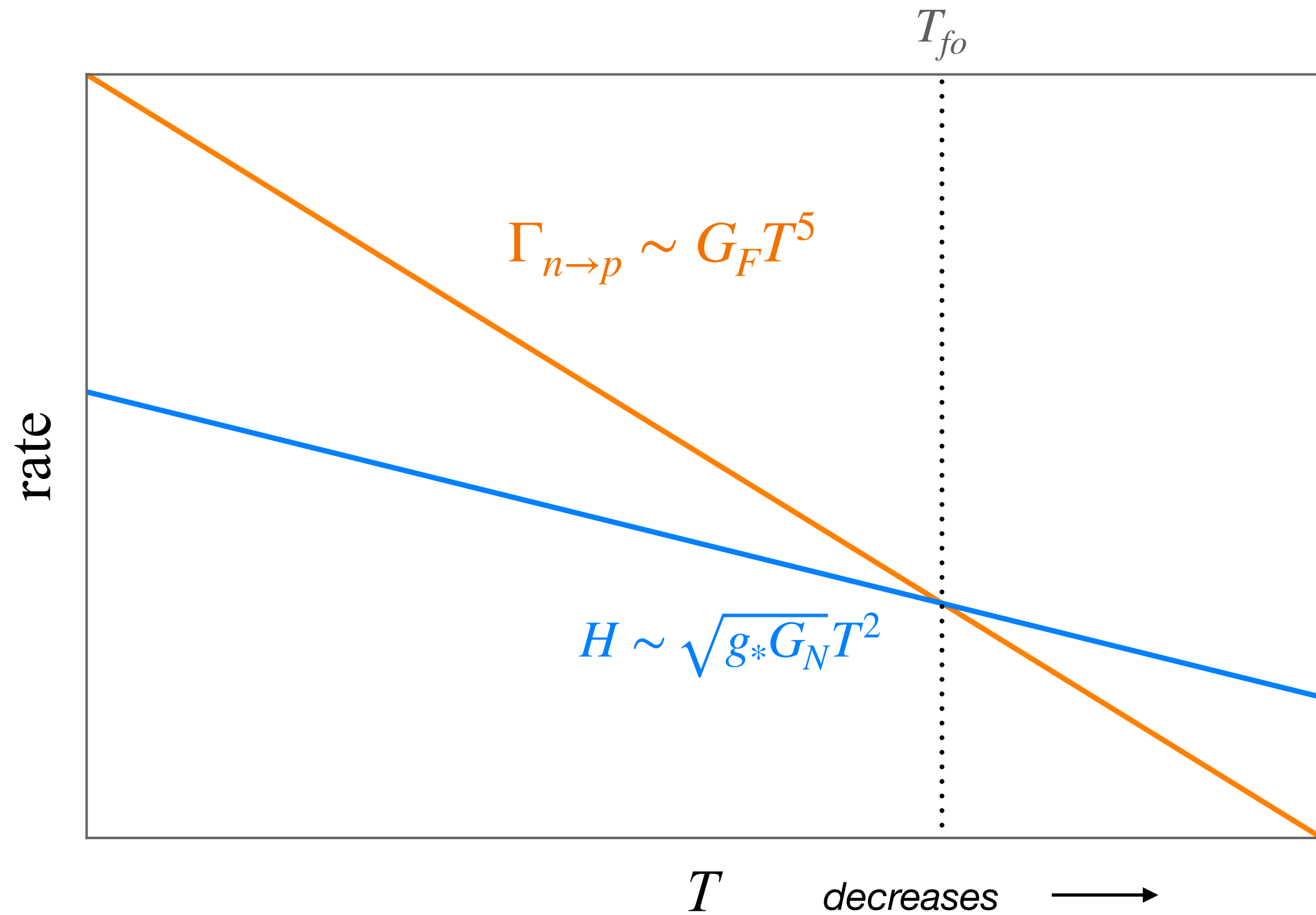
T > 1 MeV

weak interactions keep neutrons and protons in thermal equilibrium



$$\frac{n}{p} = e^{-(m_n - m_p)/T}$$

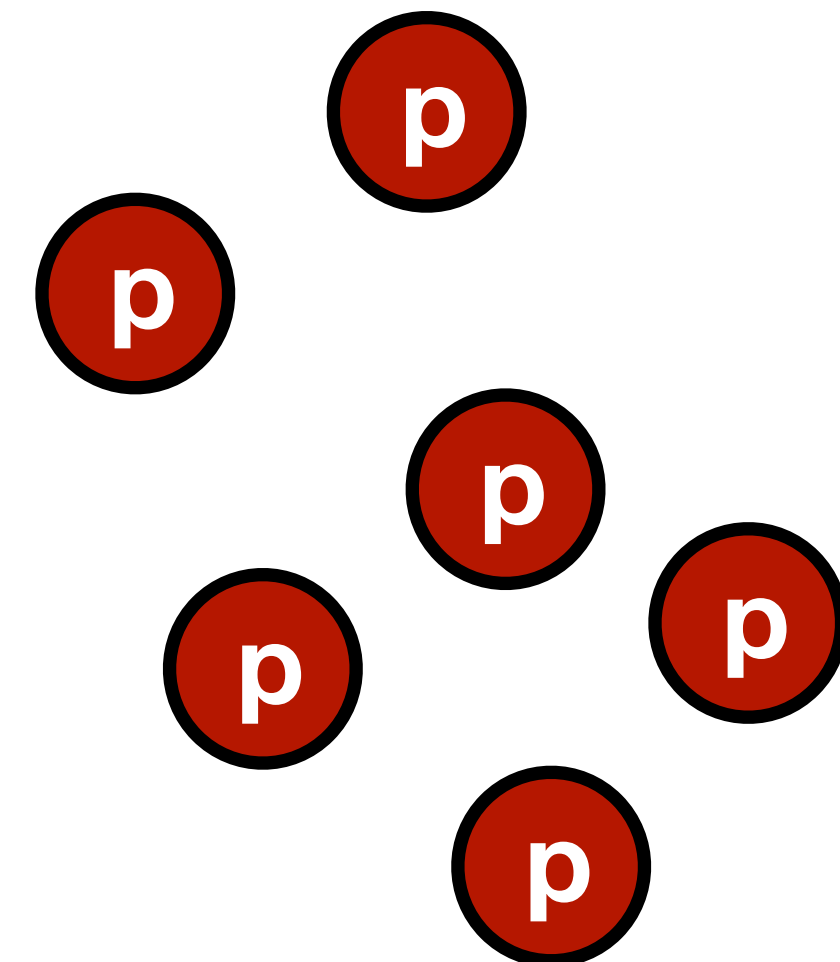
big bang nucleosynthesis



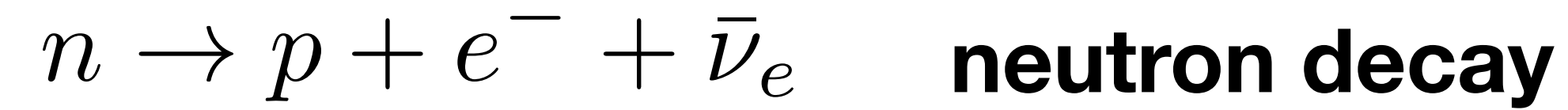
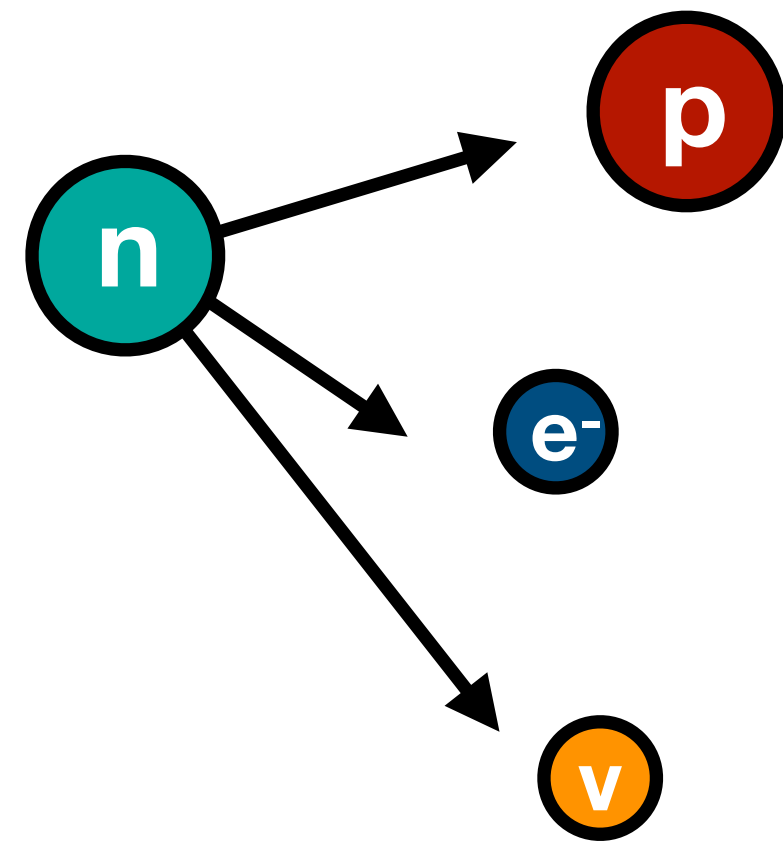
$$\Gamma_{n \rightarrow p} \sim G_F T^5 \quad \text{n} \longleftrightarrow \text{p}$$

$$T_{fo} \sim \left(\frac{g_* G_N}{G_F^4} \right)^{1/6} \simeq 1 \text{ MeV}$$

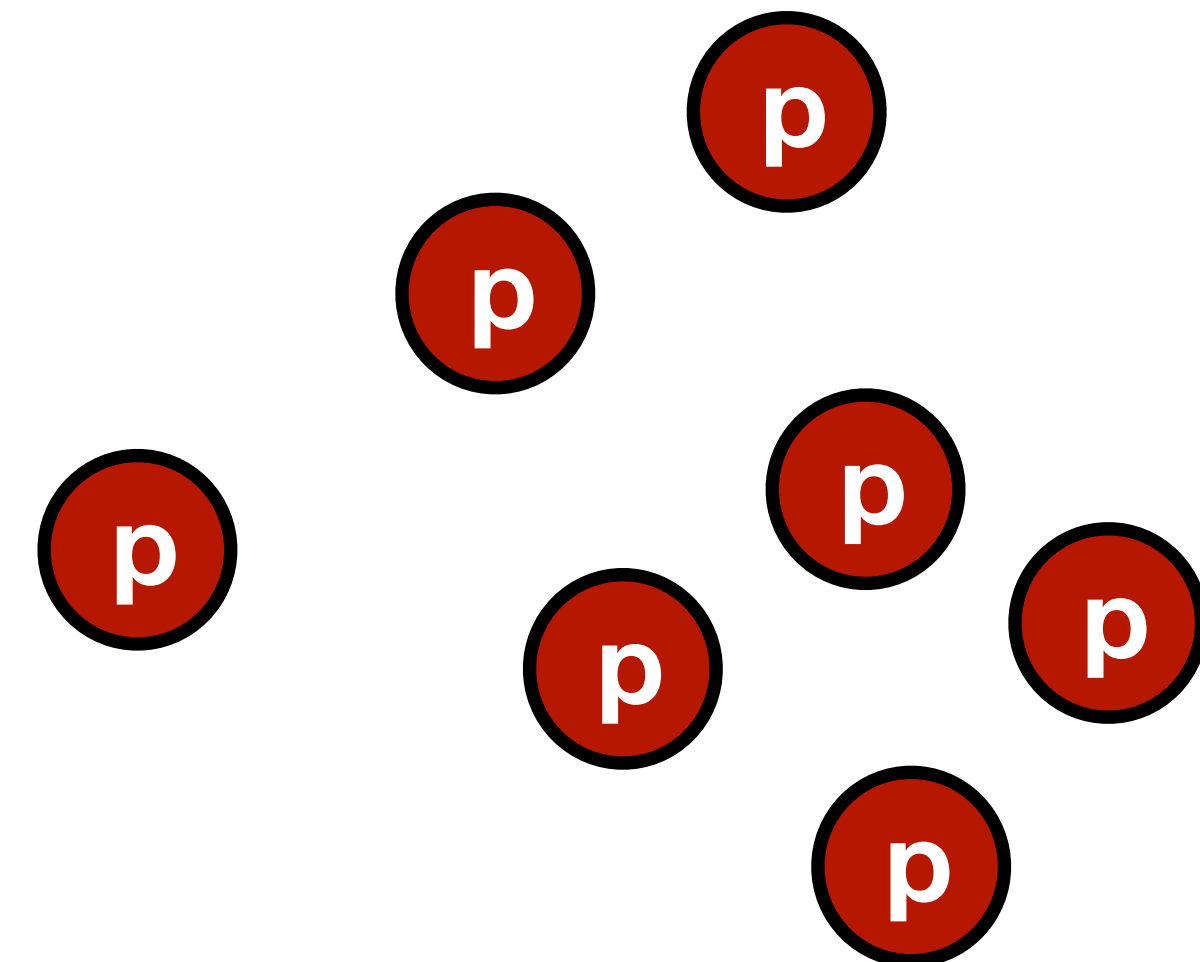
$$\left. \frac{n}{p} \right|_{fo} = e^{-(m_n - m_p)/T_{fo}} \simeq \frac{1}{6}$$



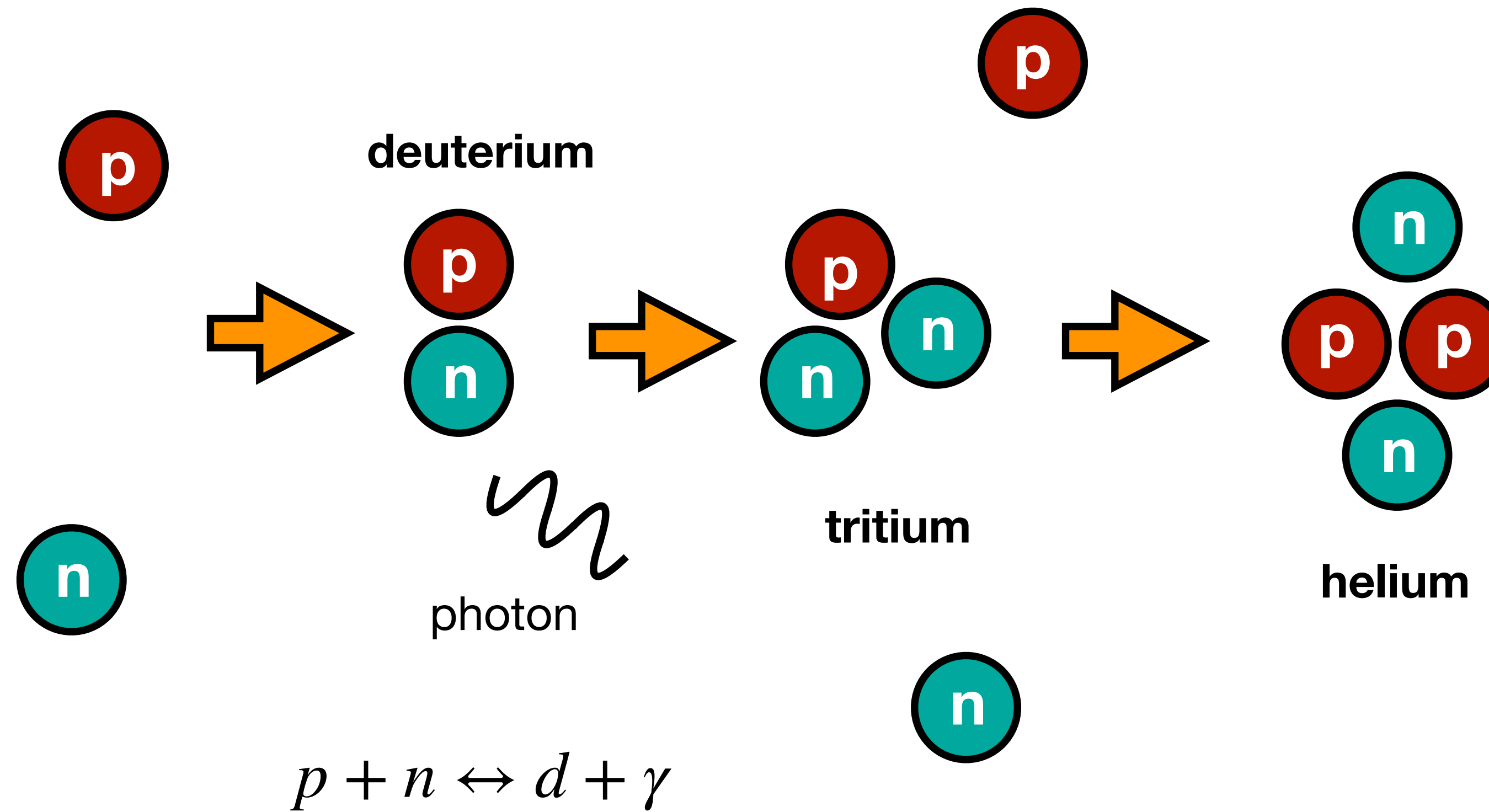
big bang nucleosynthesis



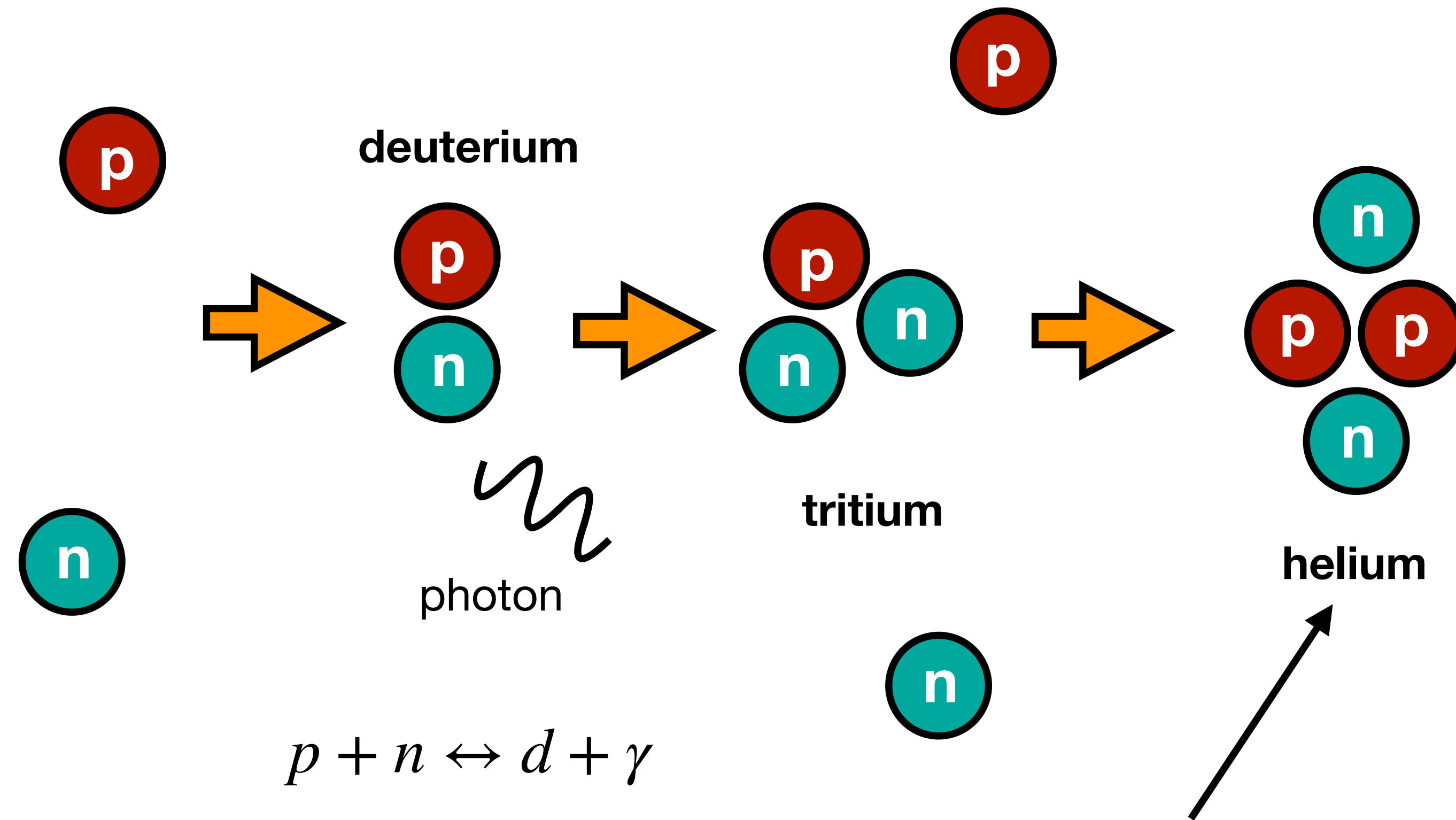
$$\left. \frac{n}{p} \right|_{f_0} = e^{-(m_n - m_p)/T_{f_0}} \simeq \frac{1}{6} \rightarrow \frac{1}{7}$$



Helium-4 formation



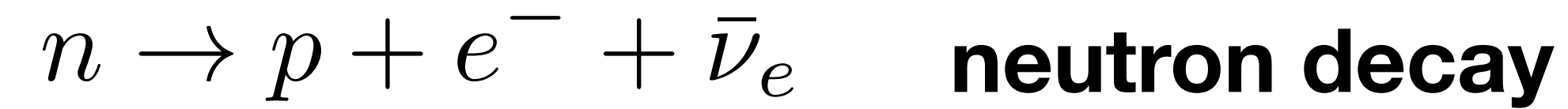
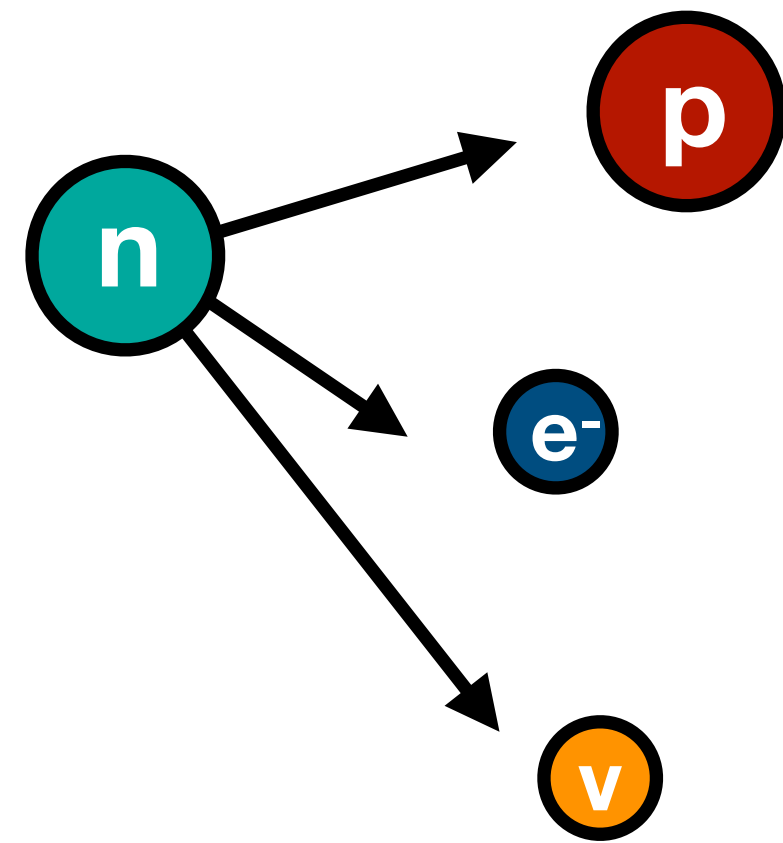
Helium-4 formation



very stable!

all free neutrons end up here

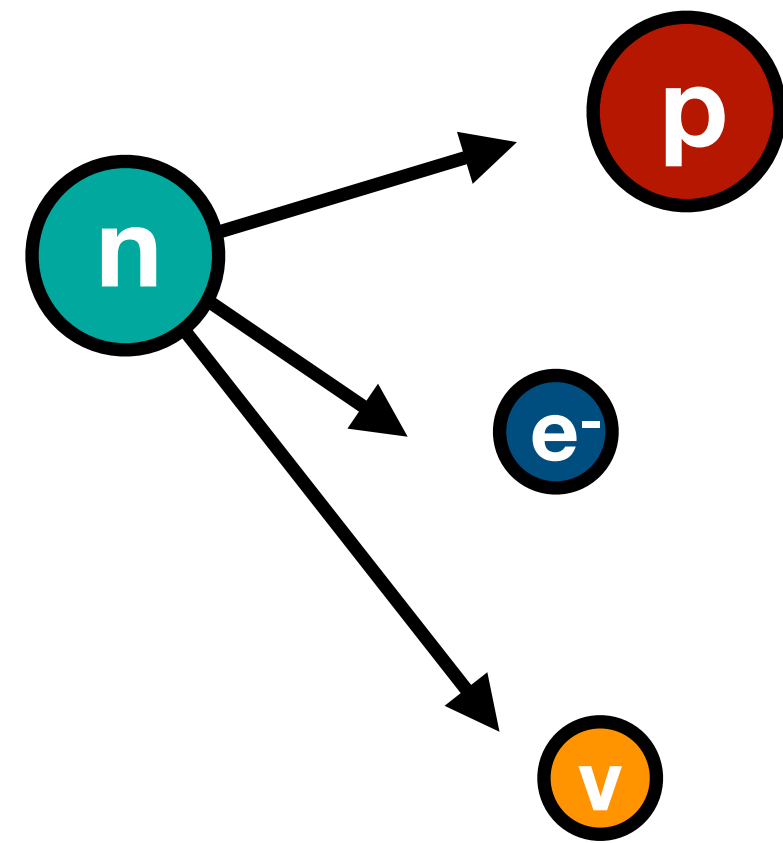
Helium-4 formation



$$\left. \frac{n}{p} \right|_{f_0} = e^{-(m_n - m_p) / T_{f_0}} \simeq \frac{1}{6} \rightarrow \frac{1}{7}$$

$${}^4\text{He} \rightarrow Y_p = \frac{2(n/p)}{1 + n/p} \simeq 0.25$$

Helium-4 formation

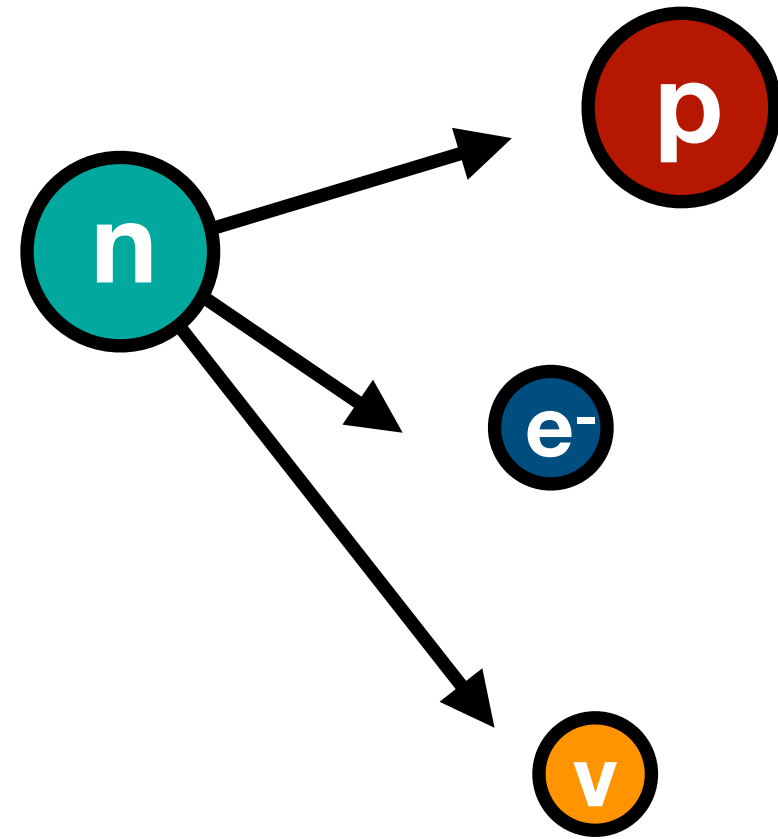


$$\left. \frac{n}{p} \right|_{f_0} = e^{-(m_n - m_p) / T_{f_0}} \simeq \frac{1}{6} \rightarrow \frac{1}{7}$$

determines 4He abundance

$${}^4\text{He} \rightarrow Y_p = \frac{2(n/p)}{1 + n/p} \simeq 0.25$$

Helium-4 formation



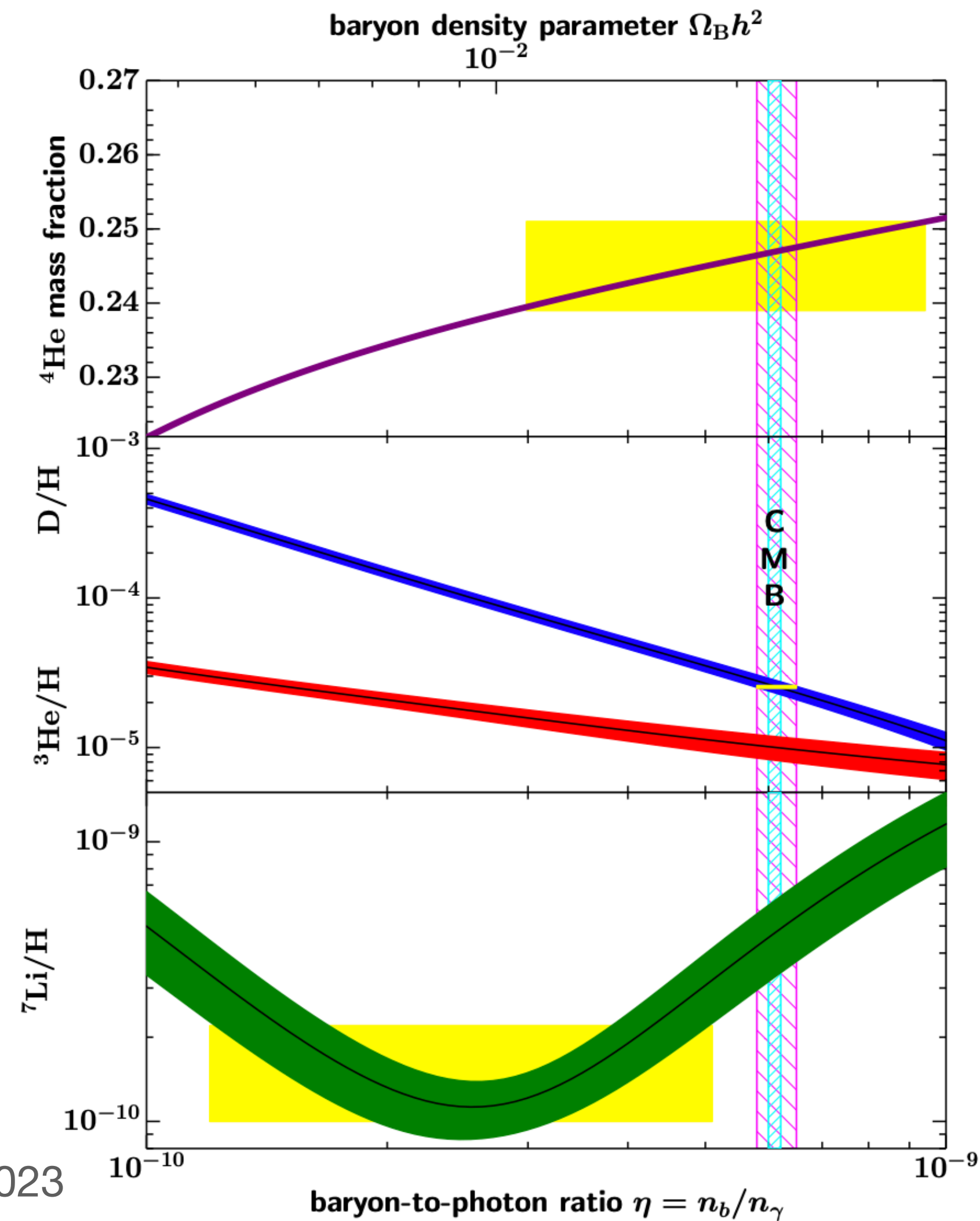
new physics can change this!

$$\left. \frac{n}{p} \right|_{f_0} = e^{-(m_n - m_p) / T_{f_0}} \simeq \frac{1}{6} \rightarrow \frac{1}{7}$$

determines ${}^4\text{He}$ abundance

$${}^4\text{He} \rightarrow Y_p = \frac{2(n/p)}{1 + n/p} \simeq 0.25$$

primordial abundances



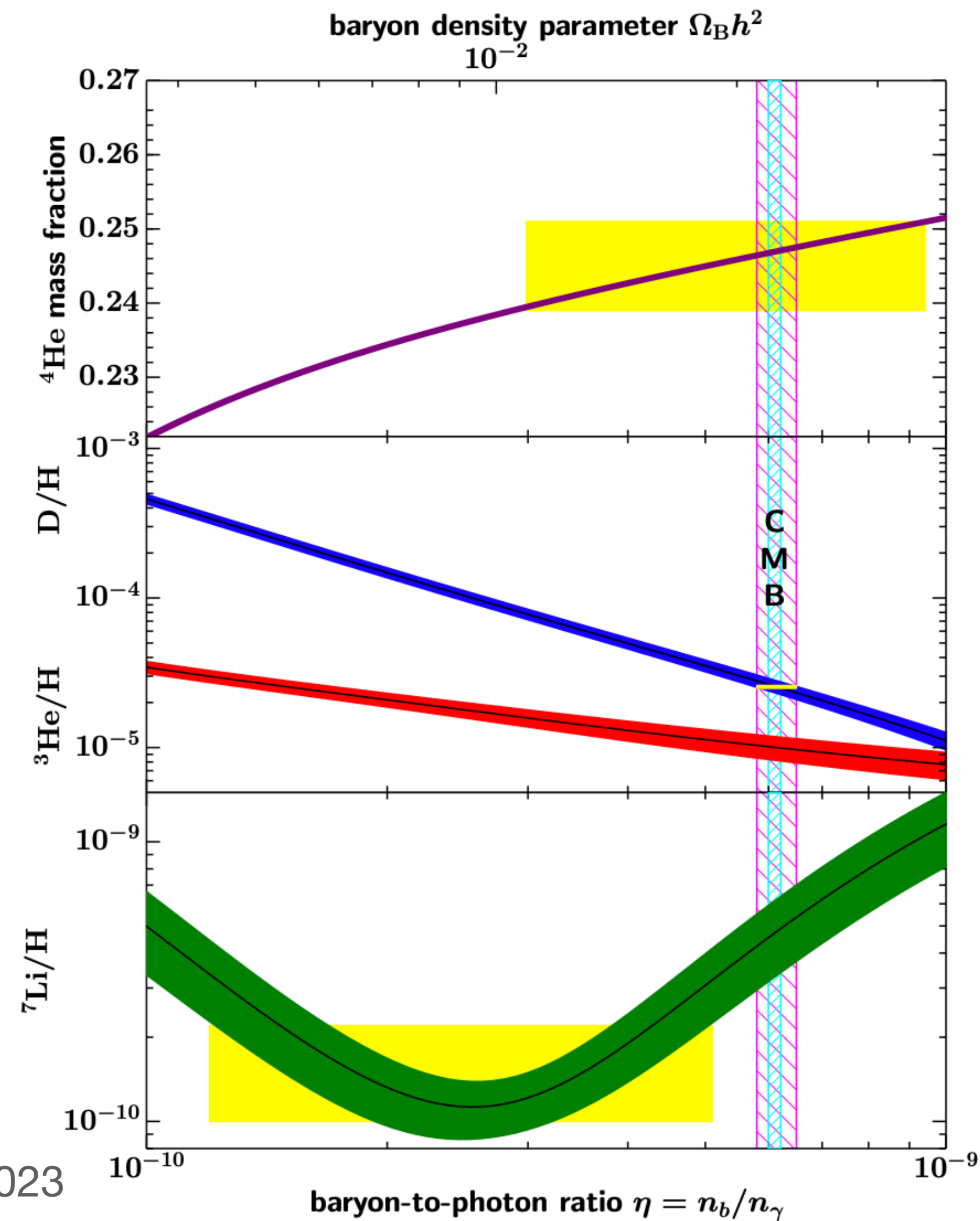
PDG 2023

Helium-4

$$Y_{\text{th}} = 0.24709 \pm 0.00025$$

$$Y_{\text{exp}} = 0.245 \pm 0.003$$

primordial abundances



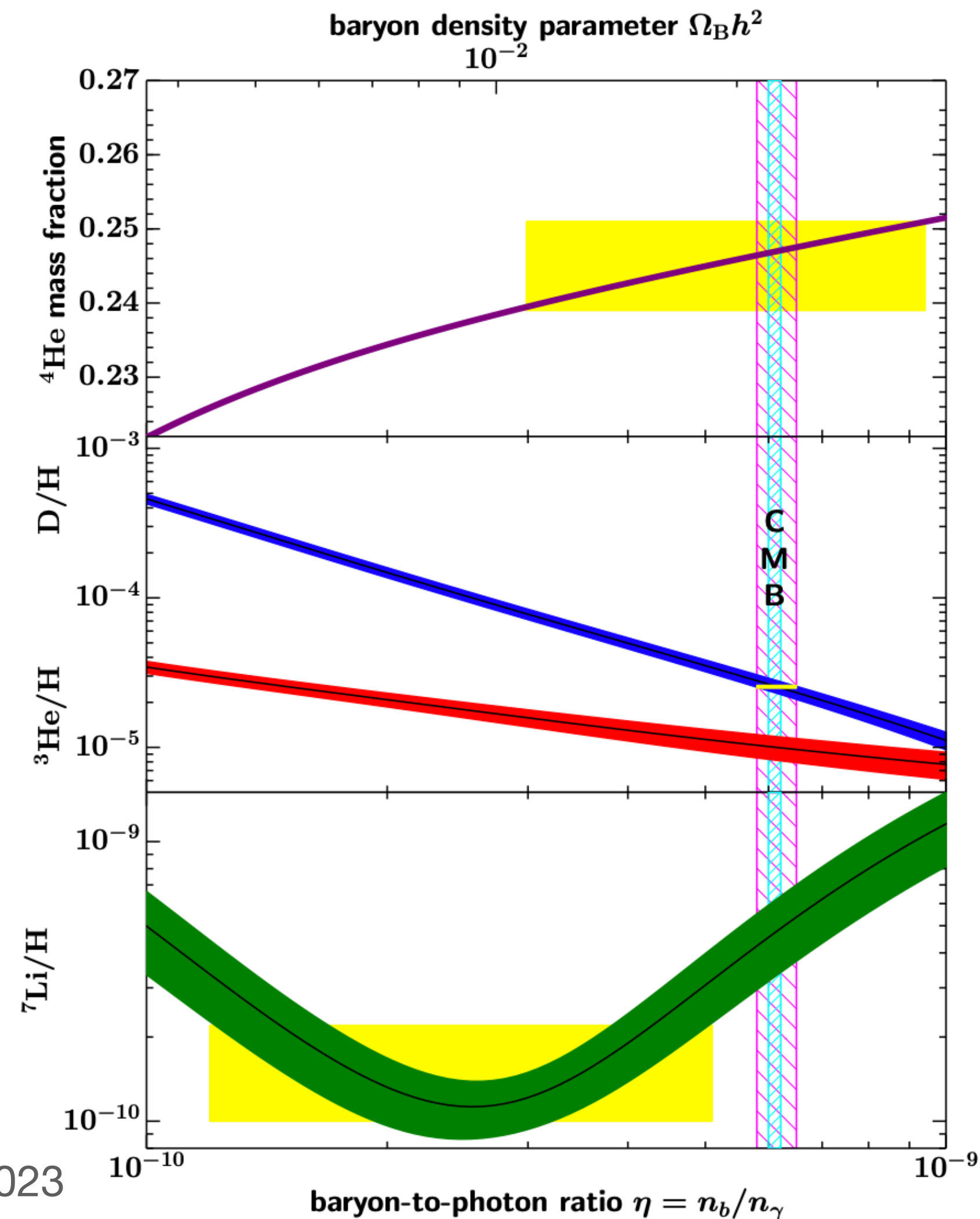
PDG 2023

Helium-4

$$Y_{\text{th}} = 0.24709 \pm 0.00025 \quad Y_{\text{exp}} = 0.245 \pm 0.003$$

$$\frac{\Delta Y_p}{Y_p} = \frac{Y_{\text{exp}} - Y_{\text{th}}}{Y_{\text{th}}} = -0.008458 \pm 0.012183$$

primordial abundances



PDG 2023

Helium-4

$$Y_{\text{th}} = 0.24709 \pm 0.00025 \quad Y_{\text{exp}} = 0.245 \pm 0.003$$

$$\frac{\Delta Y_p}{Y_p} = \frac{Y_{\text{exp}} - Y_{\text{th}}}{Y_{\text{th}}} = -0.008458 \pm 0.012183$$

$$\frac{\Delta Y_p}{Y_p} \simeq \frac{\Delta X_{n,W}}{X_{n,W}} - \Delta \left(\int_{a_W}^{a_{\text{BBN}}} \frac{da}{aH(a)} \Gamma_n(a) \right)$$

neutron abundance

neutron decay

neutron abundance at BBN

$$X_{n,BBN} = X_{n,W} \exp\left(-\int_{a_W}^{a_{BBN}} \frac{da}{aH} \Gamma_n\right)$$

$$\begin{aligned} \frac{\Delta X_{n,BBN}}{X_{n,BBN}} &= \frac{\Delta X_{n,W}}{X_{n,W}} - \left(\int_{a_W}^{a_{BBN}} \frac{da}{aH} \Delta\Gamma_n\right) - \frac{\Gamma_n}{H} \frac{\Delta a}{a} \Bigg|_{a_W}^{a_{BBN}} \\ &\approx \frac{\Delta X_{n,W}}{X_{n,W}} - \left(\int_{a_W}^{a_{BBN}} \frac{da}{aH} \Delta\Gamma_n\right) + \frac{\Gamma_n}{H} \Bigg|_{a_{BBN}} \frac{\Delta T_{BBN}}{T_{BBN}} \end{aligned}$$

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neutron abundance at BBN

$$X_{n,BBN} = X_{n,W} \exp\left(-\int_{a_W}^{a_{BBN}} \frac{da}{aH} \Gamma_n\right)$$

$$\frac{\Delta X_{n,BBN}}{X_{n,BBN}} = \frac{\Delta X_{n,W}}{X_{n,W}} - \left(\int_{a_W}^{a_{BBN}} \frac{da}{aH} \Delta\Gamma_n\right) - \frac{\Gamma_n}{H} \frac{\Delta a}{a} \Bigg|_{a_W}^{a_{BBN}}$$

$$\approx \boxed{\frac{\Delta X_{n,W}}{X_{n,W}}} - \left(\int_{a_W}^{a_{BBN}} \frac{da}{aH} \boxed{\Delta\Gamma_n}\right) + \frac{\Gamma_n}{H} \Bigg|_{a_{BBN}} \boxed{\frac{\Delta T_{BBN}}{T_{BBN}}}$$

Need to know these

$$\frac{\Delta T_{BBN}}{T_{BBN}} \approx \boxed{\frac{\Delta B_D}{B_D}}$$

neutron abundance at freeze-out

Instantaneous approximation: $X_{n,W} \simeq e^{-m_{np}/T_W}$
approximate weak freeze-out temperature

$$\frac{\Delta X_{n,W}}{X_{n,W}} \simeq -\frac{m_{np}}{T_W} \left(\frac{\Delta m_{np}}{m_{np}} - \frac{\Delta T_W}{T_W} \right)$$


however, we need to go beyond instantaneous approximation...

neutron abundance at freeze-out

neutron abundance $\frac{n_n}{n_b} = \frac{1}{1 + e^{m_{np}/T}} \equiv X_n^{\text{eq}}$

kinetic equation $\frac{dX_n}{dt} = -\lambda_{n \rightarrow p} (1 + e^{-m_{np}/T}) (X_n - X_n^{\text{eq}})$

neutron-proton conversion $\lambda_{n \rightarrow p} = \frac{1 + 3g_A^2}{\pi^3} G_F^2 T^5 J(m_{np}/T)$

phase-space integral 

Neutron abundance at Weak freeze-out

$$X_{n,W} = - \int_0^\infty da \frac{dX_n^{\text{eq}}}{da} \exp \left[- \int_a^\infty \frac{da_1}{a_1} \frac{\lambda_{n \rightarrow p}}{H(a_1)} \left(1 + e^{-\frac{m_{np}}{T}} \right) \right]$$

neutron abundance at freeze-out

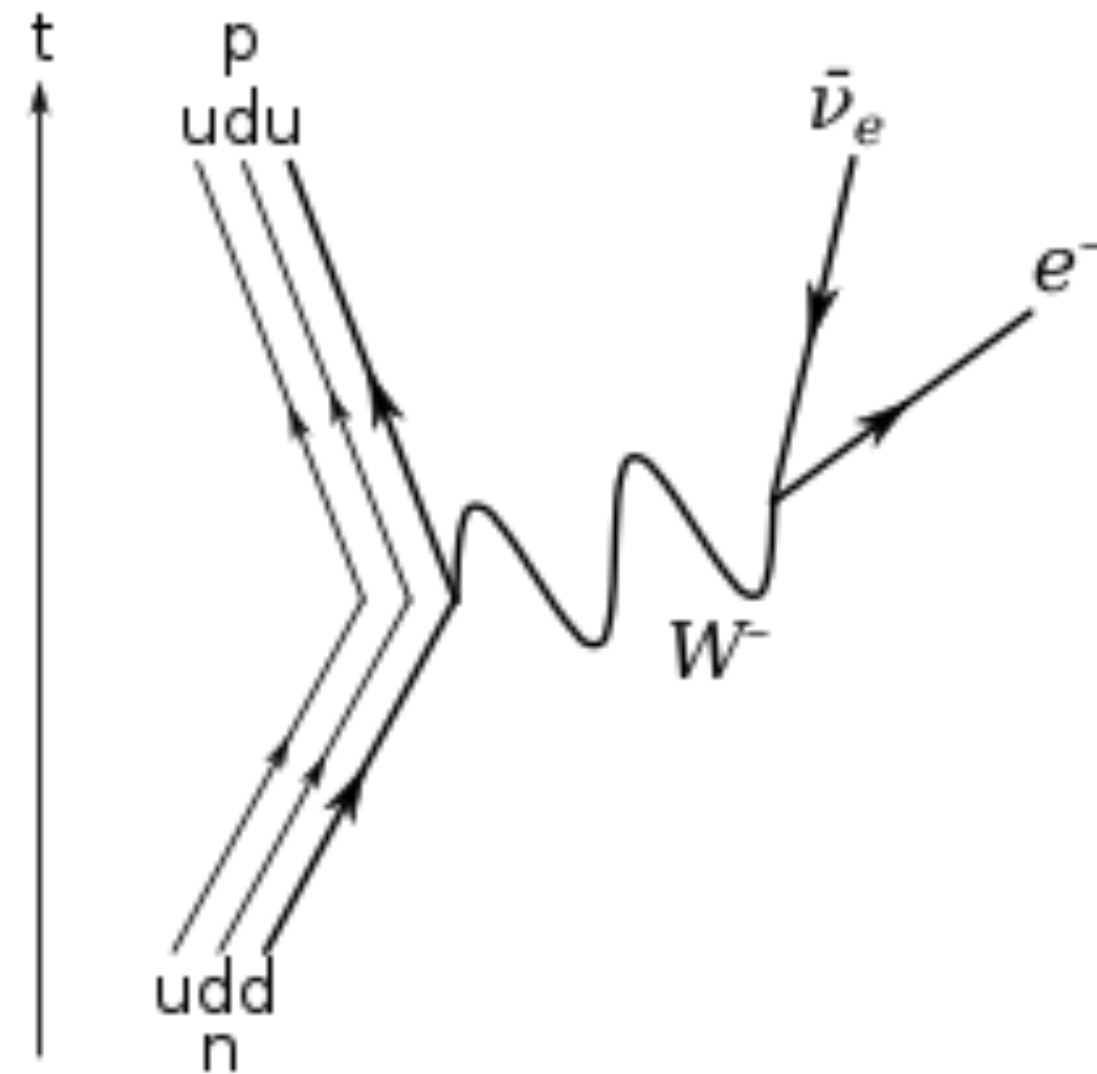
need to go beyond instantaneous approximation...

$$\frac{\Delta X_{n,W}}{X_{n,W}} \simeq -\frac{m_{np}}{T_W} \left(\frac{\Delta m_{np}}{m_{np}} - \frac{\Delta T_W}{T_W} \right)$$

instead,

$$\Delta X_{n,W} = \int_0^\infty \frac{da}{a} \frac{m_{np}}{2T(1 + \cosh(m_{np}/T))} \exp \left[- \int_a^\infty \frac{da'}{a'} \tilde{\lambda}_{n \rightarrow p} \right] \times \left\{ - \tilde{\lambda}_{n \rightarrow p} \frac{\Delta m_{np}}{m_{np}} + \int_a^\infty \frac{da'}{a'} \tilde{\lambda}_{n \rightarrow p} \left[\frac{m_{np} X_n^{eq}}{T} \frac{\Delta m_{np}}{m_{np}} - \frac{6g_{A_n}^2}{1 + 3g_{A_n}^2} \frac{\Delta g_{A_n}}{g_{A_n}} - 2 \frac{\Delta G_F}{G_F} - \frac{m_{np} J'}{TJ} \frac{\Delta m_{np}}{m_{np}} + \frac{3\zeta(3)}{2J} \frac{m_e^2}{T^2} \left(\frac{\Delta m_e}{m_e} - \frac{\Delta m_{np}}{m_{np}} \right) \right] \right\}.$$

neutron decay



$$\Gamma_n = \frac{1 + 3g_{A_n}^2}{2\pi^3} G_F^2 m_e^5 P \left(\frac{m_{np}}{m_e} \right)$$

phase space factor

$$\frac{\Delta\Gamma_n}{\Gamma_n} = \frac{6g_{A_n}^2}{1 + 3g_{A_n}^2} \frac{\Delta g_{A_n}}{g_{A_n}} + 2 \frac{\Delta G_F}{G_F} + 5 \frac{\Delta m_e}{m_e} + \frac{m_{np} P'}{m_e P} \left(\frac{\Delta m_{np}}{m_{np}} - \frac{\Delta m_e}{m_e} \right)$$

4He analytic constraint

$$\begin{aligned}
 \frac{\Delta Y_p}{Y_p} = & \overset{\substack{\text{neutron abundance@BBN} \\ \swarrow}}{\frac{\Delta X_{n, BBN}}{X_{n, BBN}}} + \overset{\substack{\text{deuterium binding energy} \\ \swarrow}}{\frac{\Gamma_n \Delta B_D}{H B_D}} \Bigg|_{a_{BBN}} - \int_{a_W}^{a_{BBN}} \frac{da}{a} \frac{\Gamma_n}{H} \left(\frac{6g_{A_n}^2}{1 + 3g_{A_n}} \overset{\substack{\text{axial coupling} \\ \swarrow}}{\frac{\Delta g_{A_n}}{g_{A_n}}} \right) \\
 & + 2 \overset{\substack{\text{Fermi constant} \\ \swarrow}}{\frac{\Delta G_F}{G_F}} + 5 \overset{\substack{\text{electron mass} \\ \swarrow}}{\frac{\Delta m_e}{m_e}} + \overset{\substack{\text{neutron-proton mass difference} \\ \swarrow}}{\frac{m_{np} P'}{m_e P}} \left(\frac{\Delta m_{np}}{m_{np}} - \frac{\Delta m_e}{m_e} \right)
 \end{aligned}$$

$$\frac{\Delta Y_p}{Y_p} = \frac{Y_p^{\text{exp}} - Y_p^{\text{th}}}{Y_p^{\text{th}}} = -0.008 \pm 0.012$$

Scalar with universal coupling

Scalar field

$$\mathcal{L} \supset \frac{1}{2} (\partial_\mu \phi)^2 - \frac{1}{2} m_\phi^2 \phi^2$$

universal coupling

$$g_{\mu\nu} \rightarrow g_{\mu\nu} (1 + 2\kappa)$$

additional contributions to SM particle masses!

$$\mathcal{L}_{\text{SM}} \rightarrow \mathcal{L}_{\text{SM}} \pm \frac{\phi^2}{\Lambda^2} m_f \bar{f} f \pm \frac{\phi^2}{\Lambda^2} m_V^2 V^2$$

$$\frac{\phi^2}{\Lambda^2}$$

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$$\frac{\phi^2}{\Lambda^2}$$

know: scalar is all of DM now $\phi_0 = \frac{\sqrt{2\rho_{\text{DM}}}}{m_\phi}$

need to know: value of scalar field at BBN ϕ_{BBN}

scalar evolution

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

$$V(\phi) = \frac{1}{2}m_{\phi}^2\phi^2 \quad \longrightarrow \quad V(\phi) = \frac{1}{2}m_{\text{eff}}^2\phi^2$$

$$\ddot{\phi} + 3H\dot{\phi} + m_{\text{eff}}^2\phi = 0$$

scalar evolution

$$\ddot{\phi} + \boxed{3H\dot{\phi}} + \left(\boxed{m_{\phi}^2} + \boxed{m_{\text{induced}}^2} \right) \phi = 0$$

$$\begin{array}{ccc} \rho_{\text{DM}} \propto \text{constant} & \rho_{\text{DM}} \propto a^{-3}(t) & ??? \\ H & B & \end{array}$$

effective mass

$$m_{f,V}^2 \rightarrow m_{f,V}^2 \left(1 \pm \frac{\phi^2}{\Lambda^2} \right)$$

$$m_{\phi}^2 \rightarrow m_{\phi,\text{eff}}^2 \equiv m_{\phi}^2 \pm \frac{2}{\Lambda^2} \Theta_{\text{SM}}$$

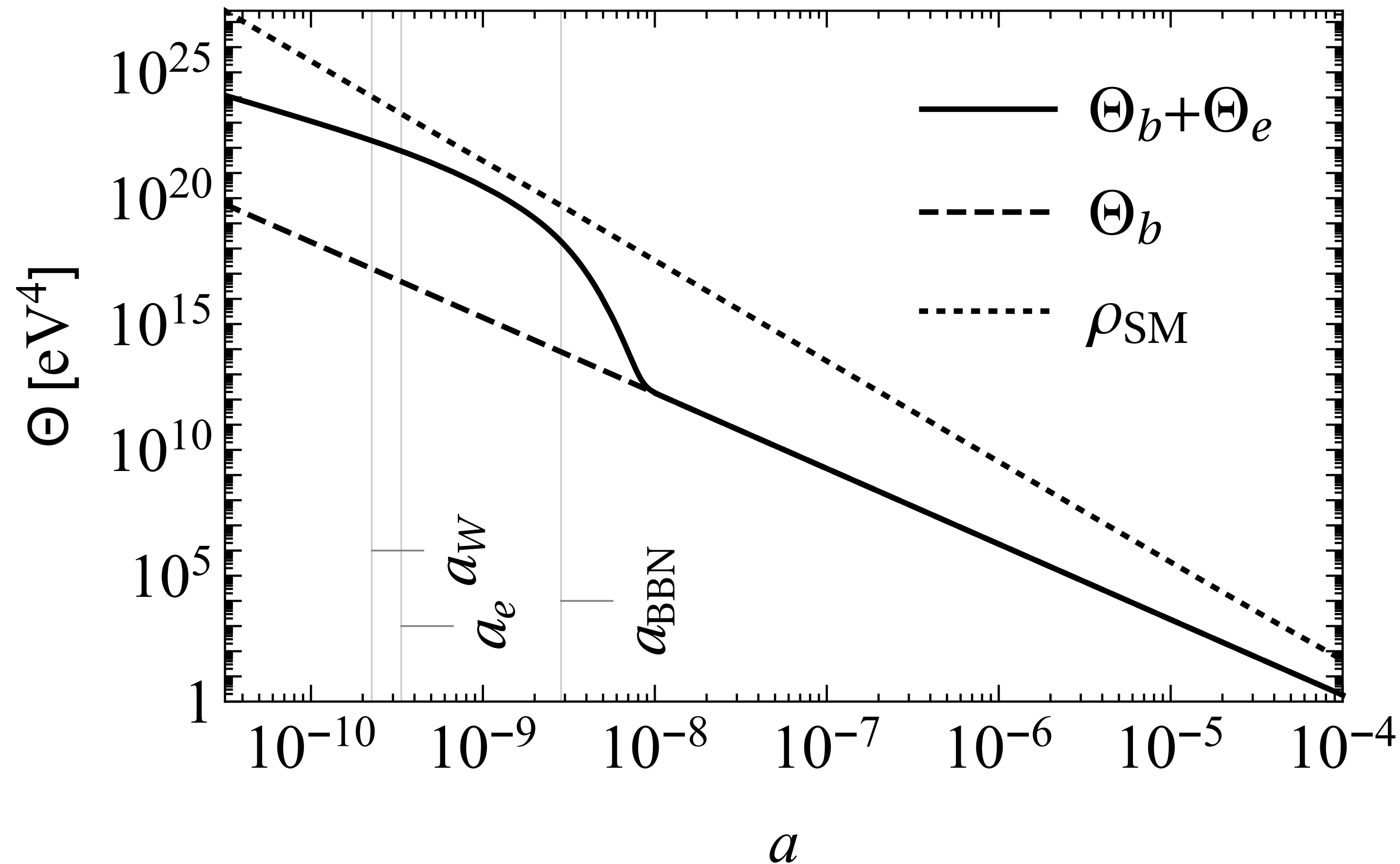


$$\Theta_{\text{SM}} = \rho_{\text{SM}} - 3p_{\text{SM}}$$

trace of SM energy-momentum tensor

SM Energy-Momentum Tensor

S. Sibiryakov, P. Sørensen, TTY *JHEP* 20 (2020) 075 [arXiv: 2006.04820]



scalar evolution

$$\ddot{\phi} + \boxed{3H\dot{\phi}} + \left(\boxed{m_{\phi}^2} + \boxed{m_{\text{induced}}^2} \right) \phi = 0$$

$$\rho_{\text{DM}} \propto \text{constant}$$

H

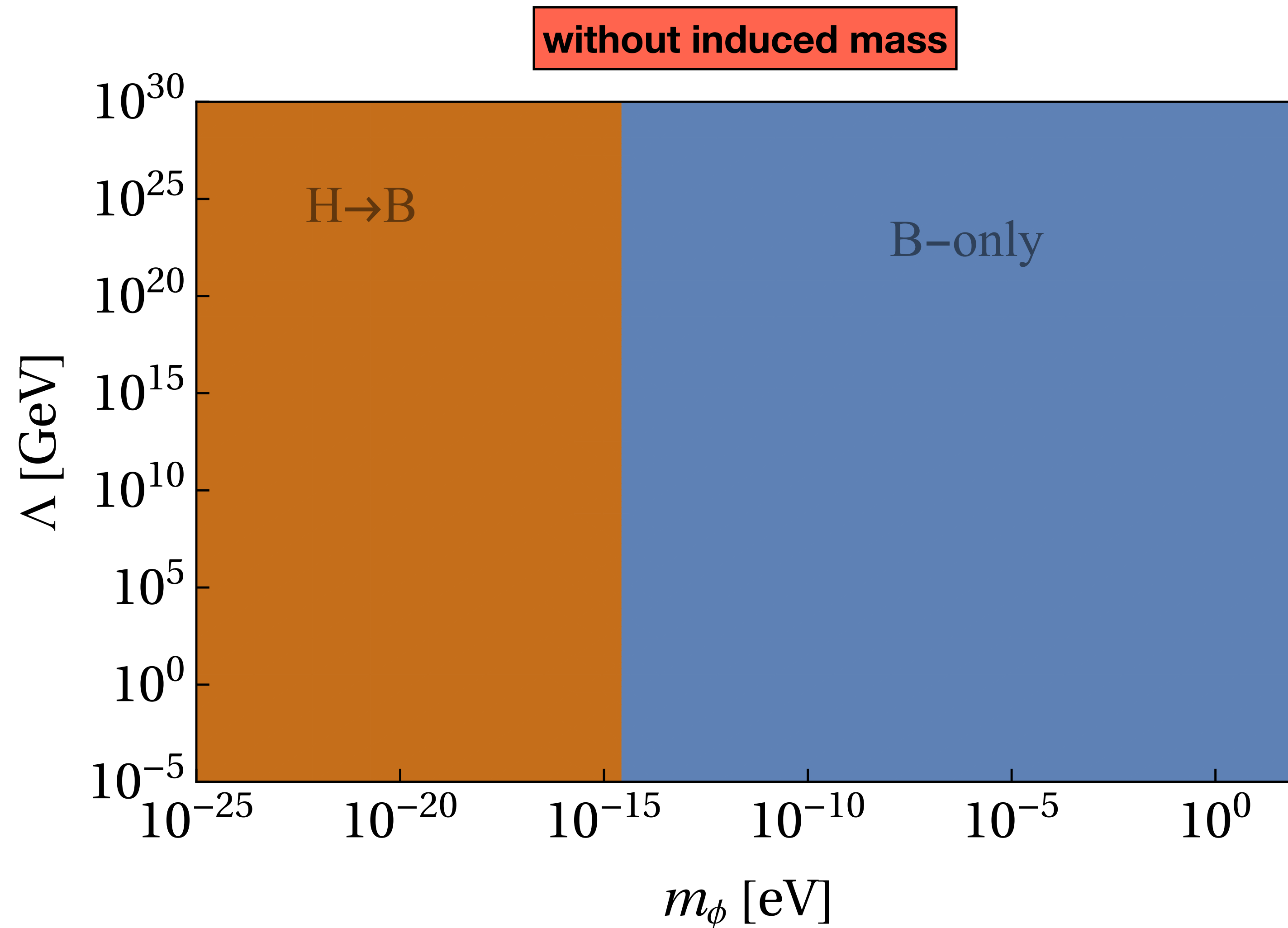
$$\rho_{\text{DM}} \propto a^{-3}(t)$$

B

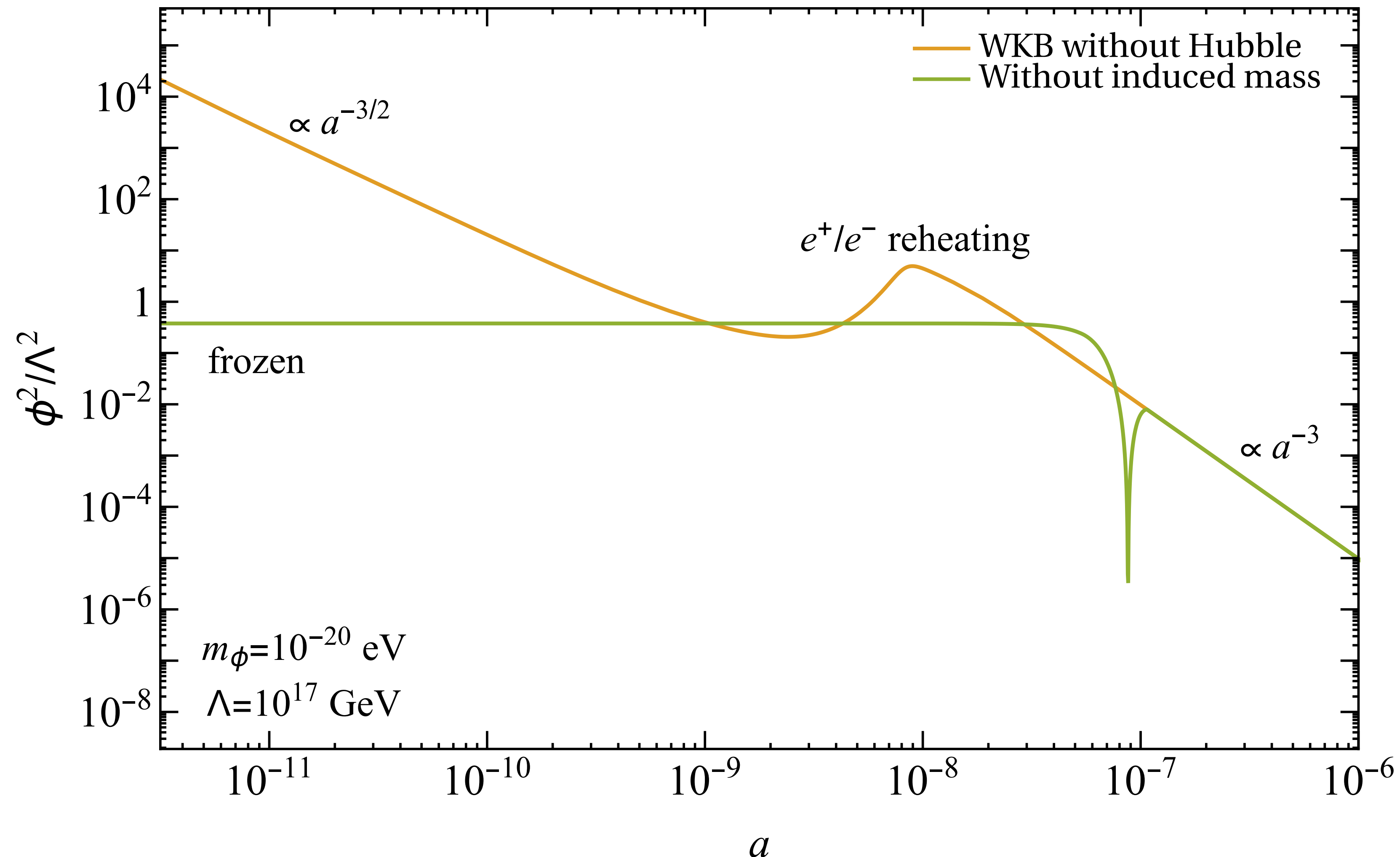
$$???$$

I

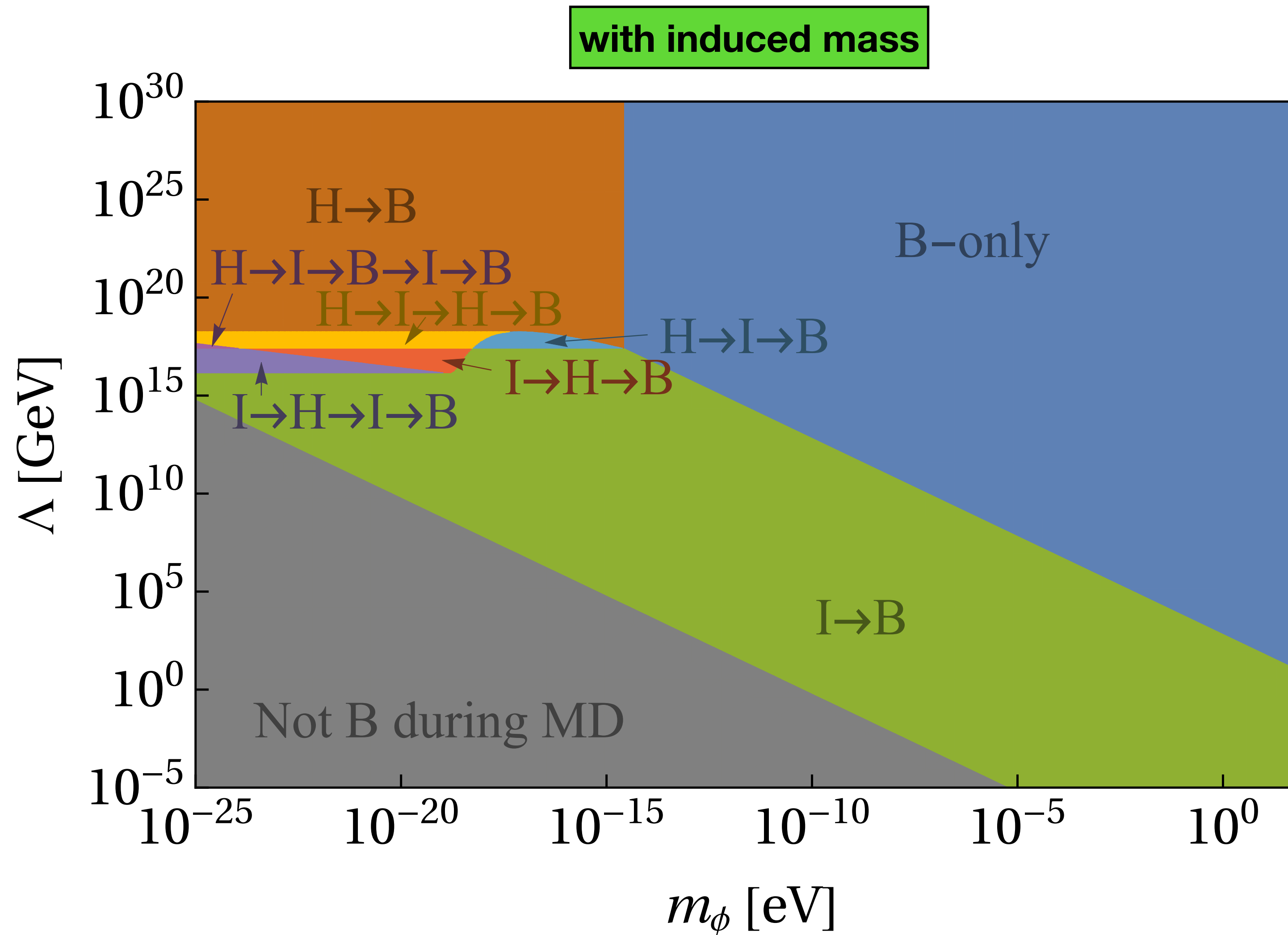
scalar evolution



scalar evolution

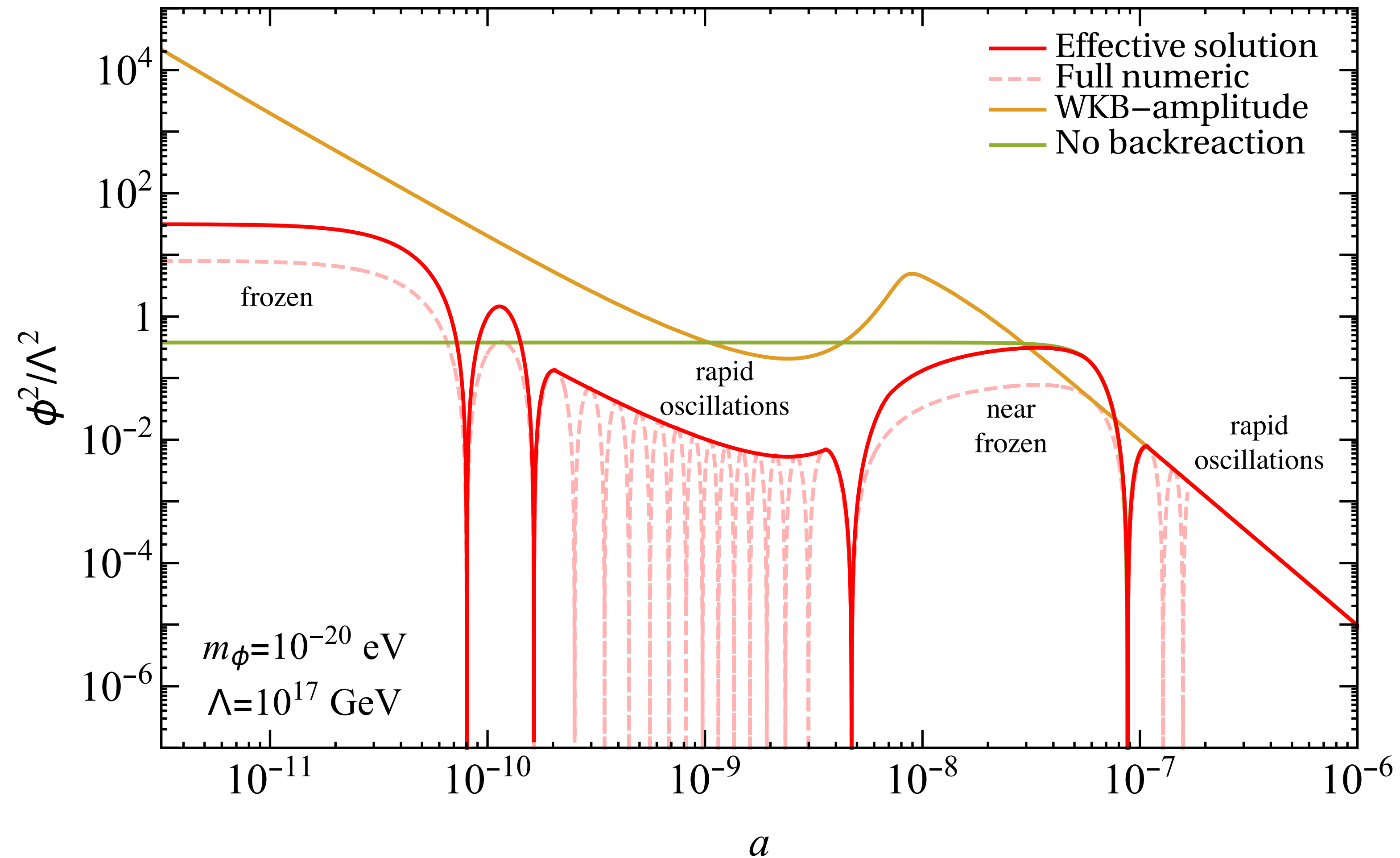


scalar evolution



scalar evolution

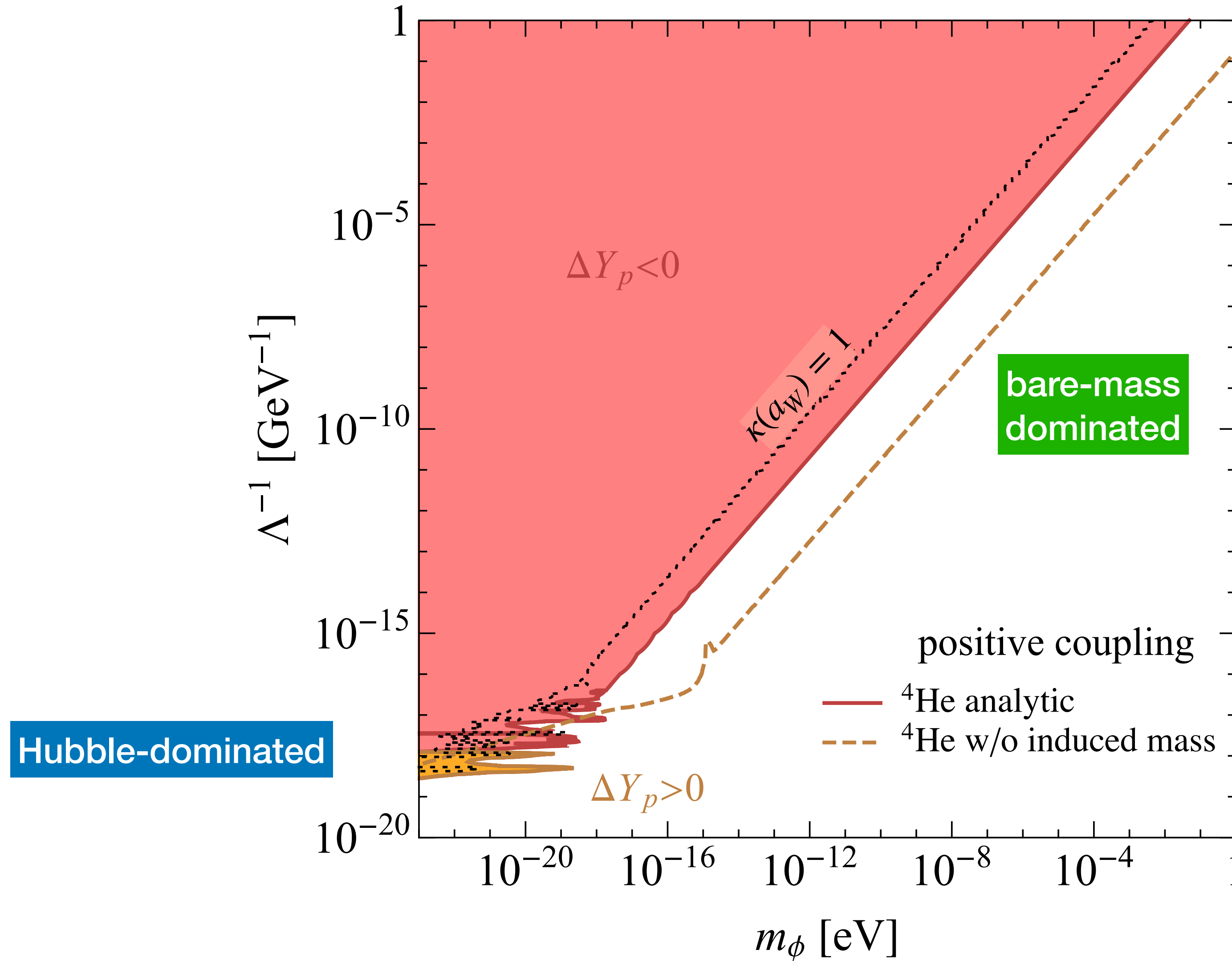
S. Sibiryakov, P. Sørensen, TTY *JHEP* 20 (2020) 075 [arXiv: 2006.04820]



4He analytic constraint

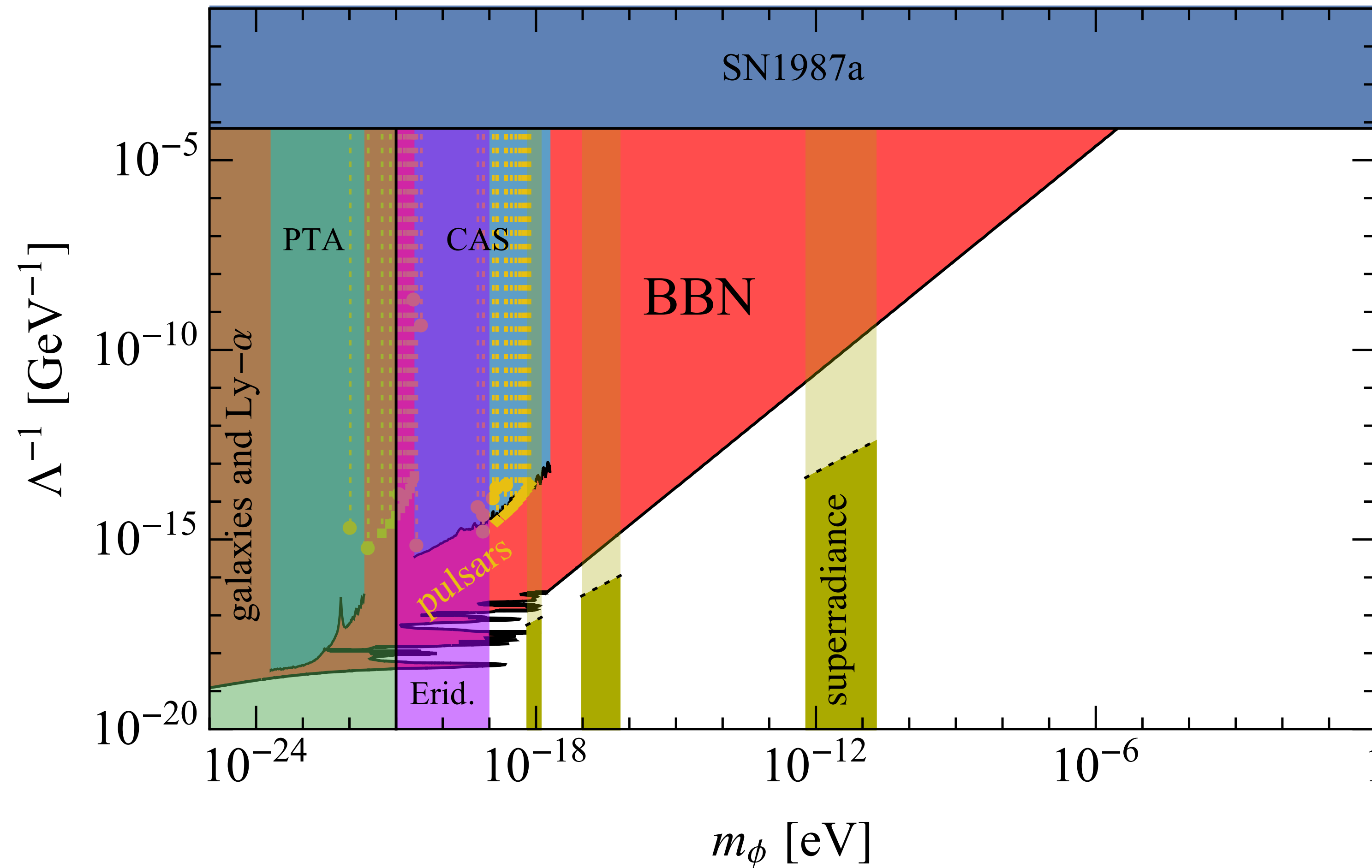
S. Sibiryakov, P. Sørensen, TTY *JHEP* 20 (2020) 075 [arXiv: 2006.04820]

see also
Stadnik, Flambaum [1503.08540]



Universally-Coupled Scalar

S. Sibiryakov, P. Sørensen, TTY *JHEP* 20 (2020) 075 [arXiv: 2006.04820]



Scalar with non-universal coupling

$$\mathcal{L} \supset 2\pi \frac{\phi^2}{M_{\text{pl}}^2} \left[\frac{d_e^{(2)}}{4e^2} F_{\mu\nu} F^{\mu\nu} - \frac{d_g^{(2)} \beta_3}{2g_3} G_{\mu\nu}^A G^{A\mu\nu} - d_{m_e}^{(2)} m_e \bar{e}e - \sum_{i=u,d} \left(d_{m_i}^{(2)} + \gamma_{m_i} d_g^{(2)} \right) m_i \bar{\psi}_i \psi_i \right]$$

$$d_{\hat{m}} \equiv \frac{d_{m_d} m_d + d_{m_u} m_u}{m_d + m_u}$$

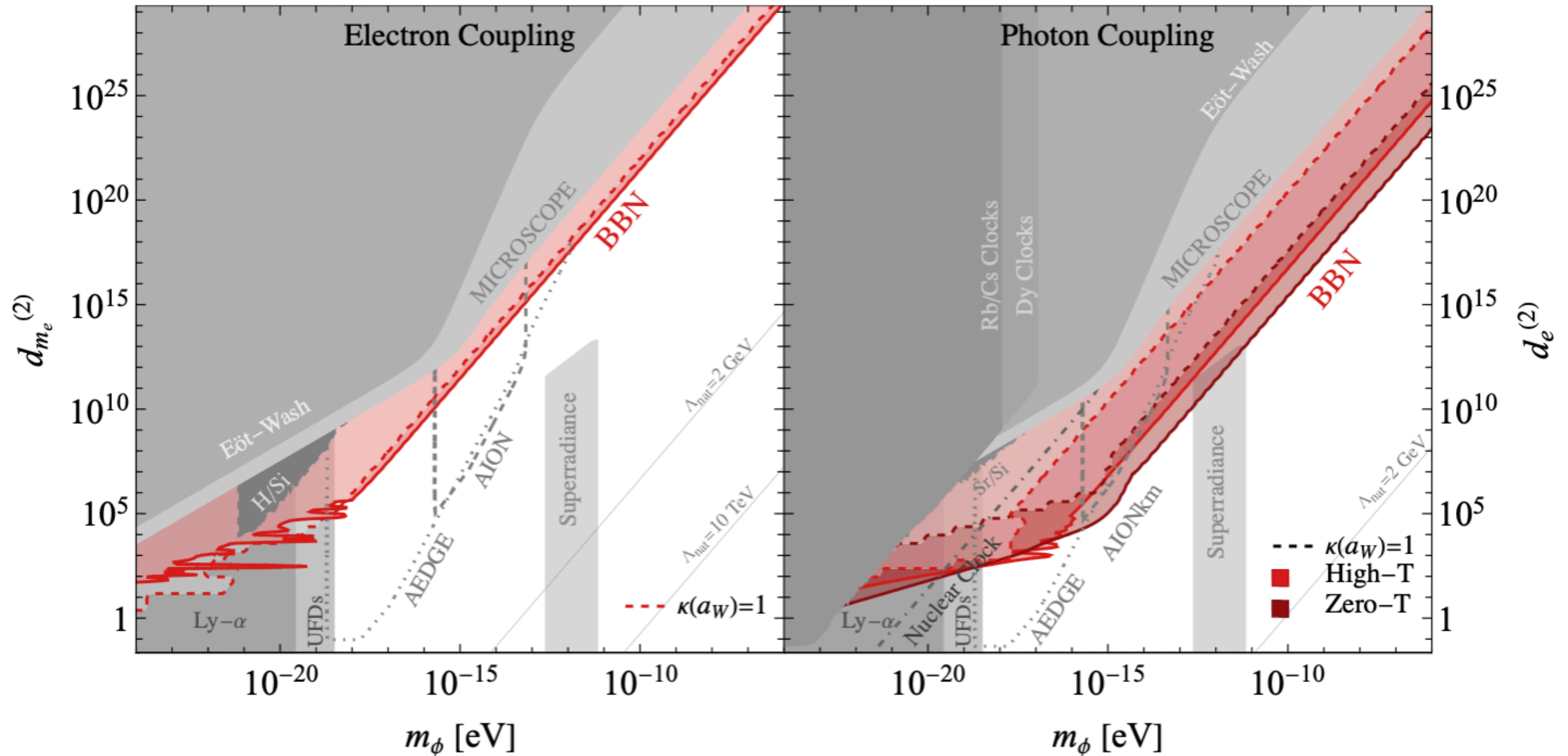
$$d_{\delta m} \equiv \frac{d_{m_d} m_d - d_{m_u} m_u}{m_d - m_u}$$

symmetric

anti-symmetric

Non-Universally coupled scalar(s)

T. Bouley, P. Sørensen, TTY *JHEP* 03 (2023) 104 [arXiv: 2211.09826]



Conclusions

- BBN is well-explained by SM physics and therefore a powerful test of BSM physics
- ultralight scalar dark matter will modify the predictions of BBN
- the induced mass leads to non-trivial DM evolution
- instantaneous neutron freeze-out approximation is insufficient and leads to qualitatively different conclusions
- a large portion of ultralight scalar DM parameter space will have noticeable effects on BBN

cảm ơn!