Gauged Global Strings

Wei Xue

BSM in Particle Physics and Cosmology

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UF UNIVERSITY of FLORIDA

with Xuce Niu and Fengwei Yang

Outline

- Introduction
  global strings and gauge strings

- Gauge $U(1)_Z \times$ global $U(1)_{PQ}$
  and string solutions

- Cosmological implication
  1) rich string structure
  2) opening up QCD axion window
  3) gauge string radiating axions?

- Conclusion

Global strings

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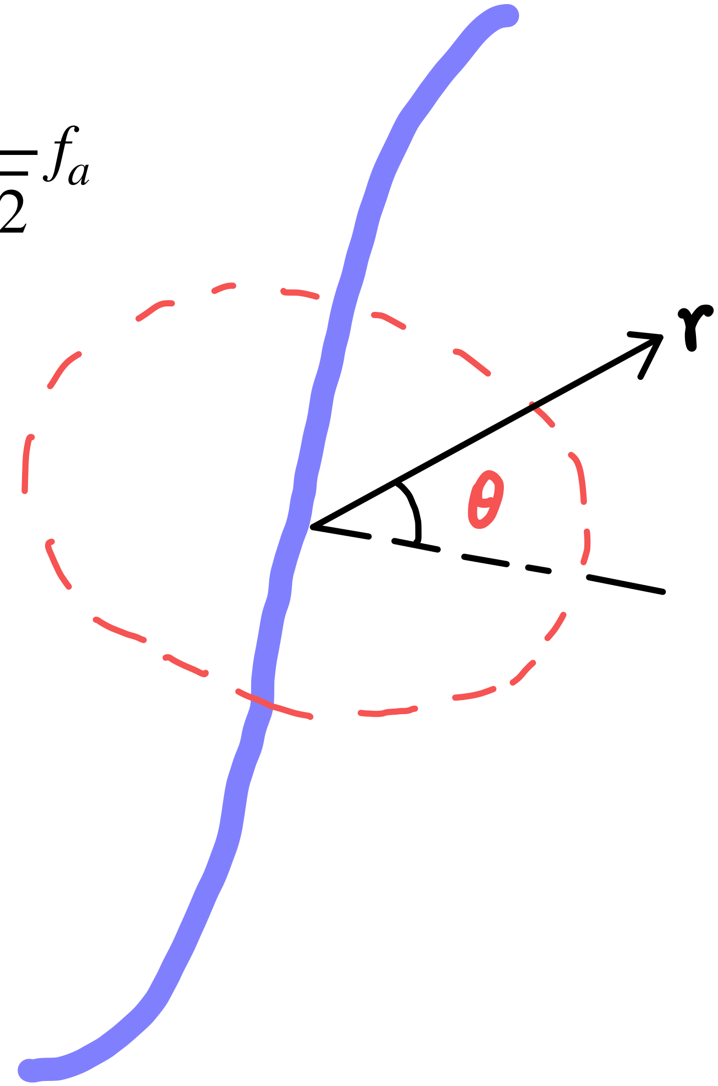

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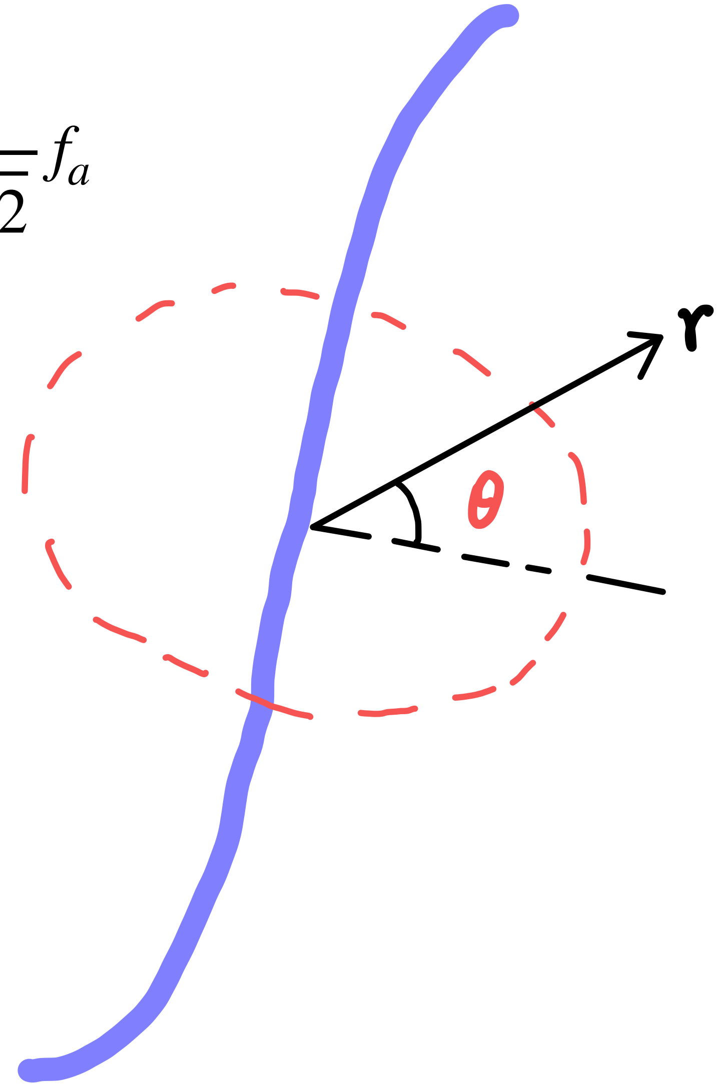
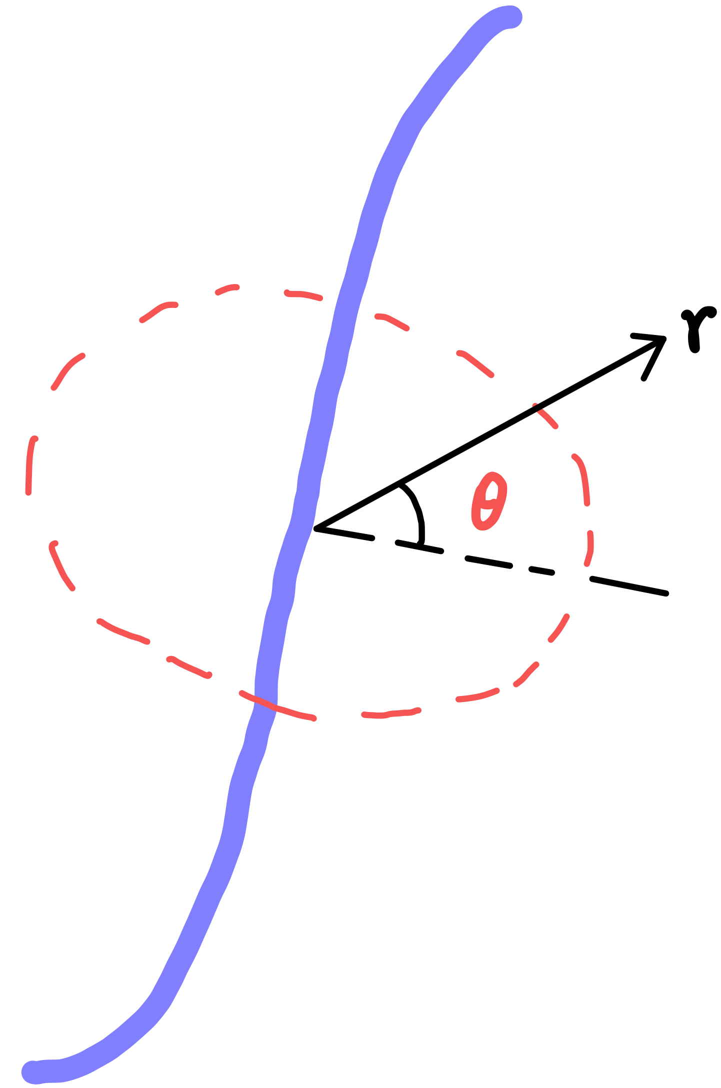
- tension

gradient term $\mu \simeq 2\pi \int_{m^{-1}}^{L} \mathrm{d}r \frac{1}{r} |\partial_\theta \Phi(r, \theta)|^2 = \pi f_a^2 \ln(mL)$



Gauge strings



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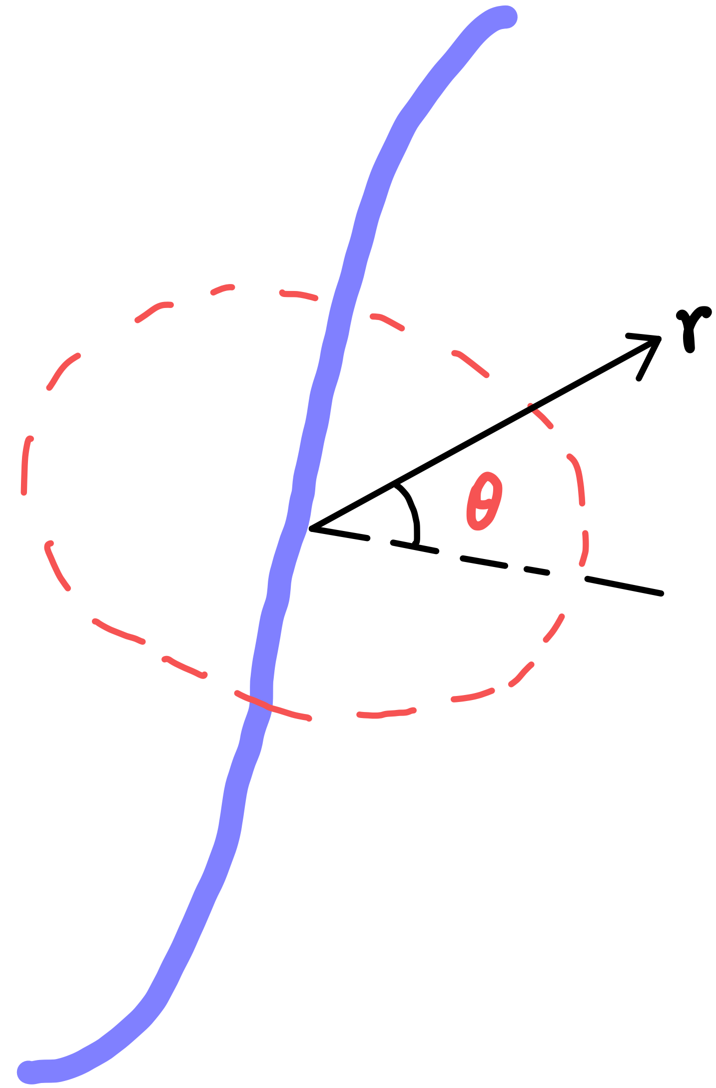
$$\Phi(r,\theta) = \frac{1}{\sqrt{2}} f_a \, \mathrm{e}^{i\theta}$$

$$Z_\mu = \frac{1}{e}\partial_\mu \theta \qquad\qquad r \to \infty$$



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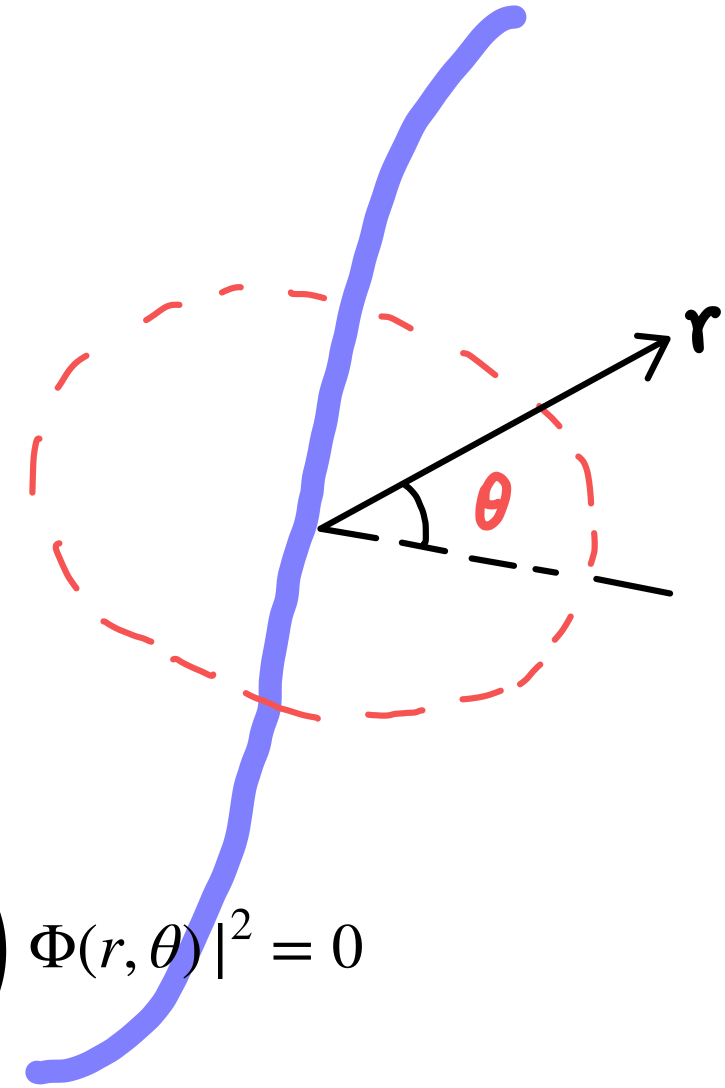

- tension

gradient term $\mu \simeq 2\pi \int_{m^{-1}}^{L} \mathrm{d}r \left| \left(\frac{1}{r}\partial_\theta - ieZ_\mu \right) \Phi(r,\theta) \right|^2 = 0$

core $\mu \simeq \mathcal{O}(1)\pi f_a^2$

Motivation of cosmic strings

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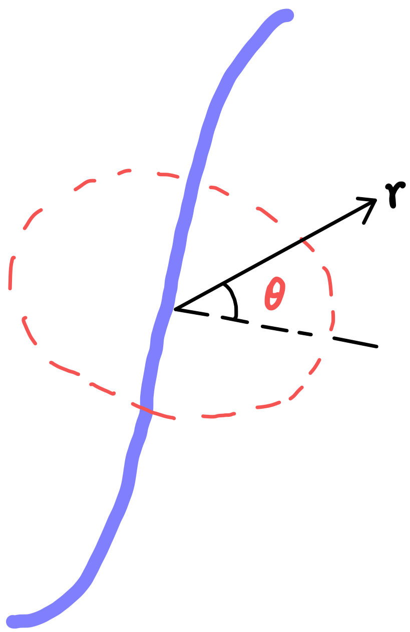

- theoretically interesting
  classical field solutions

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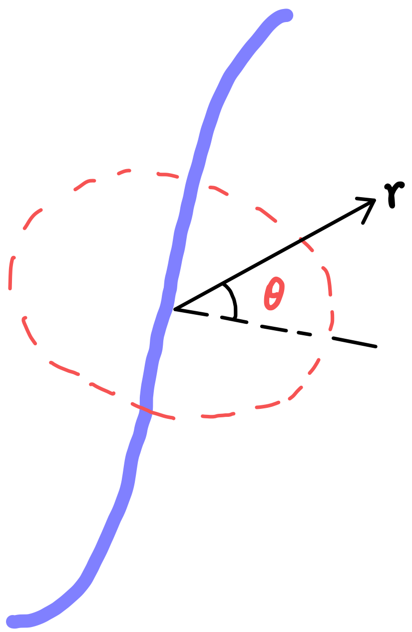

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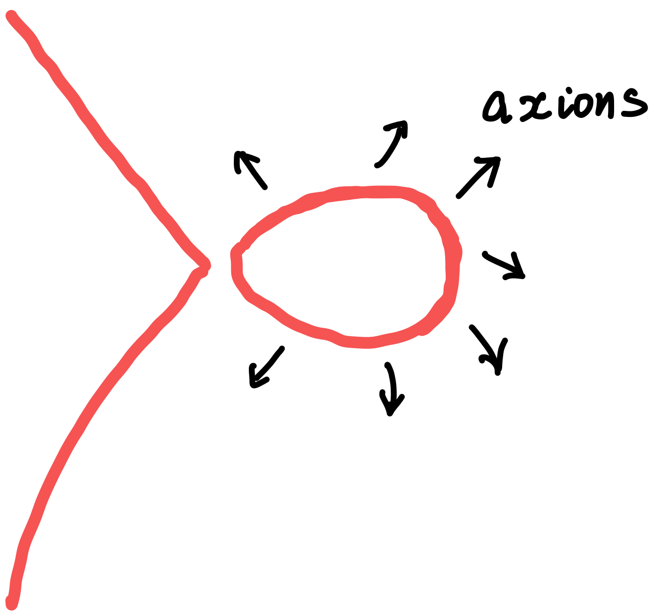

- phenomenological rich
  cosmology (Kibble mechanism)
  axion dark matter abundance
  new observables (CMB, …)

$\mathrm{U(1)_Z \times U(1)_{PQ}}$

$U(1)_Z \times U(1)_{PQ}$

- Lagrangian

$$\mathscr{L} = -\frac{1}{4}Z_{\mu\nu}Z^{\mu\nu} + D_\mu\Phi_1^\dagger D^\mu\Phi_1 - \frac{\lambda_1}{4}\left(|\Phi_1|^2 - \frac{v_1^2}{2}\right)^2 + D_\mu\Phi_2^\dagger D^\mu\Phi_2 - \frac{\lambda_2}{4}\left(|\Phi_2|^2 - \frac{v_2^2}{2}\right)^2$$

$D_\mu = \partial_\mu - ieZ_\mu$

assume that $v_1 > v_2$

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- $\Phi_1 \to \Phi_1 e^{i\alpha_Z + i\alpha_{PQ}}$

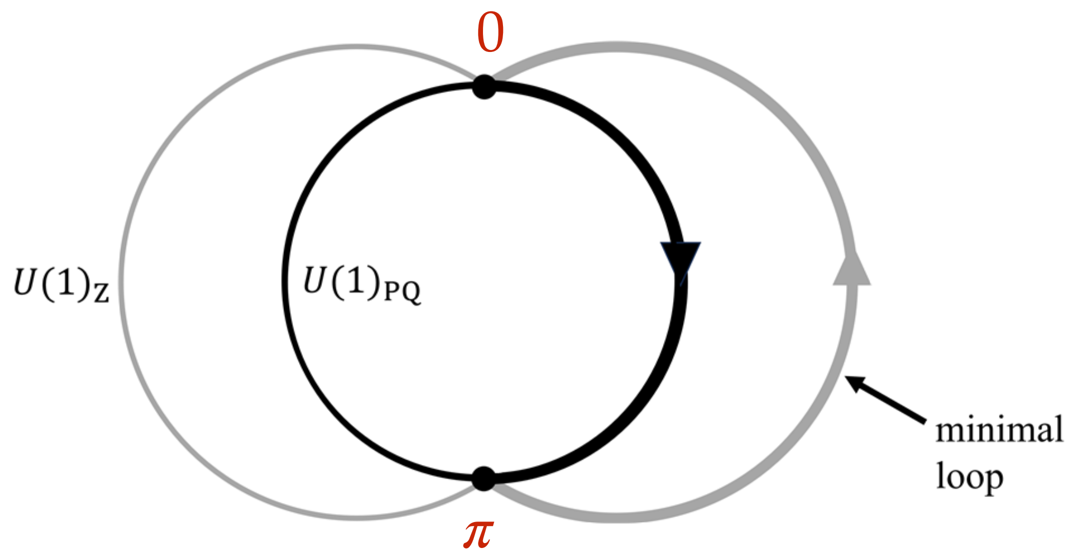
  $\Phi_2 \to \Phi_2 e^{i\alpha_Z - i\alpha_{PQ}}$

	Φ_1	Φ_2
$\mathrm{U(1)_Z}$	1	1
$\mathrm{U(1)_{PQ}}$	1	-1

Vacuum and fluctuations

Vacuum and fluctuations

- cross section of vacuum manifold

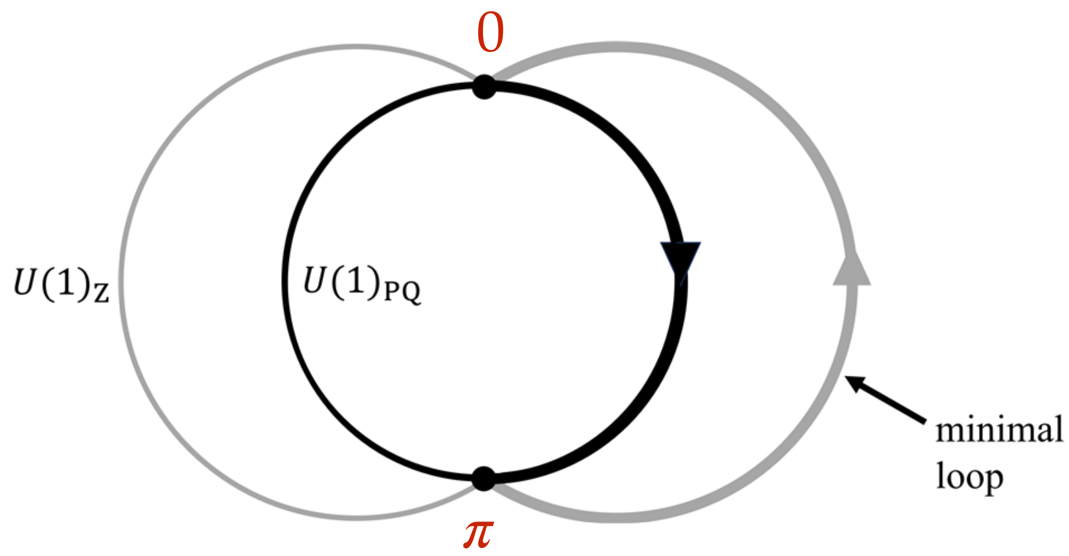


Vacuum and fluctuations

- cross section of vacuum manifold

$\alpha_Z = \pi$

$\Phi_1 \to \Phi_1 e^{i\pi}, \Phi_2 \to \Phi_2 e^{i\pi} = \Phi_2 e^{-i\pi}$



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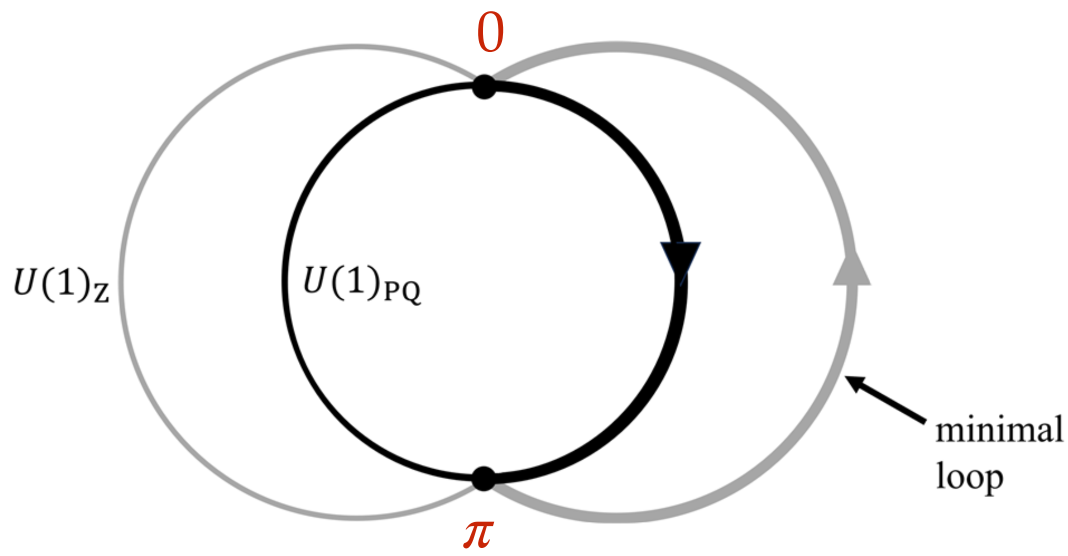

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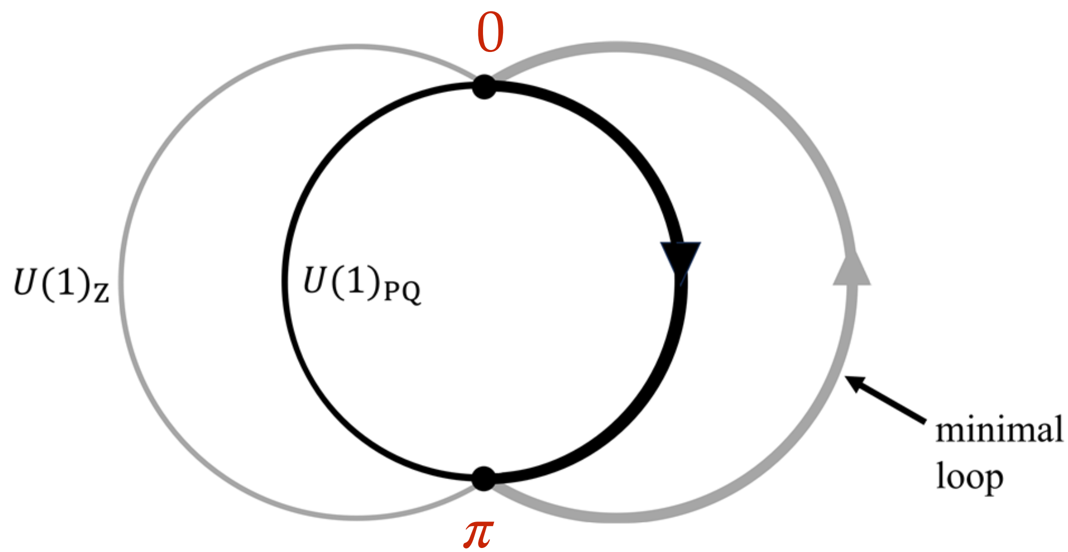

- axion direction is orthogonal to the longitudinal mode of Z^μ

$$a(x) = v_a \alpha_{\rm PQ}, \quad v_a = \frac{2v_1 v_2}{\sqrt{v_1^2 + v_2^2}} \sim 2v_2$$

Integrating with QCD axion model

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- KSVZ-like model

  introduce Q_L and Q_R with color charge and $\mathrm{U}(1)_{\mathrm{PQ}}$ charge

$$\mathscr{L} = -\frac{y}{\Lambda}\left(\Phi_1\Phi_2^*\bar{Q}_L Q_R + h.c.\right)$$

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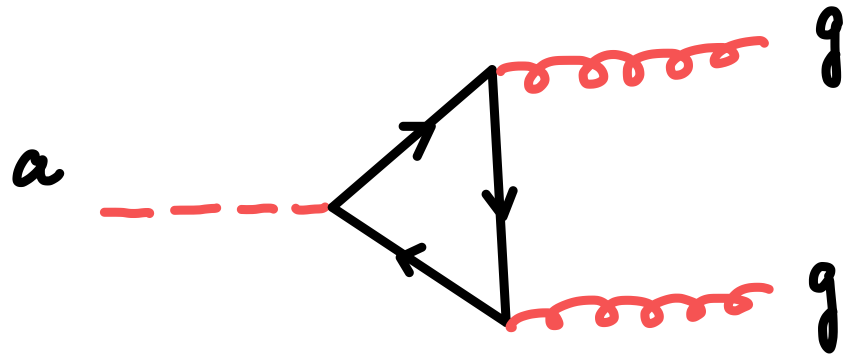

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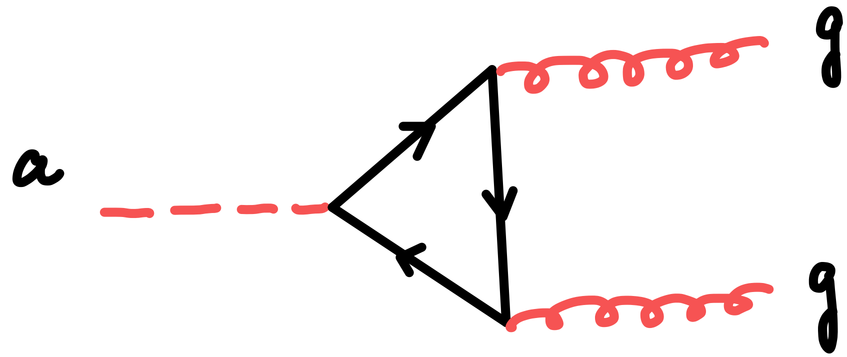
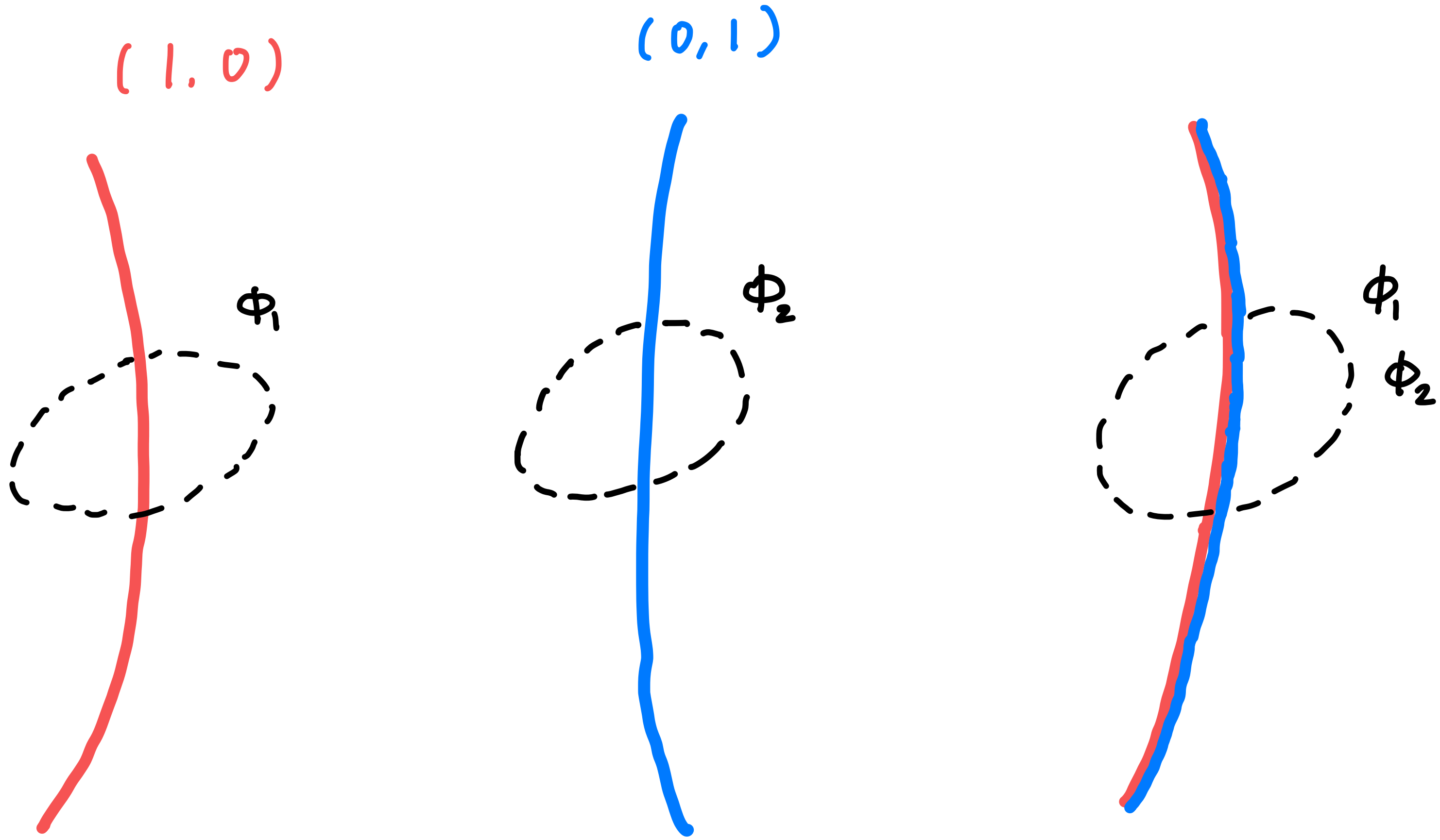


- Barr and Seckel's model
  $Q_{1L}\ Q_{1R}\ Q_{2L}\ Q_{2R}$
  color, $\mathrm{U(1)_Z}$ and $\mathrm{U(1)_{PQ}}$ charges

$$\mathscr{L} = \Phi_1 \bar{Q}_{1L} Q_{1R} + \Phi_2 \bar{Q}_{2L} Q_{2R} + h.c.$$

String Solutions



(1,0) strings

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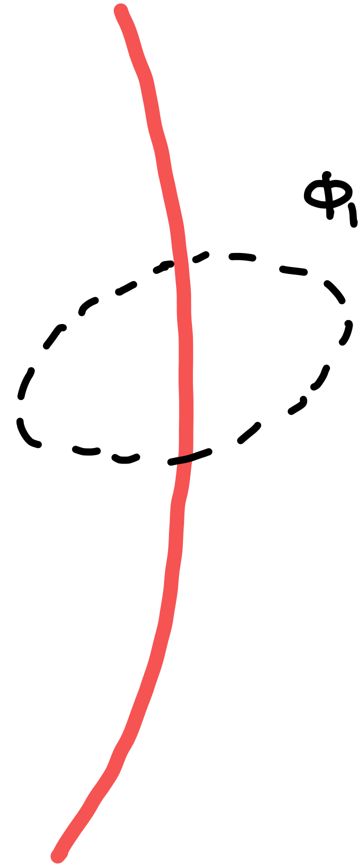
$$\Phi_1 = \frac{1}{\sqrt{2}} v_1\, e^{i\theta}, \quad \Phi_2 = \frac{1}{\sqrt{2}} v_2, \quad Z_\mu = c\,\partial_\mu \theta, \quad r \to \infty$$



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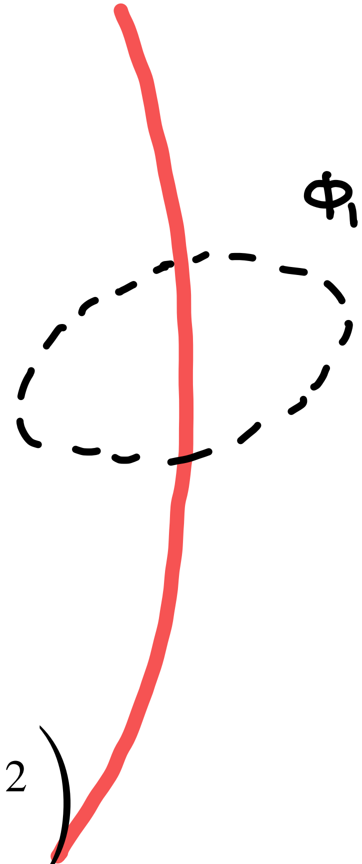

- gradient energy

$$\mu_{k,(1,0)} = \int_0^{2\pi} \mathrm{d}\theta \int_\delta^L \mathrm{d}r\, r \left(\,|\,(\frac{1}{r}\partial_\theta - ieZ_\theta)\Phi_1\,|^2 + |\,(-ieZ_\theta)\Phi_2\,|^2 \right)$$

$$= \pi \ln(\frac{L}{\delta}) \left[v_1^2 (1 - ec)^2 + v_2^2 (ec)^2 \right]$$

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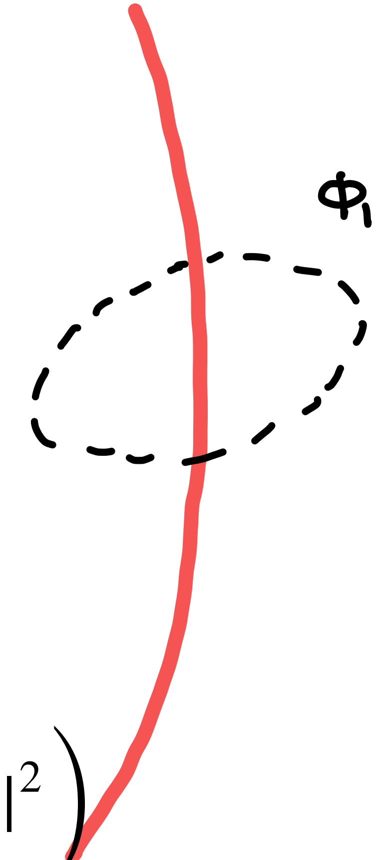

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- outside core (minimize it by varying c)

$$\mu_{k,(1,0)} = \pi \frac{v_1^2 v_2^2}{v_1^2 + v_2^2} \ln\left(\frac{L}{\delta}\right) = \pi f_a^2 \ln\left(\frac{L}{\delta}\right)$$

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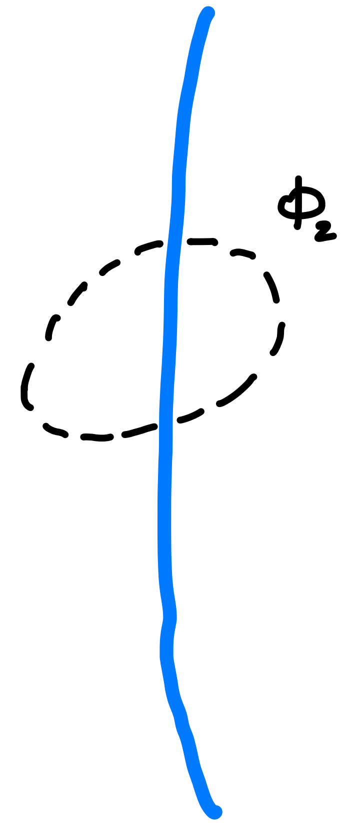

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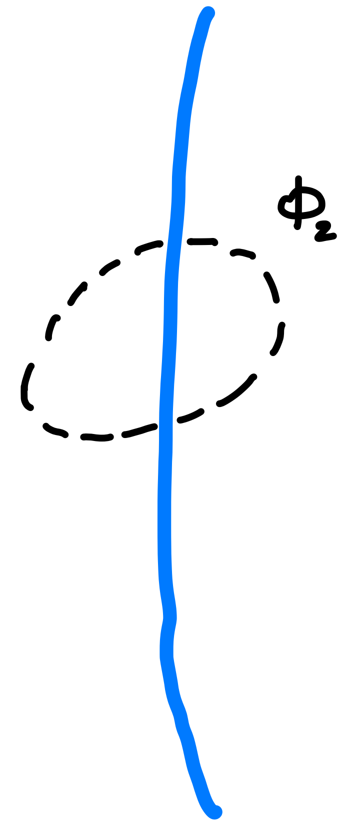
$$\mu_{k,(0,1)} = \mu_{k,(1,0)}$$



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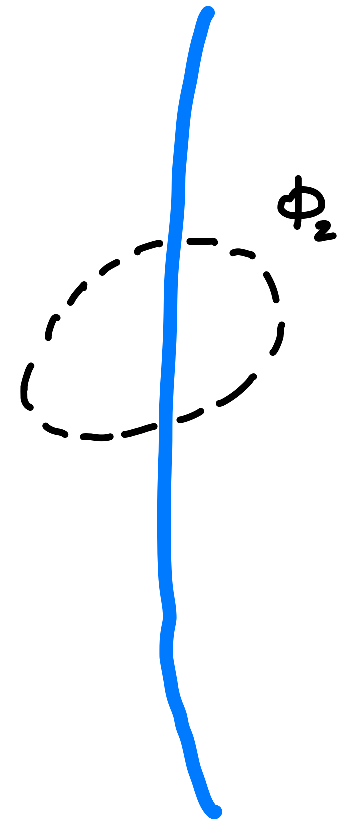

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$$\mu_{k,(0,1)} = \mu_{k,(1,0)}$$

- outside core region

  (1,0) string is equivalent to (0,-1) string through a gauge transformation

$$\left(\Phi_1 = \frac{1}{\sqrt{2}} v_1\, e^{i\theta}, \Phi_2 = \frac{1}{\sqrt{2}} v_2\right) \xrightarrow{\alpha_Z \to \alpha_Z - \theta} \left(\Phi_1 = \frac{1}{\sqrt{2}} v_1\,, \Phi_2 = \frac{1}{\sqrt{2}} v_2 e^{-i\theta}\right)$$

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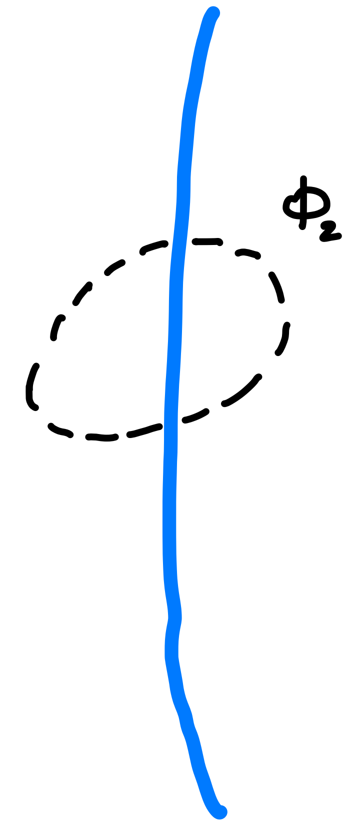

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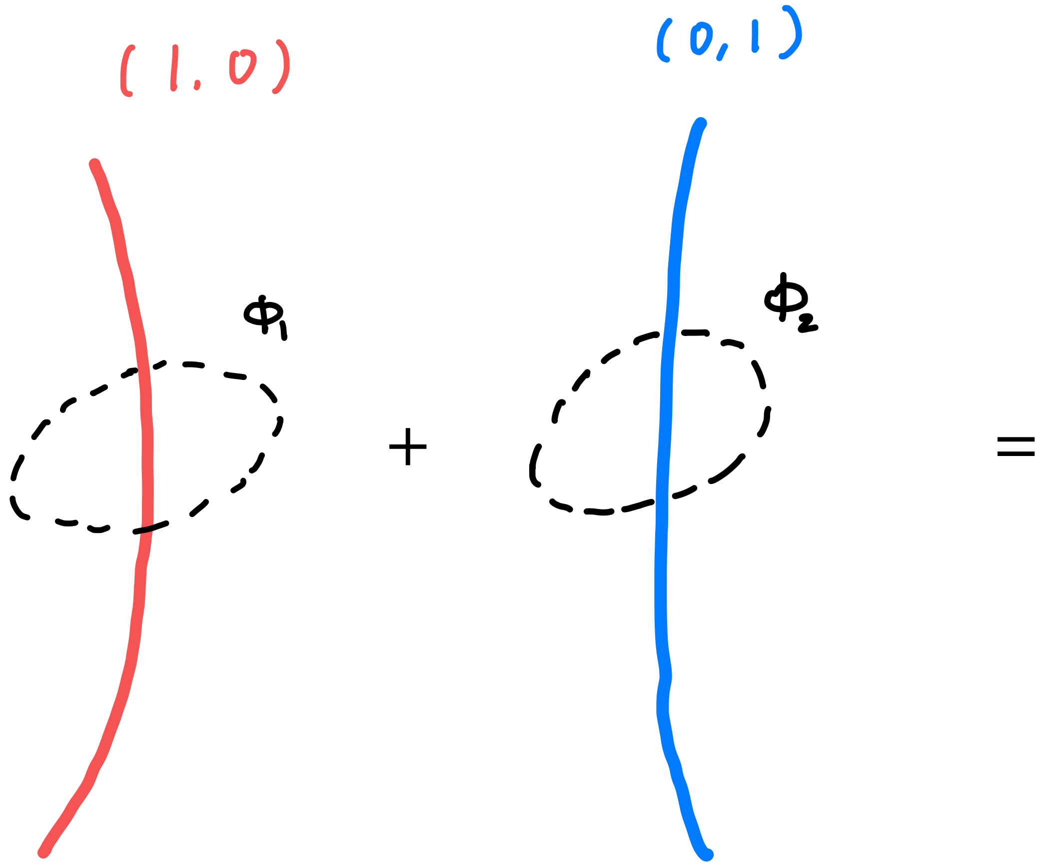
- outside core region

  (1,0) can be viewed as an anti-string of (0,1)

$(1,0) + (0,1) \to ?$



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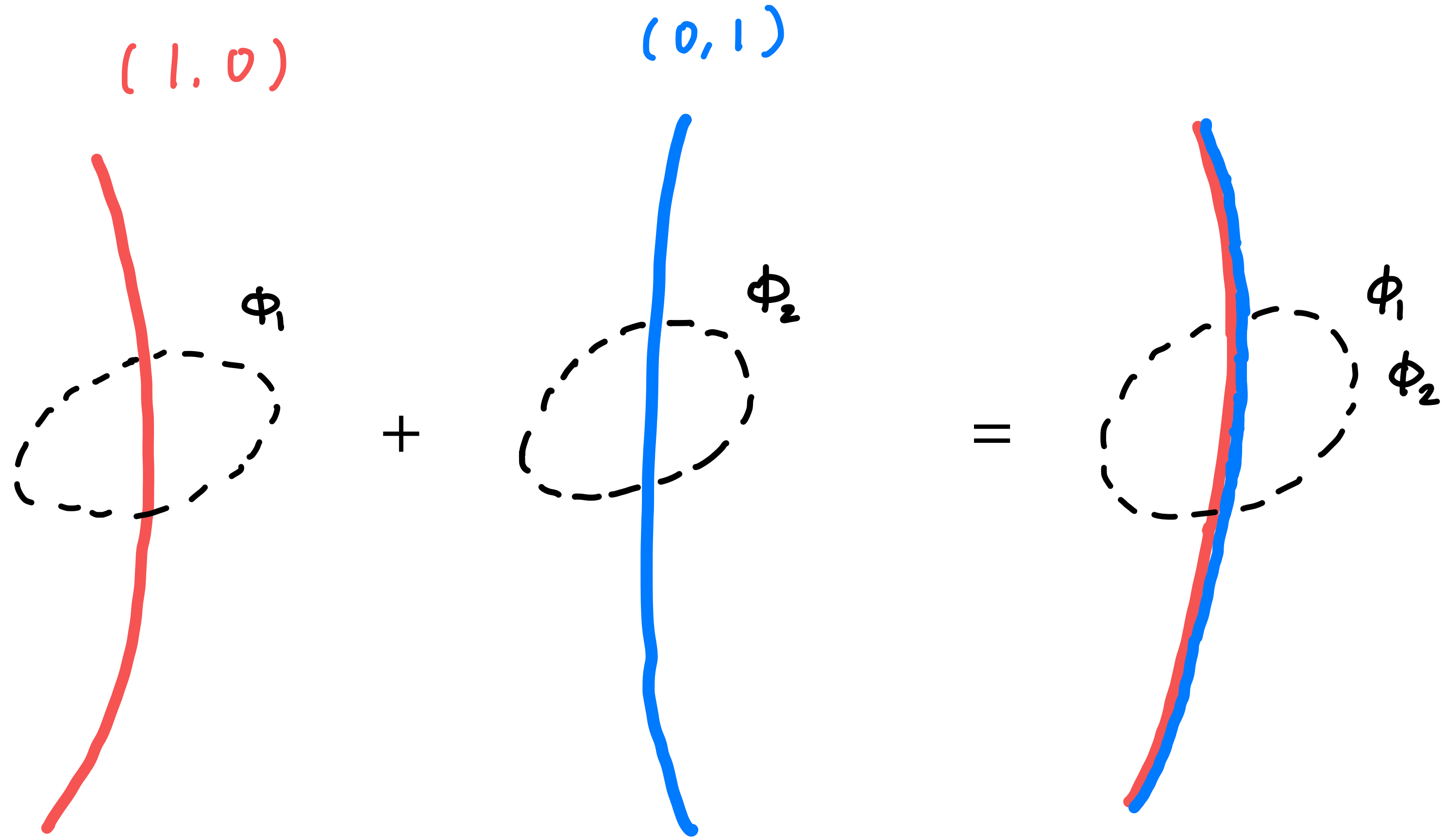

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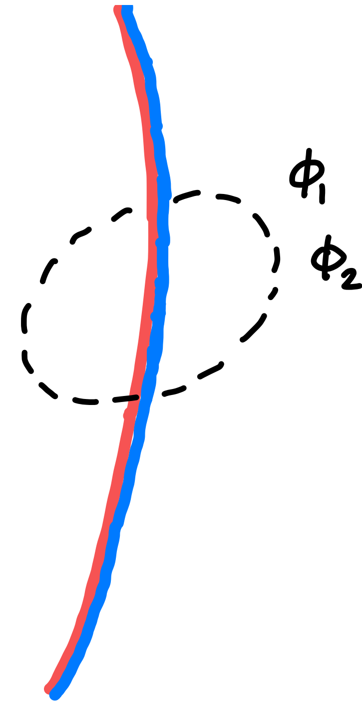

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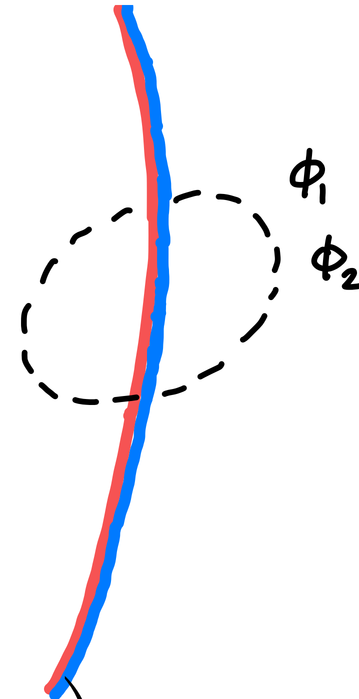

- gradient energy of (1,1) string

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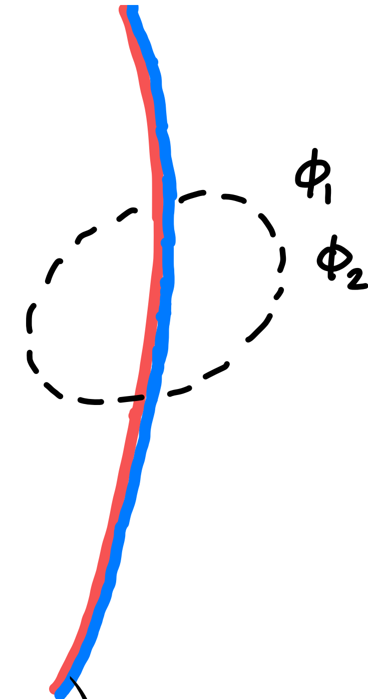

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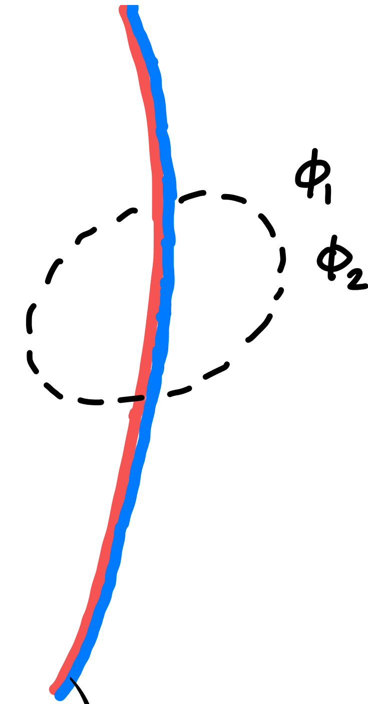
- the profile of Z_θ can simultaneously cancel the gradient energy of Φ_1 and Φ_2

$$\mu_{k,(1,1)} = 0$$

(1,1) strings



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- (1,1) gauge string

  (1,0) and (0,1) global strings

The full tension

The full tension

- magnetic self-energy, scalar potential energy, and gradient energy

The full tension

- magnetic self-energy, scalar potential energy, and gradient energy
- (1,0) string

$$\mu_{(1,0)} \simeq \pi v_1^2 + \pi v_1^2 \ln\left(\frac{m_1}{m_Z}\right) + \pi v_2^2 \ln\left(\frac{m_Z L}{2}\right)$$

(0,1) string

$$\mu_{(0,1)} \simeq \frac{\pi}{2} v_2^2 + \pi v_2^2 \ln\left(\frac{m_2}{m_Z}\right) + \pi v_2 \ln\left(\frac{m_Z L}{2}\right)$$

(1,1) string

$$\mu_{(1,1)} = \pi v_1^2 + \pi v_1^2 \ln\left(\frac{m_1}{m_Z}\right) + 0$$

The full tension

- magnetic self-energy, scalar potential energy, and gradient energy

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- heavy core of (1,0) string $\mu_{(1,0)} > \mu_{(0,1)}$

The full tension

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(1,1) string

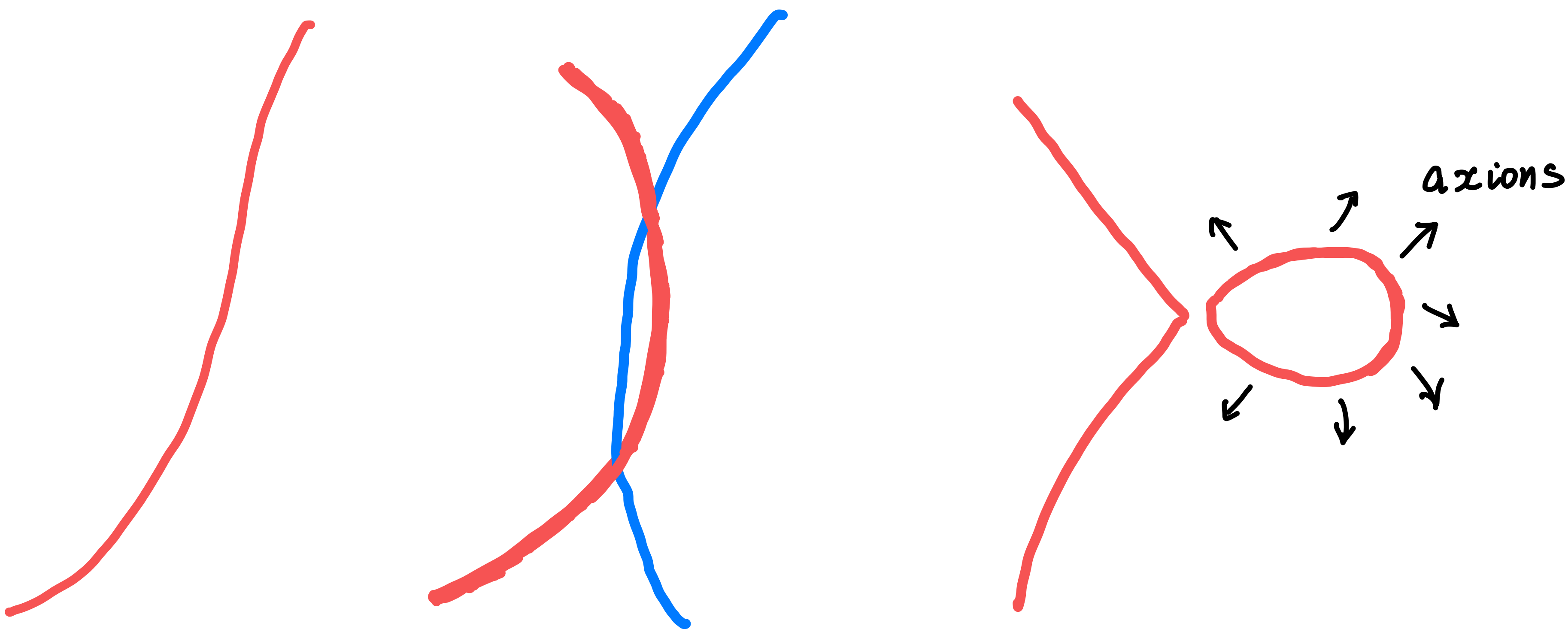
$$\mu_{(1,1)} = \pi v_1^2 + \pi v_1^2 \ln\left(\frac{m_1}{m_Z}\right) + 0$$

- heavy core of (1,0) string $\quad \mu_{(1,0)} > \mu_{(0,1)}$

- binding energy of (1,1) string

$$\mu_{(1,0)} + \mu_{(0,1)} - \mu_{(1,1)} = \pi v_2^2 \left[2 \ln\left(\frac{m_Z L}{2}\right) - 1\right]$$

Cosmological Implication

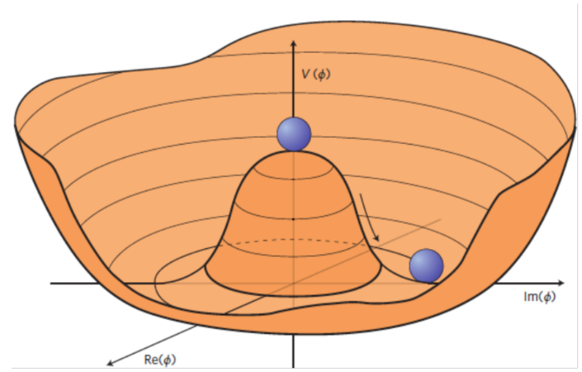


formation evolution radiation

First phase transition and string formation

First phase transition and string formation

- consider $v_1 \gg v_2$

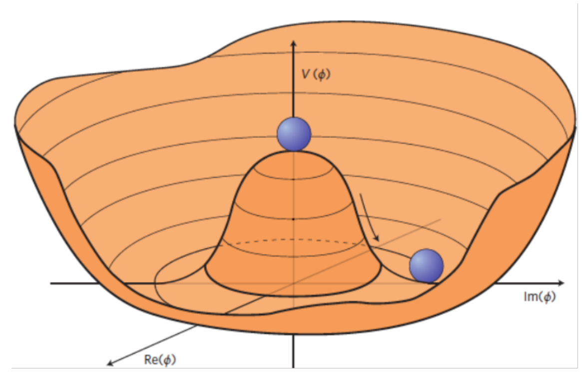


first phase transition

$$\langle \Phi_1(x) \rangle = \frac{v_1}{\sqrt{2}} \text{ and } \langle \Phi_2(x) \rangle = 0$$

First phase transition and string formation

- consider $v_1 \gg v_2$



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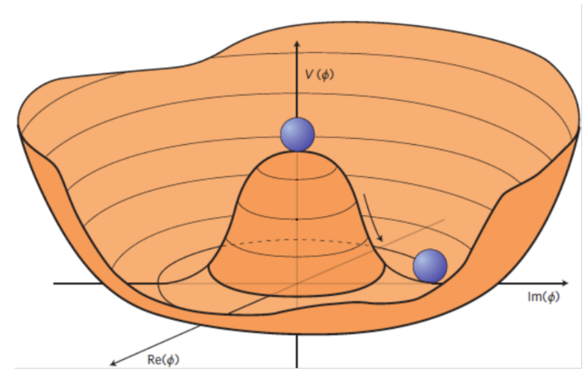

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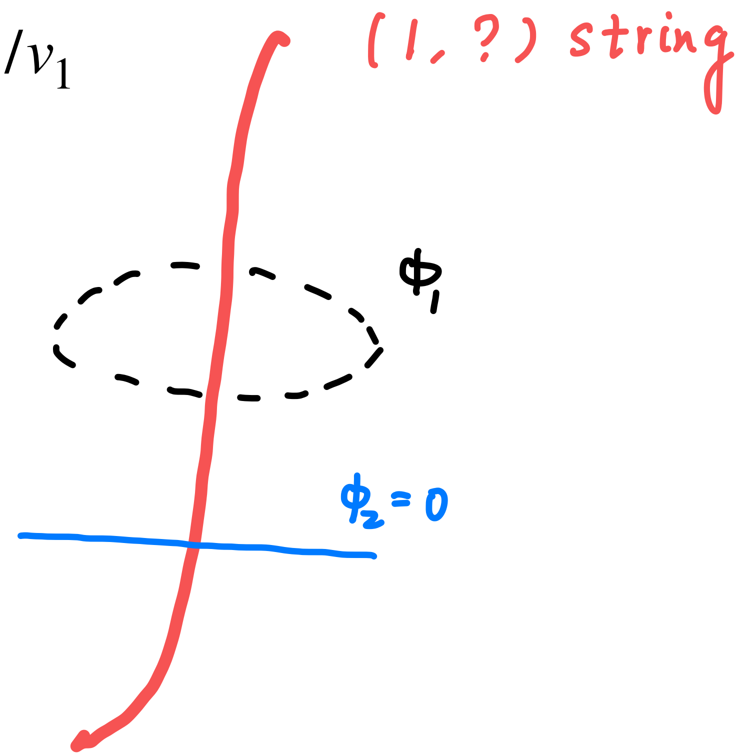

- U(1) gauge strings form

  (1, n) string

Second phase transition

Second phase transition

- second phase transition

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- string formation, the correlation length ~ $1/v_2$

- (0,1) strings form via Kibble mechanism

(1,n) string in the second phase transition

- (1, n) string $\rightarrow$?

(1,n) string in the second phase transition

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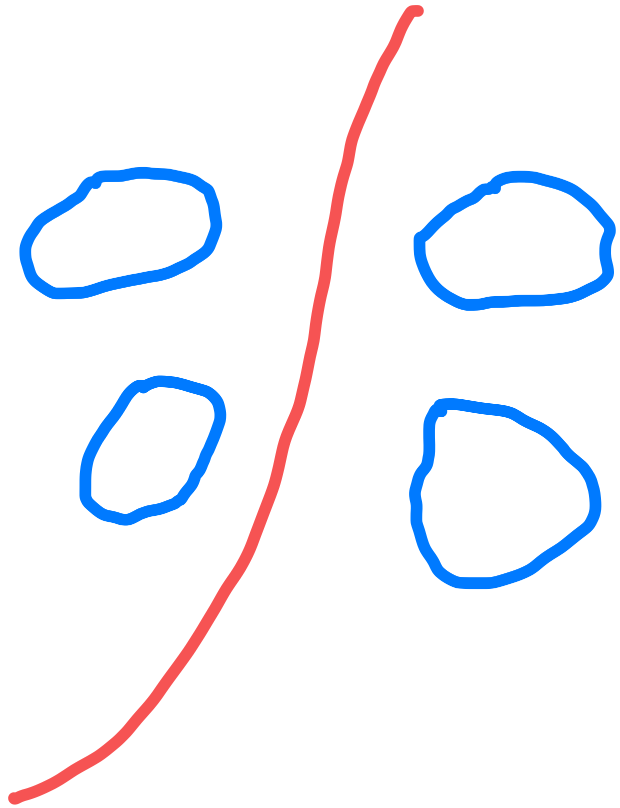

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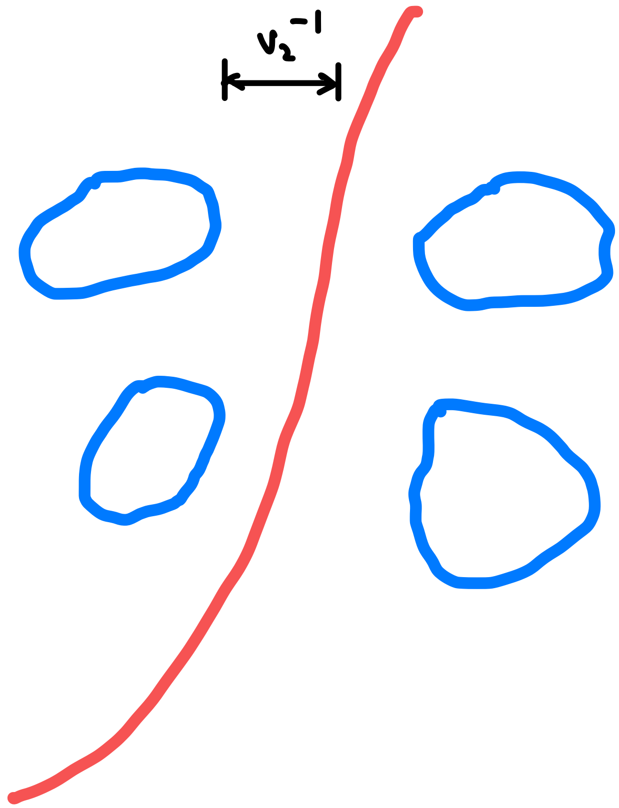

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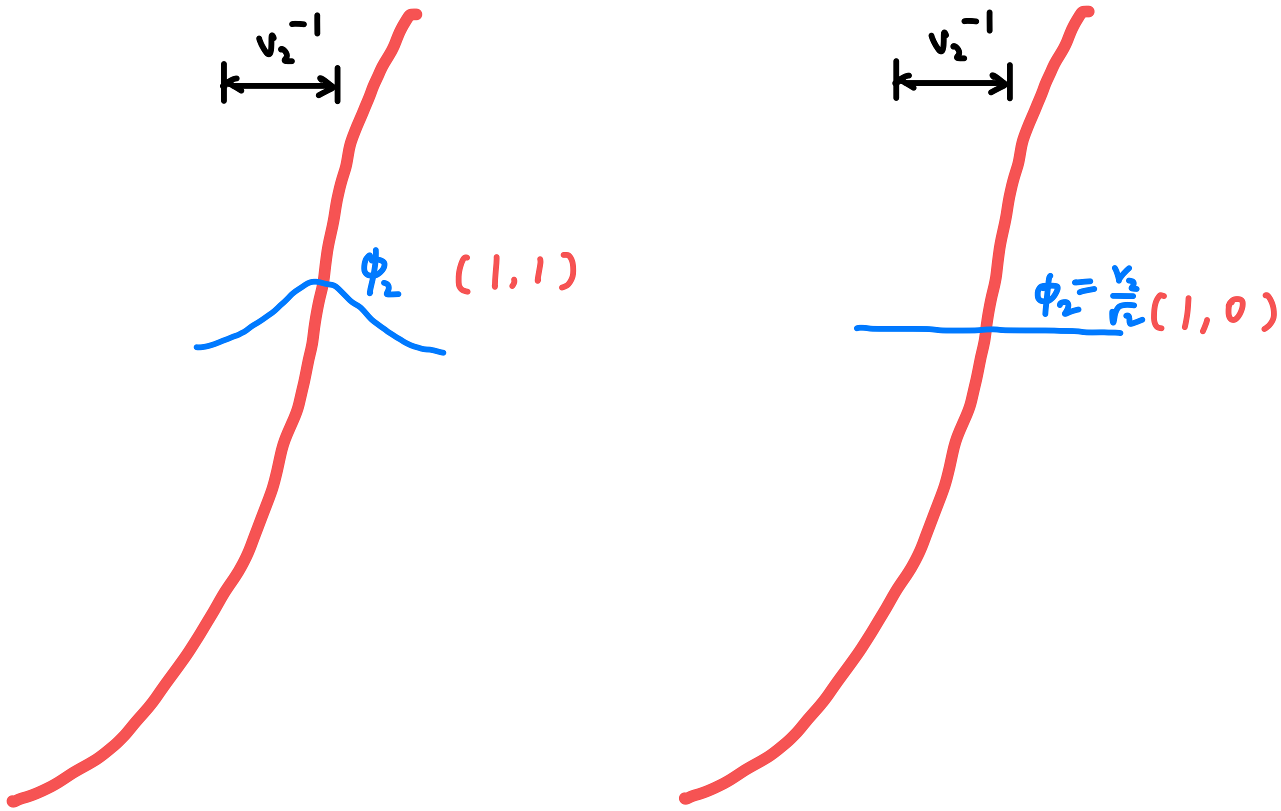
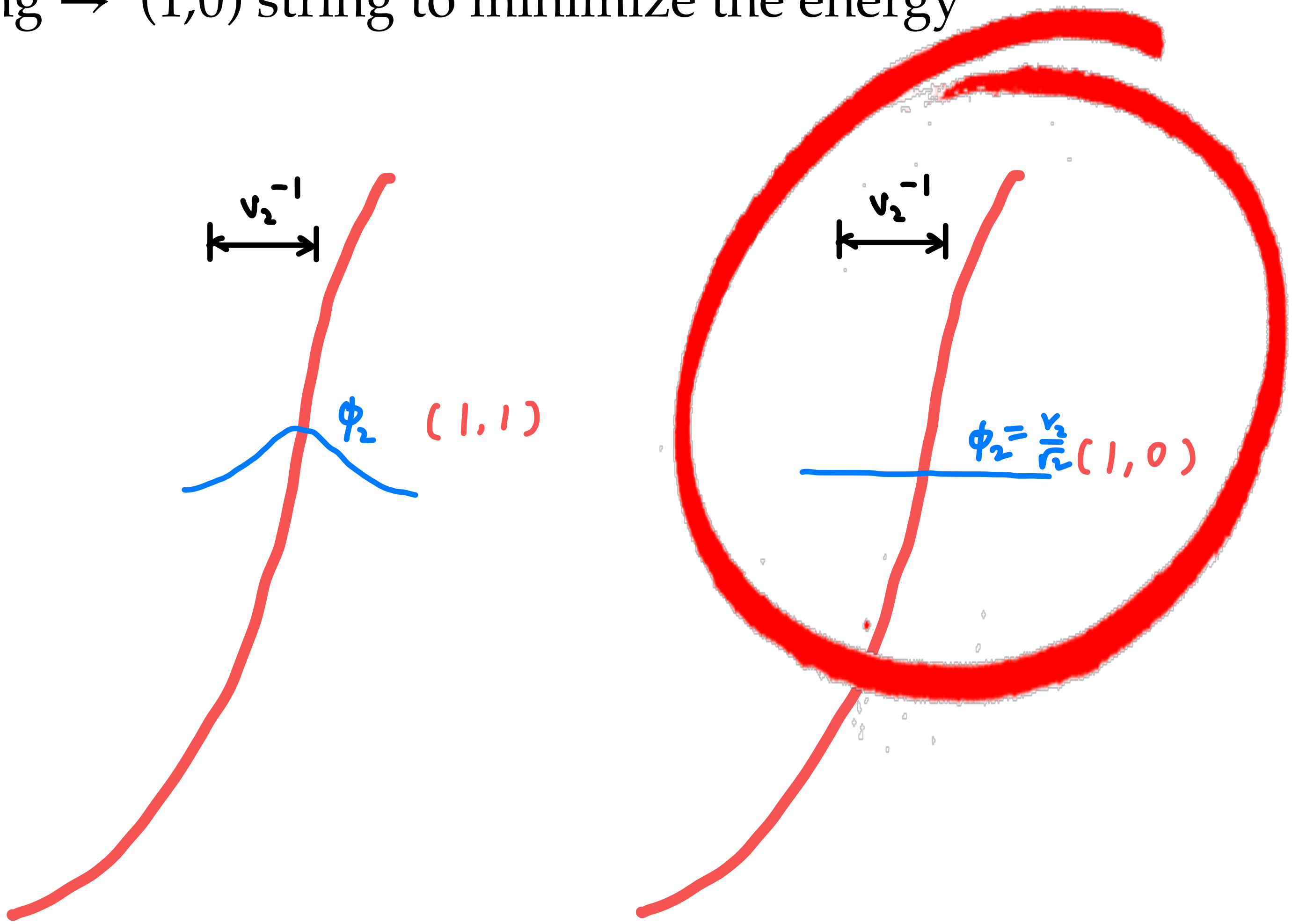


(1,n) string in the second phase transition

- (1, n) string → (1,0) string to minimize the energy



String network evolution

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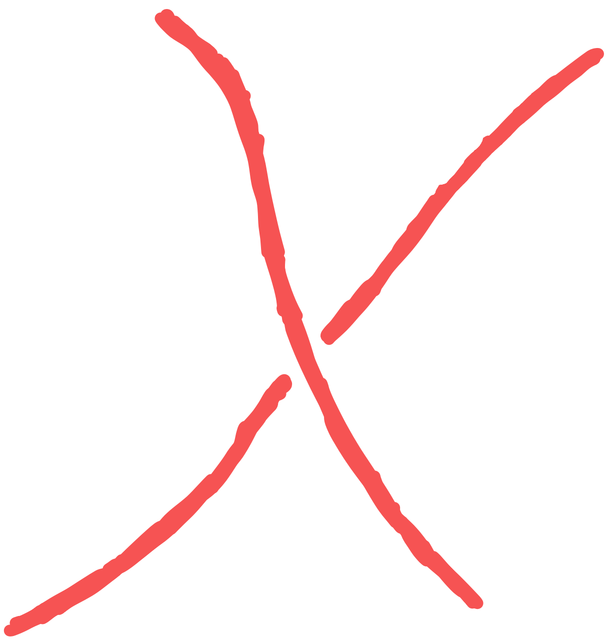

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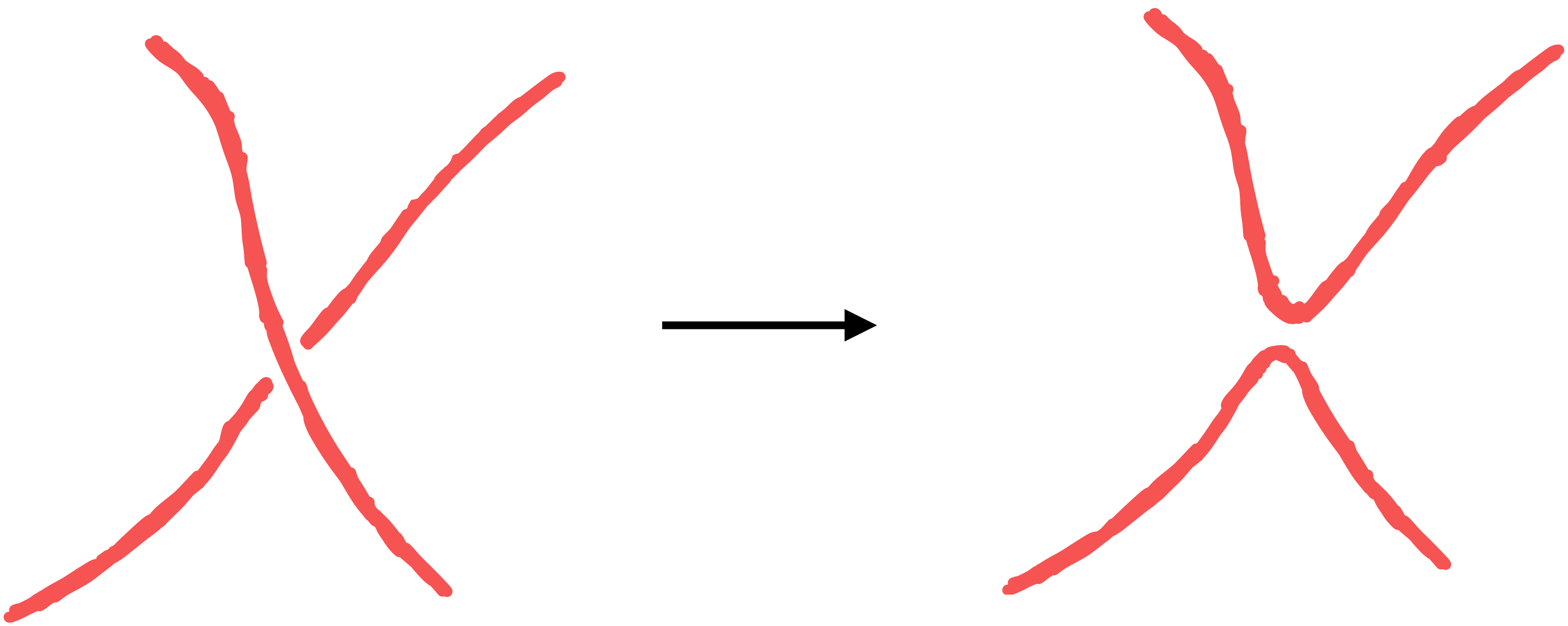

String network evolution

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  Y-junctions

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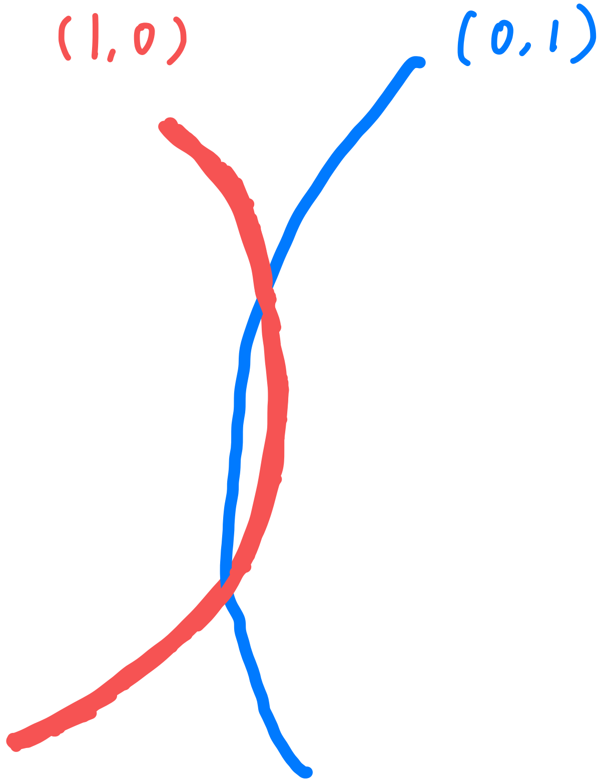

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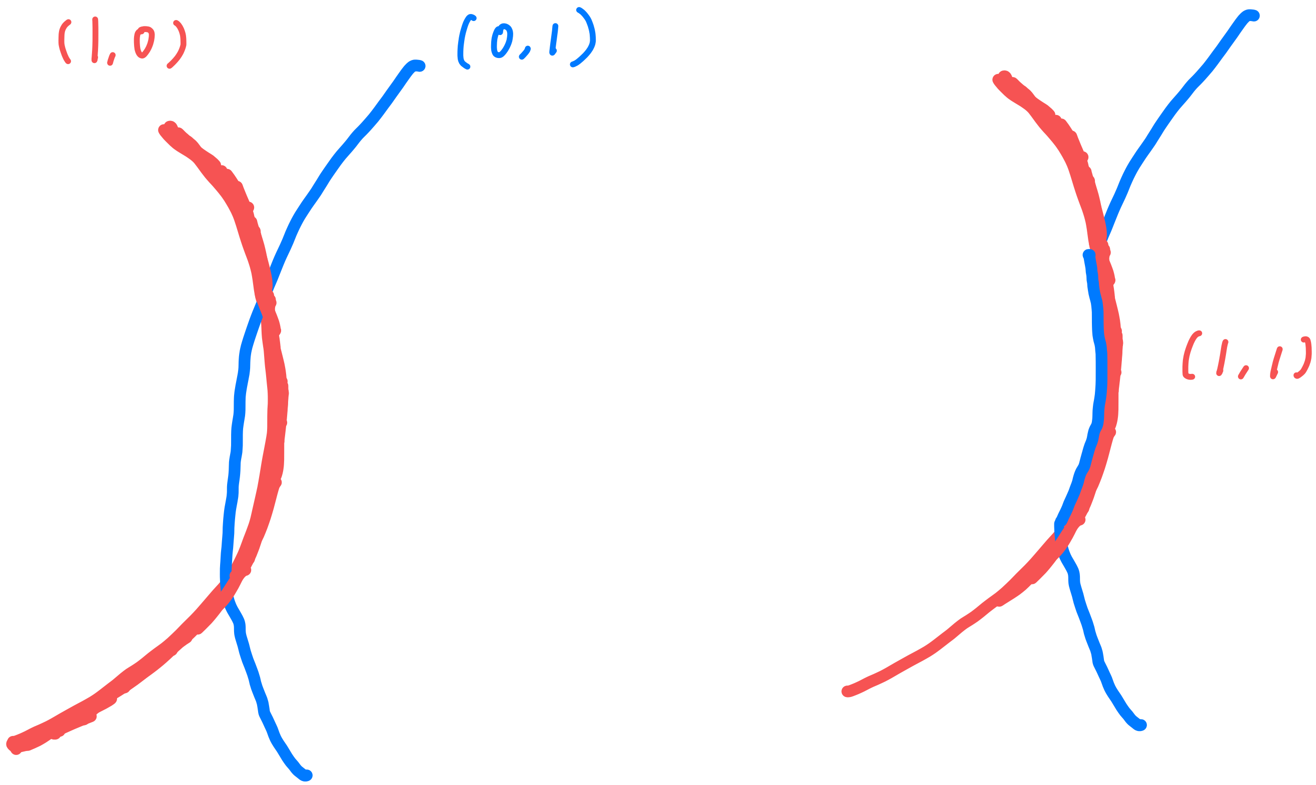

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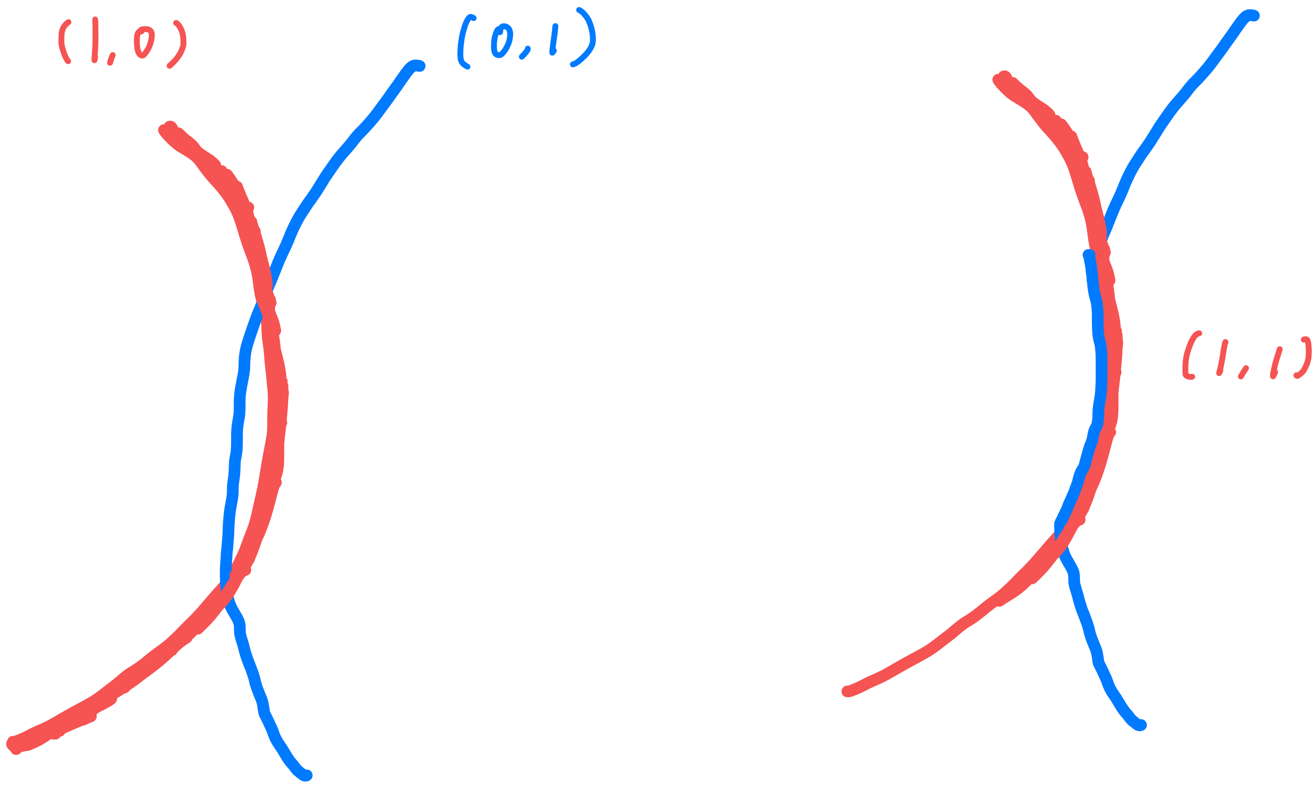

- Other works on simulations of Y-junctions
  found 1) some fraction of Y-junctions remain
  2) scaling solution

Urrestilla, Vilenkin JHEP(2008)
Rajentie, Skellariodou, Stoica, JCAP (2007)
Copeland, Saffin JHEP (2005)
…

QCD axion

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- dark matter abundance

misalignment + string radiation + domain wall collapse

$$\rho_{a,0} = \rho_a^{\mathrm{vac}}(t_0) + m_a n_a^{\mathrm{str}}(t_0) + m_a n_a^{\mathrm{DW}}(t_0)$$

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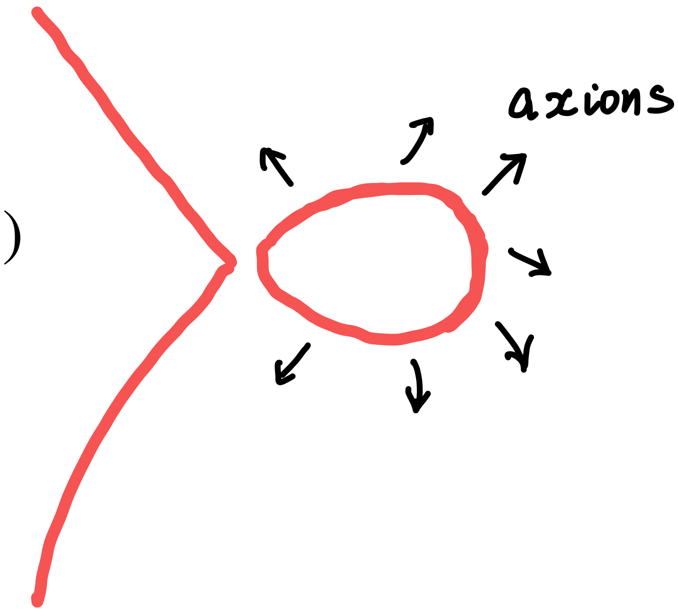

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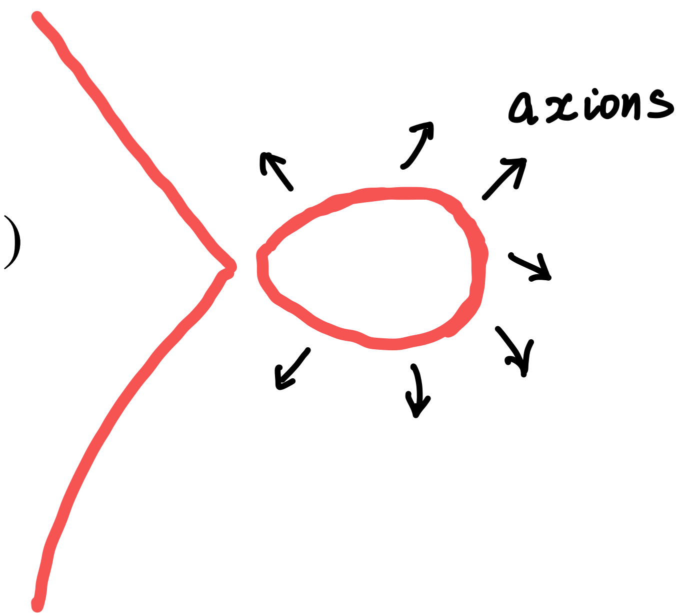

- $n_a^{\mathrm{str}} \propto \dfrac{1}{\langle \frac{dE}{d\omega} \rangle}$

Gauged global string

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- (0,1) string tension is the same as a standard QCD axion string

- heavy core of (1,0) string

$$\mu_{(1,0)}(t) \simeq \pi v_1^2 \ln\left(\frac{m_1}{m_Z}\right) + \pi f_a^2 \ln\left(\frac{m_Z t}{2}\right)$$

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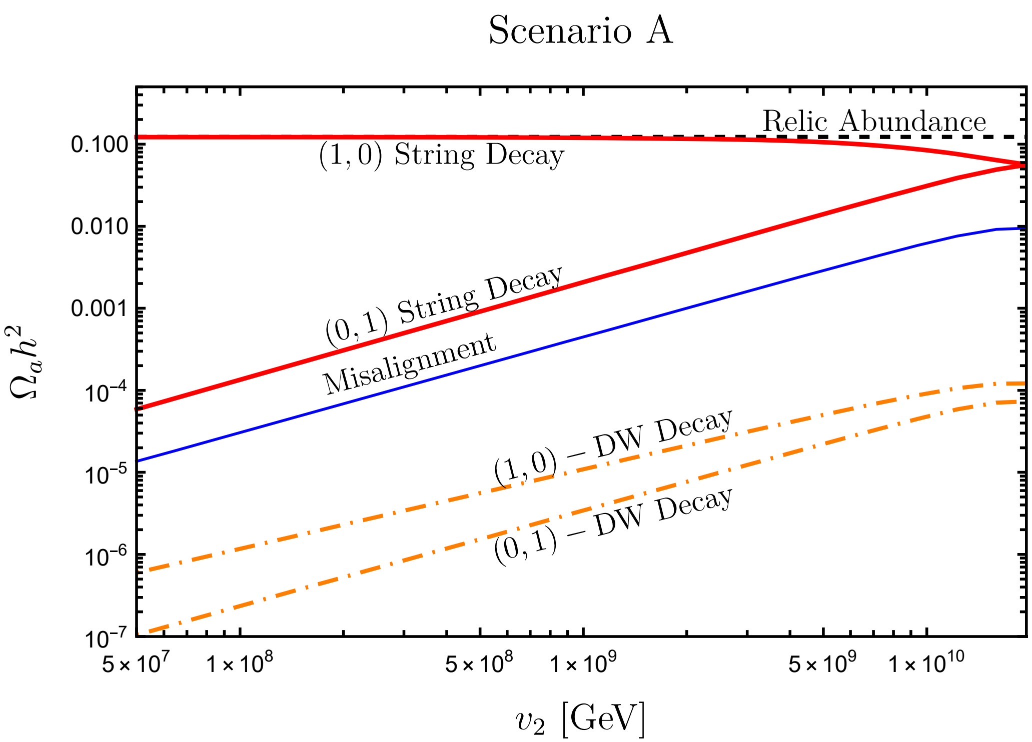

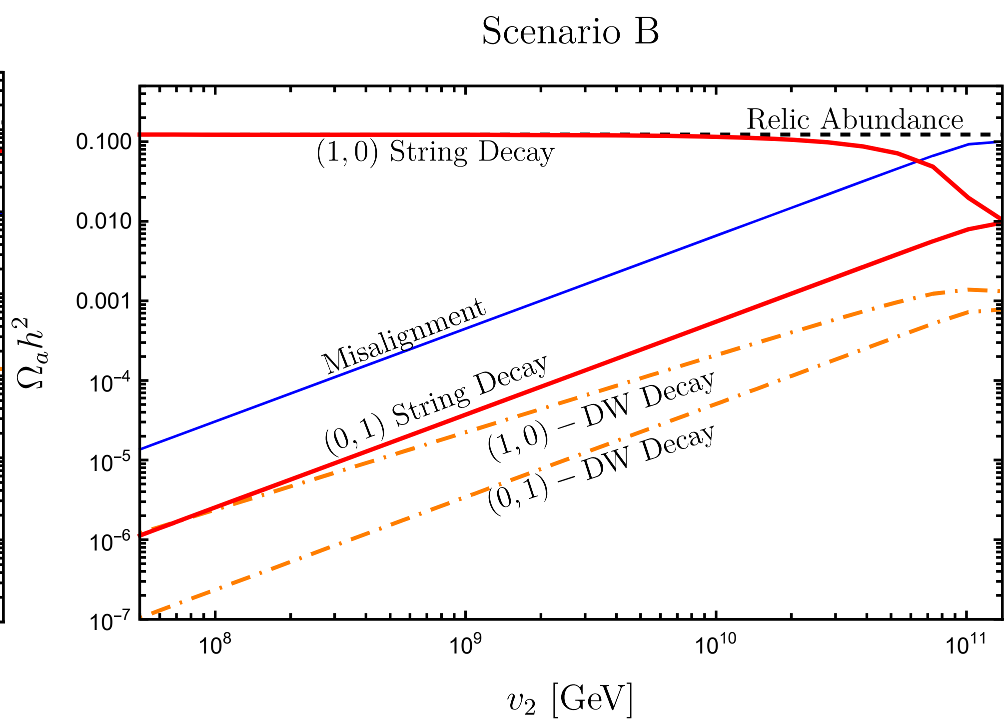

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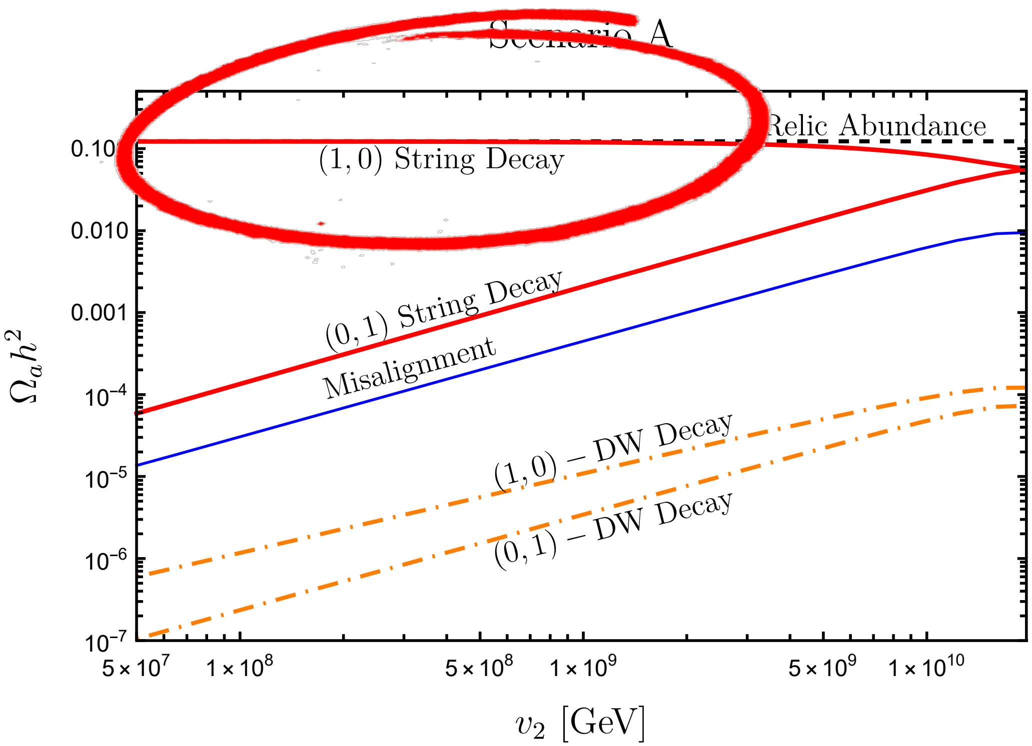
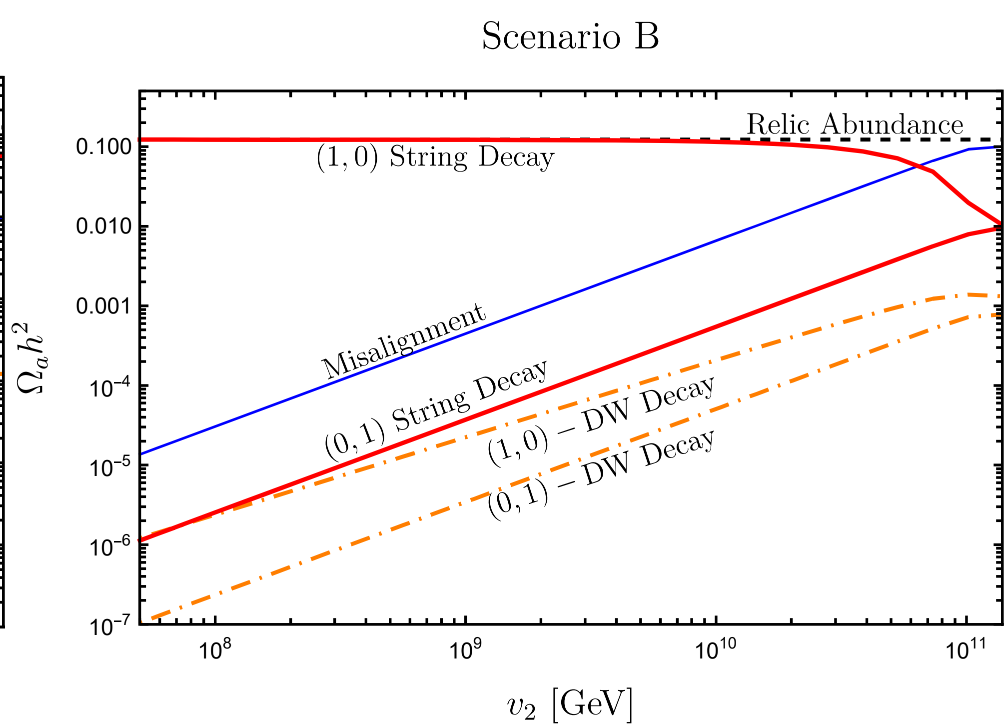
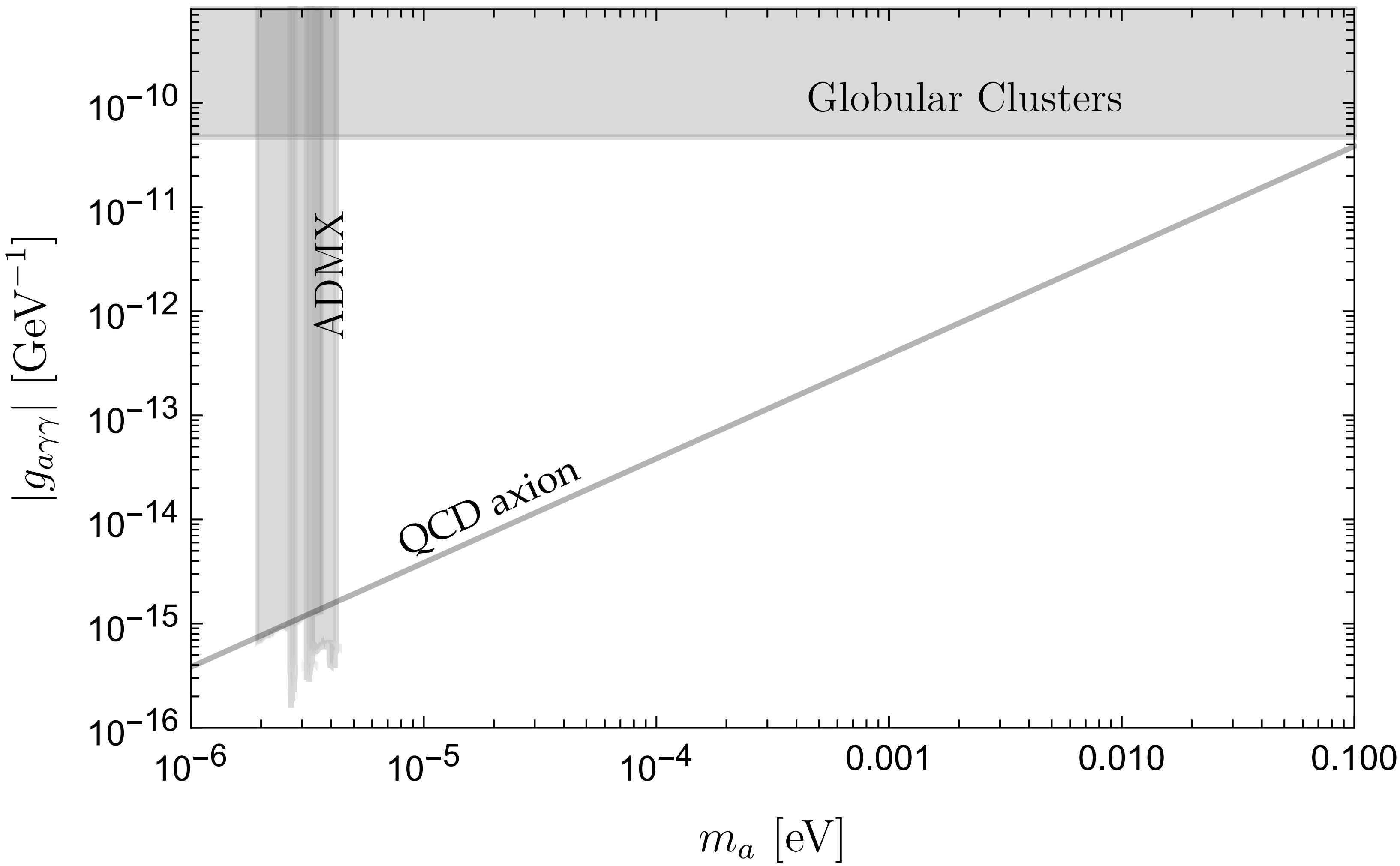
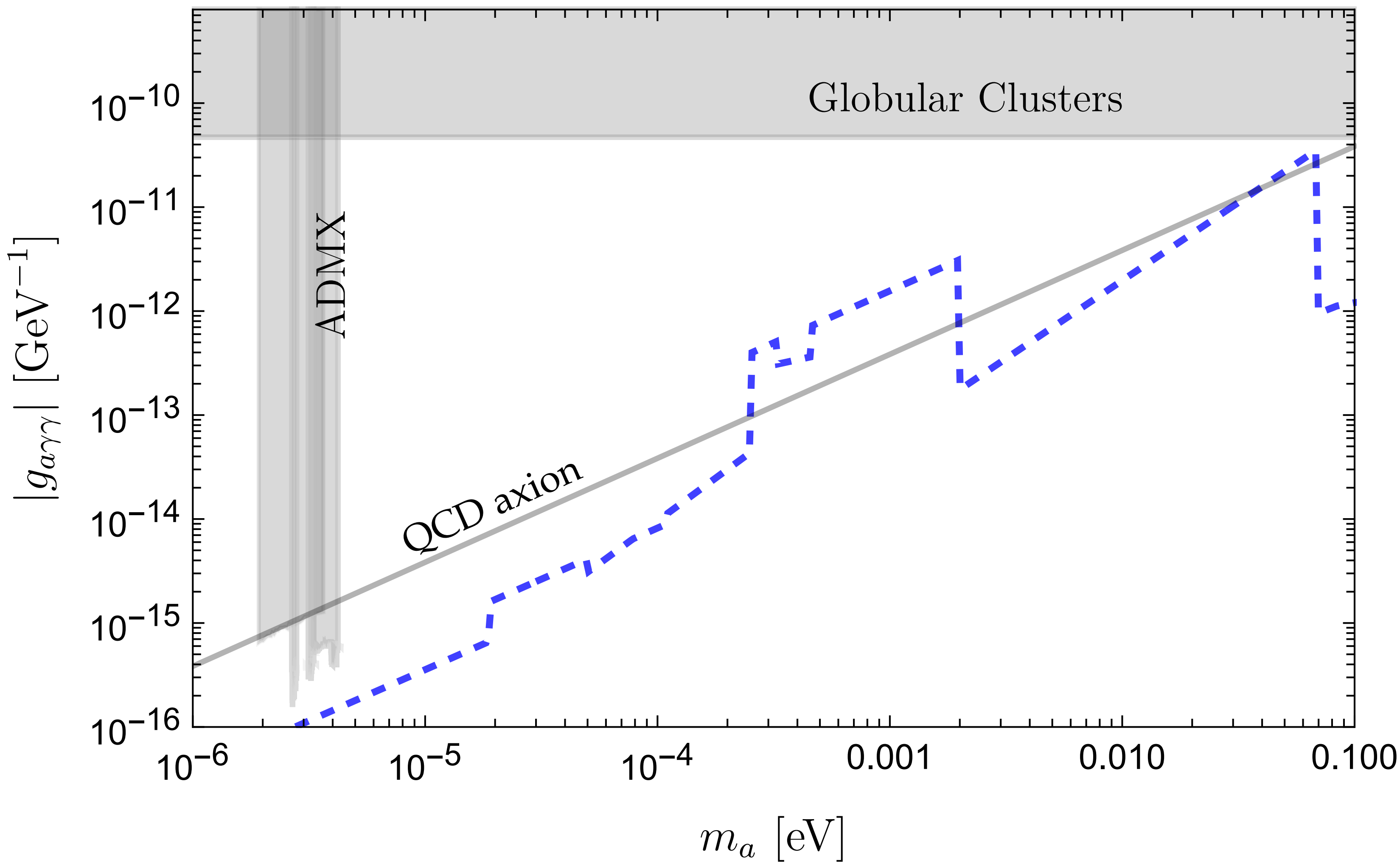




QCD axion window



Future explorations



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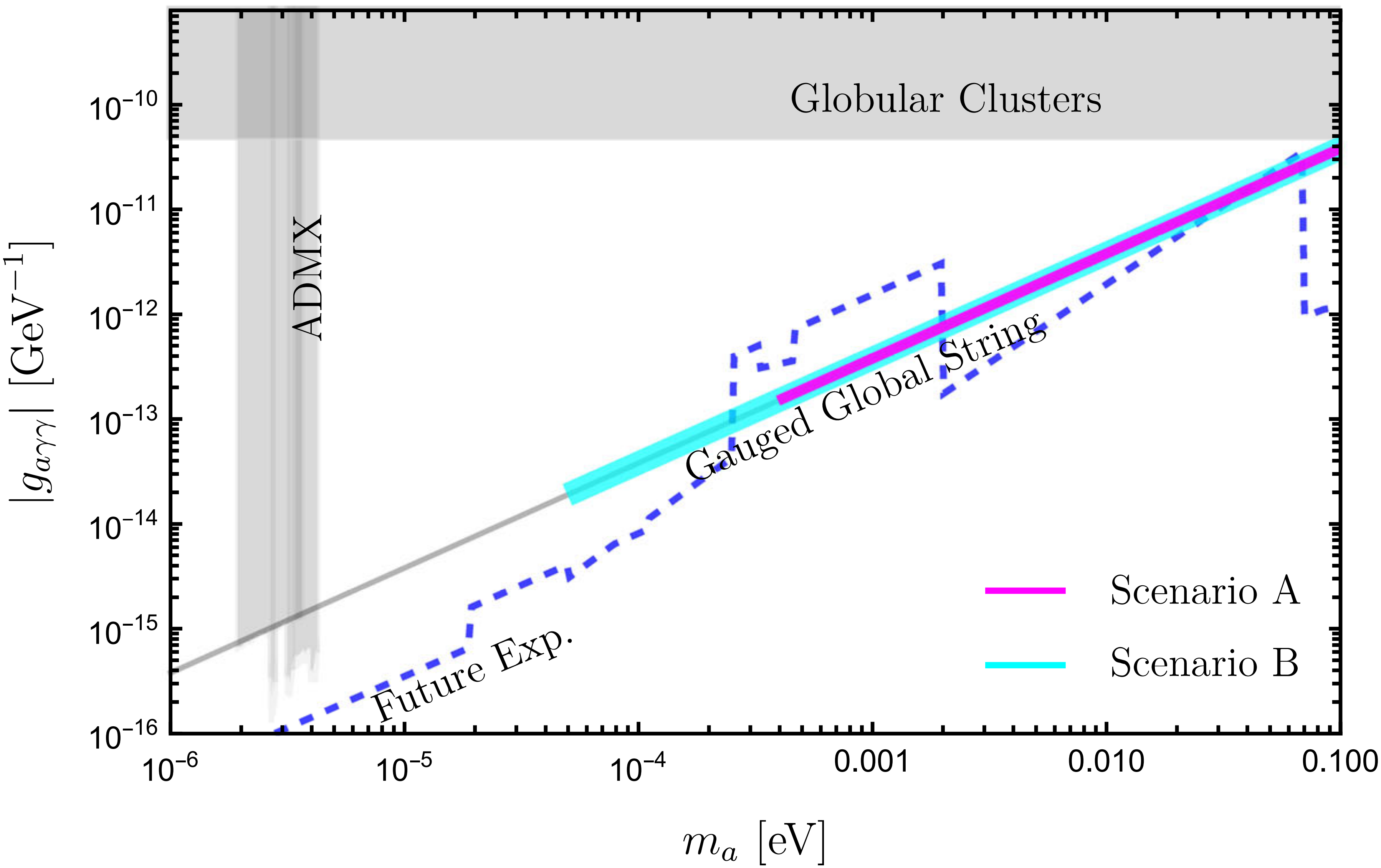

Gauge string (1,1) radiation

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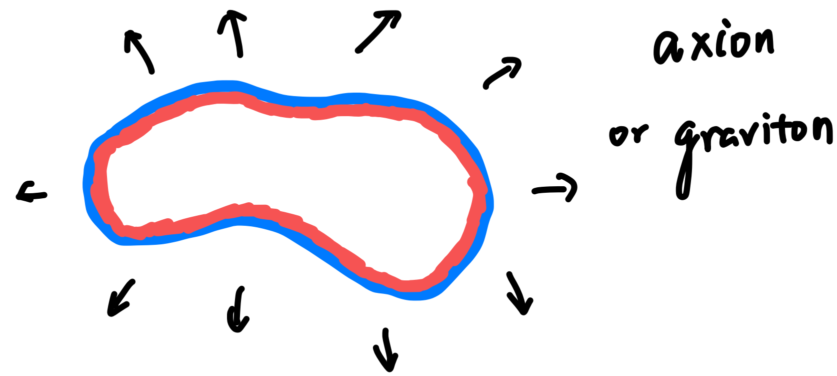
- How about (1,1) gauge strings?



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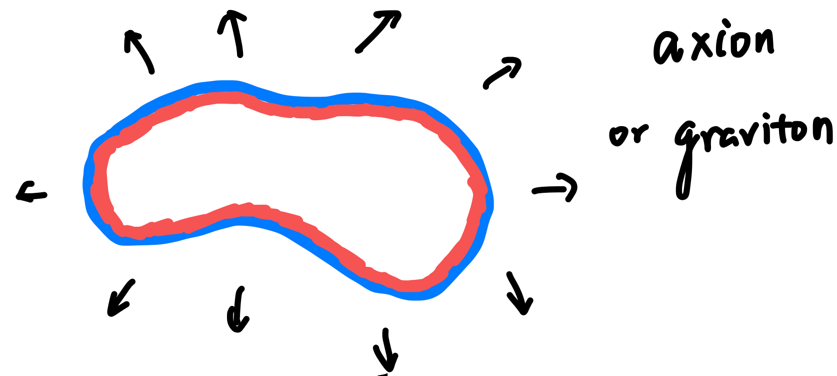

- (1,1) string is gauge string, but it also has axion as light d.o.f

$$\mathscr{L} = -\frac{1}{4}Z_{\mu\nu}Z^{\mu\nu} + \frac{1}{2}e^2(\phi_1^2 + \phi_2^2)Z_\mu^2 - g(\phi_1, \phi_2)\, e\, Z^\mu \partial_\mu a + \frac{1}{2}\, f(\phi_1, \phi_2)\, (\partial_\mu a)^2$$

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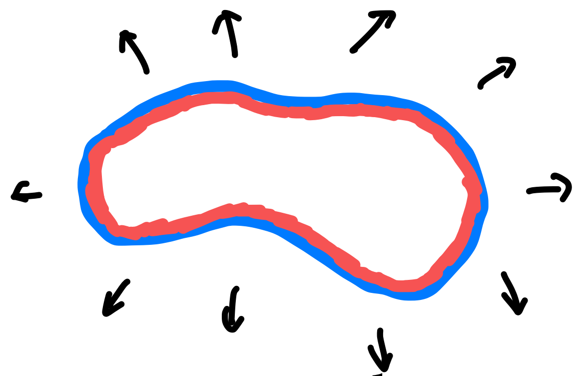

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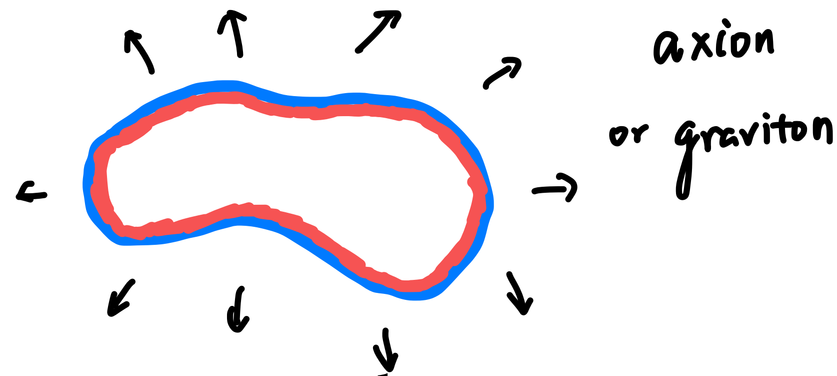

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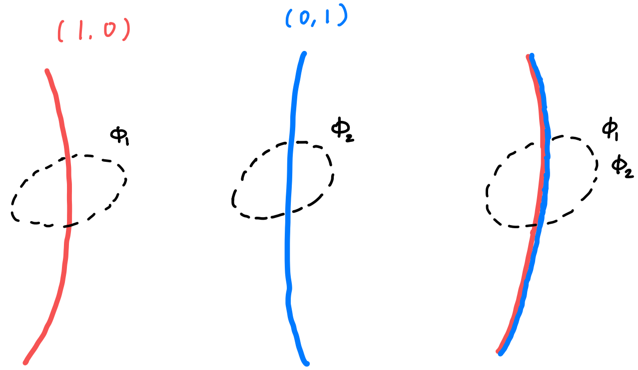
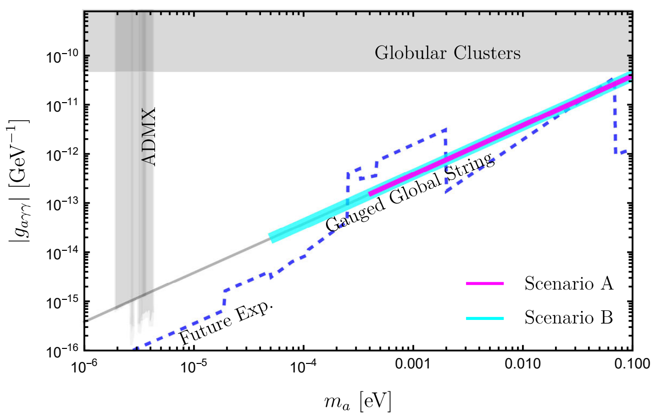
- radiation power

$$\frac{\mathrm{d}P_a}{\mathrm{d}\Omega} \sim e^2 f_a^2$$

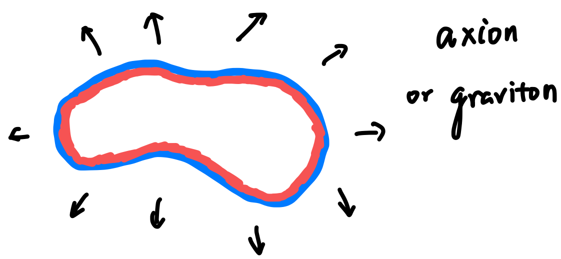
Conclusion



- $U(1)_Z \times U(1)_{PQ}$
  (1,0), (0,1) and (1,1) strings



- Cosmology
  Y-Junctions
  opening QCD axion mass windows



- (1,1) gauge string
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