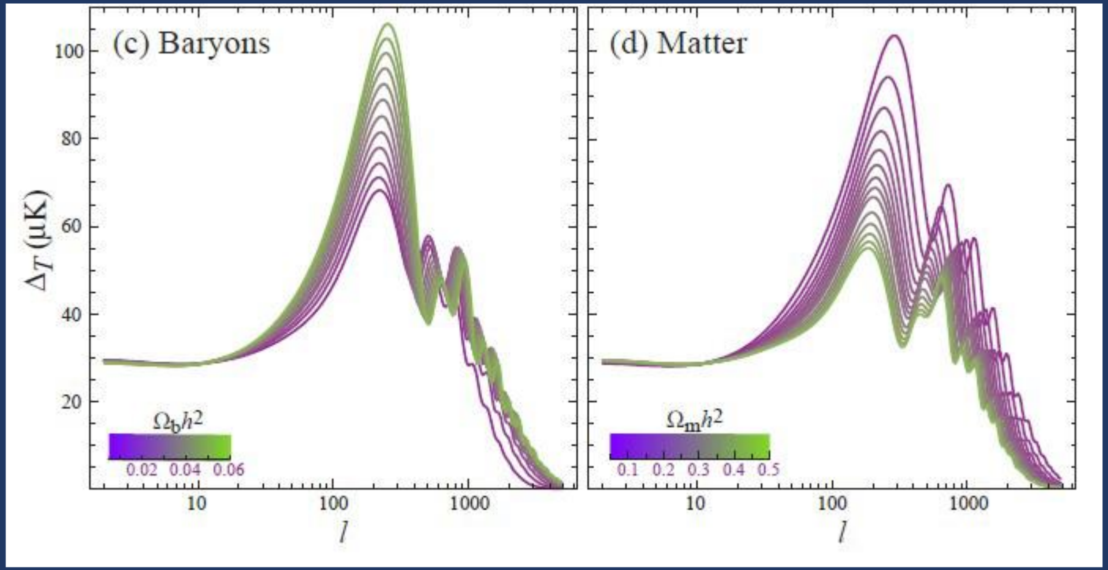
Ultralight Dark Matter and g-2

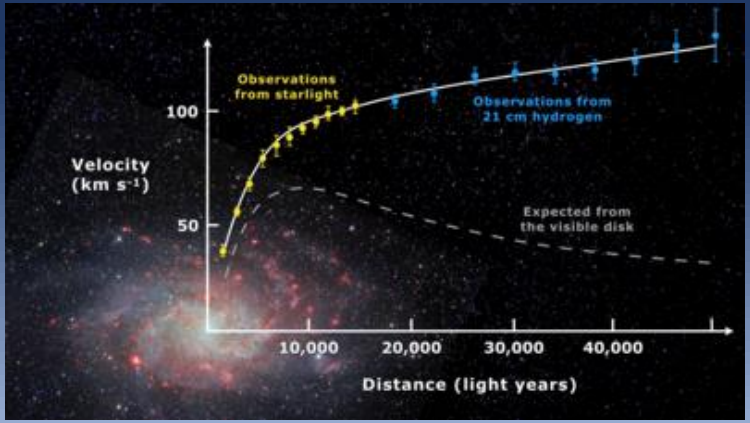
Jason L Evans

Tsung-Dao Lee Institute

Dark Matter: Evidence

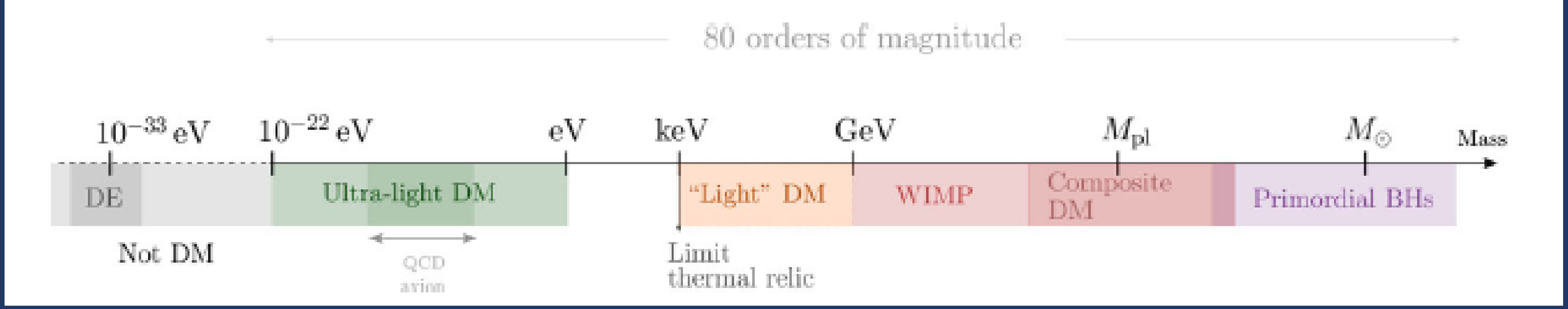
- Dark Matter
    - Gravitational Evidence





Dark Matter: What We Know

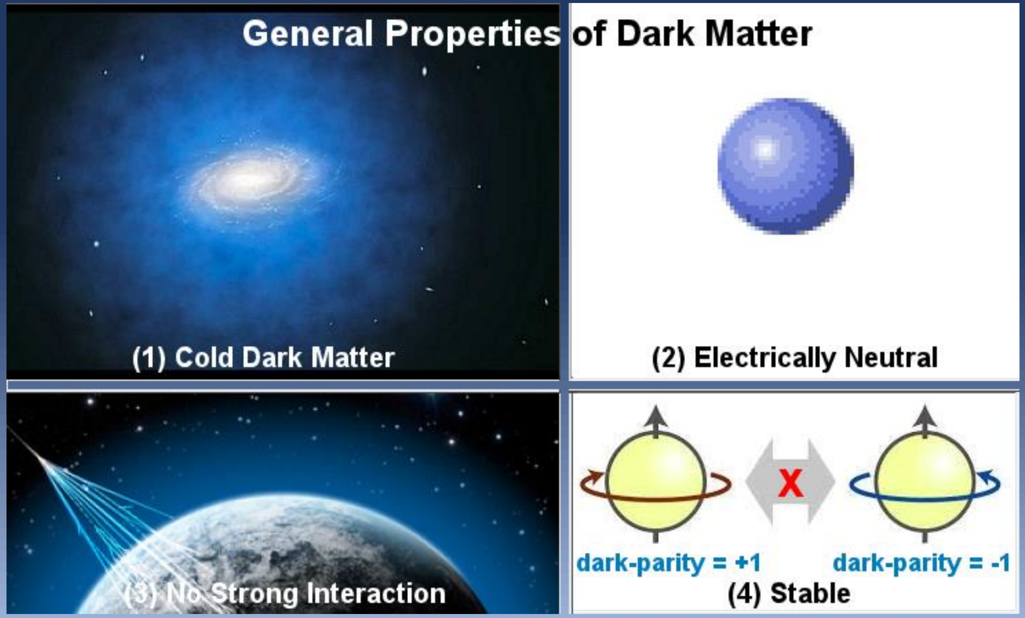
- Dark matter mass range poorly constrained



- DM should be cold

- $SU(3)XU(1)_{EM}$ Neutral

- Stable $\tau_{DM} \gg \frac{1}{H}$



Light Fermionic Dark Matter: Tremaine Gunn Bound

- Pauli-Exclusion Principle
  - No two fermions can occupy same energy state

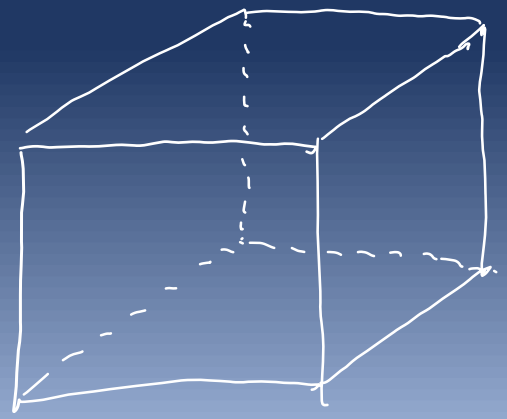
- How many fermions can occupy a volume V?
  - Pauli-Exclusion limits two per energy state (spin)
  - 3D infinite square well, count states

$$-\frac{\hbar^2}{2m}\nabla^2\psi = E\psi \qquad k_i = \frac{\pi}{L} n_i$$

Number of states

$$\psi(x,y,z) = N\sin(k_x x)\sin(k_y y)\sin(k_z z)$$

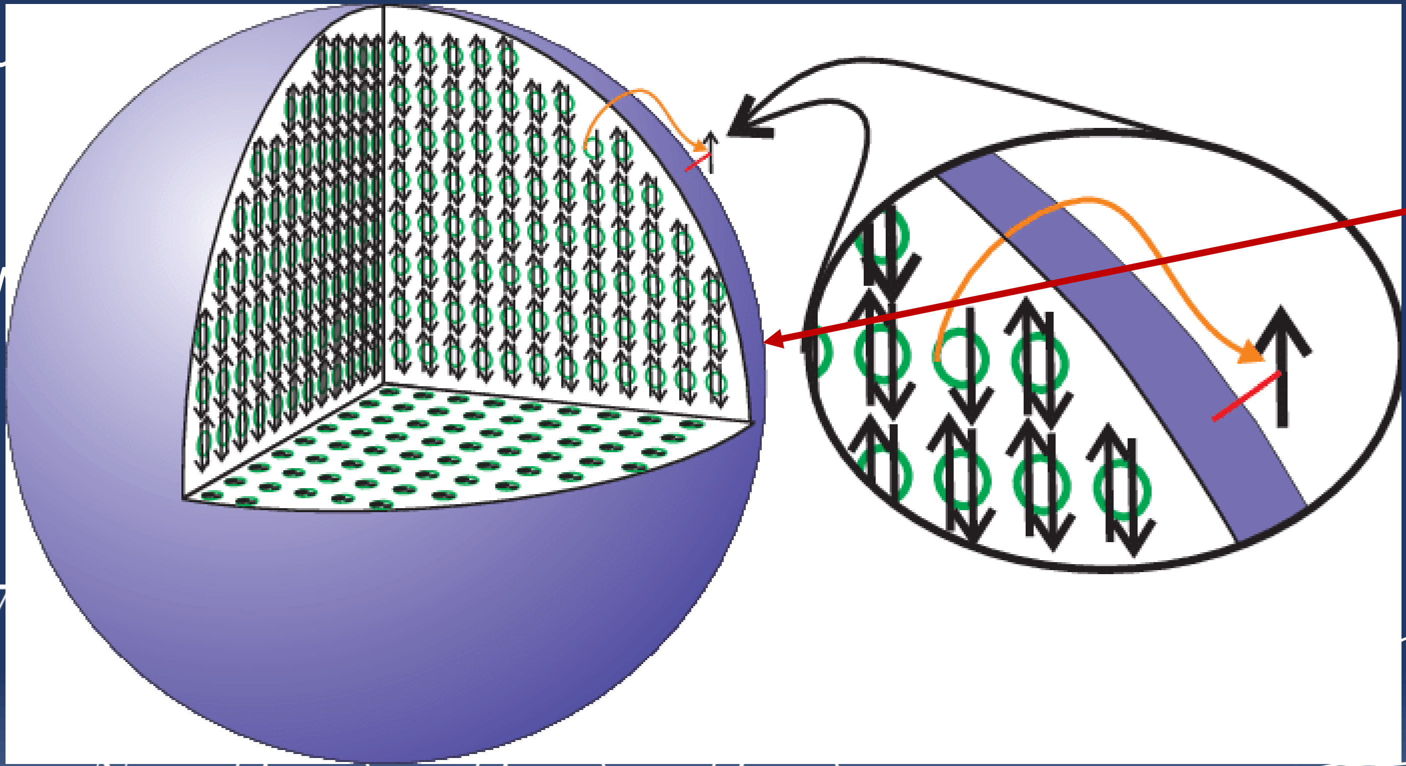
$$dN = dn_x dn_y dn_z = \frac{V}{\hbar^3} d^3p$$



Light Fermionic Dark Matter: Tremaine Gunn Bound

- Pau
  -

- How
  -
  -

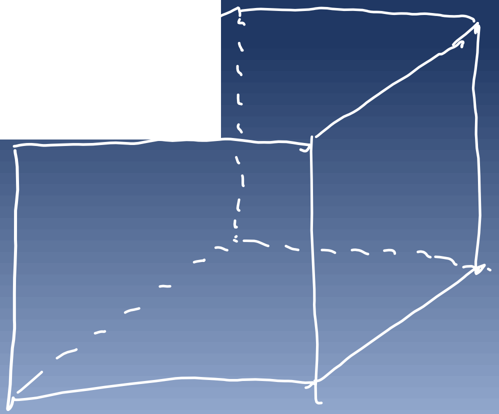


Galaxy is a sphere full of fermions

$$-\frac{\hbar^2}{2m}\nabla$$

$$\psi(x, y, z) = N \sin(k_x x) \sin(k_y y) \sin(k_z z)$$

$$dN = dn_x dn_y dn_z = \frac{V}{\hbar^3} d^3p$$



Light Fermionic Dark Matter: Tremaine Gunn Bound

- Pauli-Exclusion Principle
  - No two fermions can occupy same energy state

- How many fermions can occupy a volume V?
  - What does this say for a galaxy

Galaxy Size

Variance of velocity

$$M_V \approx \frac{\sigma^2 r}{G}$$

Virial Theorem Mass

$$N \approx \frac{V p^3}{\hbar^3} \approx \frac{r^3 m^3 \sigma^3}{\hbar^3}$$

Number of particles for a given momentum

$$M_{MAX} \approx mN \approx \frac{r^3 m^4 \sigma^3}{\hbar^3}$$

Maximum mass in galaxy

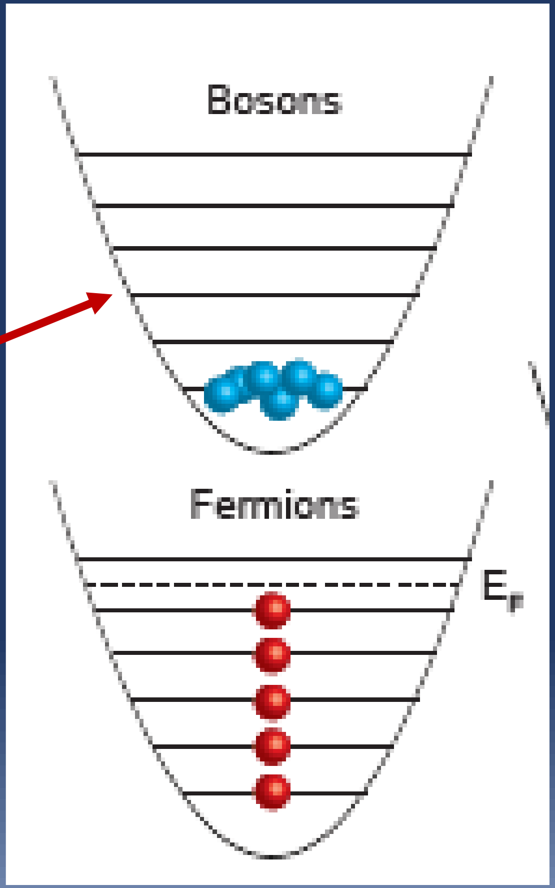
- Minimum dark matter mass $M_V > M_{MAX}$

$$m > \left(\frac{\hbar}{G r^2 \sigma}\right)^{1/4} = 20\ eV \left(\frac{r}{20\ \text{kpc}}\right)^{-1/2} \left(\frac{\sigma}{200\ \text{km/s}}\right)^{-1/4}$$

Light Bosonic Dark Matter

- Count the number of states for a boson
  - Basically, the same but can have ∞ particles per state

No lower bound on mass



Light Bosonic Dark Matter

- Count the number of states for a boson
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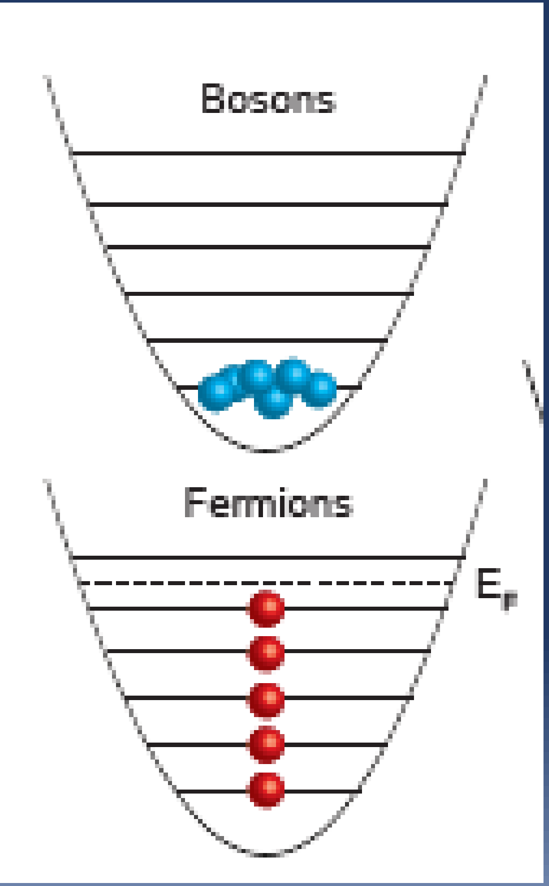
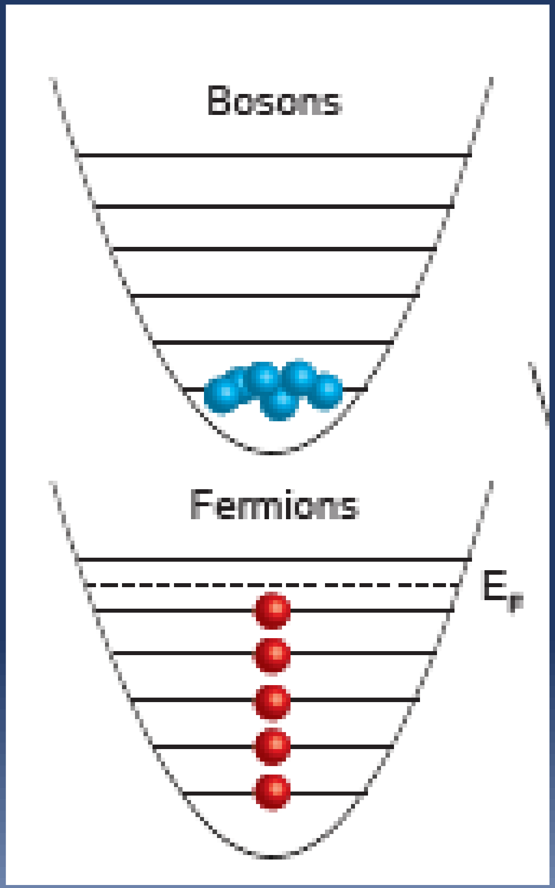
- Can there be a lower bound on the mass?



Ultralight Bosonic Dark Matter

- ❑ Count the number of states for a boson
  - ❑ Basically the same, but can have ∞ particle per state
- ❑ Can there be a lower bound on the mass?
  - ❑ ULBD must be a condensate on "small" scales
  - ❑ Condensate must be smaller than galaxy

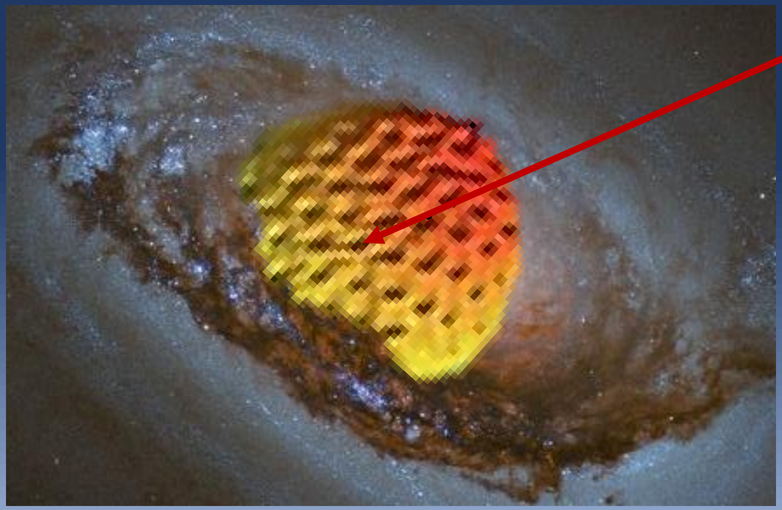


Dwarf Galaxy

$$\sim 100\ pc$$

$$\sim \frac{1}{10^{-25}\ \mathrm{eV}}$$

Real estimate $10^{-24}\ eV$ or even larger



Condensate

Ultralight Bosonic Dark Matter

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Bound not from special properties of bosons

Dwarf Galaxy

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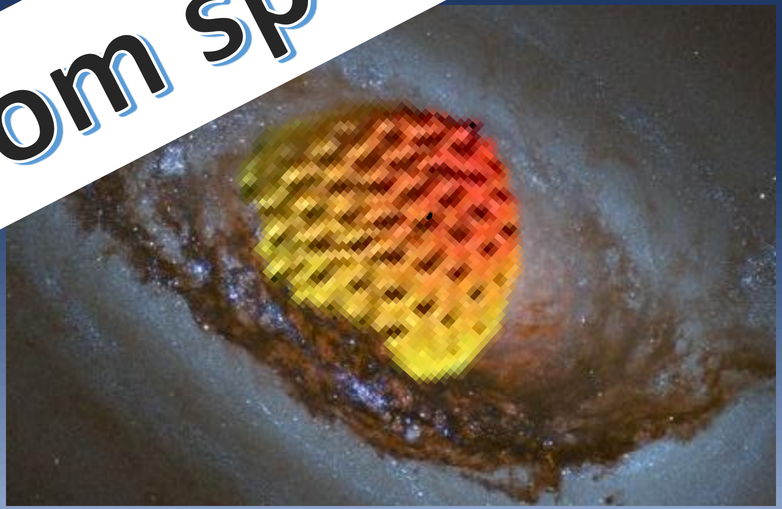

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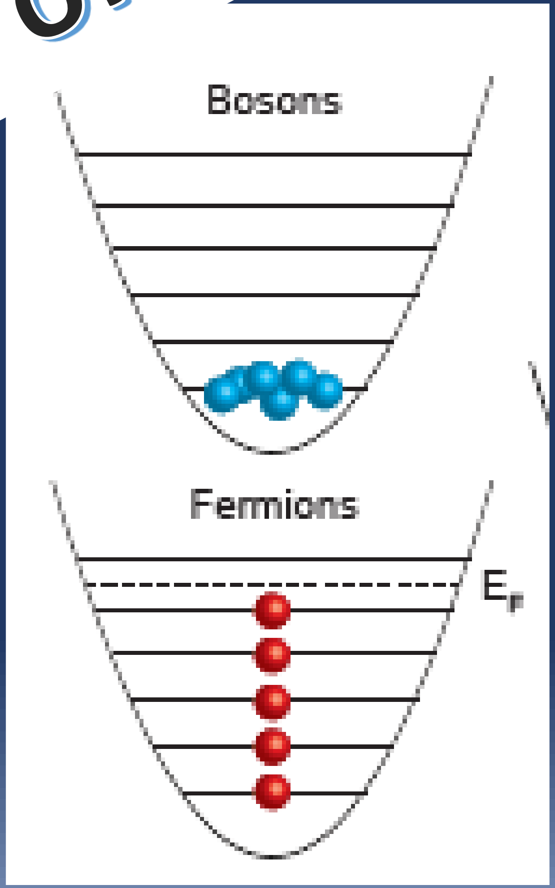

Why Are Bosons Special?

- Bosons can occupy the same energy state

$$|n(k_1), n(k_2), \ldots\rangle = \prod_i \left\{ \frac{1}{(2\pi)^3 2\omega_{k_i}[n(k_i)+1]^{1/2}} [a^\dagger(k_i)]^{n(k_i)} \right\} |0\rangle$$

$$a(k)^n \neq 0$$

- In a background, a decay process becomes

$$\langle 0, k_2, k_3 | \lambda \phi_3 \phi_2 \phi_1 | k_1, 0, 0 \rangle \quad \rightarrow \quad \langle n(k_1), n(k_2)+1, n(k_3)+1 | \lambda \phi_3 \phi_2 \phi_1 | n(k_1)+1, n(k_2), n(k_3) \rangle$$

- Giving an enhancement

$$\langle n(k_i) | \phi_i \sim \langle n(k_i)+1 | \sqrt{n(k_i)+1}$$

$$|M_0|^2 \rightarrow |M_0|^2 (1+n(k_1))(1+n(k_2))(1+n(k_3))$$

For $n(k_i) \gtrsim 1$ naively decays enhanced

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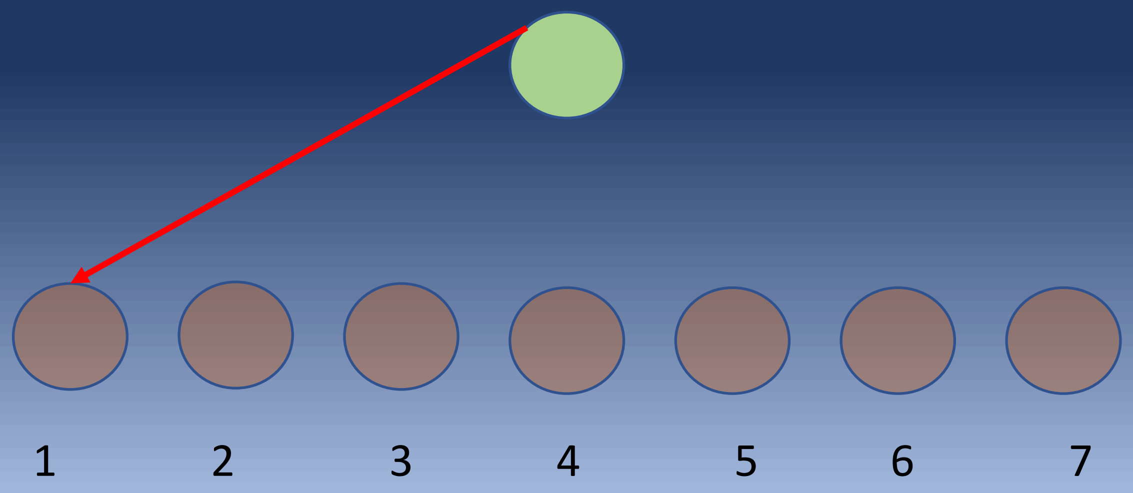
  - Where is the enhancement from?

There is one way



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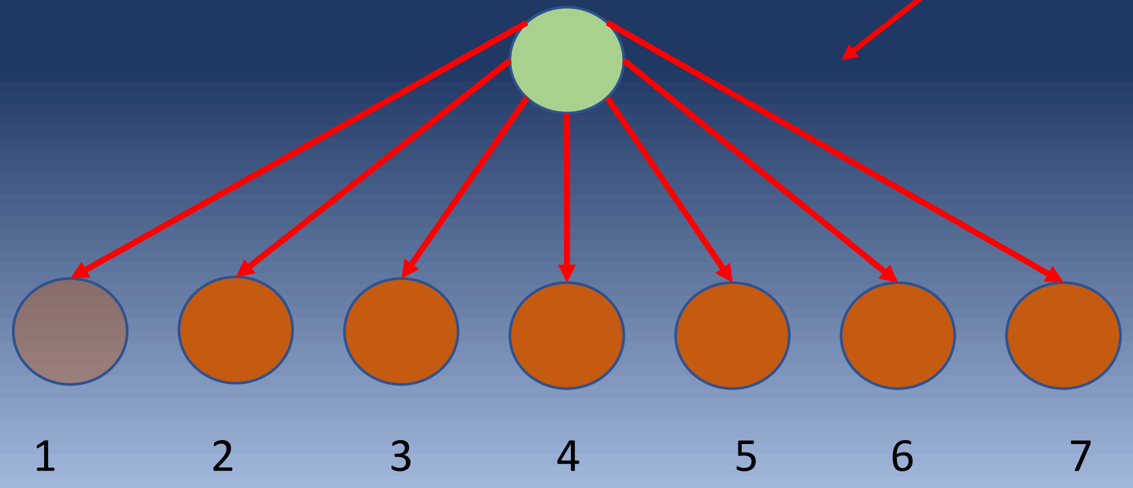
  - ❑ Where is the enhancement from?

Effectively more paths for the particle to take

There are $n = 7$ ways



Bose Enhancement

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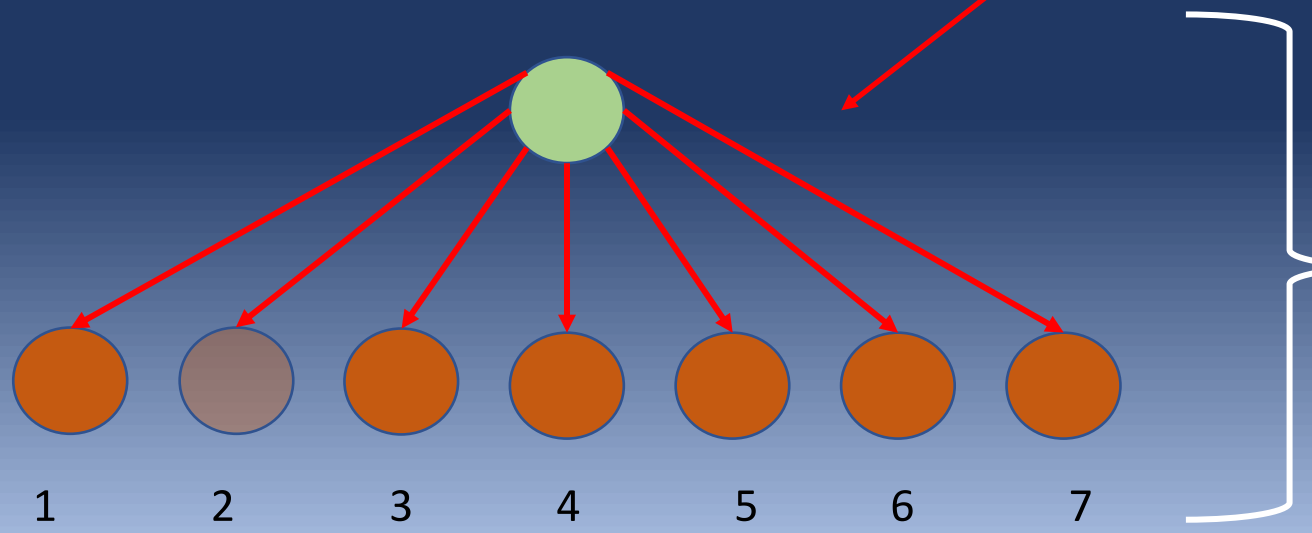

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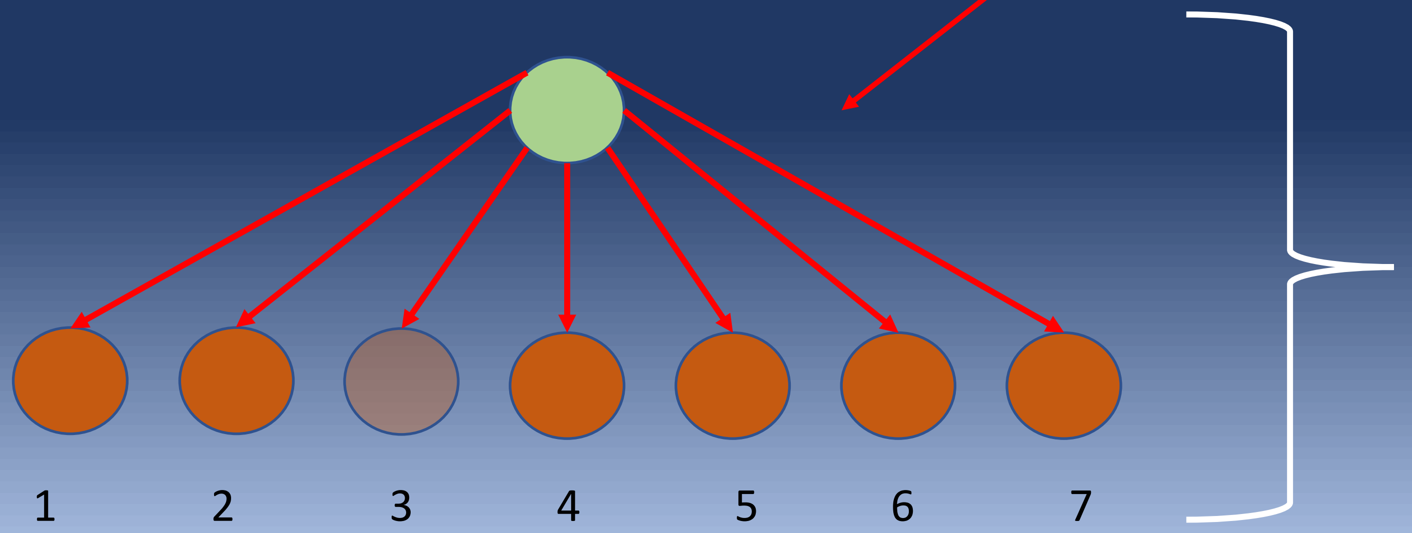

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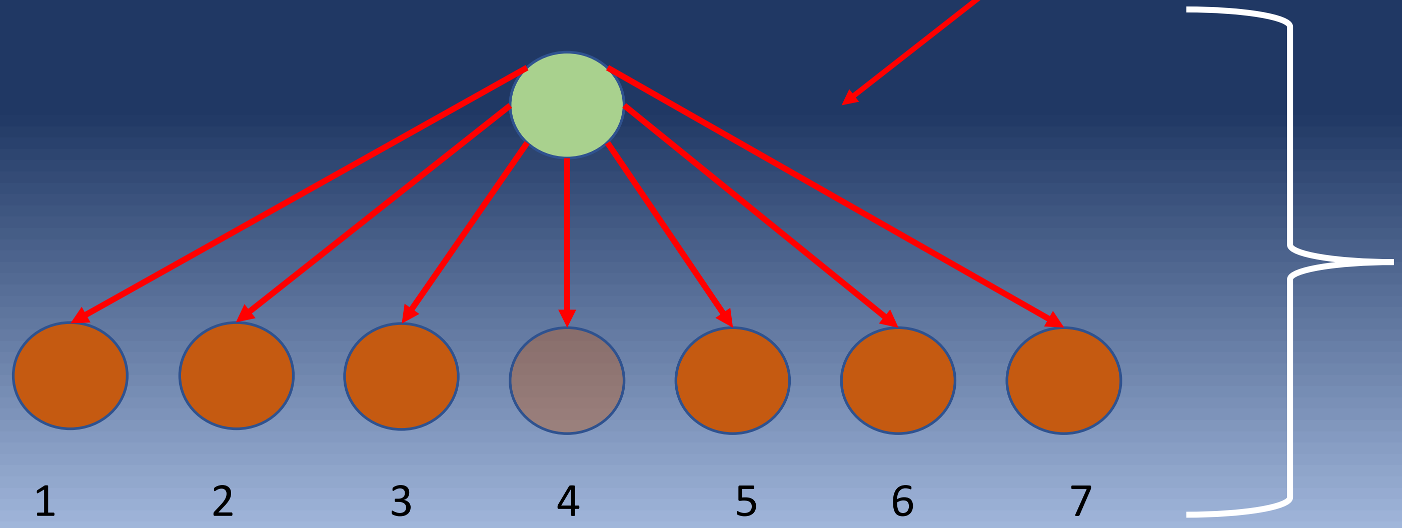

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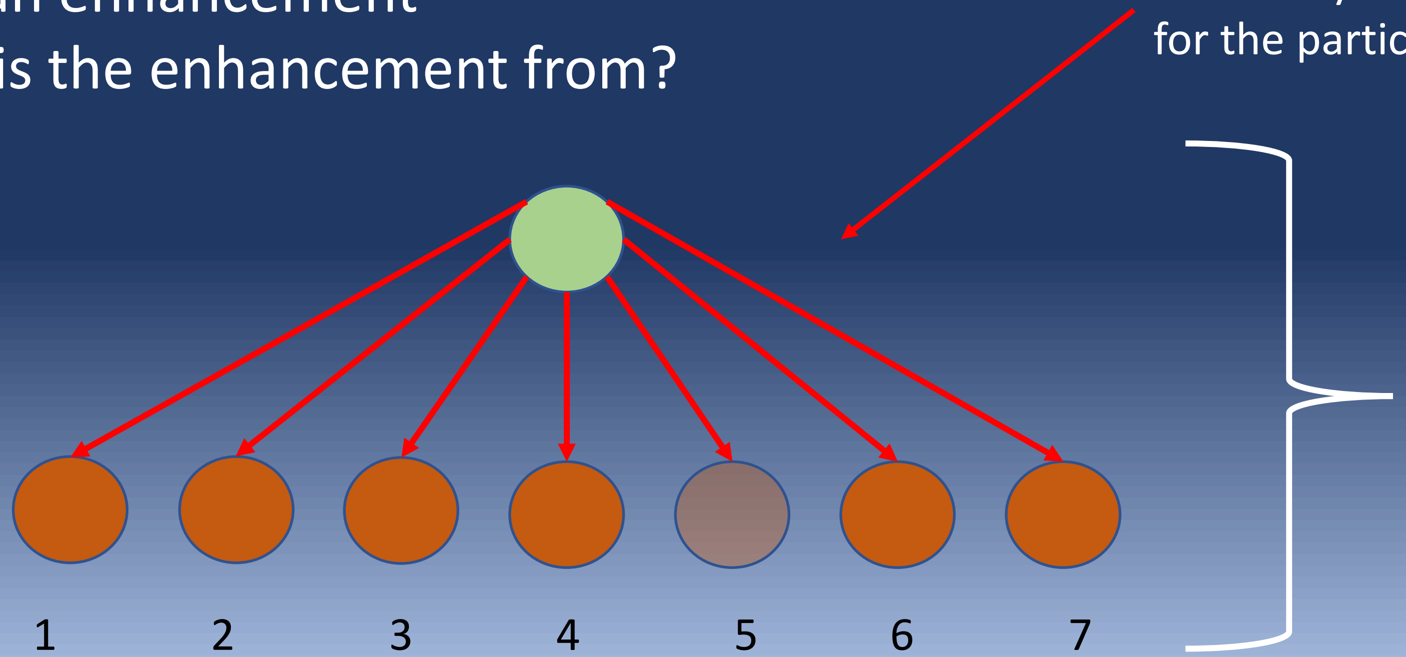

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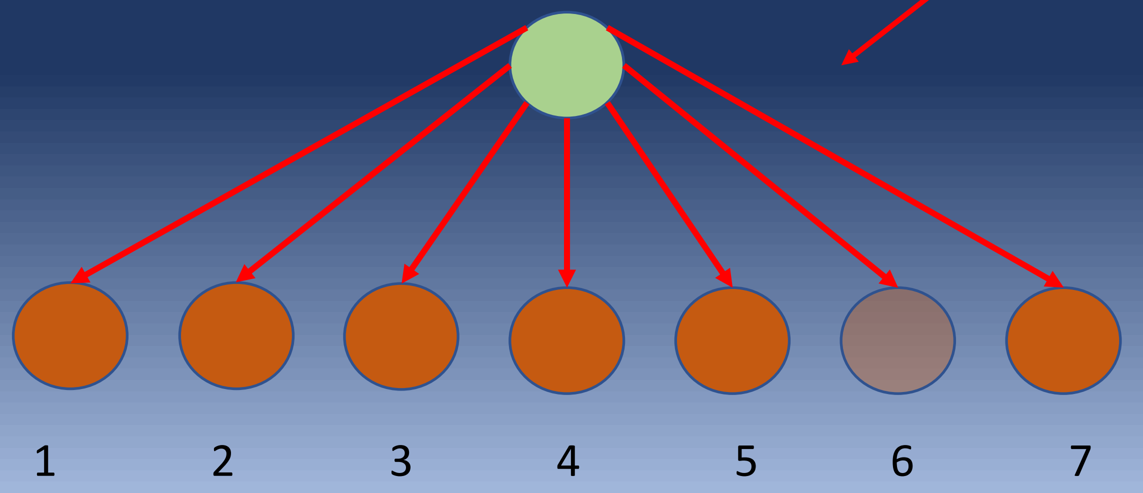

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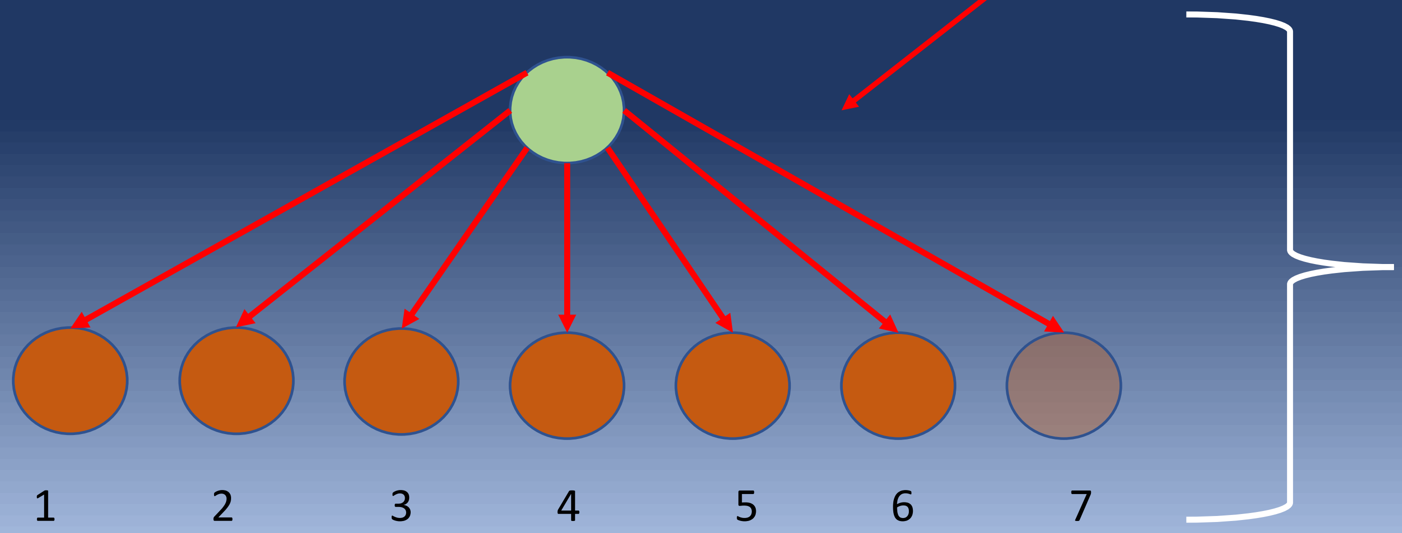

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$$a(k)^n \neq 0$$

- In a background a decay process becomes
  - Giving an enhancement
  - Where is the enhancement from?
  - This ultimately is from indistinguishable particles

Purely Quantum Mechanical

$$\psi_s = \frac{1}{\sqrt{2}}\left[\psi_\alpha(1)\psi_\beta(2) + \psi_\alpha(2)\psi_\beta(1)\right]$$

Identical particles

$$\psi_s = \sqrt{2}\psi_\alpha(1)\psi_\beta(2)$$

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- In a background a decay process becomes
- What happens to the propagator?

$$\langle 0|\phi^\dagger\phi|0\rangle \quad \rightarrow \quad \langle n|\phi^\dagger\phi|n\rangle$$

- Background provides an additional piece

$\propto \delta^3(k-k')$

Can be significantly enhanced

$\propto n\delta^3(k-k')$

No background $\langle 0|a(k)a^\dagger(k')|0\rangle = \langle 0|\left[a(k), a^\dagger(k')\right]|0\rangle$

Background $\langle n|a(k)a^\dagger(k')|n\rangle = \langle n|\left[a(k), a^\dagger(k')\right]|n\rangle + \langle n|a^\dagger(k)a(k')|n\rangle$

n Independent of distribution

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Same form as time ordered thermal propagator in real time formulation.

$$G(k) = \frac{i}{k^2 - m^2 + i\epsilon} - 2\pi n(k)\delta(k^2 - m^2)$$

Correction from background: Will affect loop processes

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$$|n(k_1), n(k_2), \ldots\rangle = \prod_i \left\{ \frac{1}{(2\pi)^3 2\omega_{k_i}[n(\ldots} \right.$$

$\neq 0$

- In a background a de…
- What happens t…

…$|n\rangle$

…itional piece

All processes potentially enhanced by background

$$\ldots \frac{\ldots}{\ldots m^2+i\epsilon} - 2\pi n(k)\delta(k^2 - m^2)$$

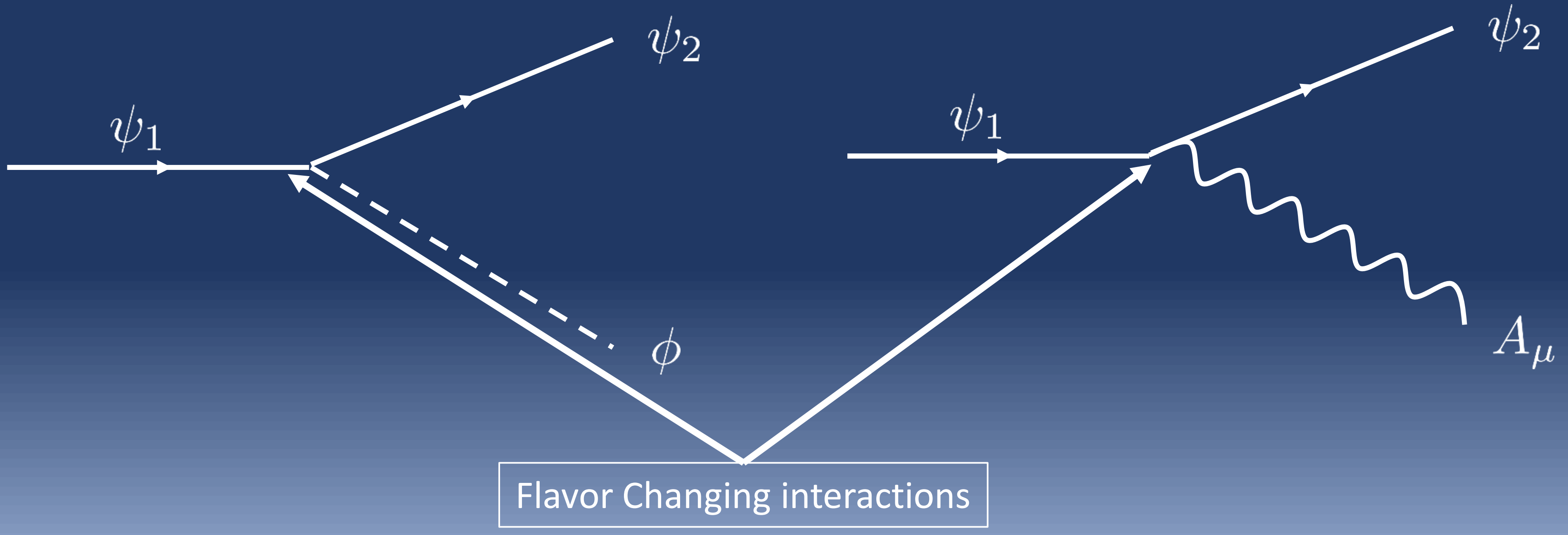
Same as time ordered thermal propagator in real time formulation.

Correction from background: Will affect loop processes

What happens for decays?

- Two body decays
  - Coupling hard to realize

$$m_{\psi_1} > m_{\psi_2}$$



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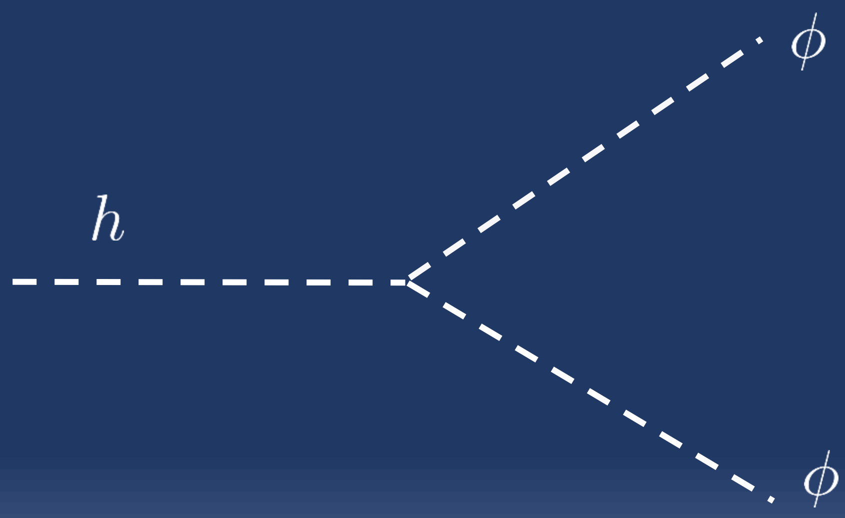
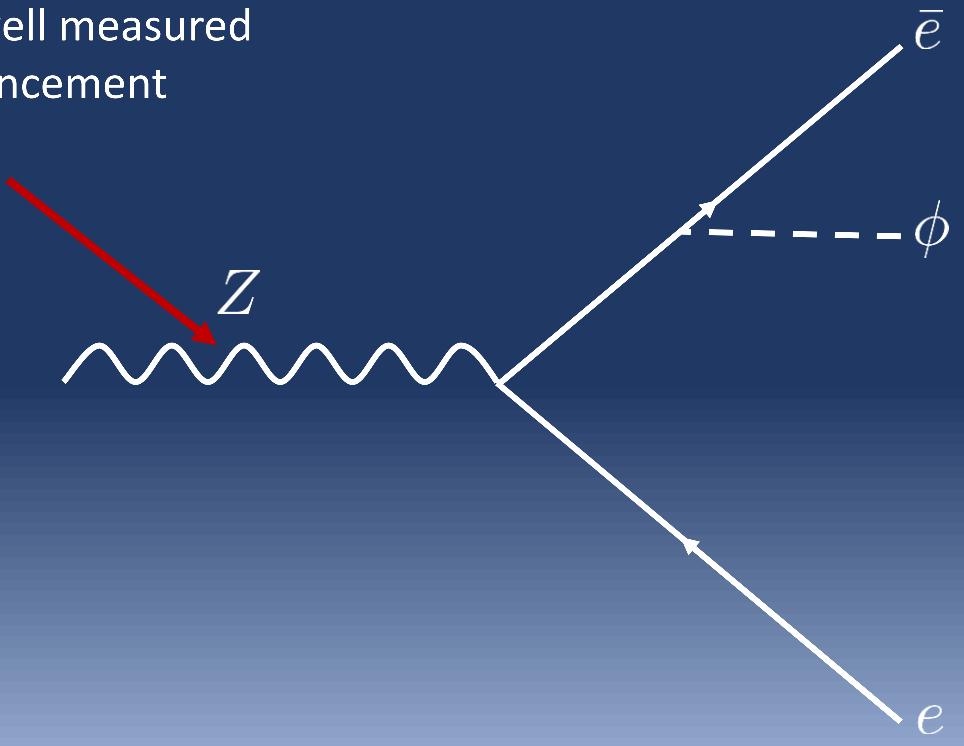


Makes it hard to keep ϕ light

What happens for decays?

- ❑ Two body decays
- ❑ Three body decays
  - ❑ Many possibilities!!

- ❑ Z decays well measured
- ❑ Small enhancement detectable



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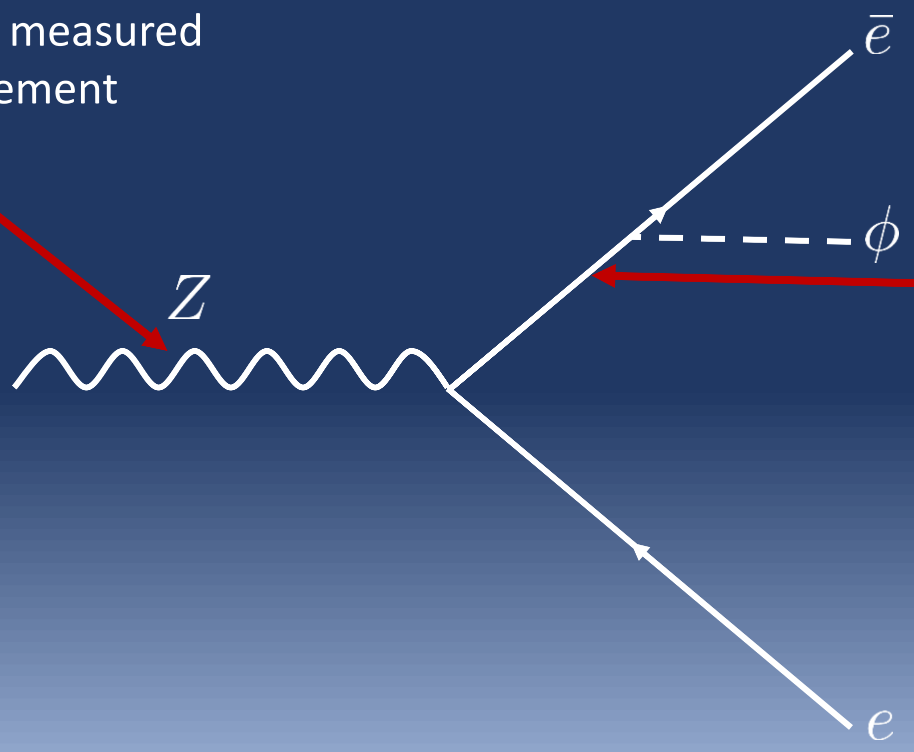

Warning

- ❑ Propagator on shell when ϕ momentum goes to zero
- ❑ IR divergences must cancel

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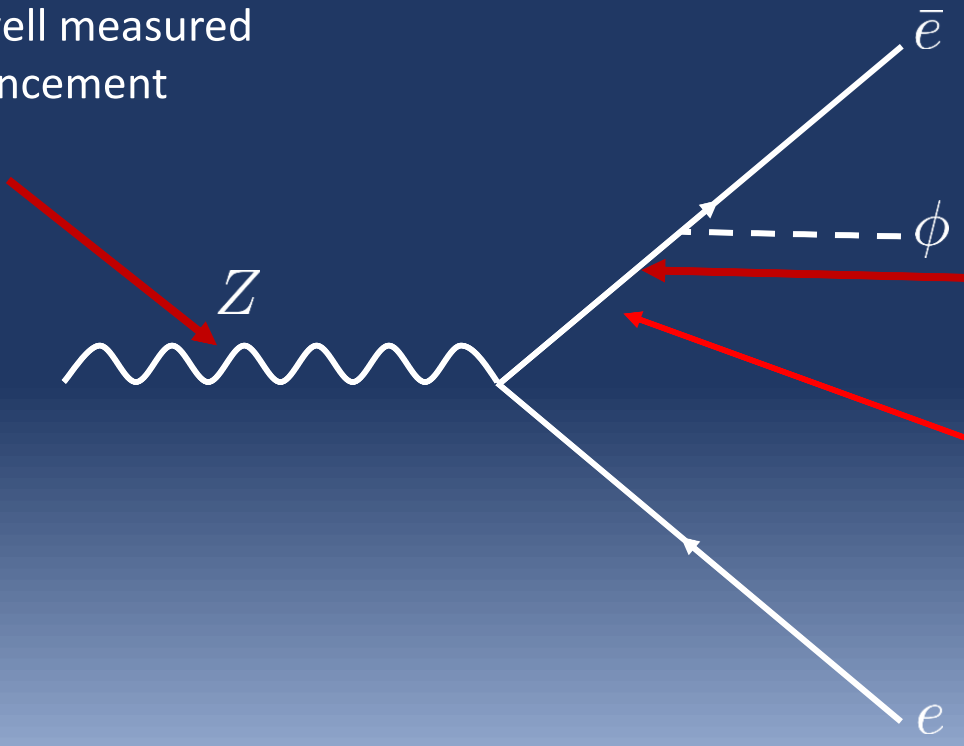

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Effectively Bremsstrahlung

What happens for decays?

- ❑ Two body decays
- ❑ Three body decays
  - ❑ Many possibilities!!
    - ❑ IR divergences in finite temperature
    - ❑ Have to include wavefunction renormalization

- ❑ Power Divergent IR singularity
- ❑ Shows up in decay too

$$\Sigma_\beta \supset \frac{\alpha}{4\pi^2} I_A(k)(\gamma \cdot k - m_e) \qquad I_A(k) = 8\pi \int \frac{dq}{q} n(E_q) \qquad \lim_{q\to 0} n(E_q) \sim \frac{1}{q}$$

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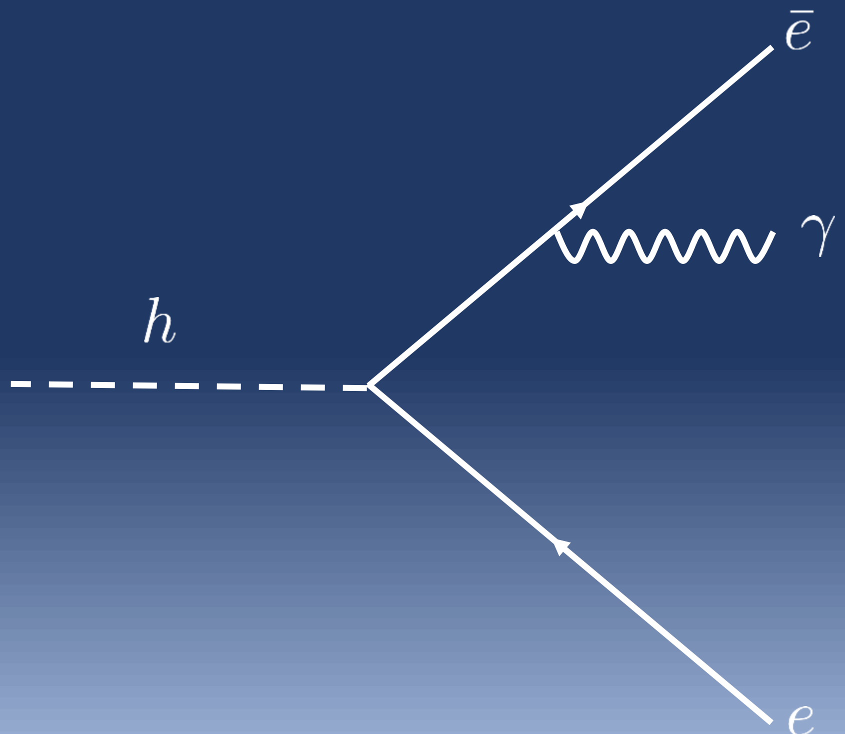
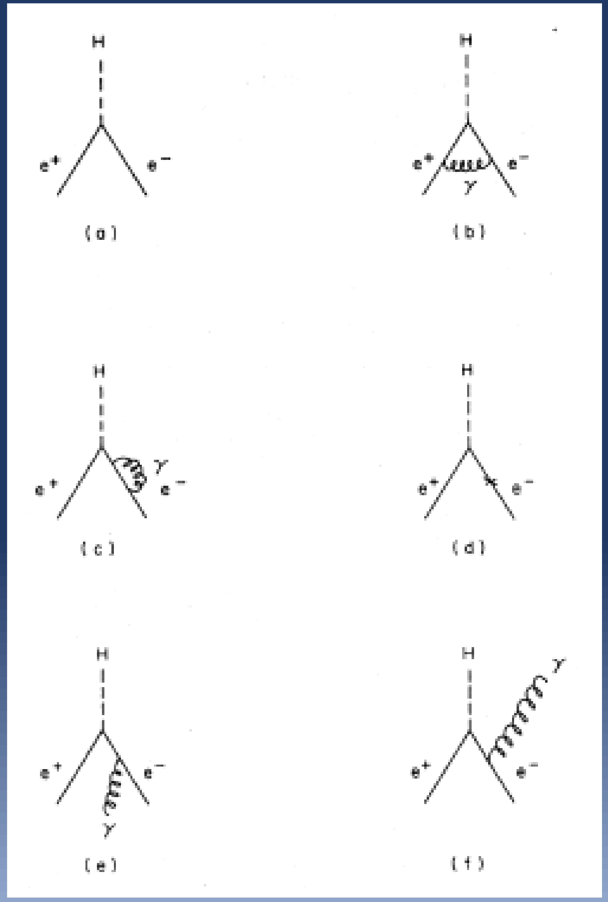
  - Example: Higgs Decay in Thermal Bath





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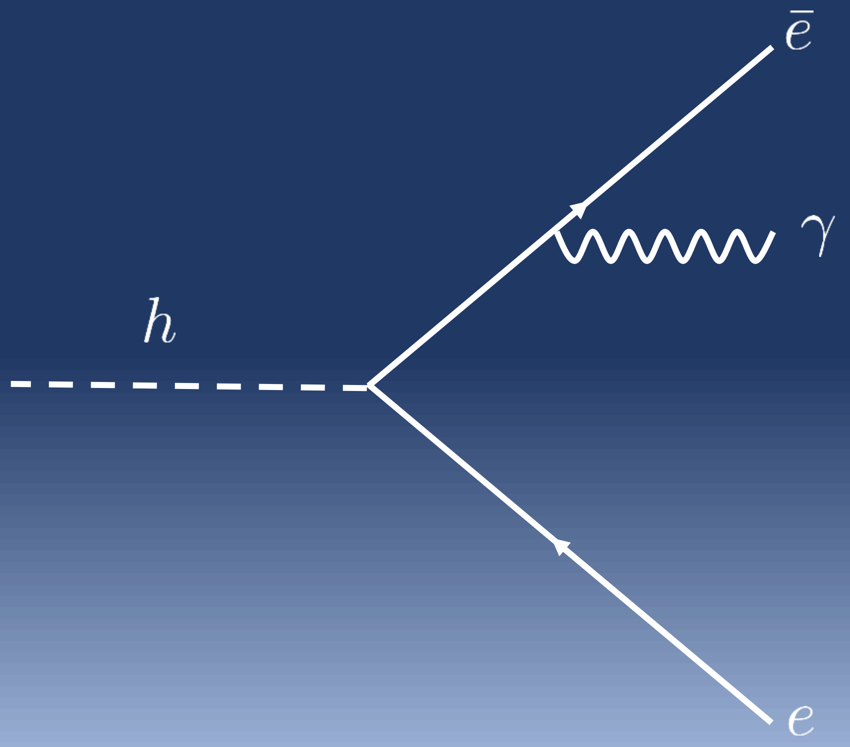
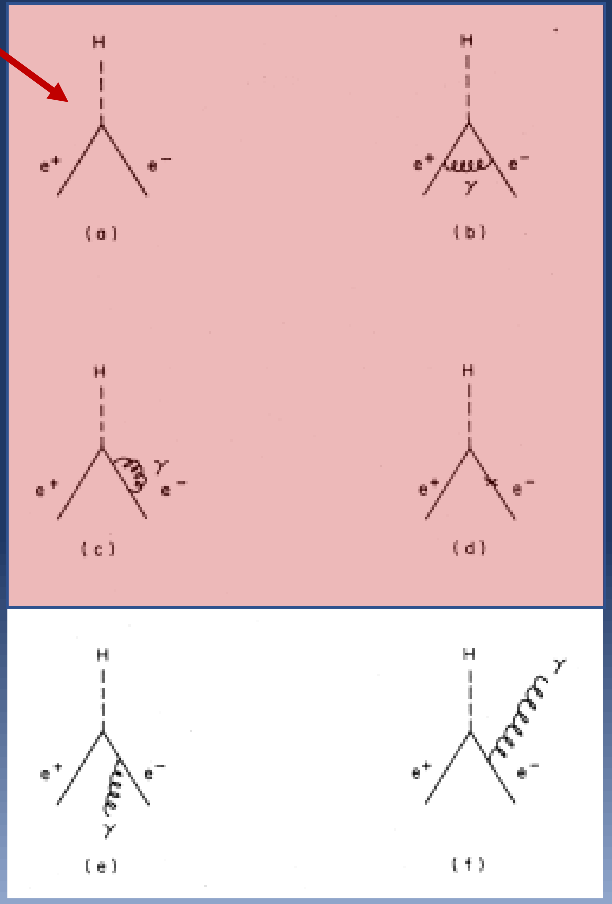


- ❑ Vertex Corrections
  - ❑ Background enhanced
- ❑ Same for $k \to 0$

Correction to this decay



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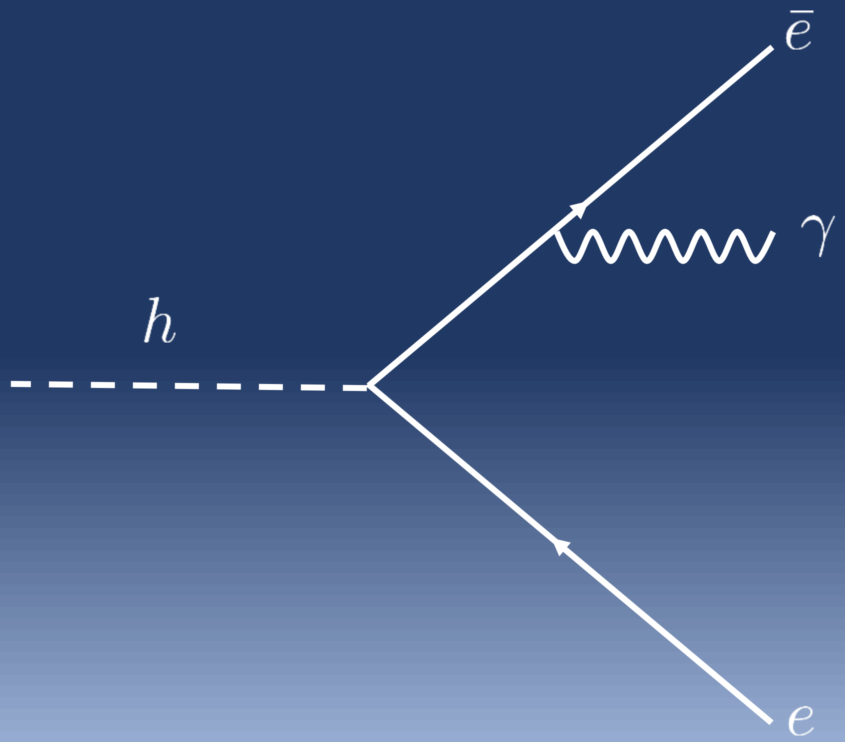
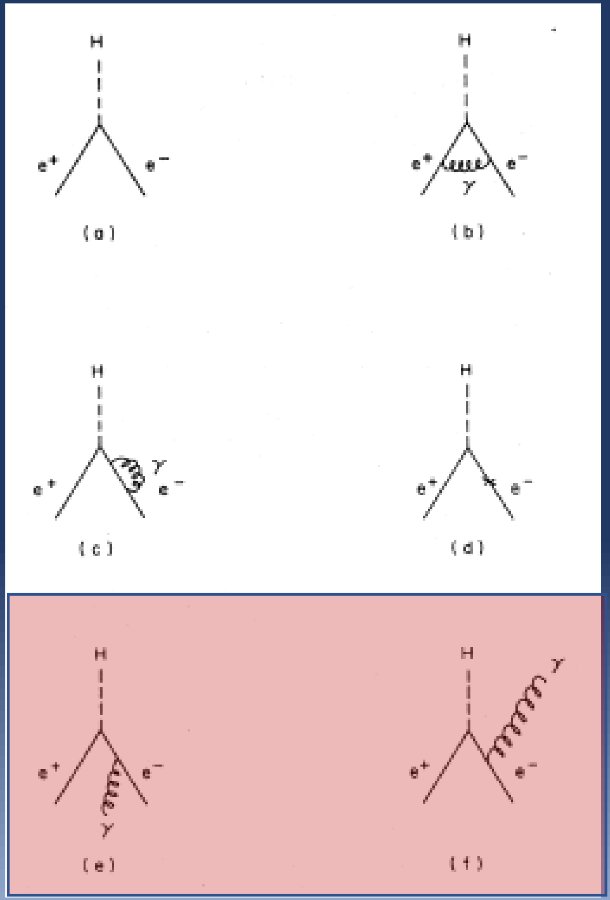


$$|M_{SE/SA}|^2 \sim M^0_{ver} M^\alpha_{ver}$$

- ❑ Stimulated Emission/Absorption
- ❑ Background enhanced



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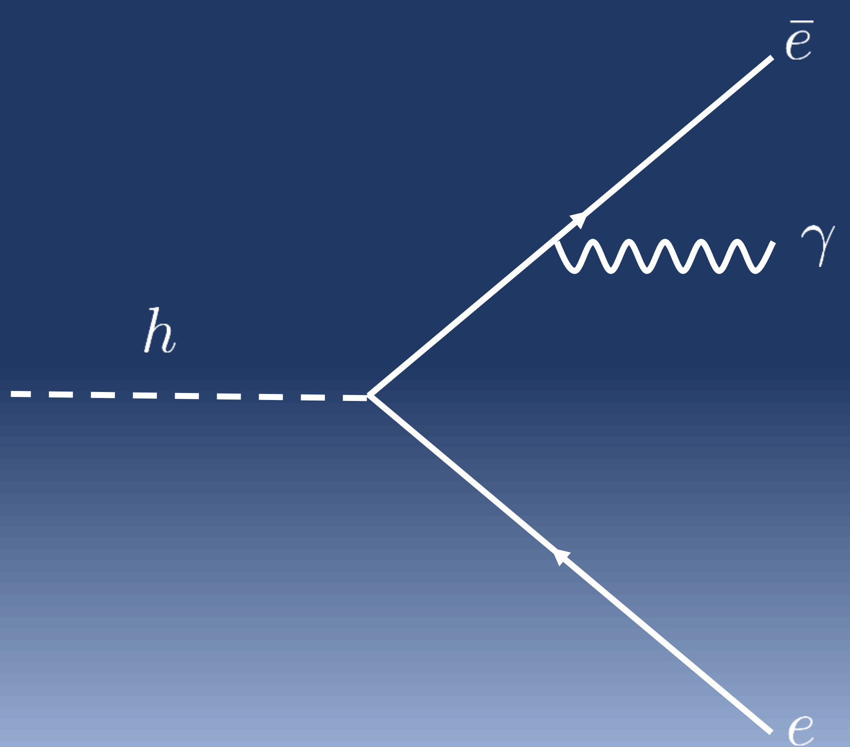


$$|M_{SE/SA}|^2 \sim M^0_{ver} M^\alpha_{ver}$$

- ❑ Enhancement completely cancels at 1-loop

Donoghue, Holstein (1983)

$$\Gamma_{T=0}(h \to e^+ e^- \gamma) = \Gamma_{T\neq 0}(h \to e^+ e^- \gamma) + \mathcal{O}(T^3)$$

- ❑ Cancellation of IR divergences expected

Bremsstrahlung cancelations

$$\frac{d\Gamma}{dk} = n_B(k)\left[\frac{1}{k}R_{-1} + R_0 + kR_1 + O(k^2)\right]$$

$$n_k(k) \sim \frac{1}{k}$$

- ❑ Finite piece goes as

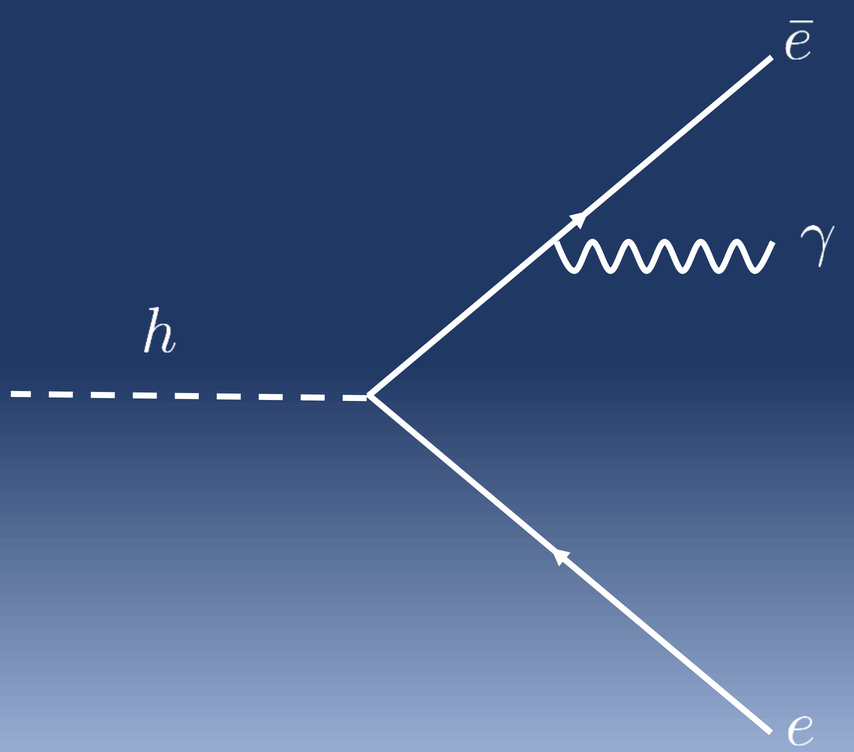
$$\int dk R_1 \left(k n_k(k)\right) \quad \sim \quad \int dk R_1 k^0$$

- ❑ What if it is a background

$$n_k(E_k) \sim \frac{\rho_{DM}}{\Delta k k^2 m_{DM}} \quad \to \int dk k^2 n_k(E_k) \delta(k^2 - m^2_{DM}) \sim \frac{\rho_{DM}}{m^2_{DM}}$$

What happens for decays?

- ❑ Two body decays
- ❑ Three body decays
  - ❑ Many possibilities!!
  - ❑ Example: Higgs Decay in Thermal Bath



$$|M_{SE/SA}|^2 \sim M^0_{ver} M^\alpha_{ver}$$

- ❑ Enhancement completely cancels at 1-loop

$$\Gamma_{T=0}(h \to e^+e^-\gamma) = \Gamma_{T\neq 0}(h \to e^+e^-\gamma) + \mathcal{O}(T^3)$$

- ❑ Cancellation of IR divergences expected

$$\frac{d\Gamma}{dk} = n_B(k)\left[\frac{1}{k}R_{-1} + R_0 + kR_1 + O(k^2)\right] \qquad \sim \mathcal{O}(T^2)$$

- ❑ Not IR divergent, why does it cancel?

This vanishing appears to be accidental, but we have also calculated the radiative corrections for the decay of a pseudoscalar H (instead of scalar) and found that to be zero also.

Donoghue, Holstein (1983)

What happens to loop processes?

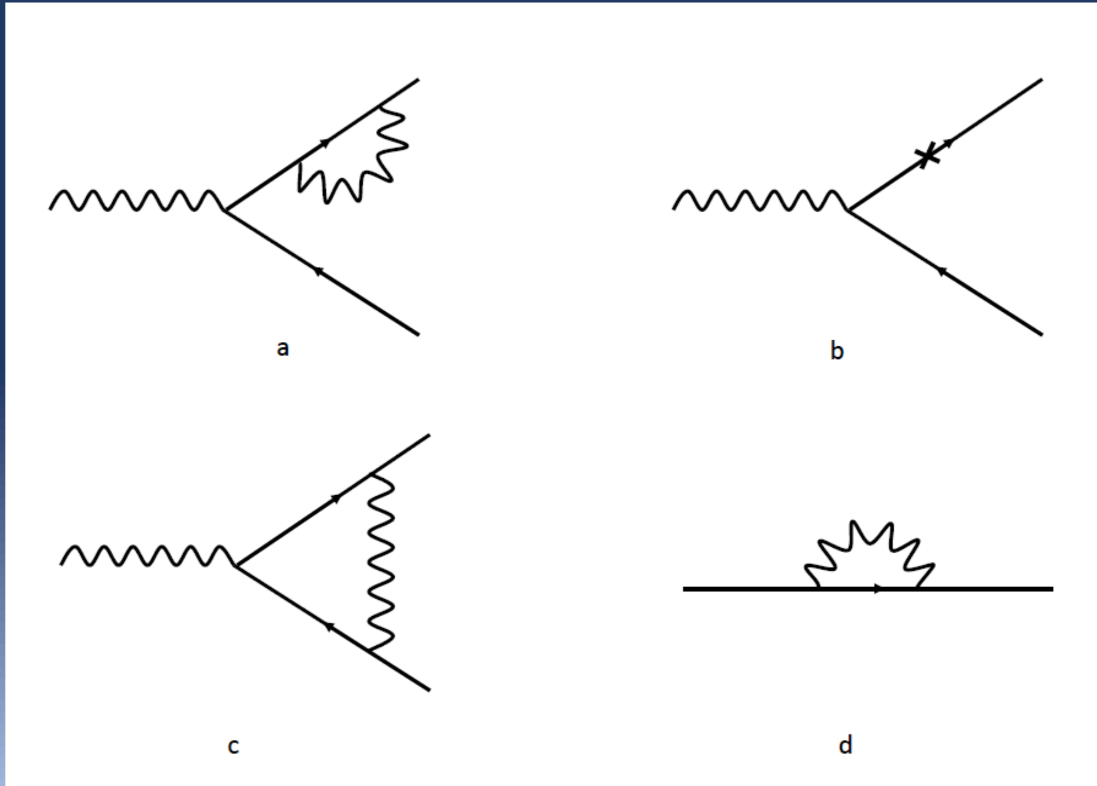
- What will give us the most “bang for our buck”?
  - First generation particles in loop (Denominator then $m_{e,d,u}^2 m_{DM}^2$)

$$\frac{\rho_{DM}}{m_i^2 m_{DM}^2}$$ ← Dimensional analysis and IR finite

  - Precision measurement, more sensitive to everything

What happens to loop processes?

- What will give us the most “bang for our buck”?
  - First generation particles in loop (Denominator then $m_{e,d,u}^2 m_{DM}^2$)
  - Precision measurement, more sensitive to everything
- $g-2$ of electron perfect
- Plan
  - Calculate $g-2$ of the electron
  - Show that charge is not renormalized in background
  - Show ward identities satisfied in background

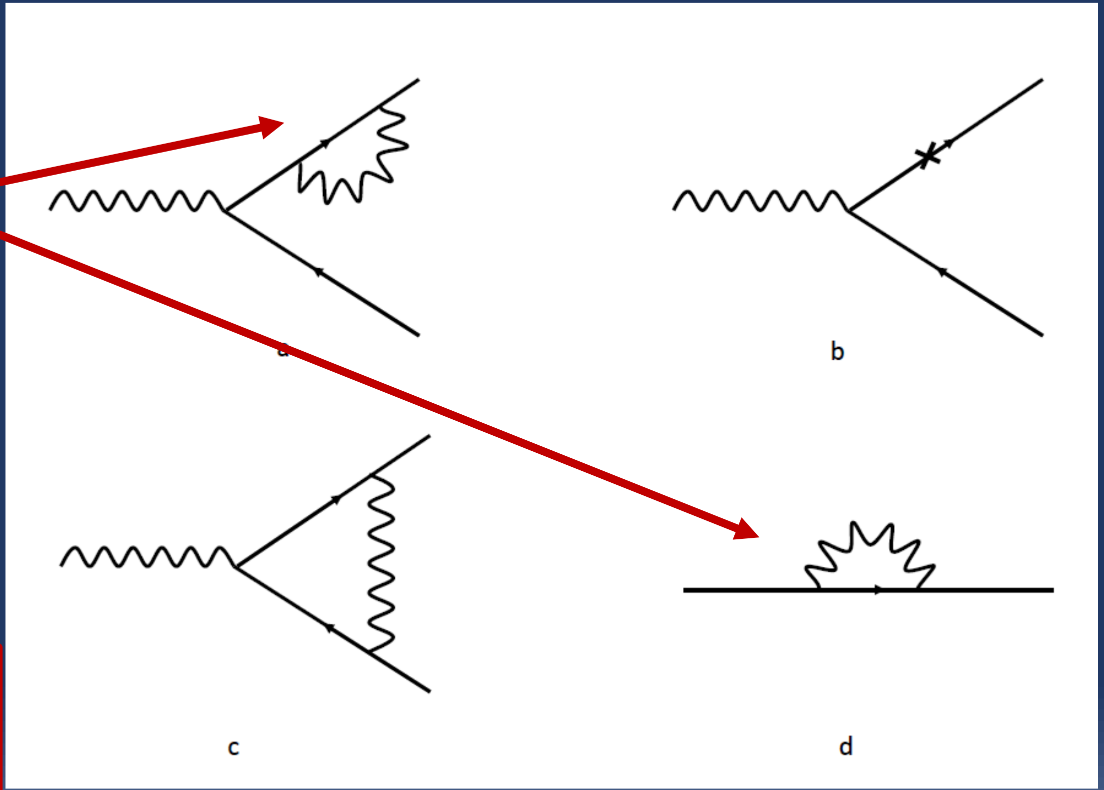


Wave Function Renormalization

- ❑ Start with wave function renormalization

$$\Sigma_n = B(k) + C(k)\,(\gamma \cdot k - m_e) + \gamma \cdot D(k)$$

- ❑ New type of contribution
  - ❑ Violates Lorentz symmetry
  - ❑ Can't be combined to C(k) or B(k)



$$D^\mu(k) = -2e^2\chi^2 \int \frac{d^4q}{(2\pi)^3} \frac{q_\mu}{q^2 + 2q\cdot k + k^2 - m_e^2} n(E_q)\delta(q^2 - m_{DM})$$

$$B(k) = \quad 2e^2\chi^2 \int \frac{d^4q}{(2\pi)^3} \frac{m_e}{q^2 + 2q\cdot k + k^2 - m_e^2} n(E_q)\delta(q^2 - m_{DM})$$

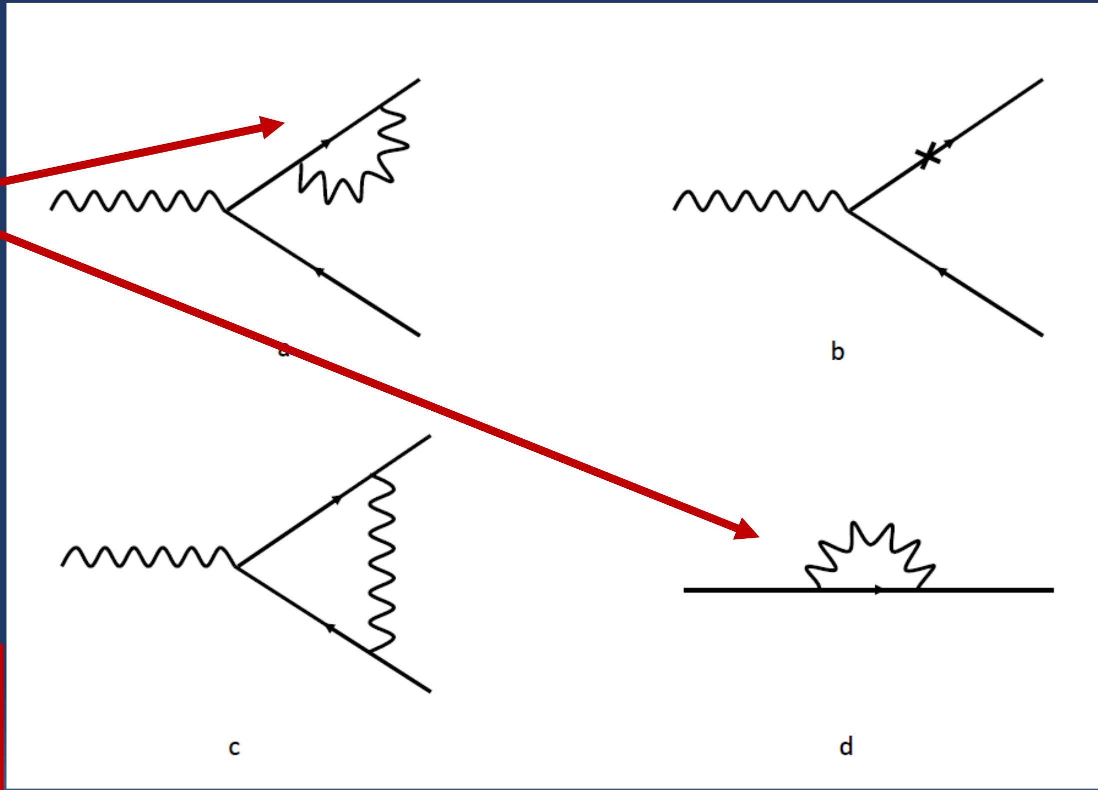
$$C(k) = -2e^2\chi^2 \int \frac{d^4q}{(2\pi)^3} \frac{1}{q^2 + 2q\cdot k + k^2 - m_e^2} n(E_q)\delta(q^2 - m_{DM})$$

Do no specify $n(E_q)$ until end

Wave Function Renormalization

- Start with wave function renormalization

$$\Sigma_n = B(k) + C(k)\,(\gamma \cdot k - m_e) + \gamma \cdot D(k)$$



- New type of contribution
  - Violates Lorentz symmetry
  - Can’t be combined to C(k) or B(k)

Lorentz violating

$$D^\mu(k) = -2e^2\chi^2 \int \frac{d^4q}{(2\pi)^3} \frac{q_\mu}{q^2 + 2q \cdot k + k^2 - m_e^2} n(E_q)\delta(q^2 - m_{DM})$$

$$B(k) = \ \ 2e^2\chi^2 \int \frac{d^4q}{(2\pi)^3} \frac{m_e}{q^2 + 2q \cdot k + k^2 - m_e^2} n(E_q)\delta(q^2 - m_{DM})$$

$$C(k) = -2e^2\chi^2 \int \frac{d^4q}{(2\pi)^3} \frac{1}{q^2 + 2q \cdot k + k^2 - m_e^2} n(E_q)\delta(q^2 - m_{DM})$$

Do no specify $n(E_q)$ until end

Gauge mixing parameter

What happens to loop processes?

- ❑ Start with wave function renormalization

$$\Sigma_n = B(k) + C(k)\,(\gamma \cdot k - m_e) + \gamma \cdot D(k)$$

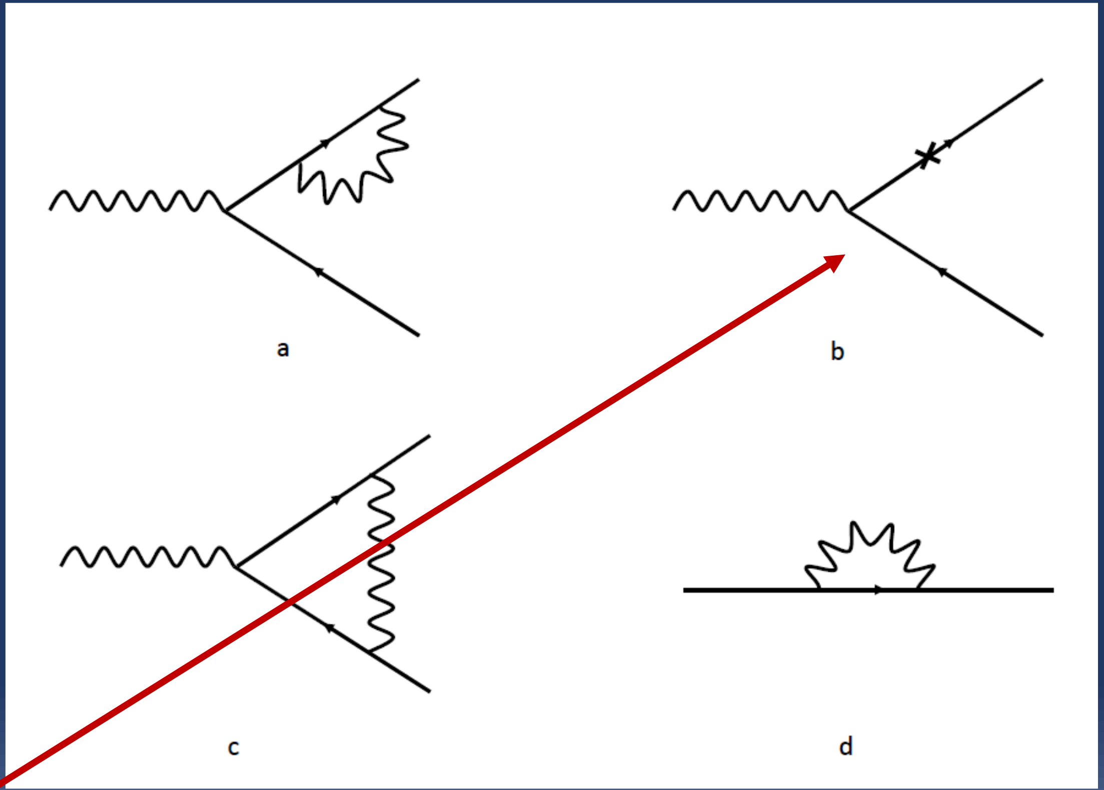
- ❑ New type of contribution
  - ❑ Violates Lorentz symmetry
  - ❑ Can't be combined to C(k) or B(k)

- ❑ Use a background dependent spinor

$$\left[\gamma \cdot k - m - \frac{\alpha}{4\pi^2}\left(\gamma \cdot D(k) + B(k)\right)\right]\psi_n = 0$$

  - ❑ So that we have "mass counterterms"

$$\text{“}\delta m\text{”} = \frac{\alpha}{4\pi^2}\left(\gamma \cdot D(k) + B(k)\right)$$



Not a real counterterm, just use it like a counterterm.
No new divergences

What happens to loop processes?

- Total Vertex correction

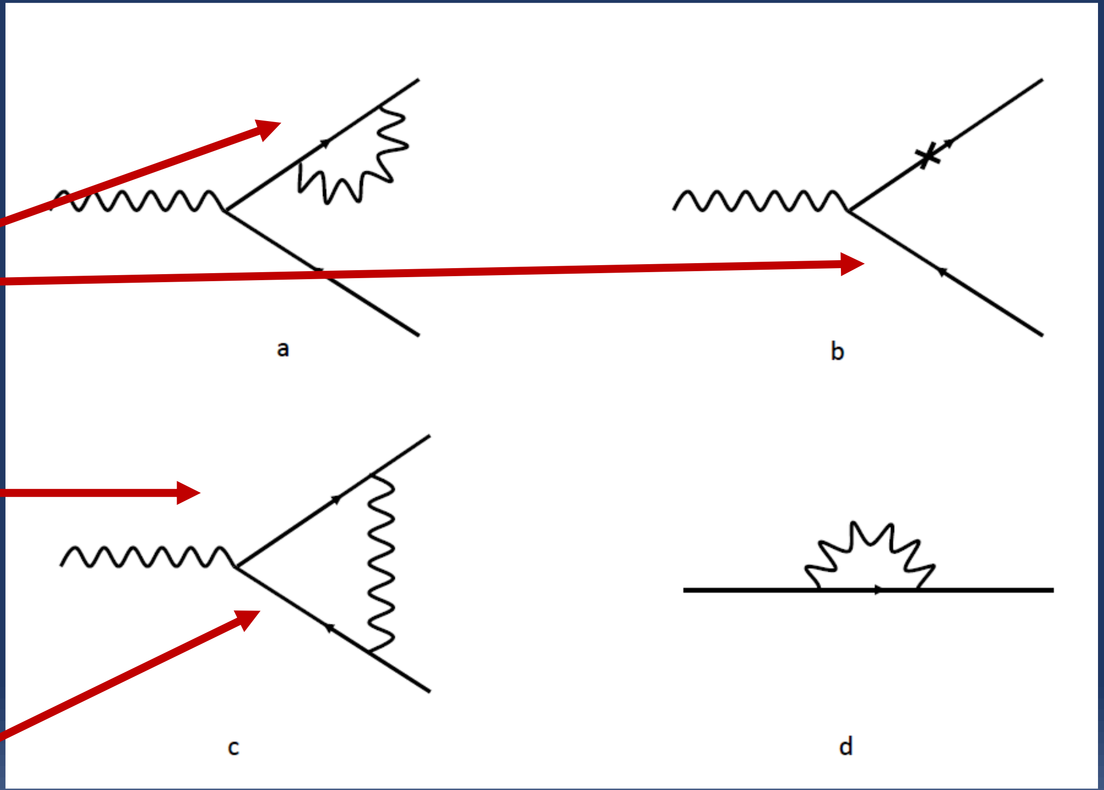
$$iM_{TOT\mu} = -ie\bar{u}_n(\bar{k})\left[\gamma_\mu\left[1\right.\right.$$

$$\left.-\frac{1}{2}\frac{1}{E}\frac{d}{dE}\left(m_e B(k) + k_\nu D^\nu(k)\right) + \frac{1}{2}\frac{D^0(k)}{E} + (k \leftrightarrow \bar{k})\right]$$

$$+\left[\frac{1}{2}\frac{d}{dk_\mu}\left[B(k) + \frac{k_\nu D^\nu(k)}{m_e}\right] - \frac{D^\mu(k)}{2m_e}\right.$$

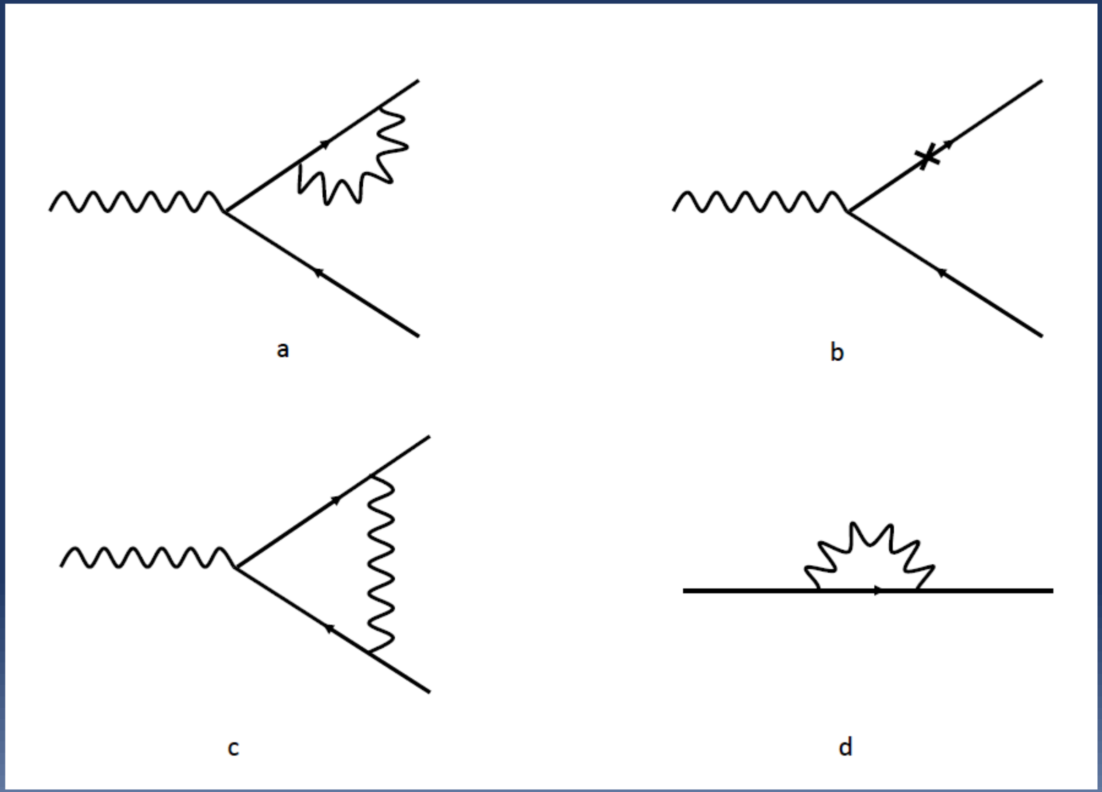
$$\left.\left.+\frac{[\gamma_\alpha, \gamma_\nu]_- \Delta k_\alpha}{8m_e}\frac{dD^\nu(k)}{dk_\mu} + (k \leftrightarrow \bar{k})\right] + F_\mu(\Delta k)\right] u_n(k)$$

$$F_\mu(\Delta k) = -e^2\chi^2 \int \frac{d^4\Pi_q}{(2\pi)^3} \frac{2\,(\gamma\cdot q + m_e)\,[\gamma\cdot\Delta k, \gamma_\mu] - 4\gamma_\mu\,[\Delta k_\alpha q^\alpha]}{(q^2 + 2q_\nu k^\nu)^2},$$



Charge Non-Renormalization

- Charge non-renormalization
  - Apply $\Delta k = 0,\ \mu = 0$ to vertex



$$i\, M_{TOT_0}|_{\Delta k=0} = -ie\bar{u}_n(\bar{k}) \left[\gamma_0 \left[1 + \left[-\frac{m_e}{E}\gamma_0 + 1\right] \left[\frac{d}{dE}\left(B(k) + \frac{k_\nu D^\nu(k)}{m_e}\right) + \frac{D^0(k)}{E}\right]\right] u_n(k)\right.$$

Cancels Due to Gordon Decomposition for $\Delta k = 0$

$$i\, M_{TOT_0}|_{\Delta k=0} = -ie$$

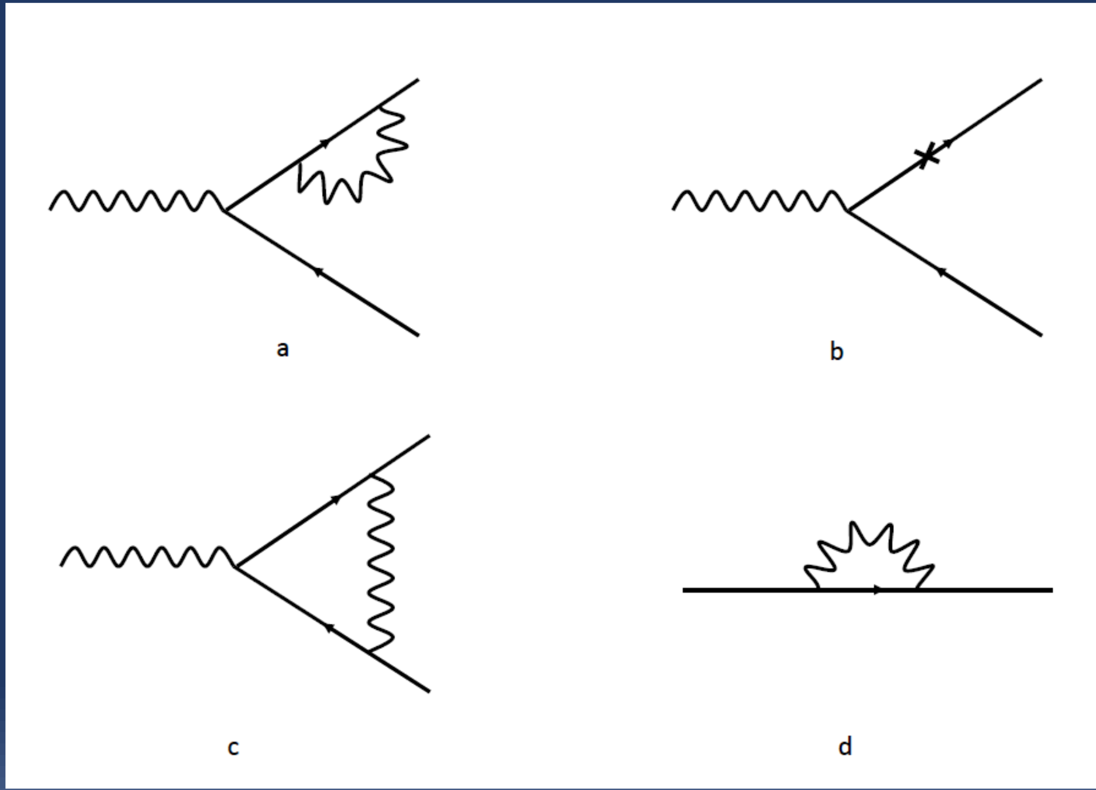
Ward Identities

- Ward Identities apply Δk^{μ} to $M_{TOT_{\mu}}$
  - Use slightly different form of $M_{TOT_{\mu}}$

$$\Delta k = \bar{k} - k$$

$$iM_{TOT\mu} = -ie\bar{u}_n(\bar{k})\left[\gamma_\mu\left[1 - \frac{1}{2}\frac{1}{E}\frac{d}{dE}\left(m_e B(k) + k_\nu D^\nu(k)\right) + \frac{1}{2}\frac{D^0(k)}{E} + (k \leftrightarrow \bar{k})\right] + \left[\frac{1}{2}\frac{d}{dk_\mu}\left[B(k) + \frac{k_\nu D^\nu(k)}{m_e}\right] - \frac{D^\mu(k)}{2m_e} + \frac{[\gamma_\alpha,\gamma_\nu]_- \Delta k_\alpha}{8m_e}\frac{dD^\nu(k)}{dk_\mu} + (k \leftrightarrow \bar{k})\right] + F_\mu(\Delta k)\right] u_n(k)$$

Come From Gordon Decomposing → $\gamma \cdot \frac{dD}{dk^{\mu}}$



Ward Identities

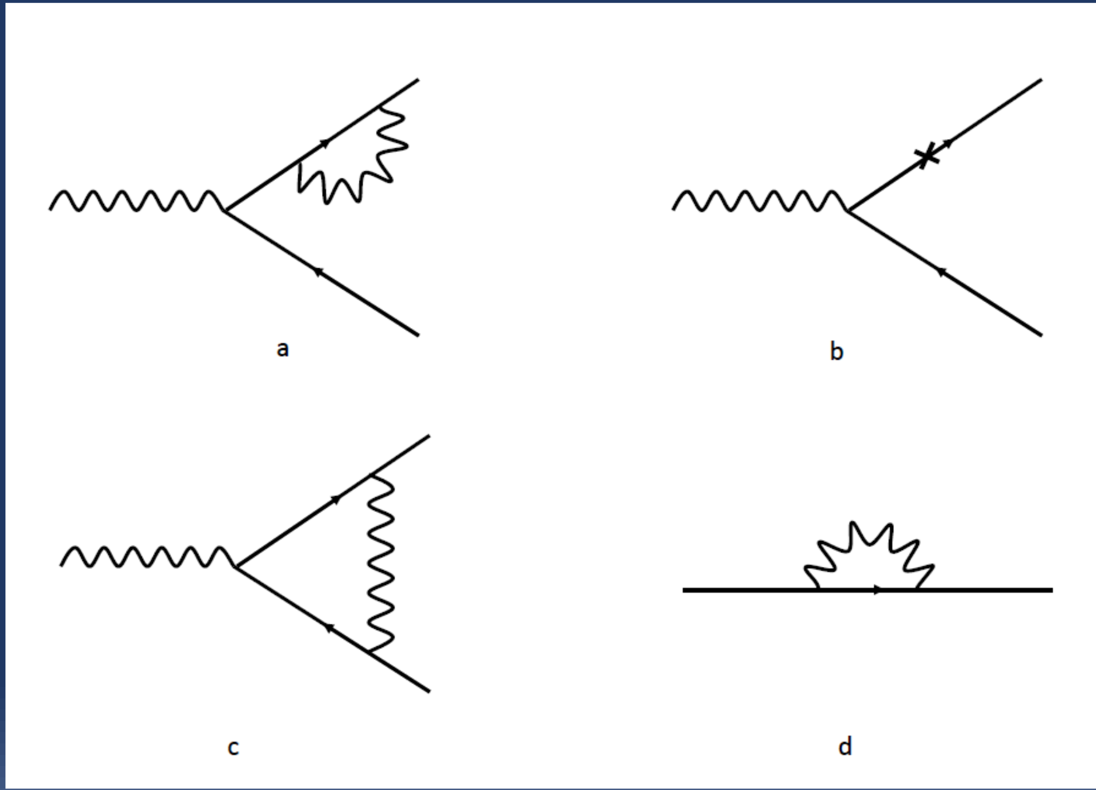
- Ward Identities apply Δk^{μ} to $M_{TOT_{\mu}}$
  - Use slightly different form of $M_{TOT_{\mu}}$
    - Generically true to order Δk^3

$$\Delta k = \bar{k} - k$$

$$\Delta k^3$$

$$\Delta k^{\mu} M_{TOT\mu} = -e\bar{u}_n(\bar{k})$$
$$\times \left[B(k) - B(\bar{k}) + \Delta k^{\mu} \left[\frac{dB(k)}{dk_{\mu}} + \frac{dB(\bar{k})}{d\bar{k}_{\mu}} \right] \right.$$
$$\left. + \gamma_{\nu} \left[D^{\nu}(k) - D^{\nu}(\bar{k}) + \Delta k^{\mu} \left[\frac{dD^{\nu}(k)}{dk_{\mu}} + \frac{dD^{\nu}(\bar{k})}{d\bar{k}_{\mu}} \right] \right] \right] u_n(k)$$

$$\Delta k^3$$



Background Contribution to g-2

- Simplified Total Vertex correction

$$iM_{TOT_\mu} = -ie\bar{u}(\bar{k})\Big[\gamma_\mu\Big[1+\frac{1}{2}\Big[\frac{D^0(k)}{E_k}+\frac{D^0(\bar{k})}{E_{\bar{k}}}\Big] - R\frac{m_e}{E_k}\bar{I}_0(k) - R\frac{m_e}{E_{\bar{k}}}\bar{I}_0(\bar{k}) - \frac{R}{2m_e}\Delta k_\nu \bar{I}^\nu(k)\Big] - \frac{D_\mu(k)}{2m_e} - \frac{D_\mu(\bar{k})}{2m_e} + R\left[\bar{I}_\mu(k)+\bar{I}_\mu(\bar{k})\right] + \Big[I_\mu^\nu - R\frac{k_\mu+\bar{k}_\mu}{m_e}\bar{I}^\nu(k) + 2R\gamma\cdot\bar{I}(k)\delta_\mu^\nu + 2m_e R\delta_\mu^\nu I_A(k)\Big]\times\frac{[\gamma_\alpha,\gamma_\nu]_-\Delta k^\alpha}{4m_e}\Big]u(k),$$

$$R = \left(\frac{m_{DM}}{m_e}\right)^2 \qquad g-2$$

$$I_A(k) = e^2\chi^2\int\frac{d^4\Pi_q}{(2\pi)^3}\frac{2m_e}{(2q\cdot k)^2} \sim \int dq\frac{N(E_q)}{q}$$

$$N(E_q)|_{\mathrm{IR}} \sim \frac{1}{q}$$

$$\bar{I}_\mu(k) = e^2\chi^2\int\frac{d^4\Pi_q}{(2\pi)^3}\frac{4q_\mu m_e^3}{(2q\cdot k)^3} \sim \int dq\frac{N(E_q)}{q}$$

$$I_{\mu,\nu} = e^2\chi^2\int\frac{d^4\Pi_q}{(2\pi)^3}\frac{q_\mu q_\nu}{(q\cdot k)^2} \sim \int dqqN(E_q)$$

$$d^4\Pi_q = d^4q\,\bar{n}(E_q)\delta(q^2-m_{DM}^2)$$

Relativistic Hamiltonian

- Magnetic Field (Approximate Penning trap)

$$A^0 = 0 \qquad \vec{A} = \frac{1}{2}\vec{B} \times \vec{r}$$

- Momentum Integrals ($\frac{|q|^2}{E_q^2} \ll 1$)

Other components suppressed by β_{DM}^2

determines size of effect

$$R I_A(k) = \frac{\delta m_n}{2m_e^2}\left(\frac{m_e}{E_k}\right)^2 \qquad R\bar{I}_0(k) = \frac{\delta m_n}{2m_e}\left(\frac{m_e}{E_k}\right)^3 \qquad \delta m_n = \frac{e^2\chi^2}{(2\pi)^3}\frac{1}{m_e m_{DM}}\int d^3q\,\bar{n}(E_q)$$

$$\sim \frac{\rho_{DM}}{m_e m_{DM}^2}$$

- Relativistic Hamiltonian
  - Must use corrections to frequencies and compare to experiment

$$H'_T = E_\beta - \frac{e}{2E_\beta}\left[\vec{L}\cdot B + \vec{\sigma}\cdot B\right]\left[1 - 2R\frac{m_e}{E_k}\bar{I}^0(k)\right] + \frac{eR}{2E_p}\left[\frac{|k|^2}{m_e^2}\bar{I}^0(k) - 2\bar{I}^0(k) - 2I_A(k)E_p\right]\vec{\sigma}\cdot\vec{B},$$

Relativistic Hamiltonian

- Magnetic Field (Approximate Penning trap)

$$A^0 = 0 \qquad \vec{A} = \frac{1}{2}\vec{B} \times \vec{r}$$

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Other components suppressed by β_{DM}^2

determines size of effect

$$RI_A(k) = \frac{\delta m_n}{2m_e^2}\left(\frac{m_e}{E_k}\right)^2 \qquad R\bar{I}_0(k) = \frac{\delta m_n}{2m_e}\left(\frac{m_e}{E_k}\right)^3 \qquad \delta m_n = \frac{e^2\chi^2}{(2\pi)^3}\frac{1}{m_e m_{DM}}\int d^3q\bar{n}(E_q)$$

- Relativistic Hamiltonian
  - Must use corrections to frequencies and compare to experiment

$$H_T' = E_\beta - \frac{e}{2E_\beta}\left[\vec{L}\cdot B + \vec{\sigma}\cdot B\right]\left[1 - 2R\frac{m_e}{E_k}\bar{I}^0(k)\right] + \frac{eR}{2E_p}\left[\frac{|k|^2}{m_e^2}\bar{I}^0(k) - 2\bar{I}^0(k) - 2I_A(k)E_p\right]\vec{\sigma}\cdot\vec{B},$$

Spin frequency corrections

- Cyclotron Frequency also corrected
- Affects $g-2$ measurement

Experimental Constraints

- Predicted spin and cyclotron frequencies

$\sim \delta m_n$

$$\omega_c = \frac{e|B|}{2E_\beta}\left[1 - \frac{2Rm_e}{E_k}\bar{I}^0(k)\right]$$

SM $\mathcal{O}(\alpha)$ prediction

$$\omega_{s\perp} = \omega_c\left[1 + \frac{\alpha}{2\pi}\frac{E_k}{m_e} + R\left(\left(2 - \frac{|k|^2}{m_e^2}\right)\bar{I}^0(k) + 2I_A(k)E_k\right)\right]$$

- Measured quantity ratio

SM prediction

Background corrections small

$$R_f = \frac{\omega_a}{\omega_c} = \frac{\omega_{s\perp} - \omega_c}{\omega_c} \simeq R_{f_0}\left[1 + \frac{\delta\omega_a}{\omega_{a_0}} - \frac{\delta\omega_c}{\omega_{c_0}}\right]$$

Because $\omega_a \simeq \frac{\alpha}{2\pi}\frac{E_k}{m_e}\omega_c$ this dominates

Experimental Constraints

- Number density of dark matter

$$n_{DM} = \frac{1}{3}\frac{\rho_{DM}}{m_{DM}}$$

Three polarizations

- Occupation number is density per $(2\pi)^3$

$$\bar{n} = \frac{1}{3}\frac{n_{DM}}{\frac{4\pi q^2 \Delta q}{(2\pi)^3}}$$

Assumes DM velocity spread small

- Integrate occupation number

Enhanced by mass squared

$$\int d^3q\,\bar{n}(E_q) \simeq \int d^3q\,\frac{\rho_{DM}}{m_{DM}}\frac{(2\pi)^3}{12\pi q^2\Delta q} \simeq \frac{(2\pi)^3\rho_{DM}}{3m_{DM}} \Rightarrow \delta m_n = \frac{4\pi}{3}\alpha\chi^2\frac{\rho_{DM}}{m_e m_{DM}^2}$$

- The variation in R_f is then

$$\frac{\Delta R_f}{R_{f0}} \simeq \frac{\delta\omega_a}{\omega_{a0}} \simeq \frac{(2\pi)^2}{3}\chi^2\frac{\rho_{DM}}{m_{DM}^2 E_k^2}$$

Experimental Constraints

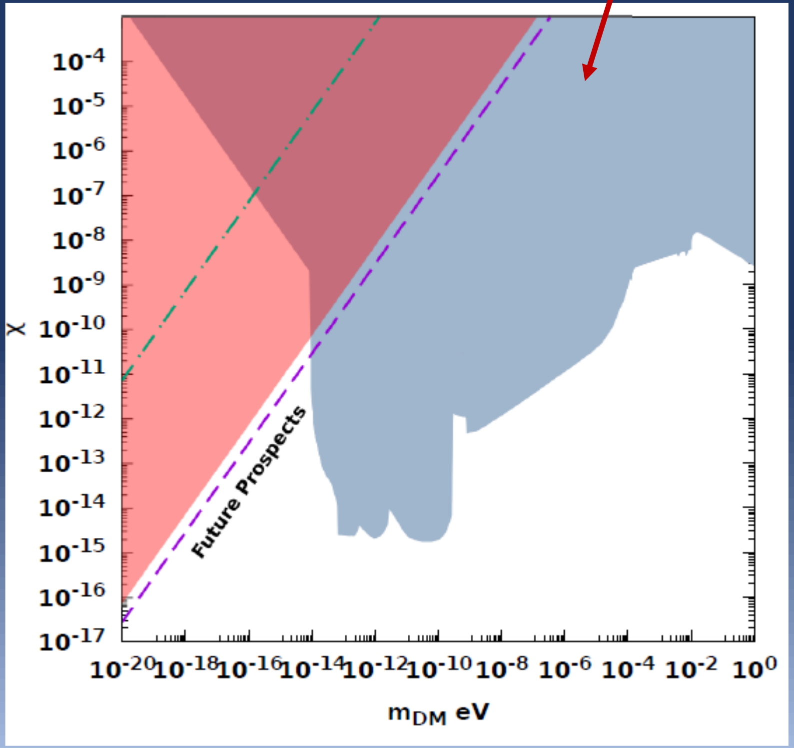
- Experimental uncertainties
- The experimental constraints on R_f

$$\frac{\Delta R_f}{R_{f0}} \simeq \frac{\delta\omega_a}{\omega_{a0}} \simeq \frac{(2\pi)^2}{3}\chi^2 \frac{\rho_{DM}}{m_{DM}^2 E_k^2} < 2\frac{\Delta\omega_c}{\omega_c} \simeq 4\times 10^{-11}$$

Being Conservative

Previous constraints from: Caputo, Millar, O'Hare and Vitagliano

- Gives a constraint on χ for a given m_{DM}
- Very strong compared to previous constraints



Experimental Constraints

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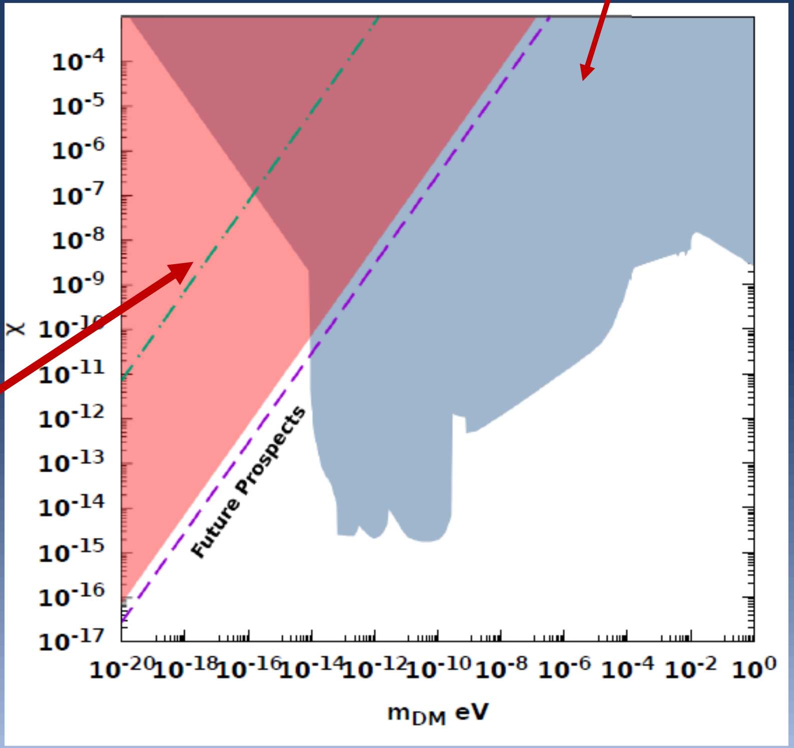
- Gives a constraint on χ for a given m_{DM}

- Very strong compared to previous constraints

- Constraints scale as ½ power of DM density

$$\frac{\Omega_A}{\Omega_{cdm}} \sim 10^{-10}$$



ALP Background Dark Matter

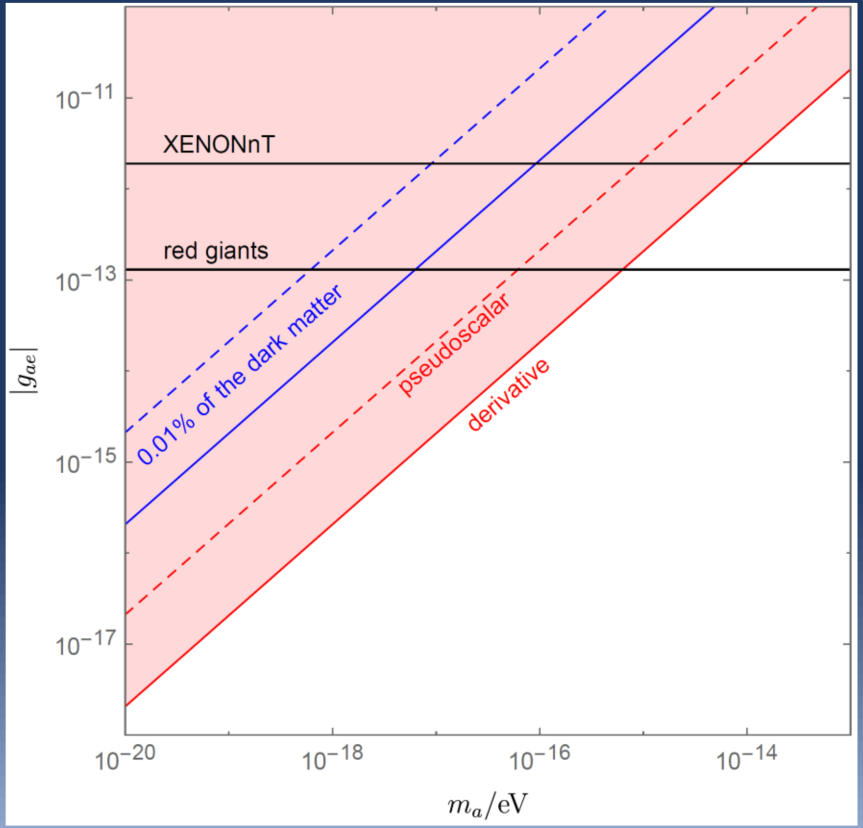
- ALP's are another motivated ultralight dark matter Background
  - Also contributes to the anomalous magnetic moment

$$\mathcal{L} \supset g_{ae} a \bar{\psi} \gamma^{\mu} \gamma_5 \psi + g_{ae} \frac{\partial_{\mu} a}{2f} \bar{\psi} \gamma^{\mu} \gamma_5 \psi$$

- Experimental constraints on its contribution

$$\frac{\Delta R_f}{R_{f0}} \simeq \frac{\delta \omega_a}{\omega_{a0}} \simeq \frac{1}{2} \frac{(2\pi)}{\alpha} \left(\frac{g_{ae} m_e}{2f} \right)^2 \frac{\vec{k}^2}{E_k m_e} \frac{\rho_{DM}}{m_{DM}^2 m_e^2} < 0.7 \times 10^{-12}$$

- For light ALP constraint quite strong

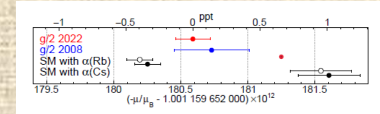


Arza, JLE 2308.05375

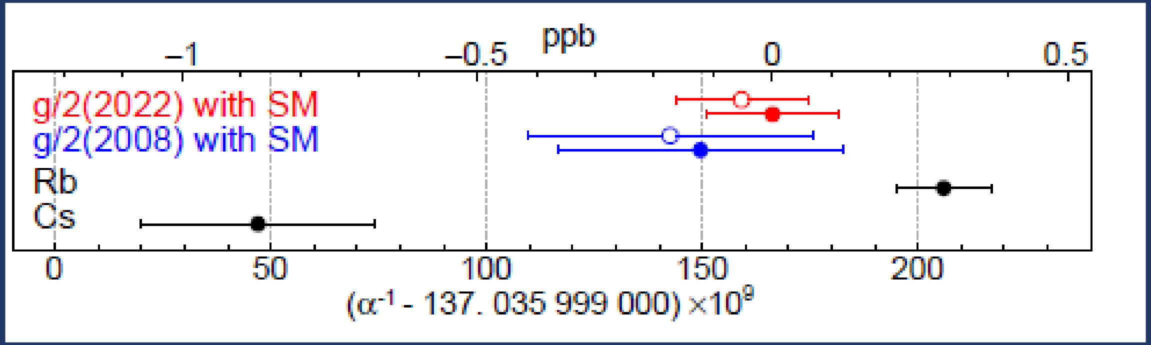
Conclusions

- ❑ Fundamental properties of dark matter can lead to constraints
  - ❑ Pauli exclusion principle prevents ultralight fermionic dark matter
  - ❑ Compton wavelength prevents super-ultralight dark matter bosons
- ❑ Production of ultralight dark matter
  - ❑ Thermal production problematic
  - ❑ Coherent condense production needed
- ❑ Ultralight dark matter is very dense throughout the universe
  - ❑ Occupation number becomes very large
  - ❑ Number of paths for processes shoots way up (Bose Enhancement)
- ❑ Bose enhancements work on loop processes
  - ❑ Anomalous magnetic contribution greatly enhanced
  - ❑ Strong constrains gauge mixing parameter for dark photon
  - ❑ Coupling of ALP to electron strongly constrained as well

Experimental Constraints



- ❑ Previous Measurements give us an average value for α



- ❑ Weighted average very close to 2022 measurement (~164)
  - ❑ Also close to theory prediction
- ❑ Allows us to call any deviation larger than experimental error a measurement

Can we treat the dark photon as a particle?

- ❑ Dark matter condensate has very long period

$$T \sim \frac{2\pi}{m_{DM}} \simeq 18 \text{ hrs} \left(\frac{10^{-20} \text{ eV}}{m_{DM}}\right)$$

- ❑ Decoherence time of condensate
  - ❑ Virilization from gravity on large object

$$\tau_{\text{dec}} \sim \frac{\Delta\beta^2}{m_{DM}}$$

- ❑ Can experiments resolve this as a particle?
  - ❑ Heisenberg uncertainty principle

$$\Delta E \Delta t > \frac{1}{2} \quad \Rightarrow \quad \Delta t > \frac{1}{2m_{DM}}$$

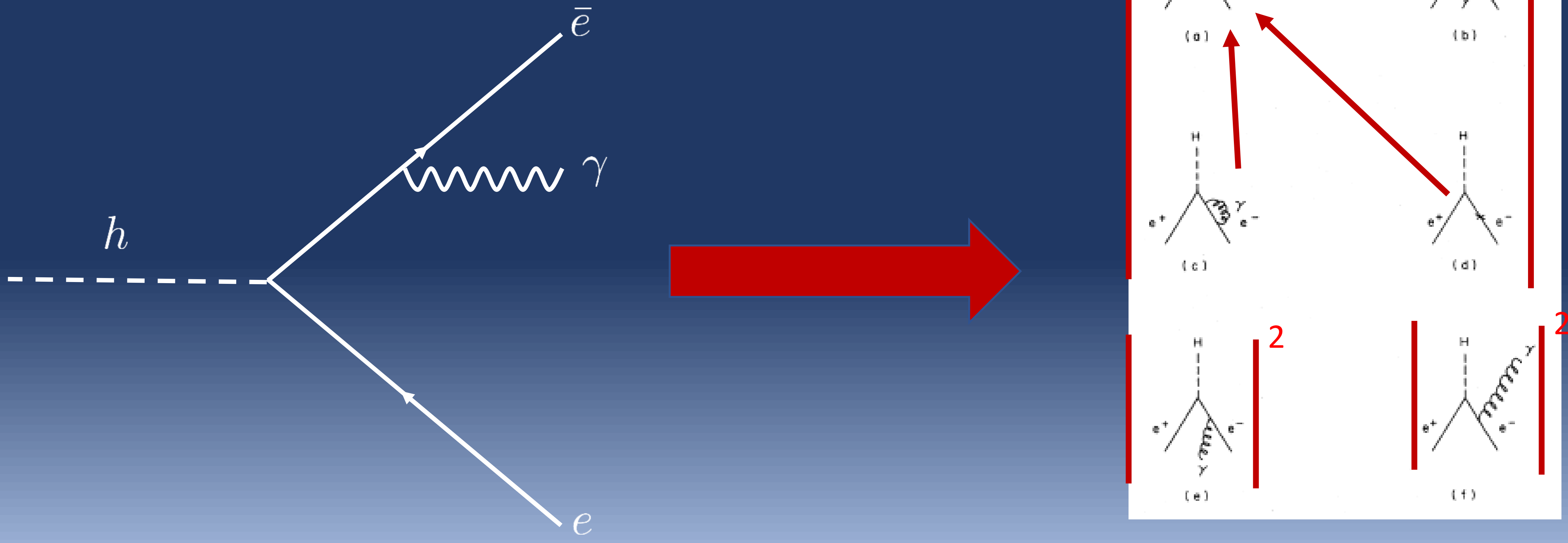
Time to resolve energy m_{DM}

run #	time	magnetic field (T)	cyclotron frequency (GHz)
1-1	2021-12-19-14:45 – 2021-12-20-13:46		
1-2	2021-12-22-12:57 – 2021-12-23-10:37	5.373	150.411
1-3	2021-12-26-13:33 – 2021-12-27-15:31		
2-1	2021-12-29-17:43 – 2021-12-30-17:37		
2-2	2021-12-31-15:15 – 2022-01-01-23:18		
2-3	2022-01-02-16:46 – 2022-01-04-11:43	5.300	148.361
2-4	2022-01-05-12:46 – 2022-01-06-10:49		
3-1	2022-01-31-21:47 – 2022-02-02-12:01		
3-2	2022-02-03-11:02 – 2022-02-04-13:58		
3-3	2022-02-04-16:13 – 2022-02-05-19:17	5.269	147.498
3-4	2022-02-06-15:44 – 2022-02-07-16:30		
3-5	2022-02-07-17:56 – 2022-02-08-21:15		
4-1	2022-02-11-18:13 – 2022-02-14-00:14		
4-2	2022-02-15-19:47 – 2022-02-17-17:15	5.326	149.091
4-3	2022-02-19-11:38 – 2022-02-21-09:50		
5-1	2022-04-07-19:37 – 2022-04-08-19:53		
5-1	2022-04-09-12:24 – 2022-04-10-21:49	4.071	113.956
5-1	2022-04-10-21:03 – 2022-04-11-14:04		
6-1	2022-04-12-17:58 – 2022-04-13-15:10		
6-1	2022-04-13-16:13 – 2022-04-14-14:32	4.245	118.822
6-1	2022-04-14-16:58 – 2022-04-15-13:38		
7-1	2022-04-17-19:26 – 2022-04-18-22:13	4.078	114.141
7-2	2022-04-18-22:16 – 2022-04-20-10:29		
8-1	2022-06-26-11:38 – 2022-06-27-14:28		
8-2	2022-06-27-15:02 – 2022-06-28-13:48		
8-3	2022-06-28-14:59 – 2022-06-29-10:19	4.969	139.097
8-4	2022-06-29-11:33 – 2022-06-30-13:38		
9-1	2022-07-01-16:05 – 2022-07-02-10:21		
9-2	2022-07-02-10:27 – 2022-07-03-11:37	5.001	139.989
9-3	2022-07-03-12:08 – 2022-07-04-11:33		
10-1	2022-07-05-09:07 – 2022-07-06-11:10		
10-2	2022-07-06-12:56 – 2022-07-07-11:57	4.537	127.007
10-3	2022-07-07-17:10 – 2022-07-08-14:04		
11-1	2022-07-11-10:59 – 2022-07-12-10:48		
11-2	2022-07-13-09:45 – 2022-07-14-11:27		
11-3	2022-07-14-11:31 – 2022-07-15-13:02	3.108	87.010
11-4	2022-07-15-13:07 – 2022-07-16-18:38		

Table 4.1: Data sets used for the g-factor determination.

What does this mean for Decays?

- ❑ Two body decays
- ❑ Three body decays
  - ❑ Many possibilities!!
  - ❑ Higgs Decay in Thermal Bath



Ward Identities

- Ward Identities

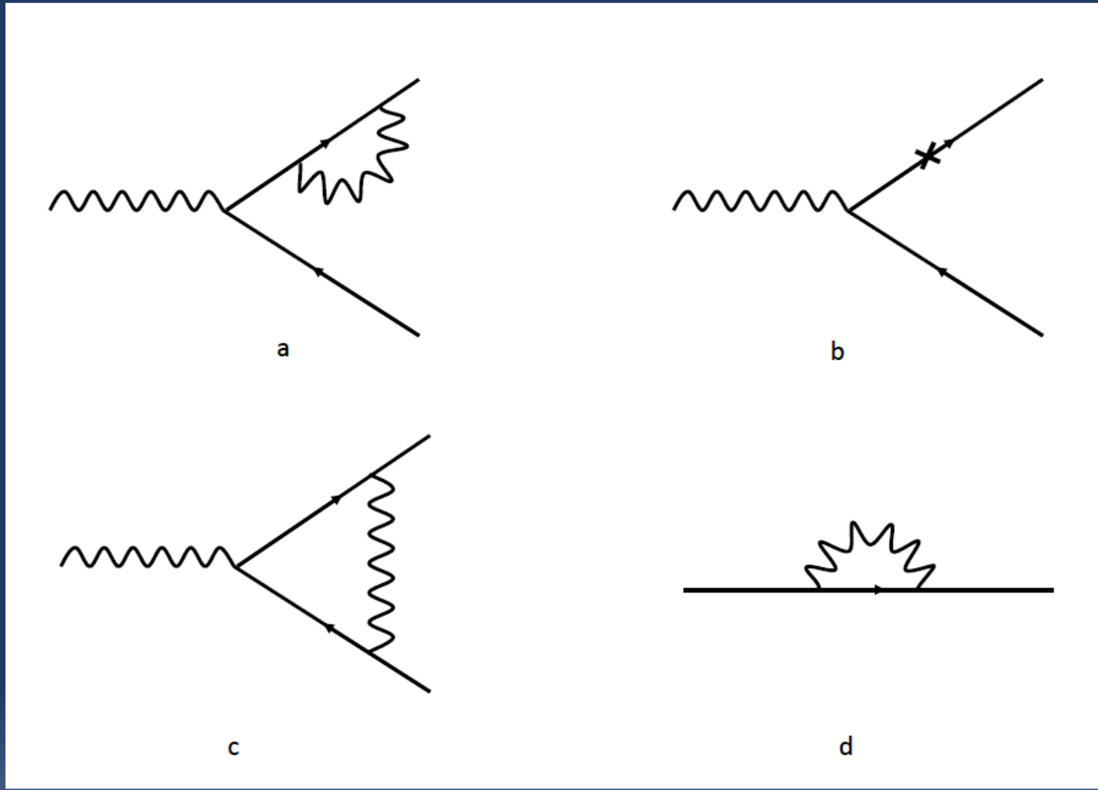
$$\Delta k^\mu M_{TOT\mu} = -e\bar{u}_n(\bar{k})$$

$$\times \left[B(k) - B(\bar{k}) + \Delta k^\mu \left[\frac{dB(k)}{dk_\mu} + \frac{dB(\bar{k})}{d\bar{k}_\mu} \right] \right. \rightarrow \Delta k^3$$

$$+ \frac{k_\mu + \bar{k}_\mu}{2m_e} \left[D_\mu(k) - D_\mu(\bar{k}) \right] - \frac{\Delta k^\mu}{2m_e} \left[D_\mu(k) + D_\mu(\bar{k}) \right] \rightarrow m_{DM}^4$$

$$+ \left[\left[\frac{dD^\nu(k)}{dk_\mu} + \frac{dD^\nu(\bar{k})}{d\bar{k}_\mu} \right] \Delta k^\mu + D_\nu(k) - D_\nu(\bar{k}) \right] \rightarrow \Delta k^3$$

$$\left. \times \frac{[\gamma_\alpha, \gamma_\nu]_- \Delta k_\alpha}{4m_e} \right] u_n(k) \ .$$

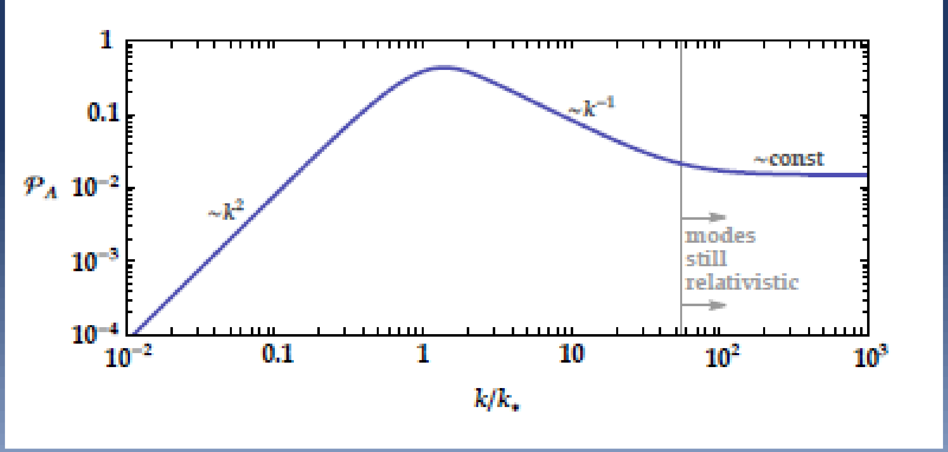


Production of Ultralight Dark Photon Dark Matter

- Thermal production out

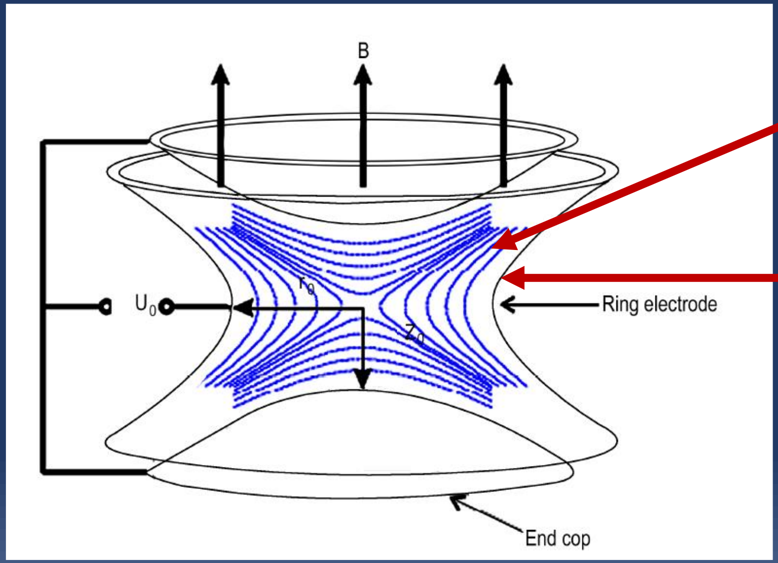
- Production of longitudinal modes from quantum fluctuations
  - longitudinal mode behaves like scalar field
  - Choose the Bunch-Davies vacuum we get
  - Power spectrum suppressed at low momentum

$$P_{A_L} \simeq \left(\frac{k}{m}\right)^2 P_\pi \simeq \left(\frac{kH_I}{2\pi m}\right)^2$$



The Penning Trap

- ❑ The Penning Trap
  - ❑ Constant magnetic Field/Quadrapole Electric Field



E field just to contain particle

Electron effectively orbits in constant B field

- ❑ Clearly not cavity since electric field non-zero inside
  - ❑ Thus, fields penetrate trap

Cavity Effects

- If the experiment were in a Cavity, this effect would cancel
  - The cavity would produce an identical background of photons
    - Except opposition spin vector

- This introduces additional enhanced propagators

$$\langle n, n' | A_\mu(x) A'_\mu(y) | n, n' \rangle \qquad \langle n, n' | A_\mu(x) A_\mu(y) | n, n' \rangle$$

$$n(E_k) = \chi n'(E_k)$$

Cavity Generated

- This then leads to a total propagator of

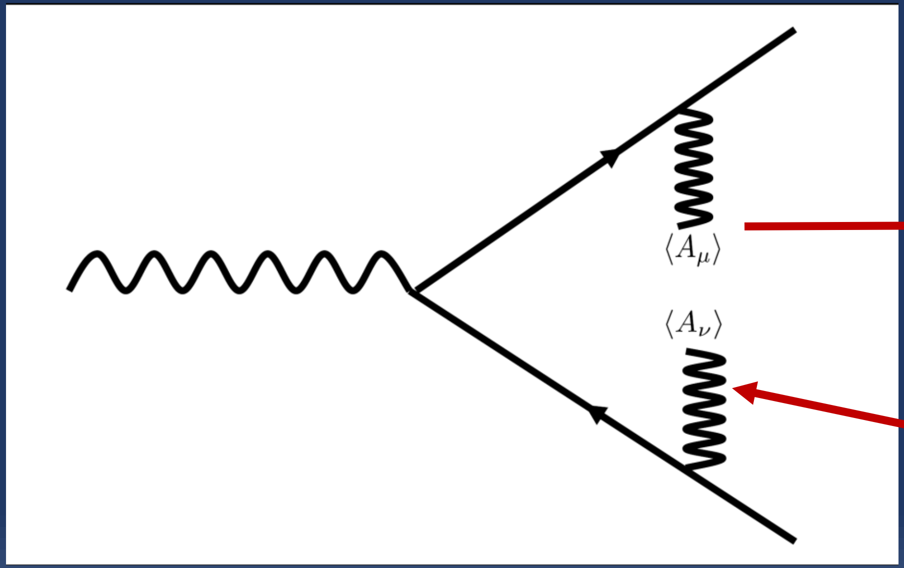
Negative: Spin sum has negative sign

$$[\chi^2 \langle n, n' | A'_\mu(x) A'_\mu(y) | n, n' \rangle + 2\chi \langle n, n' | A_\mu(x) A'_\mu(y) | n, n' \rangle + \langle n, n' | A_\mu(x) A_\mu(y) | n, n' \rangle] = 0$$

- Classically this amounts to $A + \chi A' = 0$

Is there an enhancement in the classical limit

- Can do a similar calculation with a background A'_μ



Because massive, this is not the same

Need opposite correlated momentum

- However, there is a very strong field because so light

$$\rho_{DM} \sim m_{DM}^2 A^\mu A_\mu \qquad \langle A_\mu \rangle \sim \sqrt{\frac{\rho_{DM}}{m_{DM}^2}}$$

Why is inverse scaling of dark matter mass ok?

- The integrands is expanded in k so R_i depends on k only through $n_k(E_k)$

$$\frac{d\Gamma}{dk} = n_B(k)\left[\frac{1}{k}R_{-1} + R_0 + kR_1 + O(k^2)\right]$$

- kR_1 scales as k^0
- $I_A(q), I_\mu(q)$ scale as q^{-2}
  - But $\propto R = q^2/m_e^2$
  - Effective scaling q^0
- $I_{\mu\nu}(q)$ scales as q^0
- Thus for $m_{DM} \to 0$ still well defined

$$I_A(k) = e^2\chi^2 \int \frac{d^4\Pi_q}{(2\pi)^3} \frac{2m_e}{(2q\cdot k)^2}$$

$$\sim \int dq \frac{N(E_q)}{q}$$

$$N(E_q)|_{\text{IR}} \sim \frac{1}{q}$$

$$\bar{I}_\mu(k) = e^2\chi^2 \int \frac{d^4\Pi_q}{(2\pi)^3} \frac{4q_\mu m_e^3}{(2q\cdot k)^3}$$

$$\sim \int dq \frac{N(E_q)}{q}$$

$$I_{\mu,\nu} = e^2\chi^2 \int \frac{d^4\Pi_q}{(2\pi)^3} \frac{q_\mu q_\nu}{(q\cdot k)^2}$$

$$\sim \int dq q N(E_q)$$

$$d^4\Pi_q = d^4q\, \bar{n}(E_q)\delta(q^2 - m_{DM}^2)$$

What about no background?

- Same formulas apply to no background

$$iM_{TOT\mu} = -ie\bar{u}_n(\bar{k})\Big[\gamma_\mu\Big[1 - \frac{1}{2}\frac{1}{E}\frac{d}{dE}\left(m_e B(k) + k_\nu D^\nu(k)\right) + \frac{1}{2}\frac{D^0(k)}{E} + (k \leftrightarrow \bar{k})\Big] + \left[\frac{1}{2}\frac{d}{dk_\mu}\left[B(k) + \frac{k_\nu D^\nu(k)}{m_e}\right] - \frac{D^\mu(k)}{2m_e} + \frac{[\gamma_\alpha,\gamma_\nu]_- \Delta k_\alpha}{8m_e}\frac{dD^\nu(k)}{dk_\mu} + (k \leftrightarrow \bar{k})\right] + F_\mu(\Delta k)\Big] u_n(k)$$

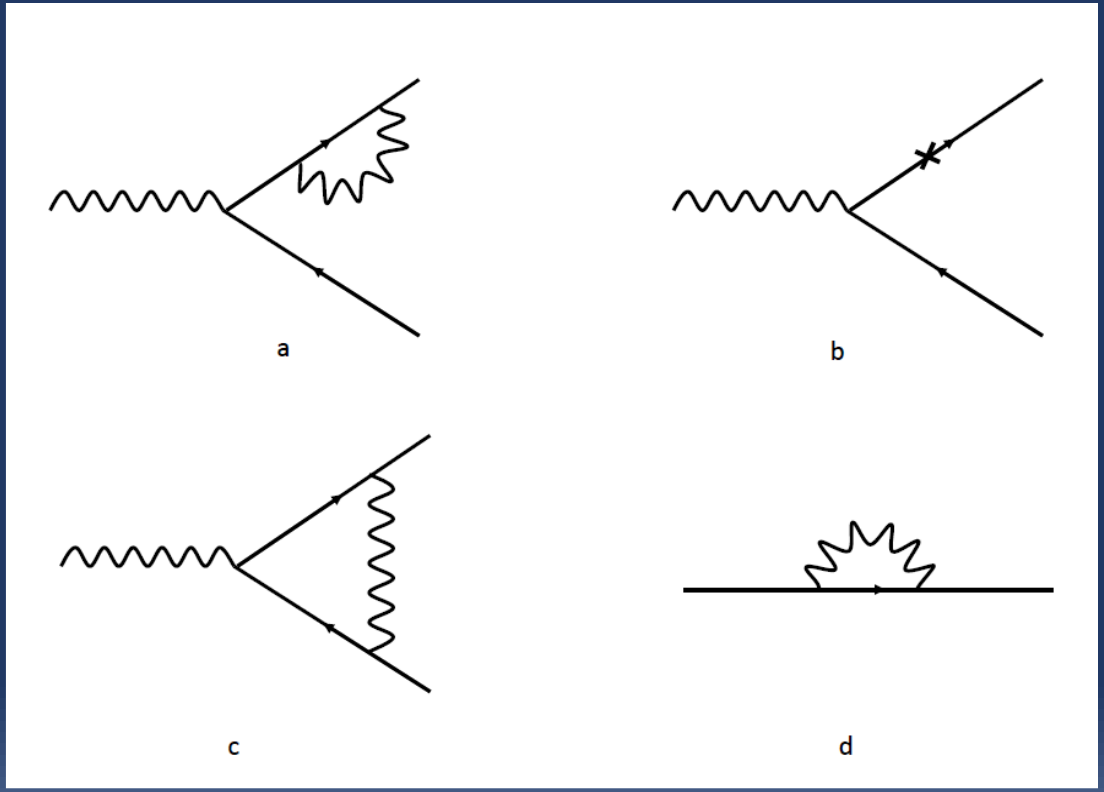
- $B(k),\ D^\mu(k),\ F(\Delta k)$ Found by

$$2\pi\delta(q^2 - m_a^2) \rightarrow \frac{i}{q^2 - m_a^2}$$

- Applied to pseudoscalar we get

$$a_e = -\left(\frac{m_e}{f_a}\right)^2 \frac{c_{ee}^2}{16\pi^2}\left[1 + 2x + x(1-x)\ln(x) - \frac{2x(x-3)\sqrt{x(x-4)}}{x-4}\ln\left(\frac{1}{2}\left[\sqrt{x} + \sqrt{x-4}\right]\right)\right]$$

Exactly what previous calculations get



Ward Identities

- Ward Identities apply Δk^{μ} to $M_{TOT_{\mu}}$
  - Use slightly different form of $M_{TOT_{\mu}}$

$$= B(k) - B(\bar{k}) + \gamma \cdot D(k) - \gamma \cdot D(\bar{k})$$

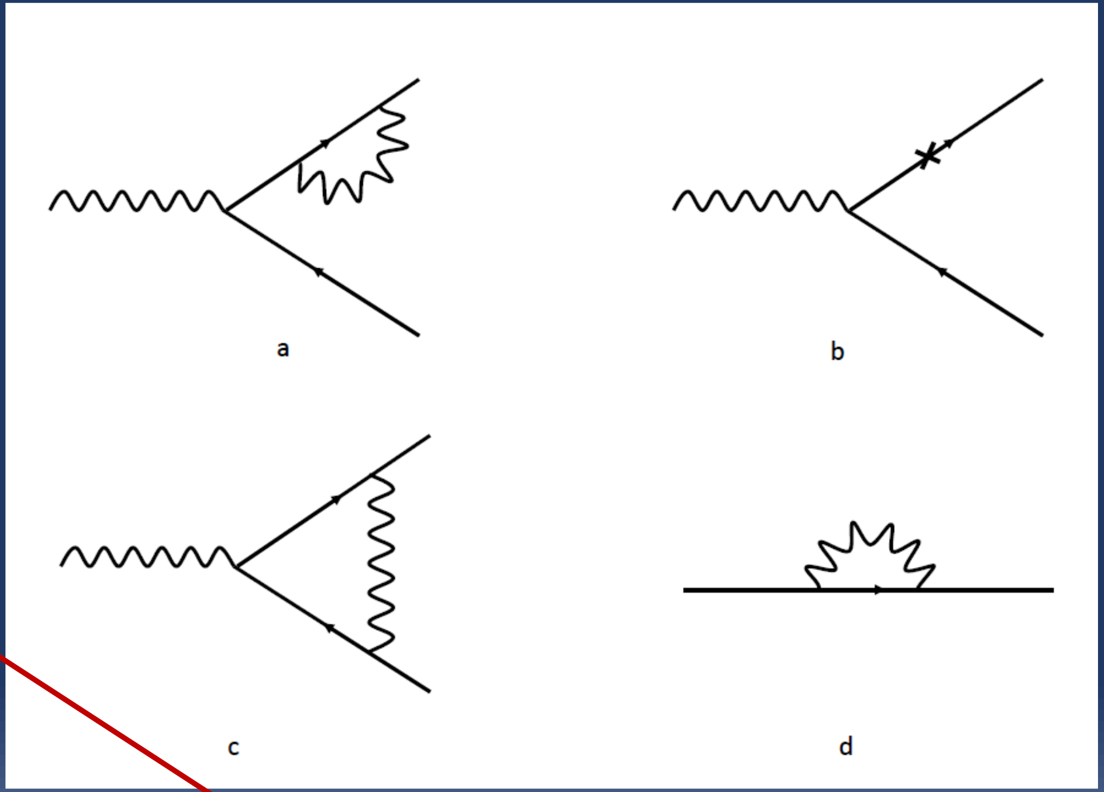
$$i\Delta k^{\mu} M_{TOT\mu} = -ie\bar{u}_n(\bar{k}) \Big[\gamma \cdot \Delta k \Big[1$$

$$-\frac{1}{2}\frac{1}{E}\frac{d}{dE}\left(m_e B(k) + k_{\nu} D^{\nu}(k)\right) + \frac{1}{2}\frac{D^0(k)}{E} + (k \leftrightarrow \bar{k})\Big]$$

$$\Delta k^{\mu}\left[\frac{1}{2}\gamma \cdot \left[\frac{d}{dk^{\mu}}D(k) + \frac{d}{d\bar{k}^{\mu}}D(\bar{k})\right]\right.$$

$$\left. + \frac{d}{dk^{\mu}}B(k) + \frac{d}{d\bar{k}^{\mu}}B(\bar{k})\right] + \Delta k^{\mu} F_{\mu}(\Delta k)\Big] u_n(k)$$

$$= 0$$



This row of order α^2

Ionosphere: Plasma Screening of Electric Field

- Simple model of Ionosphere permittivity
  - Find the displacement of the electron and then Polarization

$$m\ddot{x}(t) - e\vec{B} \times \dot{x} = -eE(\omega)e^{-i\omega t} \quad \xrightarrow{x(t) = x(\omega)e^{-i\omega t}} \quad -m\omega^2 x_i(\omega) + ie\omega\left[\vec{B} \times x(\omega)\right]_i = -eE_i(\omega)$$

Ionosphere: Plasma Screening of Electric Field

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$$m\ddot{x}(t) - e\vec{B}\times\dot{x} = -eE(\omega)e^{-i\omega t} \quad \xrightarrow{x(t)=x(\omega)e^{-i\omega t}} \quad -m\omega^2 x_i(\omega) + ie\omega\left[\vec{B}\times x(\omega)\right]_i = -eE_i(\omega)$$

- Solve for the displacement of the electron in the atom

Charge # Density

$$x_i(\omega) = \frac{1}{(\omega^2-\omega_B^2)\omega^2}\left[\omega^2\delta_{ij} + \omega_B^2 b_i b_j - i\omega_B\omega\epsilon_{ijk}b_k\right]\frac{eE_j}{m} \longrightarrow P_i = -eNx(\omega)_i$$

- The polarization can then be found and thus permittivity

$$\omega_p^2 = \frac{e^2 N}{\epsilon_0 m}$$

$$D_i = \epsilon_0 E_i + P_i = \epsilon_0\left[\delta_{ij} - \frac{\omega_p^2}{(\omega^2-\omega_B^2)\omega^2}\left[\omega^2\delta_{ij} + \omega_B^2 b_i b_j - i\omega_B\omega\epsilon_{ijk}b_k\right]\right]$$

If we take propagation parallel to $\vec{B}$

$$D = \epsilon_0\left[1 - \frac{\omega_p^2}{\omega(\omega\mp\omega_B)}\right]\vec{E}_\pm$$

Ionosphere: Plasma Screening of Electric Field

- Simple model of Ionosphere permittivity
  - Find the displacement of the electron and then Polarization
- Solve for the displacement of the electron in the atom

Charge # Density

$$x_i(\omega) = \frac{1}{(\omega^2-\omega_B^2)\omega^2}\left[\omega^2\delta_{ij} + \omega_B^2 b_i b_j - i\omega_B\omega\epsilon_{ijk}b_k\right]\frac{eE_j}{m} \longrightarrow P_i = -eNx(\omega)_i$$

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- Using this can solve for the dispersion relation

Circular Polarized

$$\vec{k}\times\vec{k}\times E = \mu_0\omega^2 D \xrightarrow{\omega \ll \omega_p, \omega_B} k_\pm^2 = \pm\frac{\omega_p^2}{\omega\omega_B \hat{b}\cdot\hat{n}}$$

B direction

Propagation Direction

$k^2 < 0$ and real then No propagation

What happens to loop processes?

- Self energy plus counterterm contribution

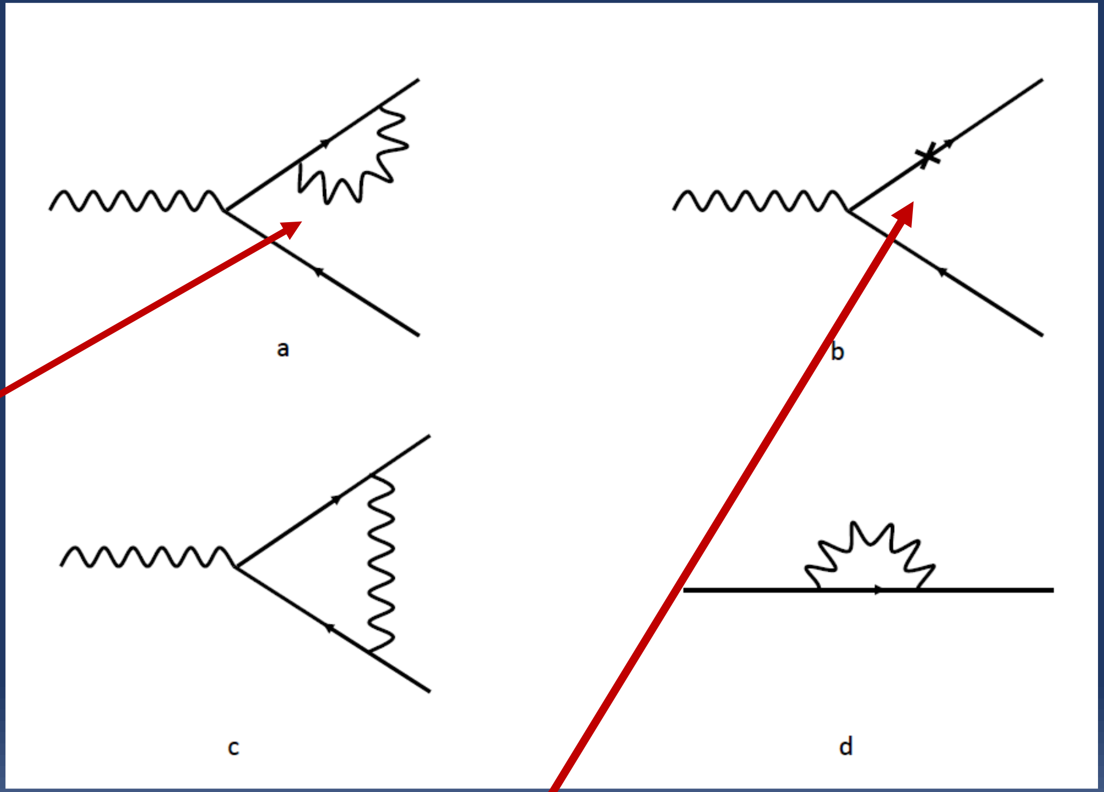
$$iM_{SE+CT} = ie\bar{u}(\bar{k})\gamma_\mu \left[C(k) + \frac{1}{E_k}\left[m_e \frac{\partial B(k)}{\partial E_k}\Big|_{k^2=m_e^2} + \frac{\partial k_\mu D^\mu(k)}{\partial E_k}\Big|_{k^2=m_e^2} - D^0(k)\right] + C(\bar{k}) + \frac{1}{E_{\bar{k}}}\left[m_e \frac{\partial B(\bar{k})}{\partial E_{\bar{k}}}\Big|_{\bar{k}^2=m_e^2} + \frac{\partial \bar{k}_\mu D^\mu(k)}{\partial E_{\bar{k}}}\Big|_{\bar{k}^2=m_e^2} - D^0(\bar{k})\right]\right] u(k)$$

- Derivatives arise because of definition

$$D^\mu(k) = -2e^2\chi^2 \int \frac{d^4q}{(2\pi)^3} \frac{q_\mu}{q^2 + 2q\cdot k + k^2 - m_e^2} n(E_q)\delta(q^2 - m_{DM})$$

$$B(k) = 2e^2\chi^2 \int \frac{d^4q}{(2\pi)^3} \frac{m_e}{q^2 + 2q\cdot k + k^2 - m_e^2} n(E_q)\delta(q^2 - m_{DM})$$

$$C(k) = -2e^2\chi^2 \int \frac{d^4q}{(2\pi)^3} \frac{1}{q^2 + 2q\cdot k + k^2 - m_e^2} n(E_q)\delta(q^2 - m_{DM})$$



Mass counterterms on shell,
Since part of EOM

Wave Function Renormalization in a Background

- Thermally corrected propagator

$\tilde{k}$ $\tilde{m}$

$$S^{-1}(k) = \gamma\cdot k - m_R + \Sigma_n = [1+C(k)]\gamma\cdot[k+D(k)] - [1+C(k)][m_R - B(k)]$$

$$\Sigma_n = B(k) + C(k)\,(\gamma\cdot k - m_R) + \gamma\cdot D(k)$$

- Renormalized propagator
  - Perform k_0 integral

Wave Function Renormalization

$$i\int \frac{d^4k}{(2\pi)^4}\frac{Z_2^{-1}(\gamma\cdot\tilde{k}+\tilde{m})e^{-ik\cdot(x-y)}}{\tilde{k}^2-\tilde{m}^2+i\epsilon}$$

Residue changed if $B(k),\ k_\mu D^\mu(k)$ depend on k

- Compare to calculation of $\langle\psi\bar{\psi}\rangle$

Background dependent spinors

$$S^R(x-y) = \int\frac{d^4k}{(2\pi)^3}\left[\theta(x_0-y_0)\frac{\tilde{k}+\tilde{m}}{2\tilde{E}}e^{-ik\cdot(x-y)} - \theta(y_0-x_0)\frac{\tilde{k}-\tilde{m}}{2\tilde{E}}e^{ik\cdot(x-y)}\right]$$

$$\sum_{\text{spin}} u_n(k)\bar{u}_n(k) = \frac{\gamma\cdot\tilde{k}+\tilde{m}}{2\tilde{E}}$$

- Wave function renormalization changed by background

Standard Wave Function Renormalization

$$Z_2^{-1} = 1 + C(k) + \frac{1}{E}\frac{d}{dE}\left(mB(k) + k_\mu D^\mu(k)\right) - \frac{D^0(k)}{E}$$

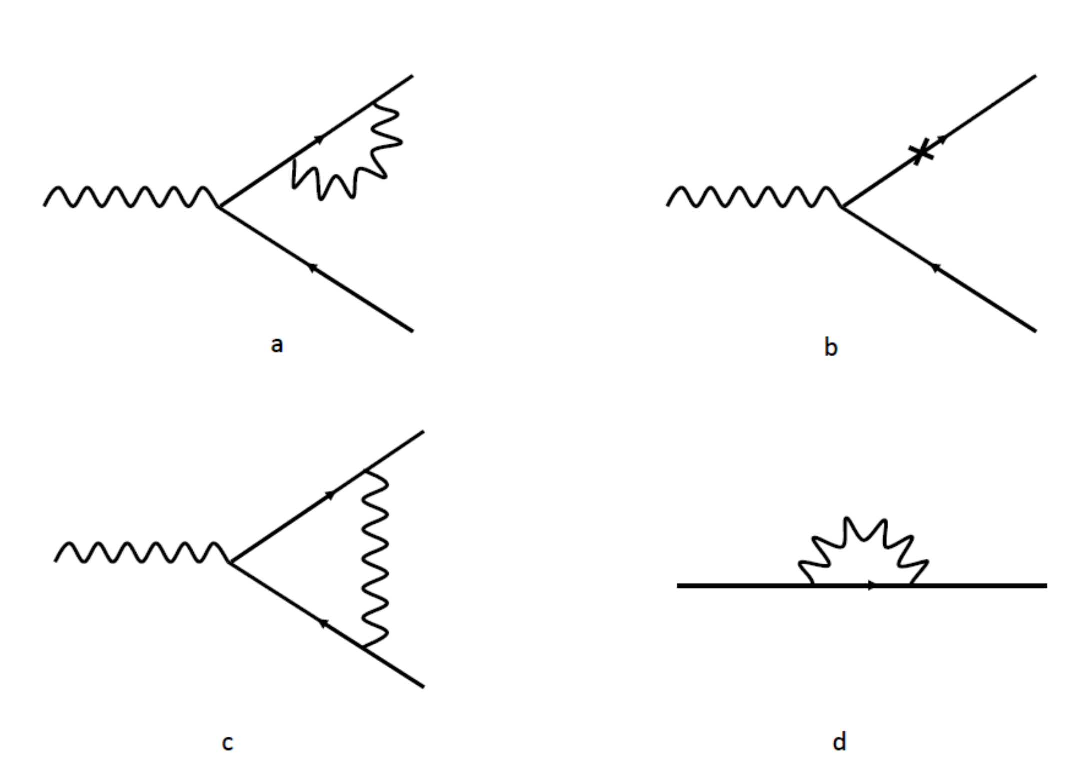
$E \to \tilde{E}$

Charge Non-Renormalization

- Charge non-renormalization
  - Apply $\Delta k = 0, \; \mu = 0$ to vertex

$$iM_{TOT\mu} = -ie\bar{u}_n(\bar{k})\Bigg[\gamma_\mu\Bigg[1$$

$$\underbrace{\overbrace{-\frac{1}{2}\frac{1}{E}\frac{d}{dE}\left(m_e B(k) + k_\nu D^\nu(k)\right) + \frac{1}{2}\frac{D^0(k)}{E} + (k \leftrightarrow \bar{k})\Bigg] + \left[\frac{1}{2}\frac{d}{dk_\mu}\left[B(k) + \frac{k_\nu D^\nu(k)}{m_e}\right] - \frac{D^\mu(k)}{2m_e}}^{\bar{k}\to k} + \frac{[\gamma_\alpha,\gamma_\nu]_-\,\Delta k_\alpha}{8m_e}\frac{dD^\nu(k)}{dk_\mu} + (k \leftrightarrow \bar{k})\right] + F_\mu(\Delta k)}_{\to 0}\Bigg] u_n(k)$$



Production of Ultralight Dark Photon Dark Matter

- Thermal production not possible
  - Dark Matter tends to be hot

$$T_{CMB} \sim 10^{-3}\ \text{eV} \qquad \left.\frac{m_{DM}}{T}\right|_{\text{today}} \ll 1$$

Production of Ultralight Dark Photon Dark Matter

- Thermal production out

- Production of longitudinal modes from quantum fluctuations
  - longitudinal mode behaves like scalar field

$$A_L \sim \partial_\mu \pi$$

$$\pi(\vec{k},t) \equiv \frac{m}{k} A_L(\vec{k},t),$$

$$S_{\rm Long} \xrightarrow{am \ll k} \int \frac{a^3 d^3k}{(2\pi)^3}\, dt \frac{1}{2}\left(|\partial_t \pi|^2 - \frac{k^2}{a^2}|\pi|^2\right) = \int a^3 d^3x\, dt \frac{1}{2}\left((\partial_t \pi)^2 - \frac{1}{a^2}|\vec{\nabla}\pi|^2\right)$$

  - Choose the Bunch-Davies vacuum we get

$$P_{A_L} \simeq \left(\frac{k}{m}\right)^2 P_\pi \simeq \left(\frac{kH_I}{2\pi m}\right)^2$$

Graham, Mardon, Rajendran

Production of Ultralight Dark Photon Dark Matter

- Thermal production out

- Production of longitudinal modes from quantum fluctuations
  - longitudinal mode behaves like scalar field
  - Choose the Bunch-Davies vacuum we get
  - Power spectrum suppressed at low momentum
  - Relation between inflation scale and mass for dark matter

$$\frac{\Omega_{\rm vector}}{\Omega_{\rm cdm}} = \sqrt{\frac{m}{6\times10^{-6}\,{\rm eV}}}\left(\frac{H_I}{10^{14}\,{\rm GeV}}\right)^2 .$$

$$m_{DM} \sim 10^{-20}\ {\rm eV} \ \rightarrow \frac{\Omega_{\rm vector}}{\Omega_{\rm cdm}} \sim 10^{-7}$$

However, can still place a strong constraint!!

Production of Ultralight Dark Photon Dark Matter

- Thermal production out
- Production of longitudinal modes from quantum fluctuations
- Production from inflaton induced tachyonic DP mass

Kitajima and Nakayama

$$\mathcal{L} = -\frac{f^2(\phi)}{4} F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m_A^2 A_\mu A^\mu - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) + \mathcal{L}_X,$$

$$V(\phi) = \frac{1}{2}m_\phi^2\phi^2,$$

$$f(\phi) = \exp\left(-\frac{\gamma}{8}\frac{\phi^2}{M_{\rm Pl}^2}\right)$$

- Equations of motion

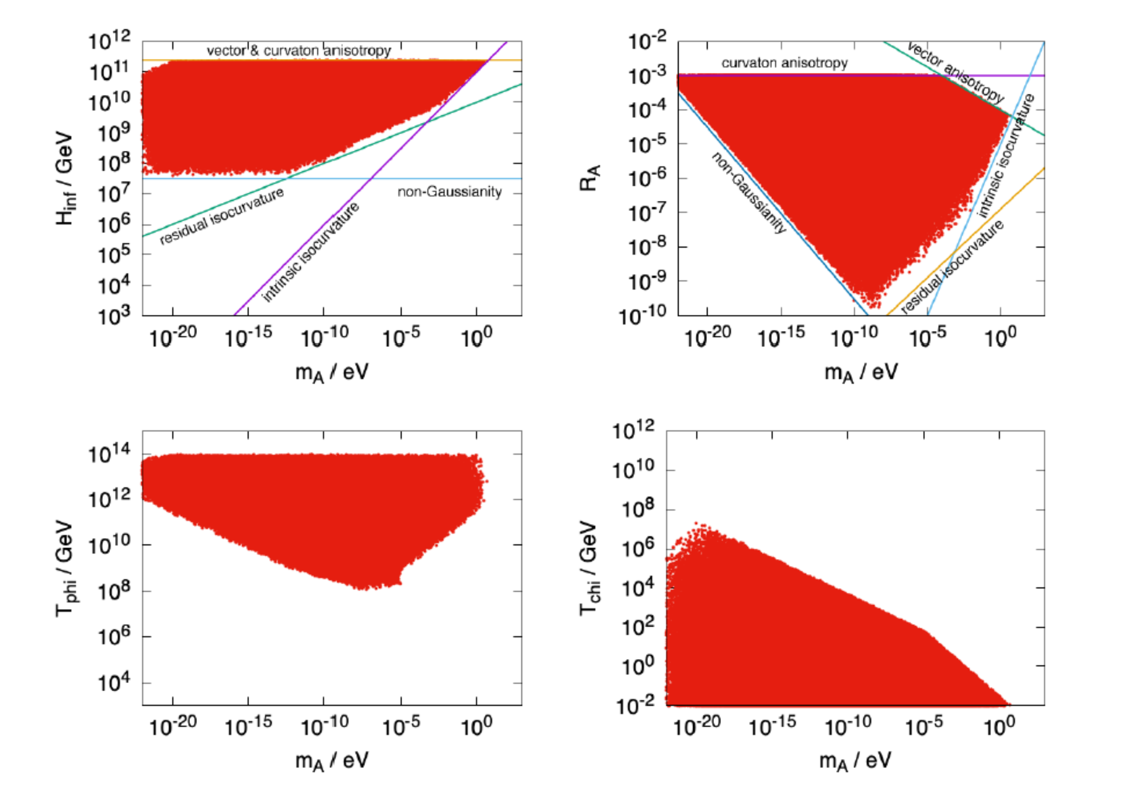
Tachyonic Mass

$$\ddot{\overline{A}}_i + 3H\dot{\overline{A}}_i + \left(\frac{m_A^2}{f^2} - \frac{(\alpha+4)(\alpha-2)}{4}H^2 + \frac{2-\alpha}{2}\dot{H}\right)\overline{A}_i = 0 \longleftarrow \alpha = \gamma < 4$$

- Have to worry about isocuvature, so need a curvaton

Production of Ultralight Dark Photon Dark Matter

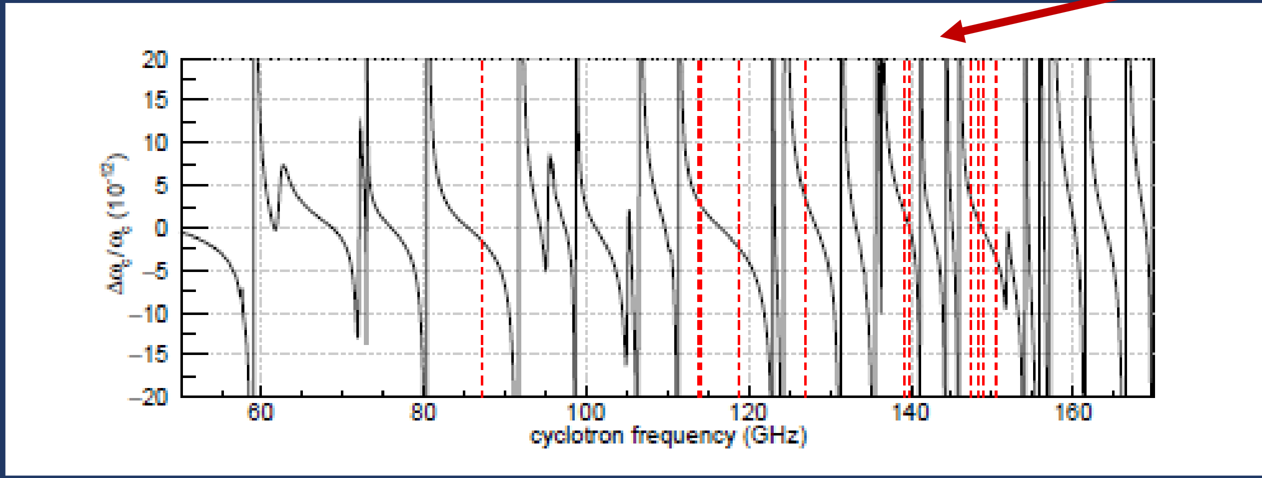
- Thermal production out
- Production of longitudinal modes from quantum fluctuations
- Production from inflaton induced tachyonic DP mass



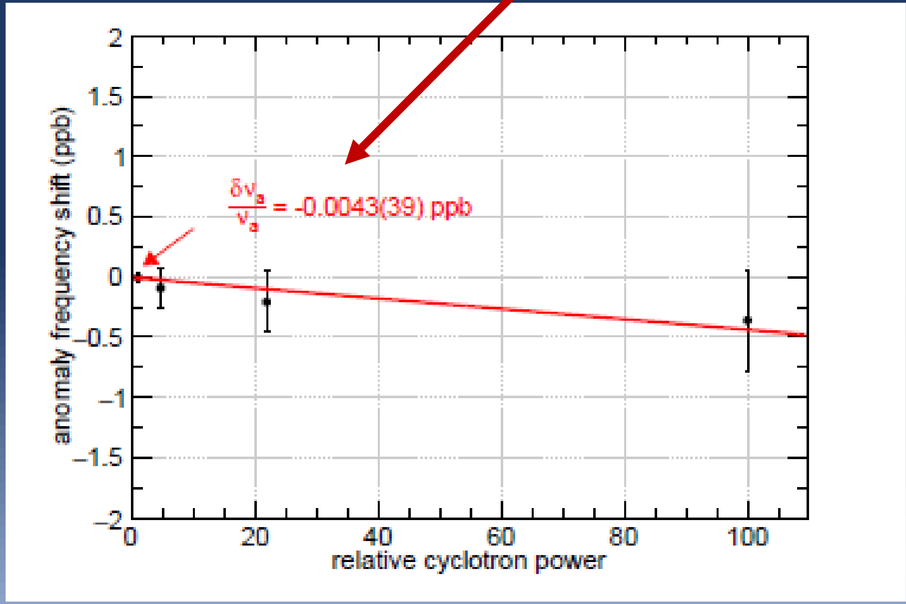
Experimental Constraints

- Experimental uncertainties

$$\frac{\Delta\omega_c}{\omega_c} \simeq \pm 2 \times 10^{-11}$$



$$\frac{\Delta\omega_a}{\omega_a} \simeq \pm 4 \times 10^{-12}$$



Fan, Xing. 2022. An Improved Measurement of the Electron Magnetic Moment. Doctoral dissertation, Harvard University Graduate School of Arts and Sciences.

Experimental Constraints

- Experimental uncertainties
- The experimental constraints on R_f

Correction to Theoretical prediction of ratio

$$\frac{\delta R_f}{R_{f0}} \simeq \frac{\delta \omega_a}{\omega_{a0}} \simeq \frac{(2\pi)^2}{3}\chi^2 \frac{\rho_{DM}}{m_{DM}^2 E_k^2}$$

$$\frac{\Delta R_f}{R_{f0}} \simeq -\frac{\Delta\omega_c}{\omega_{c0}} < 2\frac{\Delta\omega_c}{\omega_c} \simeq 4\times 10^{-11}$$

Dominant measurement error on ratio

Experimental Constraints

- Experimental uncertainties
- The experimental constraints on R_f
  - Theory<Experiment (Measured g-2 very consistent with SM)

$$\frac{(2\pi)^2}{3}\chi^2\frac{\rho_{DM}}{m_{DM}^2 E_k^2} < 2\frac{\Delta\omega_c}{\omega_c} \simeq 4 \times 10^{-11}$$

Being Conservative

- Gives a constraint on χ for a given m_{DM}

$$\chi < 7.1 \times 10^3 \frac{m_{DM}}{eV}\left(\frac{\Omega_A}{\Omega_{cdm}}\right)^{1/2}$$

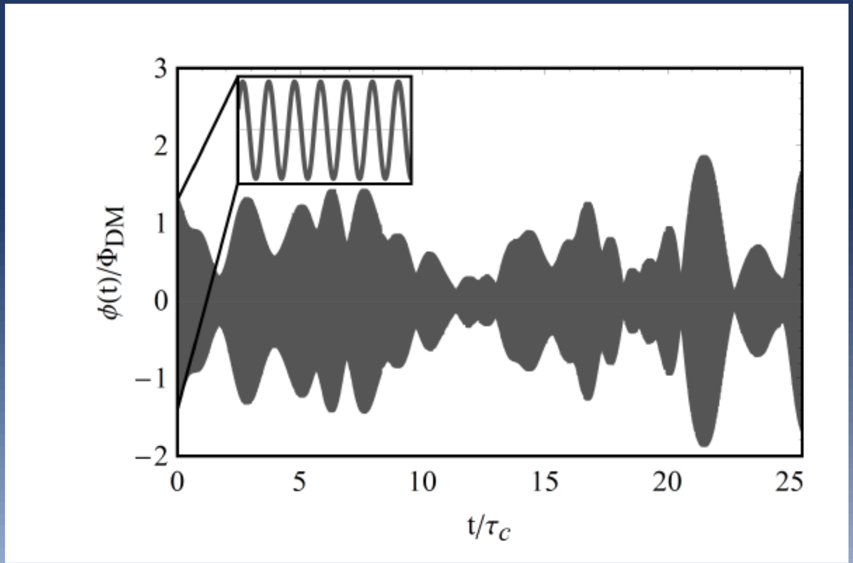
Stocastic Nature of the E-Field

- The dark photon field is virialized by the gravitational interactions

$$t_{\rm coh} \sim \frac{1}{m_{DM}\beta^2} = 2.1 \times 10^3 \text{ yr} \left(\frac{10^{-20} \text{ eV}}{m_{DM}}\right)$$

Much shorter than age of universe

- Leads to stochastic electric field



$$n_{DM} = \text{constant}$$

$$\rho_{DM} = n_{DM} m_{DM}$$

If coherent we have relation to E field

$$E \simeq \sqrt{2\rho_{DM}}$$

Broken by decoherence

IR cut off from experiment size

- The IR cut off is set by the particle the dark photon interacts with not experiment
  - The occupation number of the dark matter

$$n \sim \frac{\rho_{DM}}{4\pi k^2 \Delta k m_{DM}} \sim \frac{\rho_{DM}}{m_{DM}^4 \beta^3}$$

  - Contribution to anomalous magnetic moment scales as

$$\Delta a_e \sim \frac{\rho_{DM}}{E_e^2 m_{DM}^2}$$

Cut off by $\left(\frac{m_{DM}}{E_e}\right)^2$

  - Furthermore, can solve for propagator in constant electric field of electron
    - Contribution comes from near pole
    - Near pole propagator same so that IR divergences cancel