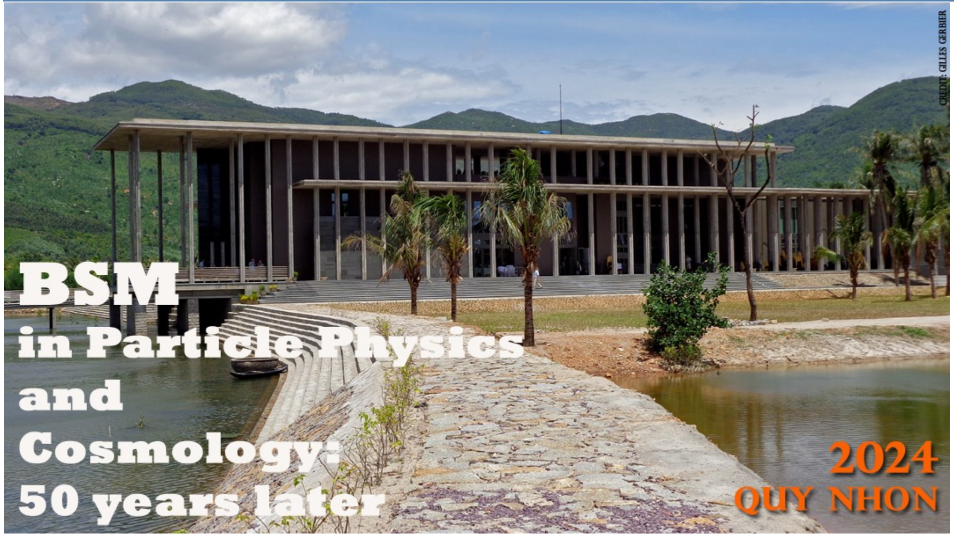

Relic QCD axions from the Early Universe

Giovanni Villadoro

The Abdus Salam International Centre for Theoretical Physics

Phys. Rev. Lett. 131 (2023) 1, 011004

with A. Notari and F. Rompineve

The (Minimal) QCD Axion

$$\mathcal{L}_{SM}^{\theta=0} + \frac{1}{2}(\partial_\mu a)^2 + \frac{a}{f_a}\frac{\alpha_s}{8\pi}G\tilde{G} + \ldots$$

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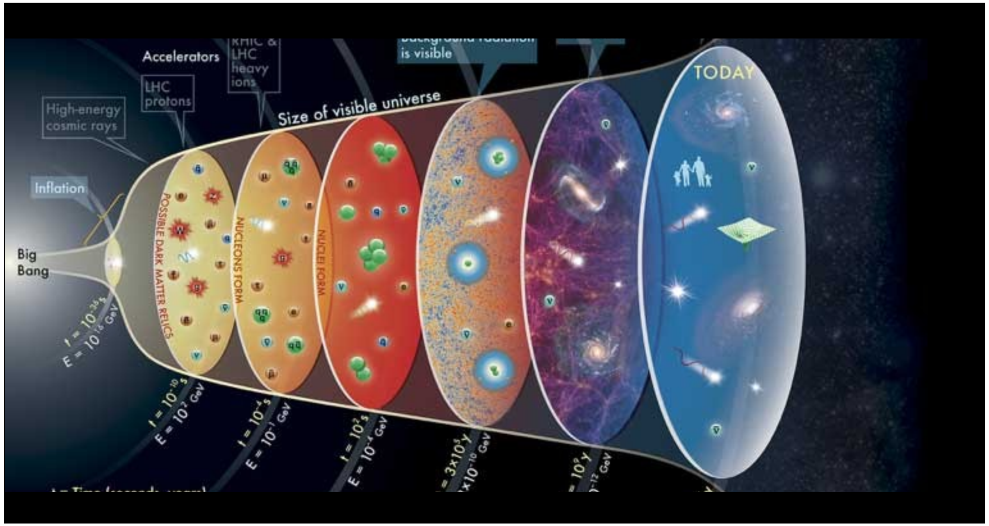

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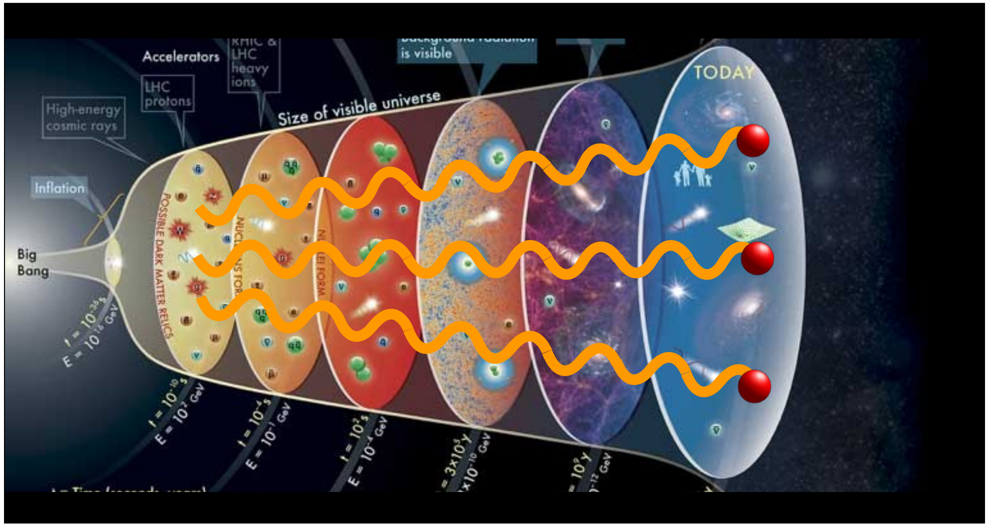

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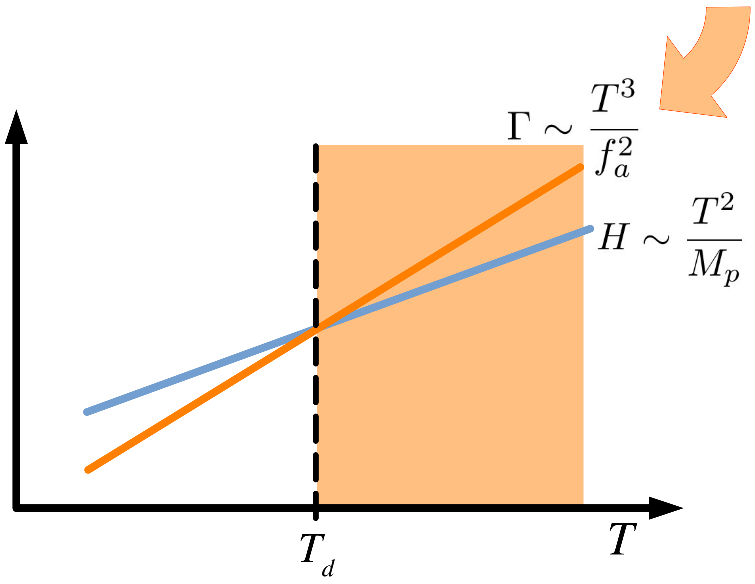

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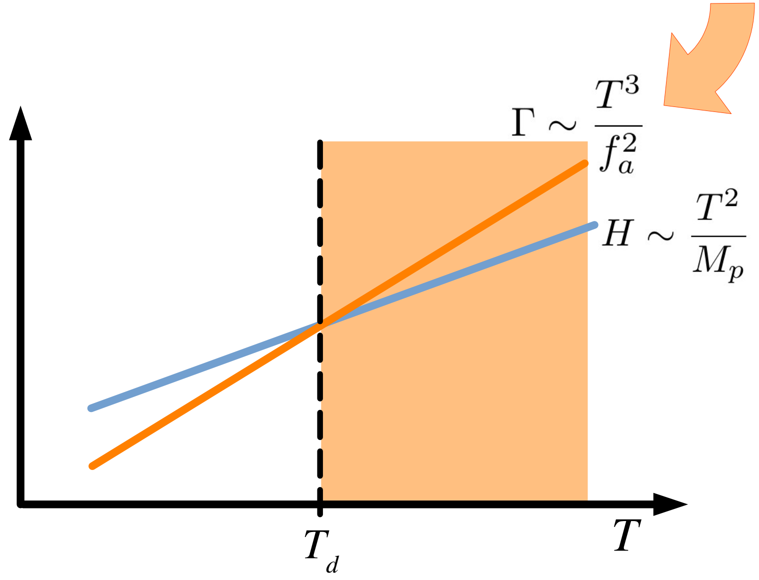

$$T_d \sim \frac{f_a^2}{M_p} \sim \Lambda_{QCD}\left(\frac{f_a}{10^8 \text{ GeV}}\right)^2$$

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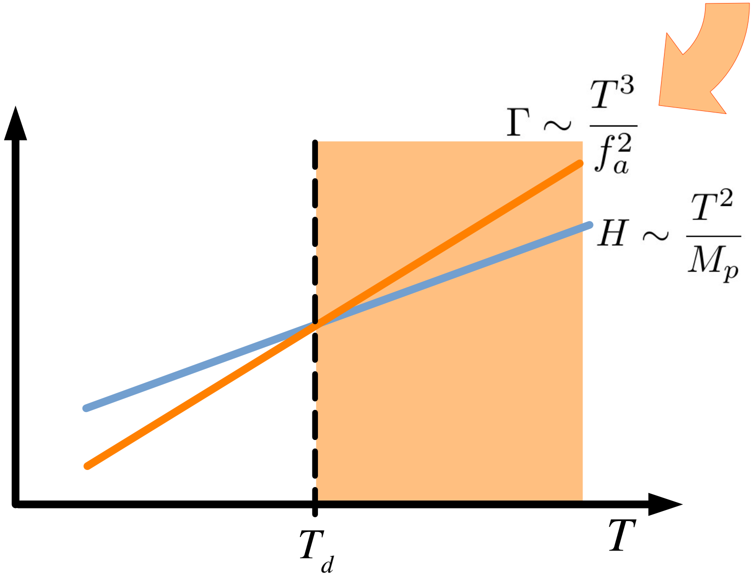

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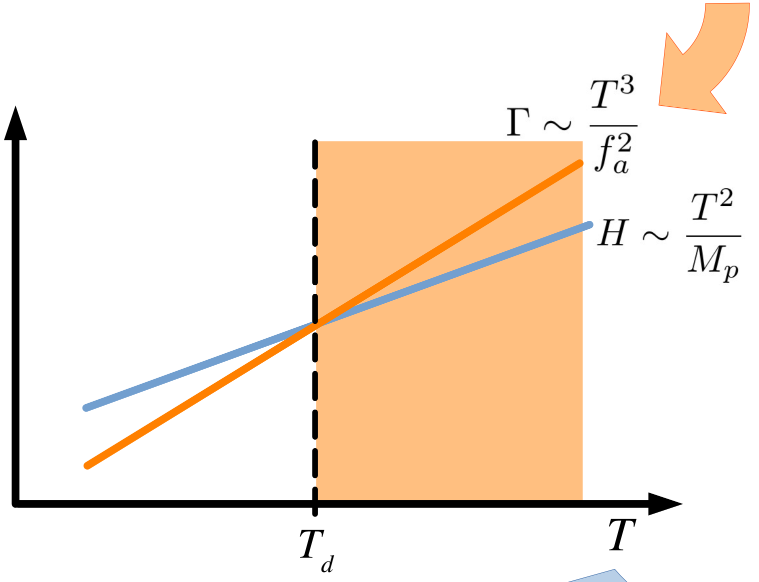

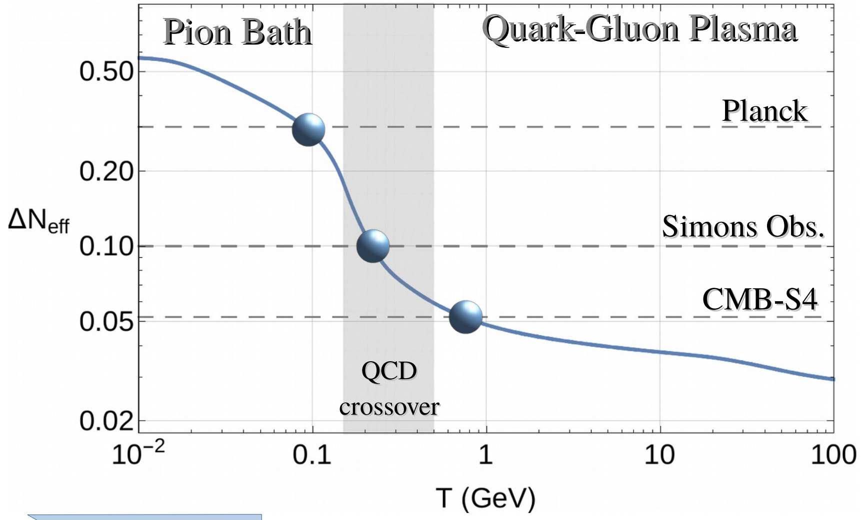

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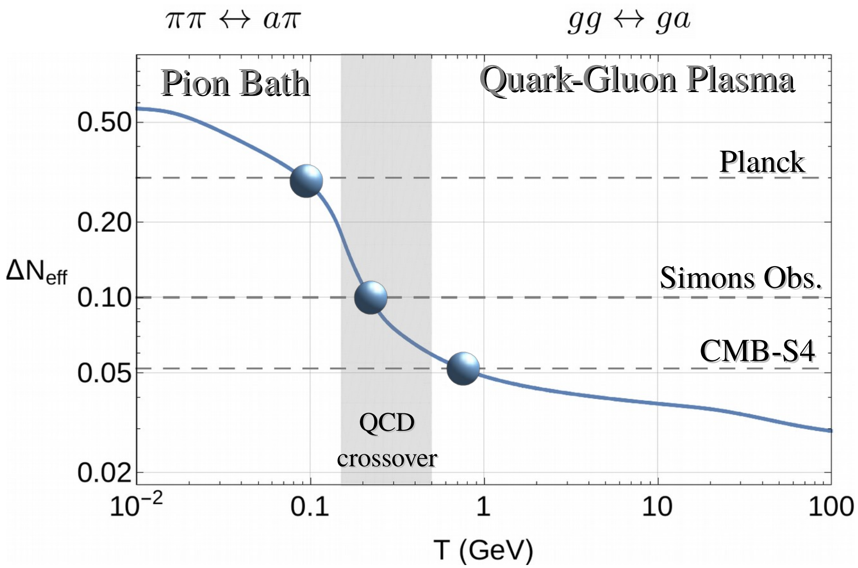
Axion $\Delta N_{\rm eff}$ has long history:



Arias-Arogon, Baumann, Bernal, Berezhiani, Chang, Choi, D'Eramo, Di Luzio, Di Valentino, Dunsky, Ferreira, Giusarma, Graf, Green, Guo, Hall, Hajkarim , Hannestad, Harigaya, Khlopov, Lattanzi, Martinelli, Masso, Melchiorri, Mena, Merlo, Mirizzi, Notari, Piazza, Raffelt, Rompineve, Rota, Sakharov, Silk, Slosar, Steffen, Wallisch, Wong, Yun, Zsembinszki ...

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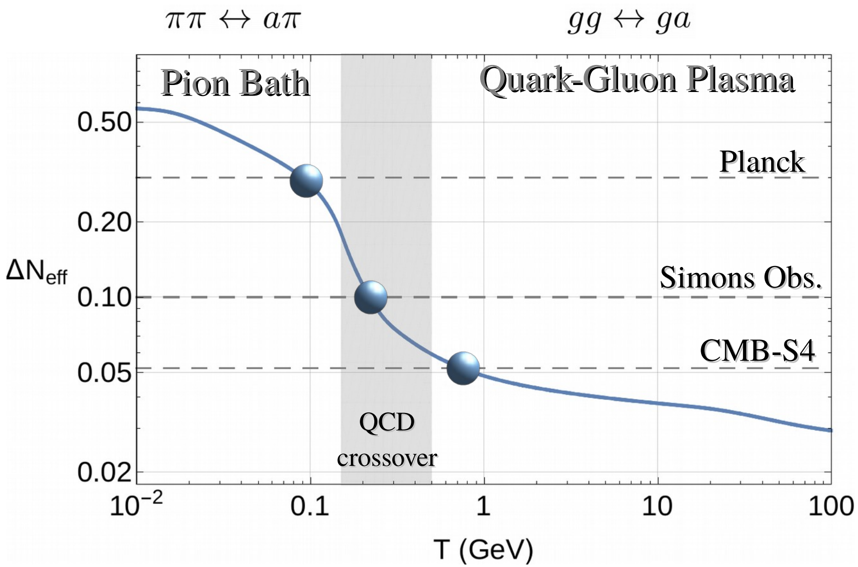
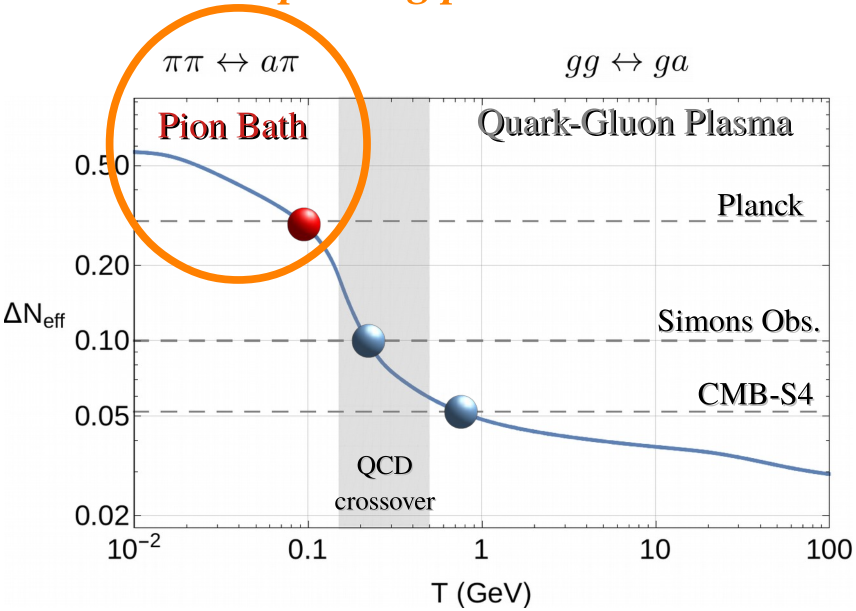


Boltzmann Eq.

$$\frac{dY}{d\log x} = (Y^{\rm eq} - Y)\frac{\overline{\Gamma}}{H}\left(1 - \frac{1}{3}\frac{d\log g_{*,S}}{d\log x}\right)$$

Axion ΔN_{eff} has long history:

Improving present bounds



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$$\frac{df_{\mathbf{p}}}{dt} = (1 + f_{\mathbf{p}})\,\Gamma^{<} - f_{\mathbf{p}}\,\Gamma^{>}$$

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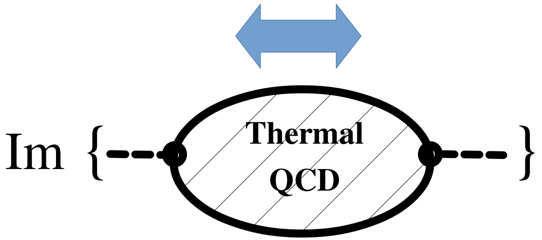

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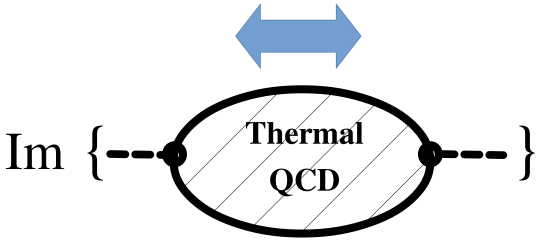

$$\Gamma^{<} = \frac{1}{2E}\int \left(\prod_{i=1}^{3} \frac{d^3\mathbf{k}_i}{(2\pi)^3 2E_i}\right) f_1^{\rm eq} f_2^{\rm eq}(1 + f_3^{\rm eq})(2\pi)^4\delta^{(4)}(k_1^\mu + k_2^\mu - k_3^\mu - k^\mu)|\mathcal{M}|^2_{2\,\leftrightarrow\,2}$$

1. The Thermalization Rate Γ

$$\pi\pi \leftrightarrow a\pi$$

LO χPT rate
(Chang Choi '93)

$$|\mathcal{M}^{\rm LO}|^2 = \theta_{a\pi}^2 \frac{s^2 + t^2 + u^2 - 3m_\pi^4}{f_\pi^4}$$

$$\theta_{a\pi} = \frac{f_\pi}{2f_a}\left(\frac{m_d - m_u}{m_d + m_u} + c_u - c_d\right)$$

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→ breaks down at T ~ 60 MeV

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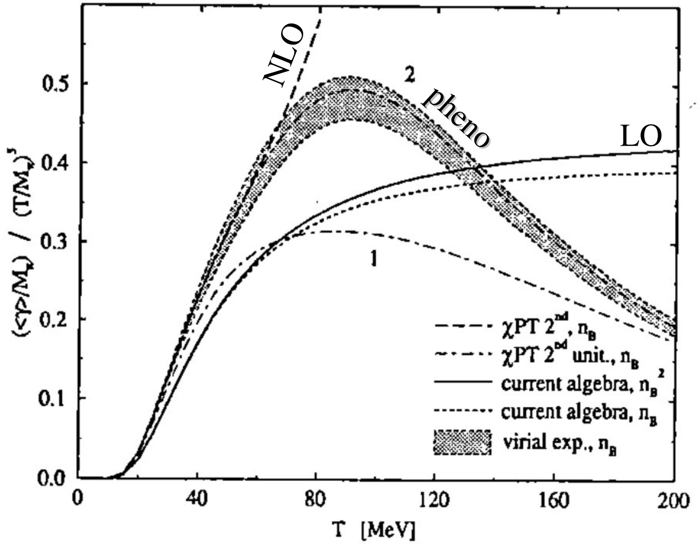

Schenk ‘94

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Strategy:

@ all orders in χPT

$$\mathcal{M}_{a\pi^i \to \pi^j \pi^k} = \theta_{a\pi} \cdot \mathcal{M}_{\pi^0 \pi^i \to \pi^j \pi^k} + \mathcal{O}\left(\frac{m_\pi^2}{s}\right)$$

e.g. @ LO

$$|\mathcal{M}^{\rm LO}|^2 = \theta_{a\pi}^2 \frac{s^2 + t^2 + u^2 - 3m_\pi^4}{f_\pi^4}$$

$$|\mathcal{M}_{\pi\pi}^{\rm LO}|^2 = \frac{s^2 + t^2 + u^2 - 4m_\pi^4}{f_\pi^4}$$

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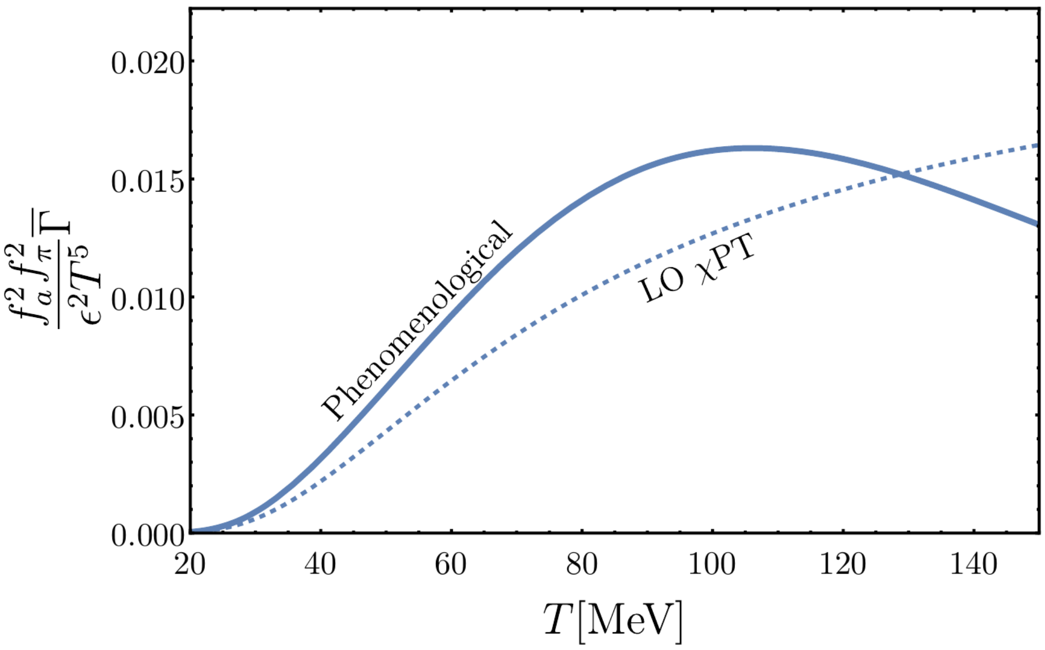

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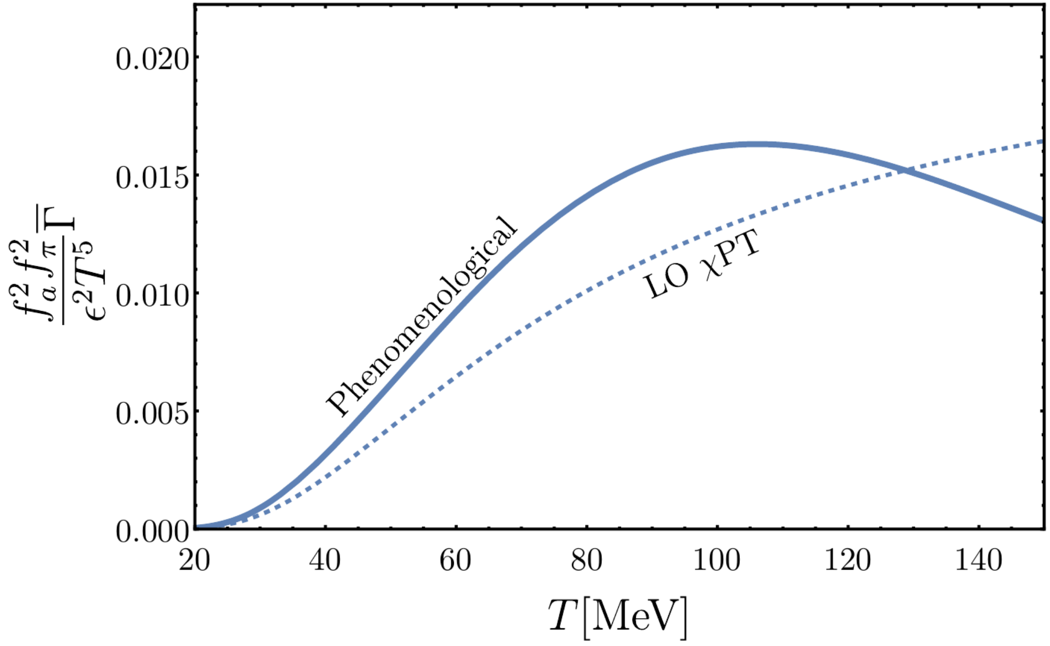

In reasonable agreement with:

Di Luzio, Camalich, Martinelli, Oller, Piazza '22 (using NLO+unitarization)

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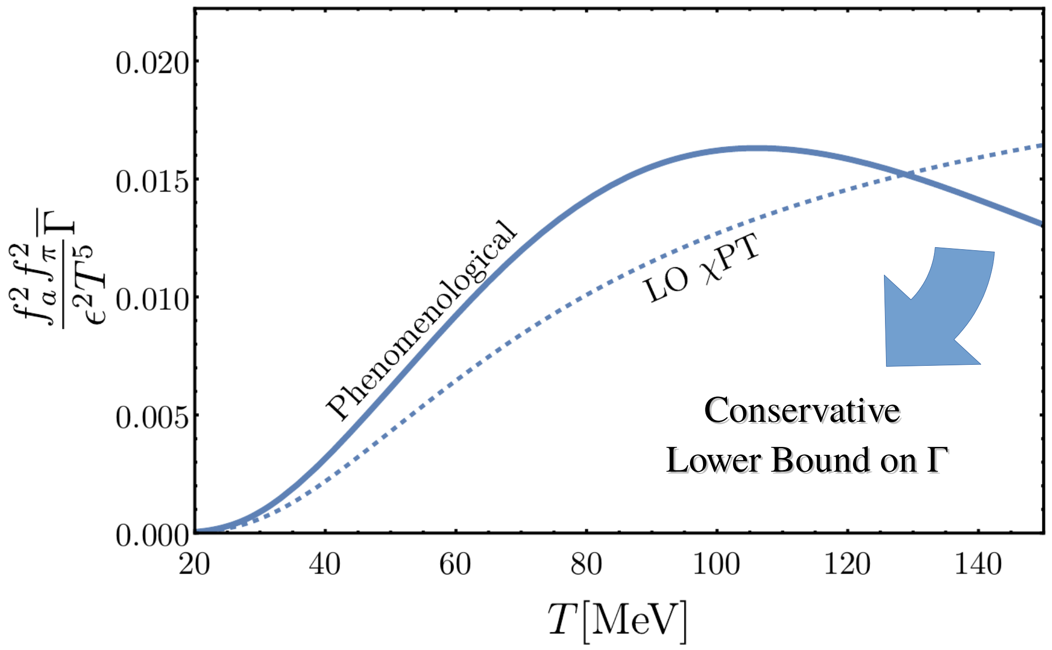

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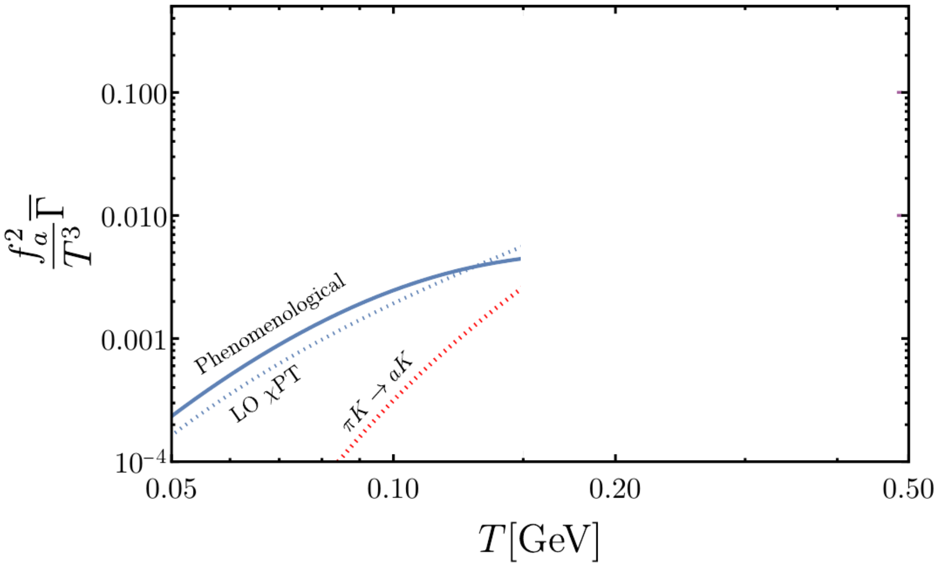

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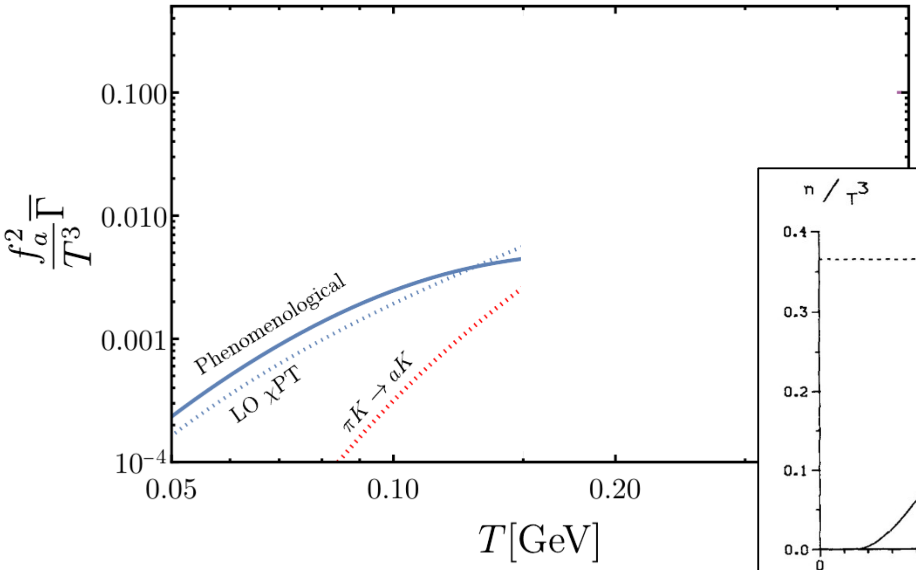
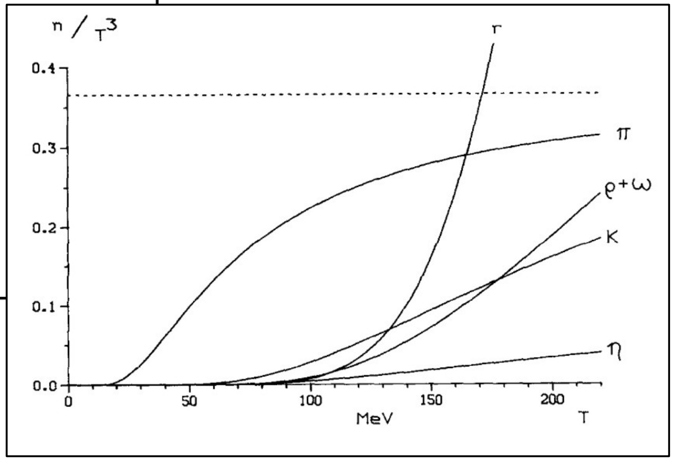
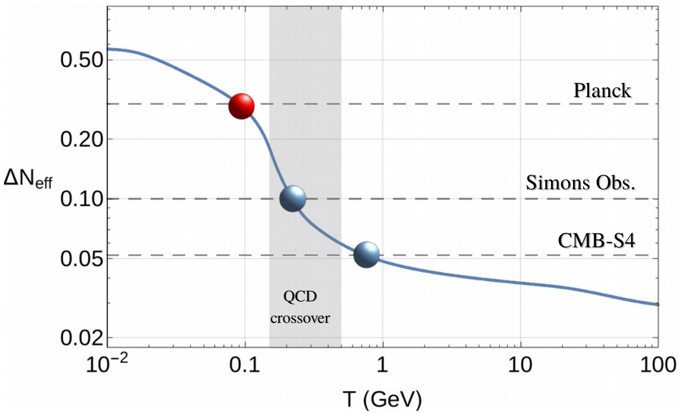




Gerber Leutwyler '89

2. Momentum Dependence



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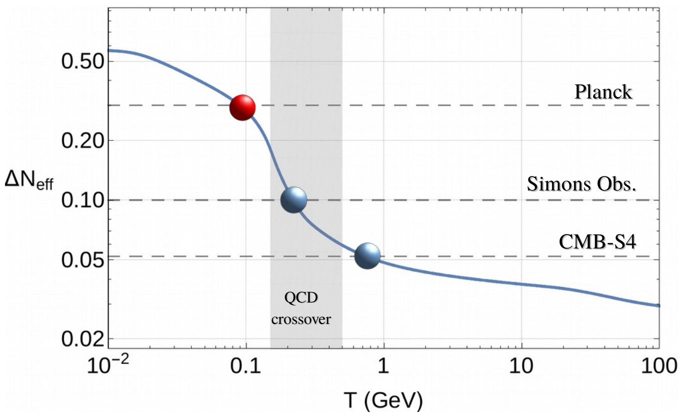

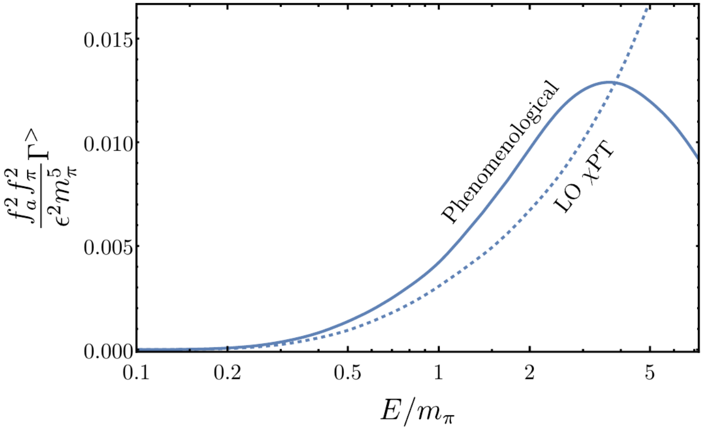

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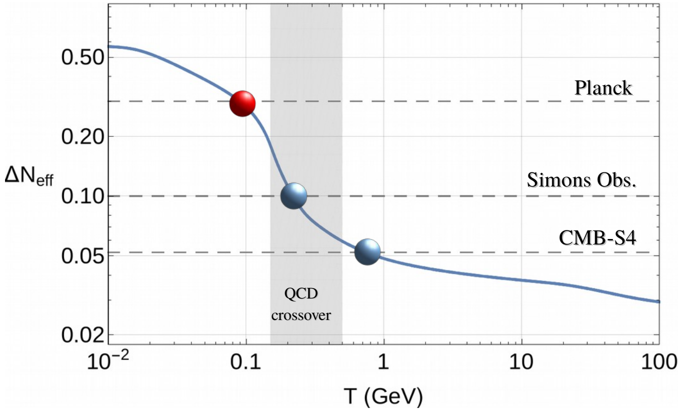

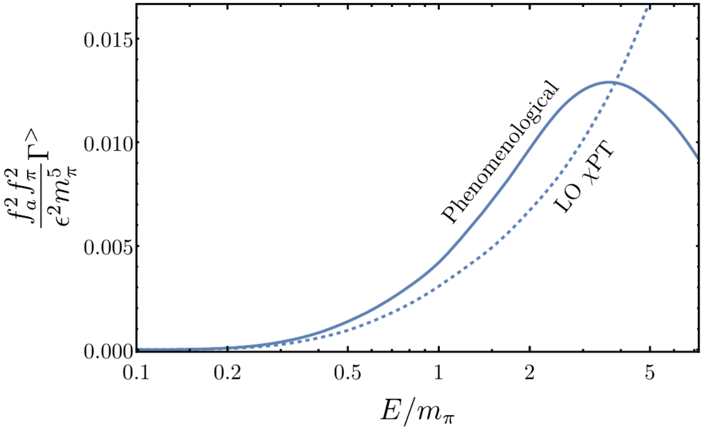

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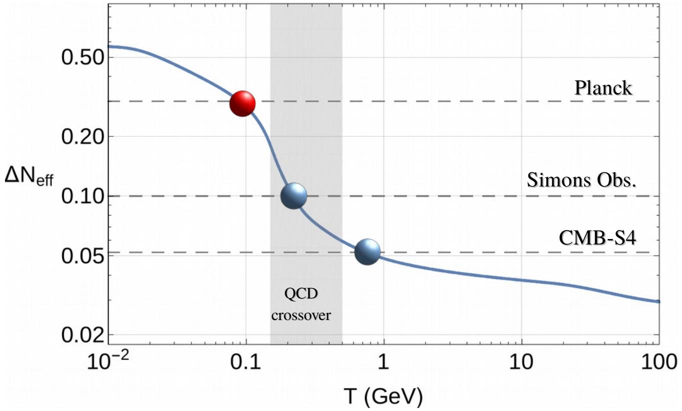

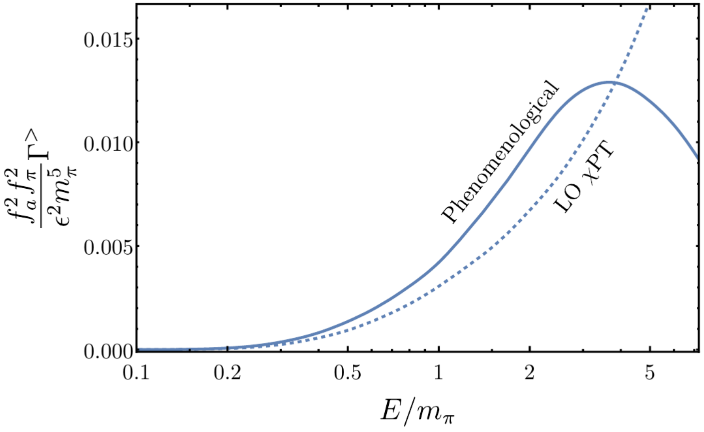

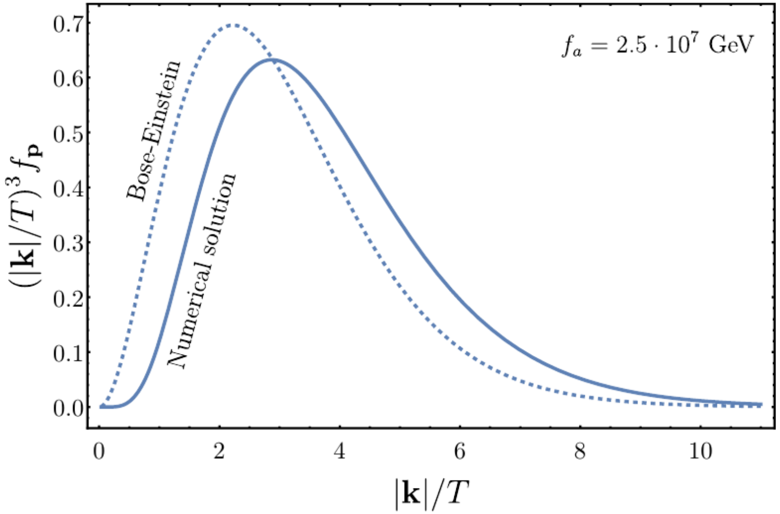

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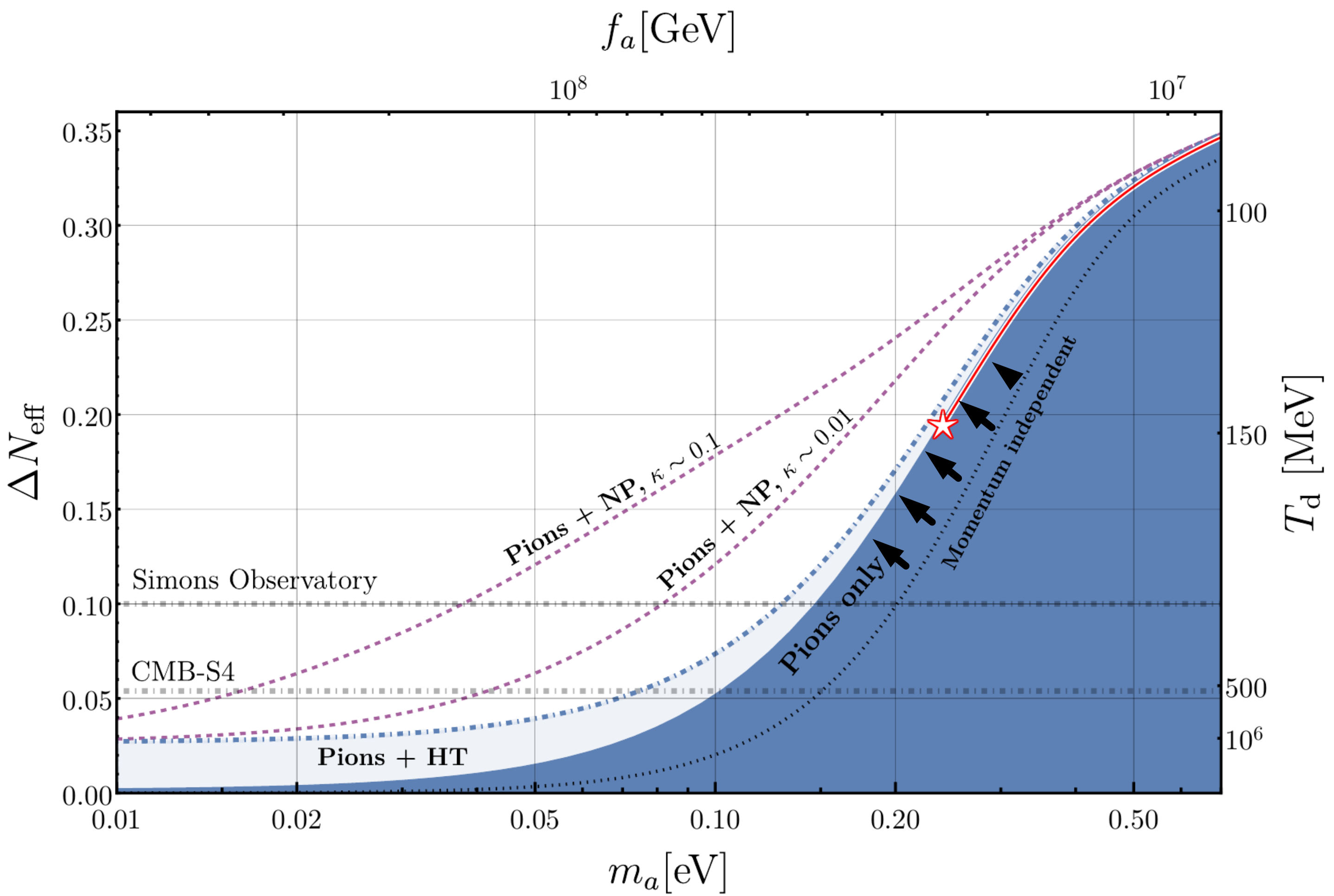
~ 40% enhancement

Future Reach



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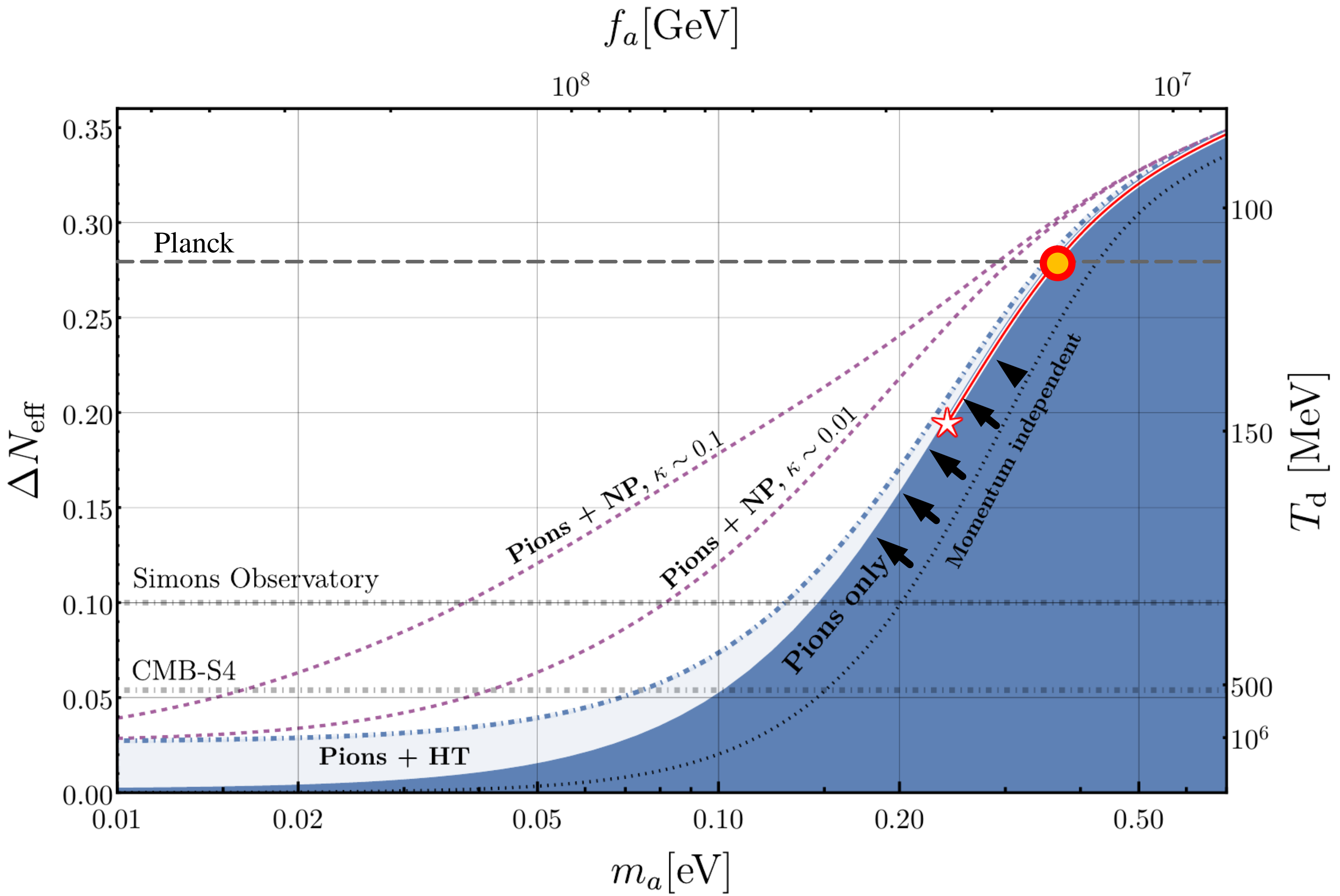

3. Cosmological Fit ($\Lambda_{\mathrm{CDM}} + \Sigma\, m_\nu + m_a$)

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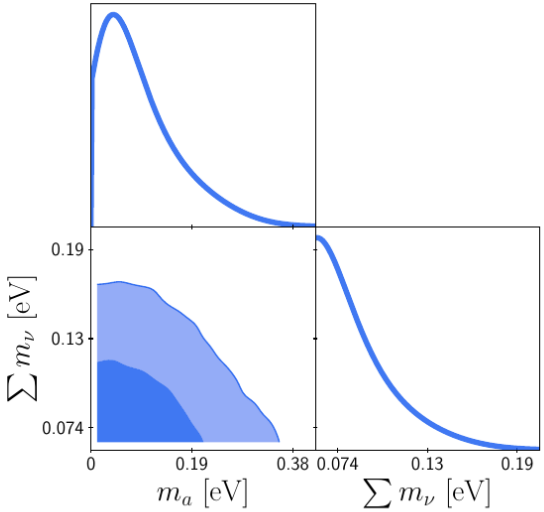

$m_a \leq 0.24$ eV

$f_a \geqslant 2.4 \cdot 10^7$ GeV

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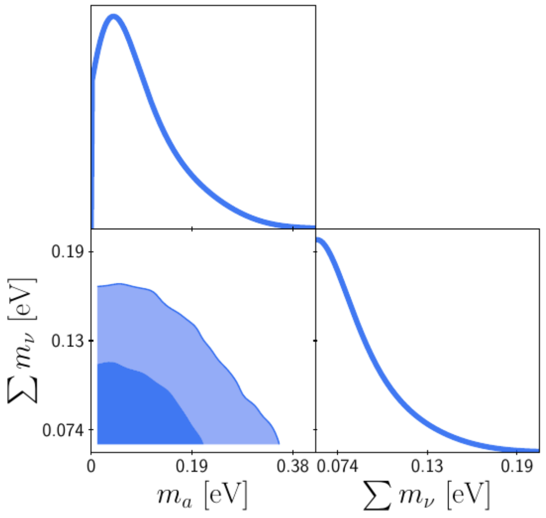

$$m_a \leq 0.24 \text{ eV} \qquad f_a \geqslant 2.4 \cdot 10^7 \text{ GeV}$$

$$\Leftrightarrow$$

$$\Delta N_{\text{eff}} \lesssim 0.19$$

finite mass effect

~ 40% enhancement

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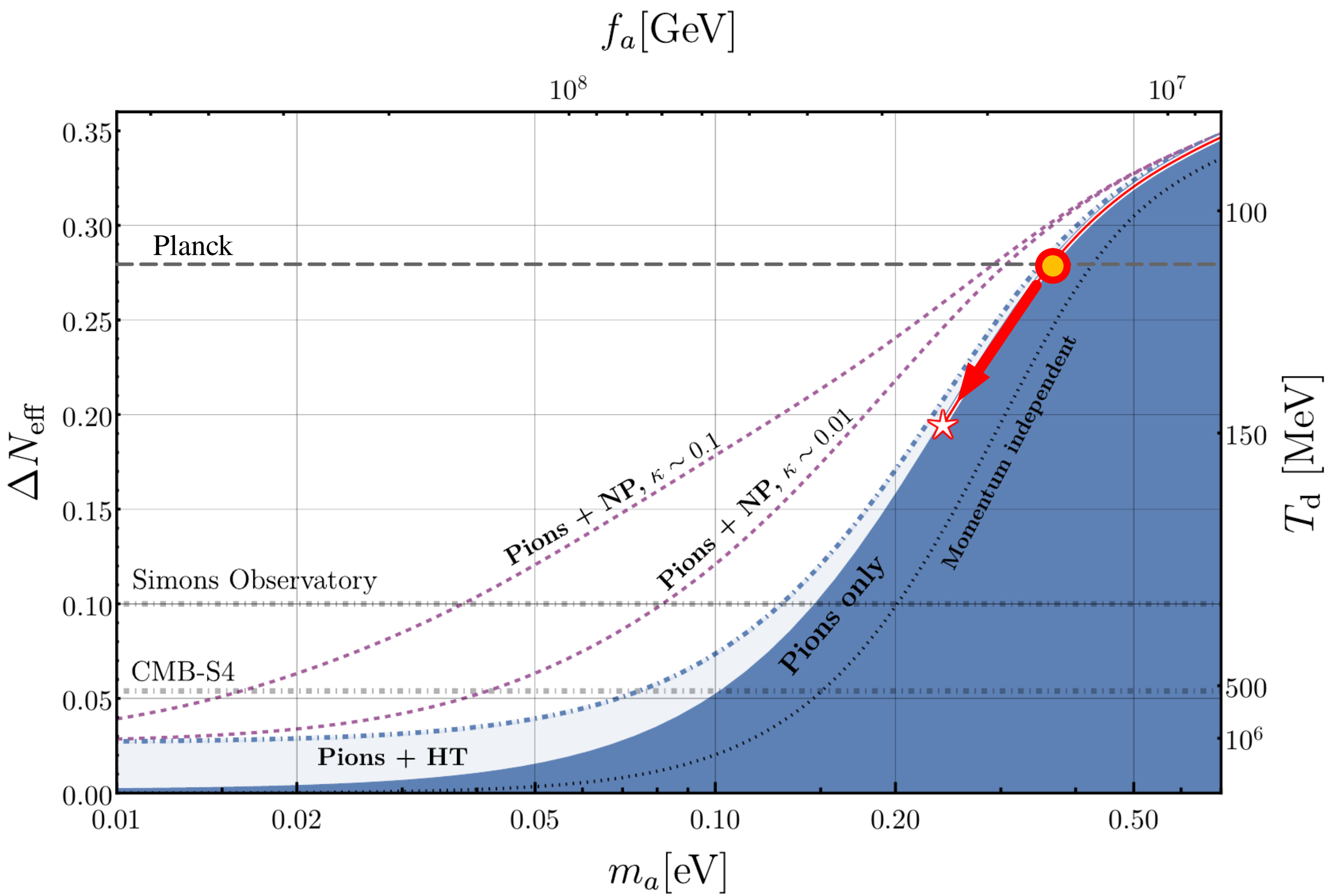

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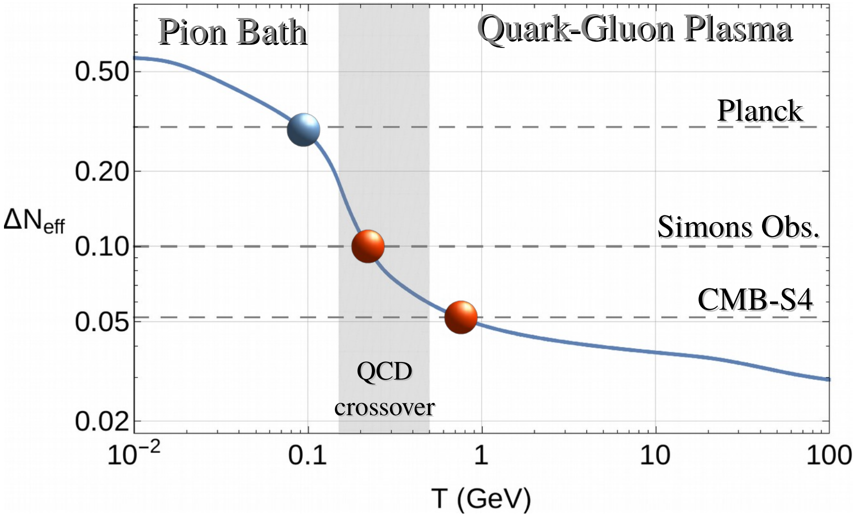

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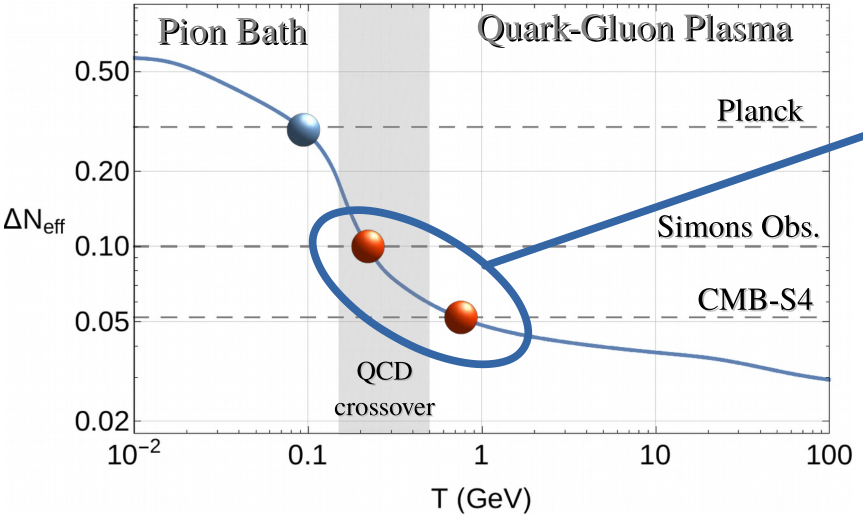

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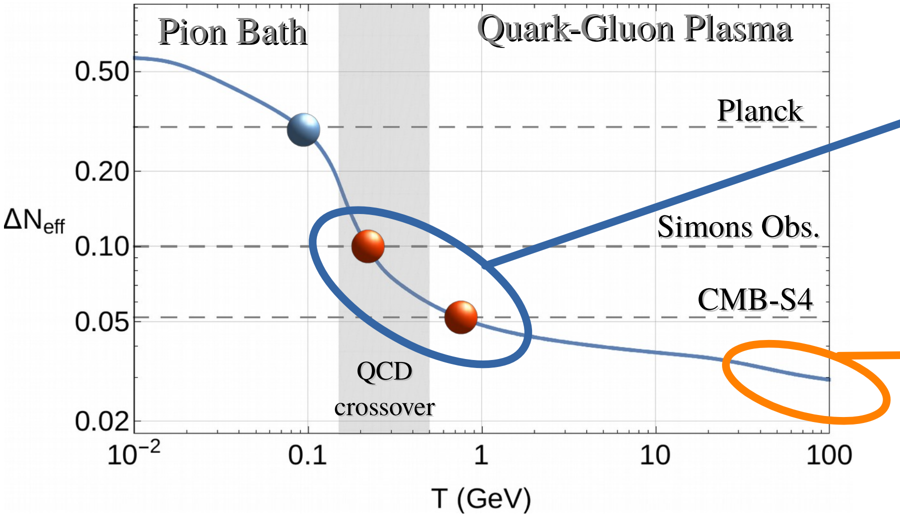
Non-Perturbative

Future Reach



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Non-Perturbative

$gg \leftrightarrow ga$

Perturbative ?

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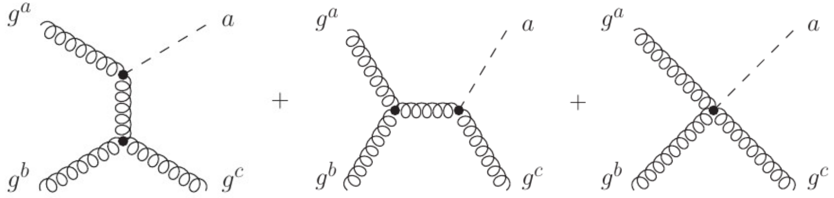
High Temperatures Regime



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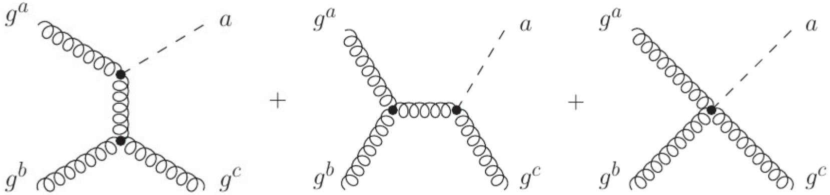

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Masso, Rota, Zsembinszki ‘02
Graf, Steffen ‘10

$$F_3 = g_s^2 \log\left(\frac{3T^2}{2m_g^2}\right) = g_s^2 \log\left(\frac{3}{2g_s}\right)^2$$

for $g_s \ll 1$

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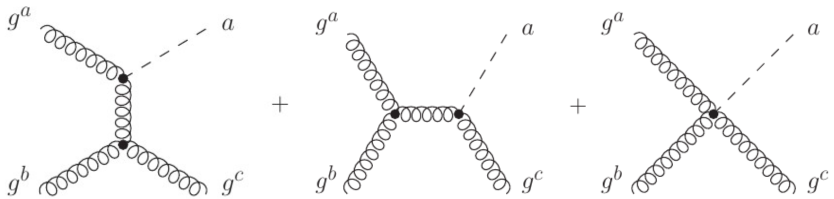

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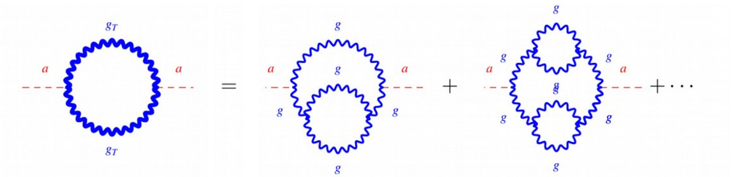
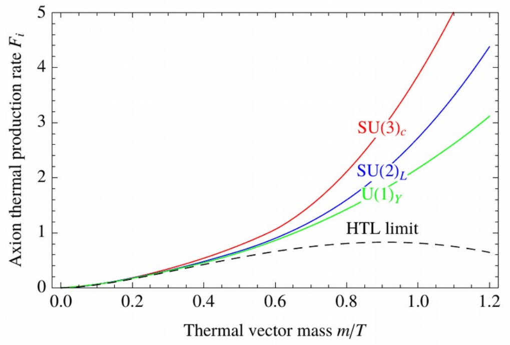
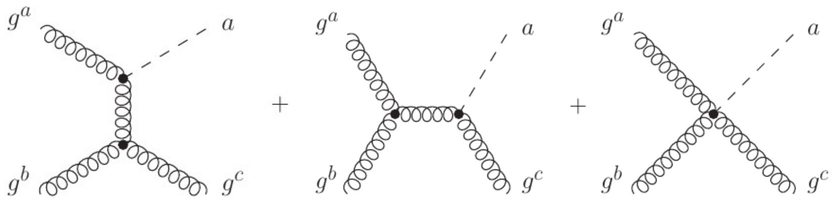
Salvio, Strumia, Xue '13





High Temperatures Regime



$$\overline{\Gamma}_{\rm pert} = \frac{\alpha_s^2 T^3}{4\pi^3 f_a^2} F_3$$

Masso, Rota, Zsembinszki ‘02
Graf, Steffen ‘10

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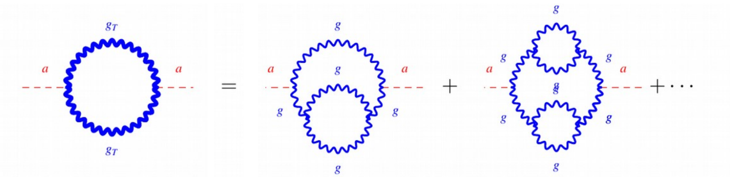
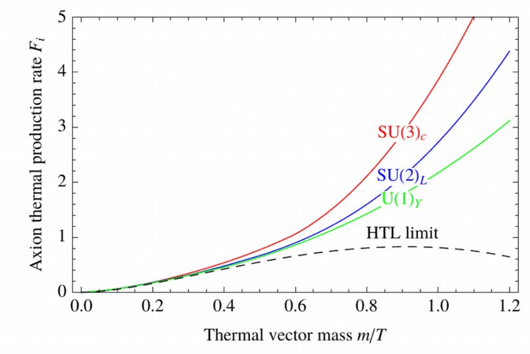
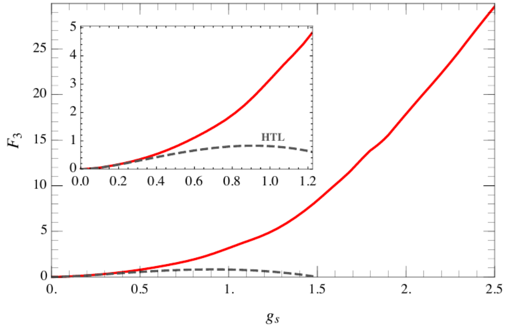




D’Eramo, Hajkarim, Yun ‘21



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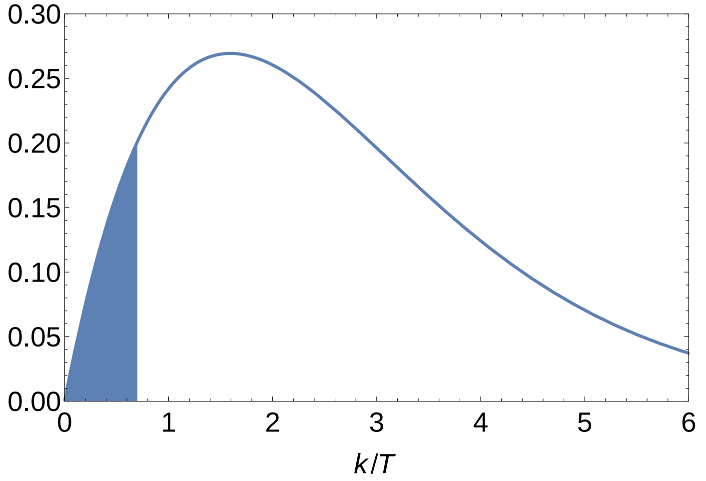

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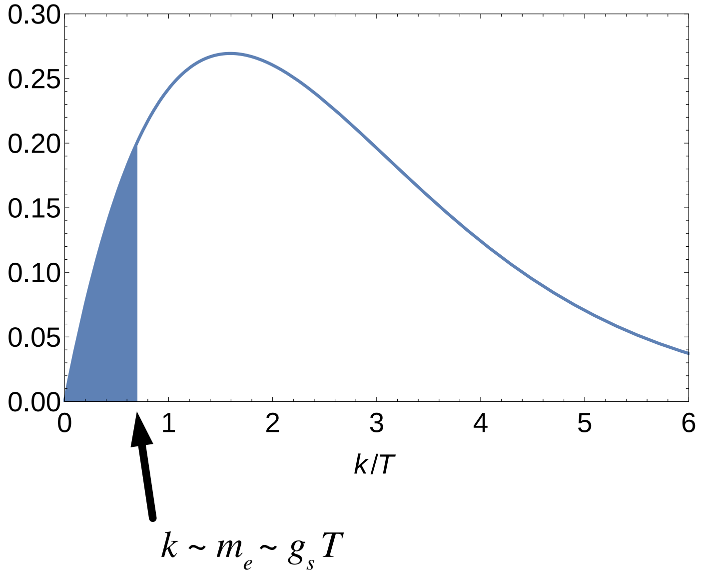

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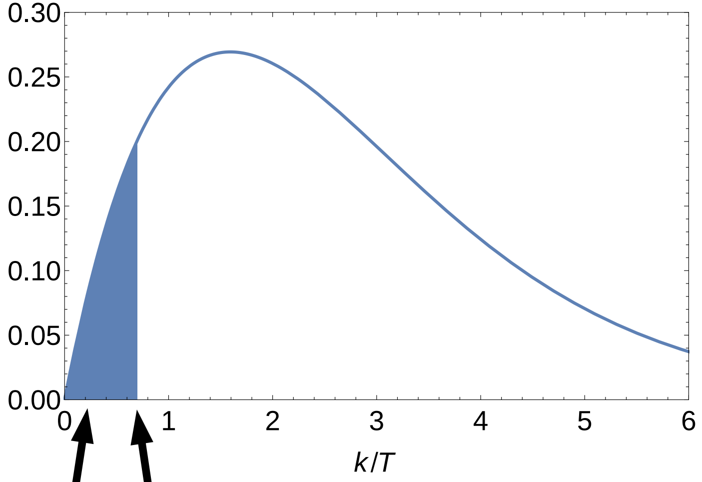


$k \sim m_e \sim g_s T$

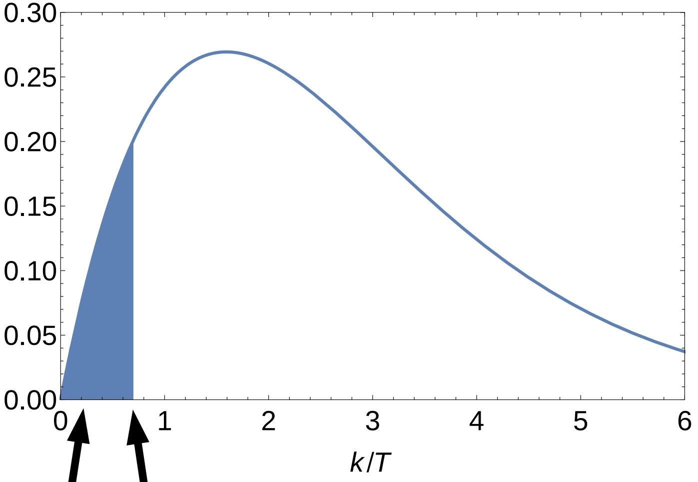
$k \sim m_m \sim g_s^2 T$

$\# \sim 1 / g_s^2$

Linde ‘80

Gross, Pisarski, Yaffe ‘81

High Temperatures Regime



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Linde ‘80

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@ $g_s \ll 1$:

collective effects are phase-space suppressed $O(g_s{}^n)$

[e.g. for free energy $O(g_s{}^6)$]

large occupation numbers → dominated by semi-classic

[non-linear YM equations - e.g. strong sphalerons]

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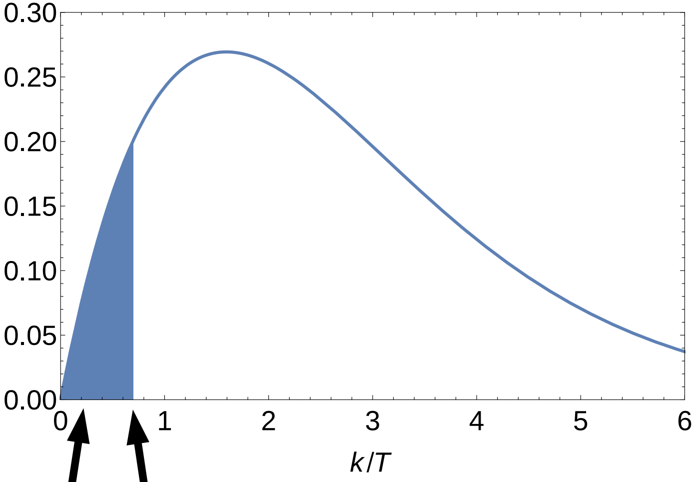

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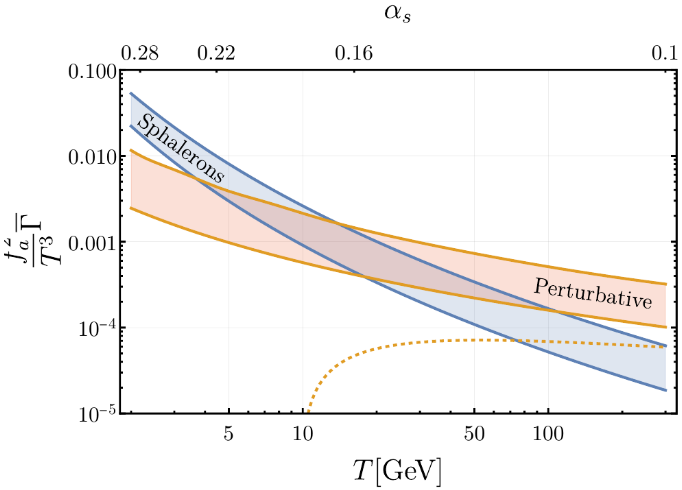
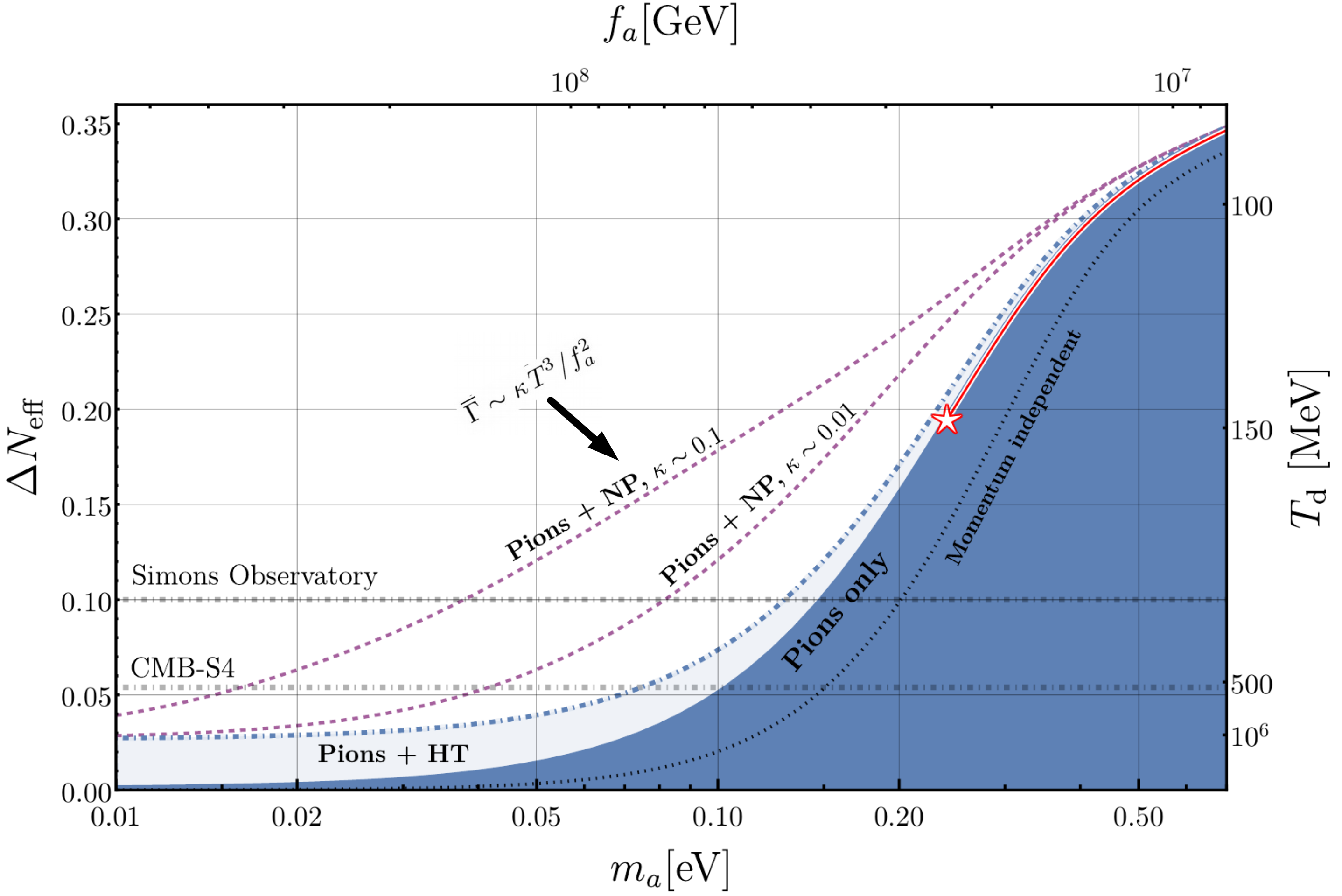


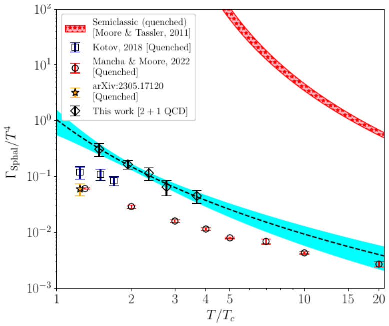
$\Gamma_{\rm sphal} \simeq (N_c \alpha_s)^5 T^4$

Moore, Tassler ‘10

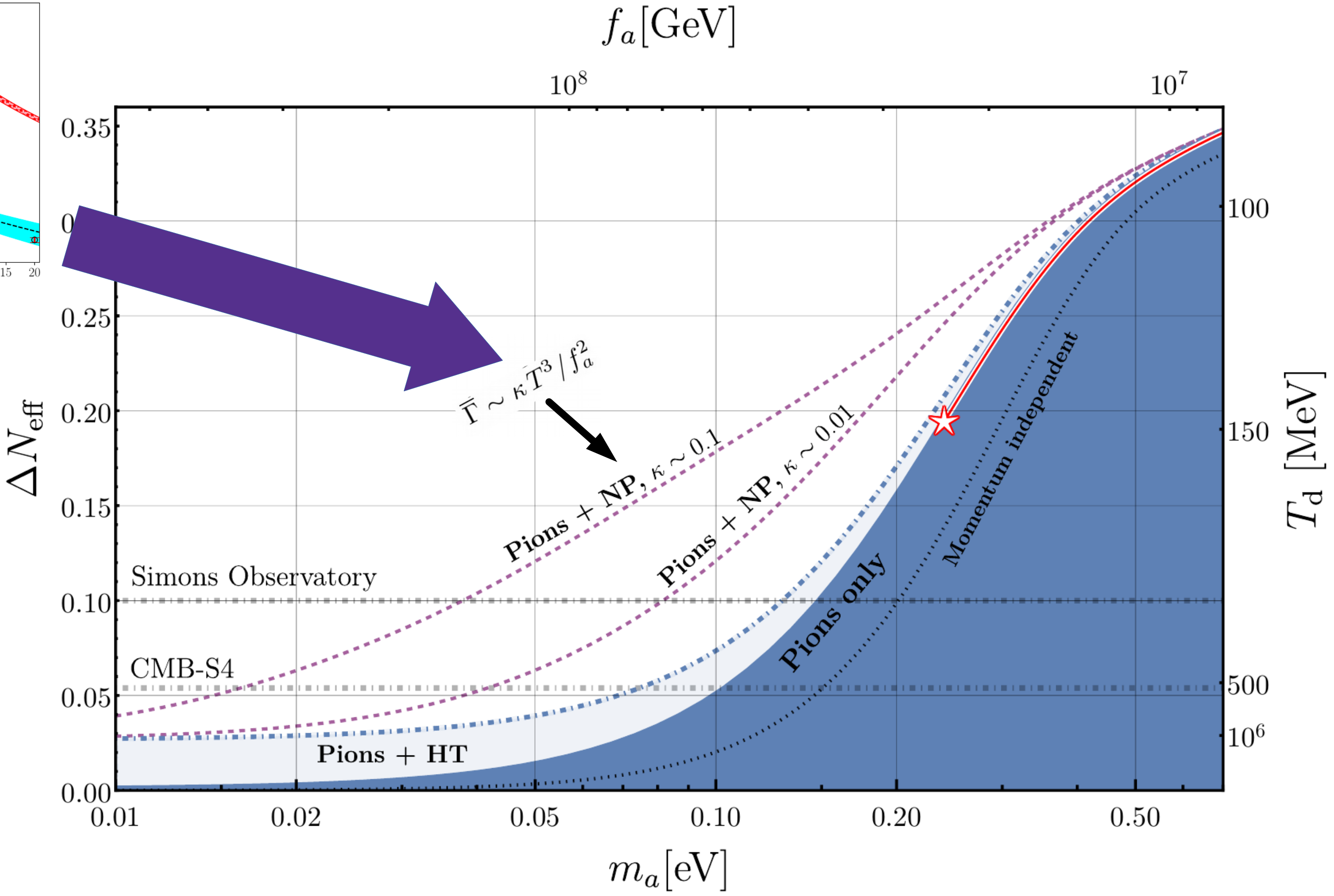
Future Reach



Future Reach



Bonanno et al.
2308.01287



Conclusions:

- More reliable upper bound on m_a (< 0.24 eV) from cosmology (for minimal KSVZ-like axions)
- Importance of momentum dependence on Boltzmann equation @ around QCD scale
- Doubts about reliability of perturbative rates above T_c
- Non-perturbative rates crucial for interpreting upcoming CMB experiments

  Promising preliminary results from Lattice QCD...

Thanks!

Back Up

1. The Thermalization Rate Γ

General form of low energy axion QCD Lagrangian:

$$\mathcal{L} = \bar{q}\left(i\partial\!\!\!/ + \frac{c_0}{2f_a}\partial\!\!\!/ a\,\gamma_5\right) q - \bar{q}_L M_a q_R + h.c.\,, \qquad M_a \equiv \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix} e^{i\frac{a}{2f_a}(1+c_3\sigma^3)}$$

$$\frac{\partial_\mu a}{2f_a} j_A^\mu \overset{\chi\mathrm{PT}}{=} \mathcal{O}(M_q)$$

$$\pi^0 = \cos(\theta_{a\pi})\,\pi^0_{\rm phys} + \sin(\theta_{a\pi})\,a_{\rm phys} \simeq \pi^0_{\rm phys} + \theta_{a\pi} a_{\rm phys}$$

<u>@ all orders in χPT</u>

$$\mathcal{M}_{a\pi^i\to\pi^j\pi^k} = \theta_{a\pi}\cdot\mathcal{M}_{\pi^0\pi^i\to\pi^j\pi^k} + \mathcal{O}\left(\frac{m_\pi^2}{s}\right)$$

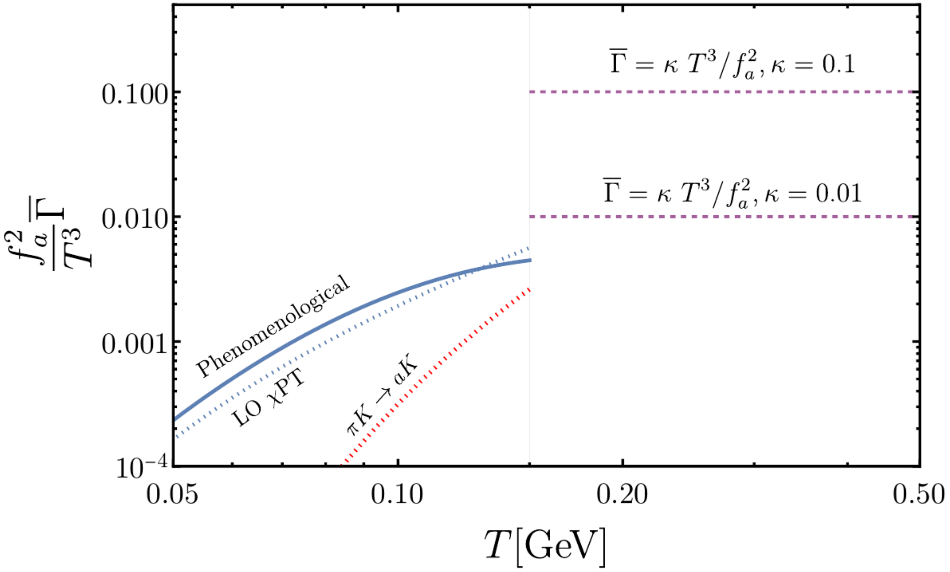
$\lesssim 10\%$

e.g. @ LO

$$|\mathcal{M}^{\rm LO}|^2 = \theta_{a\pi}^2\frac{s^2+t^2+u^2-3m_\pi^4}{f_\pi^4} \qquad |\mathcal{M}^{\rm LO}_{\pi\pi}|^2 = \frac{s^2+t^2+u^2-4m_\pi^4}{f_\pi^4}$$

Strong Sphaleron-like contribution to Axion rate

$$\overline{\Gamma}_{\rm sphal} = \frac{1}{n^{\rm eq}}\int^{|\mathbf{k}|<|\mathbf{k}_s|} \frac{d^3\mathbf{k}}{(2\pi)^3 2E}\frac{\Gamma_{\rm sphal}}{f_a^2}e^{-E/T} = \frac{(N_c\alpha_s)^5 T^3}{4\zeta_3 f_a^2}\left(1-\left(1+\frac{|\mathbf{k}_s|}{T}\right)e^{-|\mathbf{k}_s|/T}\right)$$



$$\Gamma^{>}_{\rm top}(E = |\mathbf{k}| < |\mathbf{k}_s|) \simeq \Gamma_{\rm sphal} \simeq (N_c\alpha_s)^5 T^4$$

$$|\mathbf{k}_s| \sim N_c\alpha_s T$$

The Thermal Width:

Challenge for Lattice QCD: <u>Compute Γ_k for $T > T_c$</u>

Existing Attempts (at k=0) e.g.

Moore, Tassler '10 : Classical SU(N) simulations

Kotov '18 : Quantum Euclidean (anal. cont.)

Altenkort et al. '20 : Quantum Euclidean (anal. cont.)

Mancha, Moore '22 : Quantum Euclidean (plus modeling)

$$\Gamma_{\text{sphal}} = 2T \lim_{\omega \to 0} \frac{\rho(\omega)}{\omega}$$

$$G(\tau) = \int \mathrm{d}^3x \, \langle q(\vec{0},0) q(\vec{x},\tau) \rangle$$

$$= -\int_0^\infty \frac{\mathrm{d}\omega}{\pi} \, \rho(\omega) \frac{\cosh[\omega(1/2T - \tau)]}{\sinh(\omega/2T)}$$

Important to exploit upcoming experiments!