M. A. Buen-Abad, Z. Chacko, C. Kilic, G. Marques-Tavares, TY
[arXiv:2208.05984, 2306.01844]

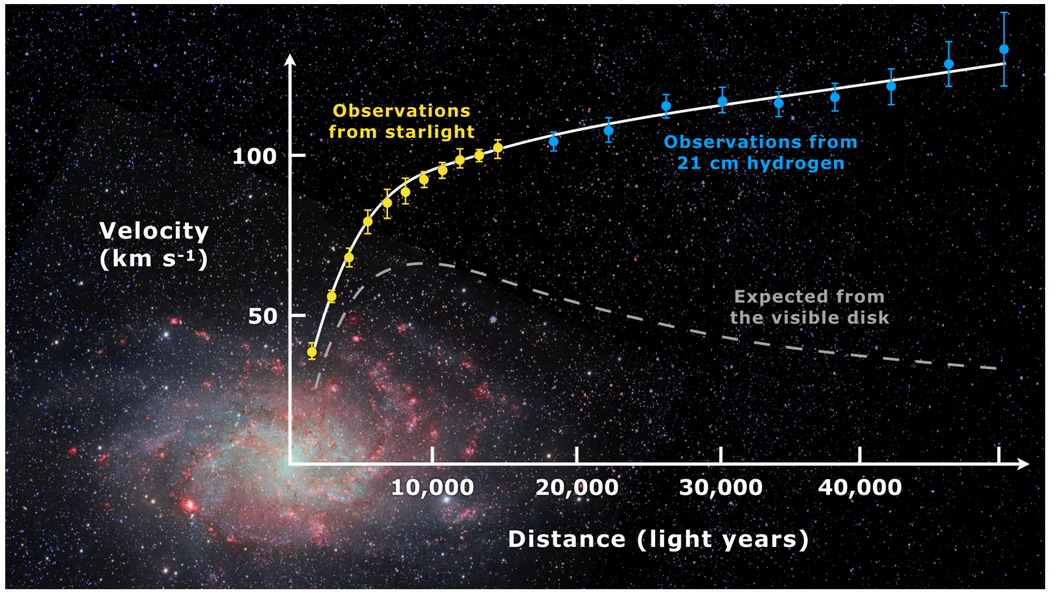
Interactions in Dark Sector and Tensions in Cosmological data

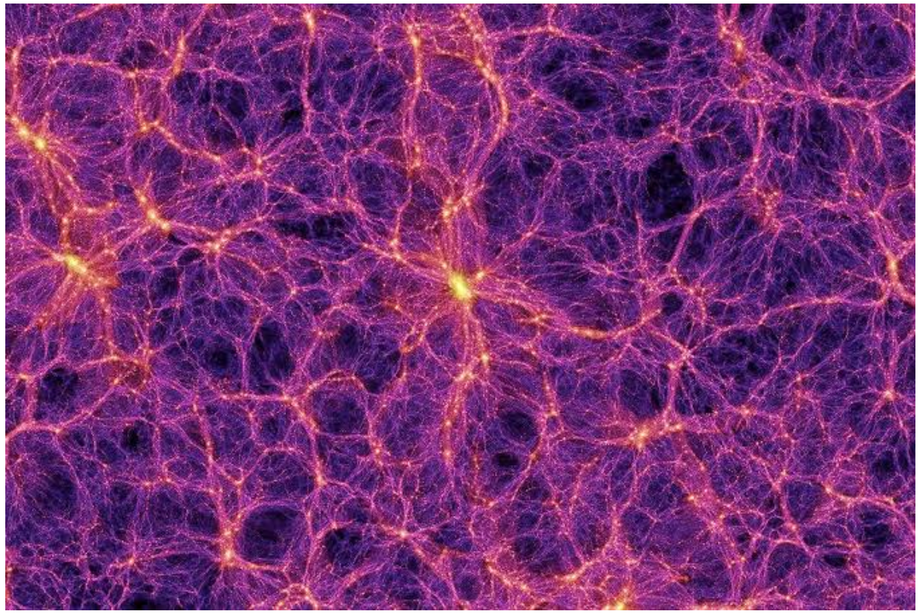
Taewook Youn
Cornell U, LEPP / Korea U

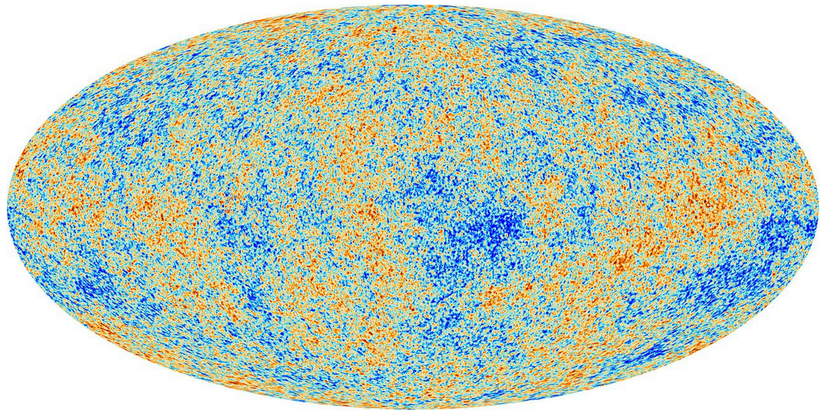
Jan 12th 2024

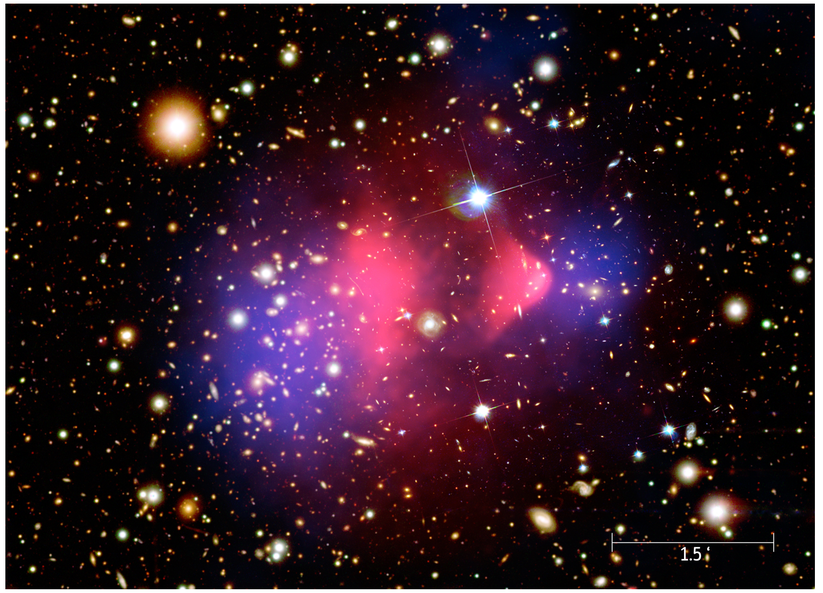
Dark Matter

Evidence









Dark Matter

What we know and don't know about DM

What we know

DM is singlet under the SM gauge groups

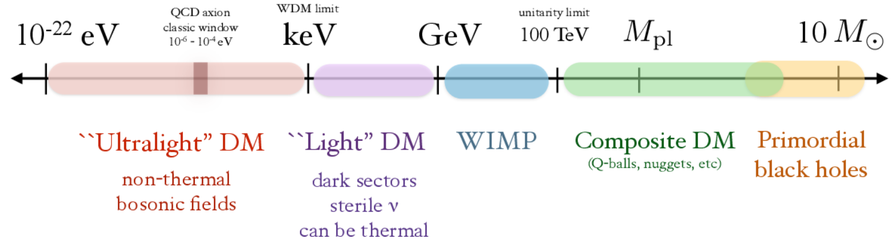
Energy density

~27% (total); ~0.4 $\mathrm{GeV/cm^3}$ (local)

DM is cold ($v/c \sim 10^{-3}$) and collisionless ($\sigma_{\rm SI}/m_{\rm DM} \lesssim 1\ \mathrm{cm^2/g}$)

What we don't know

Mass?



Production mechanism?

Origin?

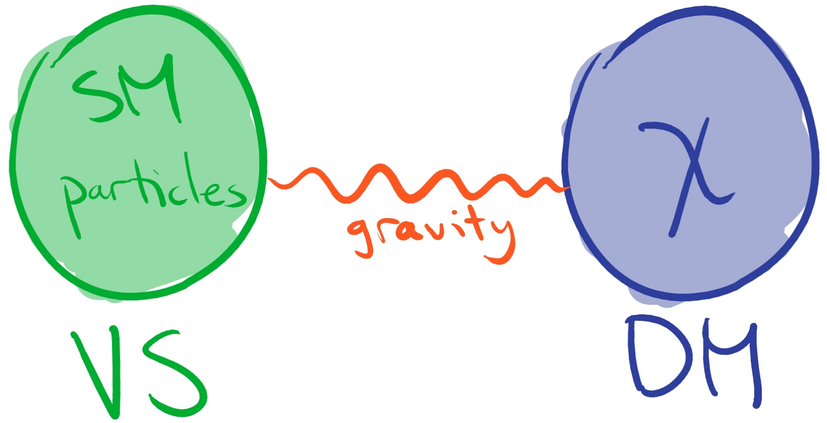
SUSY? Extra Dimension? Mirror Sector?

etc…

Dark Matter

Dark Sector

DM could be just one particle, only interacting with SM via gravitation



Dark Matter

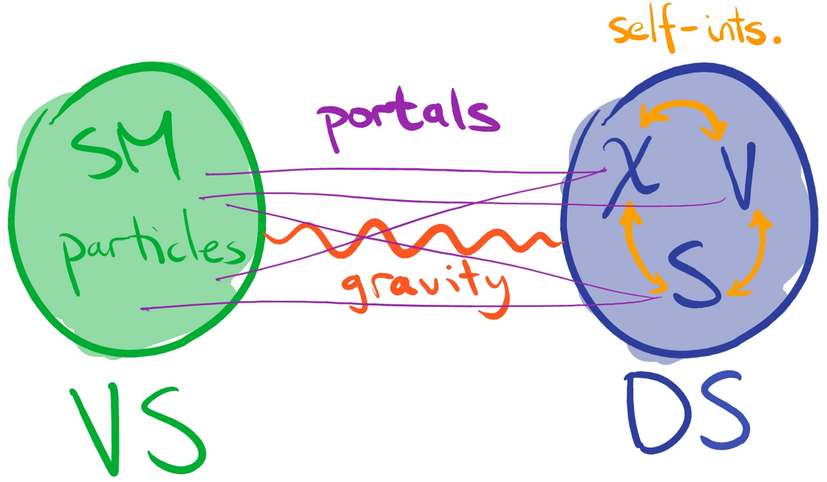
Dark Sector

DM could be just one particle, only interacting with SM via gravitation

Not necessarily!

Multiple States (eg. dark proton, dark photon, dark neutrino, etc.)

Various interactions within DS (self-interactions) and/or btw SM and DS (portals)



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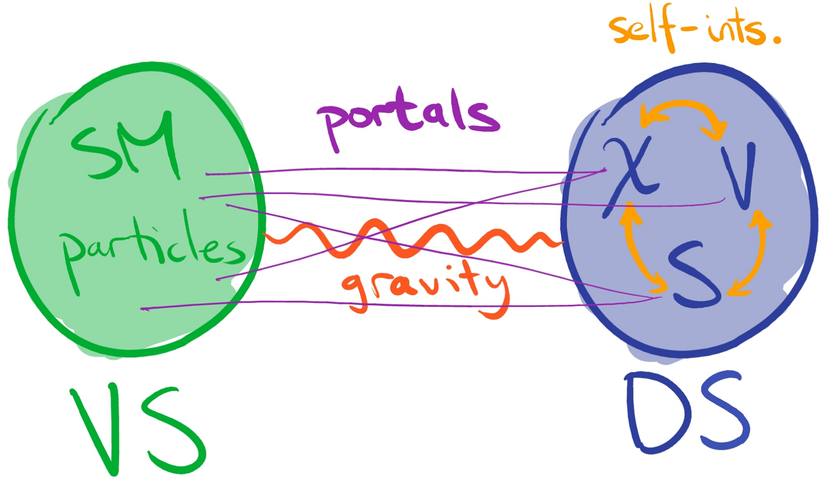

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Not necessarily!

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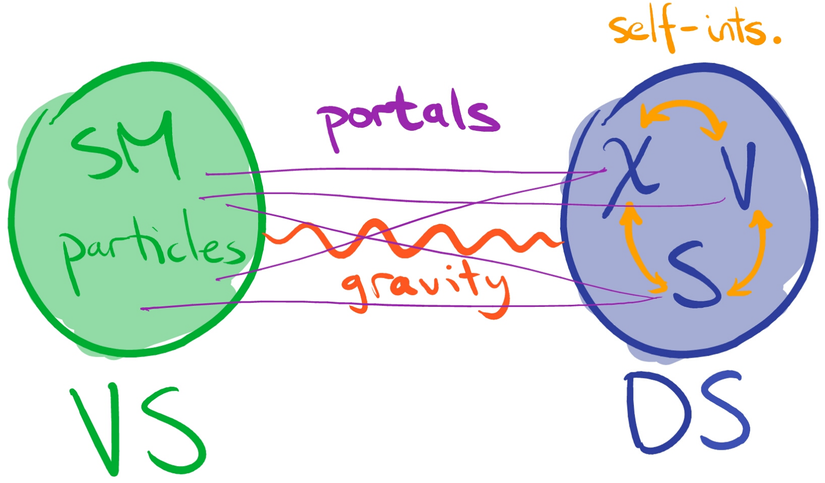
Imprint on Cosmological Observations

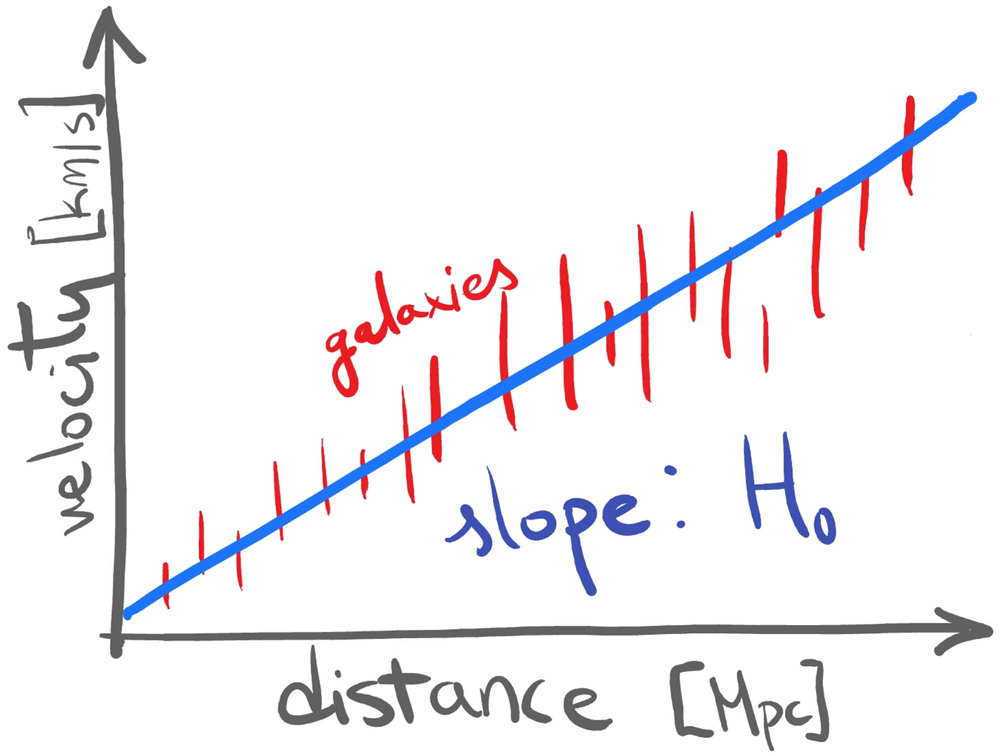
Possibly in H_0 and S_8 tensions

Even though gone, worth investigating



Cosmological Tensions

Hubble tension (~4-6 σ)



$$v = H_0 D$$

Estimate the size and age of universe

Cosmological Tensions

Hubble tension (~4-6 σ)

Early Universe

CMB fit to ΛCDM

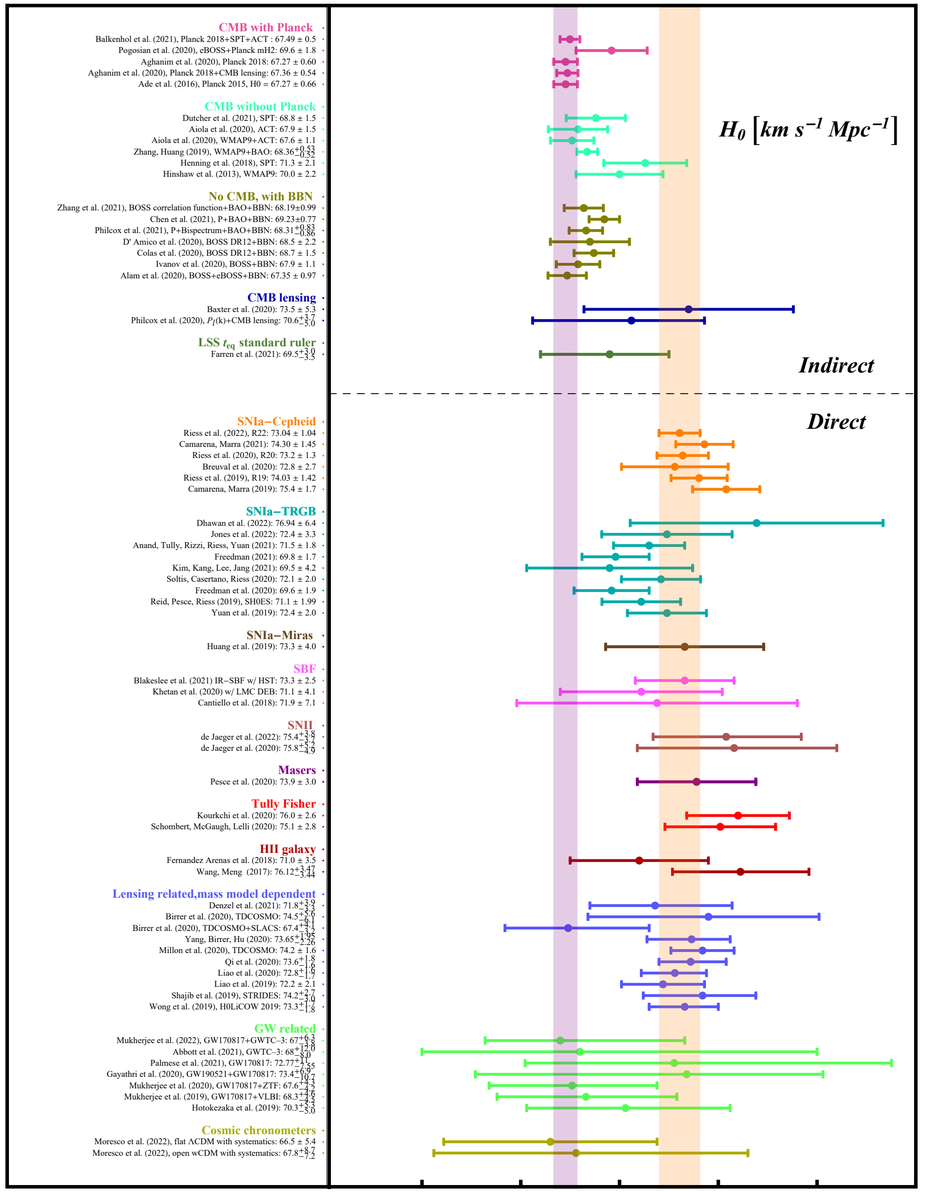
~68 km/s/Mpc **Planck '18 [arXiv:1807.06209]**

Late Universe

Cosmic Distance Ladder

~72 km/s/Mpc **A. G. Riess et al. [arXiv:2112.04510]**

Snowmass [arXiv:2203.06142]



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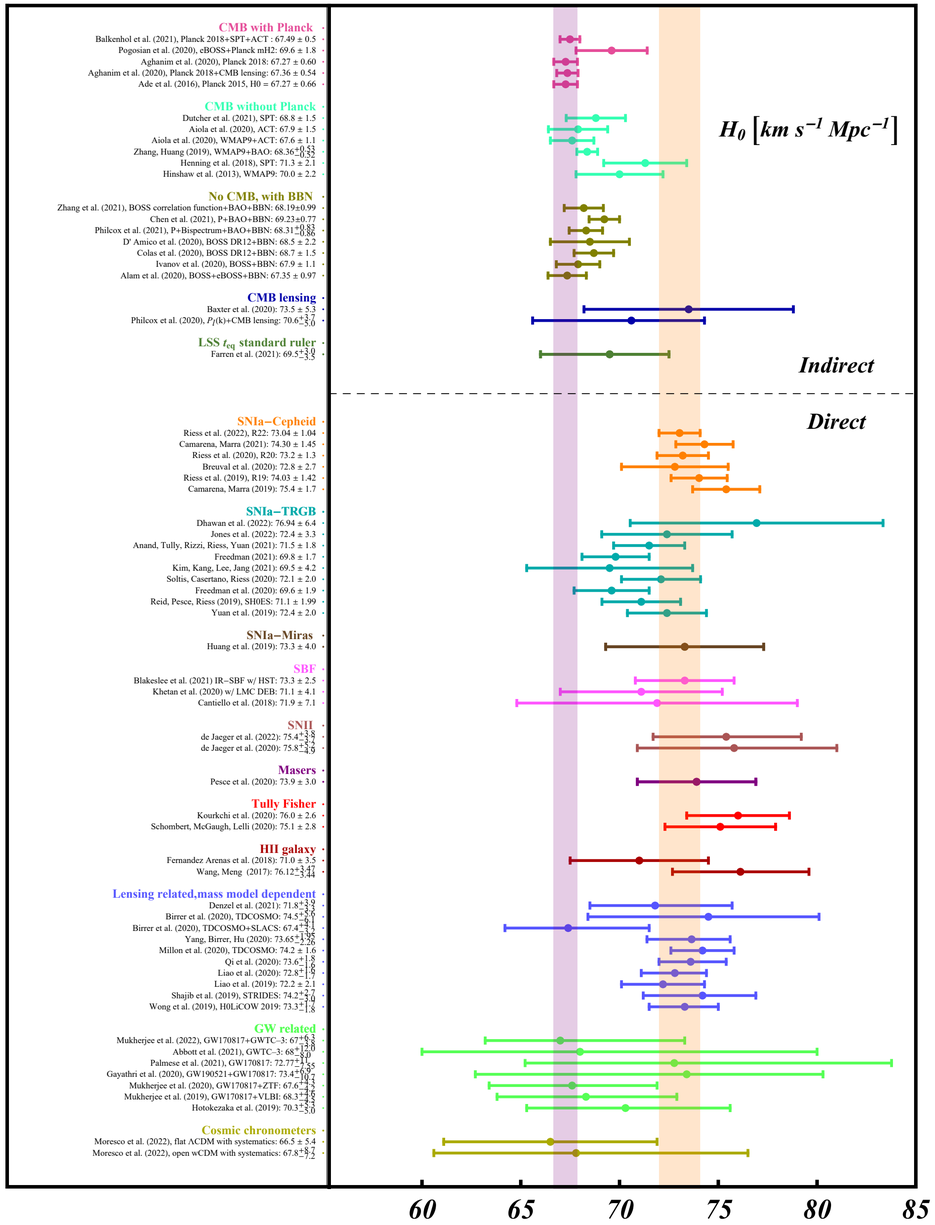
Crack in Lambda CDM?

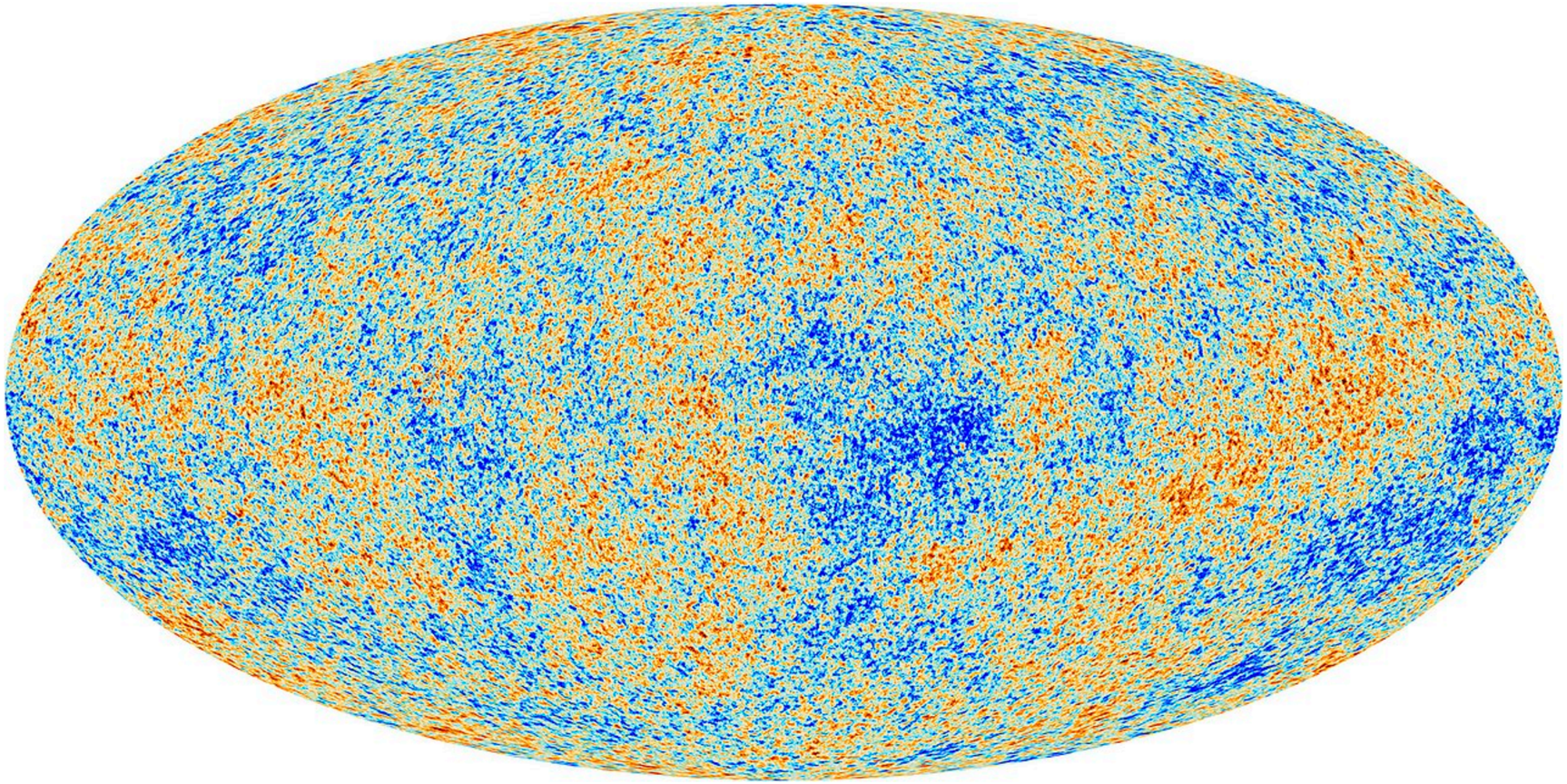
Snowmass [arXiv:2203.06142]



Hubble Tension

Early Universe Measurement

CMB measures H_0 tightly by sound horizon angle



Hubble Tension

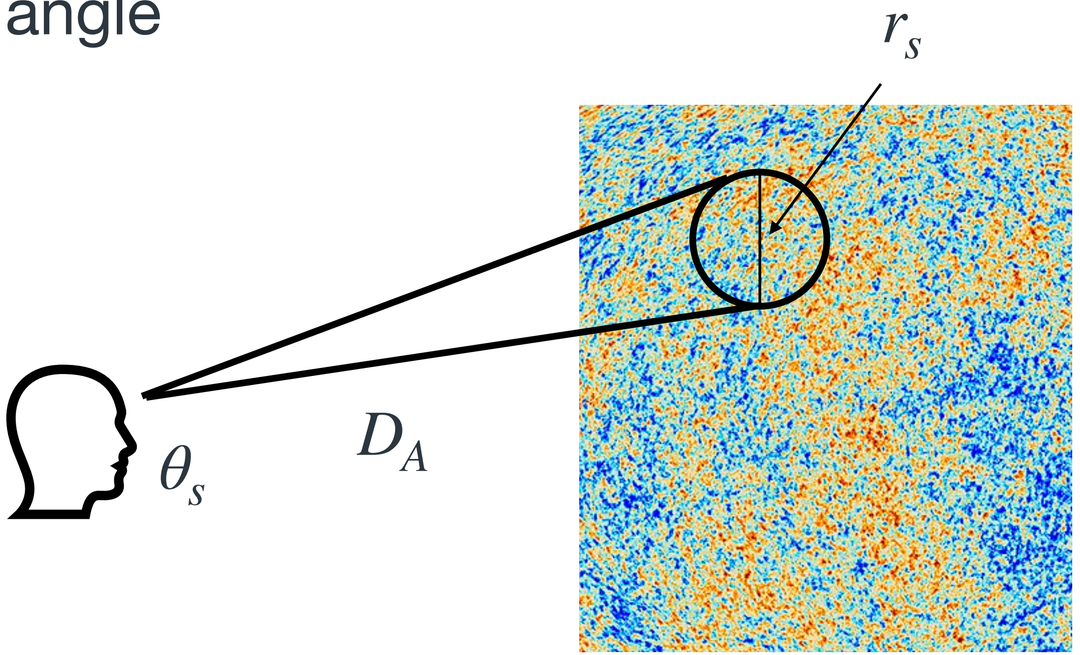
Early Universe Measurement

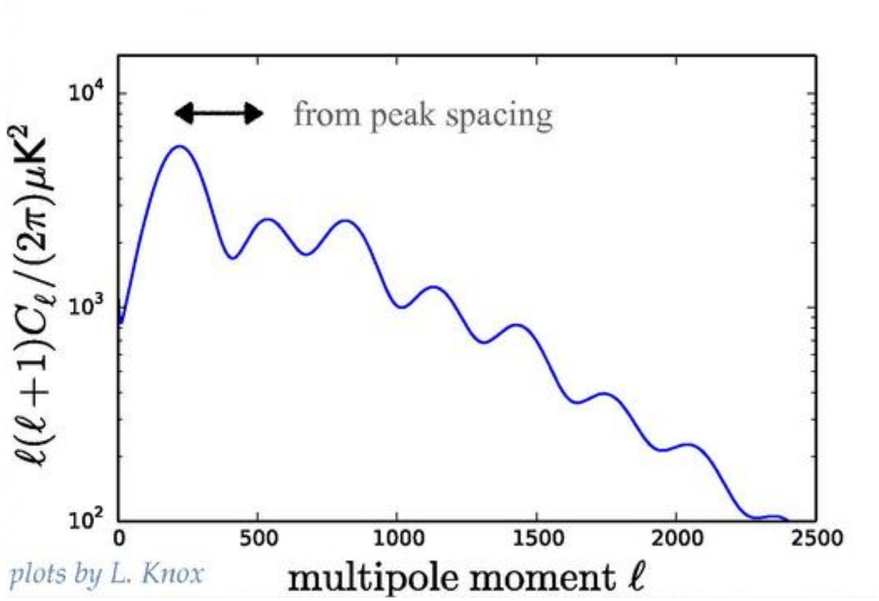
c_s: sound speed of baryon-photon plasma

$$r_s \sim \frac{c_s}{H_{\rm early}} \sim \frac{c_s}{H_{\rm rec}(\rho_{\rm early}/\rho_{\rm rec})^{1/2}}$$

CMB measures H_0 tightly by sound horizon angle

$$\theta_s = \frac{r_s}{D_A}$$





$$D_A \sim \frac{c}{H_{\rm late}} \sim \frac{c}{H_0(\rho_{\rm late}/\rho_{\rm today})^{1/2}}$$

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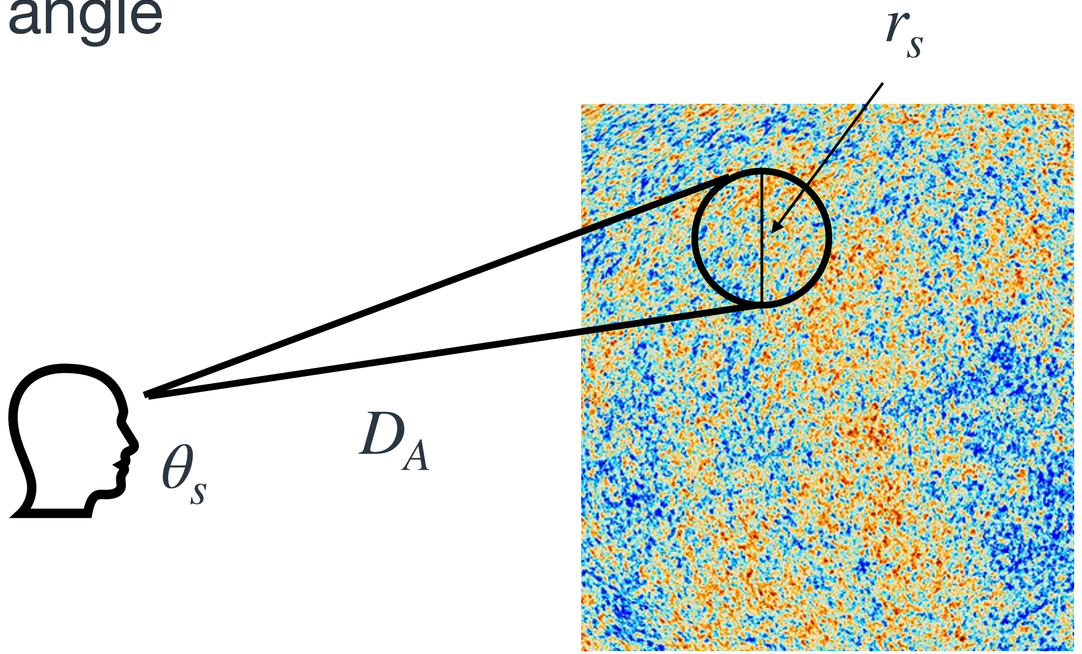


$$H_0 \sim H_{\rm rec}\theta_s \frac{c/(\rho_{\rm late}/\rho_{\rm today})^{1/2}}{c_s/(\rho_{\rm early}/\rho_{\rm rec})^{1/2}}$$

$$D_A \sim \frac{c}{H_{\rm late}} \sim \frac{c}{H_0(\rho_{\rm late}/\rho_{\rm today})^{1/2}}$$

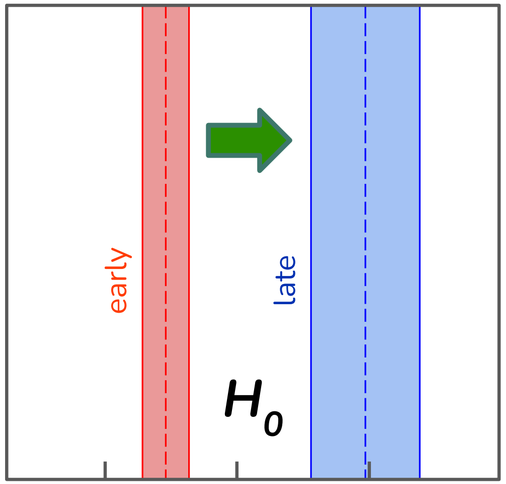
$$H_0^{\rm Planck2018} = 67.36 \pm 0.54 \ {\rm km/s/Mpc}$$

Cosmological Tensions

Hubble tension (~4-6 σ)

$$H_0 \sim H_{\rm rec}\theta_s \frac{c/(\rho_{\rm late}/\rho_{\rm today})^{1/2}}{c_s/(\rho_{\rm early}/\rho_{\rm rec})^{1/2}}$$

To increase H_0,



Cosmological Tensions

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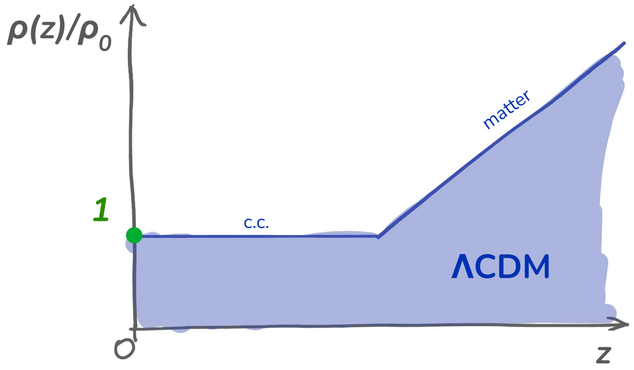
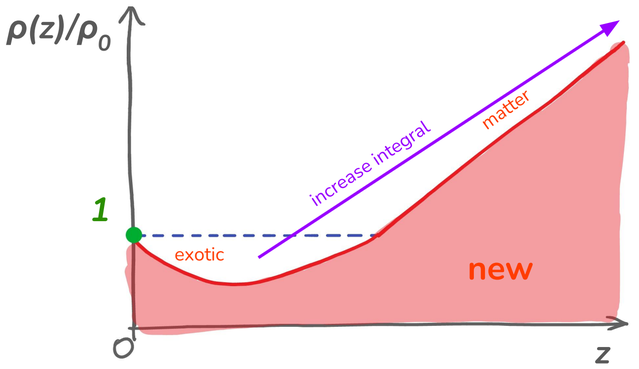




To increase H_0,

Decrease energy density at late times (late-time solutions)

Increasing $\rho(t)$ over time with ρ_0 fixed → energy is created out of nowhere

Hard to square with low-redshift astronomical observations

Cosmological Tensions

Hubble tension (~4-6 σ)

$$H_0 \sim H_{\rm rec}\theta_s \frac{c/(\rho_{\rm late}/\rho_{\rm today})^{1/2}}{c_s/(\rho_{\rm early}/\rho_{\rm rec})^{1/2}}$$

To increase H_0,

Increase energy density at early times (early-time solutions)

P. Agrawal et al. [arXiv:1904.01016]

Early Dark Energy $\rightarrow V(\phi) = \Lambda_{\rm EDE}^4[1-\cos(\phi/f_{\rm EDE})]^n,\ V(\phi) = V_0\left(\frac{\phi}{M_{pl}}\right)^{2n} + V_\Lambda$

V. Poulin et al. [arXiv:1806.10608]

Cosmological Tensions

Hubble tension (~4-6 σ)

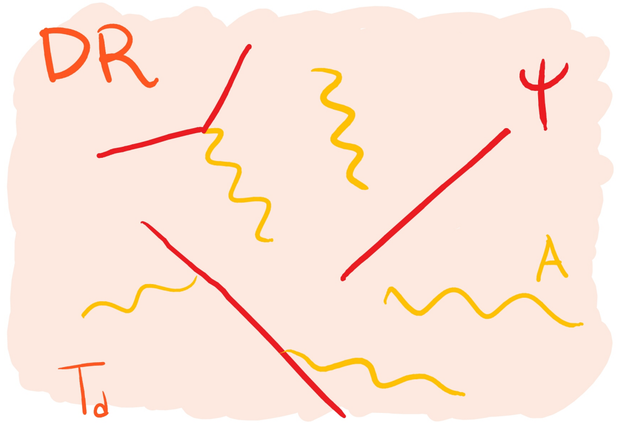
$$H_0 \sim H_{\rm rec}\theta_s \frac{c/(\rho_{\rm late}/\rho_{\rm today})^{1/2}}{c_s/(\rho_{\rm early}/\rho_{\rm rec})^{1/2}}$$

To increase H_0,

Increase energy density at early times (early-time solutions)

Early Dark Energy

Dark Radiation → Massless states in Dark Sector

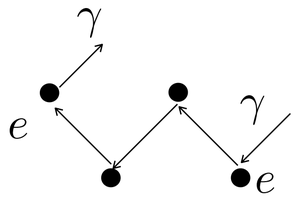


Dark Radiation

A Class of Solutions to Hubble tension

To increase H_0,

Increase energy density at early times (early-time solutions)



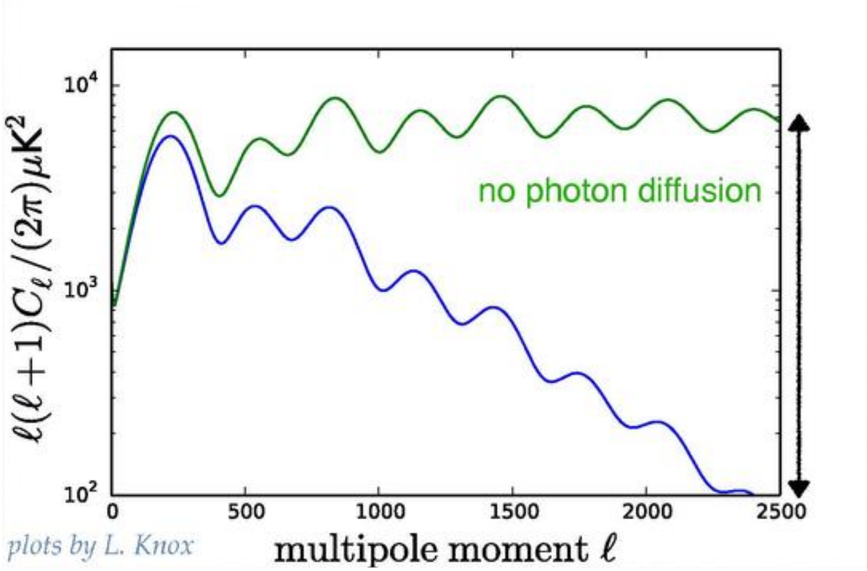
$$r_d \propto H_{\rm early}^{-1/2}$$

Free-streaming (non-interacting) Dark Radiation (DR)

Silk damping (diffusion) + Drag effect

$$\theta_d = \frac{r_d}{D_A} \rightarrow \frac{\theta_d}{\theta_s} = \frac{r_d}{r_s} \propto H_{\rm early}^{1/2}$$

$$r_s \propto H_{\rm early}^{-1}$$



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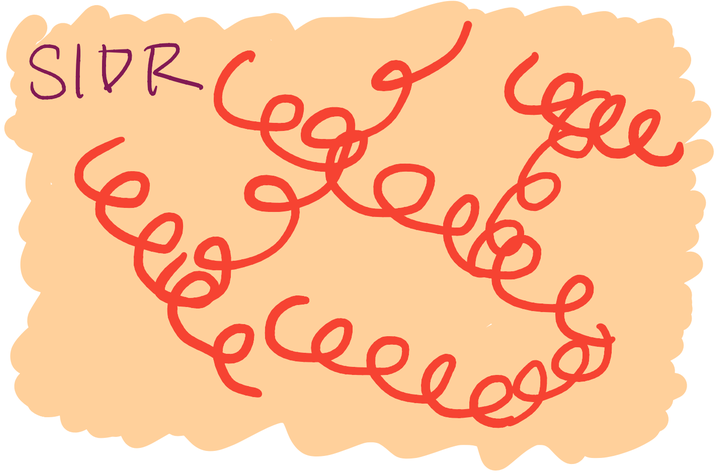
Good: Self-interacting DR **N. Blinov et al. [arXiv:2003.08387]**

Silk damping (diffusion)



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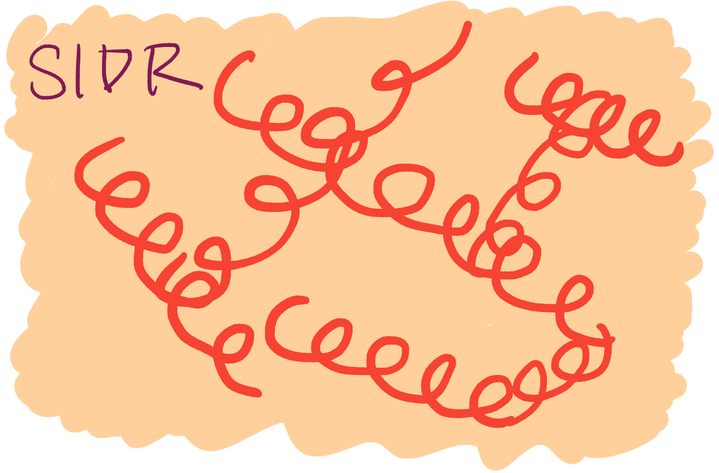
Silk damping (diffusion)

Better: Stepped DR **D. Aloni et al. [arXiv:2111.00014]**



Dark Radiation

A Class of Solutions to Hubble tension

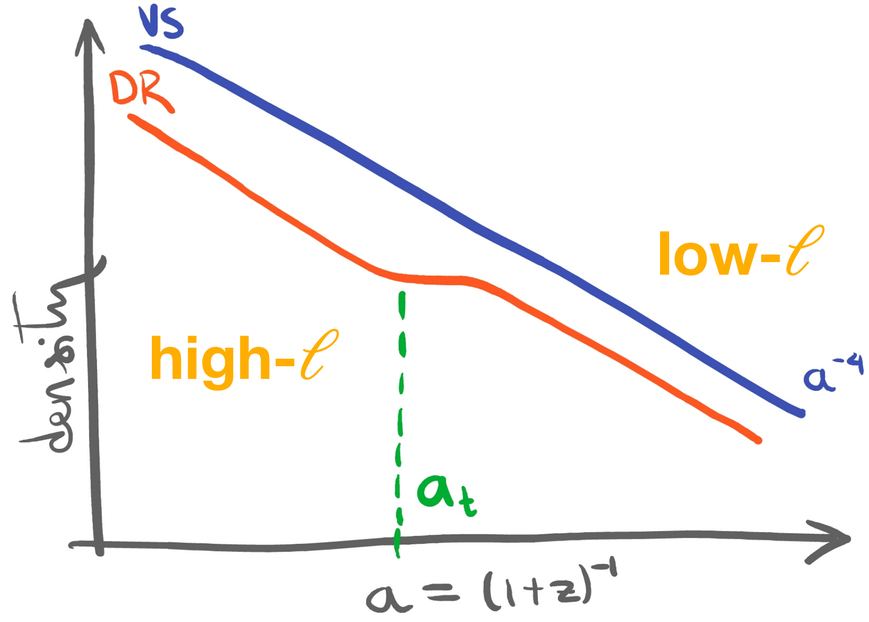
Stepped DR D. Aloni et al. [arXiv:2111.00014]

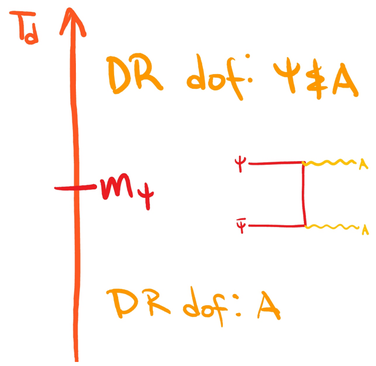
Multicomponent (e.g. ψ & A)

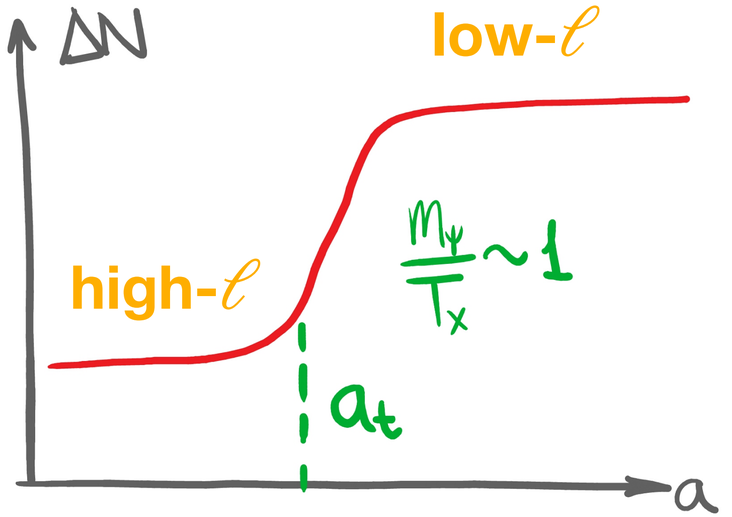
decaying / annihilating and self-interacting DR

Mass threshold around $m \sim$ eV

Entropy dump / Reheating in DS







H_0	$\Delta\chi^2$	$Q_{\rm DMAP}$
71.4	-15.1	2.4 σ

D. Aloni et al. [arXiv:2111.00014]

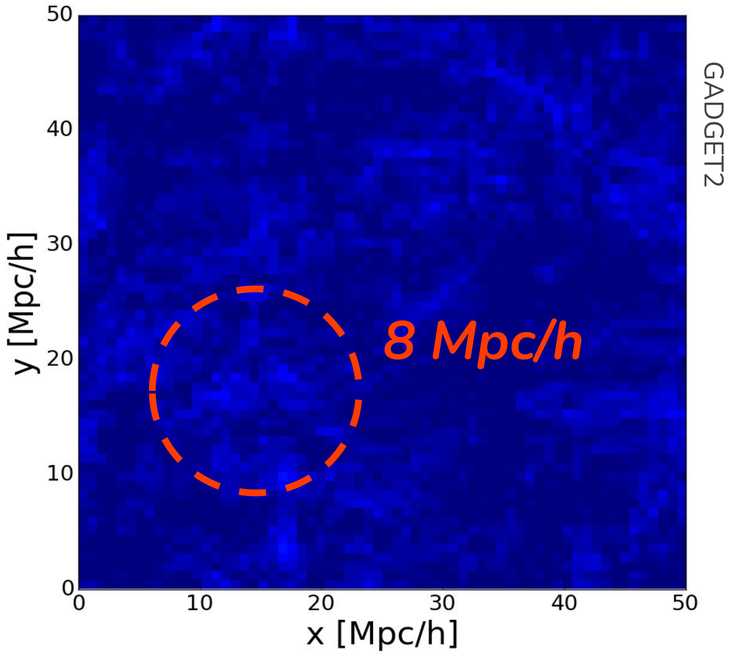
Cosmological Tensions

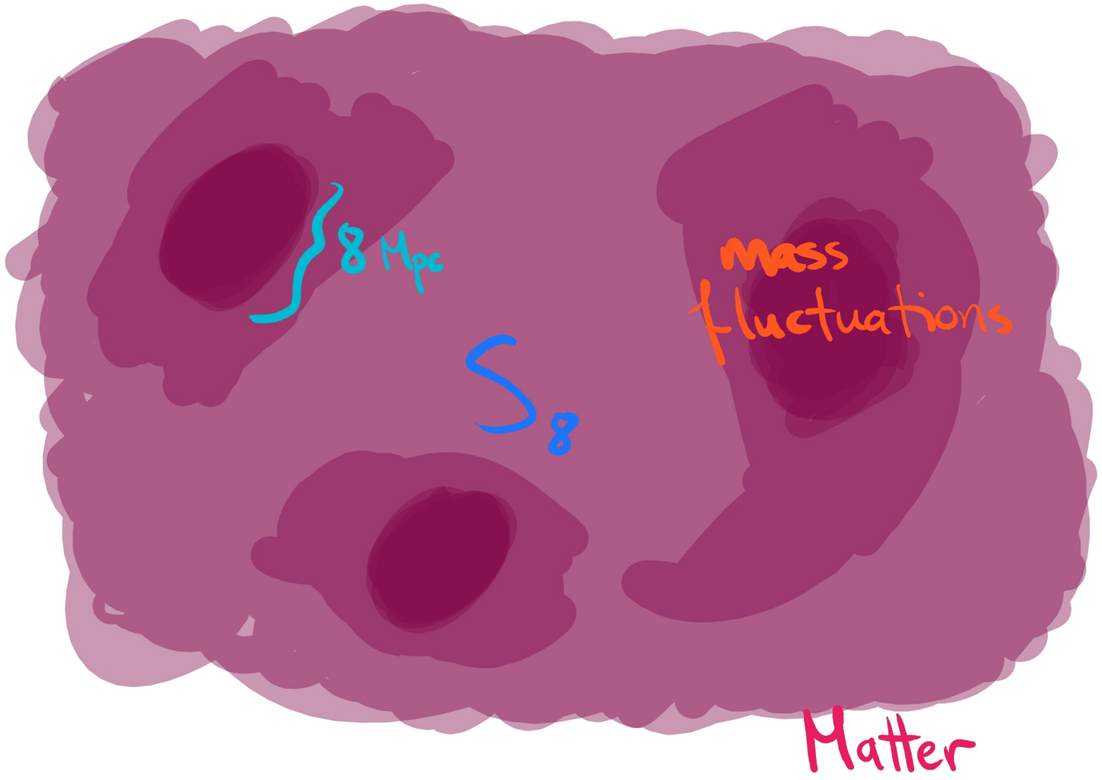
Snowmass [arXiv:2203.06142]

S_8 tension (~2-3 σ)

σ_8: amplitude of matter density fluctuations on the scale of 8 Mpc/h
(~ galaxy cluster scale)

$S_8 \equiv \sigma_8(\Omega_m/0.3)^{1/2}$:





Cosmological Tensions

S_8 tension (~2-3 σ)

Early Universe

CMB fit to ΛCDM

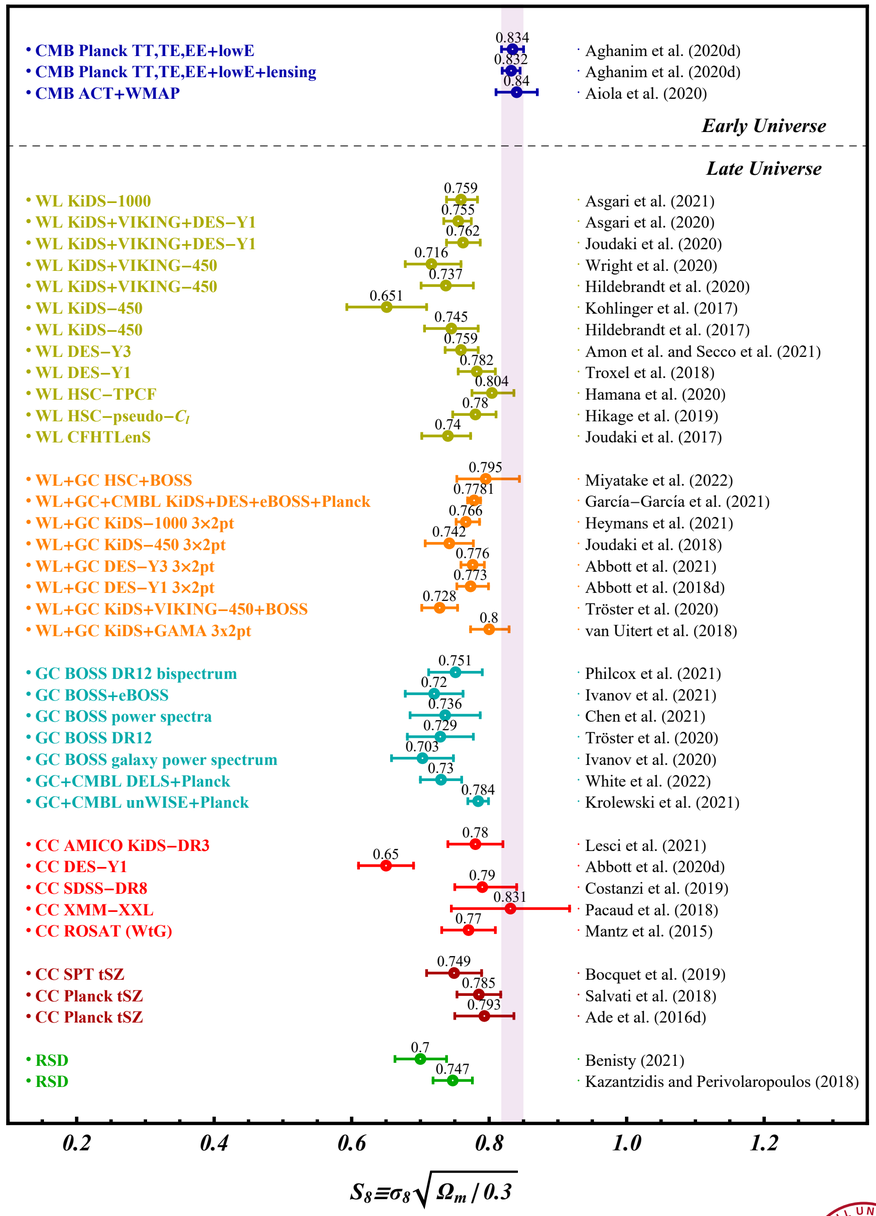
~0.83 **Planck '18 [arXiv:1807.06209]**

Late Universe

Local measurements

~0.76 **DES '21 [arXiv:2105.13544, 2105.13543]**

Snowmass [arXiv:2203.06142]



Cosmological Tensions

S_8 tension (~2-3 σ)

More likely systematic errors

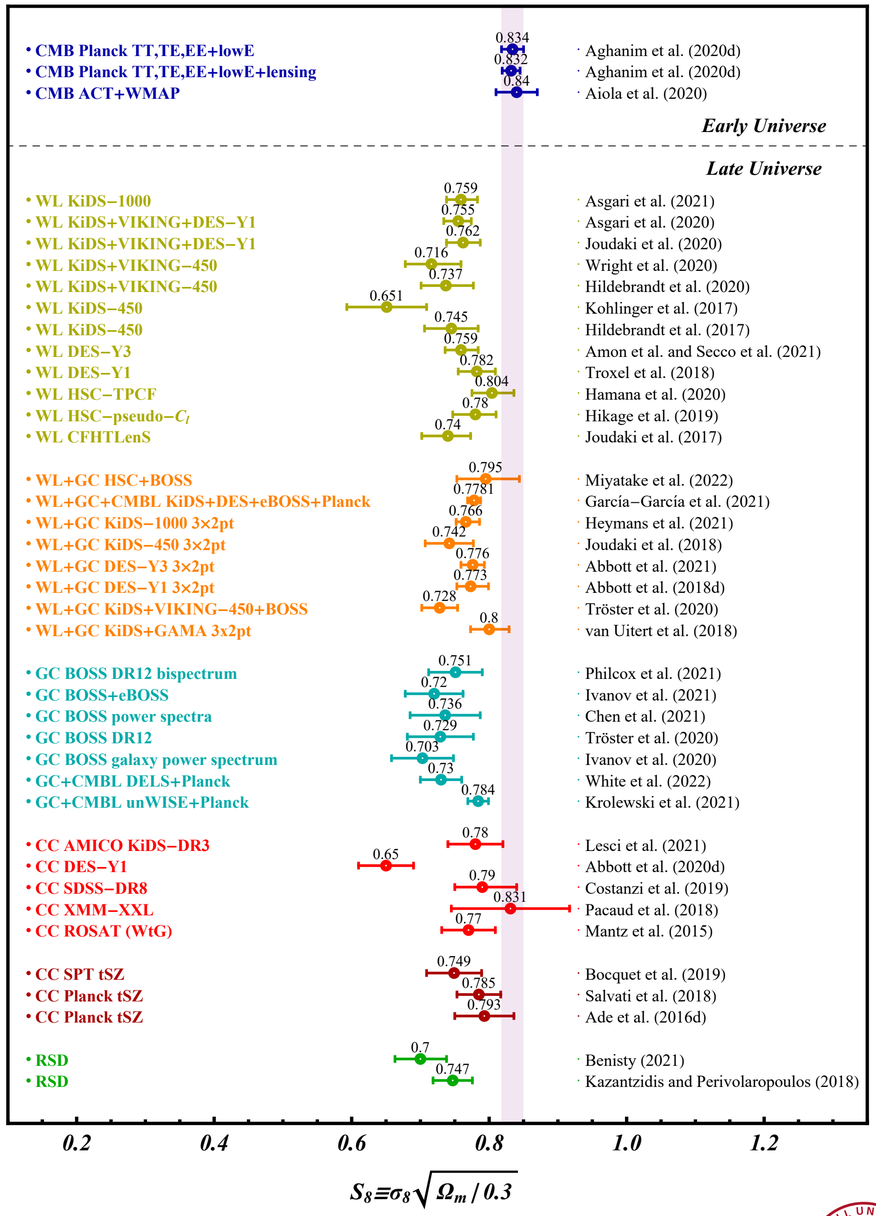
H. G. Escudero et al. [arXiv:2208.14435]
M. Tristram et al. [arXiv:2309.10034]

Early universe solutions worsen S_8 tension

with fixed $z_{\rm eq}$, $\Omega_r \uparrow \rightarrow \Omega_m \uparrow$

Early-time solutions need to deal with S_8

Snowmass [arXiv:2203.06142]



Dark Matter interaction with DR

A Class of Solutions to S_8 tension

Dark Radiation worsens S_8 tension

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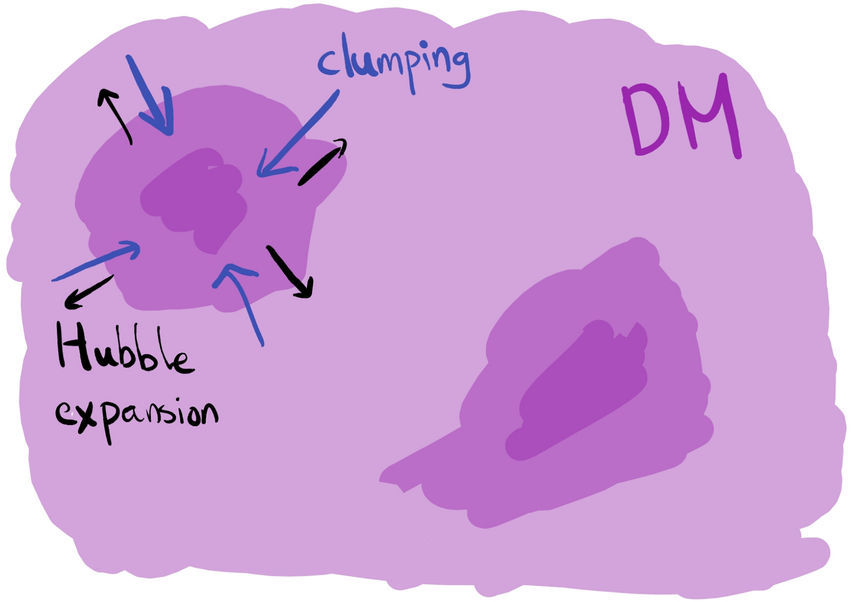

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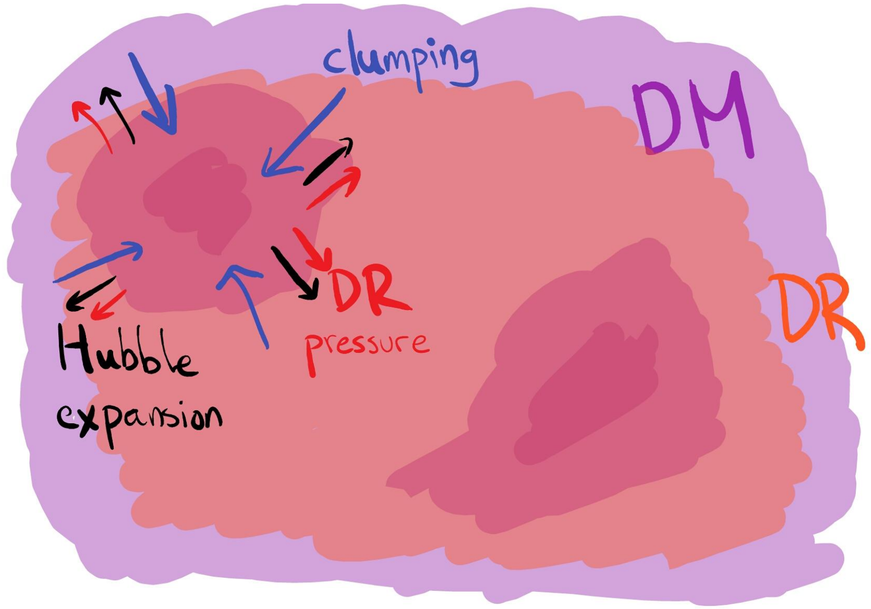
Solution: Dark Matter interaction with Dark Radiation



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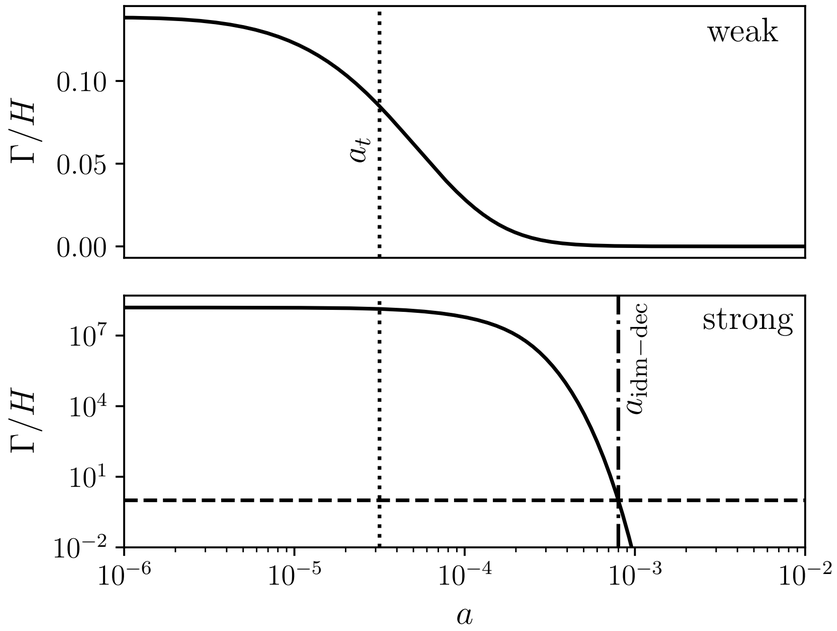
Weak interaction + entire dark matter interacting

M. Joseph et al. [arXiv:2207.03500]

Strong interaction + partial dark matter interacting

M. A. Buen-Abad, Z. Chacko, C. Kilic, G. Marques-Tavares, TY [arXiv:2208.05984, 2306.01844]



N. Schöneberg et al. [arXiv:2306.12469]

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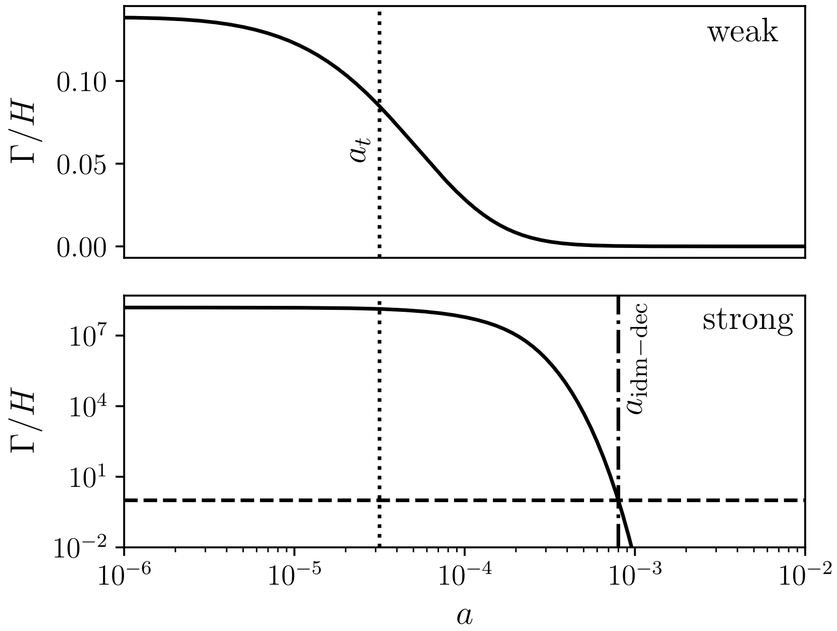
Stepped Partially Acoustic Dark Matter



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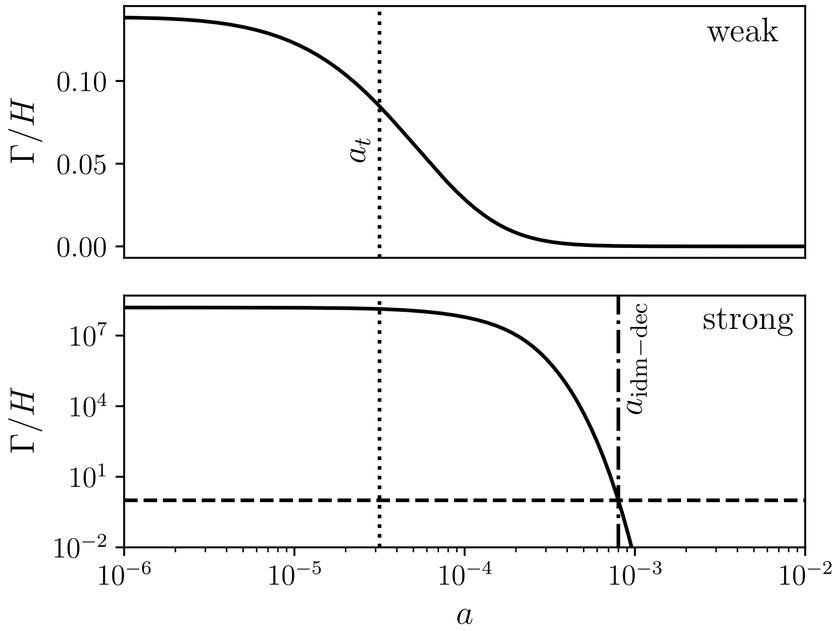

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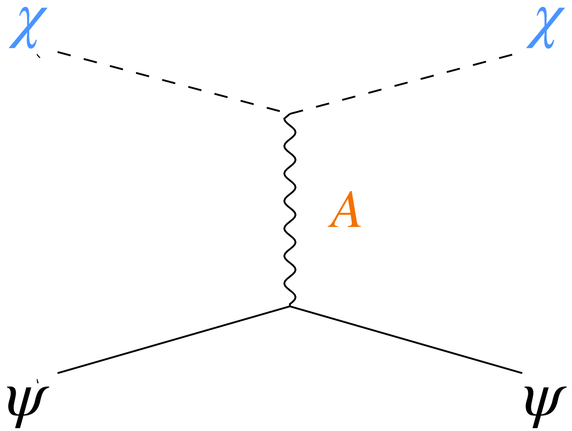
Dark Matter interaction with DR

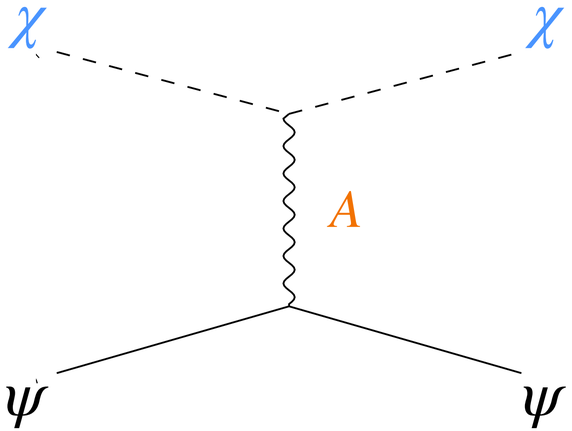
Impacts on Matter / CMB Power Spectrum

Weak

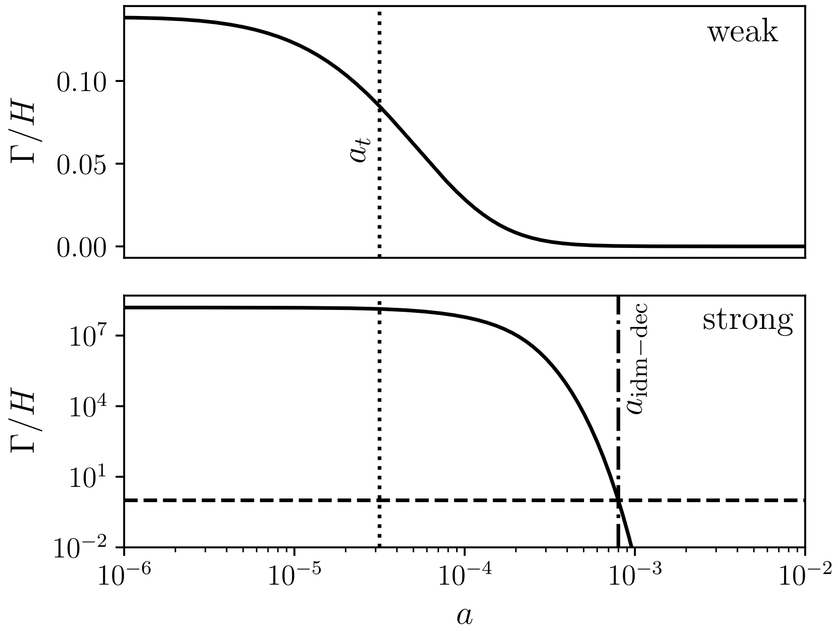


Massive A

Strong



Massive ψ

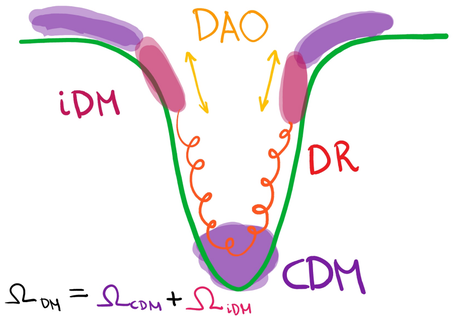


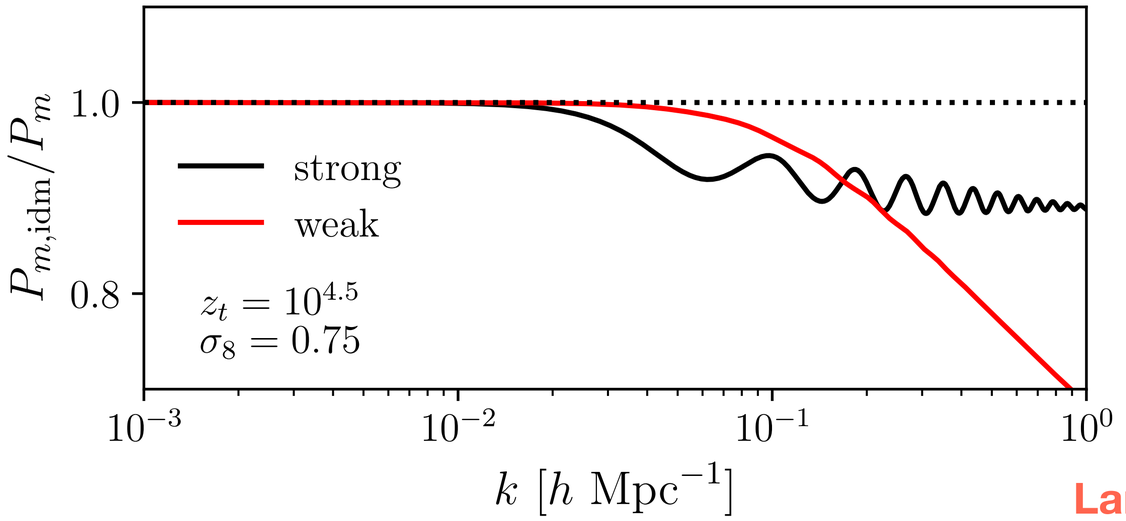
N. Schöneberg et al. [arXiv:2306.12469]

ψ, $A \in$ Stepped DR

Dark Matter interaction with DR

Impacts on Matter / CMB Power Spectrum





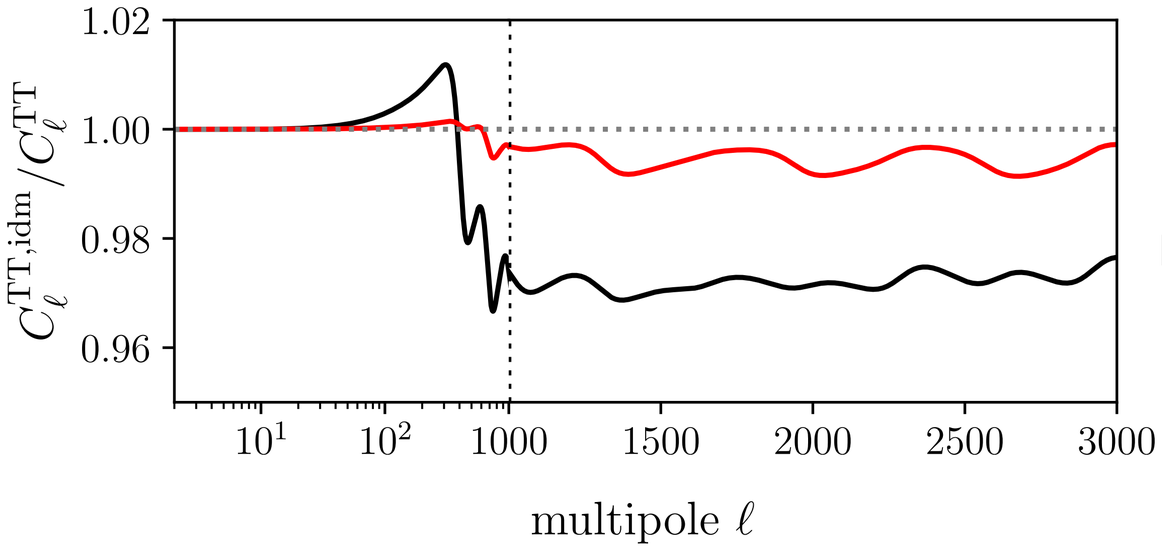
Dark Acoustic Oscillations

Large suppression at high k

N. Schöneberg et al. [arXiv:2306.12469]

Dark Matter interaction with DR

Impacts on Matter / CMB Power Spectrum



Large Impact on high ℓ

N. Schöneberg et al. [arXiv:2306.12469]

Dark Matter interaction with DR

Impacts on Matter / CMB Power Spectrum

Strong interaction btw DM and DR is not compatible with step in $\Delta N_{\rm eff}$

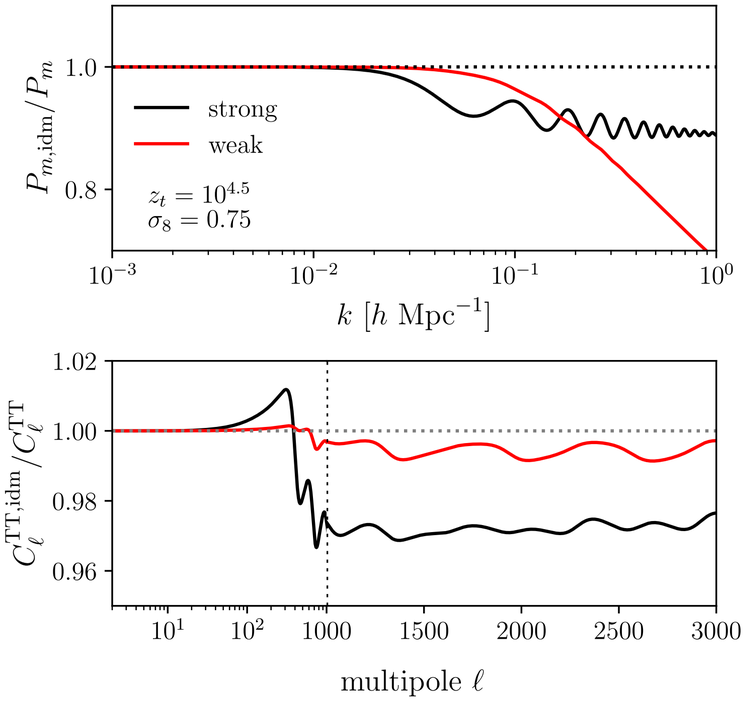
→ No reheating needed in strong interaction

Generalizing Stepped DR

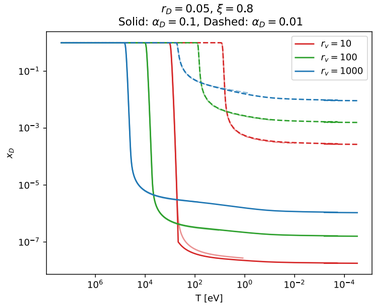
Recombination (currently in progress)

Dark atom model with self-interacting dark photon (gluon)

Short r_{mfp} → no Peeble's correction: case A recombination



N. Schöneberg et al. [arXiv:2306.12469]



Generalized Dark Radiation

Details of Model

Standard CDM

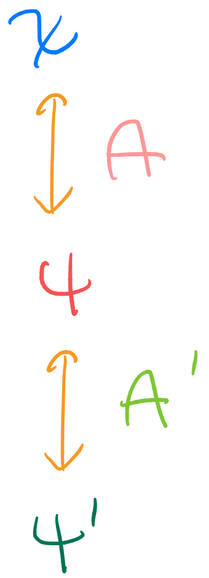
Interacting Dark Matter (iDM): χ

$$f_{\rm CDM} + f_\chi = 1$$

Self-interacting Dark Radiation

ψ, A

ψ', A'



	$U(1)_A$	$U(1)_{A'}$
χ	1	0
ψ	1	1
ψ'	0	1

Strong

0, 3 ψ' flavors: ~40, 7% jump in $\Delta N_{\rm eff}$

Weak

Markov Chain Monte Carlo (MCMC)

Setup

Data:

Baseline $\mathscr{D}$: Plank high ℓ TTTEEE, Planck low ℓ EE, Planck low ℓ TT, Plank lensing, BAO BOSS DR12, BAO small z, PANTHEON

Hubble tension $\mathscr{H}$: SH0ES

S_8 tension $\mathscr{S}$: KiDS-1000x, DES-Y3

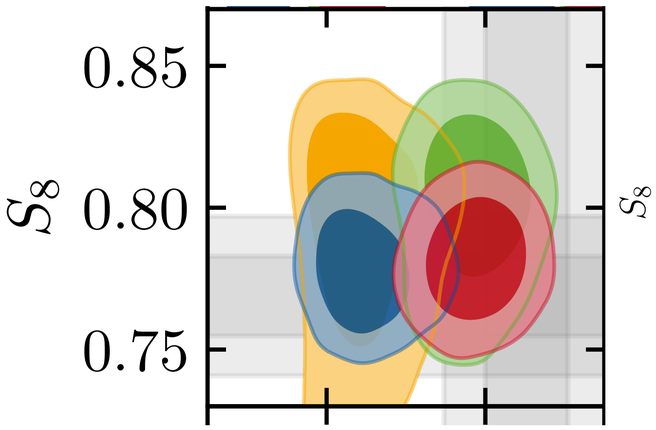
Markov Chain Monte Carlo (MCMC)

Results

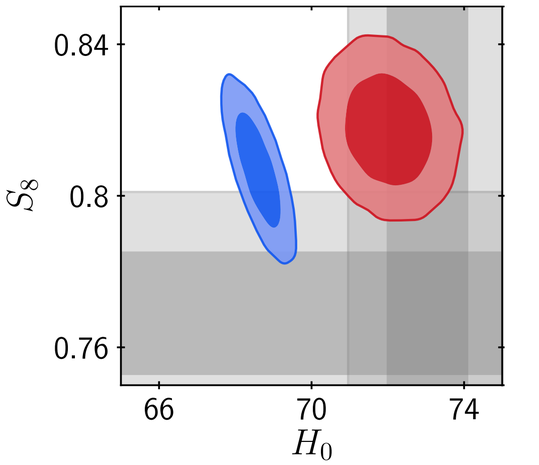
Dataset $\mathscr{DH}$

Weak

N. Schöneberg et al. [arXiv:2306.12469]



Strong



Best fit

Model	$\Delta\chi^2$	$Q_{\rm DMAP}$	H_0
LCDM	0	5.57 σ	68.64
Weak	-26.65	2.45 σ	71.83
Strong	-26.89	2.19 σ	71.98

M. Joseph et al. [arXiv:2207.03500]

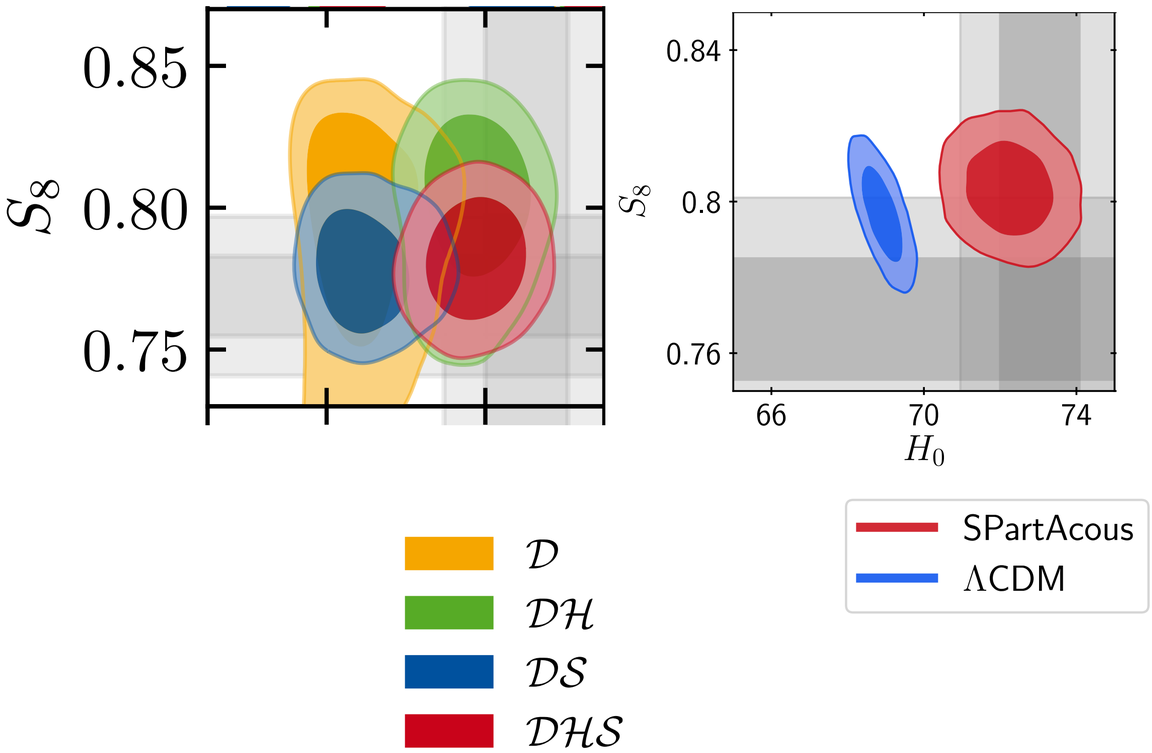
Markov Chain Monte Carlo (MCMC)

Results

Dataset $\mathcal{DHS}$

Weak **Strong**

N. Schöneberg et al. [arXiv:2306.12469]

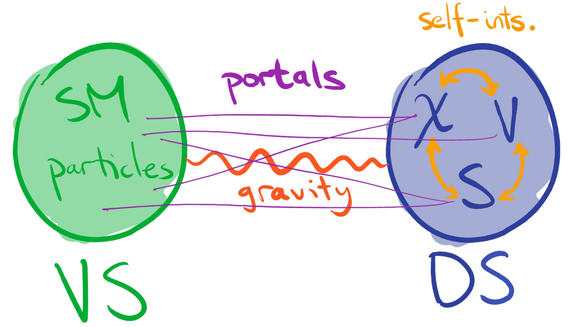


Best fit

Model	$\Delta\chi^2$	Q_{DMAP}	H_0	S_8
LCDM	0	5.80 σ	68.94	0.7972
Weak	-25.78	3.20 σ	71.84	0.792
Strong	-24.56	3.38 σ	72.26	0.8036

Conclusions

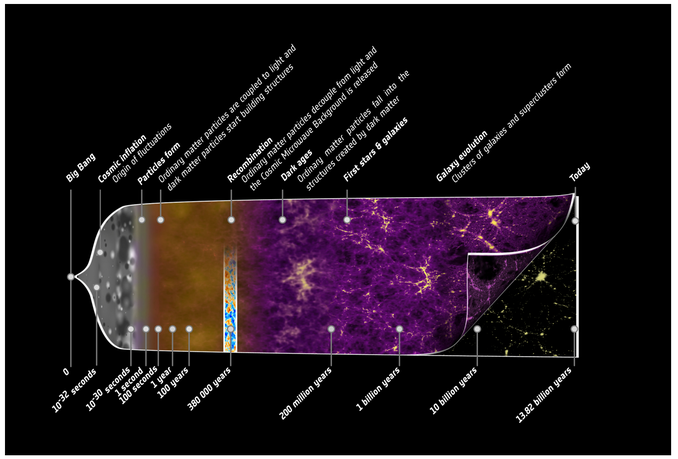
Summary and Outlook



Non-trivial Dark Sector is highly motivated

May leave imprints on cosmological data

CMB anisotropy, Large Scale Structure,
Stochastic GW, 21cm line, Lyman **α** forest, etc.

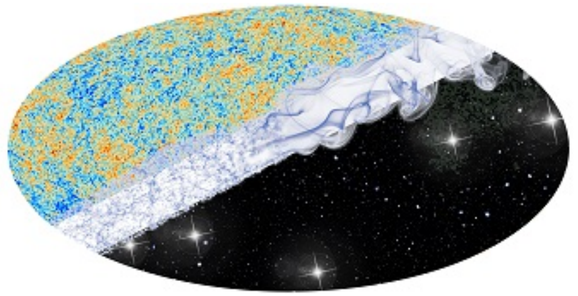


Possible solutions to Hubble / S_8 tensions in ΛCDM

Stepped DR

Interaction btw DM and DR

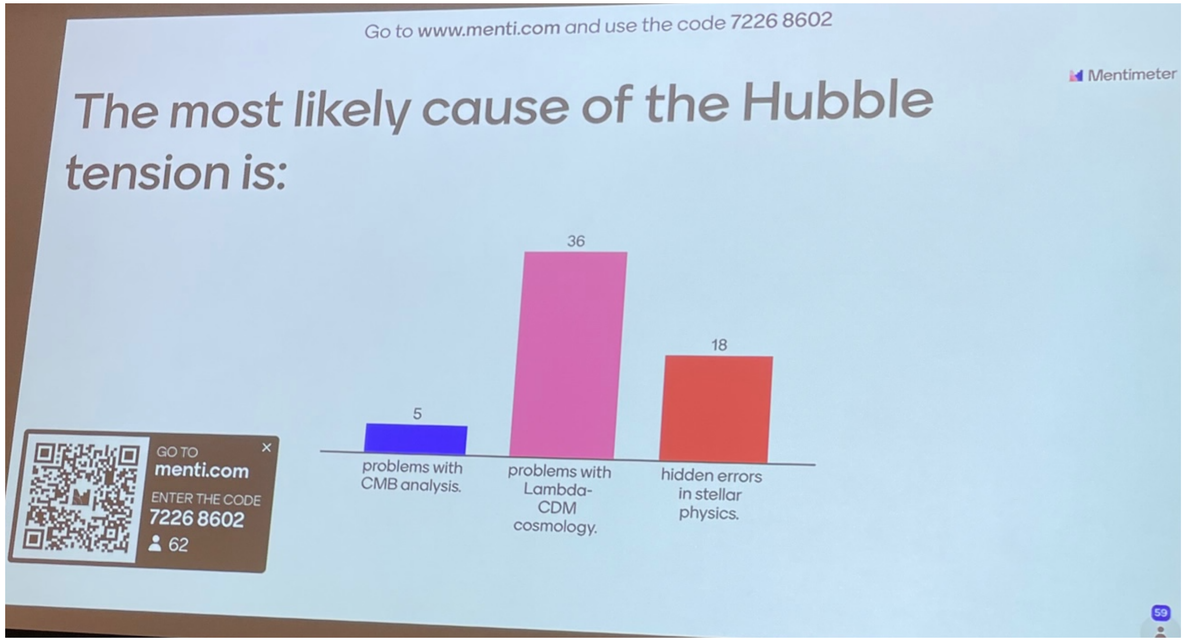
Will be probed in the future experiments!



Conclusions

Summary and Outlook

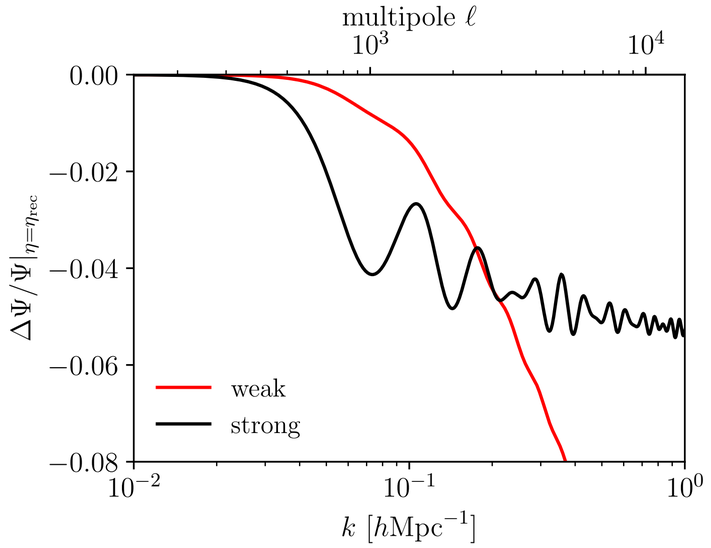
IAU Symposium 376

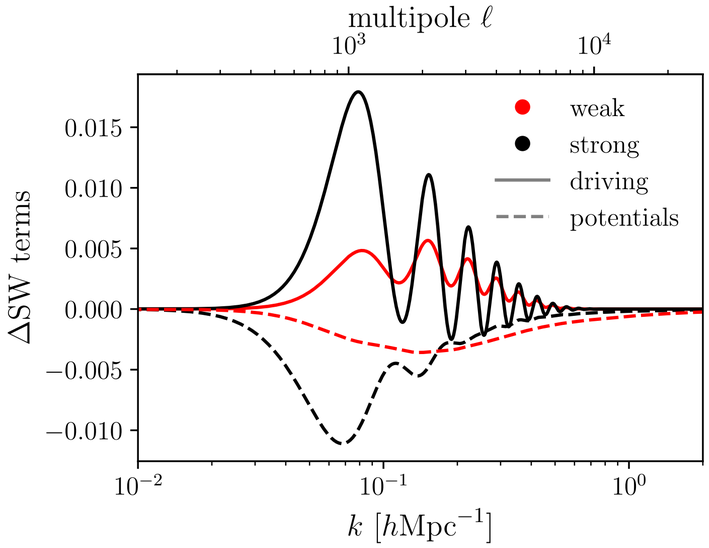


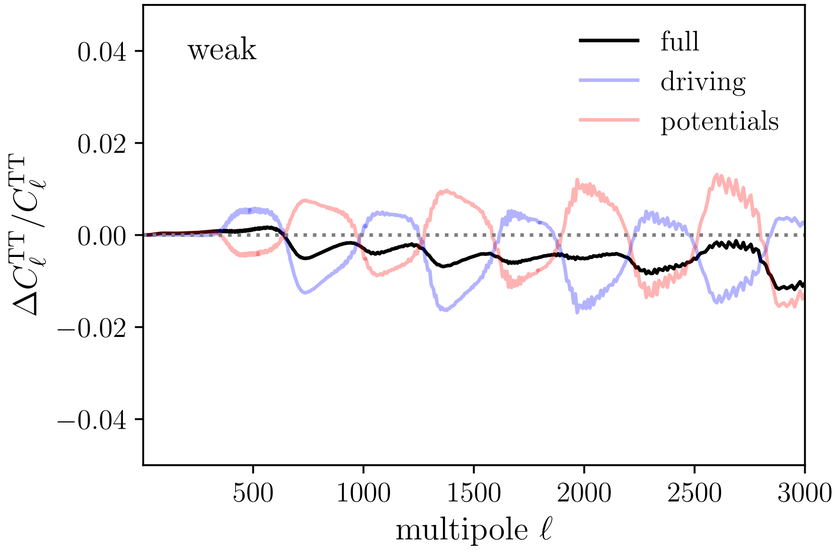
A. Riess and L. Breuval [arXiv:2308.10954]

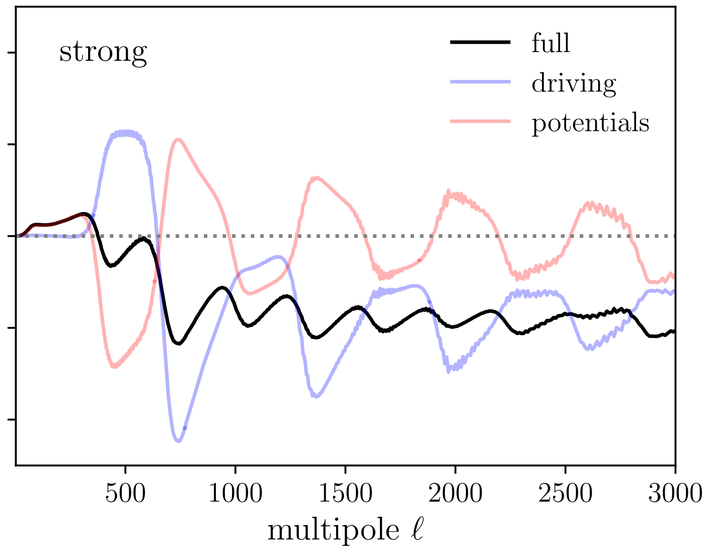
Thank You for Listening!

Supplements


multipole ℓ
10^3
10^4
0.00
−0.02
−0.04
−0.06
−0.08
$\Delta\Psi/\Psi|_{\eta=\eta_{\rm rec}}$
weak
strong
10^{-2}
10^{-1}
10^0
k [hMpc^{-1}]


multipole ℓ
10^3
10^4
0.015
0.010
0.005
0.000
−0.005
−0.010
ΔSW terms
weak
strong
driving
potentials
10^{-2}
10^{-1}
10^0
k [hMpc^{-1}]


weak
full
driving
potentials
0.04
0.02
0.00
−0.02
−0.04
$\Delta C_\ell^{\rm TT}/C_\ell^{\rm TT}$
500
1000
1500
2000
2500
3000
multipole ℓ


strong
full
driving
potentials
500
1000
1500
2000
2500
3000
multipole ℓ

Hubble Tension

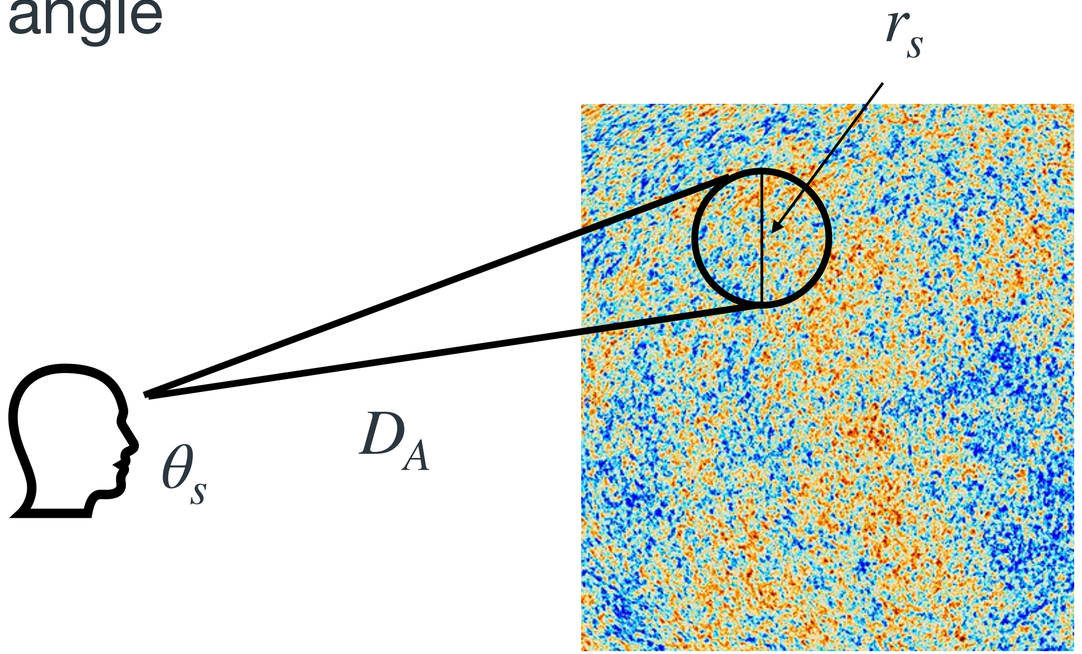
Early Universe Measurement

c_s: sound speed of baryon-photon plasma

$$r_s \sim \frac{c_s}{H_{\rm early}} \sim \frac{c_s}{H_{\rm rec}(\rho_{\rm early}/\rho_{\rm rec})^{1/2}}$$

CMB measures H_0 tightly by sound horizon angle

$$\theta_s = \frac{r_s}{D_A}$$



$$H_0 \sim H_{\rm rec}\theta_s \frac{c/(\rho_{\rm late}/\rho_{\rm today})^{1/2}}{c_s/(\rho_{\rm early}/\rho_{\rm rec})^{1/2}}$$

$$D_A \sim \frac{c}{H_{\rm late}} \sim \frac{c}{H_0(\rho_{\rm late}/\rho_{\rm today})^{1/2}}$$

$$H_0^{\rm Planck2018} = 67.36 \pm 0.54 \ {\rm km/s/Mpc}$$

Early Universe Measurement

CMB again

$$r_s = \frac{1}{H_{\rm rec}} \int_0^{t_{\rm rec}} \frac{c_s(t) {\rm d}t/t_{\rm rec}}{[\rho(t)/\rho(t_{\rm rec})]^{1/2}}$$

$$D_A = \frac{c}{H_0} \int_{t_{\rm rec}}^{t_0} \frac{{\rm d}t/t_0}{[\rho(t)/\rho_0]^{1/2}}$$

$$H_0 = H_{\rm rec}\theta_s \frac{\int_{t_{\rm rec}}^{t_0} \frac{{\rm d}t/t_0}{[\rho(t)/\rho_0]^{1/2}}}{\int_0^{t_{\rm rec}} \frac{c_s(t){\rm d}t/t_{\rm rec}}{[\rho(t)/\rho(t_{\rm rec})]^{1/2}}}$$

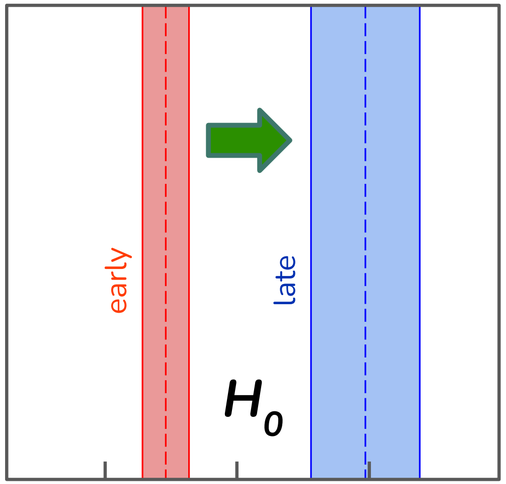
$$H_0^{\rm Planck2018} = 67.36 \pm 0.54 \ {\rm km/Mpc/s}$$

Cosmological Tensions

Hubble tension (~4-6 σ)

$$H_0 \sim H_{\rm rec}\theta_s \frac{c/(\rho_{\rm late}/\rho_{\rm today})^{1/2}}{c_s/(\rho_{\rm early}/\rho_{\rm rec})^{1/2}}$$

To increase H_0,



Cosmological Tensions

Hubble tension (~4-6 σ)

$$H_0 \sim H_{\rm rec}\theta_s \frac{c/(\rho_{\rm late}/\rho_{\rm today})^{1/2}}{c_s/(\rho_{\rm early}/\rho_{\rm rec})^{1/2}}$$

To increase H_0,

Decrease energy density at late times (late-time solutions)

Increasing $\rho(t)$ over time with ρ_0 fixed $\rightarrow$ energy is created out of nowhere

Cosmological Tensions

Hubble tension (~4-6 σ)

$$H_0 \sim H_{\rm rec}\theta_s \frac{c/(\rho_{\rm late}/\rho_{\rm today})^{1/2}}{c_s/(\rho_{\rm early}/\rho_{\rm rec})^{1/2}}$$

To increase H_0,

Decrease energy density at late times (late-time solutions)

Decrease sound speed in early universe (sounds crazy)

Cosmological Tensions

Hubble tension (~4-6 σ)

$$H_0 \sim H_{\rm rec}\theta_s \frac{c/(\rho_{\rm late}/\rho_{\rm today})^{1/2}}{c_s/(\rho_{\rm early}/\rho_{\rm rec})^{1/2}}$$

To increase H_0,

Decrease energy density at late times (late-time solutions)

Decrease sound speed in early universe (sounds crazy)

Increase energy density at early times (early-time solutions)

SPartAcous

Boltzmann equations

$$\dot{\delta}_{\rm idm} = -\theta_{\rm idm} + 3\dot{\phi}$$

$$\dot{\theta}_{\rm idm} = -\mathscr{H}\theta_{\rm idm} + k^2\psi + a\Gamma(\theta_{\rm dr} - \theta_{\rm idm})$$

$$\dot{\delta}_{\rm dr} = -(1+w)(\theta_{\rm dr} - 3\dot{\phi}) - 3\mathscr{H}(c_s^2 - w)\delta_{\rm dr}$$

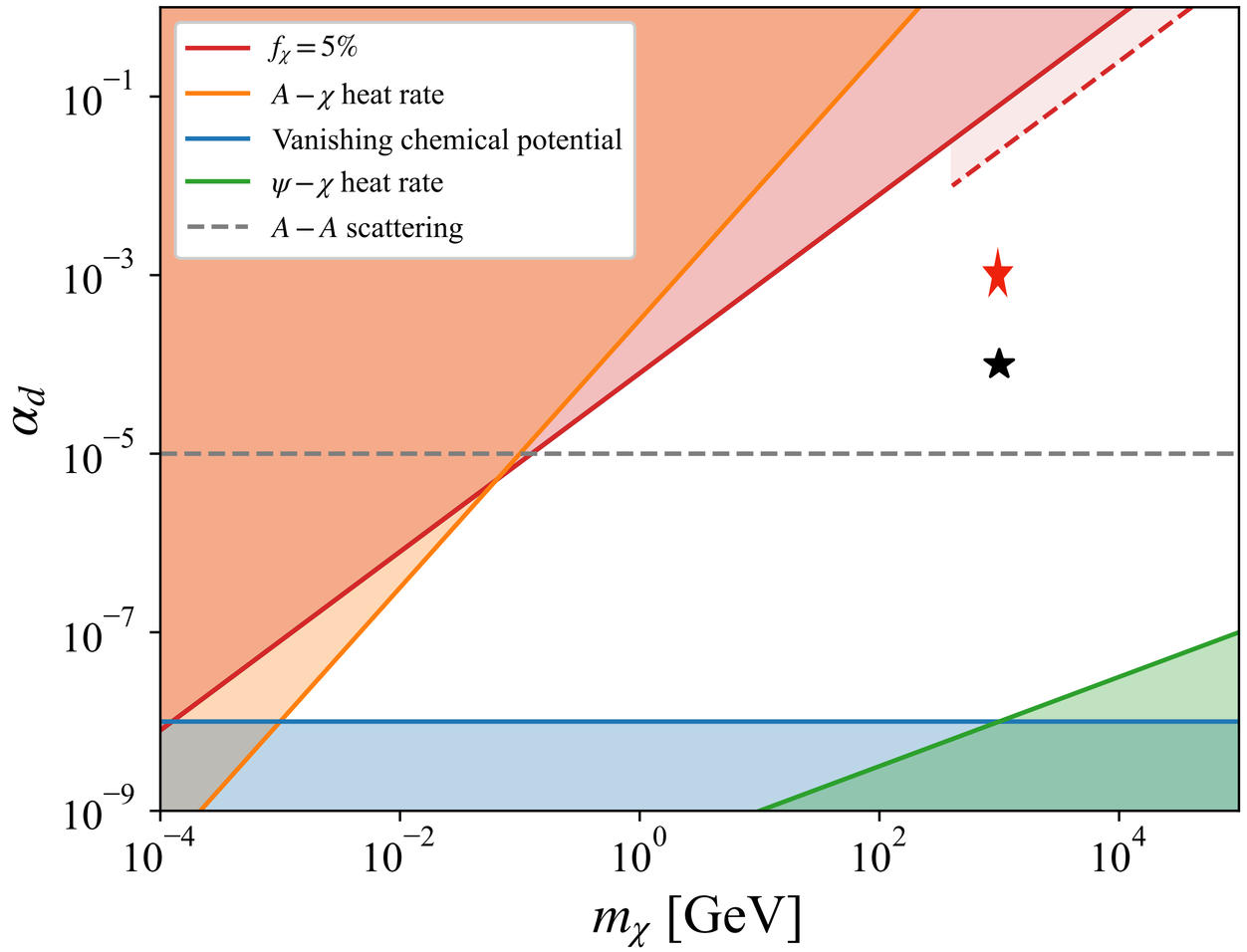
$$\dot{\theta}_{\rm dr} = -\left[(1-3w)\mathscr{H} + \frac{\dot{w}}{1+w}\right]\theta_{\rm dr} + k^2\left(\frac{c_s^2}{1+w}\delta_{\rm dr} + \psi\right) + \frac{\rho_{\rm idm}}{\rho_{\rm dr}(1+w)}a\Gamma(\theta_{\rm idm} - \theta_{\rm dr})$$

$$\Gamma = \frac{4}{3\pi}\alpha_d^2 \log(\star)\frac{T_d^2}{m_\chi}e^{-m_\psi/T_d}\left[2 + \frac{m_\psi}{T_d}\left(2 + \frac{m_\psi}{T_d}\right)\right]$$

$$r_g = \frac{g_*^{\rm UV} - g_*^{\rm IR}}{g_*^{\rm IR}} = \left(\frac{\Delta N_{\rm eff}^{\rm IR}}{\Delta N_{\rm eff}^{\rm UV}}\right)^3 - 1$$

SPartAcous

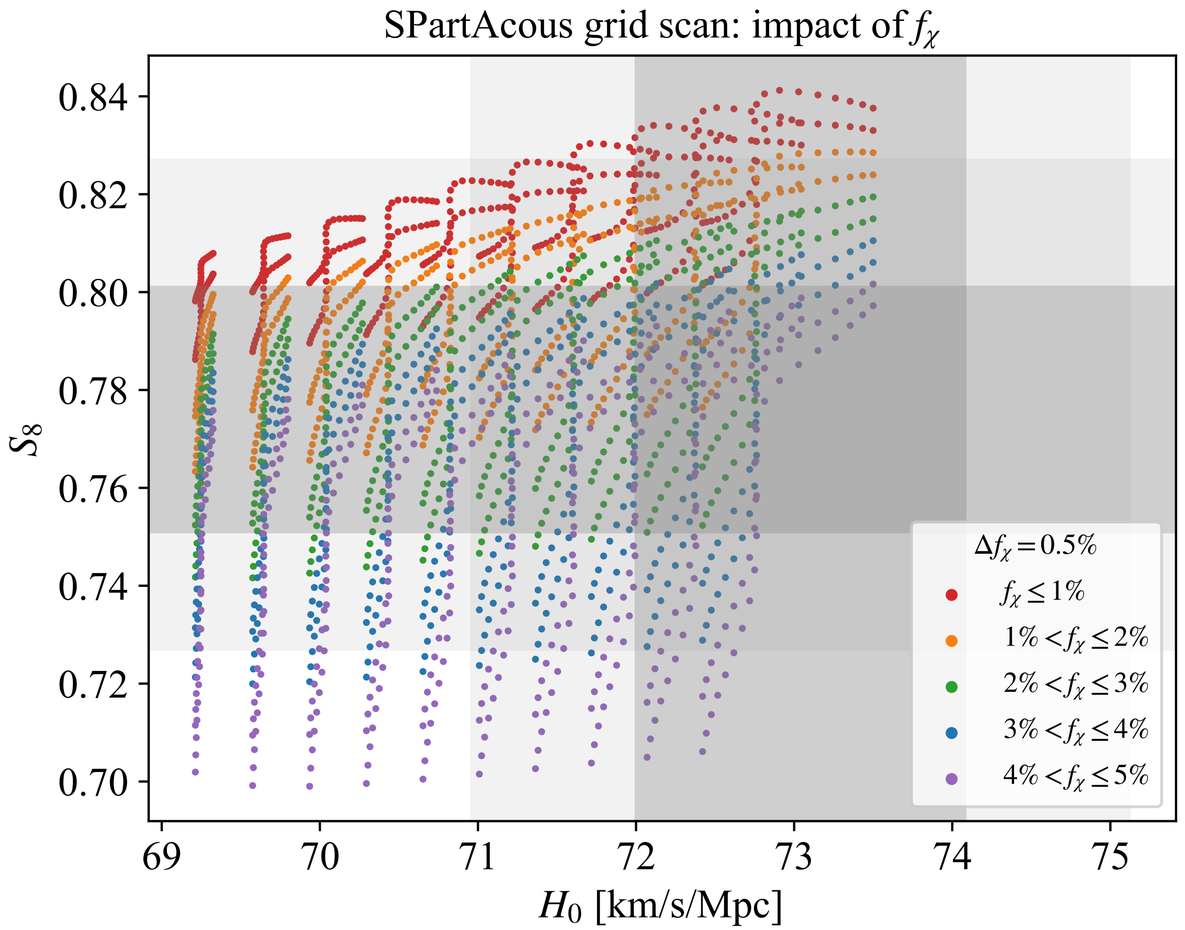
Parameter Space



H_0 and S_8 values

Using WZDR model best-fit parameters

Aloni et al. [arXiv:2111.00014]



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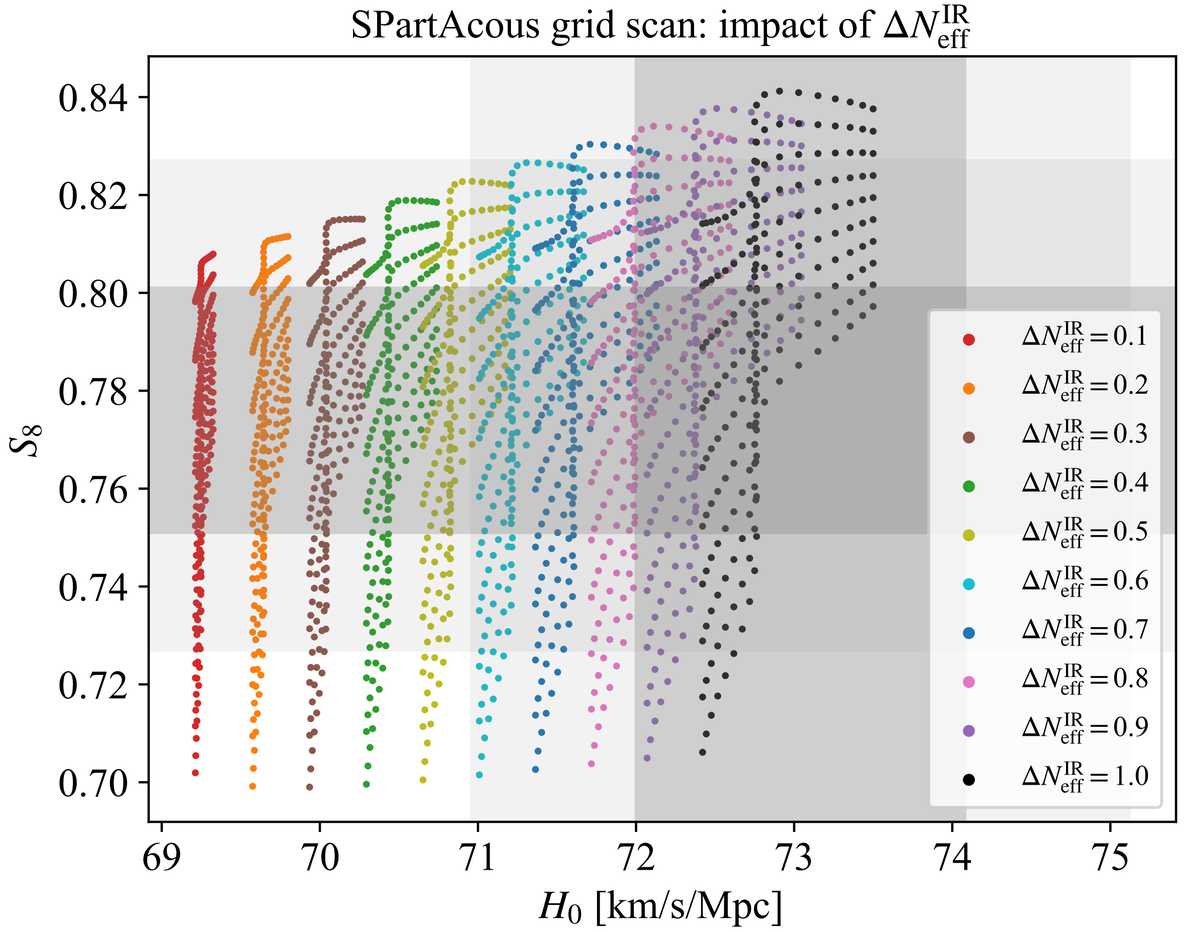

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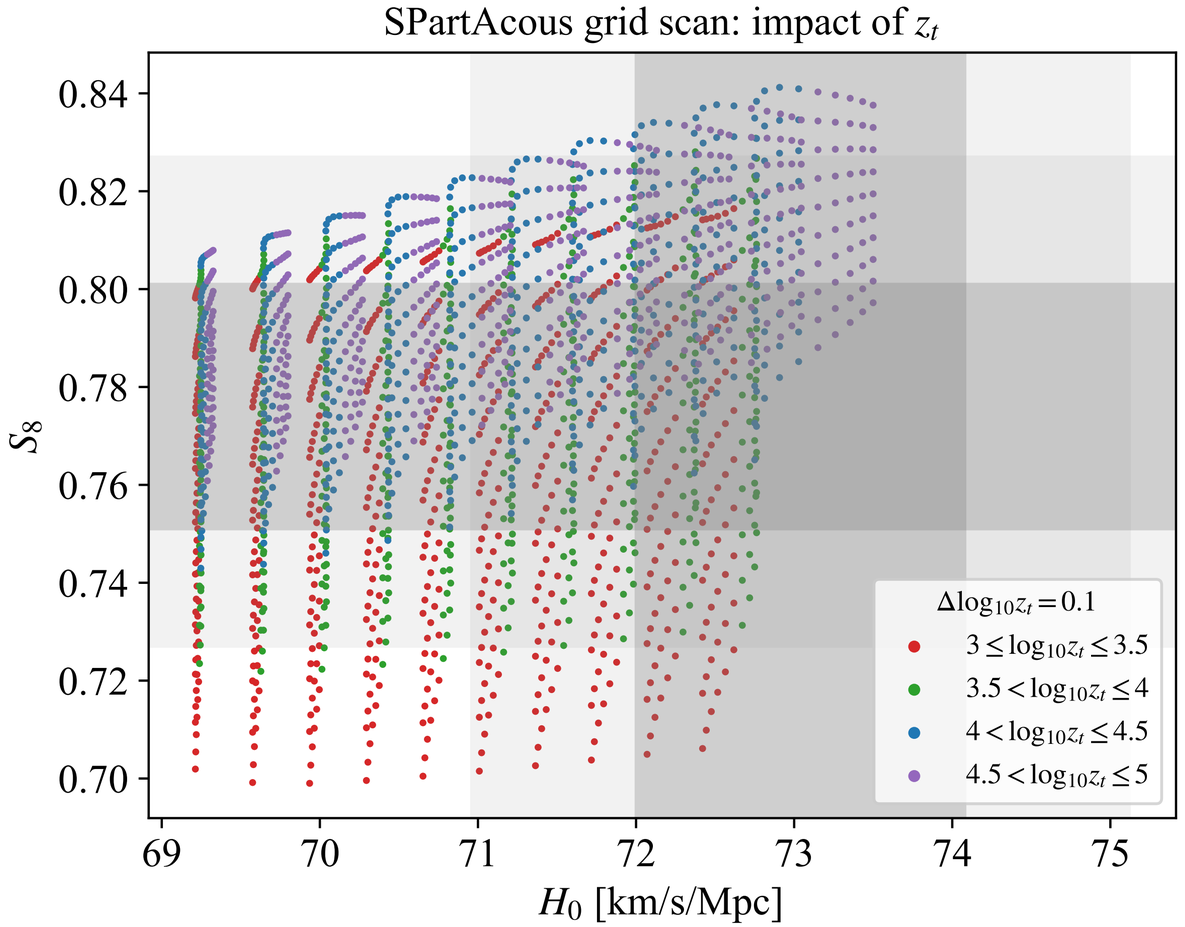

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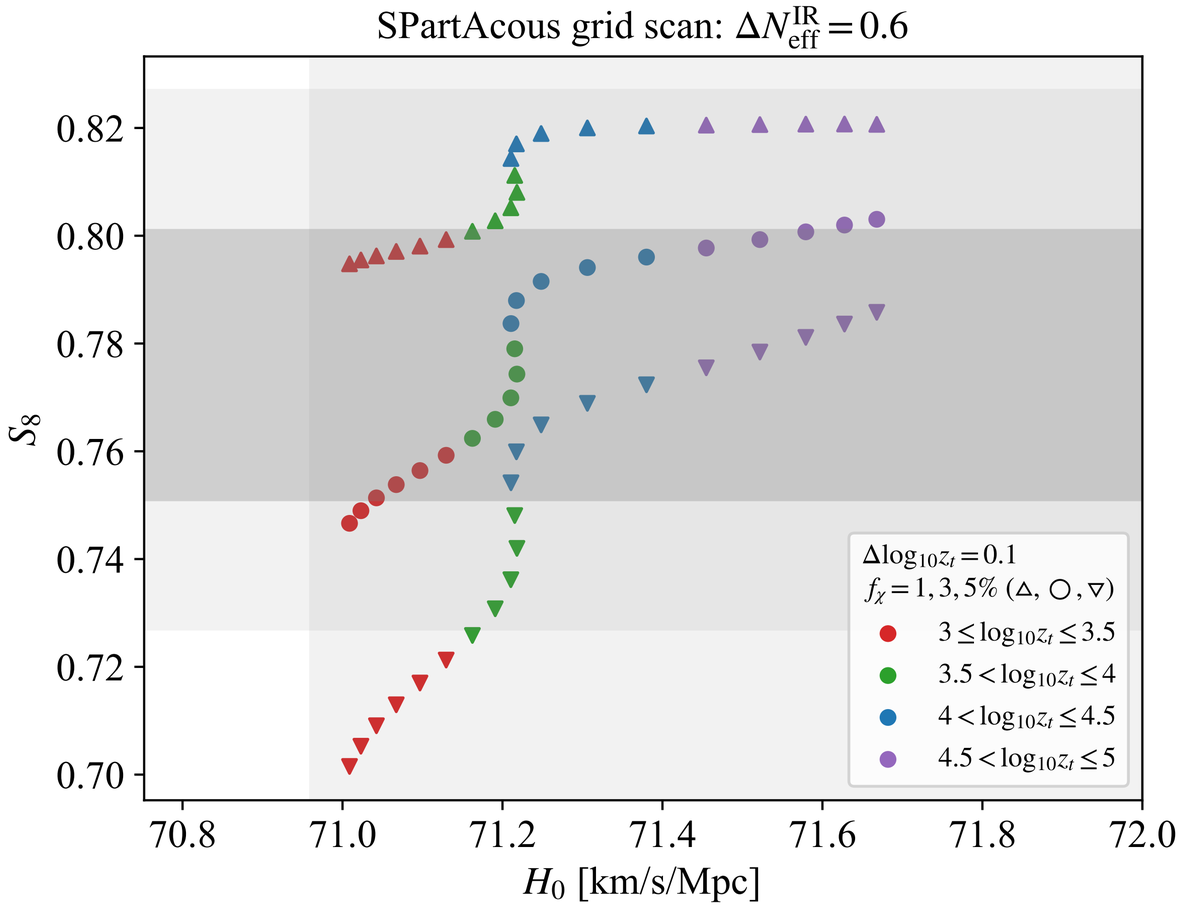
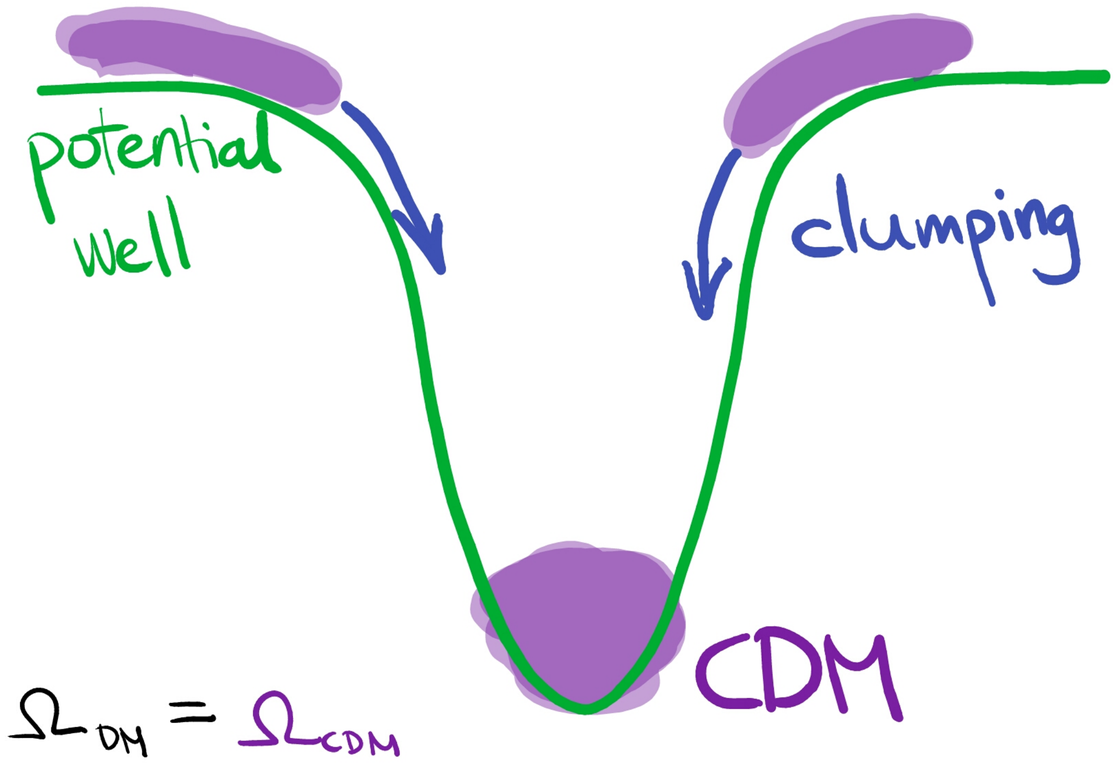


Dark Matter interaction with DR

Tightly-coupled DM-DR



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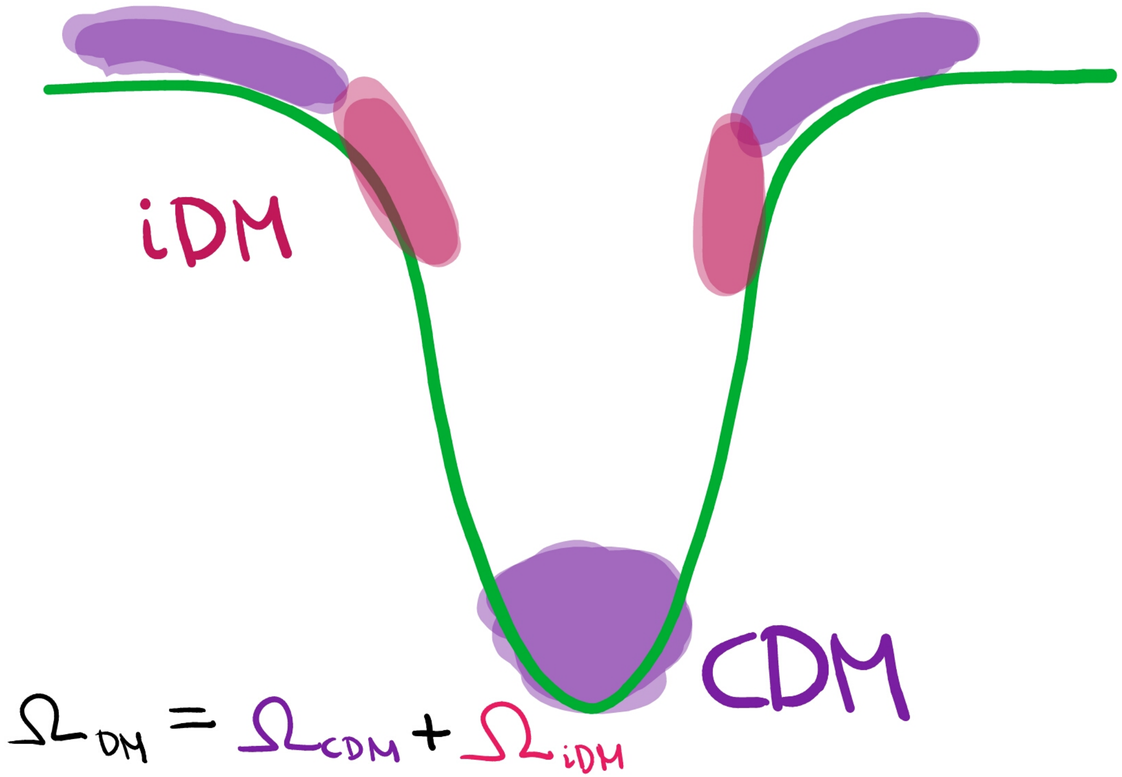

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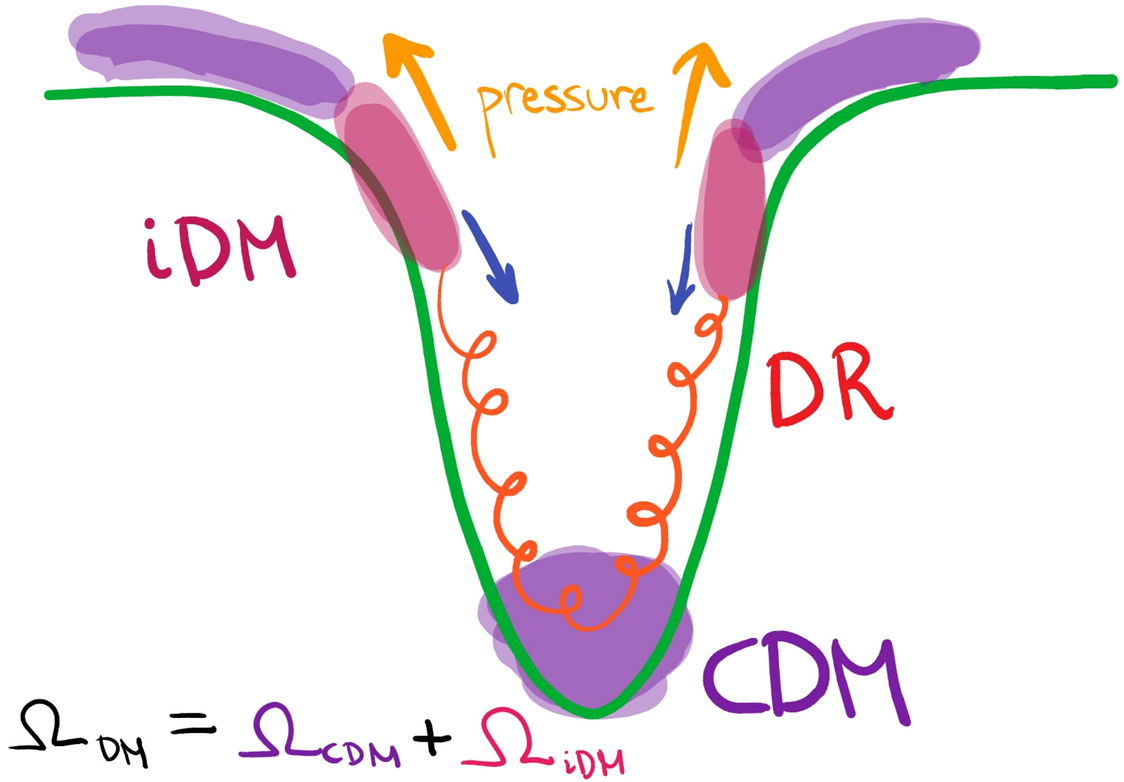
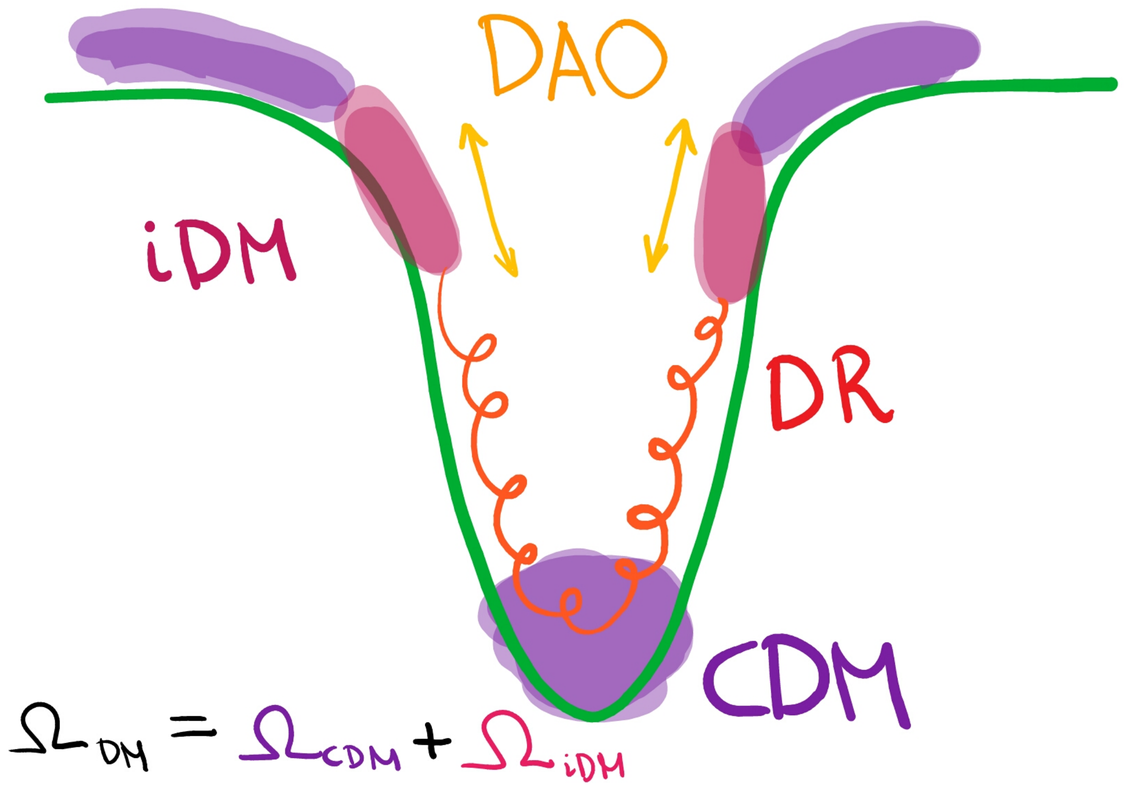


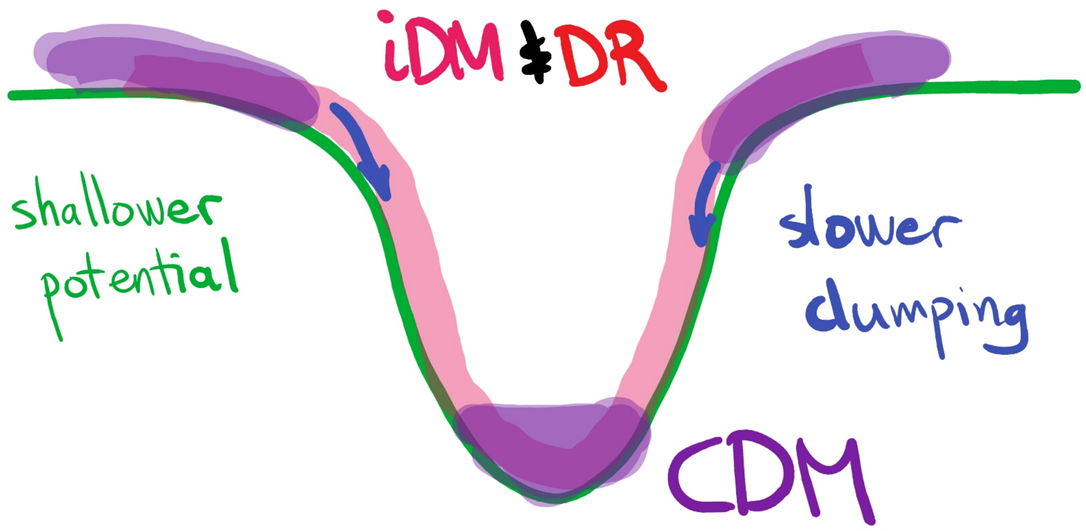
Dark Matter interaction with DR

Dark Acoustic Oscillations



Dark Matter interaction with DR

Structure Suppression

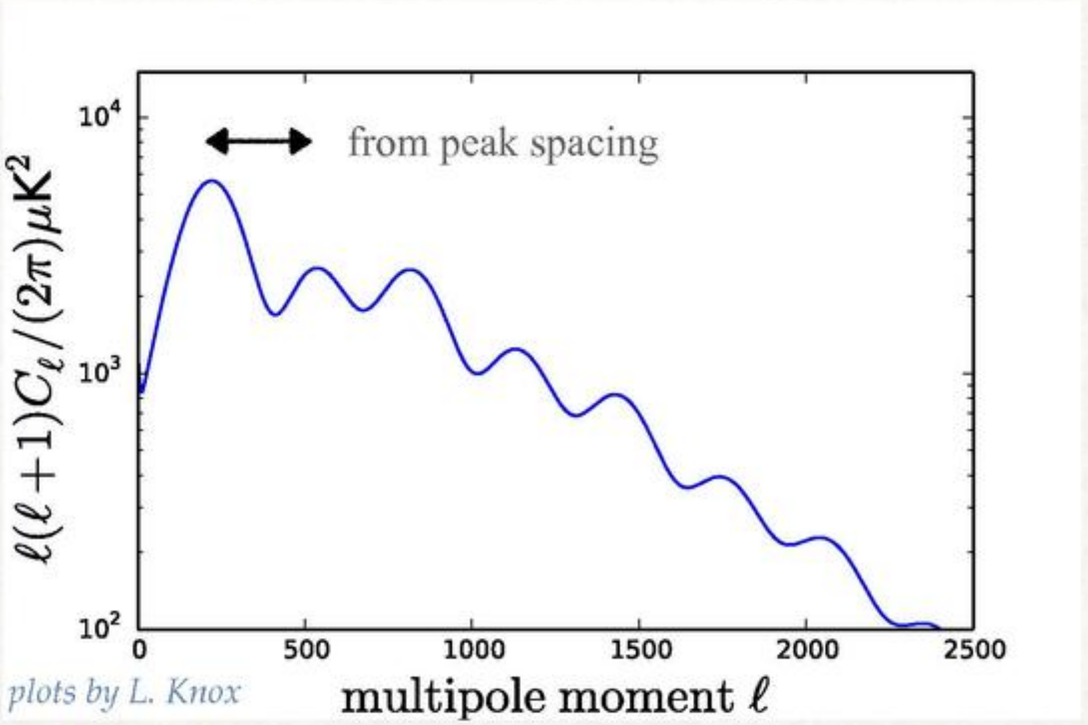


$$\Omega_{DM} = \Omega_{CDM} + \Omega_{iDM}$$

How does CMB data measure H0?

- Inference of H_0 from the CMB is model dependent.
- It comes from the measurement of three angular scales $\theta_s, \theta_d, \theta_{eq}$.

θ_s sound horizon at last scattering ~1.0404



How does CMB data measure H0?

- Inference of H_0 from the CMB is model dependent.
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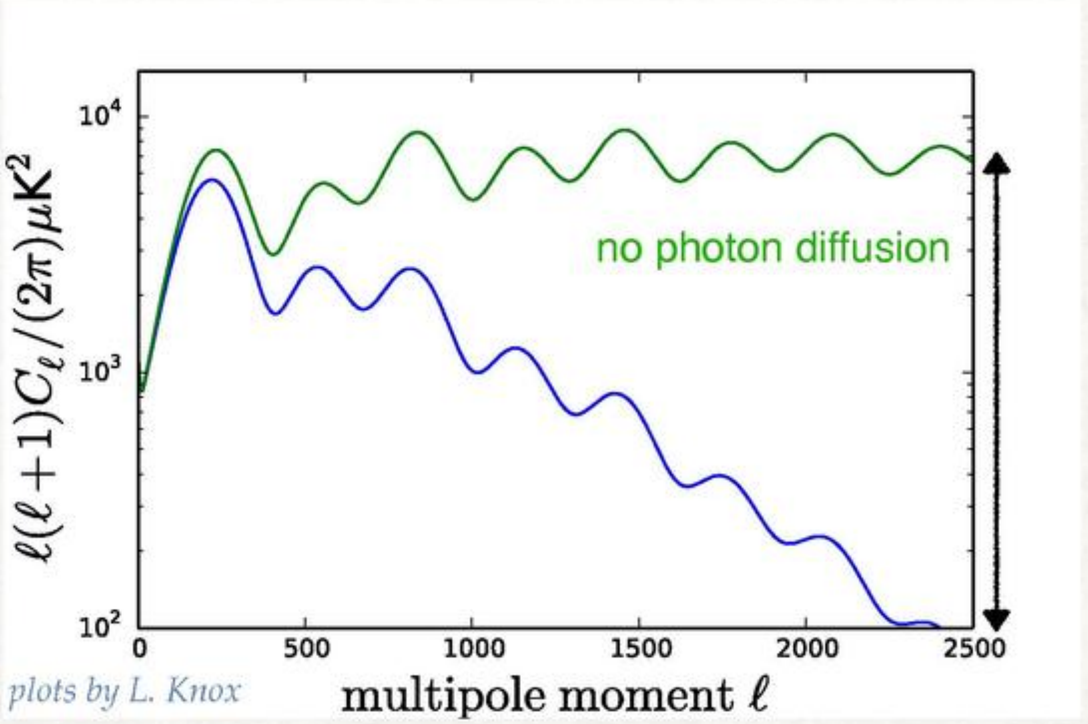
θ_d photon diffusion length at last scattering ~ 0.1609



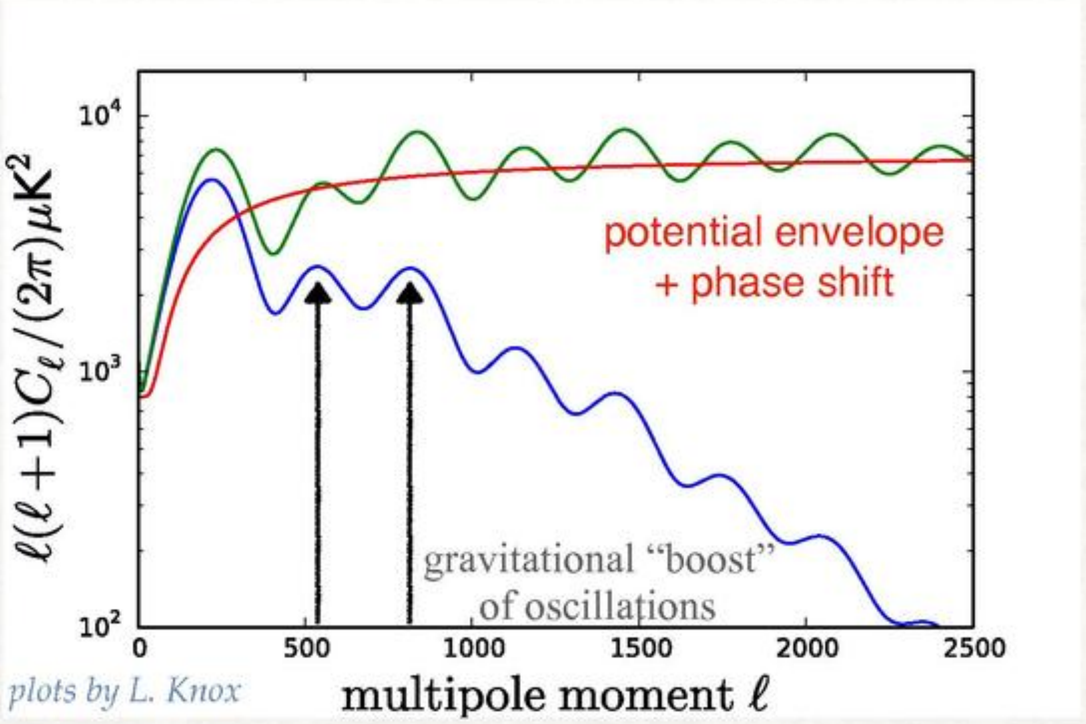
"Silk Damping"

e.g. Hu&White astro-ph/9609079, Hu++astro-ph/0006436

How does CMB data measure H0?

- Inference of H_0 from the CMB is model dependent.
- It comes from the measurement of three angular scales $\theta_s, \theta_d, \theta_{eq}$.

θ_{eq} horizon size at matter-radiation equality ~ 0.81



e.g. Hu&White astro-ph/9609079, Hu++astro-ph/0006436