Positivity bounds on Higgs-portal dark matter

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Ref: S.-S. Kim, HML, K. Yamashita, JHEP 06 (2023) 124; JHEP 11 (2023) 119

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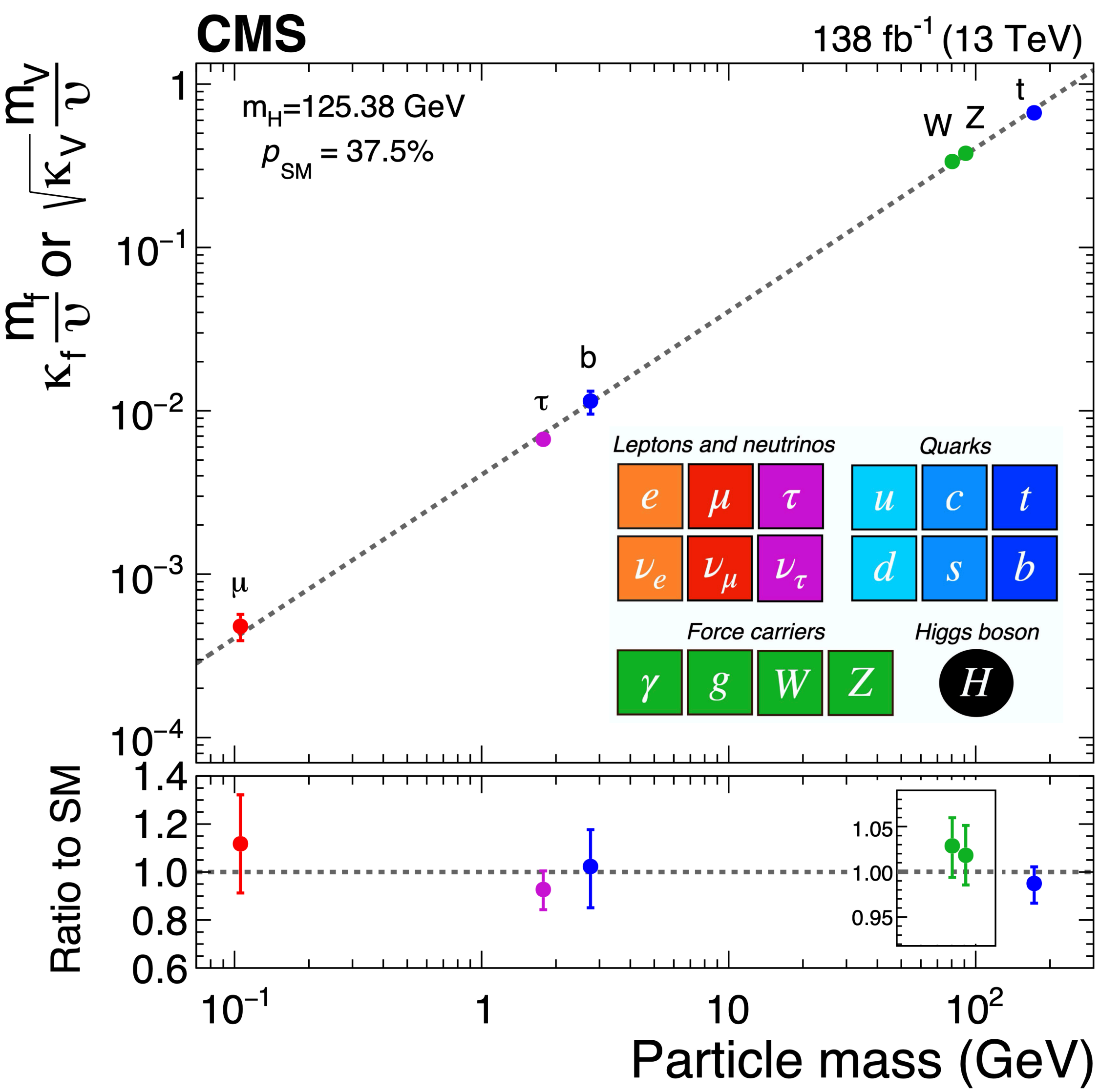
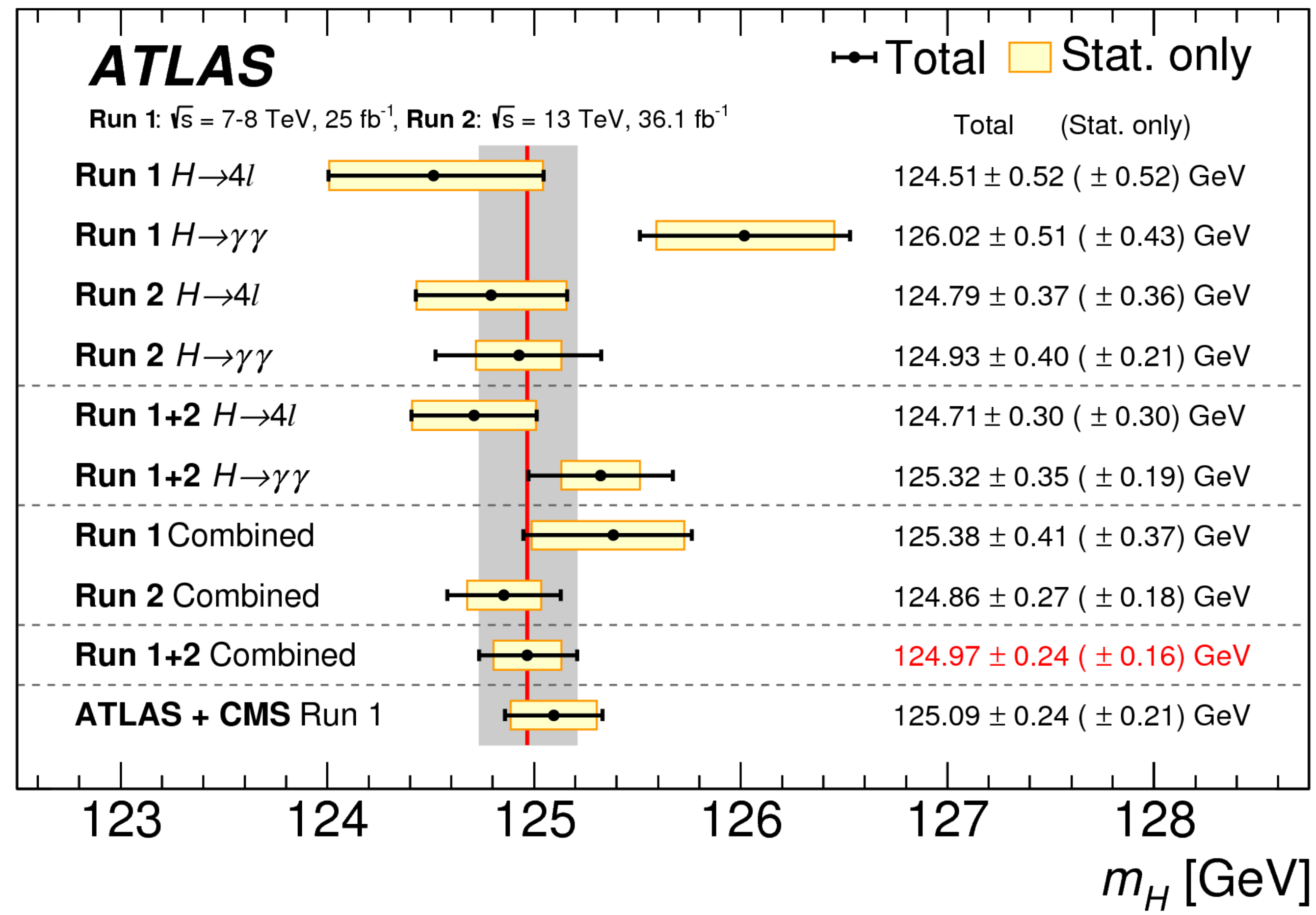
Outline

- Introduction
- Positivity on Higgs-portal couplings
- Interplay with dark matter abundance
- Conclusions

Introduction

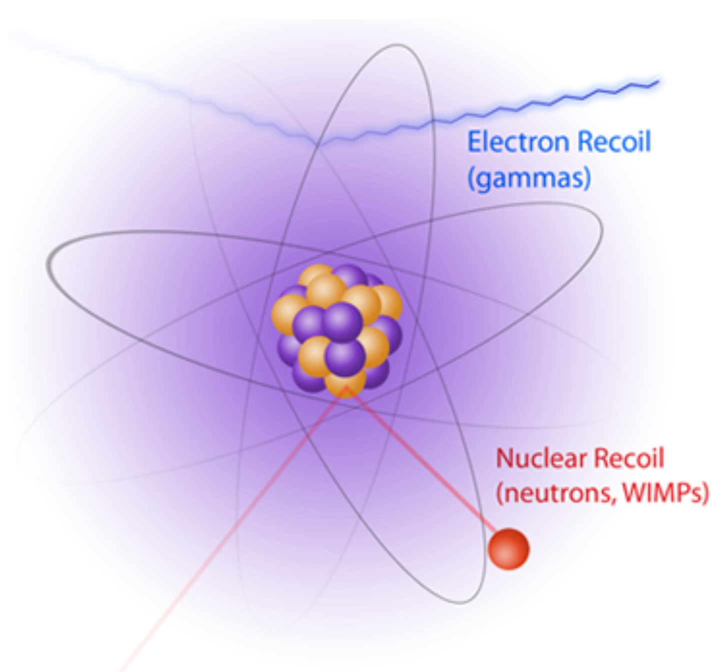
Higgs @ LHC

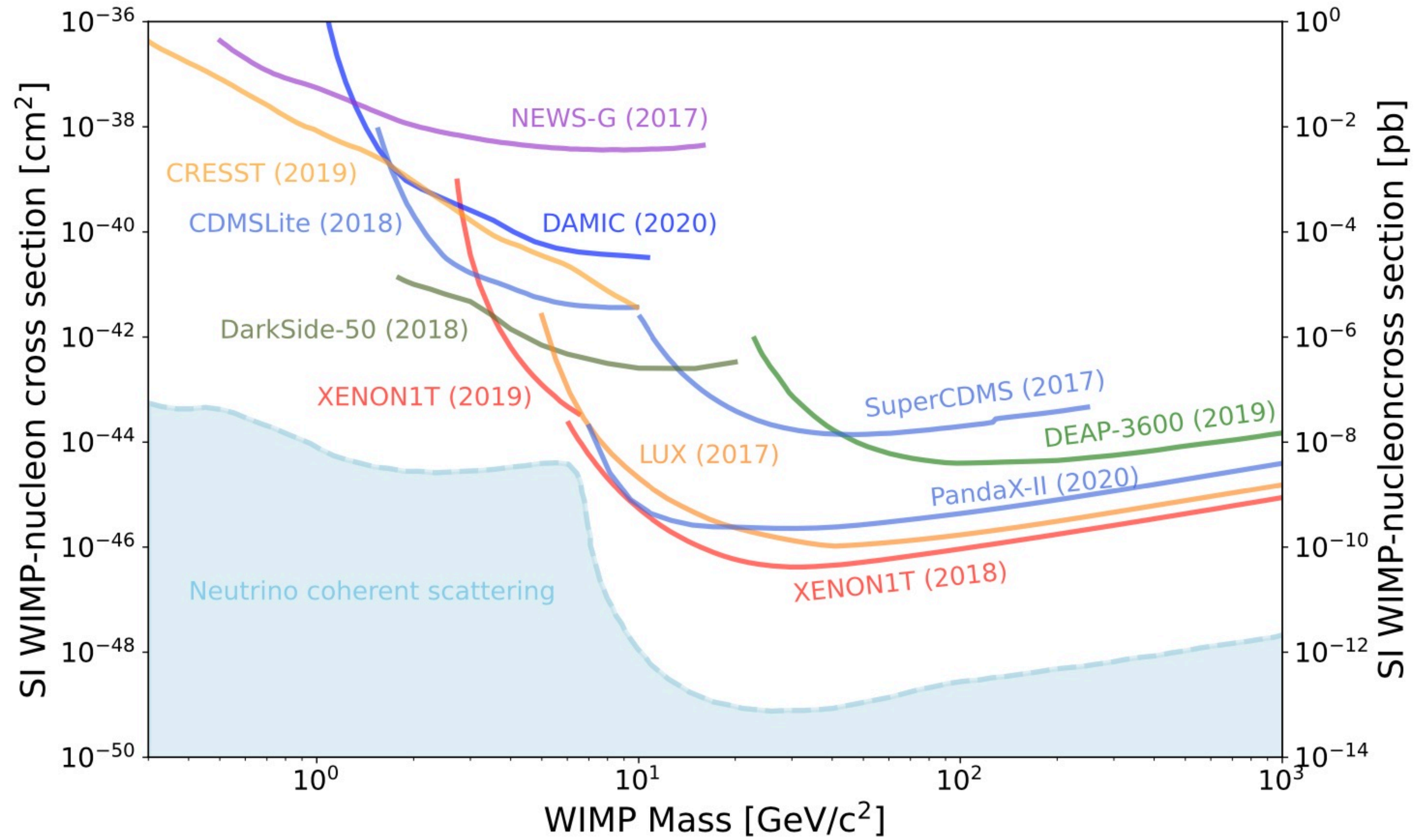


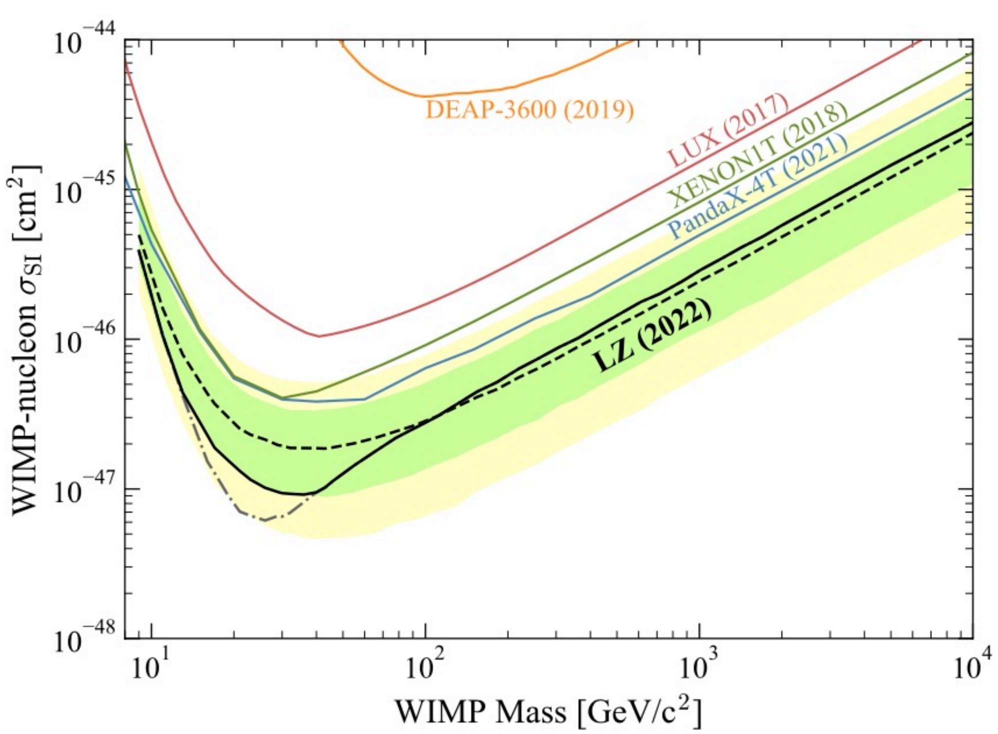


- Measured Higgs couplings are consistent with the SM.
- There is no convincing hint for new physics at TeV scale.
- Higgs precision test is crucial for EFT framework.

Dark matter @ DD







- Vanilla WIMP is constrained by direct detection, more strongly bounded recently by LZ.
- General EFT description for WIMP dark matter is desirable.

The SM EFT

- New physics beyond SM is encoded by higher order terms.

$$\mathcal{L} = \mathcal{L}_{SM} + \sum_{i=1}^{n_d} \frac{C_i^{(d)}}{\Lambda^{d-4}} Q_i^{(d)}$$

[Buchmuller, Wyler (1986); Grzadkowski et al (2010)]

Dimension-6 operators

[J. Ellis et al (2020)]

X^3		H^6 and H^4D^2		$\psi^2 H^3$	
$\mathcal{O}_G$	$f^{ABC}G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	$\mathcal{O}_H$	$(H^\dagger H)^3$	$\mathcal{O}_{eH}$	$(H^\dagger H)(\bar{l}_p e_r H)$
$\mathcal{O}_{\widetilde{G}}$	$f^{ABC}\widetilde{G}_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	$\mathcal{O}_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$	$\mathcal{O}_{uH}$	$(H^\dagger H)(\bar{q}_p u_r \widetilde{H})$
$\mathcal{O}_W$	$\varepsilon^{IJK}W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$	$\mathcal{O}_{HD}$	$(H^\dagger D^\mu H)^\star(H^\dagger D_\mu H)$	$\mathcal{O}_{dH}$	$(H^\dagger H)(\bar{q}_p d_r H)$
$\mathcal{O}_{\widetilde{W}}$	$\varepsilon^{IJK}\widetilde{W}_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$				

X^2H^2		$\psi^2 XH$		$\psi^2 H^2 D$	
$\mathcal{O}_{HG}$	$H^\dagger H\, G^A_{\mu\nu}G^{A\mu\nu}$	$\mathcal{O}_{eW}$	$(\bar{l}_p\sigma^{\mu\nu}e_r)\tau^I H W^I_{\mu\nu}$	$\mathcal{O}_{Hl}^{(1)}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{l}_p\gamma^\mu l_r)$
$\mathcal{O}_{H\widetilde{G}}$	$H^\dagger H\, \widetilde{G}^A_{\mu\nu}G^{A\mu\nu}$	$\mathcal{O}_{eB}$	$(\bar{l}_p\sigma^{\mu\nu}e_r)H B_{\mu\nu}$	$\mathcal{O}_{Hl}^{(3)}$	$(H^\dagger i\overleftrightarrow{D}^I_\mu H)(\bar{l}_p\tau^I\gamma^\mu l_r)$
$\mathcal{O}_{HW}$	$H^\dagger H\, W^I_{\mu\nu}W^{I\mu\nu}$	$\mathcal{O}_{uG}$	$(\bar{q}_p\sigma^{\mu\nu}T^A u_r)\widetilde{H}\, G^A_{\mu\nu}$	$\mathcal{O}_{He}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{e}_p\gamma^\mu e_r)$
$\mathcal{O}_{H\widetilde{W}}$	$H^\dagger H\, \widetilde{W}^I_{\mu\nu}W^{I\mu\nu}$	$\mathcal{O}_{uW}$	$(\bar{q}_p\sigma^{\mu\nu}u_r)\tau^I\widetilde{H}\, W^I_{\mu\nu}$	$\mathcal{O}_{Hq}^{(1)}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{q}_p\gamma^\mu q_r)$
$\mathcal{O}_{HB}$	$H^\dagger H\, B_{\mu\nu}B^{\mu\nu}$	$\mathcal{O}_{uB}$	$(\bar{q}_p\sigma^{\mu\nu}u_r)\widetilde{H}\, B_{\mu\nu}$	$\mathcal{O}_{Hq}^{(3)}$	$(H^\dagger i\overleftrightarrow{D}^I_\mu H)(\bar{q}_p\tau^I\gamma^\mu q_r)$
$\mathcal{O}_{H\widetilde{B}}$	$H^\dagger H\, \widetilde{B}_{\mu\nu}B^{\mu\nu}$	$\mathcal{O}_{dG}$	$(\bar{q}_p\sigma^{\mu\nu}T^A d_r)H\, G^A_{\mu\nu}$	$\mathcal{O}_{Hu}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{u}_p\gamma^\mu u_r)$
$\mathcal{O}_{HWB}$	$H^\dagger\tau^I H\, W^I_{\mu\nu}B^{\mu\nu}$	$\mathcal{O}_{dW}$	$(\bar{q}_p\sigma^{\mu\nu}d_r)\tau^I H\, W^I_{\mu\nu}$	$\mathcal{O}_{Hd}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{d}_p\gamma^\mu d_r)$
$\mathcal{O}_{H\widetilde{W}B}$	$H^\dagger\tau^I H\, \widetilde{W}^I_{\mu\nu}B^{\mu\nu}$	$\mathcal{O}_{dB}$	$(\bar{q}_p\sigma^{\mu\nu}d_r)H\, B_{\mu\nu}$	$\mathcal{O}_{Hud}$	$i(\widetilde{H}^\dagger D_\mu H)(\bar{u}_p\gamma^\mu d_r)$

$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
$\mathcal{O}_{ll}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$	$\mathcal{O}_{ee}$	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$	$\mathcal{O}_{le}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{e}_s\gamma^\mu e_t)$
$\mathcal{O}_{qq}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu q_t)$	$\mathcal{O}_{uu}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma^\mu u_t)$	$\mathcal{O}_{lu}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{u}_s\gamma^\mu u_t)$
$\mathcal{O}_{qq}^{(3)}$	$(\bar{q}_p\gamma_\mu\tau^I q_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	$\mathcal{O}_{dd}$	$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma^\mu d_t)$	$\mathcal{O}_{ld}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{d}_s\gamma^\mu d_t)$
$\mathcal{O}_{lq}^{(1)}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)$	$\mathcal{O}_{eu}$	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$	$\mathcal{O}_{qe}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma^\mu e_t)$
$\mathcal{O}_{lq}^{(3)}$	$(\bar{l}_p\gamma_\mu\tau^I l_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	$\mathcal{O}_{ed}$	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$	$\mathcal{O}_{qu}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{u}_s\gamma^\mu u_t)$
		$\mathcal{O}_{ud}^{(1)}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{d}_s\gamma^\mu d_t)$	$\mathcal{O}_{qu}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{u}_s\gamma^\mu T^A u_t)$
		$\mathcal{O}_{ud}^{(8)}$	$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)$	$\mathcal{O}_{qd}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma^\mu d_t)$
				$\mathcal{O}_{qd}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{d}_s\gamma^\mu T^A d_t)$

$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$		B-violating	
$\mathcal{O}_{ledq}$	$(\bar{l}_p^j e_r)(\bar{d}_s q_t^j)$	$\mathcal{O}_{duq}$	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
$\mathcal{O}_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\varepsilon_{jk}(\bar{q}_s^k d_t)$	$\mathcal{O}_{qqu}$	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$\mathcal{O}_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\varepsilon_{jk}(\bar{q}_s^k T^A d_t)$	$\mathcal{O}_{qqq}$	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jn}\varepsilon_{km}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$\mathcal{O}_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\varepsilon_{jk}(\bar{q}_s^k u_t)$	$\mathcal{O}_{duu}$	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$
$\mathcal{O}_{lequ}^{(3)}$	$(\bar{l}_p^j\sigma_{\mu\nu}e_r)\varepsilon_{jk}(\bar{q}_s^k\sigma^{\mu\nu}u_t)$		

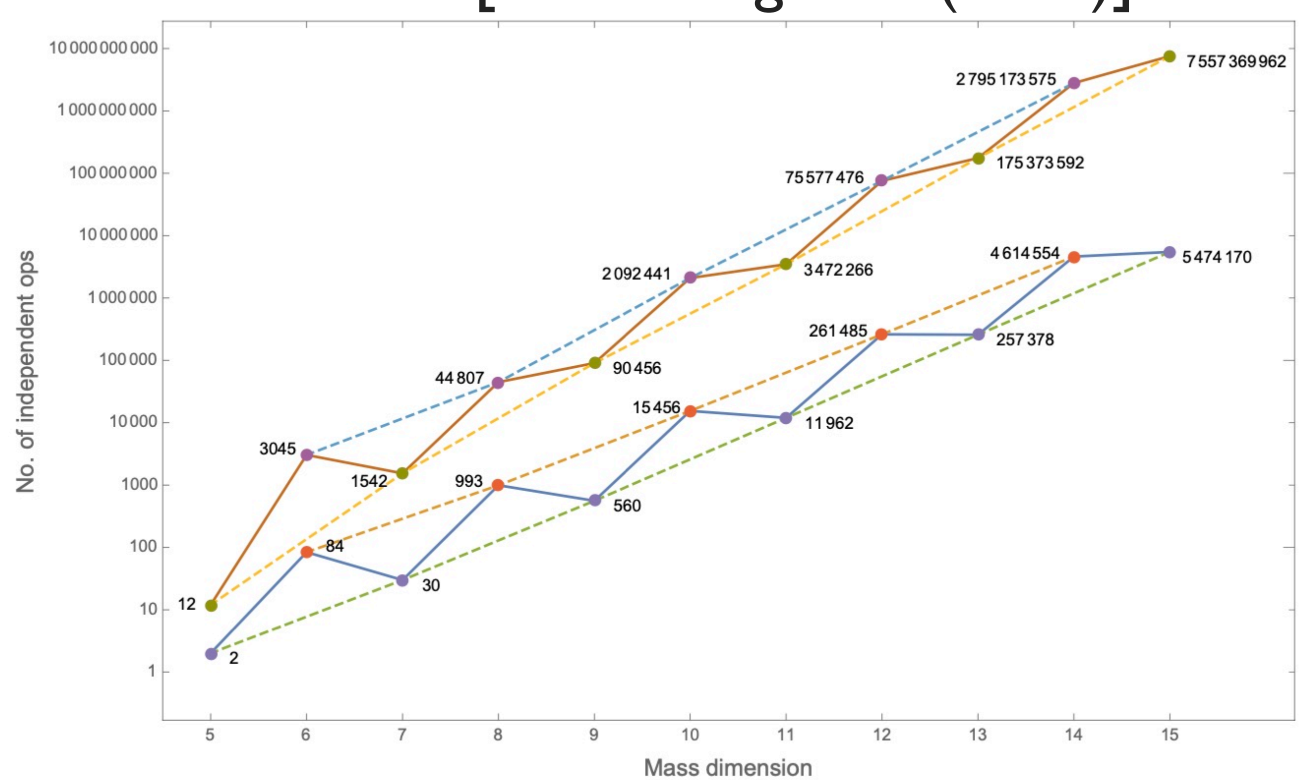
Dimension-8 (Higgs only)

[C.W. Murphy (2020); Hao-Lin Li et al (2020)]

$\mathbf{2:H^8}$		$\mathbf{3:H^6D^2}$		$\mathbf{4:H^4D^4}$	
Q_{H^8}	$(H^\dagger H)^4$	$Q_{H^6}^{(1)}$	$(H^\dagger H)^2(D_\mu H^\dagger D^\mu H)$	$Q_{H^4}^{(1)}$	$(D_\mu H^\dagger D_\nu H)(D^\nu H^\dagger D^\mu H)$
		$Q_{H^6}^{(2)}$	$(H^\dagger H)(H^\dagger\tau^I H)(D_\mu H^\dagger\tau^I D^\mu H)$	$Q_{H^4}^{(2)}$	$(D_\mu H^\dagger D_\nu H)(D^\mu H^\dagger D^\nu H)$
				$Q_{H^4}^{(3)}$	$(D^\mu H^\dagger D_\mu H)(D^\nu H^\dagger D_\nu H)$

Terms increases rapidly with dim.

[B. Henning et al (2015)]



Consistent EFTs

Higgs sector EFT: $$\Delta\mathcal{L}_H = \sum_{n+m\geq 3} \frac{c_{n,m}}{\Lambda^{2(n+m)-4}} H^{2n} D^{2m}$$

H^6 and $H^4 D^2$	
$\mathcal{O}_H$	$(H^\dagger H)^3$
$\mathcal{O}_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$
$\mathcal{O}_{HD}$	$\left(H^\dagger D^\mu H\right)^\star \left(H^\dagger D_\mu H\right)$

2 : H^8	
Q_{H^8}	$(H^\dagger H)^4$

3 : $H^6 D^2$	
$Q_{H^6}^{(1)}$	$(H^\dagger H)^2(D_\mu H^\dagger D^\mu H)$
$Q_{H^6}^{(2)}$	$(H^\dagger H)(H^\dagger \tau^I H)(D_\mu H^\dagger \tau^I D^\mu H)$

4 : $H^4 D^4$	
$Q_{H^4}^{(1)}$	$(D_\mu H^\dagger D_\nu H)(D^\nu H^\dagger D^\mu H)$
$Q_{H^4}^{(2)}$	$(D_\mu H^\dagger D_\nu H)(D^\mu H^\dagger D^\nu H)$
$Q_{H^4}^{(3)}$	$(D^\mu H^\dagger D_\mu H)(D^\nu H^\dagger D_\nu H)$

[e.g. C. Burgess, HML, M. Trott (2009)]

- Adiabaticity: $\dot{\phi}/\phi \ll \Lambda$ No excitation of new states
- Perturbativity: $\Gamma_{\rm eff} = \Gamma_{\rm tree} + \Gamma_{\rm loops}, \qquad |\Gamma_{\rm loops}| \ll |\Gamma_{\rm tree}|$
- Unitarity: S-matrix $S = 1 + iT, \quad |S| \leq 1$
- Positivity: Reduced S-matrix $$\frac{\partial^2 \mathcal{M}}{\partial s^2} = \frac{4}{\pi}\int_{s>0} ds\, \frac{s\sigma(s)}{s^3} > 0$$

Higgs inflation

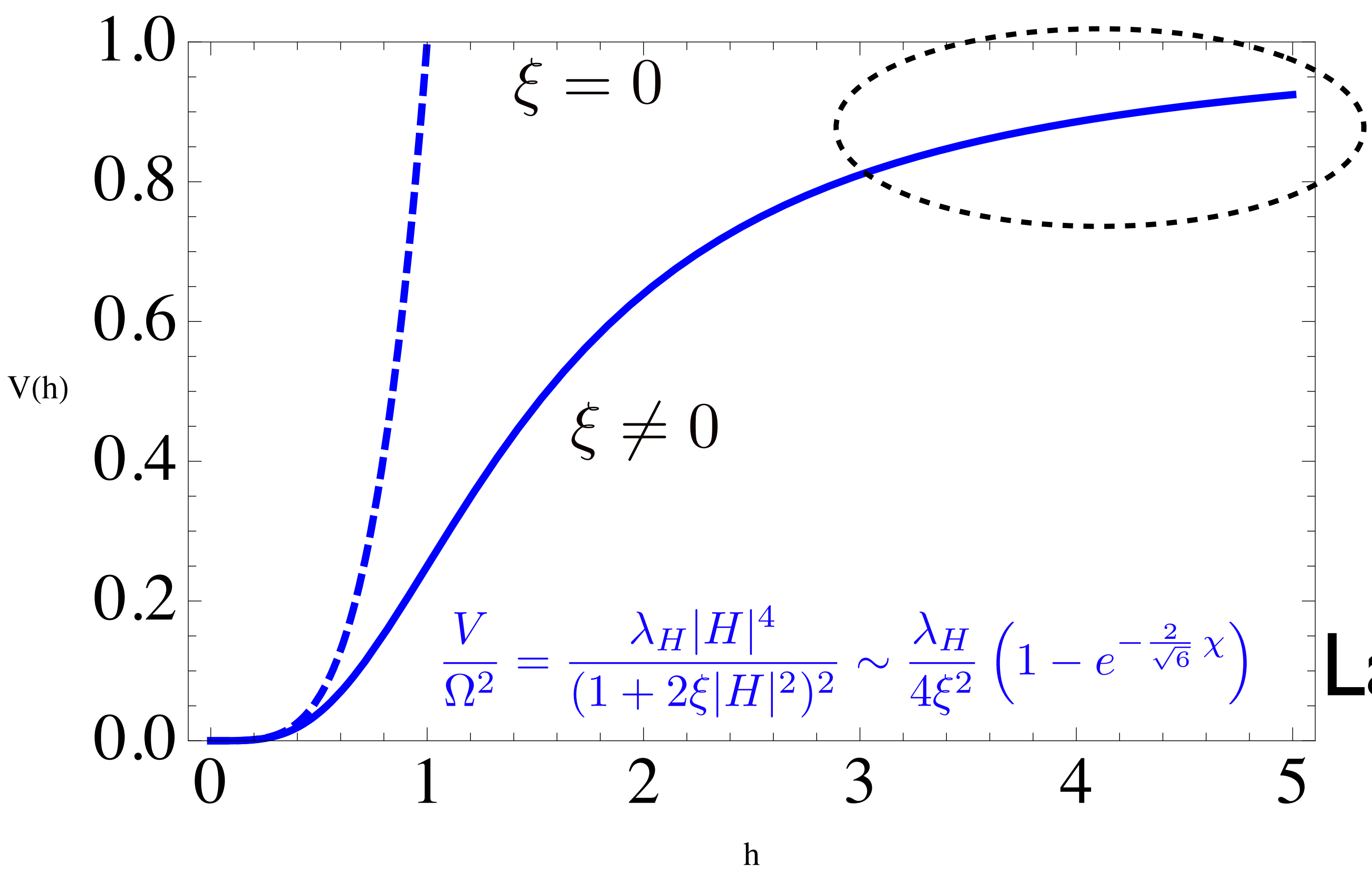
- Einstein gravity + SM Higgs + non-minimal gravity coupling

[Bezrukov, Shaposhnikov (2007)]

$$\mathcal{L} = \sqrt{-g}\Big(\frac{1}{2}\mathcal{R} + \xi|H|^2\mathcal{R} - |D_\mu H|^2 - \lambda_H(|H|^2 - v^2/2)^2\Big)$$

$$\Rightarrow \quad \mathcal{L}_E = \sqrt{-g_E}\Big(\frac{1}{2}\mathcal{R}(g_E) - \frac{3\xi^2}{\Omega^2}(\partial_\mu|H|^2)^2 - \frac{1}{\Omega}|D_\mu H|^2 - \frac{V}{\Omega^2}\Big)$$

$$g_{\mu\nu} = \Omega^{-1} g_{E,\mu\nu}, \quad \Omega = 1 + 2\xi|H|^2$$



Well consistent with CMB data due to flat Higgs potential

$$n_s = 0.966, \quad r = 0.0033$$

Amplitude of CMB anisotropies $\quad \frac{\xi}{\sqrt{\lambda_H}} = 5 \times 10^4$

Large non-minimal coupling lowers cutoff scale. $\quad E \lesssim \frac{M_P}{\xi}$

Higgs inflation as EFTs

- Higgs inflation as SM EFTs: how to linearize the non-linear Higgs kinetic term for a larger cutoff ?

$$\mathcal{L}_{\rm kin} = \frac{1}{2(1+\xi_H \vec{\phi}^2/M_P^2)}\left(\delta_{ij} + \frac{6\xi_H^2 \phi_i\phi_j/M_P^2}{1+\xi_H\vec{\phi}^2/M_P^2}\right)\partial_\mu\phi_i\partial^\mu\phi_j$$

$$\longrightarrow \mathcal{L}_{\rm linear} = \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\phi_i)^2$$

"Linear sigma models"

[G. Giudice, HML (2010); Ema et al (2020); HML, A. Menkara (2021)]

$$\mathcal{L}_{\rm eff} = \frac{3\xi_H^2}{M_P^2}\frac{(\partial_\mu|H|^2)^2}{(1+2\xi_H|H|^2/M_P^2)^2} - \frac{K_\mu K^\mu}{48M_P^2} = 0$$

"Local Weyl gravity"

[D. Ghilencea, HML (2018); S. Aoki, HML (2022)]

Conformal coupling limit "Higgs pole inflation"

[S. Clery, HML, A. Menkara (2023)]

This talk

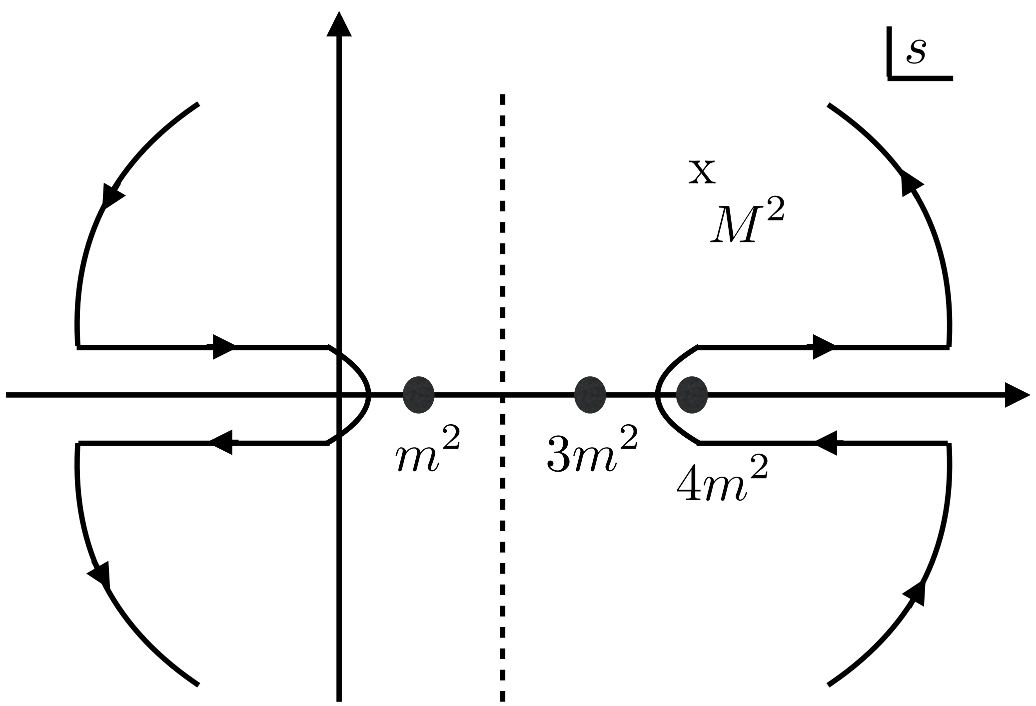
- Consistent effective Higgs-portals for dark matter.

[S.-S. Kim, HML, K. Yamashita (2023)]

Positivity bounds

- Analyticity, locality, unitarity of S-matrix

$\phi\phi \to \phi\phi$ scattering : Forward limit, $t \to 0$ [A. Adams et al (2006)]



$$A(s) = \mathcal{M}(s,t,u) = \mathcal{M}(s,0,4m^2-s)$$

: symmetric wrt s=2m²

$$\lim_{|s|\to\infty} \frac{A(s)}{s^2} = 0,$$

Polynomial bounded, e.g. Froissart-Martin bound

$$I = \oint \frac{ds}{2\pi i} \frac{A(s)}{(s-M^2)^3} \longrightarrow A''(s=M^2) = \frac{4}{\pi}\int_{\rm cuts} ds \, \frac{{\rm Im}A}{(s-M^2)^3}$$

${\rm Im}A = s\sigma(s)$: optical theorem $\longrightarrow$ $A''(s=M^2) = \frac{4}{\pi}\int_{s>0} ds \, \frac{s\sigma(s)}{s^3} > 0$

$s \ll \Lambda^2$: EFT amplitude, $A(s) = g\sum_{n=1}^{\infty} c_n \left(\frac{s^2}{\Lambda^2}\right)^n$

Similar contour integrals ➡ $c_n > 0$ "Positivity" for dim-8 ops

Higgs EFT for dark matter

- Extend the SM EFT with Higgs-portal dark matter.

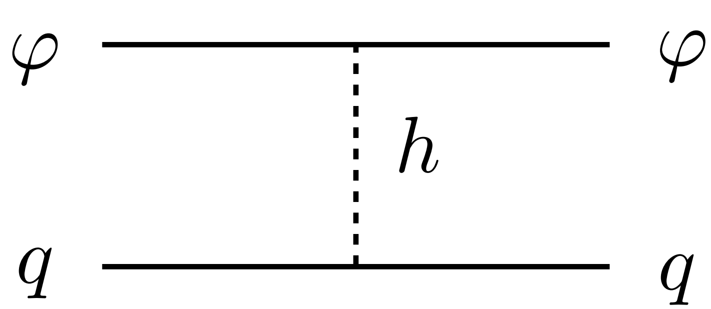
Scalar dark matter with Z_2 symmetry: [S.-S. Kim, HML, K. Yamashita (2023)]

Dimension-4	Dimension-6	Dimension-8
$c_3\, \varphi^2 \vert H\vert^2$	$\varphi^2 \vert D_\mu H\vert^2$	$(D_\mu H^\dagger D_\nu H)(\partial^\mu\varphi\partial^\nu\varphi)$
	$d_4\, \vert H\vert^2(\partial_\mu\varphi)^2$	$(D_\mu H^\dagger D^\mu H)(\partial_\nu\varphi\partial^\nu\varphi)$
	$c_3'\, \varphi^2\vert H\vert^4,\ \varphi^4\vert H\vert^2$	$d_4'\, \vert H\vert^4(\partial_\mu\varphi)^2,\ \varphi^4\vert H\vert^4$

+ DM self-interactions $\partial_\mu\varphi\partial^\mu\varphi\partial_\nu\varphi\partial^\nu\varphi\ ,\ \varphi^4,\ \varphi^6,\ \varphi^8, \cdots$

Dim-4 & Dim-6 couplings and direct detection:

$$\mathcal{L}_{h,\rm linear} = \frac{1}{3\Lambda^4}\, h\Big[2(c_3 - c_3')\lambda_H v^3 m_\varphi^2 \varphi^2 - (d_4 - d_4')\lambda_H v^3 (\partial_\mu\varphi)^2\Big], \qquad c_3 = c_3',\ d_4 = d_4'$$



Positivity bounds and UV completion for WIMP and FIMP?

Higgs-portal dark matter and positivity

Higgs-portal dark matter

- Consider a singlet scalar dark mater in SM.
- Effective Higgs-portal interactions up to dim-8:

Assumption: Z_2 symmetry, couplings suppressed by masses.

$$\mathcal{L}_{\rm Higgs-portal} = \mathcal{L}_1 + \mathcal{L}_2$$

[S.-S. Kim, HML, K. Yamashita(2023)]

Up to 2-derivatives

$$\begin{aligned}\mathcal{L}_1 = &-\frac{1}{6\Lambda^4}\Big(c_1 m_\varphi^4\varphi^4 + 4c_2 m_H^4|H|^4 + 8c_2'\lambda_H m_H^2|H|^6 + 4c_2''\lambda_H^2|H|^8 \\ &+ 4c_3 m_\varphi^2 m_H^2\varphi^2|H|^2 + 4c_3'\lambda_H m_\varphi^2\varphi^2|H|^4\Big) \\ &+ \frac{1}{6\Lambda^4}\Big(d_1 m_\varphi^2\varphi^2(\partial_\mu\varphi)^2 + 4d_2 m_H^2|H|^2|D_\mu H|^2 + 4d_2'\lambda_H|H|^4|D_\mu H|^2 \\ &+ 2d_3 m_\varphi^2\varphi^2|D_\mu H|^2 + 2d_4 m_H^2|H|^2(\partial_\mu\varphi)^2 + 2d_4'\lambda_H|H|^4(\partial_\mu\varphi)^2\Big),\end{aligned}$$

4-derivatives

$\mathcal{L}_2 \supset$

$$O^{(1)}_{H^2\varphi^2} = (D_\mu H^\dagger D_\nu H)(\partial^\mu\varphi\partial^\nu\varphi) \qquad O^{(2)}_{H^2\varphi^2} = (D_\mu H^\dagger D^\mu H)(\partial_\nu\varphi\partial^\nu\varphi)$$

$$O_{\varphi^4} = \partial_\mu\varphi\partial^\mu\varphi\partial_\nu\varphi\partial^\nu\varphi$$

$$O^{(1)}_{H^4} = (D_\mu H^\dagger D_\nu H)(D^\nu H^\dagger D^\mu H) \qquad O^{(2)}_{H^4} = (D_\mu H^\dagger D_\nu H)(D^\mu H^\dagger D^\nu H)$$

$$O^{(3)}_{H^4} = (D_\mu H^\dagger D^\mu H)(D_\nu H^\dagger D^\nu H)$$

Positivity for multiple fields

- Scattering matrix elements in the forward limit

$$A(s) = c_0 + c_1 \frac{s}{\Lambda^2} + c_2 \frac{s^2}{\Lambda^4} + \cdots \longrightarrow c_2 > 0$$

- Scattering amplitudes for superposed states, ab→ab,

$$|a\rangle = u^i\,|i\rangle\,, |b\rangle = v^i\,|i\rangle\,,\ i = \underbrace{\phi_a}_{\text{Higgs}}(a=1,2,3,4),\ \underbrace{\varphi}_{\text{Dark matter}}$$

Positivity bounds for ab→ab: [S.-S. Kim, HML, K. Yamashita(2023)]

$$u^i v^j u^{*k} v^{*l} M^{ijkl} \geq 0, \qquad M^{ijkl} = \frac{1}{2}\frac{d^2}{ds^2} M(ij \to kl)(s, t=0)\Big|_{s\to 0}.$$

Positivity for multiple fields

Bounds	Channels $(\|1\rangle + \|2\rangle \to \|1\rangle + \|2\rangle)$
$C^{(1)}_{H^4} + C^{(2)}_{H^4} \geq 0$	$\|1\rangle = \|\phi_1\rangle\,,\ \|2\rangle = \|\phi_3\rangle$
$C^{(1)}_{H^4} + C^{(2)}_{H^4} + C^{(3)}_{H^4} \geq 0$	$\|1\rangle = \|\phi_1\rangle\,,\ \|2\rangle = \|\phi_1\rangle$
$C^{(2)}_{H^4} \geq 0$	$\|1\rangle = \|\phi_1\rangle\,,\ \|2\rangle = \|\phi_2\rangle$
$C^{(1)}_{H^2\varphi^2} \geq 0$	$\|1\rangle = \|\phi_1\rangle\,,\ \|2\rangle = \|\varphi\rangle$
$C_{\varphi^4} \geq 0$	$\|1\rangle = \|\varphi\rangle\,,\ \|2\rangle = \|\varphi\rangle$
★ $2\sqrt{(C^{(1)}_{H^4} + C^{(2)}_{H^4} + C^{(3)}_{H^4})C_{\varphi^4}} \geq -\left(C^{(1)}_{H^2\varphi^2} + C^{(2)}_{H^2\varphi^2}\right)$	$\|1\rangle = 2\sqrt{C_{\varphi^4}}\,\|\phi_1\rangle + \sqrt{-(C^{(1)}_{H^2\varphi^2} + C^{(2)}_{H^2\varphi^2})}\,\|\varphi\rangle,$ $\|2\rangle = \|1\rangle$
★ $2\sqrt{(C^{(1)}_{H^4} + C^{(2)}_{H^4} + C^{(3)}_{H^4})C_{\varphi^4}} \geq C^{(2)}_{H^2\varphi^2}$	$\|1\rangle = 2\sqrt{C_{\varphi^4}}\,\|\phi_1\rangle + \sqrt{C^{(2)}_{H^2\varphi^2}}\,\|\varphi\rangle,$ $\|2\rangle = -2\sqrt{C_{\varphi^4}}\,\|\phi_1\rangle + \sqrt{C^{(2)}_{H^2\varphi^2}}\,\|\varphi\rangle$

[S.-S. Kim, HML, K. Yamashita(2023)]

- Nontrivial bounds★ on the Higgs-portal couplings with a combination of self-interactions for Higgs and dark matter.

Massive graviton or radion

- Effective Higgs-portal interactions are matched to UV complete models: [S.-S. Kim, HML, K. Yamashita(2023)]

Massive graviton $\mathcal{L}_G = -\frac{c_H}{M} G^{\mu\nu} T^H_{\mu\nu} - \frac{c_\varphi}{M} G^{\mu\nu} T^\varphi_{\mu\nu} \longrightarrow \mathcal{L}_{G,\text{eff}} = \frac{1}{4m_G^2 M^2}\left(2T_{\mu\nu}T^{\mu\nu} - \frac{2}{3}T^2\right)$

$$\frac{C^{(2)}_{H^2\varphi^2}}{\Lambda^4} = -\frac{1}{3}\frac{C^{(1)}_{H^2\varphi^2}}{\Lambda^4} = -\frac{2c_H c_\varphi}{3m_G^2 M^2}, \quad \frac{c_3^{(\prime)}}{\Lambda^4} = \frac{d_3}{\Lambda^4} = \frac{d_4^{(\prime)}}{\Lambda^4} = \frac{c_H c_\varphi}{m_G^2 M^2} = \frac{1}{2}\frac{C^{(1)}_{H^2\varphi^2}}{\Lambda^4}.$$

Radion $\mathcal{L}_r = \frac{c^r_H}{\sqrt{6}M} r T^H + \frac{c^r_\varphi}{\sqrt{6}M} r T^\varphi \longrightarrow \mathcal{L}_{r,\text{eff}} = \frac{1}{12m_r^2 M^2} T^2$

$$C^{(1)}_{H^2\varphi^2} = 0, \quad \frac{C^{(2)}_{H^2\varphi^2}}{\Lambda^4} = \frac{c^r_H c^r_\varphi}{3m_r^2 M^2}, \quad \frac{c_3^{(\prime)}}{\Lambda^4} = \frac{d_3}{\Lambda^4} = \frac{d_4^{(\prime)}}{\Lambda^4} = -\frac{2c^r_H c^r_\varphi}{m_r^2 M^2} = -6\frac{C^{(2)}_{H^2\varphi^2}}{\Lambda^4}.$$

- Positivity bounds are satisfied for $c_H c_\varphi > 0$ & $c^r_H c^r_\varphi > 0$

  ➡ Attractive forces due to massive graviton or radion.

- Zero DM-nucleon cross section at tree level: $c_3 = c_3', \ d_4 = d_4'$.

Disformal graviton

- Generalized metric tensor in Finsler geometry

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu \, F(I,H,\varphi), \quad I = L^2 g^{\alpha\beta}\partial_\alpha\varphi\partial_\beta\varphi, \quad H = L^2 \frac{(\partial_\alpha\varphi\, dx^\alpha)^2}{g_{\rho\sigma}dx^\rho dx^\sigma}$$

$$\boxed{\tilde{g}_{\mu\nu} = C g_{\mu\nu} + D\partial_\mu\varphi\partial_\nu\varphi}$$

$$C = 1 + c^2\frac{\varphi^2}{M_{Pl}^2} + c_X \frac{\partial_\mu\varphi\partial^\mu\varphi}{M_{Pl}^4}, \quad \text{“conformal”}$$

$$D = \frac{d}{M^4} + \frac{d}{M^4}\tilde{c}^2\frac{\varphi^2}{M_{Pl}^2}. \quad \text{“disformal”}$$

- Causality: sub-luminal propagation of graviton

→ $d > 0$

[J. Bekenstein (1993)]

- Effective interactions to Higgs in Finsler geometry:

$$\mathcal{L}_{\rm eff} = -\frac{1}{2}(\tilde{g}_{\mu\nu} - g_{\mu\nu})T_H^{\mu\nu}$$
$$= -\frac{1}{2}(C-1)T_\mu^{H,\mu} - \frac{1}{2}D\,\partial_\mu\varphi\partial_\nu\varphi\,T^{H,\mu\nu}$$

[P. Brax, K. Kaneta, Y. Mambrini, M. Pierre (2023)]

→ Positivity bounds: $C^{(1)}_{H^2\varphi^2} = -d > 0, \quad C^{(2)}_{H^2\varphi^2} = \frac{1}{2}d + \tilde{c}_X$ ← either signs

$c_X = \tilde{c}_X M_{Pl}^4/\Lambda^4$

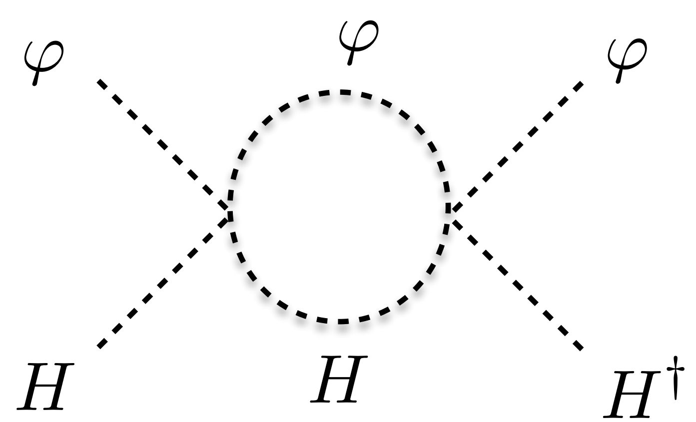
[S.-S. Kim, HML, K. Yamashita(2023)]

Positivity at loops

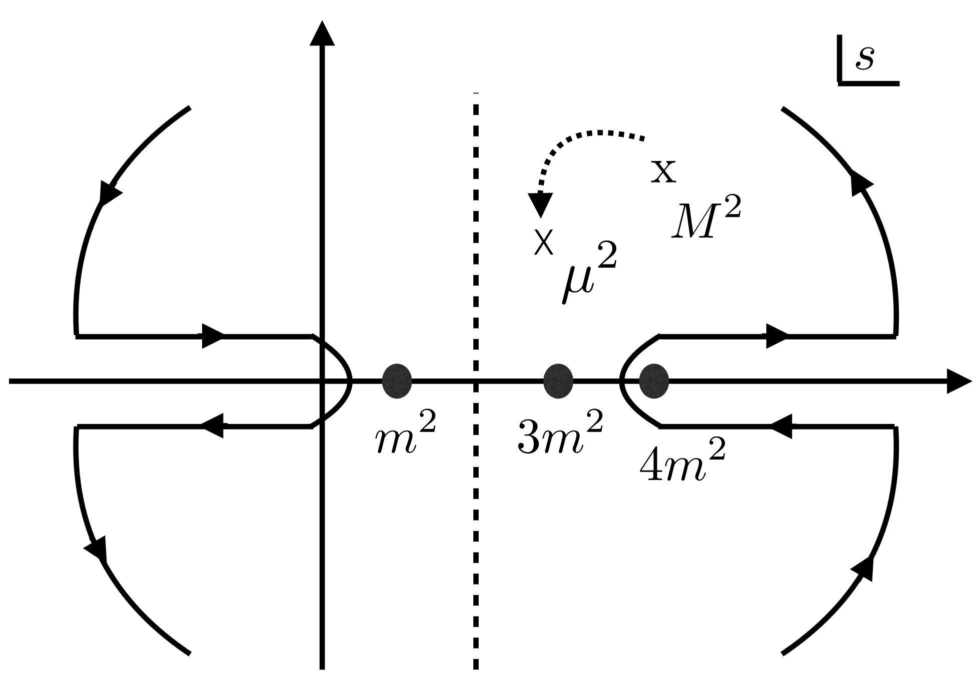
- Lower dimensional operators correct positivity bounds.

  "Running of positivity bounds"

e.g. Dimension-6 Higgs portal couplings $\tilde{d}_3 \equiv d_3 m_\varphi^2, \quad \tilde{d}_4 \equiv d_4 m_H^2$



$$\left\{ \begin{aligned} \hat{C}^{(1)}_{H^2\varphi^2} &= C^{(1)}_{H^2\varphi^2} + \frac{1}{648\pi^2\Lambda^4}\left(13(\tilde{d}_3^2 + \tilde{d}_4^2) + 20\tilde{d}_3\tilde{d}_4\right) \\ &\quad + \frac{1}{108\pi^2\Lambda^4}(\tilde{d}_3 + \tilde{d}_4)^2 \ln\frac{\mu^2}{|s|}, \\ \hat{C}^{(2)}_{H^2\varphi^2} &= C^{(2)}_{H^2\varphi^2} - \frac{5}{1296\pi^2\Lambda^4}(\tilde{d}_3 + \tilde{d}_4)^2 \\ &\quad - \frac{1}{432\pi^2\Lambda^4}(\tilde{d}_3 + \tilde{d}_4)^2 \ln\frac{\mu^2}{|s|} \end{aligned} \right.$$

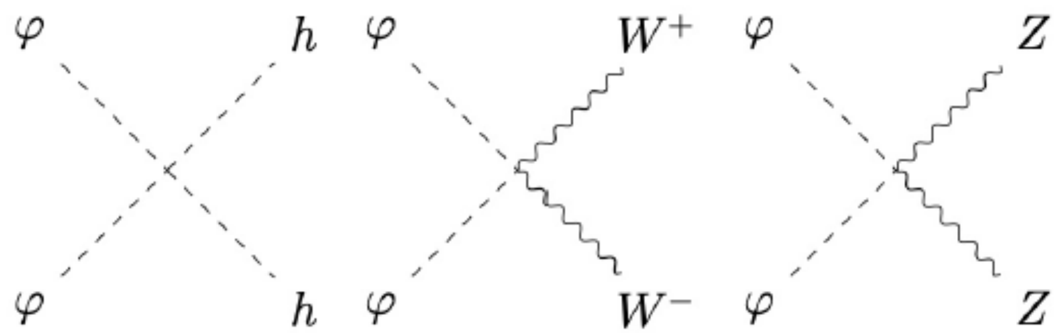


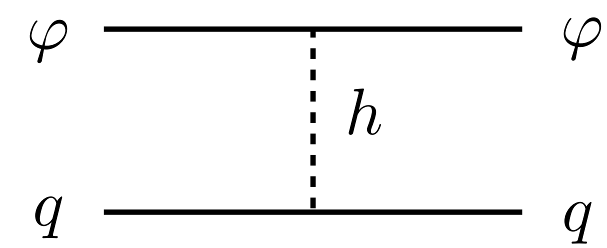
cf. X. Li, 2212.12227, similar corrections for dim-8 Higgs self-interactions

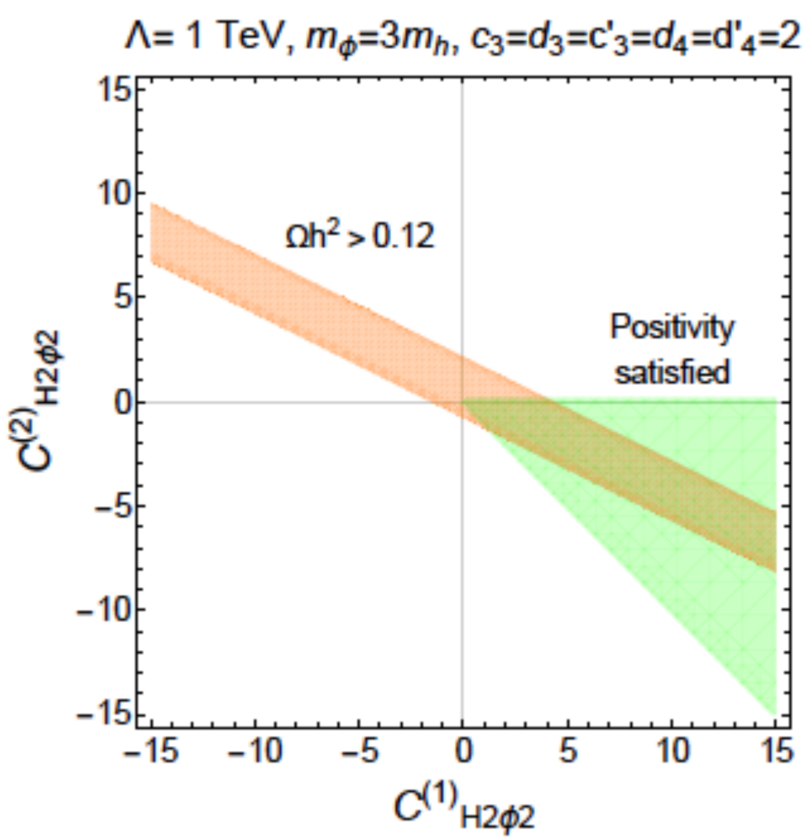
Interplay with dark matter abundance

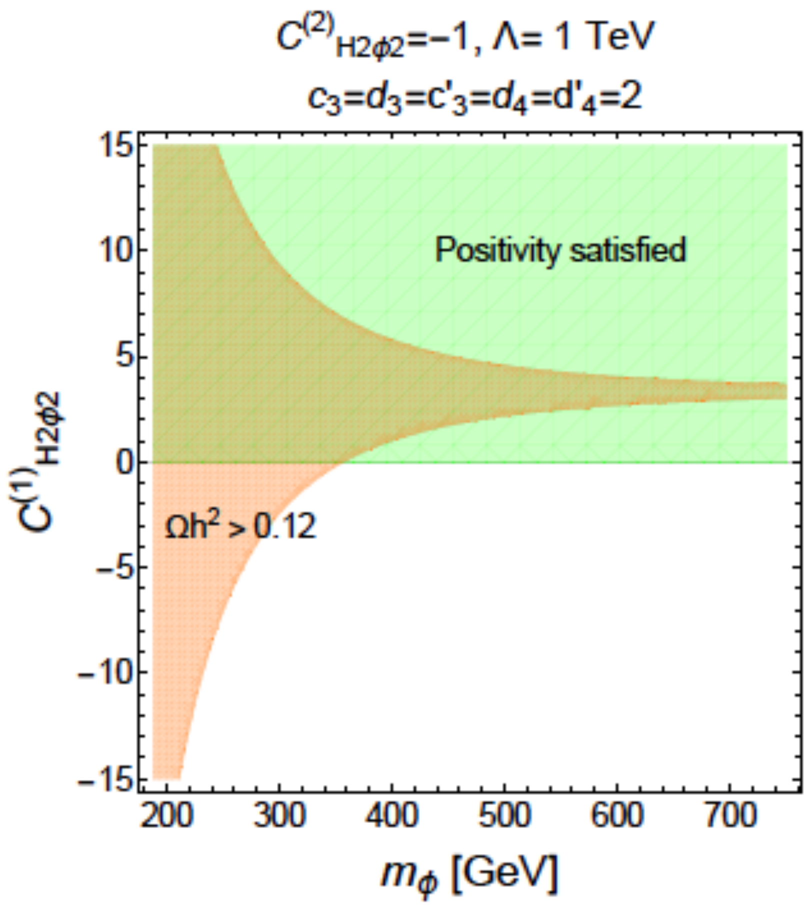
WIMP vs Positivity

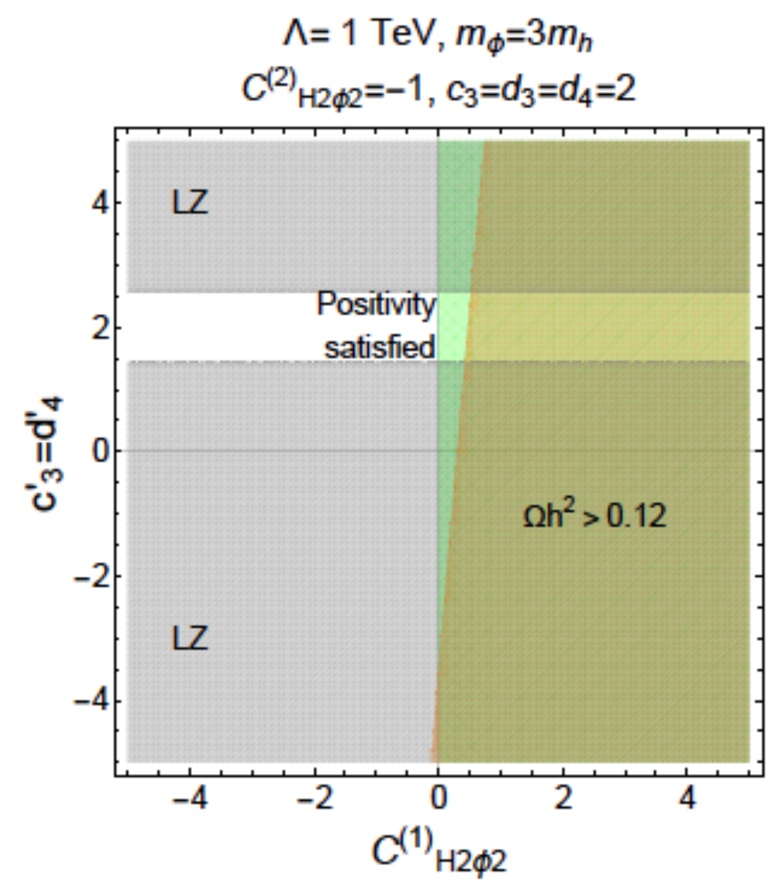
- Positivity bounds complementary to relic density & DD.









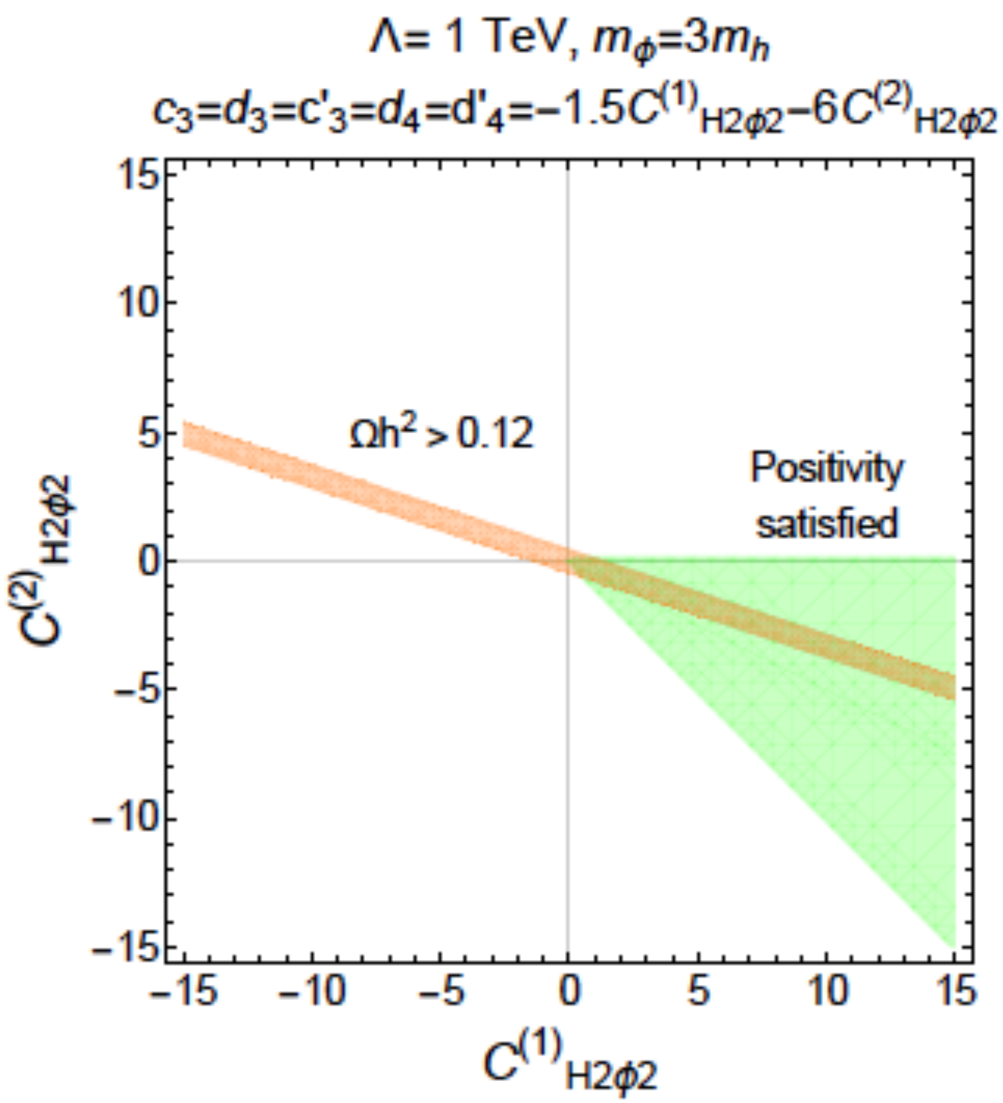


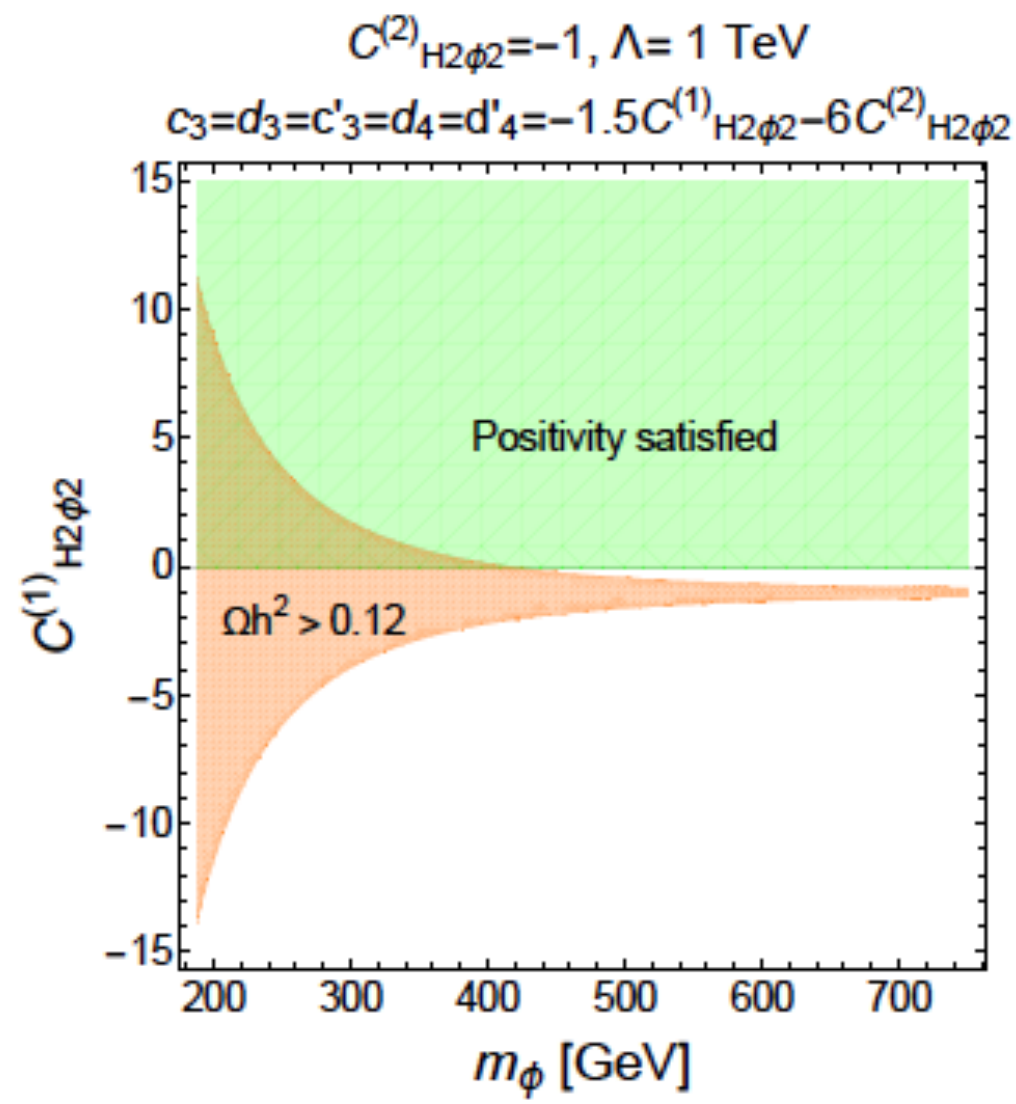
$\sqrt{(C^{(1)}_{H^4}+C^{(2)}_{H^4}+C^{(3)}_{H^4})C_{\varphi^4}}=0.1$ imposed for self-interactions.

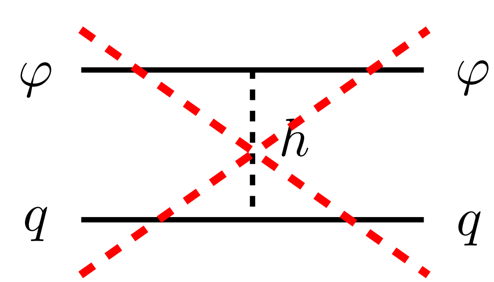
Fermion channels & direct detection relevant for $c_3\neq c_3',\ d_4\neq d_4'$.

Graviton as UV completion

- Parameter space is more restricted in the graviton case.







$$c_3 = c_3', \ d_4 = d_4'$$

No fermion channels.

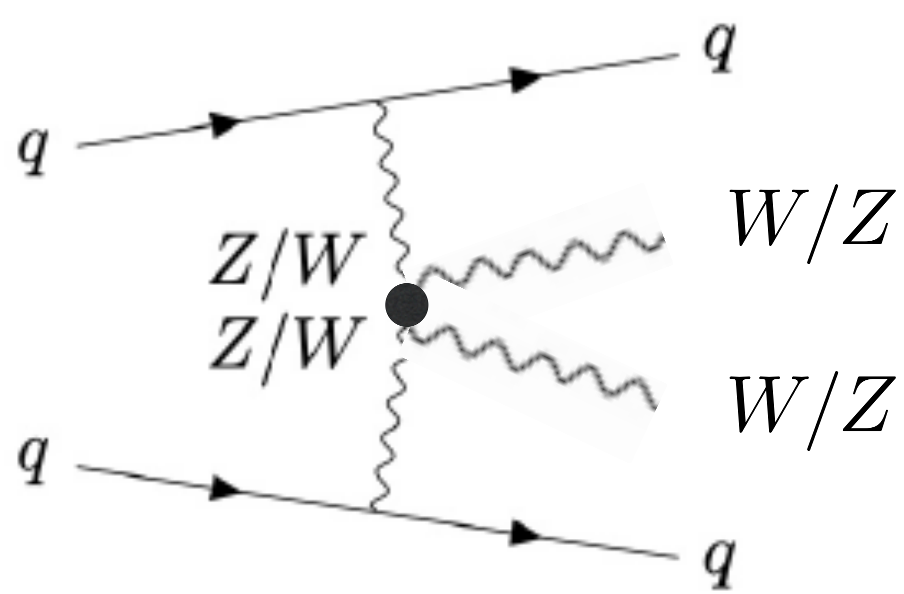
Indirect signals from WW/ZZ/hh.

No direct detection at tree level.

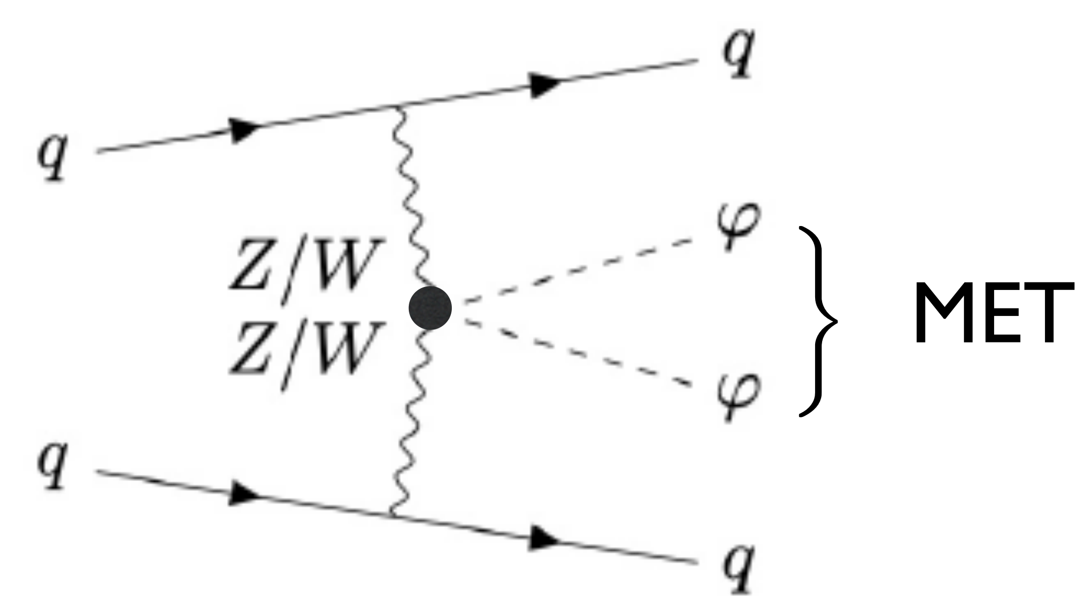
LHC limits on dim-8

- Dibosons & MET with two jets are searchable at LHC.

Dim-8 Higgs-self interactions



Dim-8 Higgs-portals



WW (or WZ): (95% C.L.)

$$C^{(2)}_{H^4}/\Lambda^4 = [-7.7, 7.7]\,\mathrm{TeV}^{-4}$$

$$C^{(3)}_{H^4}/\Lambda^4 = [-21.6, 21.8]\,\mathrm{TeV}^{-4}$$

WW, WZ, ZZ + two jets:

$$C^{(2)}_{H^4}/\Lambda^4 = [-2.7, 2.7]\,\mathrm{TeV}^{-4}$$

$$C^{(3)}_{H^4}/\Lambda^4 = [-3.4, 3.4]\,\mathrm{TeV}^{-4}$$

MET + two jets: $|C^{(1)}_{H^2\varphi^2}|/\Lambda^4 = |C^{(2)}_{H^2\varphi^2}|/\Lambda^4 < 32, \quad m_\varphi = 375\,\mathrm{GeV}.$

Freeze-in in EFTs

- Assume Scalar dark matter is never in thermal equilibrium.

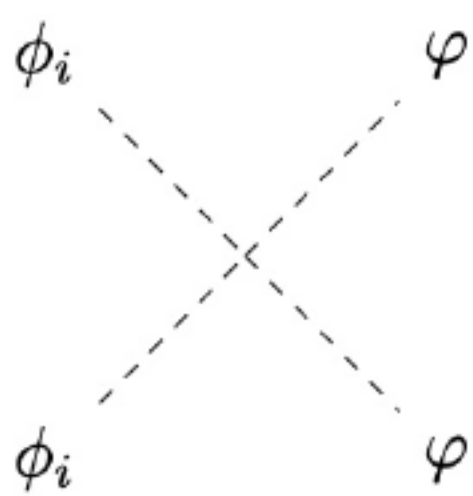
"Freeze-in DM" Dim-4: $c_3 m_H^2 m_\varphi^2/\Lambda^4 \lesssim 10^{-7}$

Dim-6: $d_3 m_\varphi^2/\Lambda^4, d_4 m_H^2/\Lambda^4 \lesssim 1/(T_{\rm reh}^3 M_{Pl})^{1/2}$

Dim-8: $C^{(1,2)}_{H^2\varphi^2}/\Lambda^4 \lesssim 1/(T_{\rm reh}^7 M_{Pl})^{1/2}$

$c_3, d_4, d_3, C^{(1,2)}_{H^2\varphi^2} = \mathcal{O}(1)$ Maximum temperature: $T_{\rm reh} \lesssim \left(\frac{\Lambda^8}{M_{Pl}}\right)^{1/7}$

- Scalar dark matter is produced by Higgs-Higgs scattering with Dim-8 Higgs portal.



Scattering amplitude square: $s, t \gg m_\varphi^2, m_H^2,$

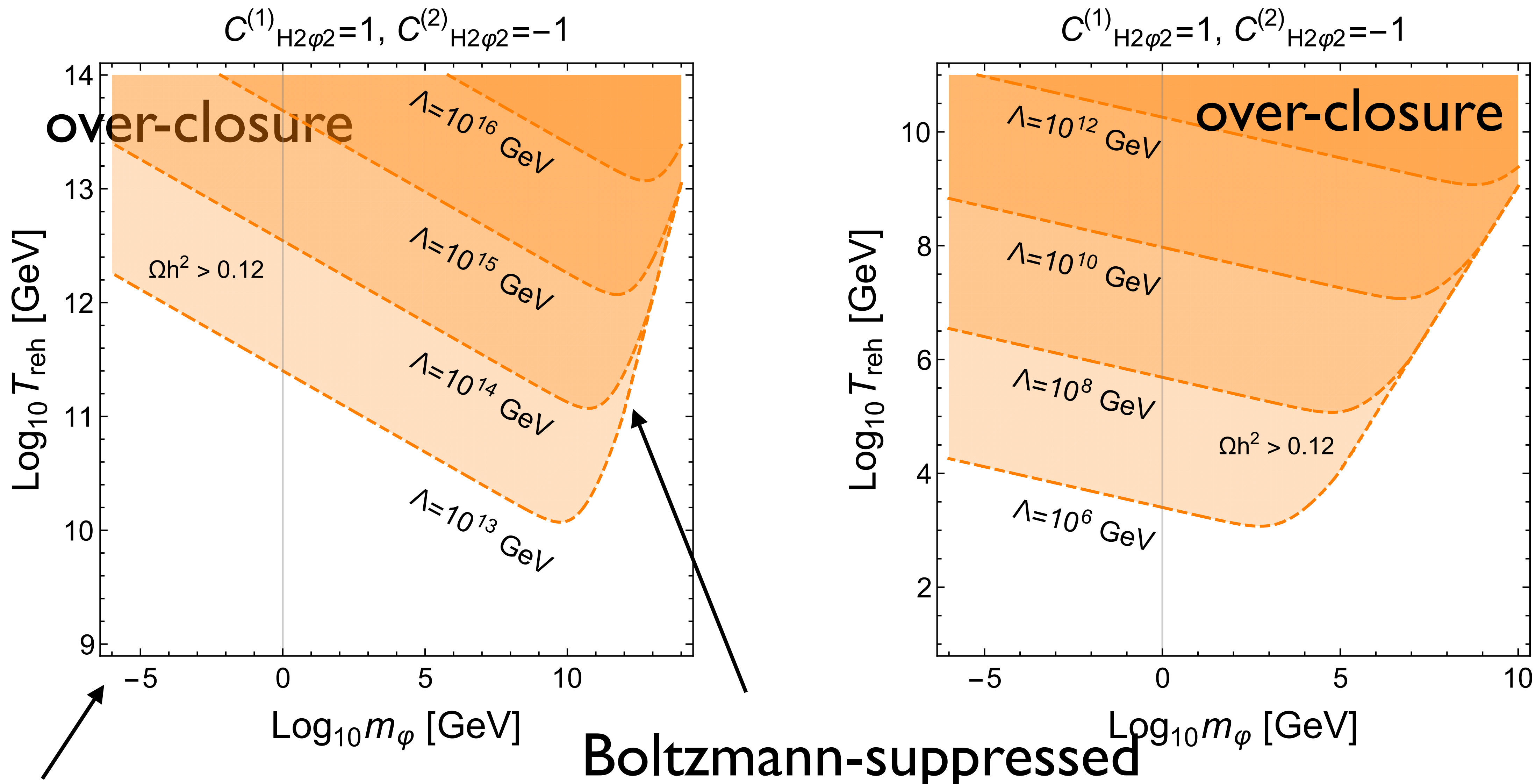
$$|\mathcal{M}_{\phi_i\phi_i\to\varphi\varphi}|^2 \simeq \frac{1}{576\Lambda^8}\left[3(C^{(1)}_{H^2\varphi^2} + 2C^{(2)}_{H^2\varphi^2})s^2 + 6C^{(1)}_{H^2\varphi^2}\, t(t+s)\right]^2$$

Dark matter produced until T=m$_H$: $T_{\rm reh} \gg m_\varphi, m_H,$

$$Y_\varphi(m_H) \simeq \frac{g_\phi T_{\rm reh}^7}{\sqrt{g_*(T_{\rm reh})}}\,\frac{2\sqrt{\frac{2}{5}}\pi^6 M_{Pl}\left(7(C^{(1)}_{H^2\varphi^2})^2 + 40C^{(1)}_{H^2\varphi^2}C^{(2)}_{H^2\varphi^2} + 60(C^{(2)}_{H^2\varphi^2})^2\right)}{138915\Lambda^8}$$

FIMP relic density

- Scalar dark matter mass vs reheating temperature



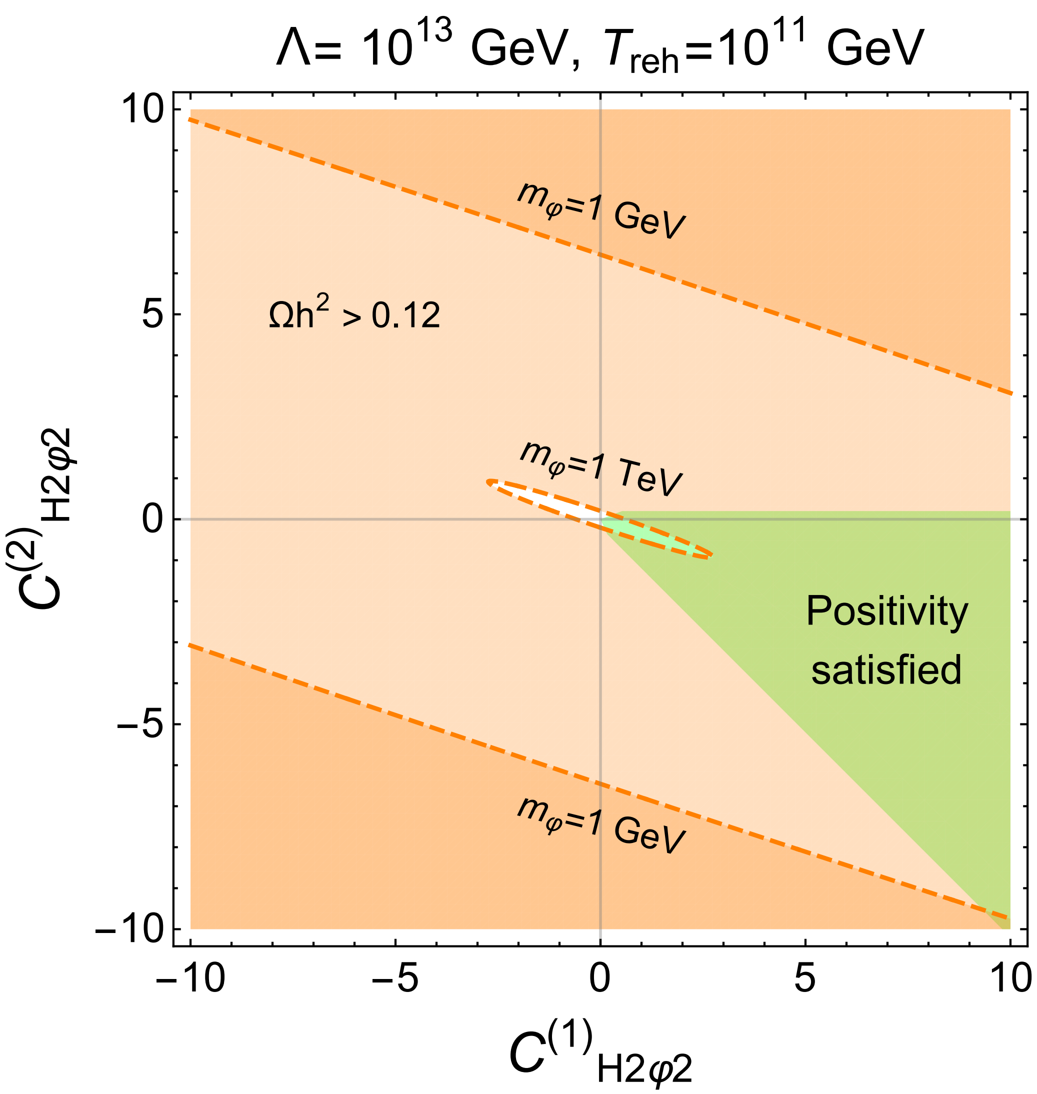
$m_\varphi > 1\,\mathrm{keV}$

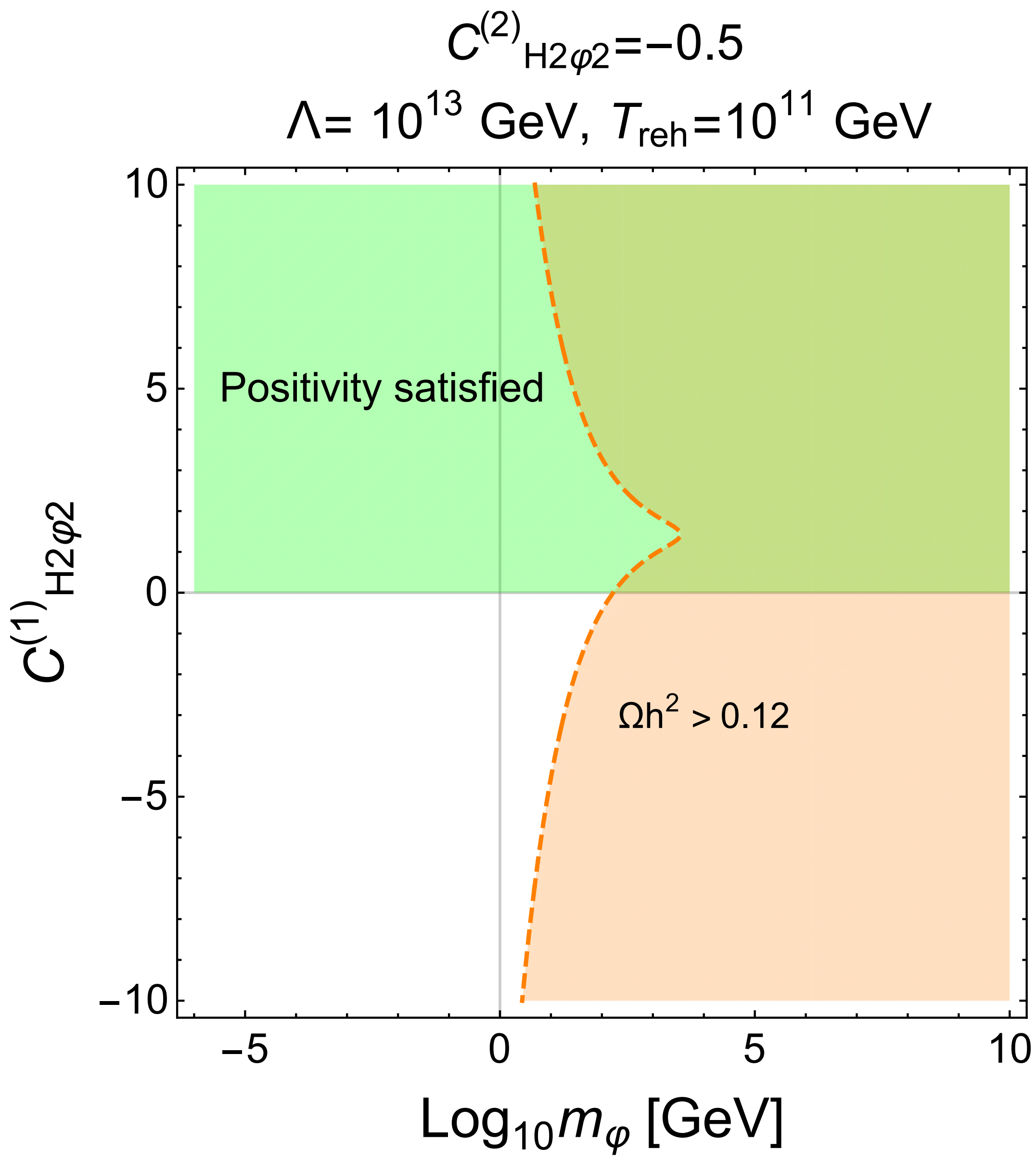
safe from Lyman- forest bound

$$\Omega_\varphi h^2 \simeq 0.12 \left(\frac{m_\varphi}{1\ \mathrm{TeV}}\right)\left(\frac{T_{\rm reh}}{10^{11}\ \mathrm{GeV}}\right)^7 \left(\frac{10^{13}\ \mathrm{GeV}}{\Lambda}\right)^8 \times \left(\frac{14}{5}(C^{(1)}_{H^2\varphi^2})^2 + 16 C^{(1)}_{H^2\varphi^2} C^{(2)}_{H^2\varphi^2} + 24 (C^{(2)}_{H^2\varphi^2})^2\right)$$

High reheating

- High T_{reh} favors relatively large DM masses, 1GeV-1TeV for $T_{reh}=10^{11}$ GeV; Positivity rules out part of parameter space.

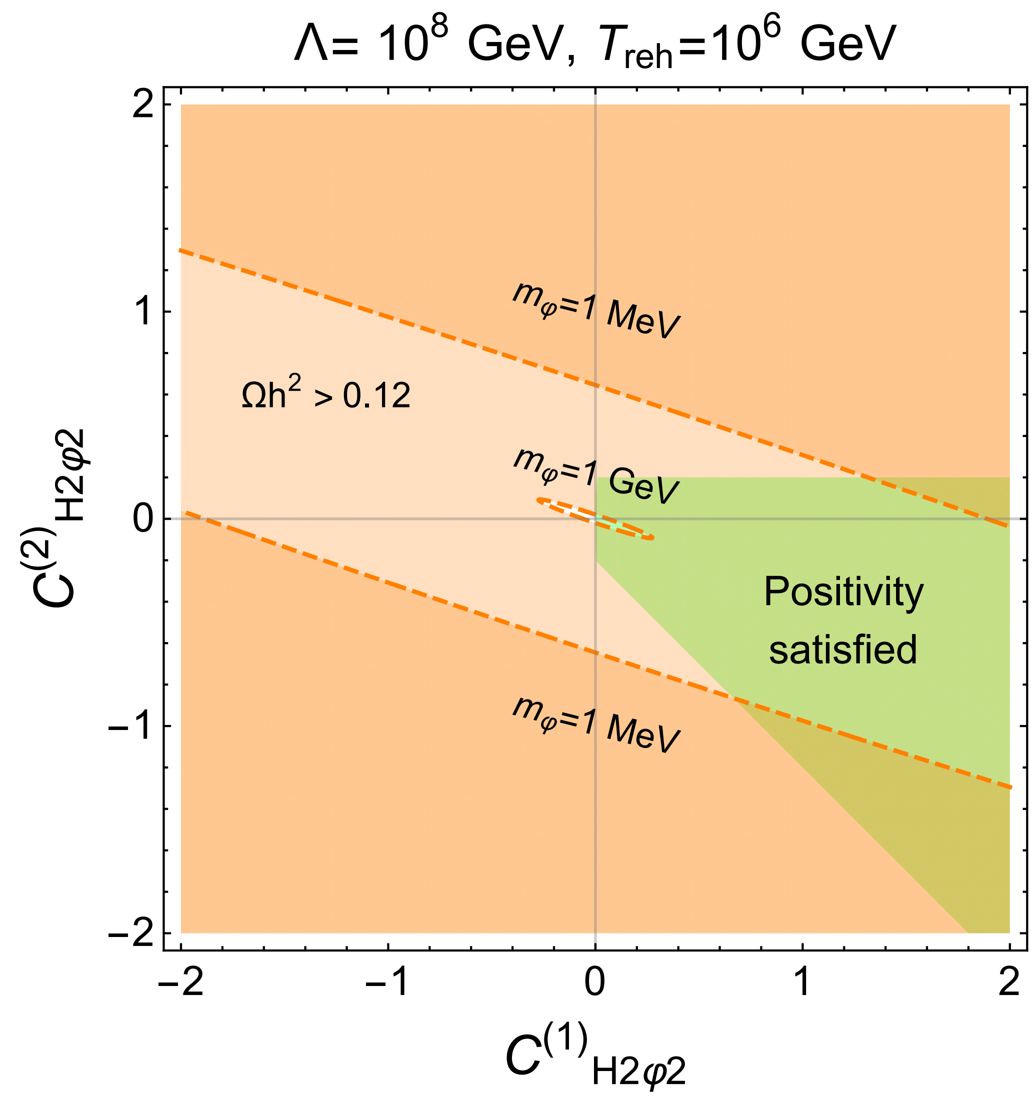


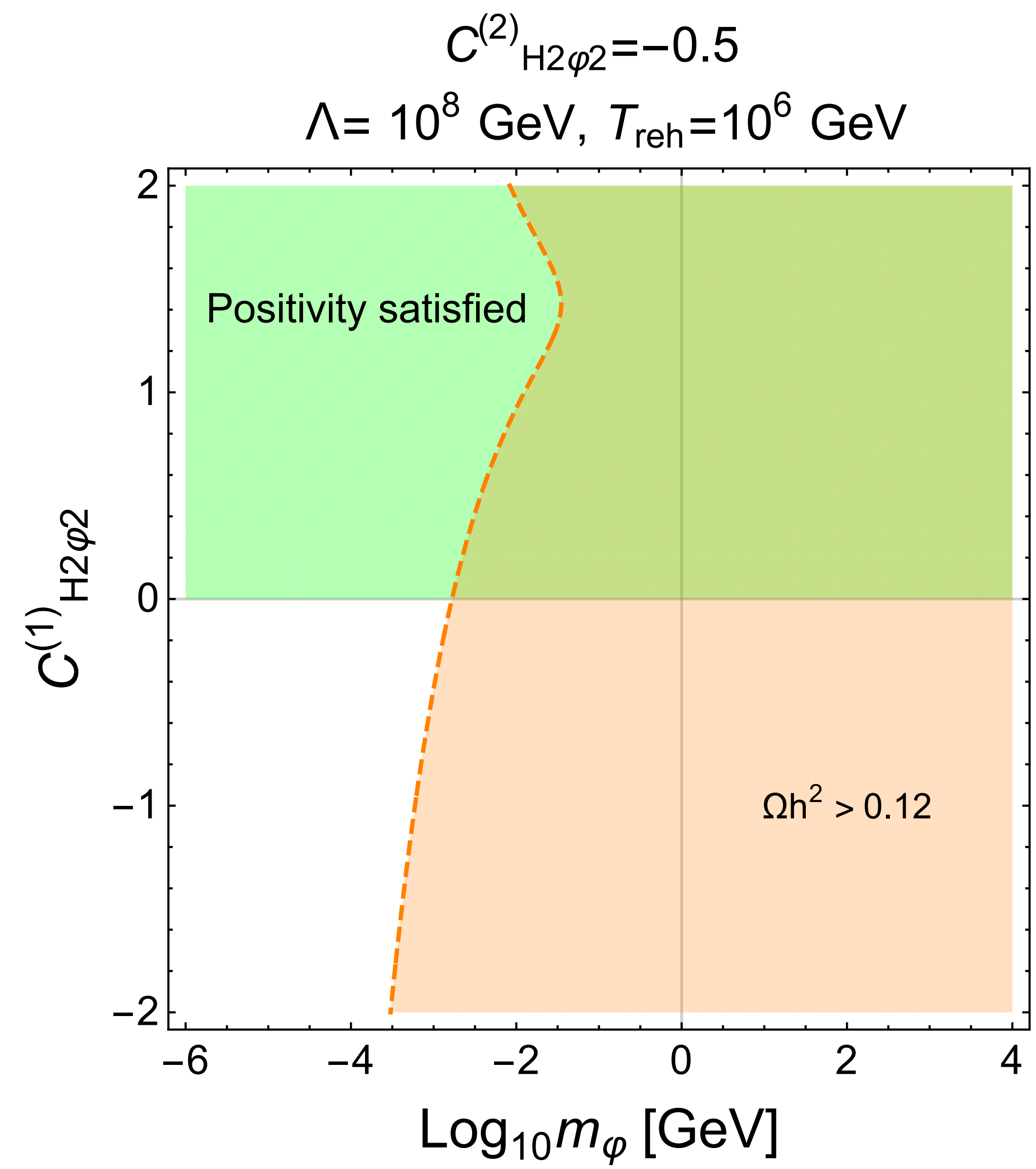


$\sqrt{(C^{(1)}_{H^4}+C^{(2)}_{H^4}+C^{(3)}_{H^4})C_{\varphi^4}} = 0.1$ imposed for self-interactions.

Low reheating

- Low T_{reh} favors small DM masses, e.g. 1MeV-1GeV for $T_{reh}=10^6$ GeV; Positivity rules out part of parameter space.





$\sqrt{(C_{H^4}^{(1)}+C_{H^4}^{(2)}+C_{H^4}^{(3)})C_{\varphi^4}}=0.1$ imposed for self-interactions.

Conclusions

- Positivity bounds lead to interesting hints through higher dimensional operators beyond the SM.

- Positivity bounds constrain dimension-8 Higgs-portal interactions for WIMP dark matter, being complementary to relic density, direct/indirect detection and collider bounds.

- When dim-4 & dim-6 operators for Higgs-portal can be suppressed by mass squares as for graviton-like resonances, dim-8 derivative operators are most important for freeze-in, being bounded by dark matter production & positivity bounds.