Axion Dark Matter from Inflation

Bingrong Yu (Cornell U. & Korea U.)

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To appear soon

in collaboration with **Ameen Ismail** and **Seung J. Lee**

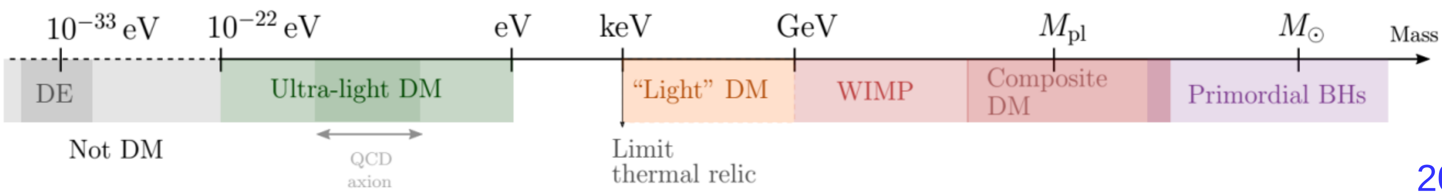
Outline

- Motivation
- Framework
- Phenomenology
- Conclusion

1 Motivation

Ultralight dark matter

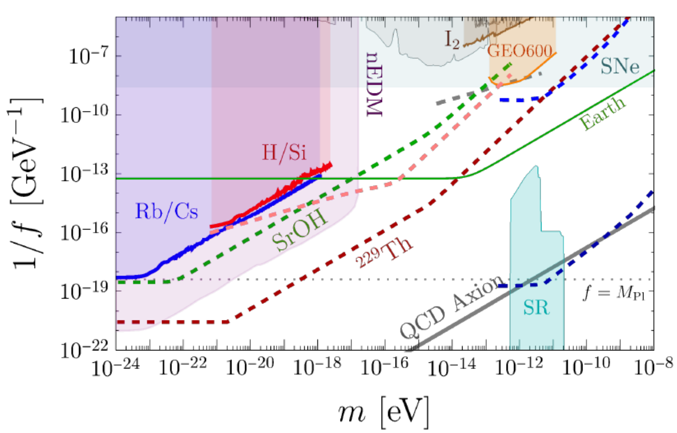
- DM models span many orders of magnitude in mass



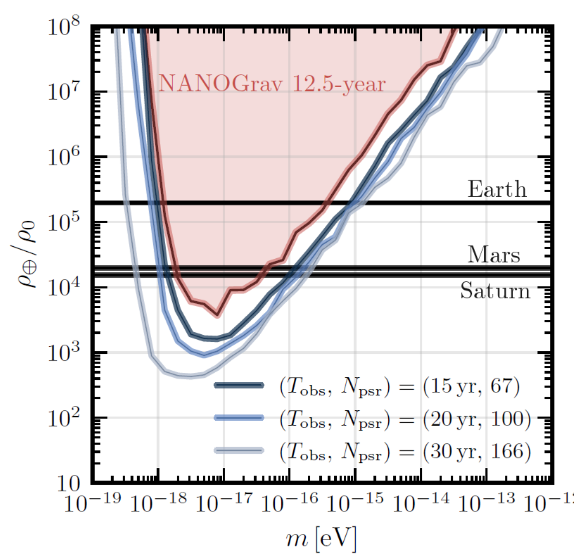
2005.03254

- Ultralight (wave) DM: 10^{-22} eV $< m <$ eV

Future atomic-/astro-physics experiments: $m < 10^{-10}$eV



Quantum sensor, 2205.12988



PTA, 2312.12225

Misalignment mechanism

➢ Axion: well-motivated ultralight DM (protected by shift symmetry)

$$\ddot{\eta} + 3H\dot{\eta} + m_\eta^2 \eta^2 = 0 \qquad V(\eta) = \Lambda_\eta^4 \left[1 - \cos\left(\frac{\eta}{f_\eta}\right)\right] \Rightarrow m_\eta = \Lambda_\eta^2 / f_\eta$$

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- Misalignment mechanism: axion starts to oscillate when $H \sim m_\eta$, and behaves as matter after then, $\rho_\eta \sim a^{-3}$

$$\left(\frac{\Omega_\eta h^2}{0.12}\right)_{\text{ALP, misalignment}} \sim \left(\frac{m_\eta}{10^{-10}\ \text{eV}}\right)^{1/2}\left(\frac{f_\eta}{10^{14}\ \text{GeV}}\right)^2$$

$$\left(\frac{\Omega_\eta h^2}{0.12}\right)_{\text{QCD axion, misalignment}} \sim \left(\frac{10^{-6}\ \text{eV}}{m_\eta}\right)^{3/2}$$

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  For QCD axion DM: $10^{-6}\text{eV} < m_\eta < 10^{-4}$ eV

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Any new windows?
Need new mechanism beyond misalignment!

2 Framework

Kinetic coupling to inflaton

- We assume the PQ symmetry has broken during inflation, $f_\eta > H_{\rm inf}$. Axion is effectively massless during inflation if $m_\eta / F < H_{\rm inf}$

$$\mathcal{S} = \int \mathrm{d}^4 x \sqrt{-g} \left[\frac{M_{\rm Pl}^2}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) - \frac{1}{2} F^2(\phi) g^{\mu\nu} \partial_\mu \eta \partial_\nu \eta \right]$$

ϕ: inflaton

η: axion

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- Flat FLRW metric

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t) \delta_{ij} \mathrm{d}x^i \mathrm{d}x^j = a^2(\tau) \left(-\mathrm{d}\tau^2 + \delta_{ij} \mathrm{d}x^i \mathrm{d}x^j \right)$$

conformal time: $\mathrm{d}\tau \equiv \frac{\mathrm{d}t}{a}$

de Sitter background: $a = -\frac{1}{H\tau}$

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- Abundance of axion is sufficiently produced through $F(\phi)$ *during inflation*

Equation of motion

$$f\left(\tau, \mathbf{k}\right) = \int \frac{\mathrm{d}^3\mathbf{k}}{\left(2\pi\right)^3} \left[f_k(\tau)\hat{a}_{\mathbf{k}} e^{\mathrm{i}\mathbf{k}\cdot\mathbf{x}} + f_k^*(\tau)\hat{a}_{\mathbf{k}}^\dagger e^{-\mathrm{i}\mathbf{k}\cdot\mathbf{x}}\right]$$

- From the action, one obtains the EOM of axion:

$$f \equiv aF\eta$$

$$f_k'' + \left(k^2 - \frac{a''}{a} - \frac{F''}{F}\right) f_k = 0$$

$$f' \equiv \mathrm{d}f/\mathrm{d}\tau$$

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2004.10743 (for dark photon DM)

- Assume the kinetic coupling: $F(\phi) = (a/a_{\mathrm{e}})^n$ ⟸ $F(\phi) \sim e^{-n\phi^2/M_{\mathrm{Pl}}^2}$

slow roll

$$\partial_x^2 f_k + \left[1 - \frac{2 + n(n+1)}{x^2}\right] f_k = 0$$

$x \equiv -k\tau = k/(aH_{\mathrm{inf}})$

$x < 1$: superhorizon

$x > 1$: subhorizon

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- Solution of axion field during inflation:

$$\boxed{f_k(\tau) = \frac{\sqrt{\pi}}{2} \sqrt{-\tau} H_\nu^{(1)}(-k\tau)}$$

$$\nu \equiv \tfrac{1}{2}\sqrt{4n^2 + 4n + 9}$$

Axion produced during inflation

- Energy density: $T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta \mathcal{S}_\eta}{\delta g^{\mu\nu}}$

$$x \equiv -k\tau$$

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$$\rho_\eta = T_{00} = \frac{1}{2a^4}\left[\left(f' + \frac{1+n}{\tau}f\right)^2 + (\partial_i f)^2\right]$$

$$\langle \rho_\eta \rangle \equiv \langle 0|\rho_\eta|0\rangle = \frac{H_{\rm inf}^4}{16\pi}\int_0^\infty \mathrm{d}x\, x^2 \left[\left|xH^{(1)}_{\nu-1}(x) + \left(n+\frac{3}{2}-\nu\right)H^{(1)}_\nu(x)\right|^2 + x^2\left|H^{(1)}_\nu(x)\right|^2\right]$$

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- The spectrum is red and peaked at super-horizon mode if $\nu > 3/2$ (i.e., $n > 0$ or $n < -1$):

$$\langle \rho_\eta \rangle \approx \frac{H_{\rm inf}^4}{16\pi^3} 2^{2\nu}\left(n + \frac{3}{2} - \nu\right)^2 \Gamma^2(\nu) \int_{-k_{\rm min}\tau}^{\rm UV} \frac{{\rm d}x}{x} x^{3-2\nu}$$

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IR cutoff set by the initial horizon: $k_{\rm min} \propto -1/\tau_{\rm i} \propto a_{\rm i} H_{\rm inf}$

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3 Phenomenology

Relic abundance

- Axion is relativistic when produced, with the peaked mometum at:

$$q_{\rm e} = k_{\rm min}/a_{\rm e} \propto e^{-N} H_{\rm inf}$$

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$$\langle \rho_\eta \rangle_{t_0} = \langle \rho_\eta \rangle_{t_{\rm e}} \left(\frac{a_{\rm e}}{a_{\rm NR}} \right)^4 \left(\frac{a_{\rm NR}}{a_0} \right)^3 \qquad \nu \equiv \tfrac{1}{2}\sqrt{4n^2+4n+9} > 3/2$$

$$\Omega_\eta h^2 = \frac{\langle \rho_\eta \rangle_{t_0}}{\rho_c} h^2 = \frac{1}{4320\pi} \left(\frac{90}{\pi^2} \right)^{1/4} g_{*\rm NR}^{1/4} g_{*\rm eq}^{3/4} g_{*\rm reh}^{-1/4} \frac{T_0^3 m_\eta H_{\rm inf}^{3/2}}{M_{\rm Pl}^{7/2} H_0^2/h^2} \frac{2^{2\nu} \left(n+3/2-\nu\right)^2 \Gamma^2(\nu)}{2\nu-3} \frac{1}{(-k_{\rm min}\tau_{\rm end})^{2\nu-2}}$$

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small axion mass compensated by exponential enhancement!

Constraints

- Back-reaction constraint

$$\ddot{\phi} + 3H_{\rm inf}\dot{\phi} + \frac{\partial V}{\partial \phi} + F\frac{\partial F}{\partial \phi} g^{\mu\nu}\partial_\mu \eta \partial_\nu \eta = 0$$

axion dynamics should not affect inflaton dynamics (single-field inflation)

$$\left| F\frac{\partial F}{\partial \phi} \langle g^{\mu\nu}\partial_\mu \eta \partial_\nu \eta \rangle \right| \ll \left| 3H_{\rm inf}\dot{\phi} \right| \qquad \langle \rho_\eta \rangle \ll 3M_{\rm Pl}^2 H_{\rm inf}^2$$

$$\Rightarrow \frac{1}{-k_{\rm min}\tau_e} = e^N \ll \left[\frac{6\pi}{|n|} \frac{2\nu - 3}{2^{2\nu}\left(n + 3/2 - \nu\right)^2 \Gamma^2(\nu)} A_{\rm s}^{-1} \right]^{\frac{1}{2\nu - 3}}$$

$$\nu \equiv \frac{1}{2}\sqrt{4n^2 + 4n + 9} > 3/2$$

$$A_{\rm s} \equiv H_{\rm inf}^2 / \left(8\pi^2 \epsilon_V M_{\rm Pl}^2\right) = 2.2 \times 10^{-9}$$

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- Isocurvature bound

Longest axion wavelength produced during inflation: $k_{\rm min}^{-1} = \dfrac{1}{\sqrt{2 + n(n+1)}} \dfrac{e^N}{H_{\rm inf}} \dfrac{a_0}{a_e}$

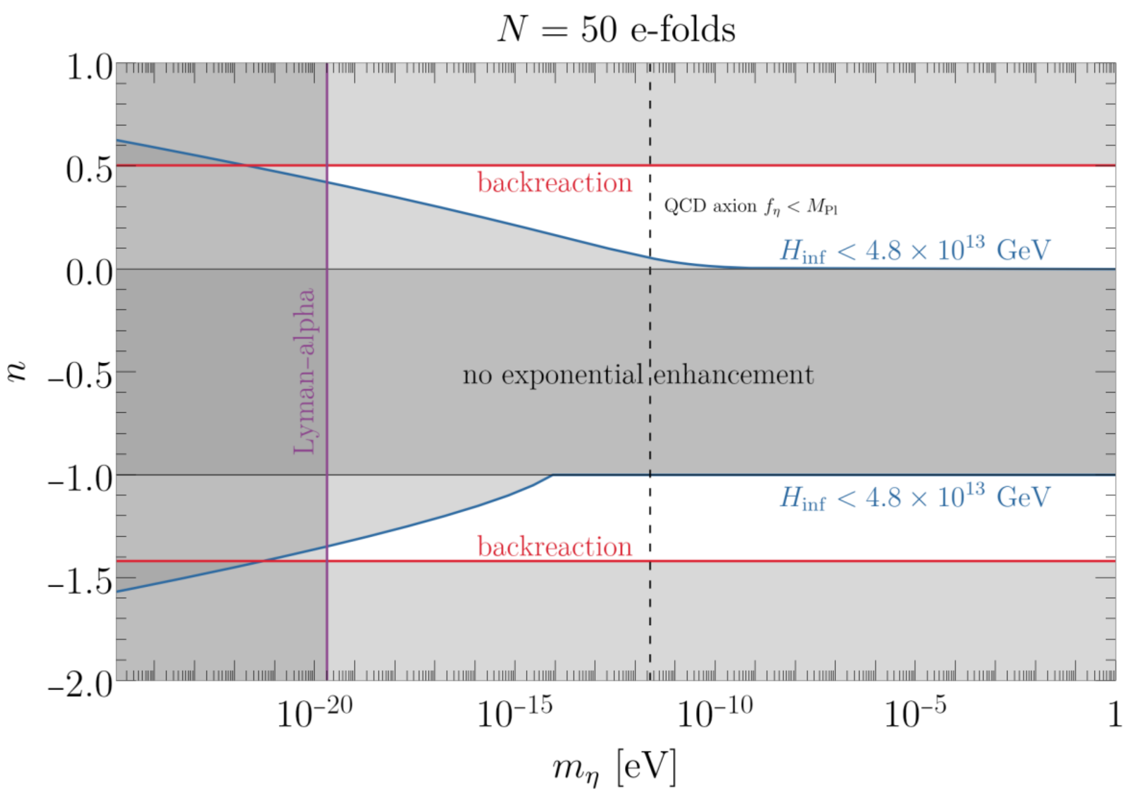
To avoid possible isocurvature bound, we require $k_{\rm min}^{-1} < k_*^{-1}$

$k_* \equiv 0.05~{\rm Mpc}^{-1}$: pivot scale measured by Planck

$$\Rightarrow N \lesssim 56 + \frac{1}{2}\log\left[2 + n\left(n+1\right)\right] + \frac{1}{2}\log\left(\frac{H_{\rm inf}}{10^{13}~{\rm GeV}}\right)$$

larger number of e-folds can be probed by future isocurvature measurement

Parameter space for DM



$$\mathcal{L}_{\rm kin} = -\frac{1}{2}F^2(\phi)g^{\mu\nu}\partial_\mu\eta\partial_\nu\eta$$

$$H_{\rm inf} = 2\pi M_{\rm pl}\sqrt{A_{\rm s}r_{\rm T}/8}$$

$$A_{\rm s} = 2.2 \times 10^{-9} \qquad r_{\rm T} < 0.036$$

$$\Rightarrow H_{\rm inf} < 4.8 \times 10^{13}\ \text{GeV}$$

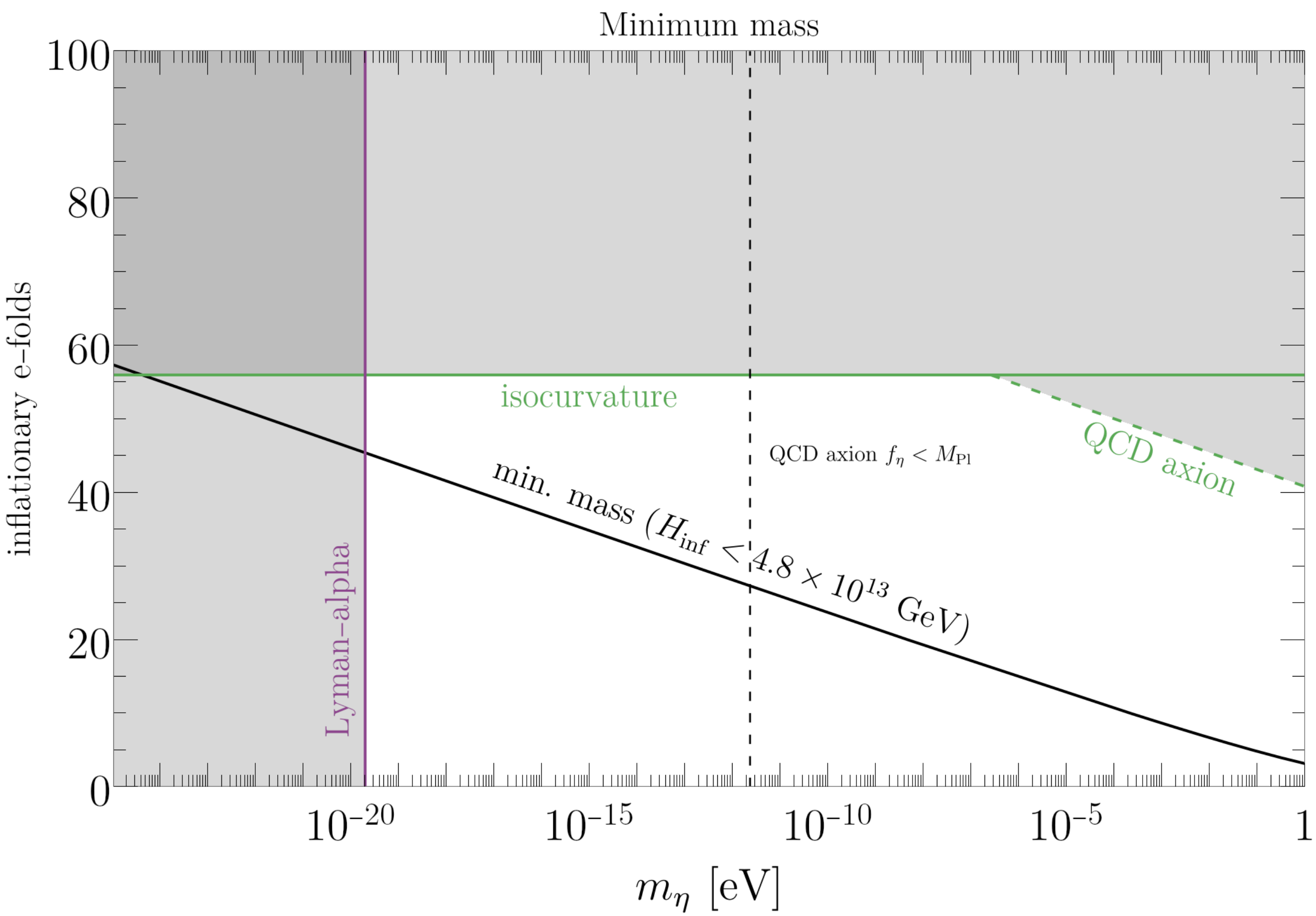
Lyman-alpha bound: 2007.12705

$$m_\eta < 2 \times 10^{-20}\ \text{eV}$$

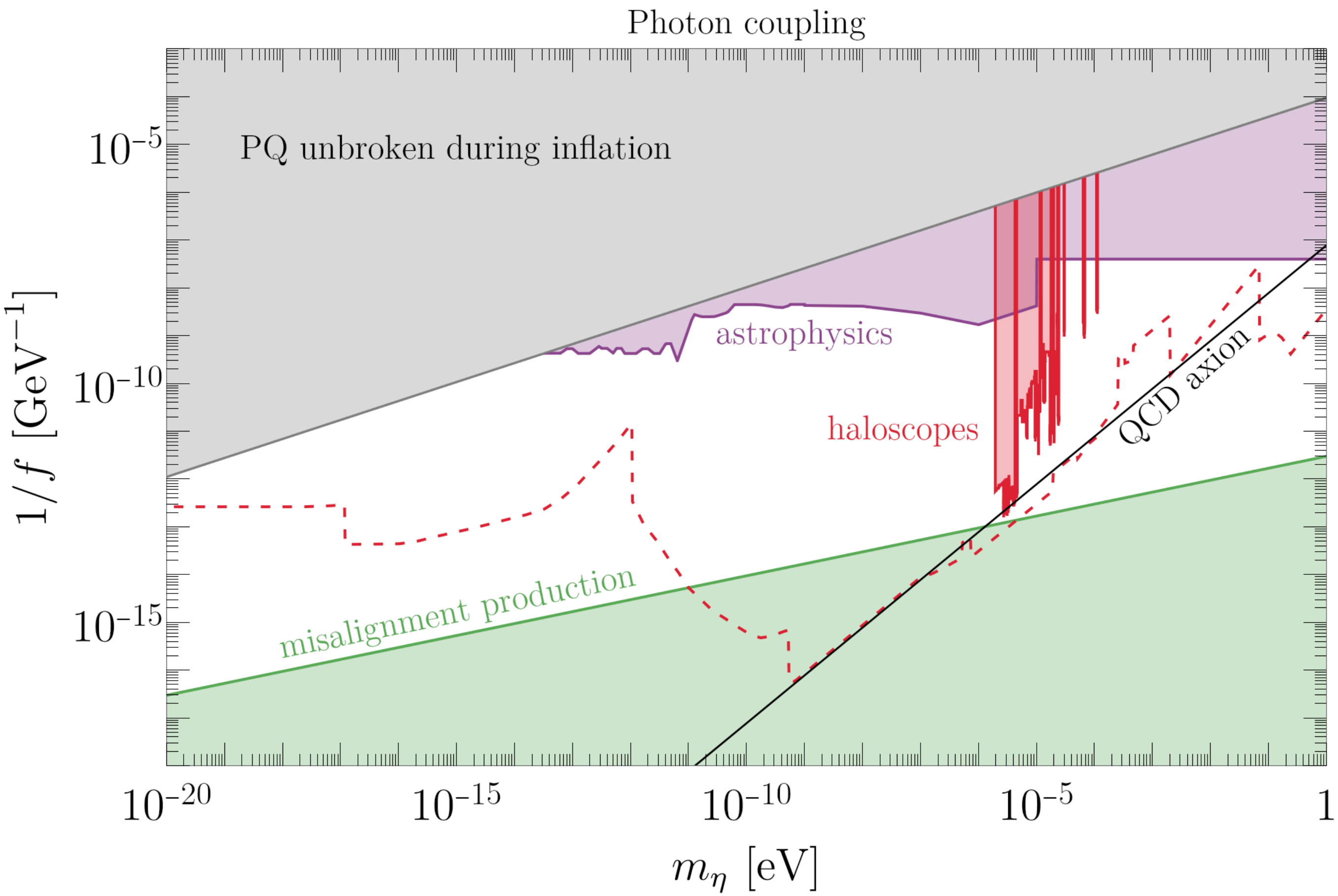
viable power index of kinetic coupling $F(\phi) = (a/a_{\rm e})^n$:

$$0 < n \lesssim 0.5 \text{ or } -1.4 \lesssim n < -1$$

Minimum mass of axion to be DM



Probed by future experiments

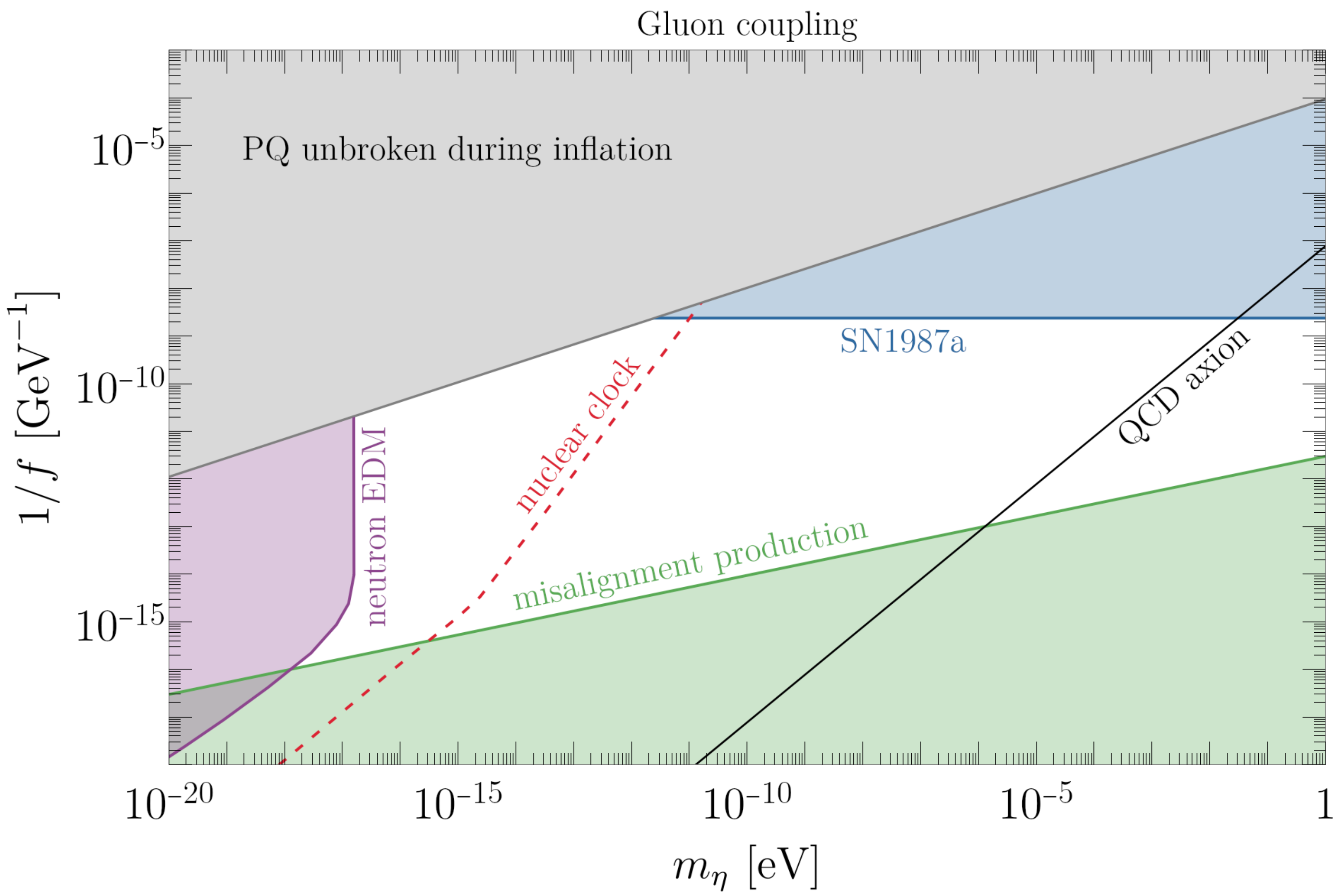


$$\mathcal{L} \supset \frac{e^2}{32\pi^2}\frac{\eta}{f}F\widetilde{F}$$

Future haloscopes (dashed line):

DANCE, SRF-m3, DMRadio, etc.

Probed by future experiments



$$\mathcal{L} \supset \frac{g_s^2}{32\pi^2}\frac{\eta}{f}G\widetilde{G}$$

Future nuclear clock (dashed line):

SrOH, Thorium, etc.

4 Conclusion

Comparison with misalignment

Axion dark matter	Inflationary fluctuation	Misalignment
Production mechanism	kinetic coupling to inflaton	oscillation due to Hubble friction
Production era	during inflation	much later, when $H \sim m_\eta$
Kinematics	relativistic when produced	NR when produced
Power spectrum	red spectrum, peaked at super-horizon scale	nearly scale-invariant spectrum
Relic abundance	insensitive to breaking scale $\Omega_\eta \propto m_\eta H_{\text{inf}}^{3/2}$	depend on breaking scale $\Omega_\eta \propto m_\eta^{1/2} f_\eta^2$
Parameter space (ALP DM)	for sub-eV axion, $f_\eta > H_{\text{inf}} > 10^4 \text{GeV}$	for sub-eV axion, $f_\eta > 10^{12}$ GeV
Parameter space (QCD axion DM)	$10^{-6}\text{eV} < m_\eta < \text{eV}$	$m_\eta \sim 10^{-6}\text{eV}$

Thank you!

Q&A

Backup slides

Origin of kinetic coupling

$$\mathcal{L}_{\text{kin}} = -\frac{1}{2} F^2(\phi) g^{\mu\nu} \partial_\mu \eta \partial_\nu \eta$$

- Realization of kinetic coupling: $F(\phi) = (a/a_{\text{e}})^n$

↑ slow-roll approximation

$$F(\phi) = \exp\left[-\frac{n}{M_{\text{Pl}}^2} \int_{\phi_{\text{e}}}^{\phi} \mathrm{d}\phi' \frac{V(\phi')}{V_\phi(\phi')}\right]$$

- Slow-roll with quadratic potential: $V(\phi) = m_\phi^2 \phi^2 / 2$

$$\begin{aligned} F(\phi) &= \exp\left[-\frac{n}{4M_{\text{Pl}}^2}\left(\phi^2 - \phi_{\text{e}}^2\right)\right] \\ &= 1 - \frac{n}{4M_{\text{Pl}}^2}\left(\phi^2 - \phi_{\text{e}}^2\right) + \frac{n^2}{32M_{\text{Pl}}^4}\left(\phi^2 - \phi_{\text{e}}^2\right)^2 + \cdots \end{aligned}$$

resummation of effective operators coupled to inflaton

Kinetic coupling from effective operator

- Exponential enhancement could also be realized from effecive operator

$$F(\phi) = 1 + \frac{C_5}{M_{\rm Pl}}(\phi - \phi_{\rm e}) \qquad |C_5\phi| < M_{\rm Pl} \qquad V(\phi) = m_\phi^2\phi^2/2$$

- EOM of axion during inflation depends only on F''/F

$$f_k'' + \left(k^2 - \frac{a''}{a} - \frac{F''}{F}\right) f_k = 0 \qquad a''/a = 2/\tau^2$$

- For $F = (a/a_{\rm e})^n$: $\quad F''/F = n(n+1)/\tau^2$

  For $F = 1 + C_5\,(\phi - \phi_{\rm e})\,/M_{\rm Pl}$: $\quad \dfrac{F''}{F} = -\sqrt{\dfrac{2}{3}} C_5 \dfrac{m_\phi}{H_{\rm inf}} \dfrac{1}{\tau^2}$

The effective Wilson coefficient $C_5 m_\phi / H_{\rm inf}$ plays the role of n

Similar conclusion valid for higher-ordered effective operators