

Genesis of the inflationary universe with a transient bouncing/cyclic era

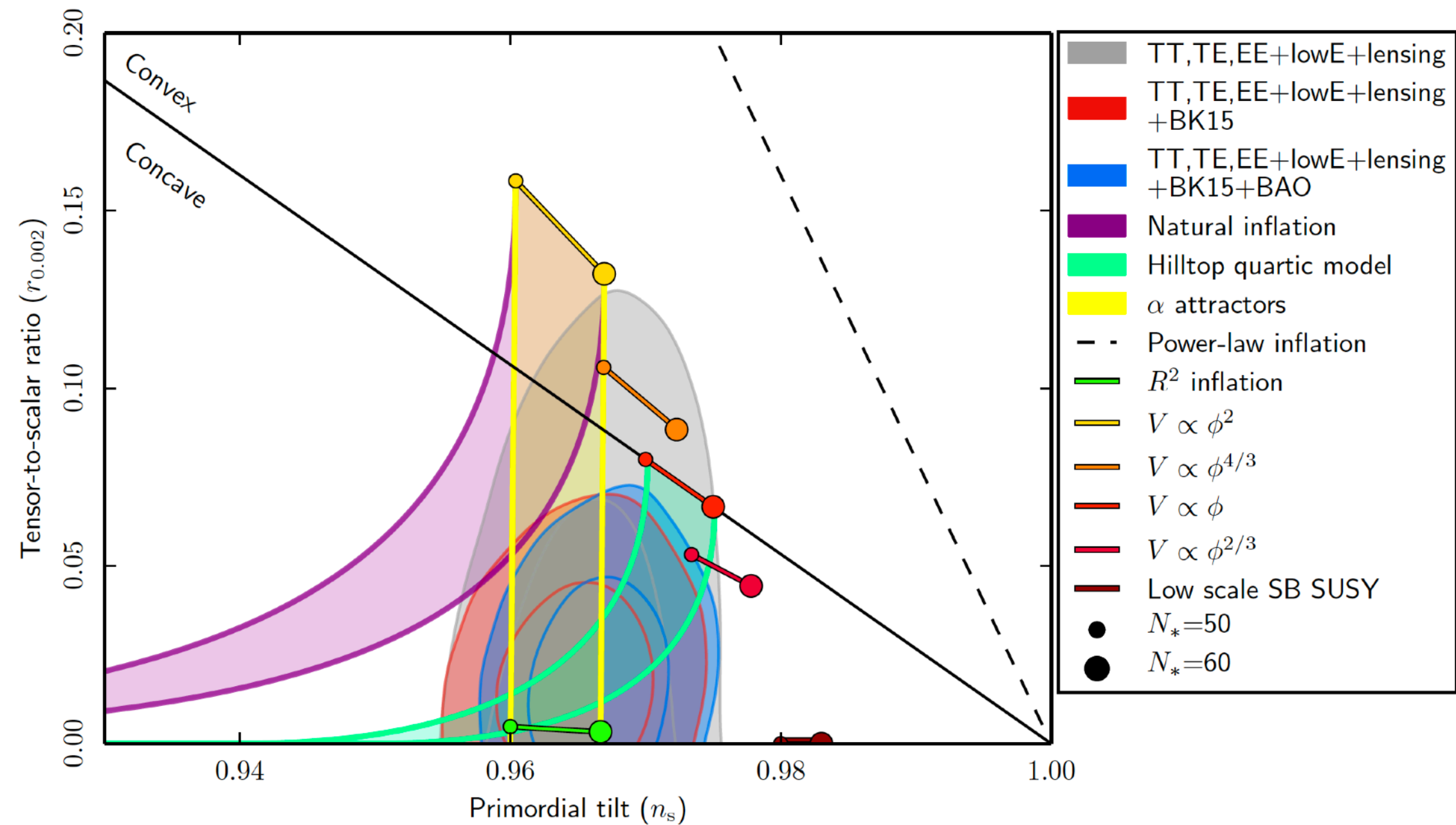


Takahiro Terada
(PTC, CTPU, IBS)

Hiroki Matsui, Fuminobu Takahashi, and Takahiro Terada, PLB 795 (2019) 152, 1904.12312

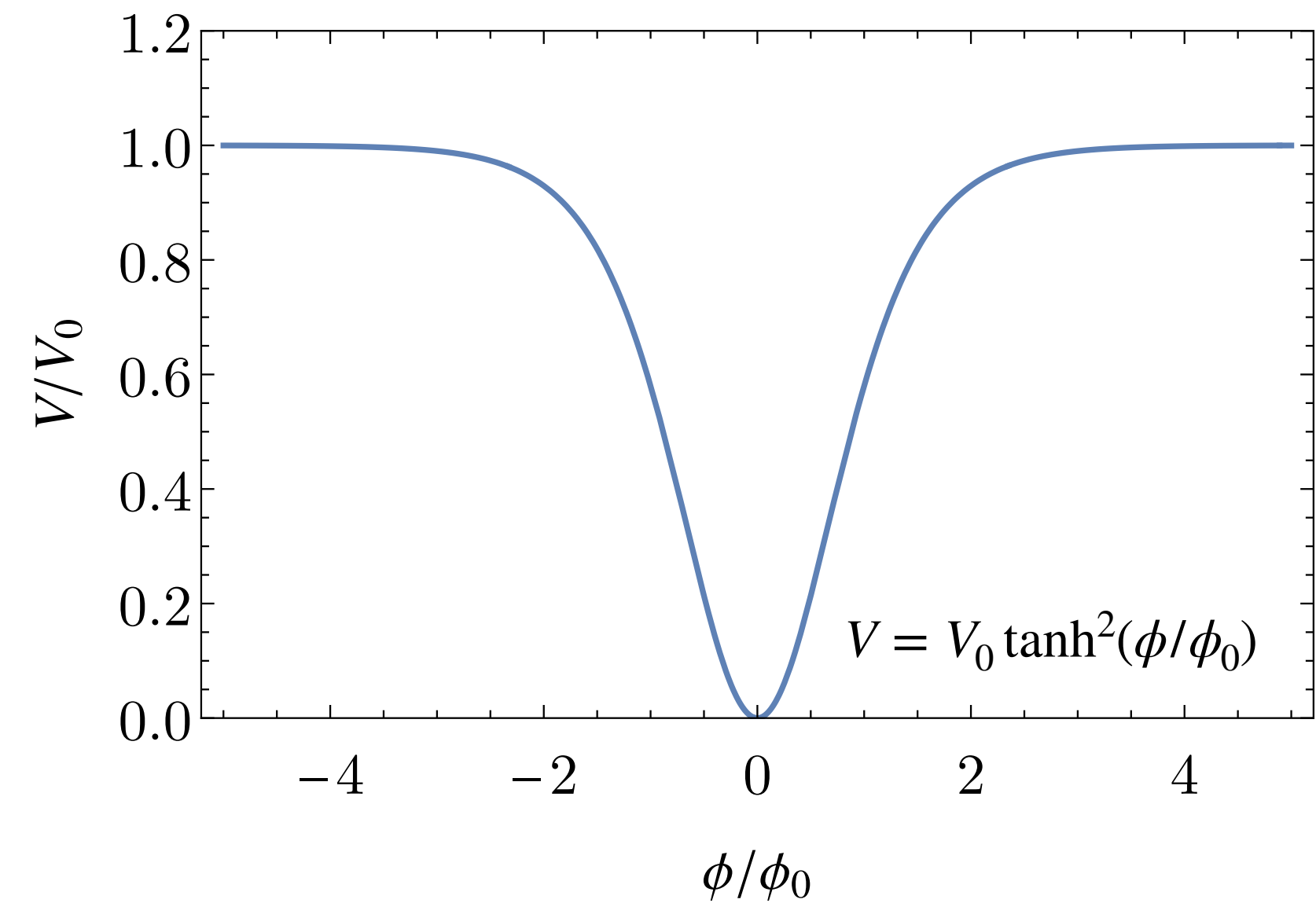
Hiroki Matsui, Alexandros Papageorgiou, Fuminobu Takahashi, and Takahiro Terada, 2305.02366; 2305.02367

Inflation



[Akrami et al. (Planck 2018), 1807.06211]. See also [Ade et al. (BICEP/Keck), 2110.00483], [Tristram et al., 2112.7961], and [Paoletti et al., 2208.10482].

A **flat** potential fits the data well.



However, the Big Bang singularity is not necessarily resolved by inflation.

[Borde, Guth, Vilenkin, gr-qc/0110012]. See also [Lesnfsky Easson, Davies, 2207.00955] for critical discussions.

Creation of the Universe from Nothing

Path integral, No-boundary proposal

[Hawking, *Pontif. Acad. Sci. Scr. Varia* 48 (1982) 563]

[Hartle, Hawking, *PRD*28 (1983) 2960]

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[Linde, *Rept. Prog. Phys.* 47 (1984) 925]

Wheeler-DeWitt eq., Tunneling effect

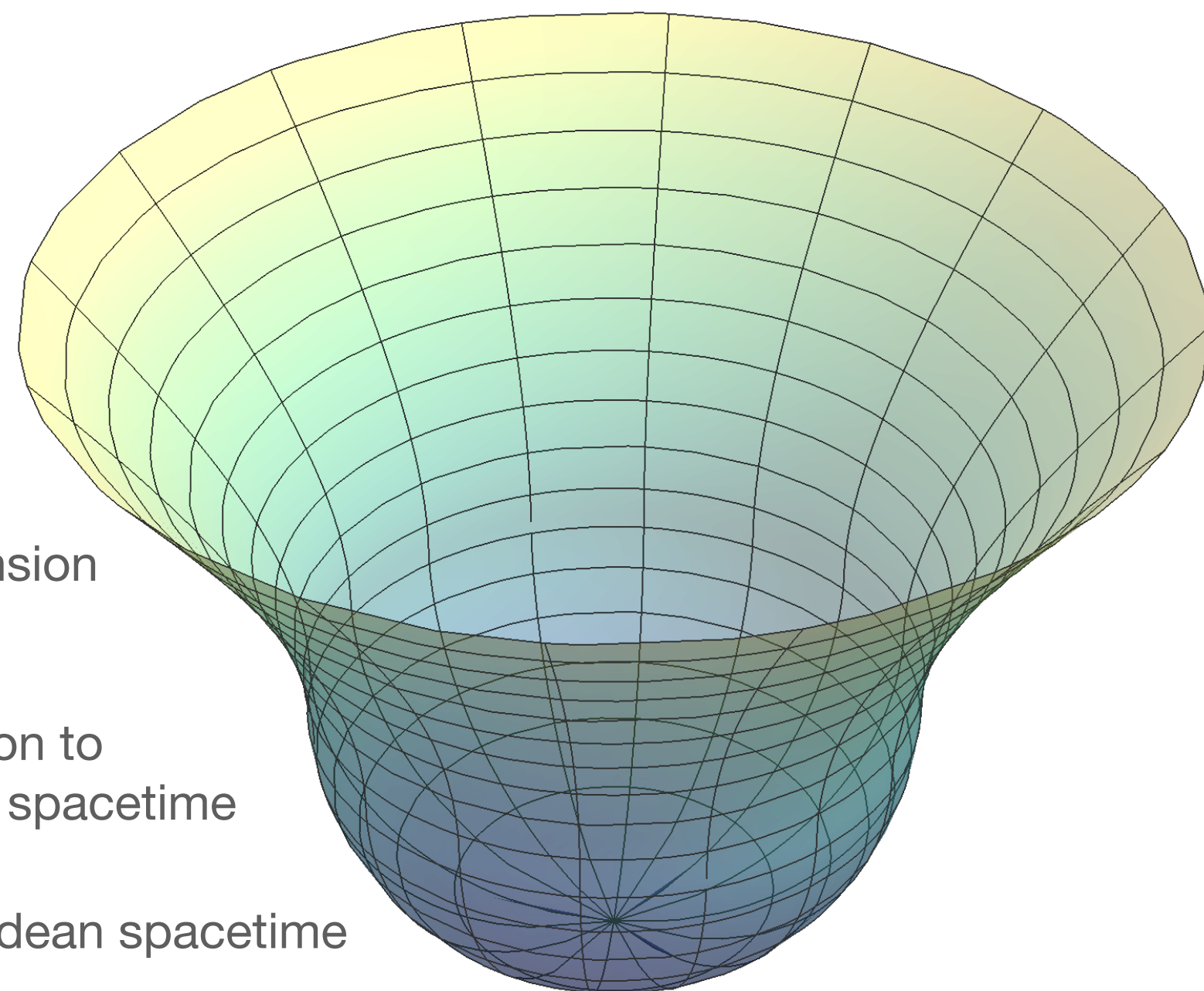
[Vilenkin, *PLB*117 (1982)]

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[Vilenkin, *PRD*37 (1988) 888]

Closed Universe (**Positive** spatial curvature)



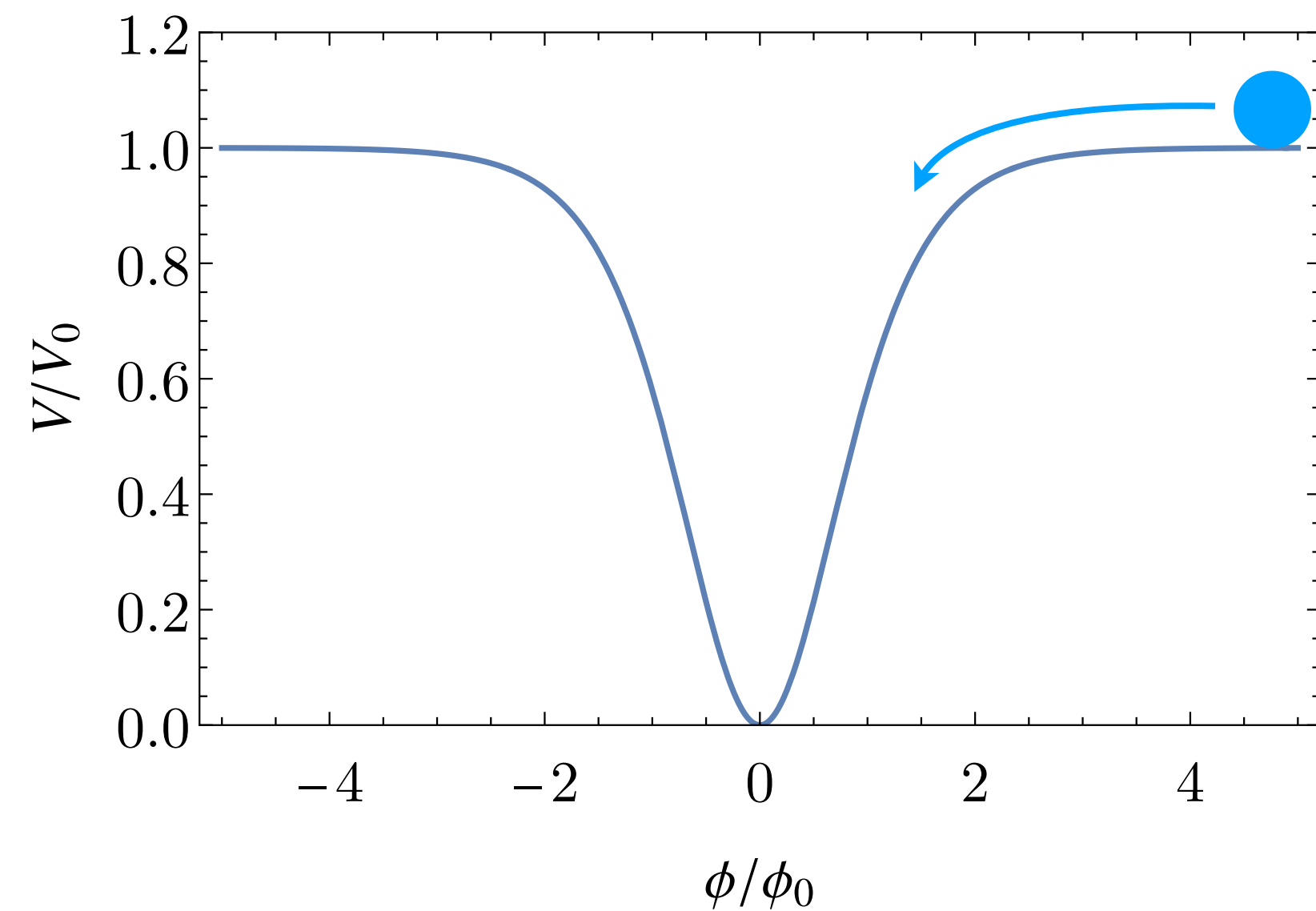
(Quasi) de Sitter expansion

Analytic continuation to
Lorentzian spacetime

Compact Euclidean spacetime

Non-singular creation
from "Nothing"

Sufficiently long ($N_e \gtrsim 50$) inflation



Creation of the Universe from Nothing

Path integral, No-boundary proposal

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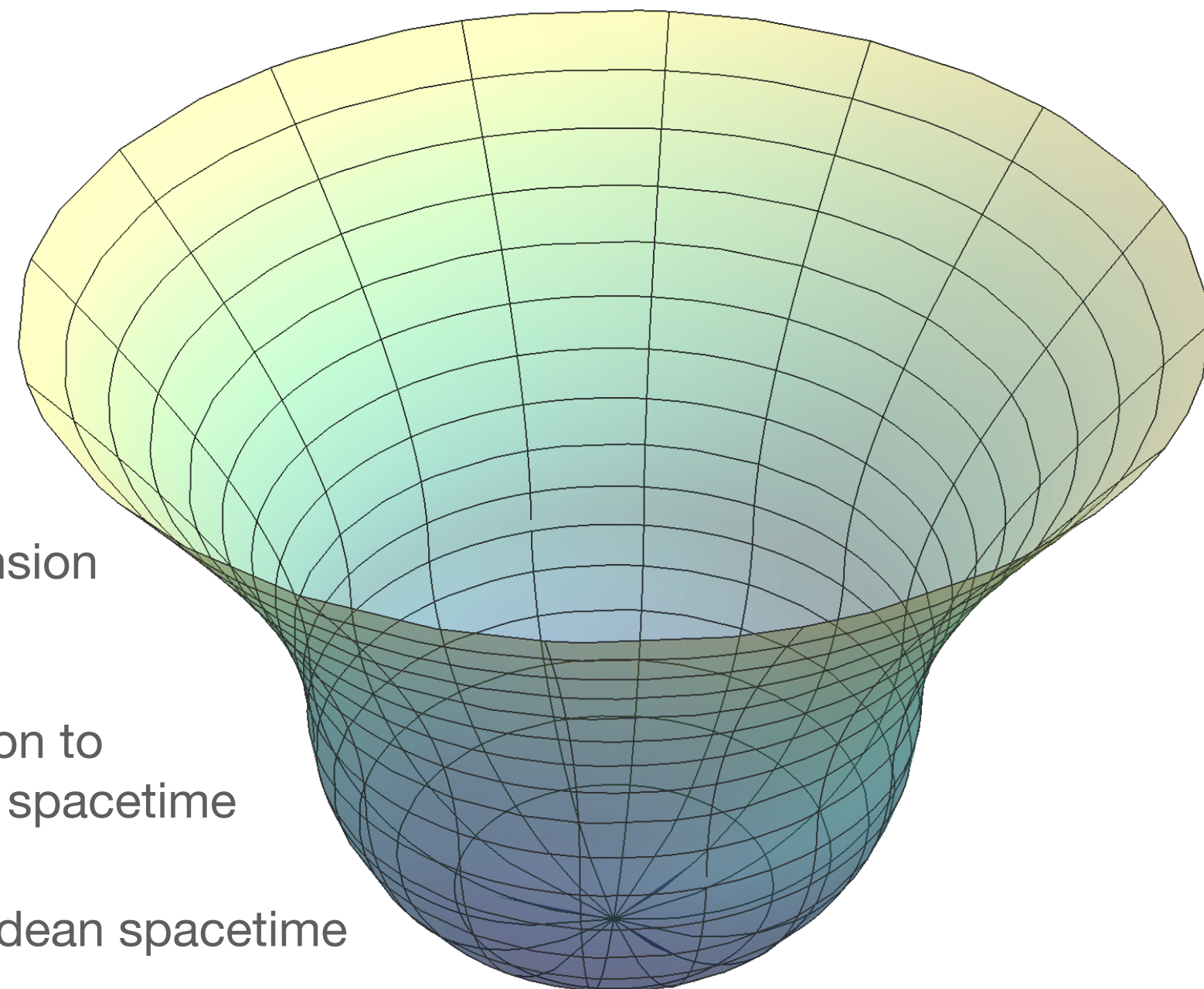
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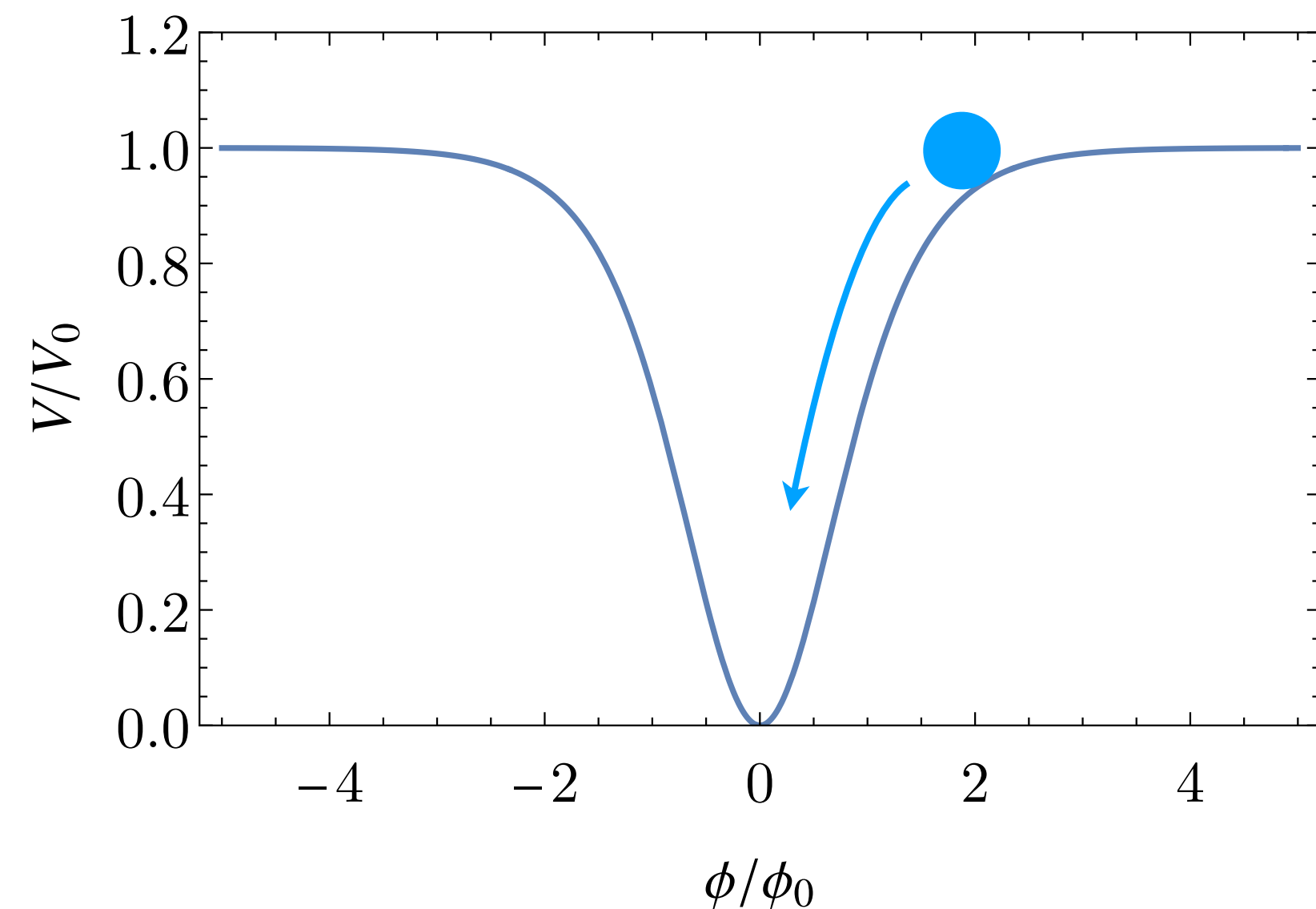
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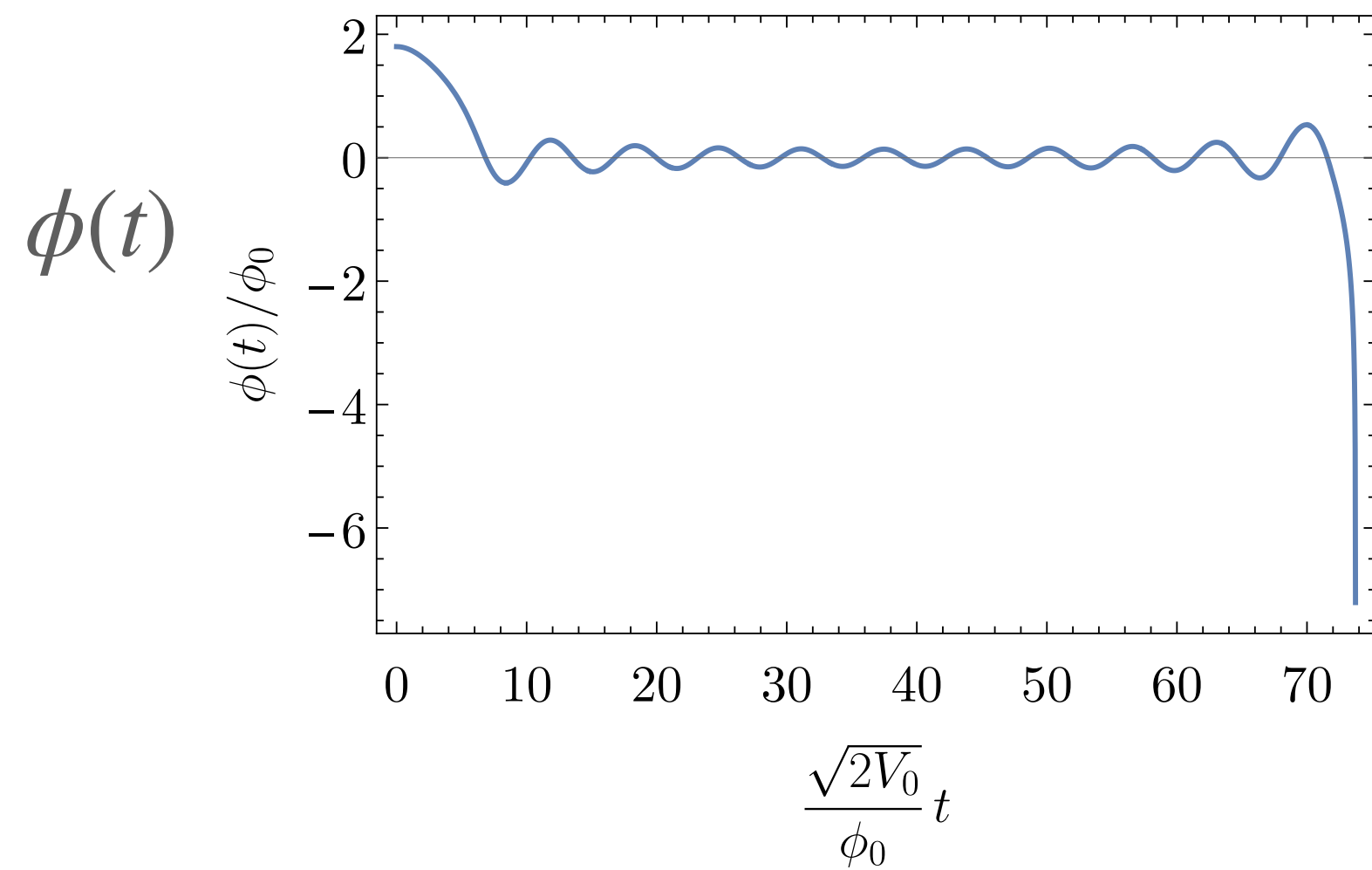
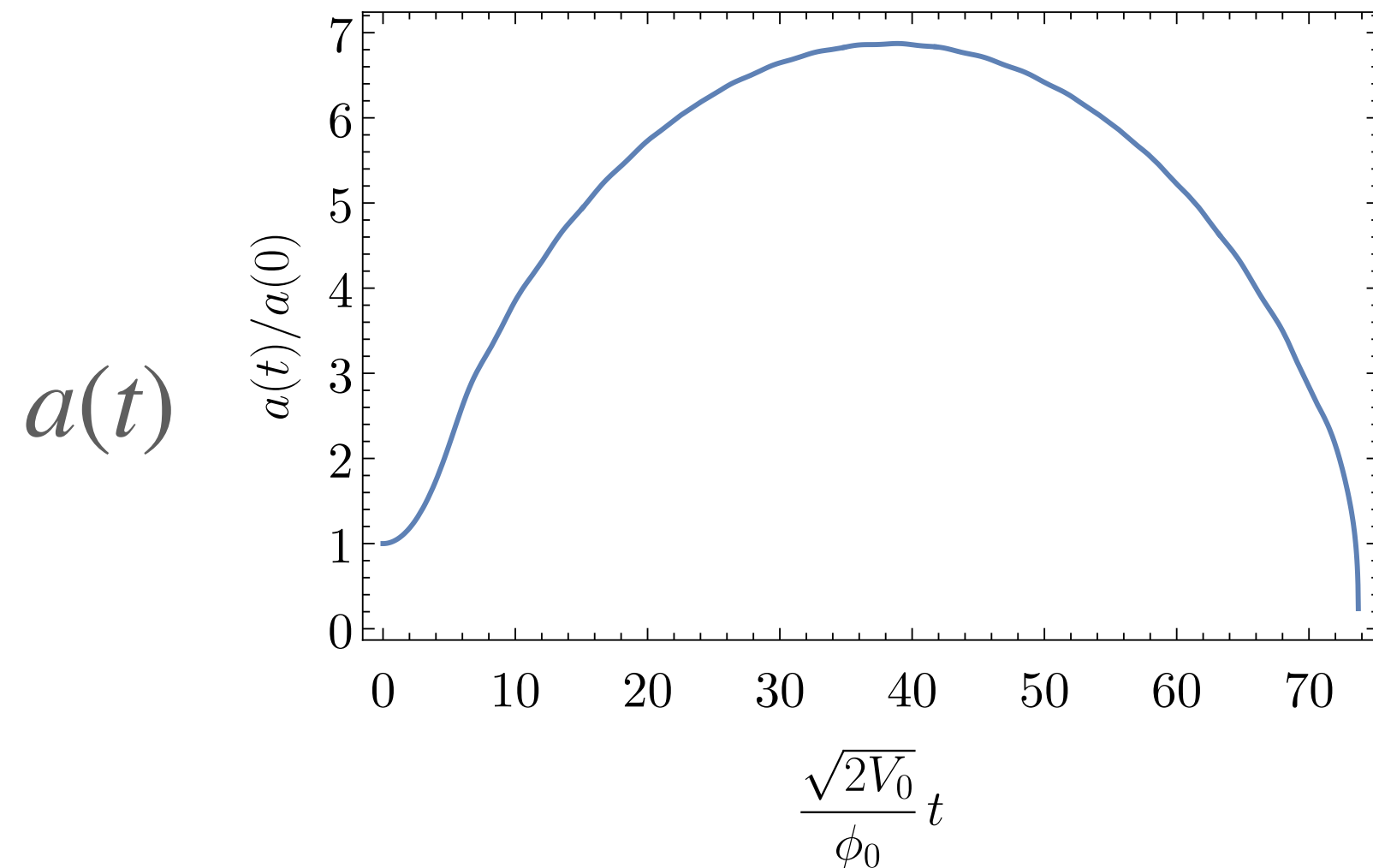
How about this case?



Big Crunch!?

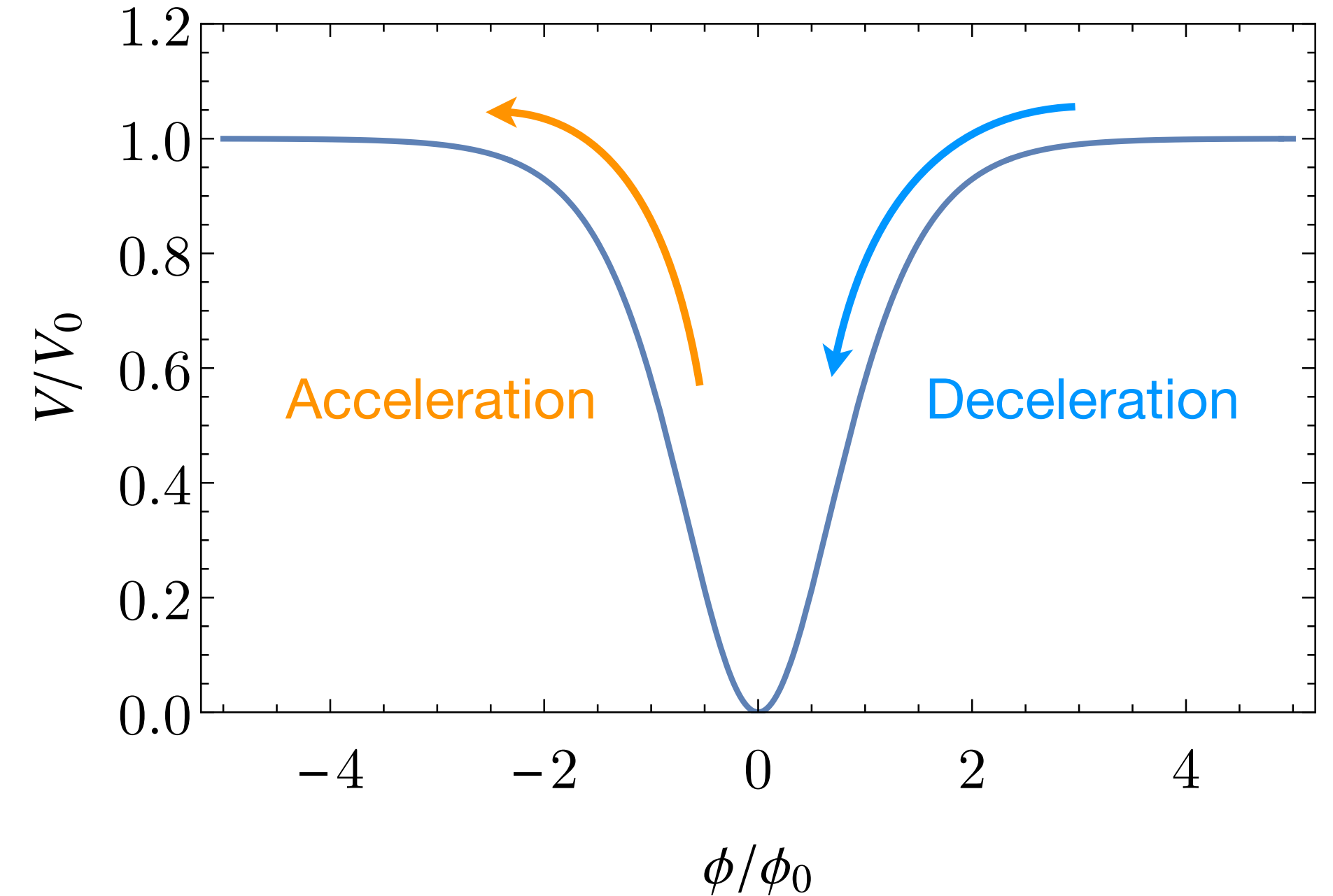
Ex. 1) Big Crunch

$$\phi_0 = \sqrt{0.6}, \quad \phi_{\text{ini}} \equiv \phi(0) = 1.8 \phi_0$$



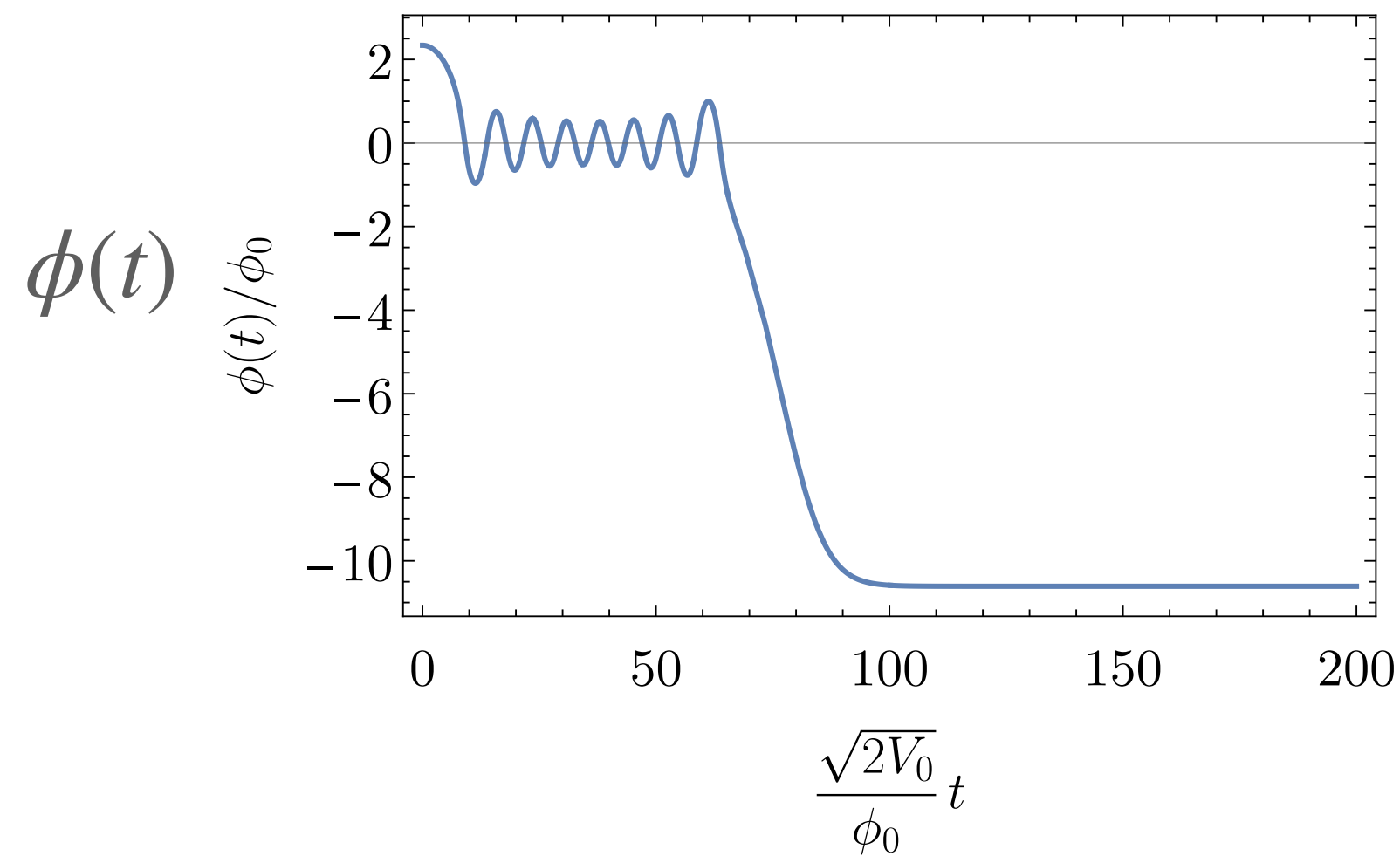
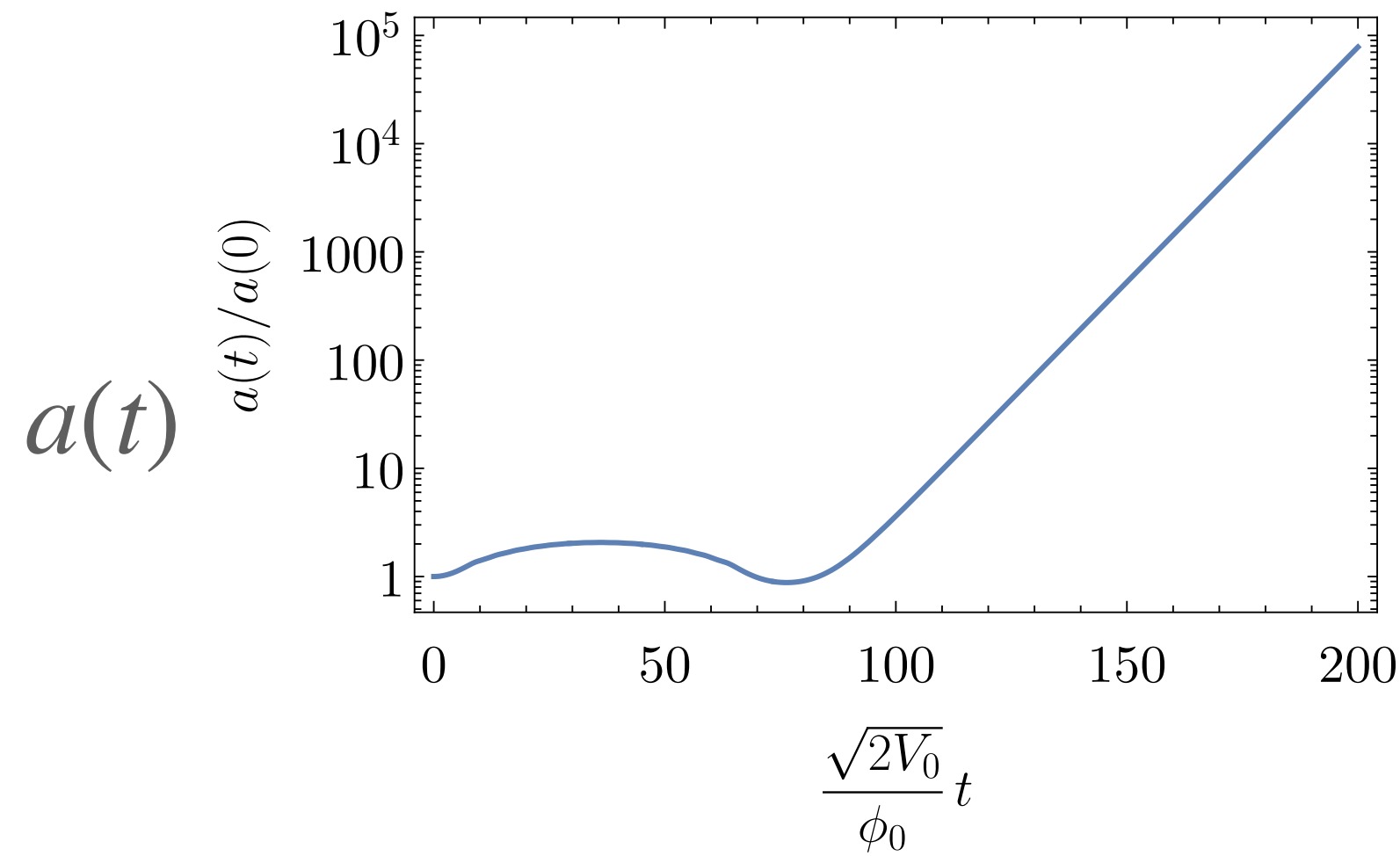
$$H^2 = \frac{\rho}{3M_{\text{P}}^2} - \frac{K}{a^2}$$

$$\ddot{\phi} + 3H\dot{\phi} + V_{\phi} = 0$$

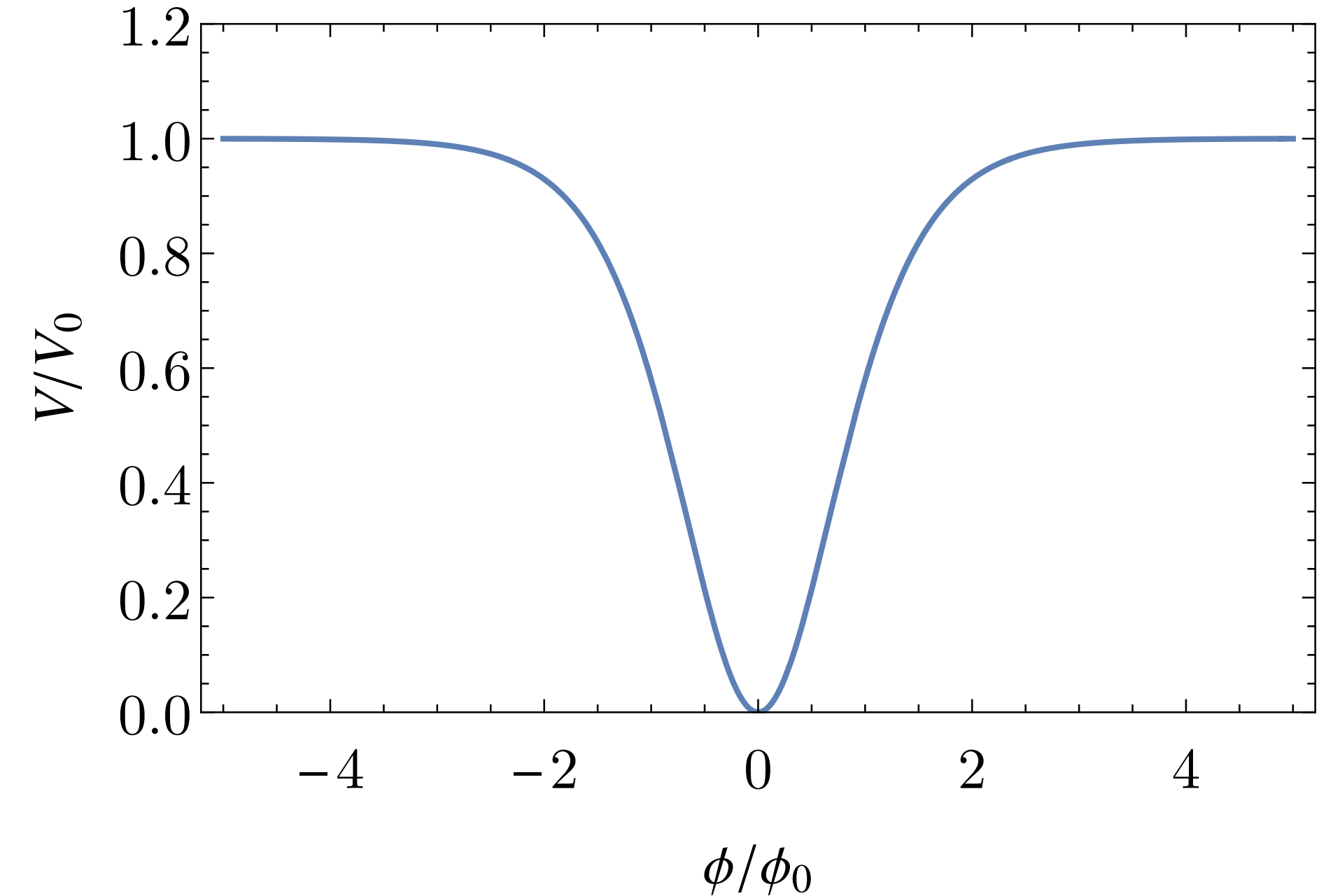


Ex. 2) Bounce & Inflation

$$\phi_0 = \sqrt{0.06}, \quad \phi_{\text{ini}} \equiv \phi(0) = 2.34 \phi_0$$



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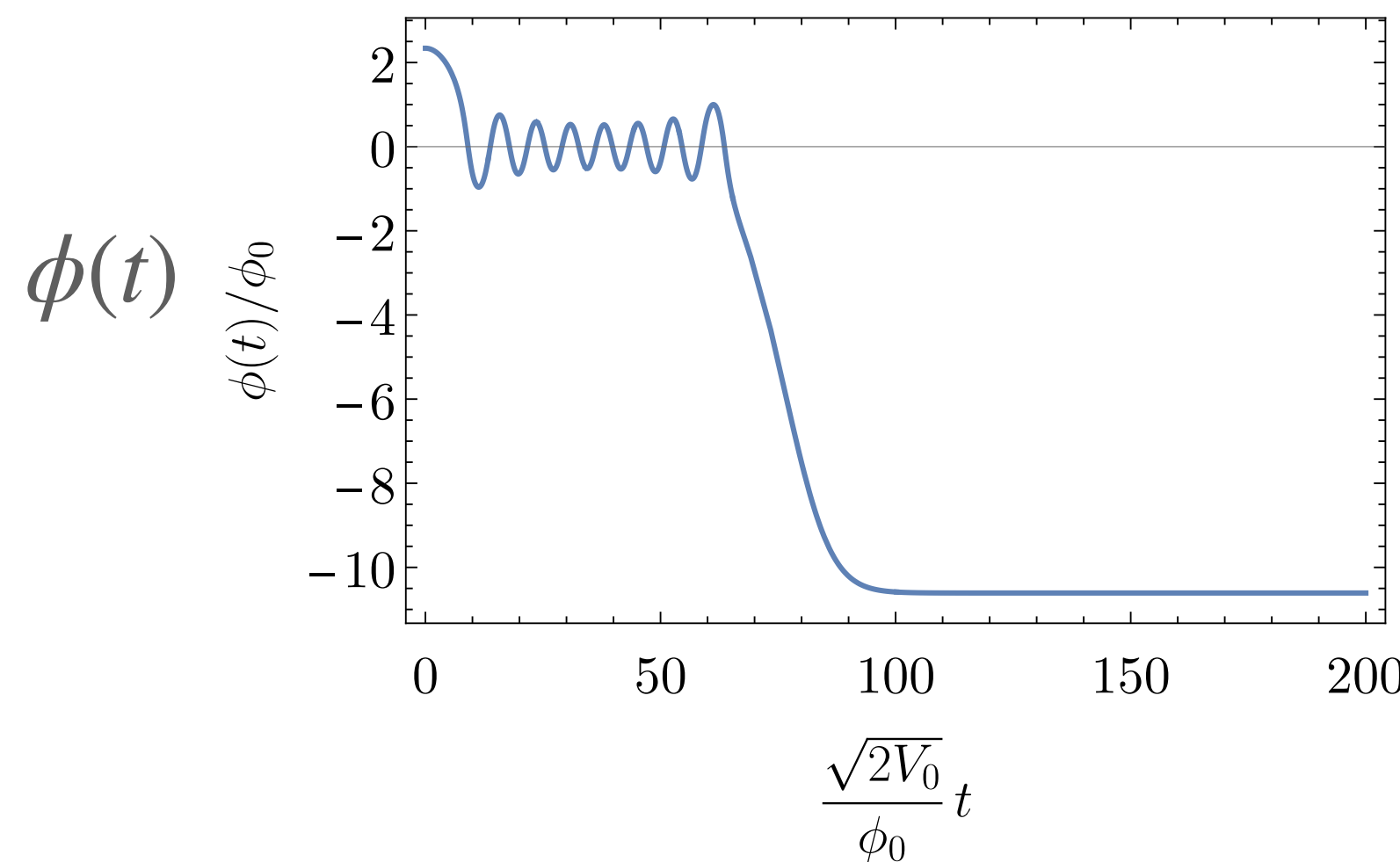
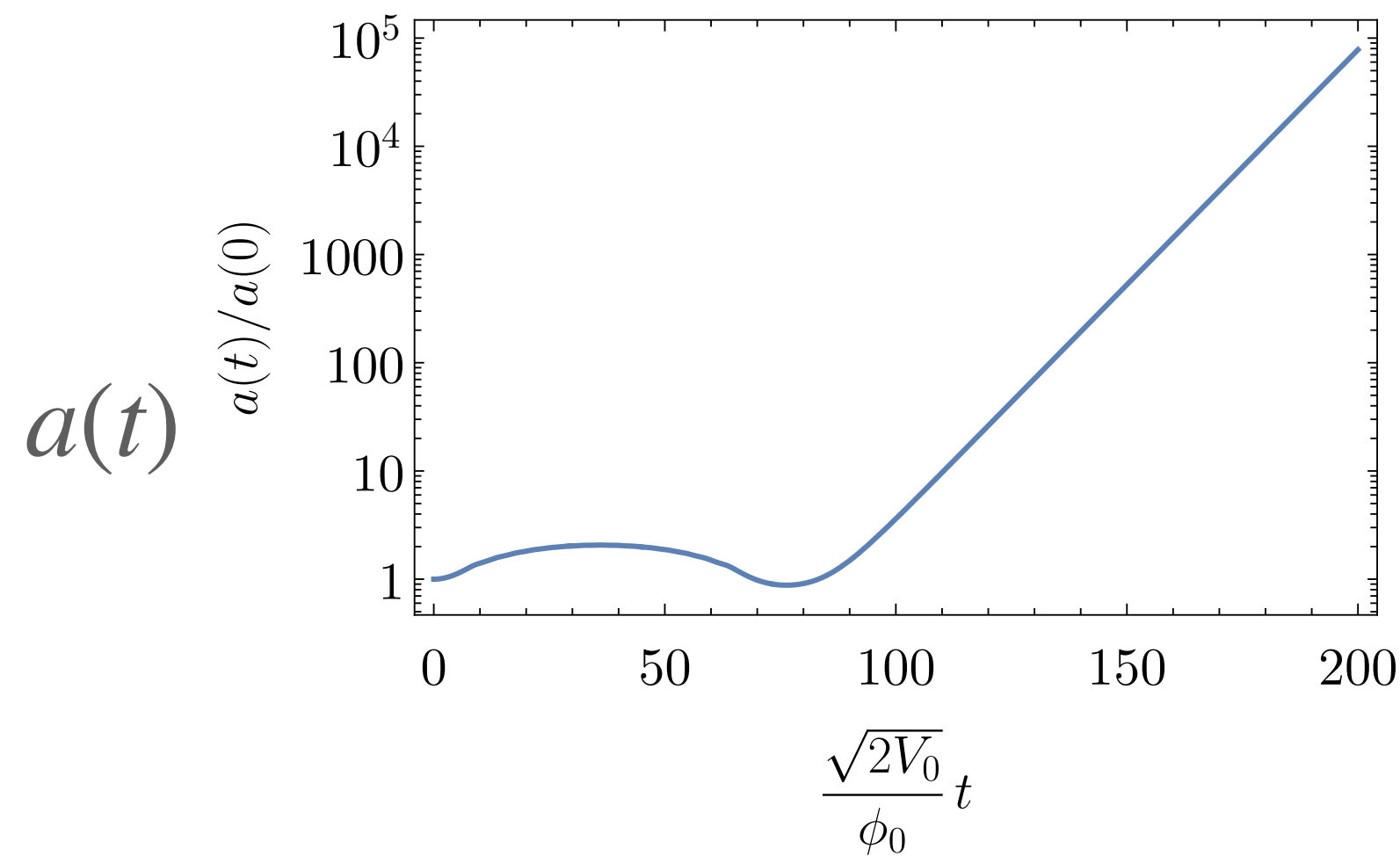


[Matsui, Takahashi, Terada, 1904.12312]

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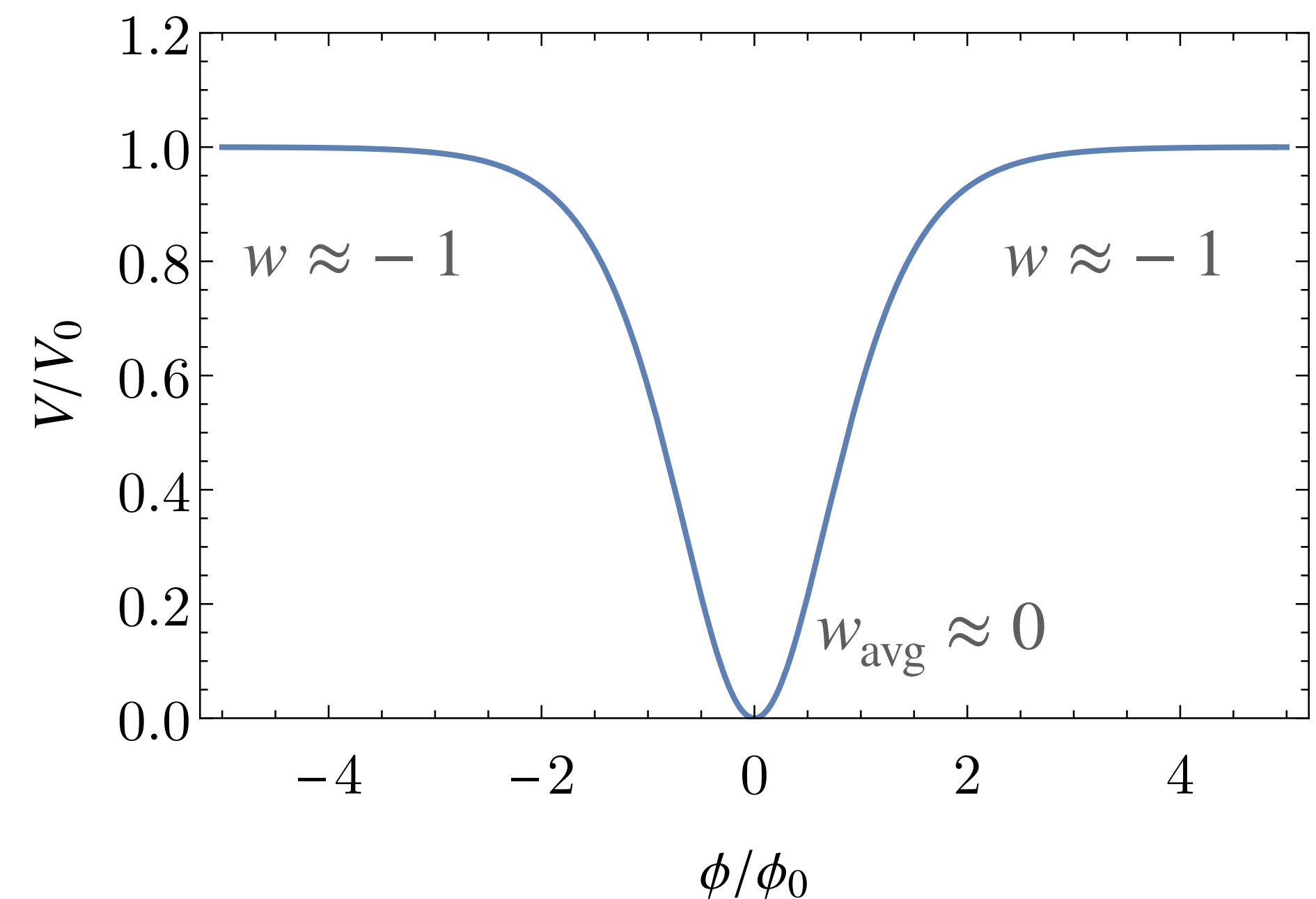
Conditions for bounce/turn-around

1. $H = 0$ is satisfied.
2. $\ddot{a}/a = -(\rho + 3P)/6$ is positive (bounce) or negative (turn-around) at the moment of $H = 0$.

$$H^2 = \frac{\rho}{3M_{\text{P}}^2} - \frac{K}{a^2}$$

$$w = \frac{P}{\rho} = \frac{\dot{\phi}^2/2 - V}{\dot{\phi}^2/2 + V}$$

$$w_{\text{avg}} = \langle w \rangle_{\text{osc}}$$

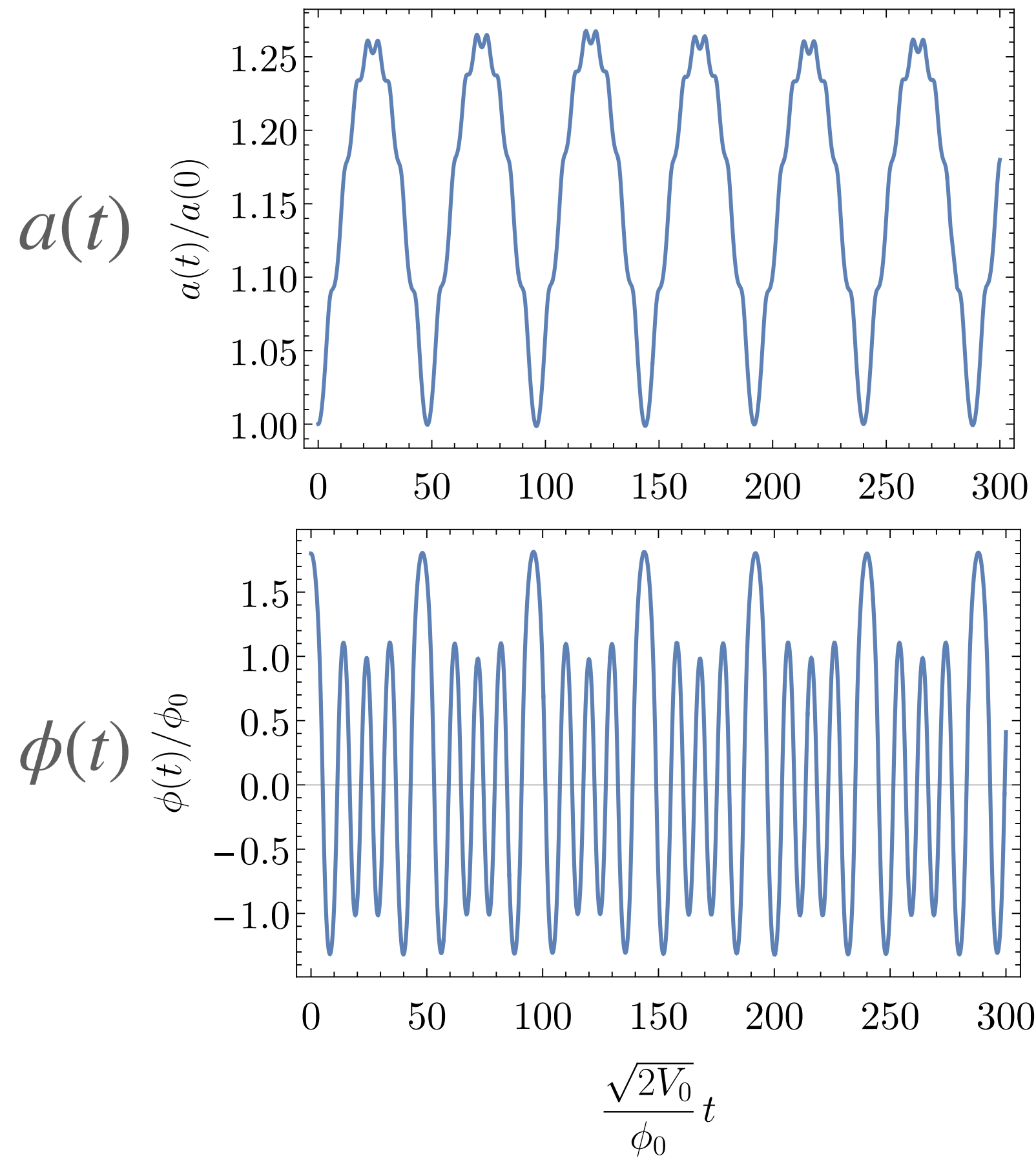


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Ex. 3) Cyclic Universe

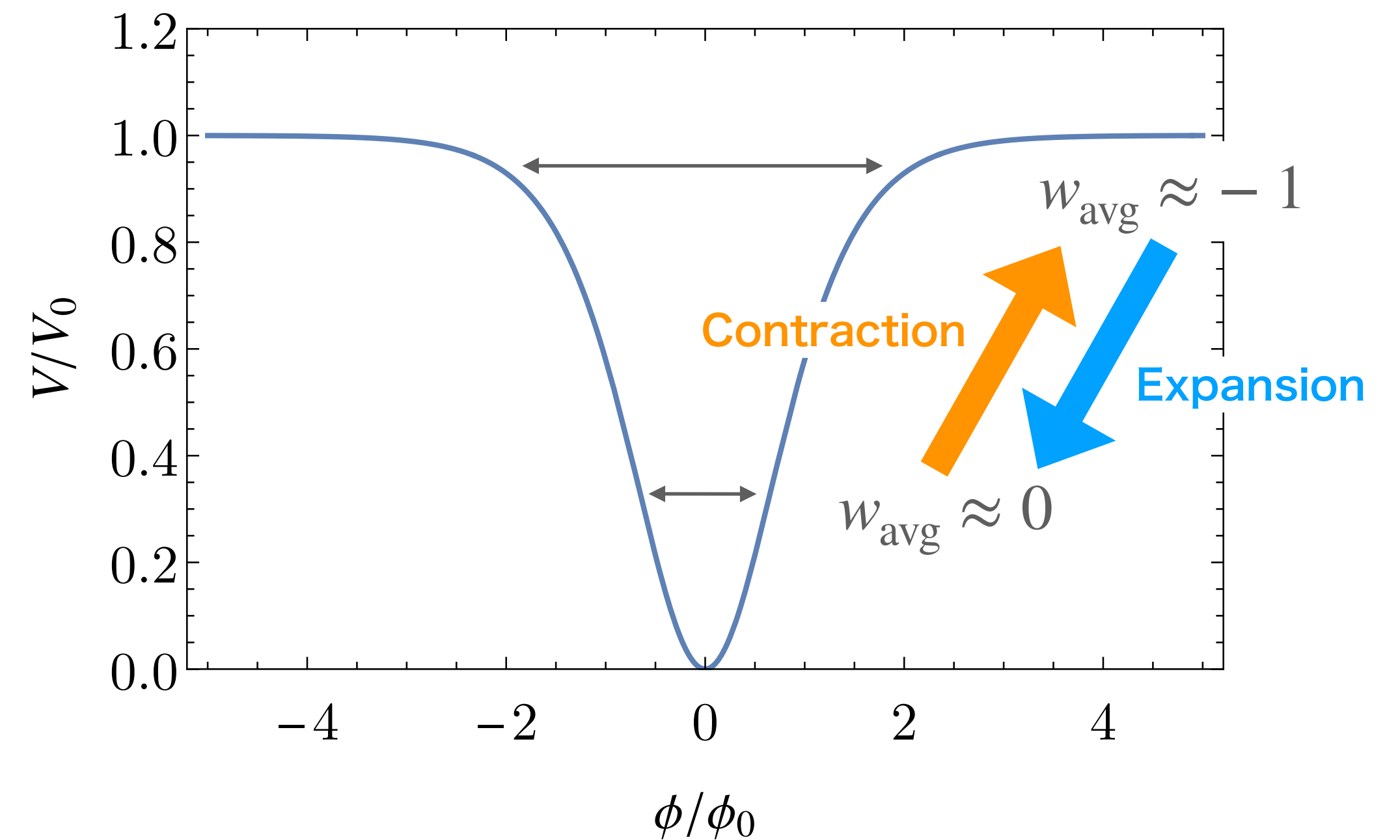
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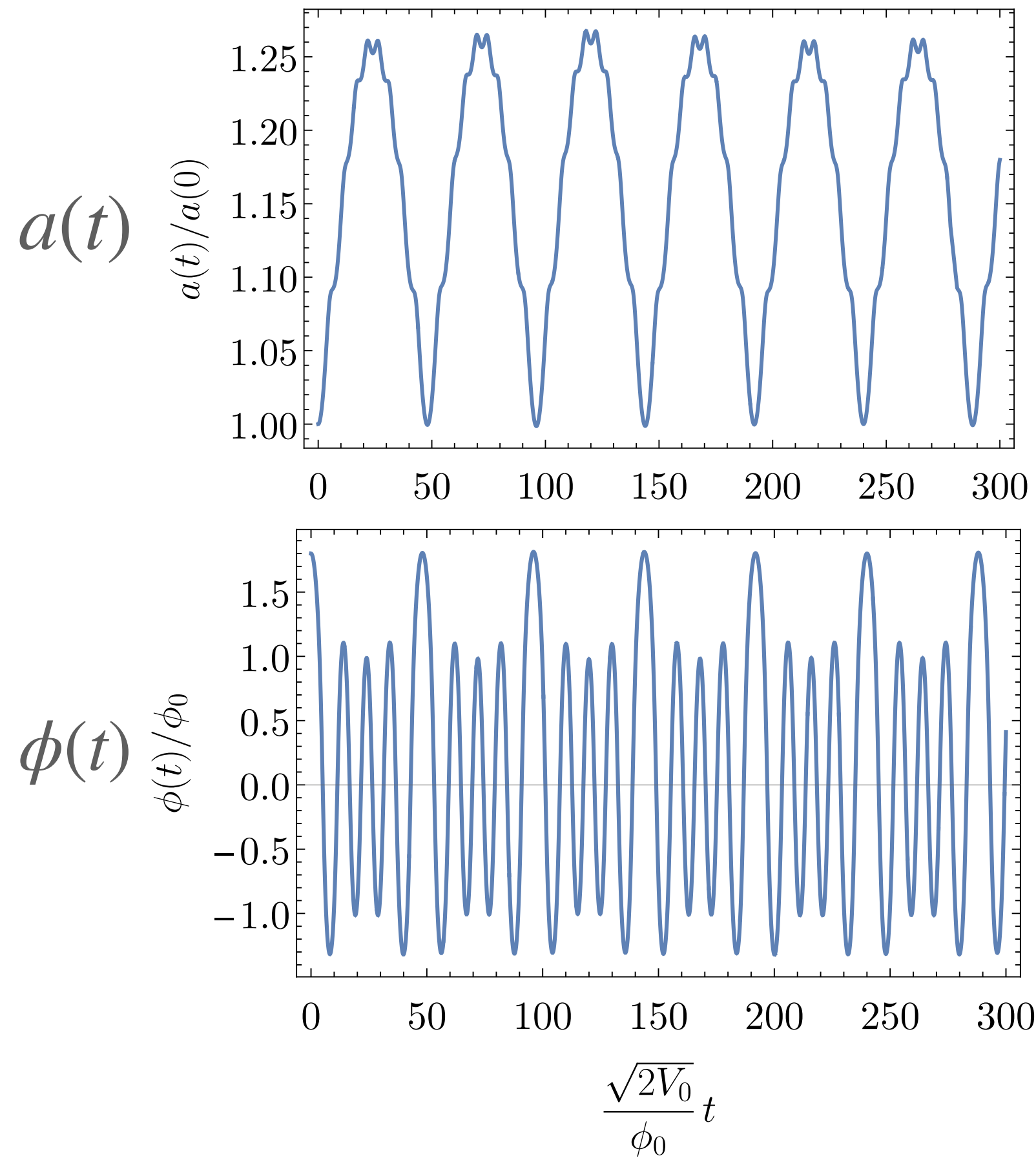


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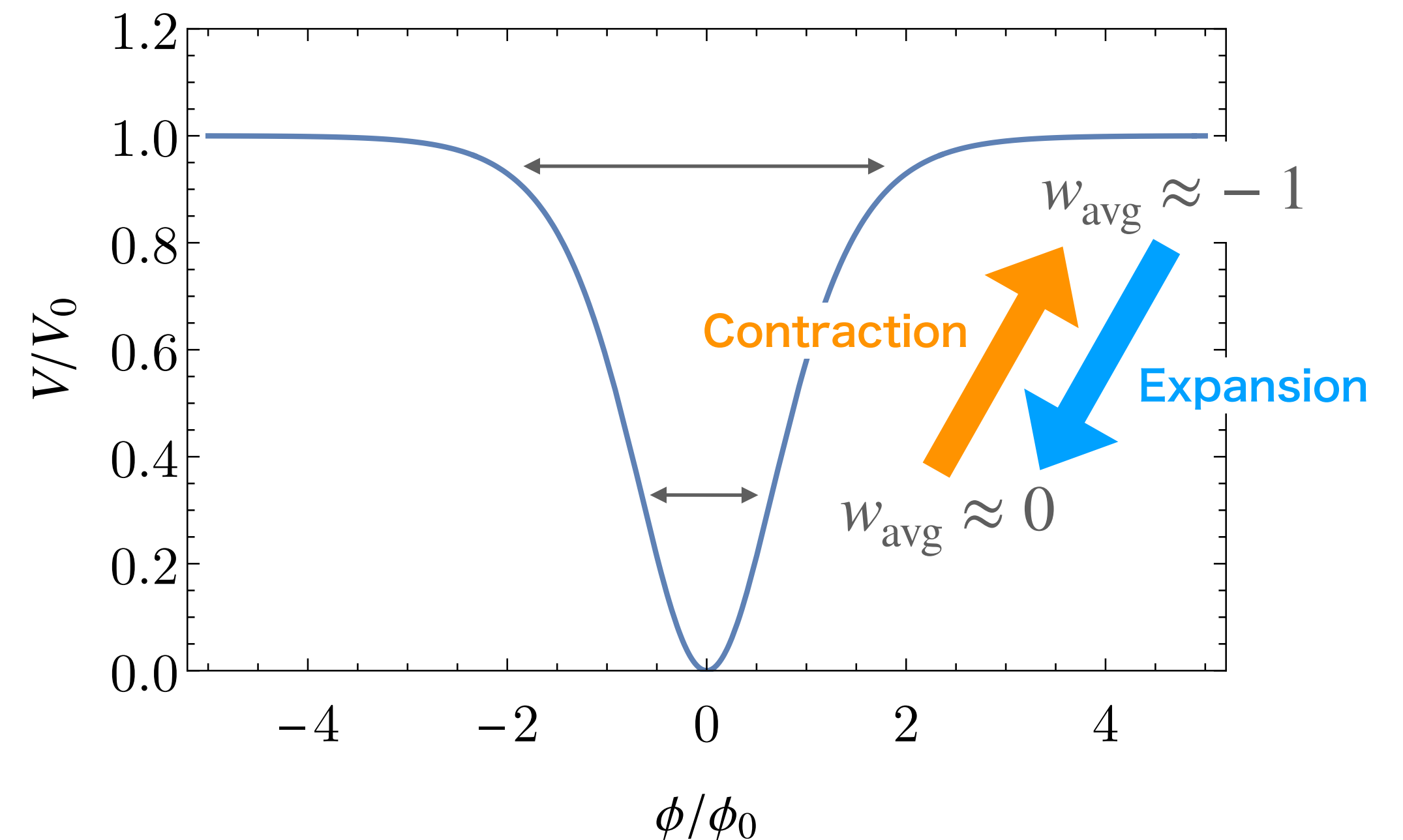
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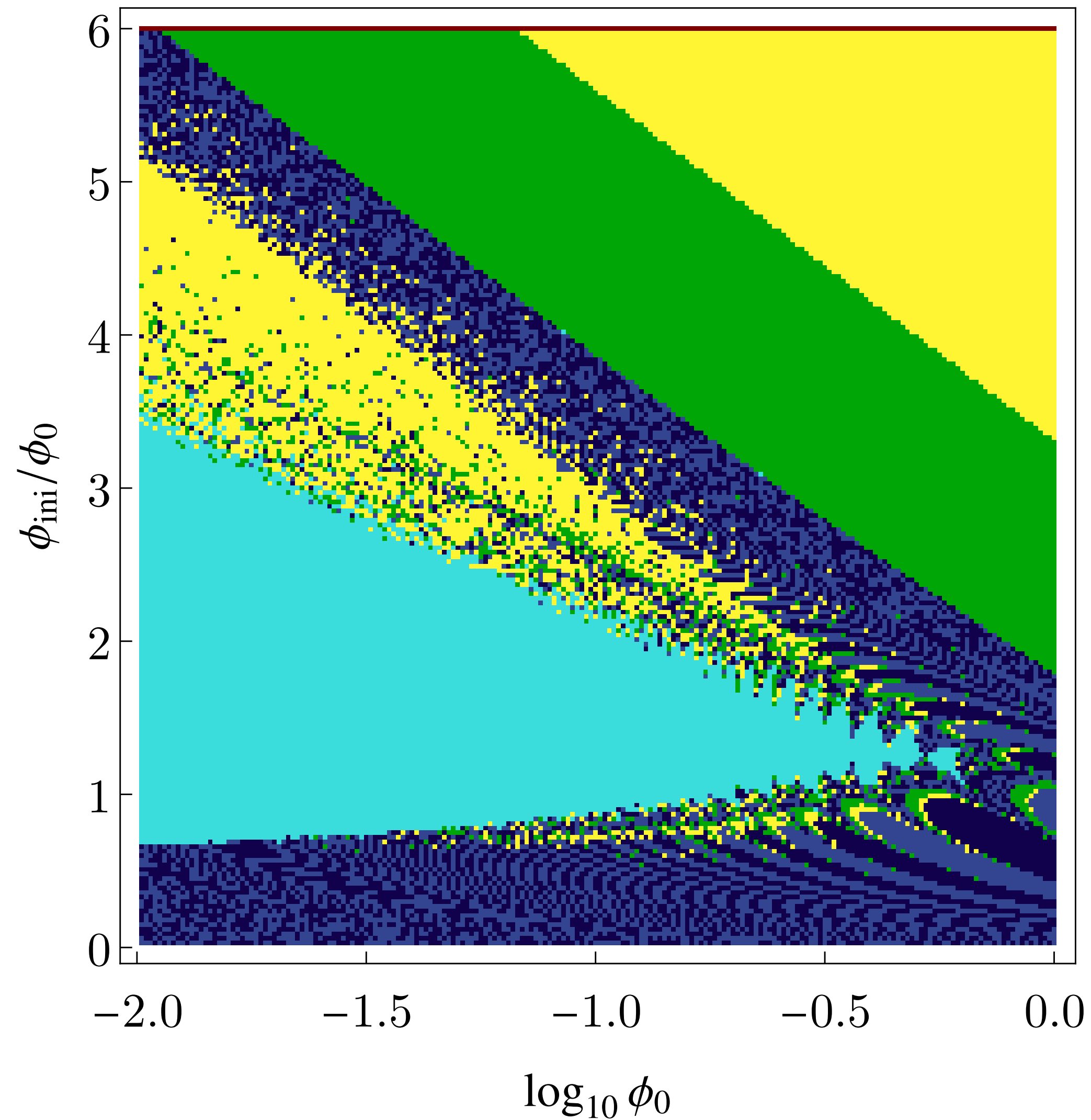
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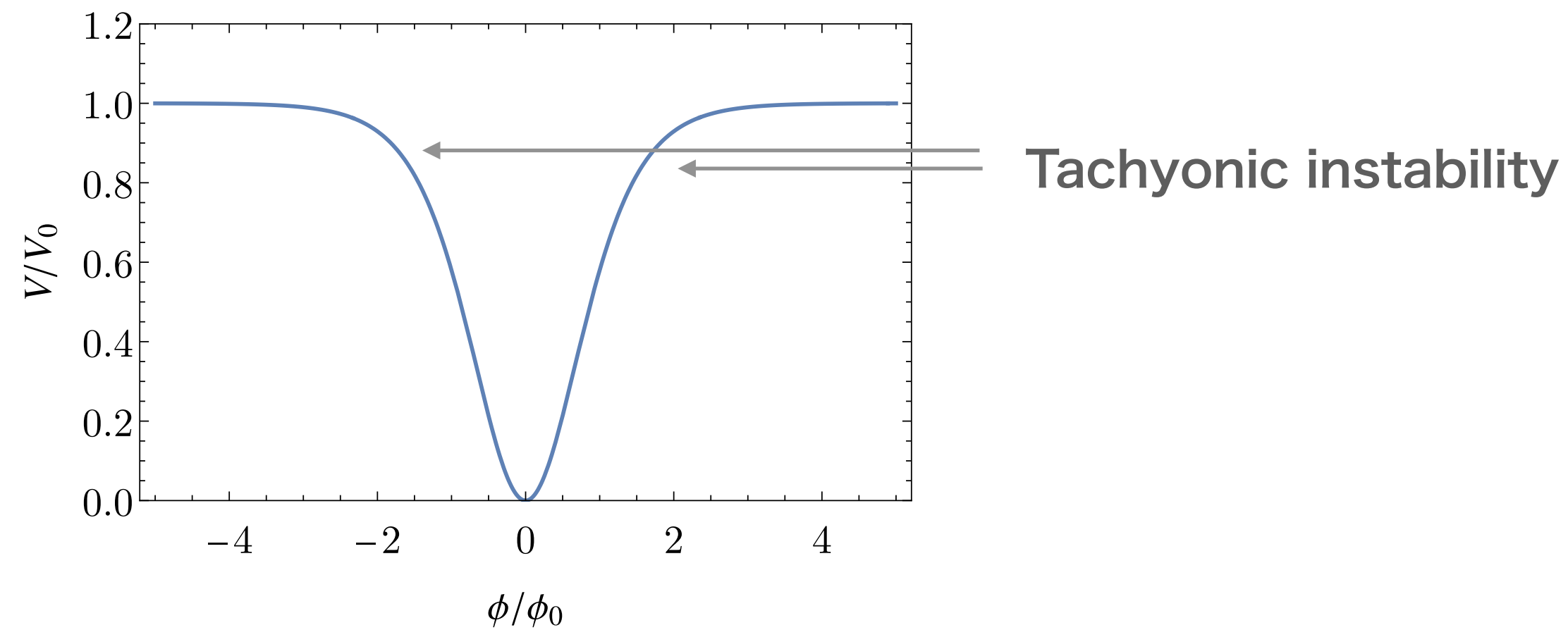
“Phase Diagram” of the Universe



- Big Crunch with $\dot{\phi} > 0$
- Big Crunch with $\dot{\phi} < 0$
- Cyclic Universe
- Short inflation ($N < 50$)
- Long inflation ($N \geq 50$)

Are the cyclic solutions relevant to our Universe?

Tachyonic Instability



Analyses in the context of preheating [Tomberg, Veermäe, 2108.10767]

Energy density fluctuation

$$\delta\rho \approx \frac{(k_{\text{peak}}/a)^4}{4\pi^{3/2}\sqrt{2\mu_{\text{peak}}t}} e^{2\mu_{\text{peak}}t}$$

$k_{\text{peak}} \approx \pi a / \Delta t_\phi$: wavenumber of the most unstable mode

$\Delta t_\phi \simeq \pi\phi_0 / \sqrt{2(V_0 - \rho)}$: half-period of ϕ oscillations

Fragmentation time

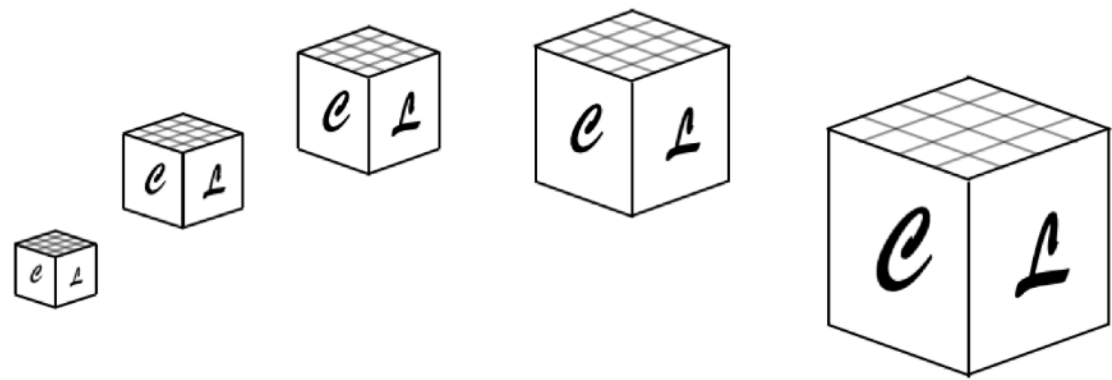
$$t_{\text{frag}} \approx \frac{1}{2\mu_{\text{peak}}} \left(\ln \left(\frac{4\pi^{3/2}c\rho}{(k_{\text{peak}}/a)^4} \right) + \frac{1}{2} \ln \ln \left(\frac{4\pi^{3/2}c\rho}{(k_{\text{peak}}/a)^4} \right) \right)$$

$\mu_{\text{peak}} = \mathcal{O}(1/\Delta t_\phi)$: the peak rate of tachyonic instability

$c = \mathcal{O}(1)$: the energy-density fraction of fluctuations and the background when fragmentation happens

Tachyonic Instability

Public classical lattice calculation tool



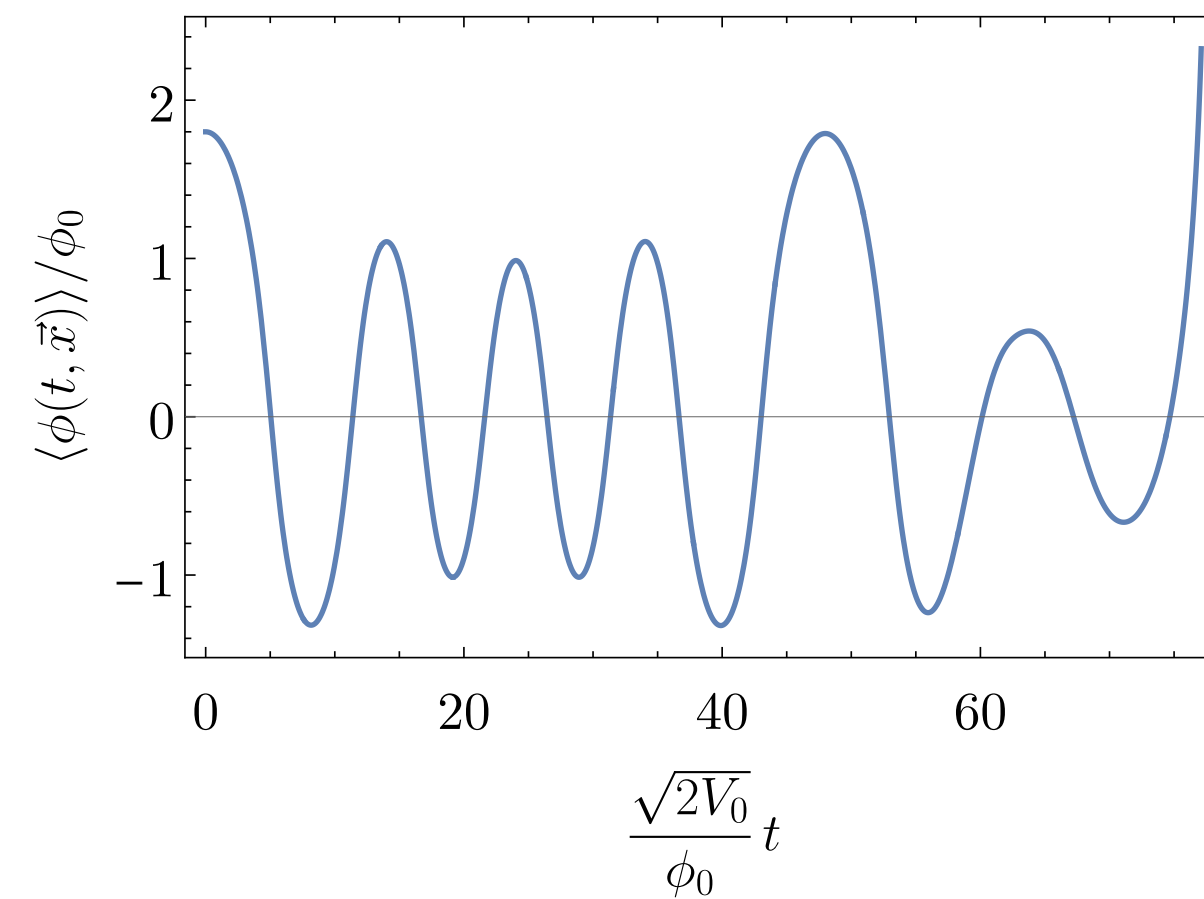
CosmoLattice

A modern code for lattice simulations of scalar and gauge field dynamics in an expanding universe

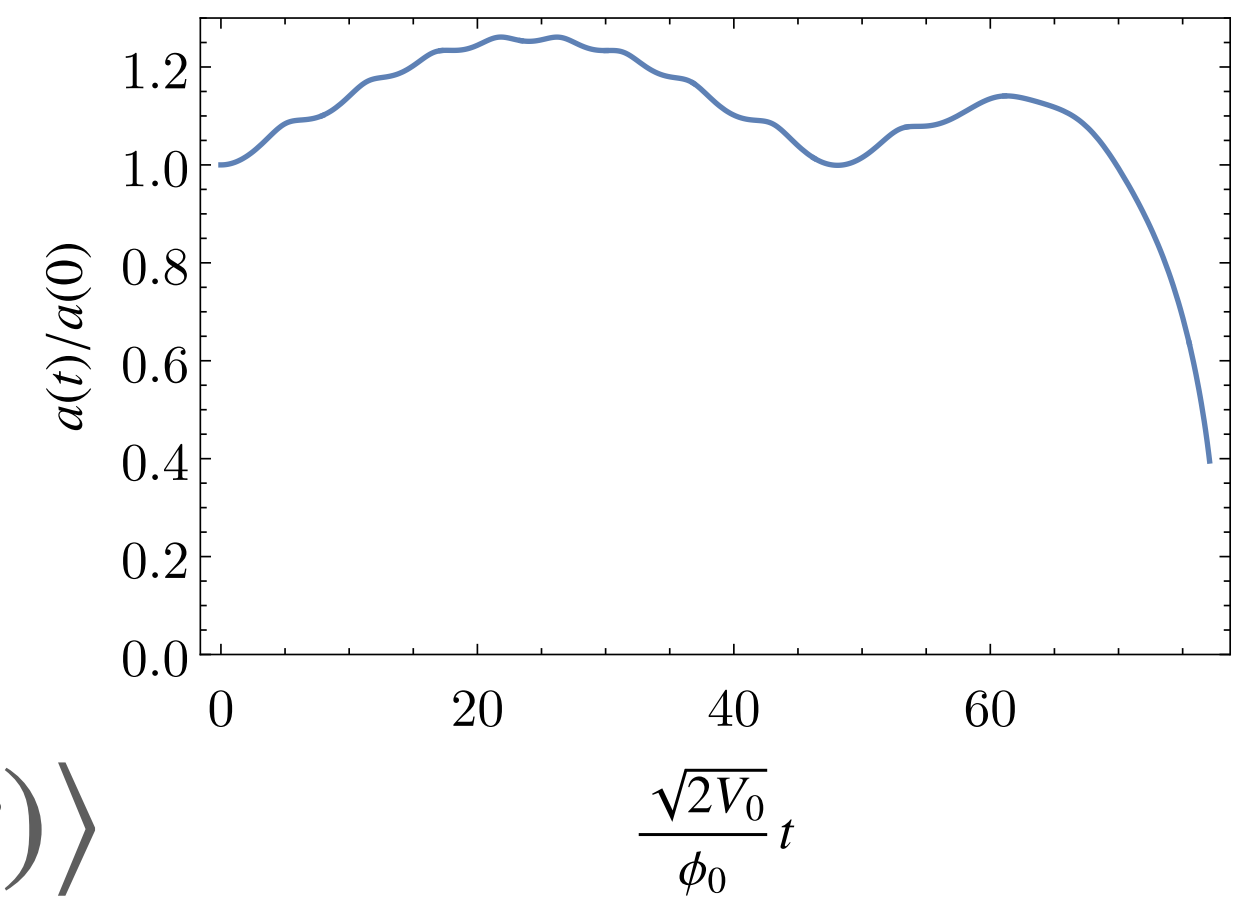
[Figueroa, Florio, Torrenti, Valkenburg, 2006.15122; 2102.01031]

<https://cosmolattice.net/>

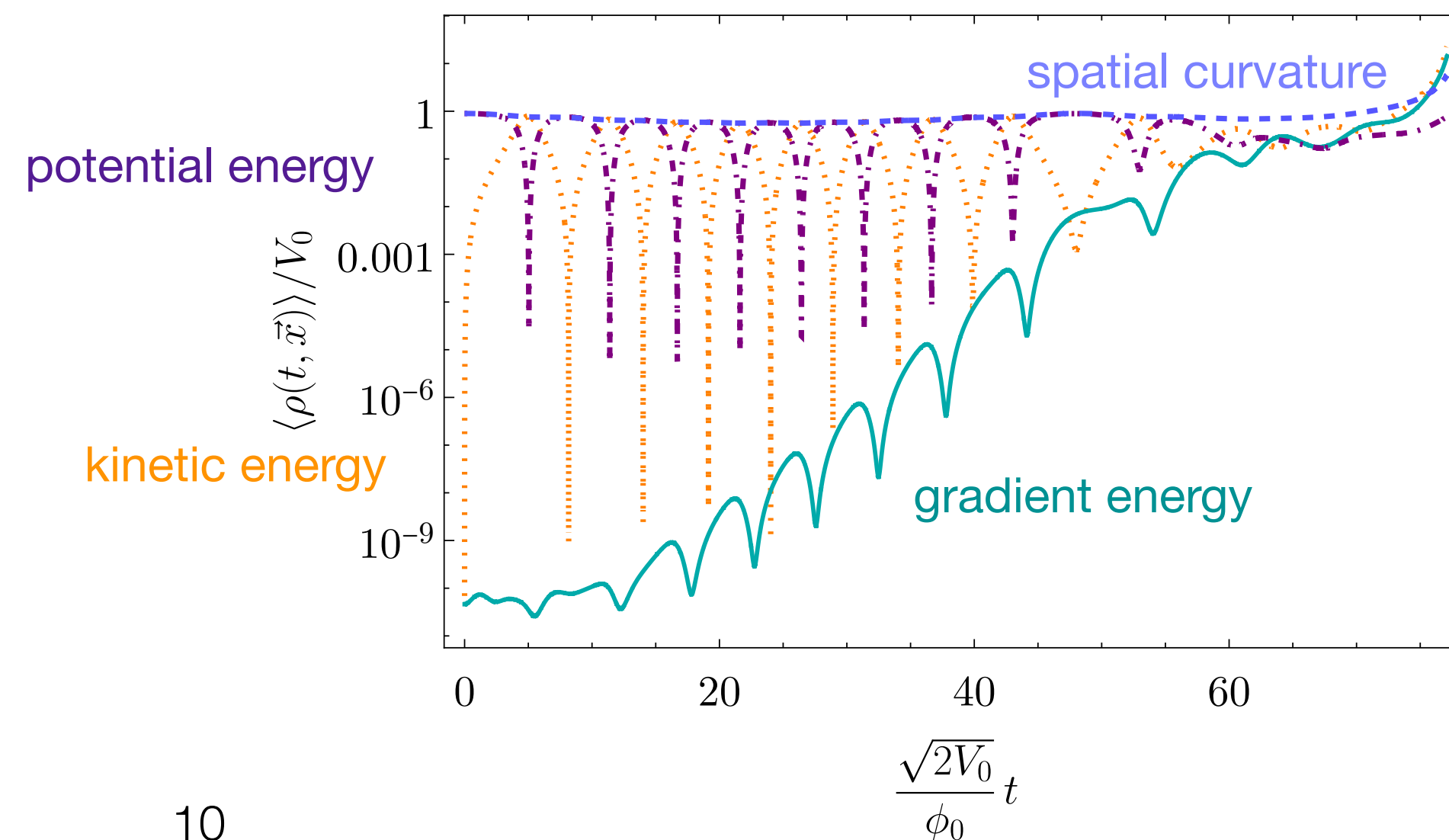
$\langle \phi(t, x) \rangle$



$a(t)$



$\langle \rho_X(t, x) \rangle$



The Question

Is it possible to turn a quasi-cyclic universe into an inflationary universe by dissipative effects *before the fragmentation time*?

Setup

(Toy model for demonstration)

Lagrangian density

$$\mathcal{L} = \frac{1}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) + \mathcal{L}_{\text{int}} + \mathcal{L}_{\text{matter}}$$

Scalar potential

$$V(\phi) = V_0 \tanh^2(\phi/\phi_0)$$

Metric Ansatz

$$ds^2 = -dt^2 + a(t)^2 \left(\frac{dr^2}{1 - Kr^2} + r^2 d\Omega_2^2 \right)$$

with the positive spatial curvature $K > 0$.

Equations of motion

$$\dot{H} = -\frac{1}{2}\dot{\phi}^2 - \frac{2}{3}\rho_r + \frac{K}{a^2},$$

$$\ddot{\phi} + (3H + \Gamma)\dot{\phi} + V' = 0,$$

$$\dot{\rho}_r + 4H\rho_r = \Gamma\dot{\phi}^2.$$

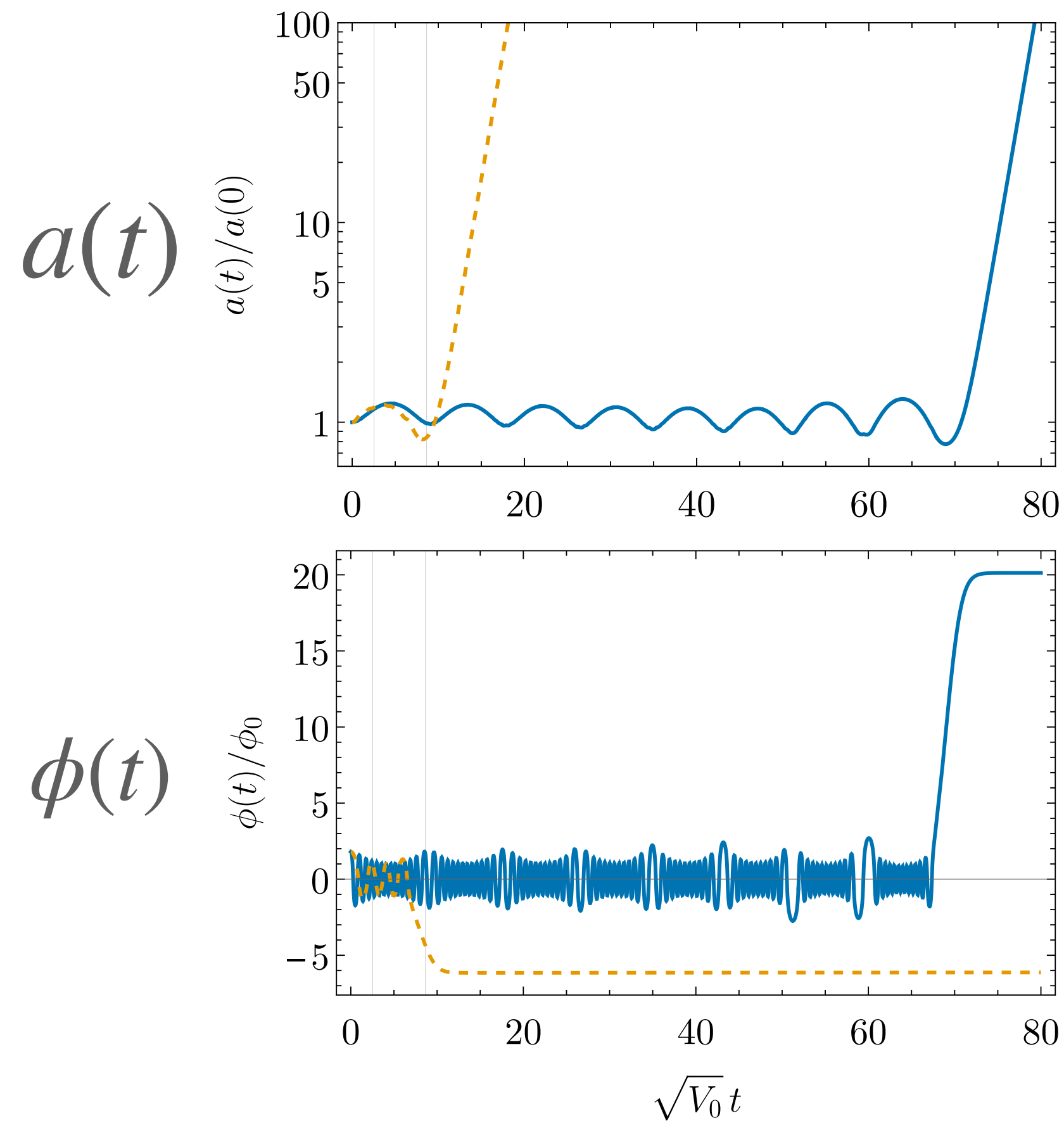
For a more concrete model,
see our longer paper: 2305.02367.

Initial conditions

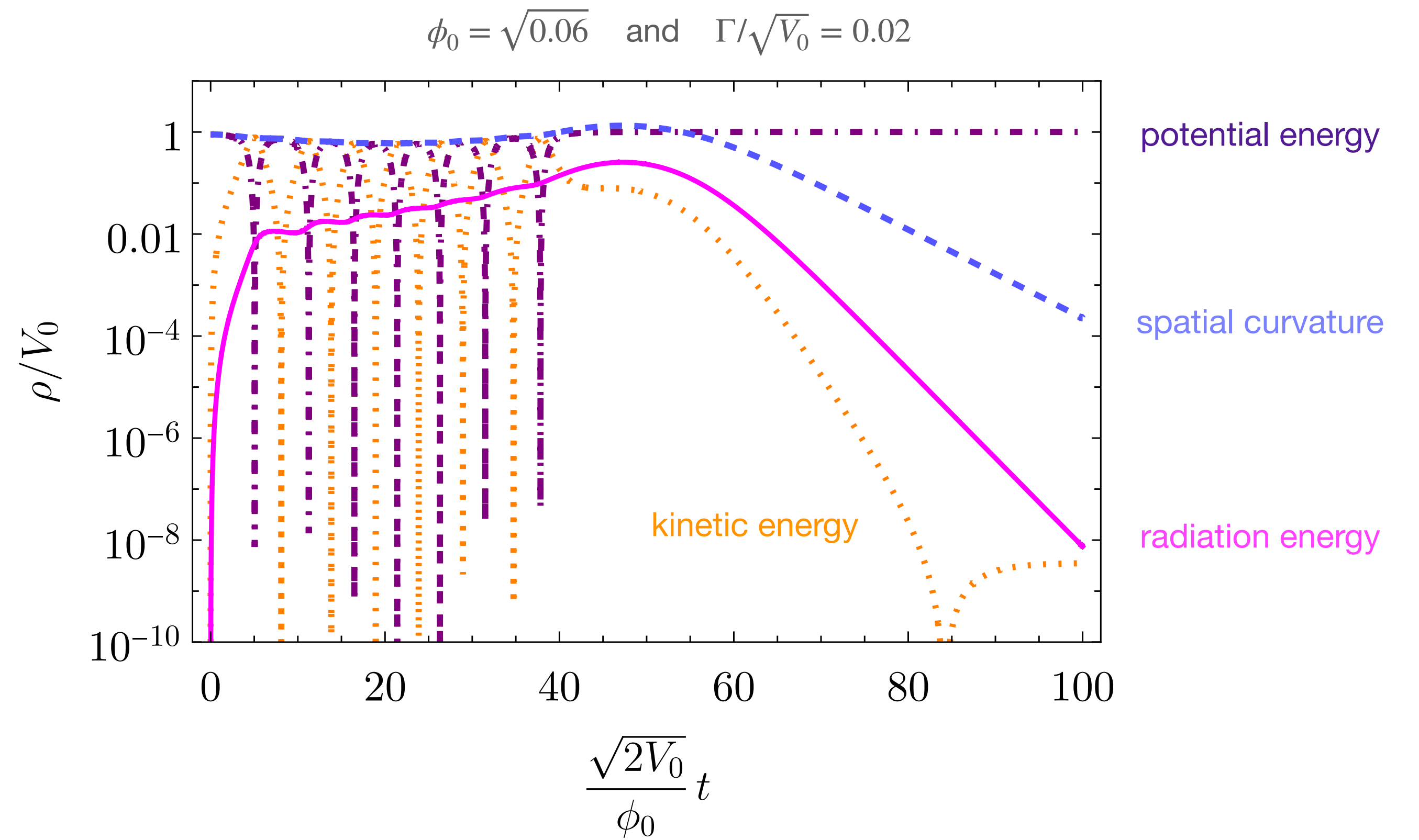
$$a(0) = \sqrt{\frac{3K}{V(\phi(0))}}, \quad \dot{a}(0) = 0, \quad \dot{\phi}(0) = 0, \quad \text{and} \quad \rho_r(0) = 0.$$

[Vilenkin, PRD 37 (1988) 888]

Emergence of Inflation

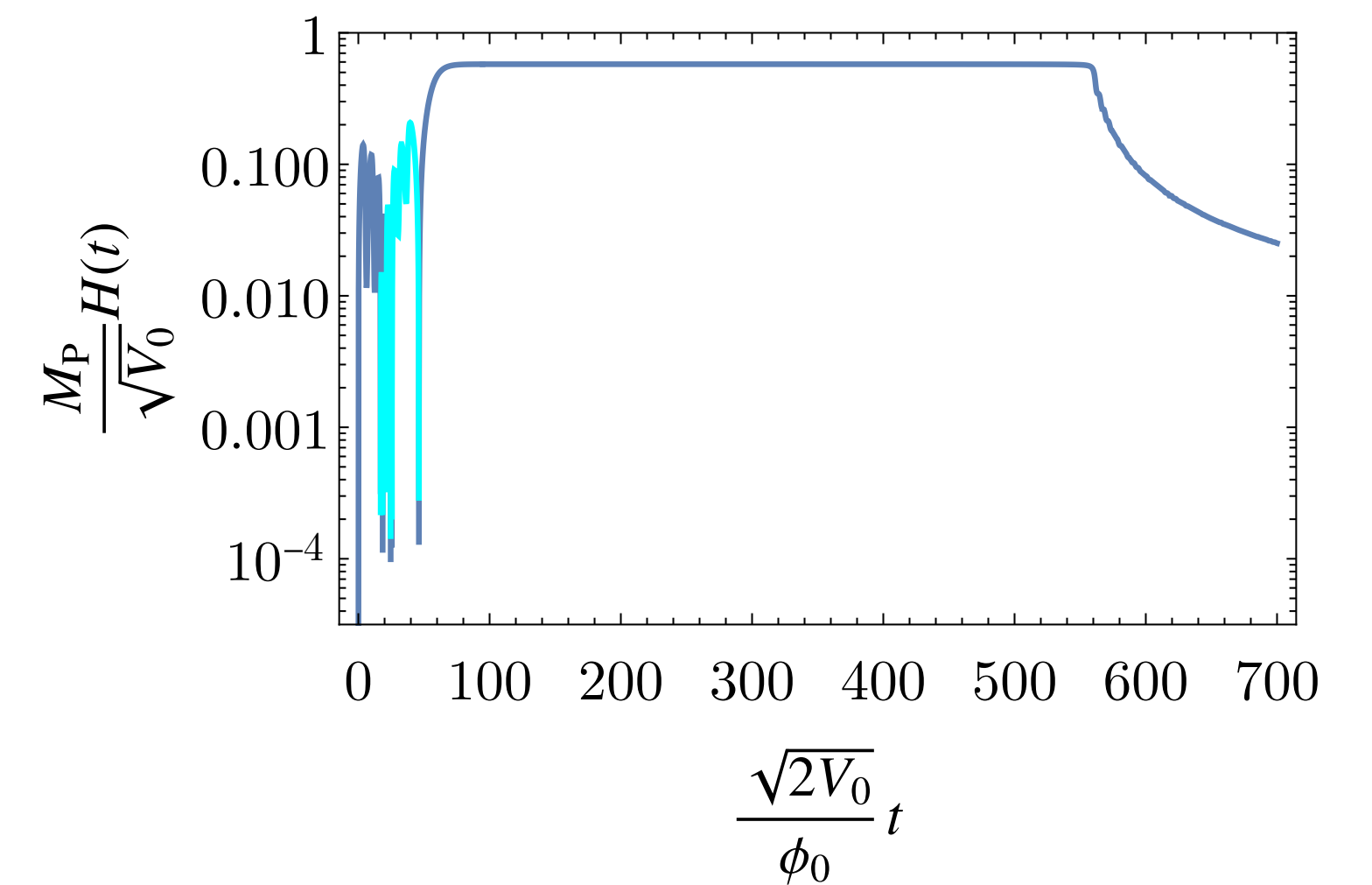
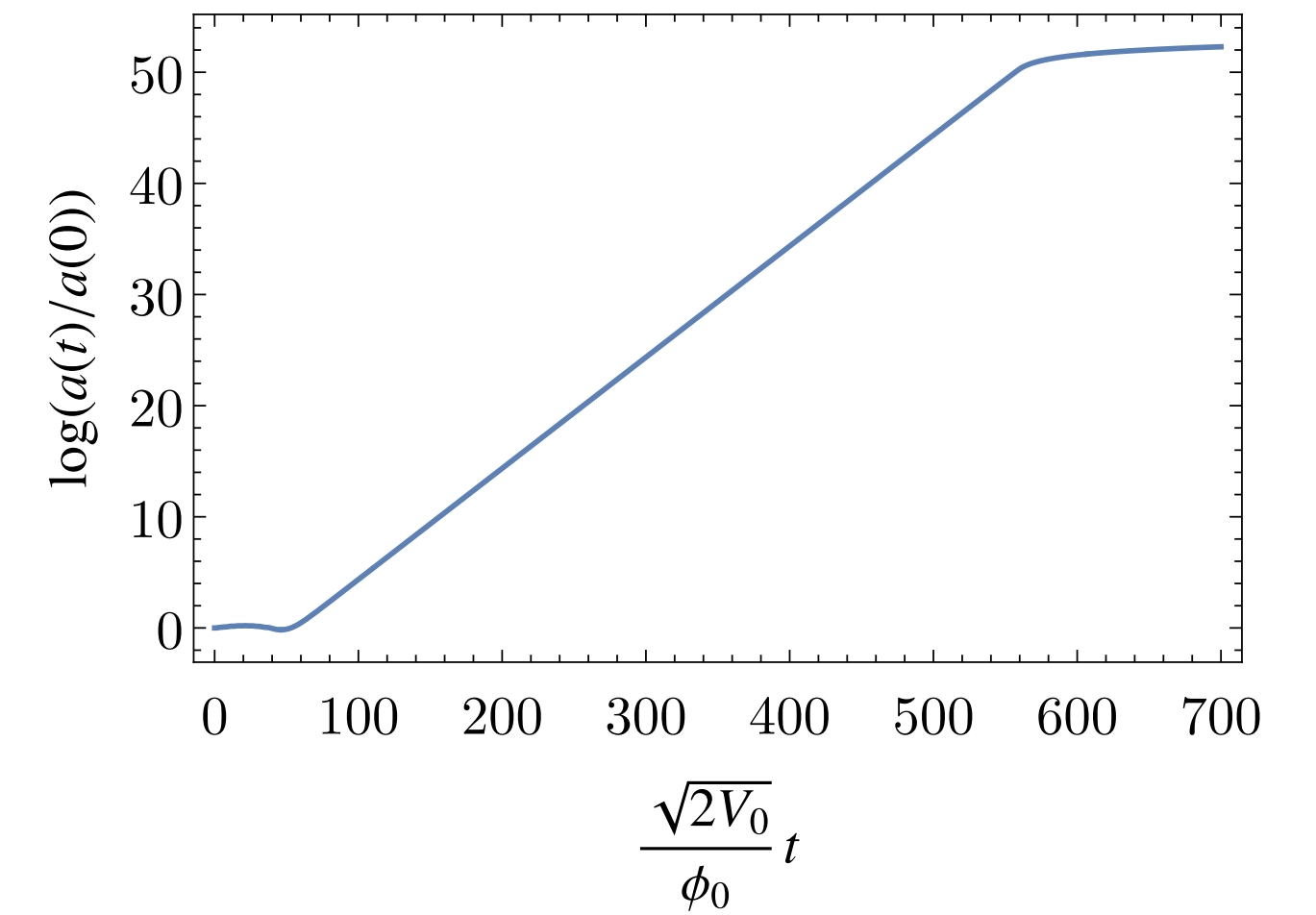
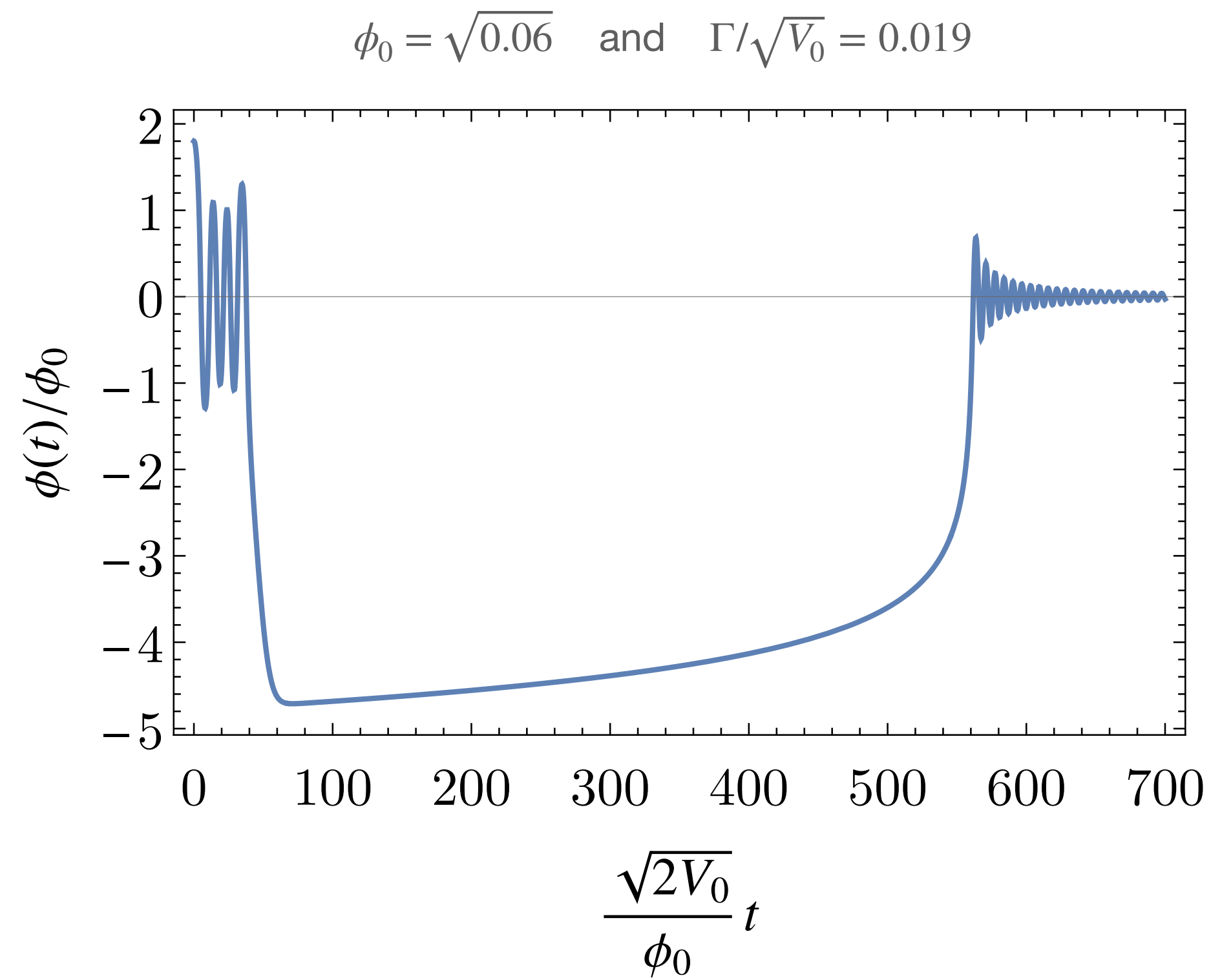


Solid lines: $\phi_0 = \sqrt{0.006}$, $\Gamma/\sqrt{V_0} = 0.003$.
 Dashed lines: $\phi_0 = \sqrt{0.06}$, $\Gamma/\sqrt{V_0} = 0.02$.



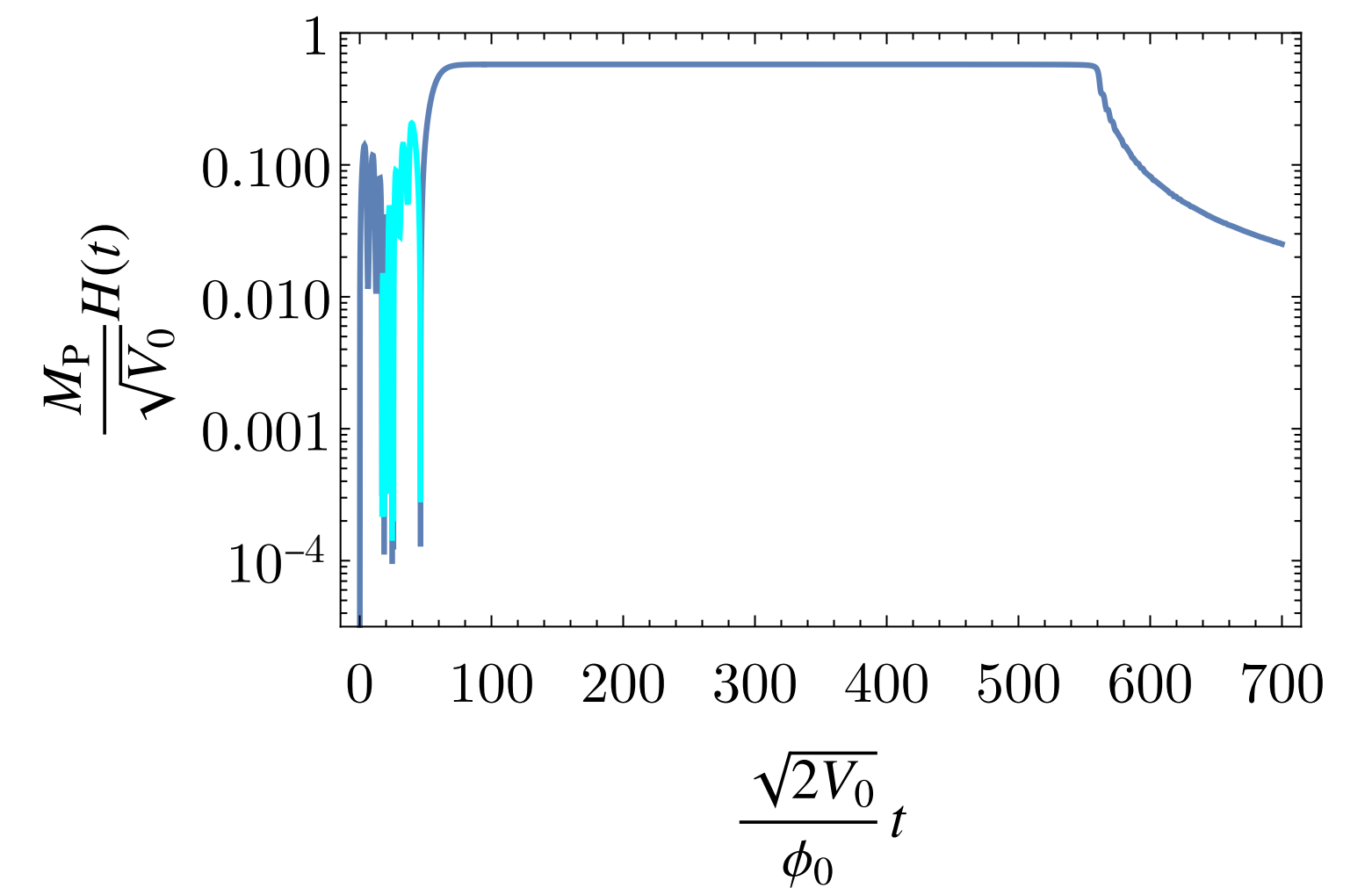
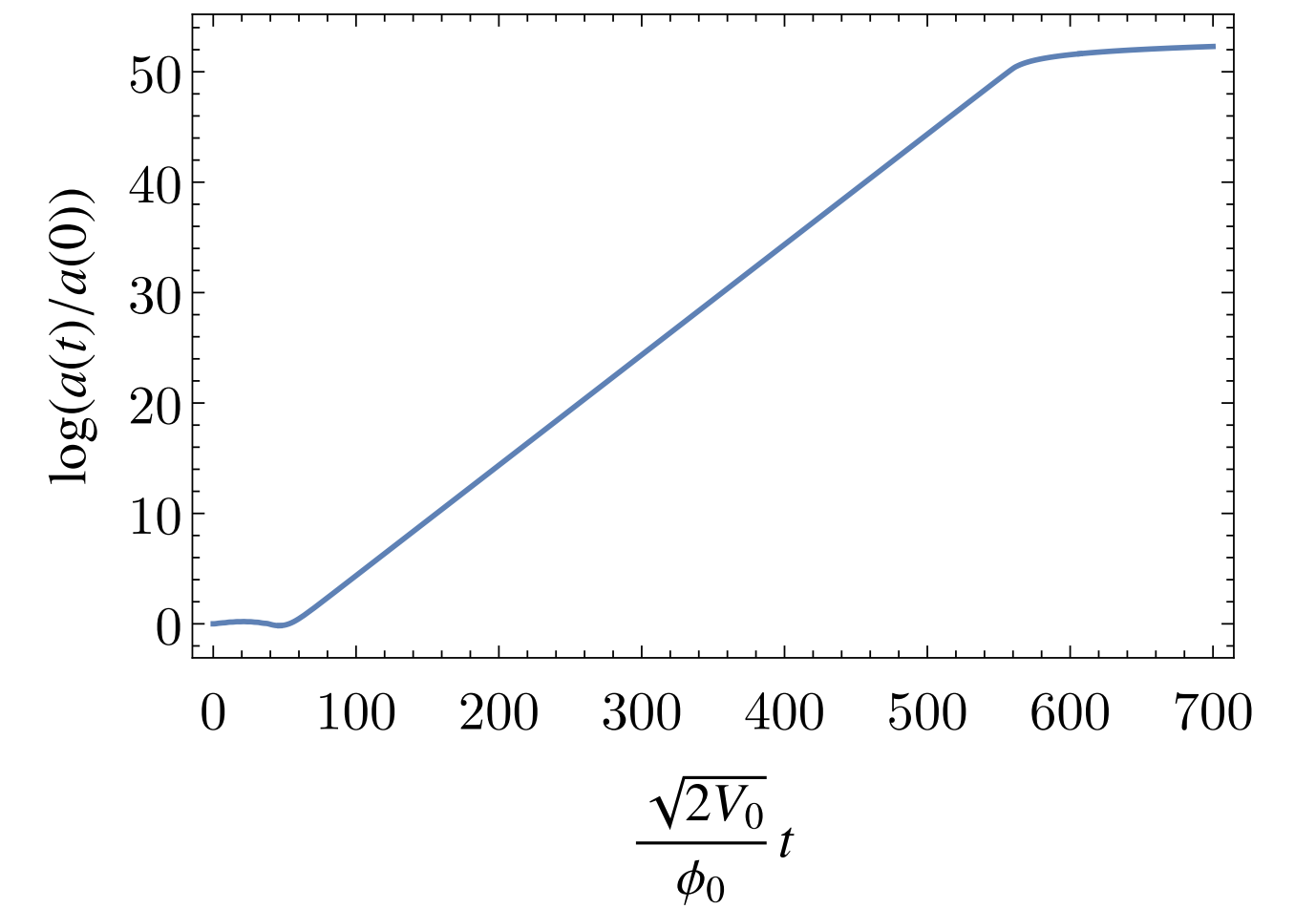
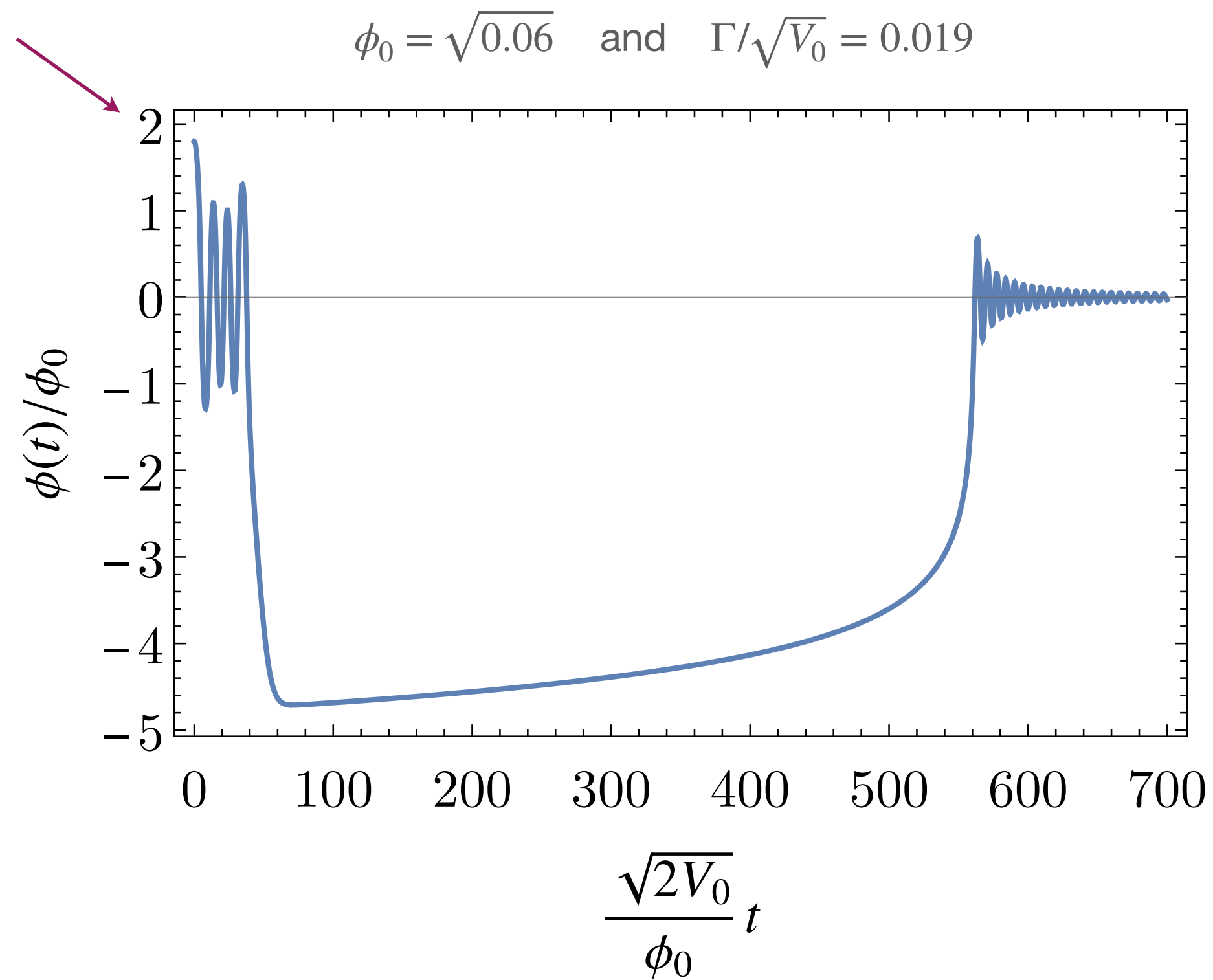
$\rho_r > 0 \rightarrow$ Bounce delayed $\rightarrow \rho_\phi$ increases $\rightarrow \phi$ reaches the plateau

Full Picture



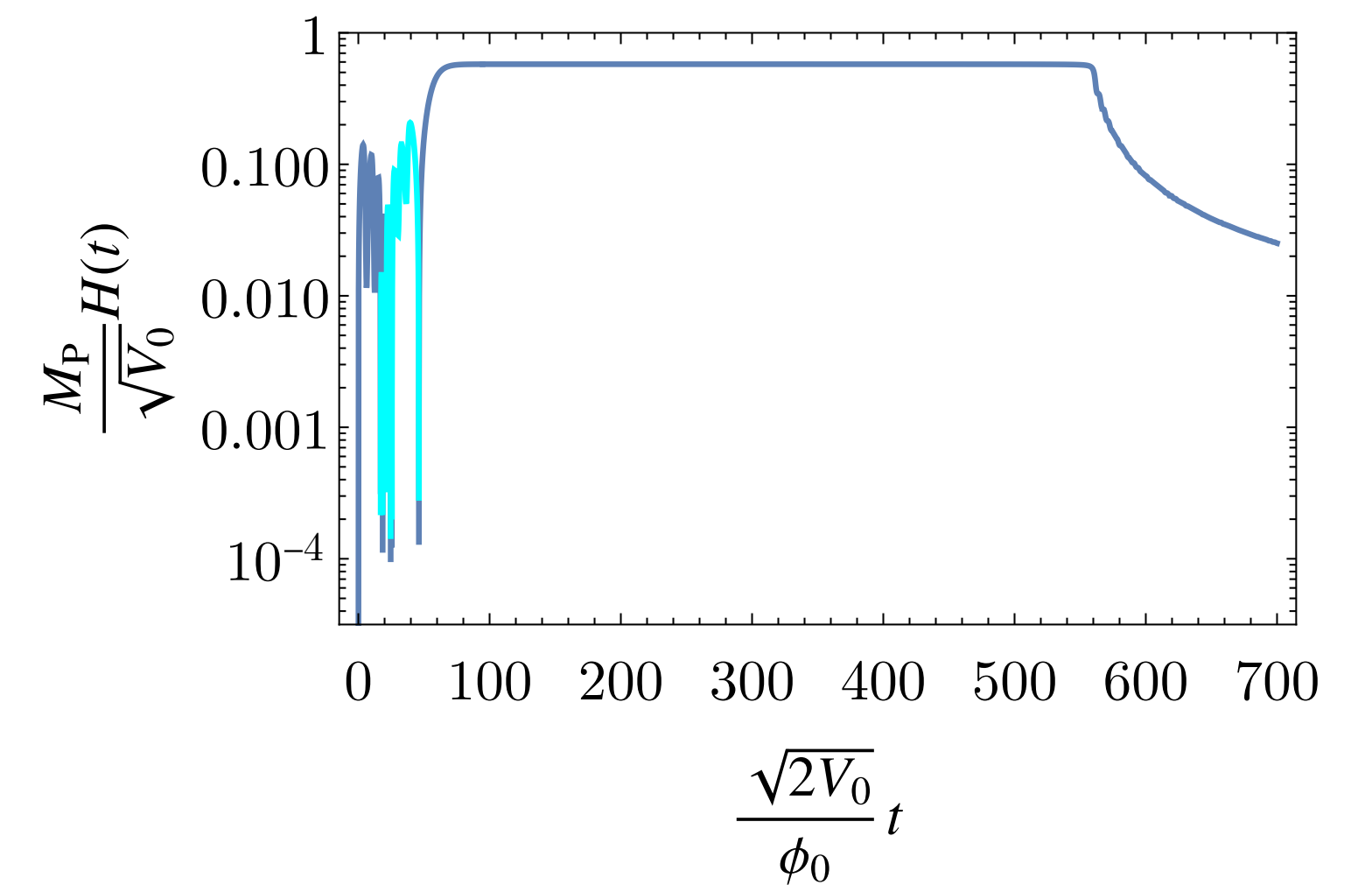
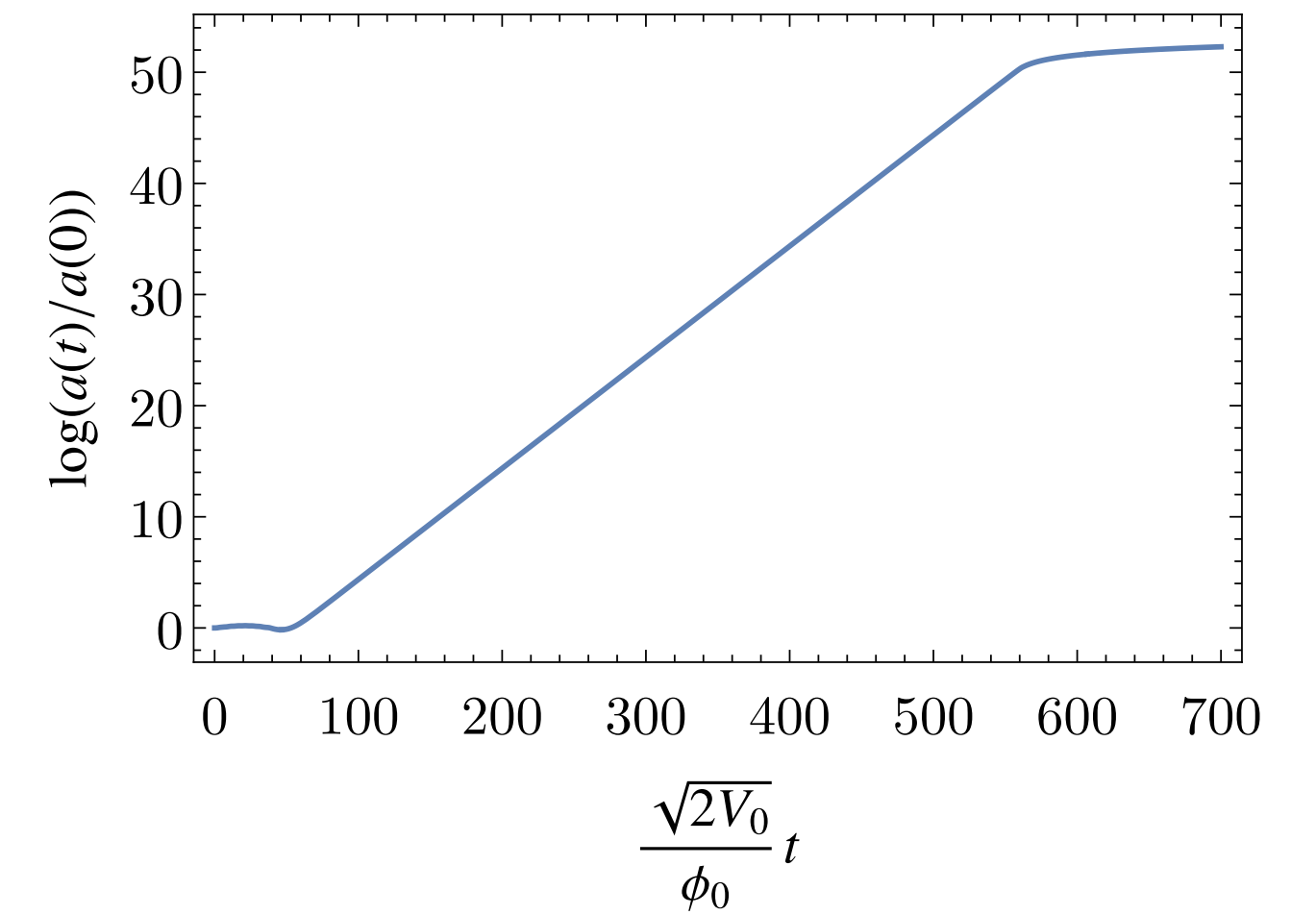
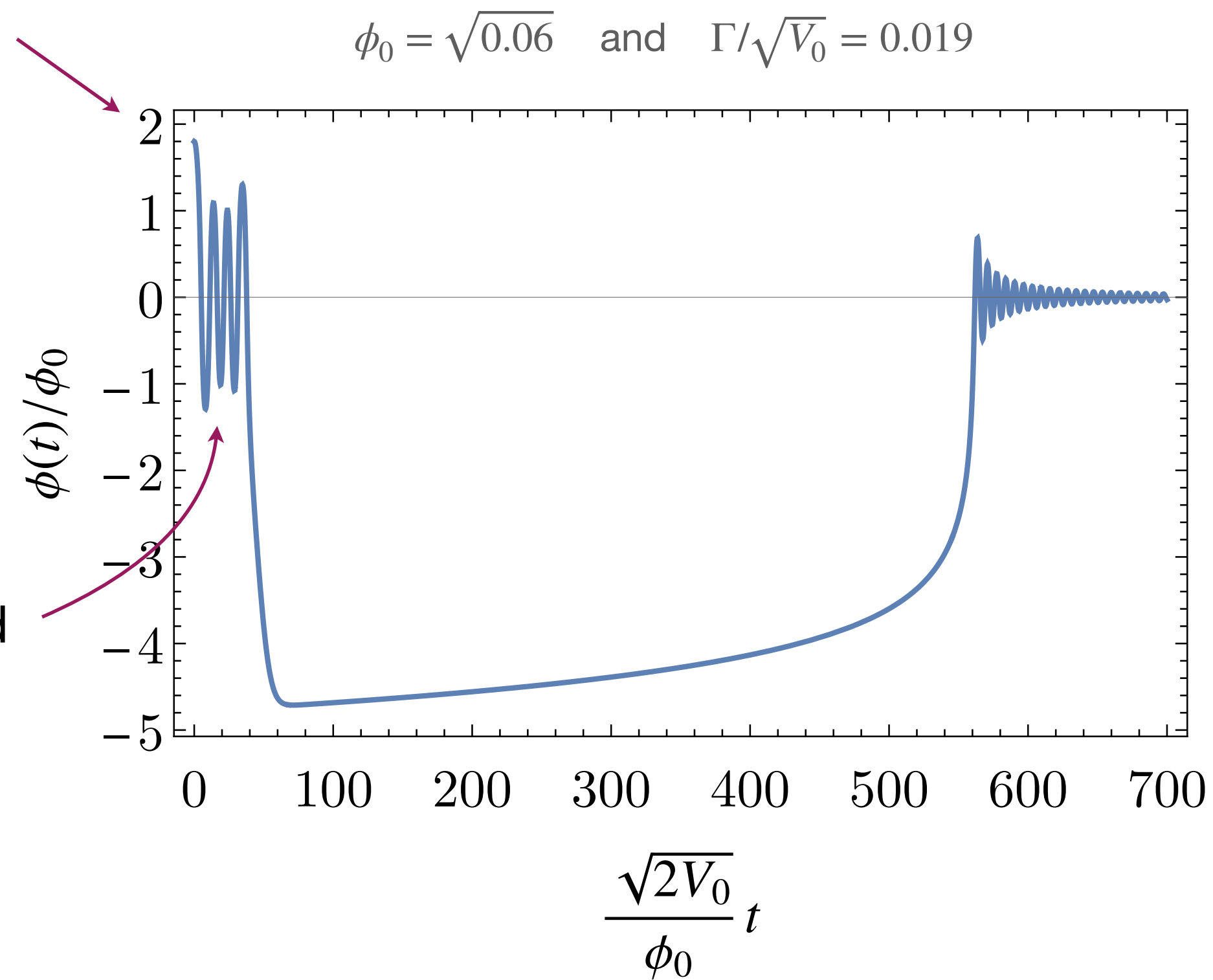
Full Picture

Quantum creation
(initial conditions)

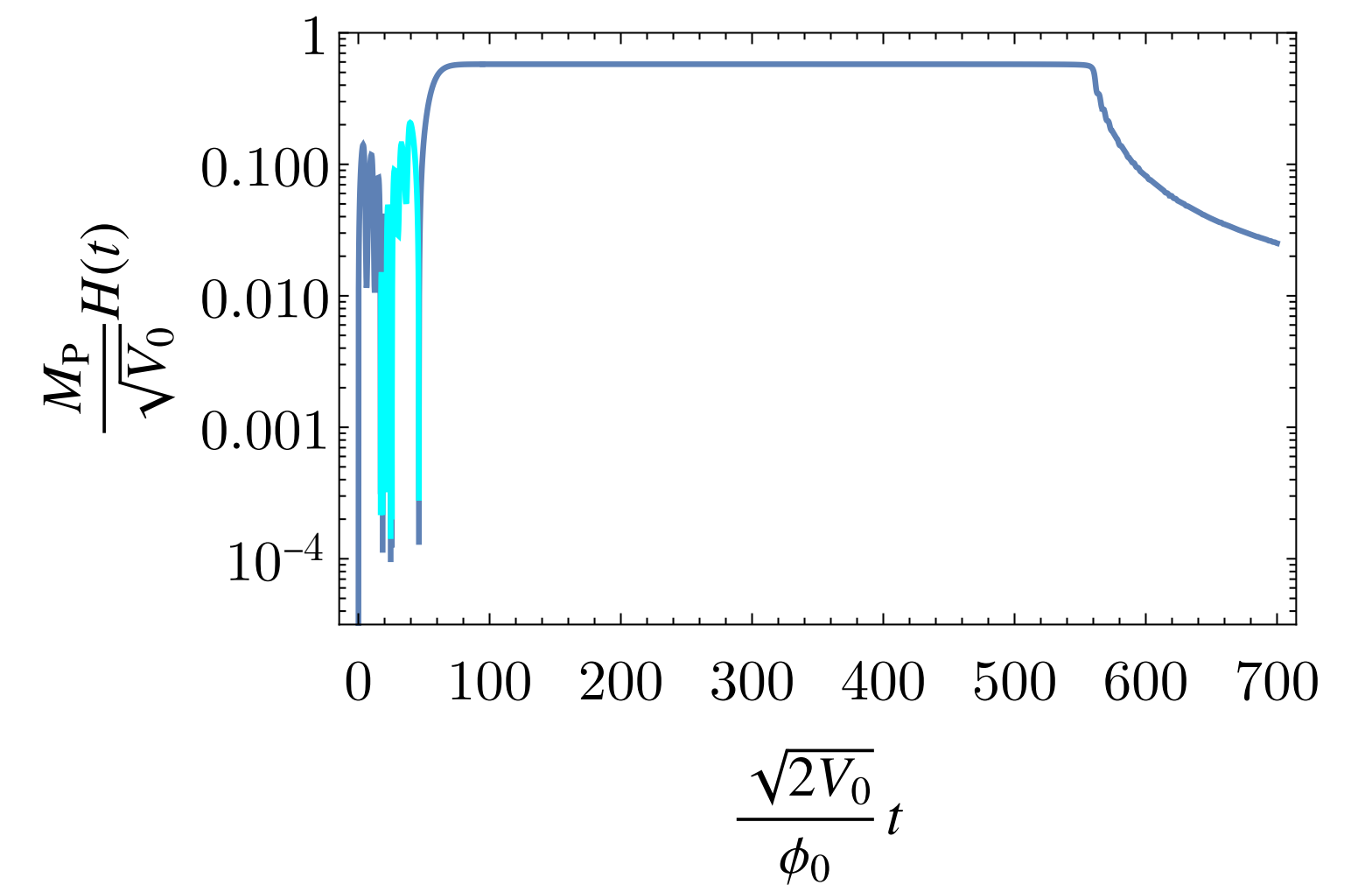
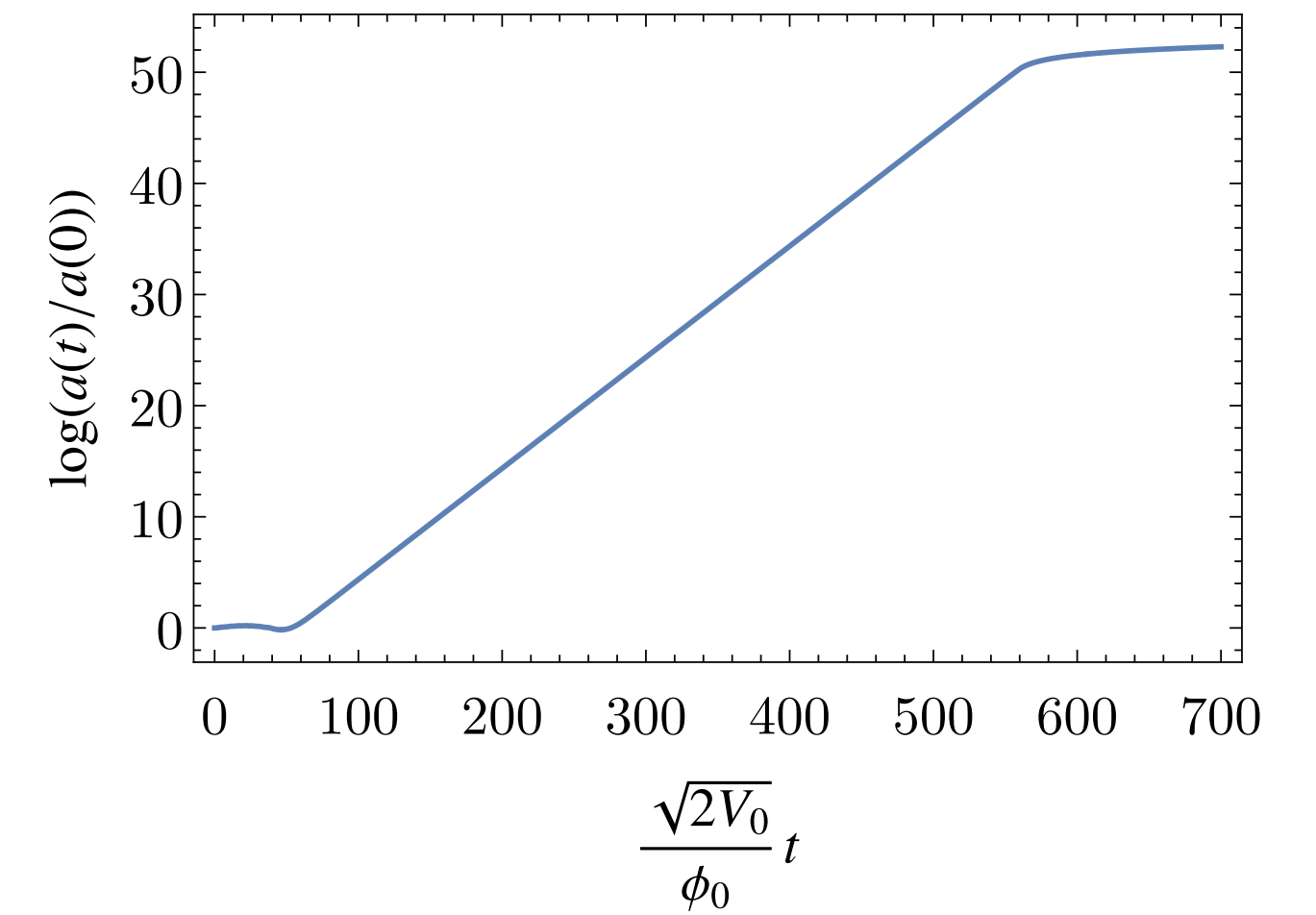
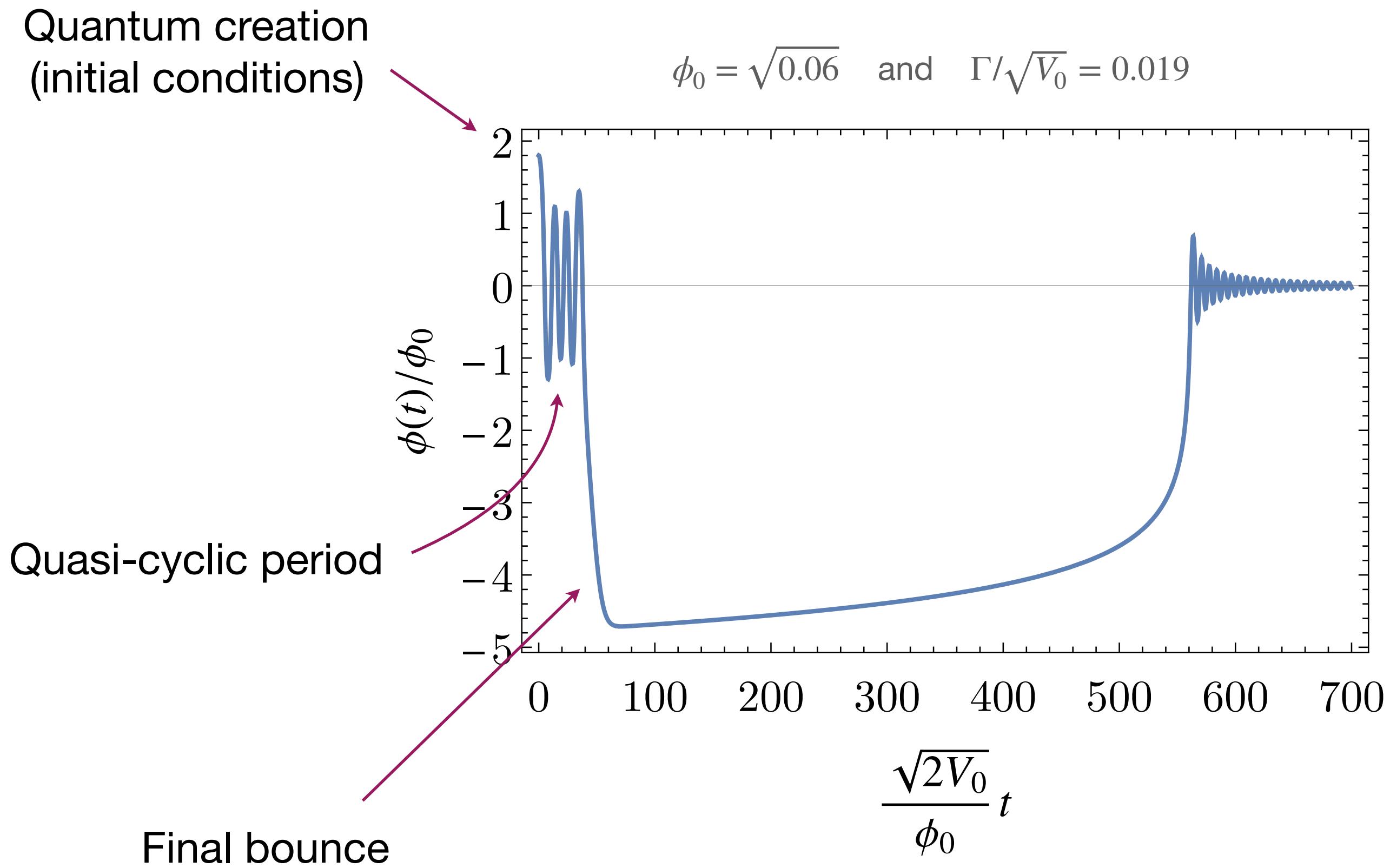


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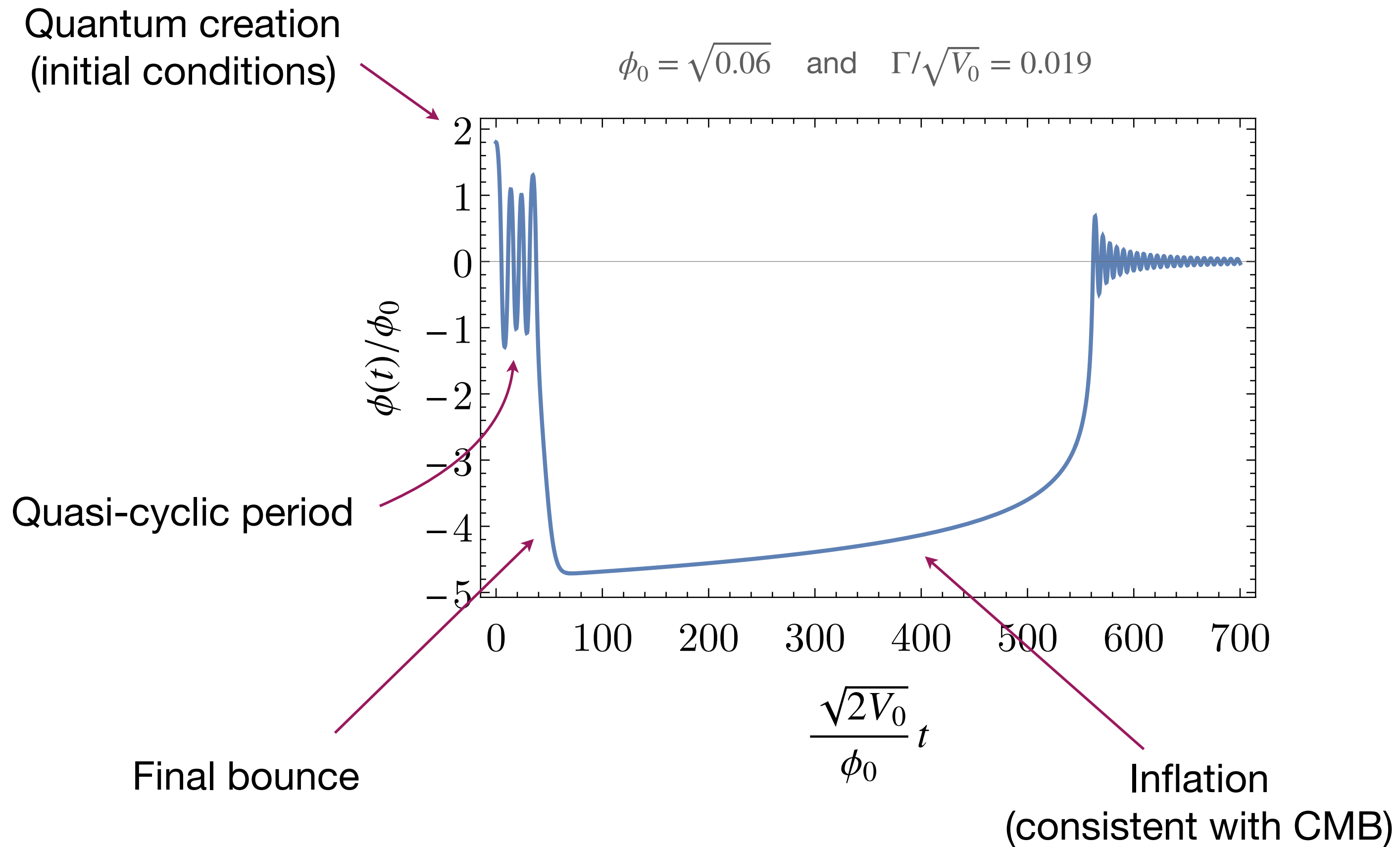
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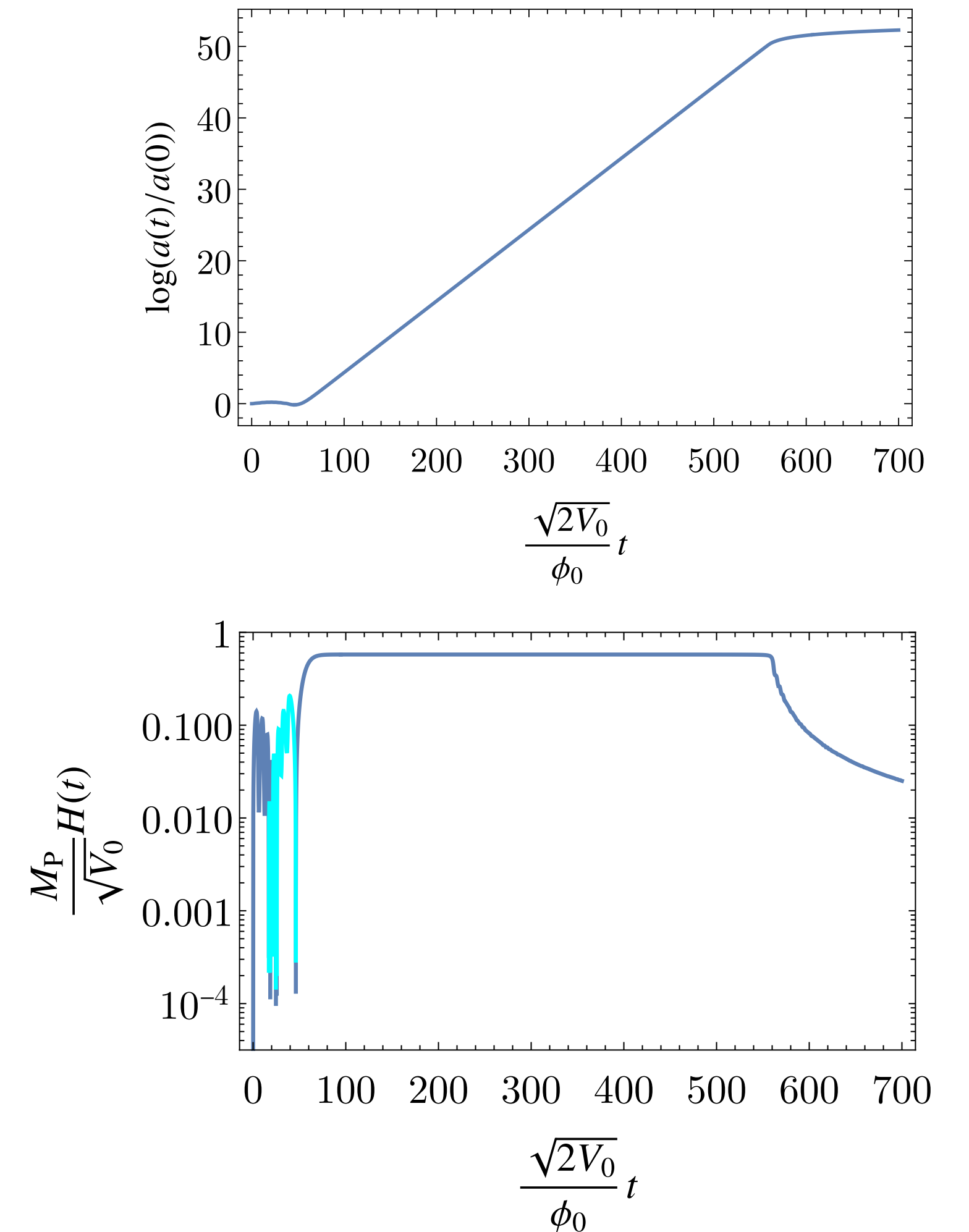
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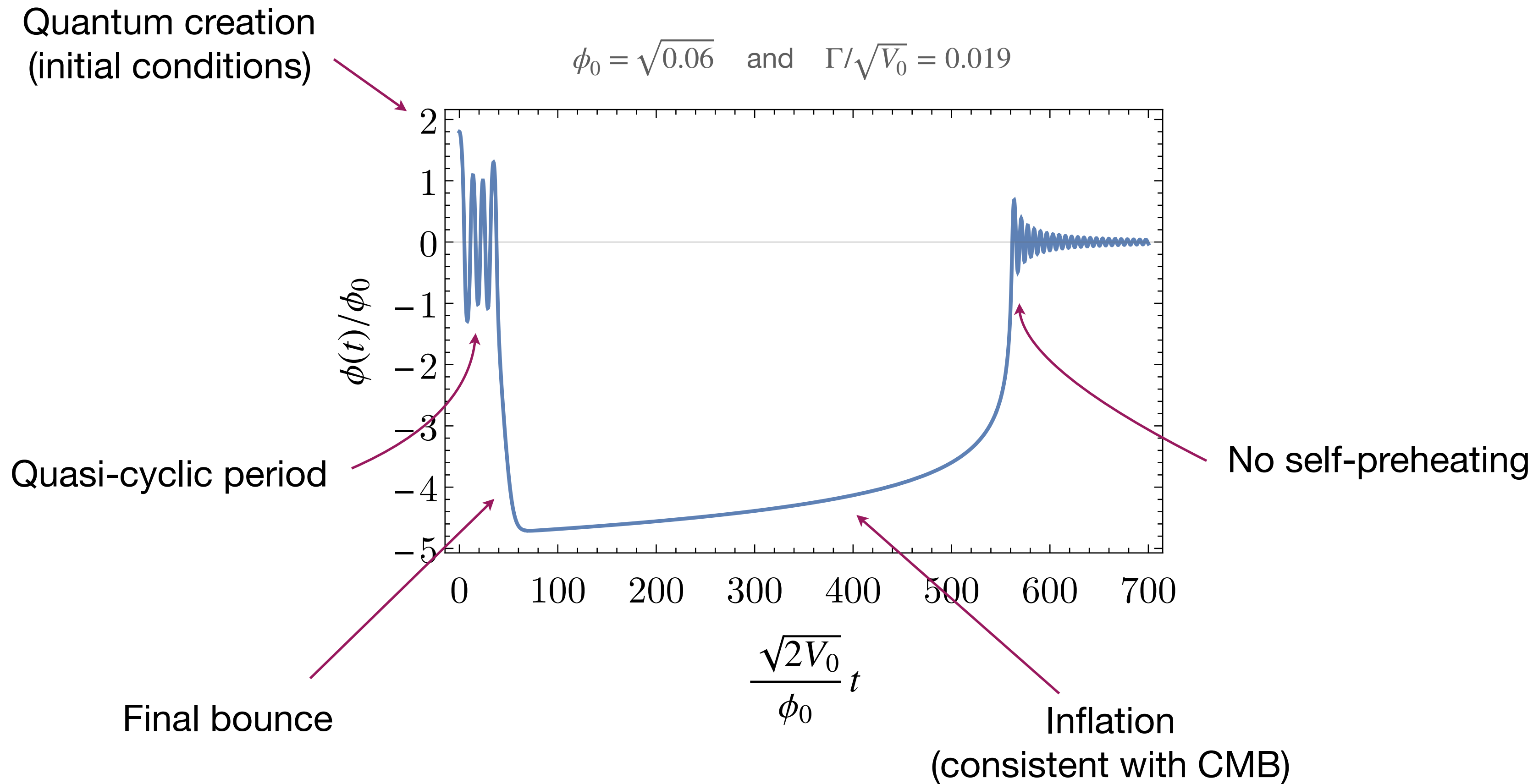
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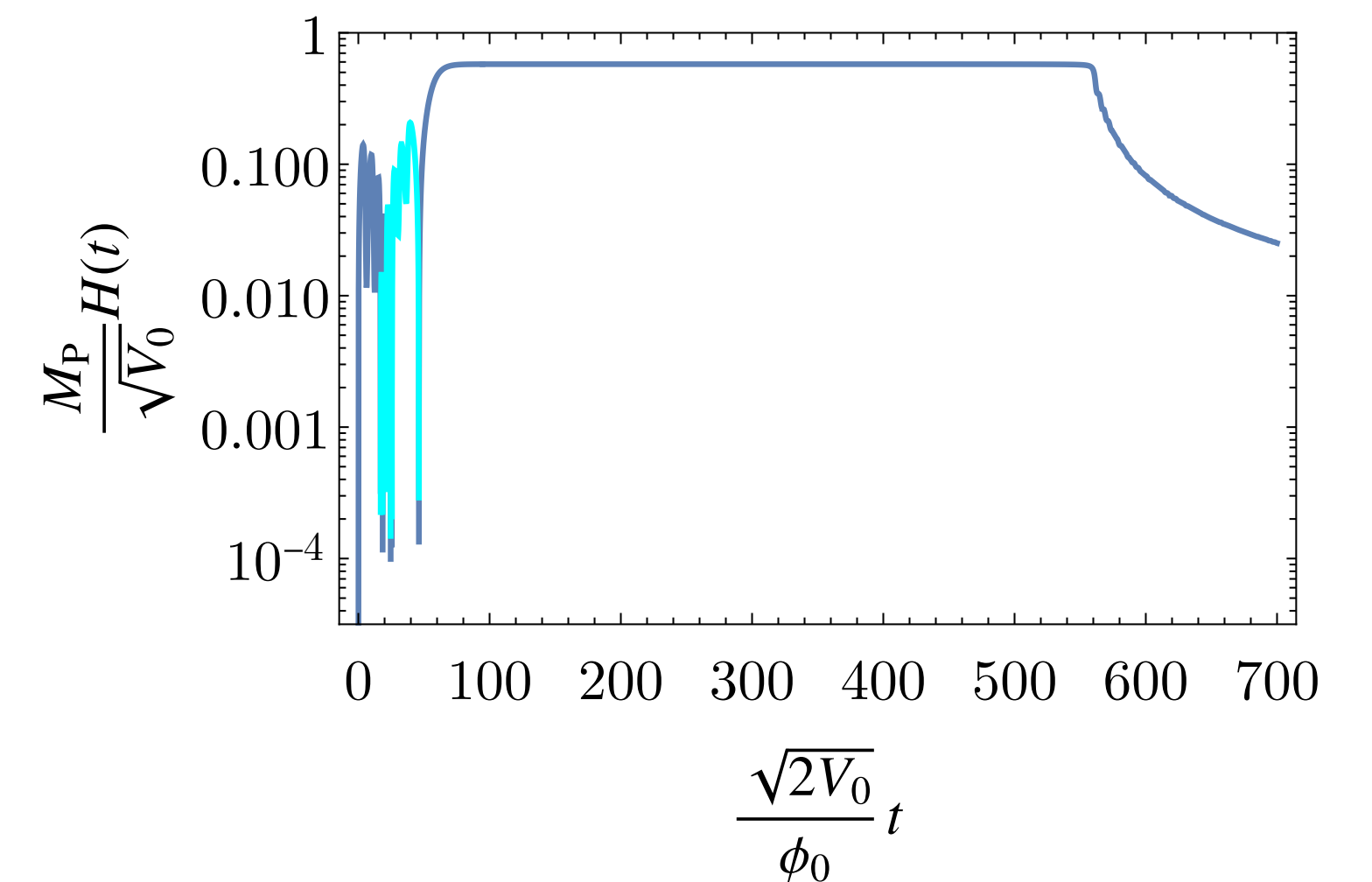
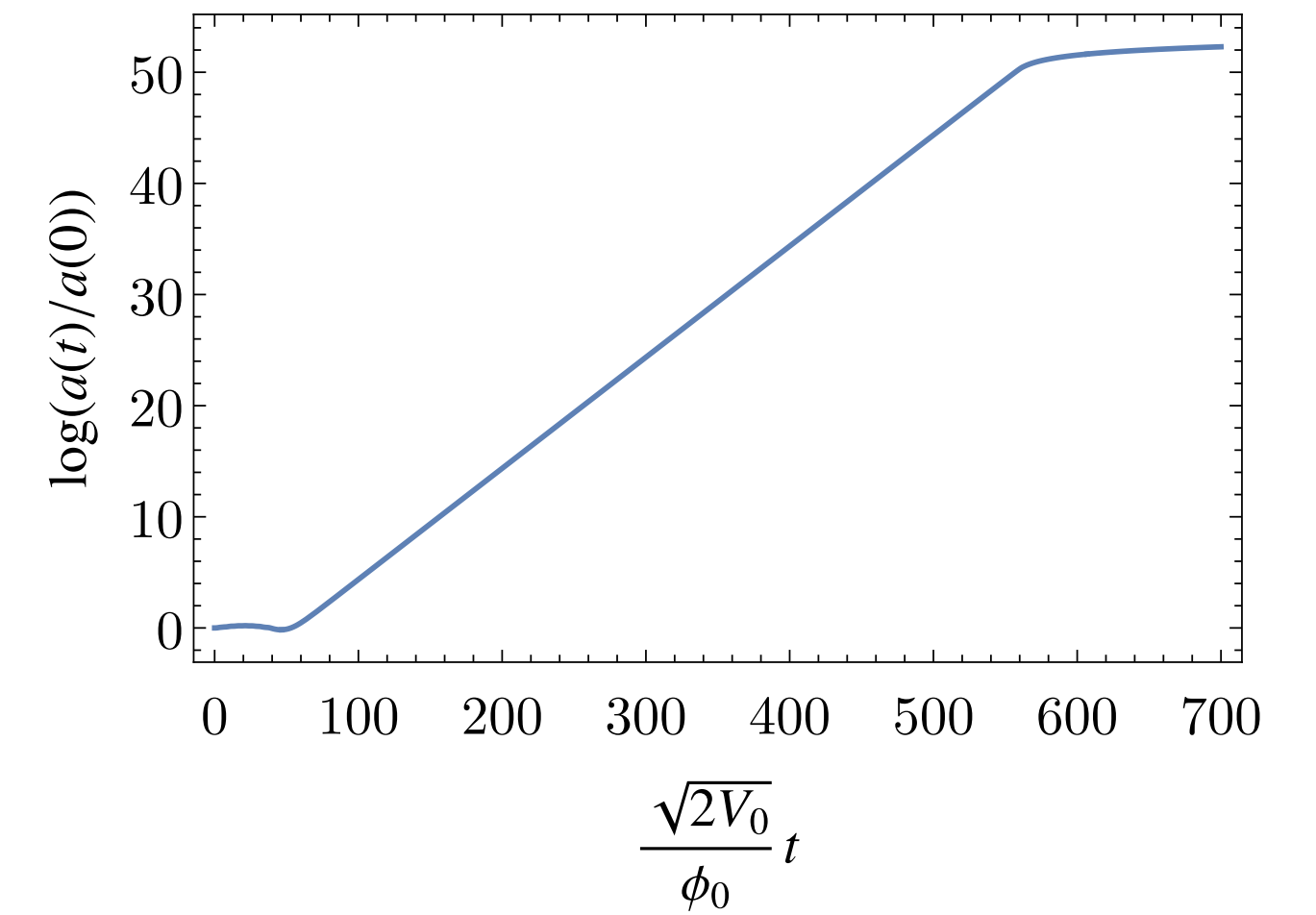
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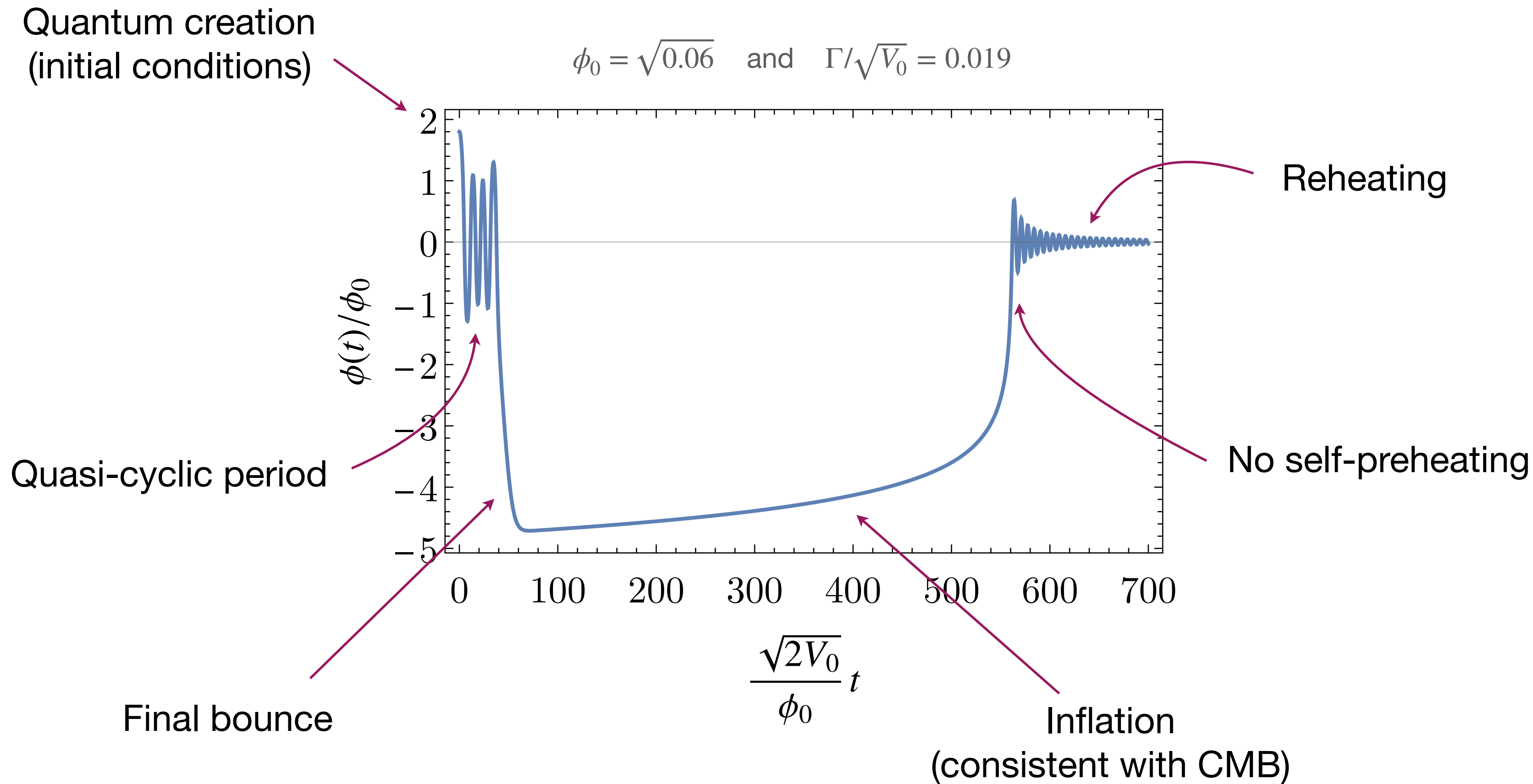
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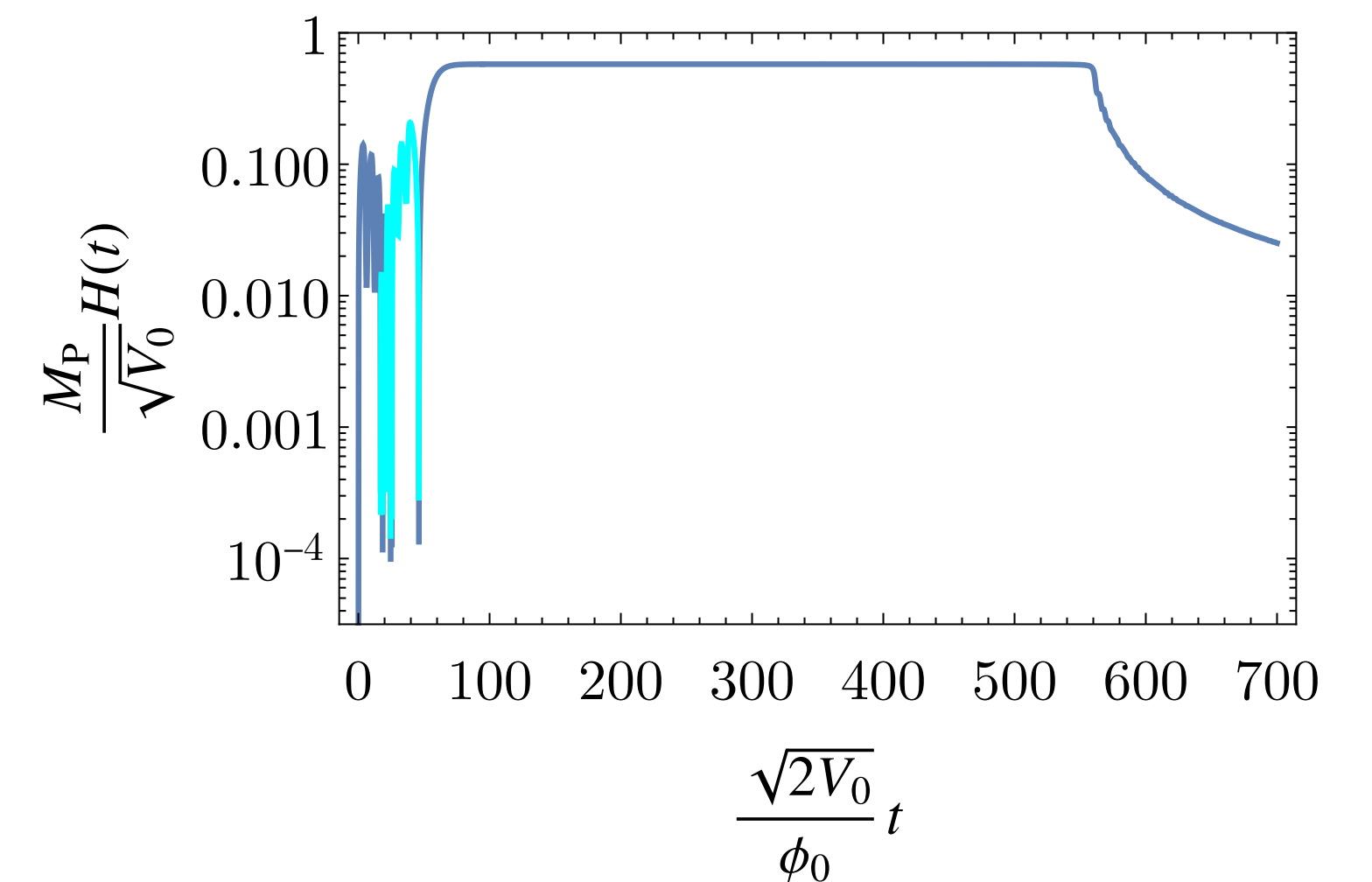
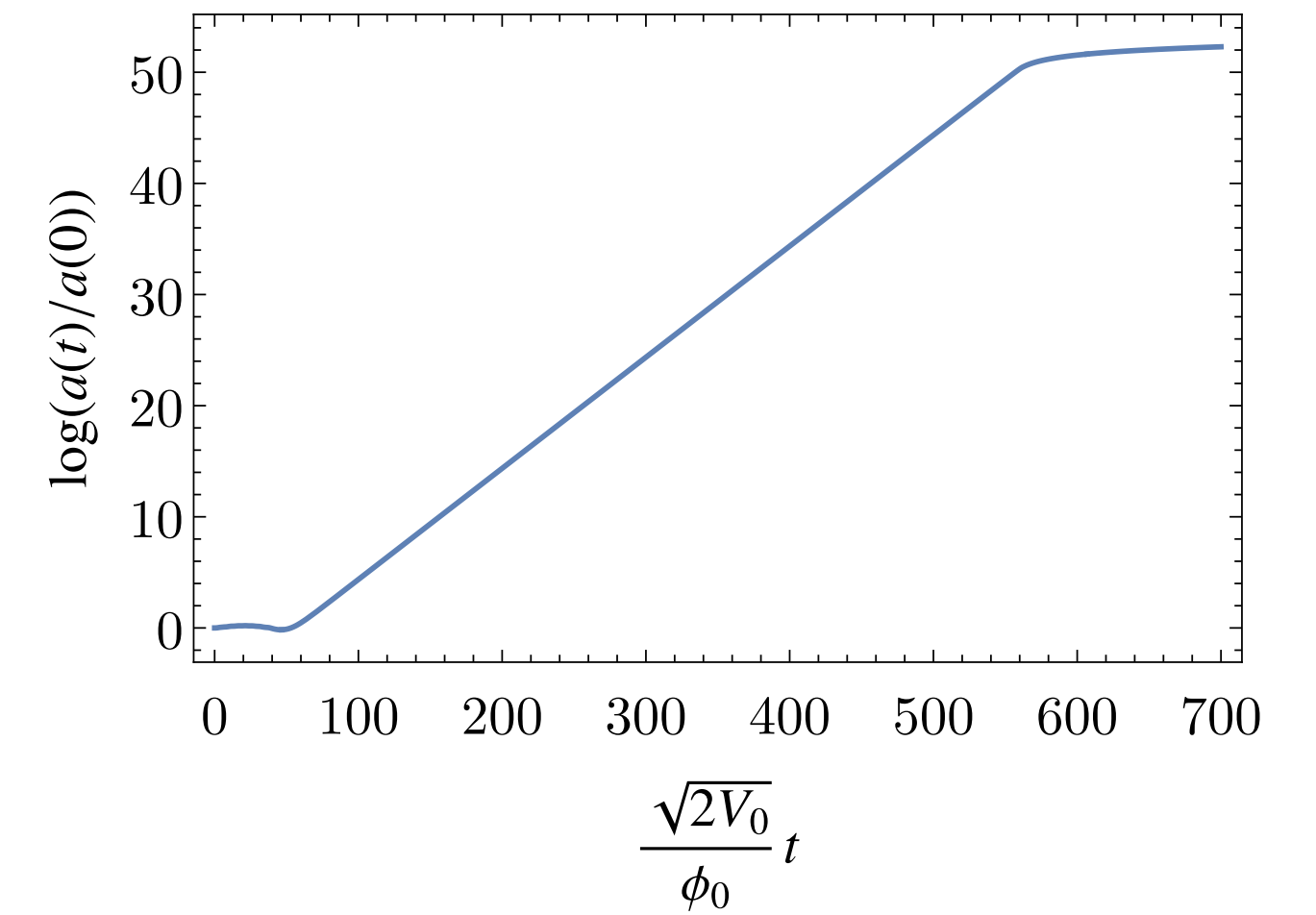
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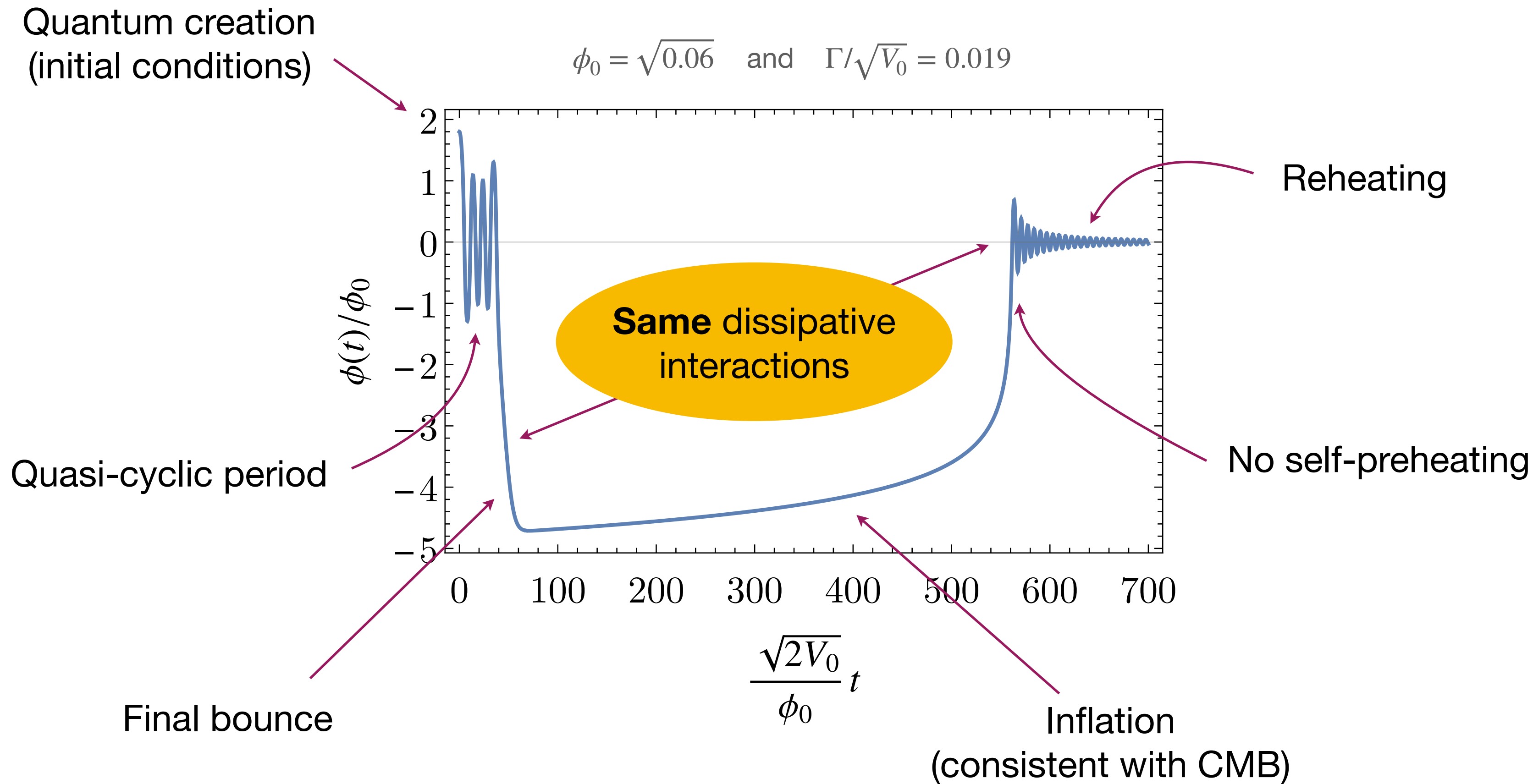
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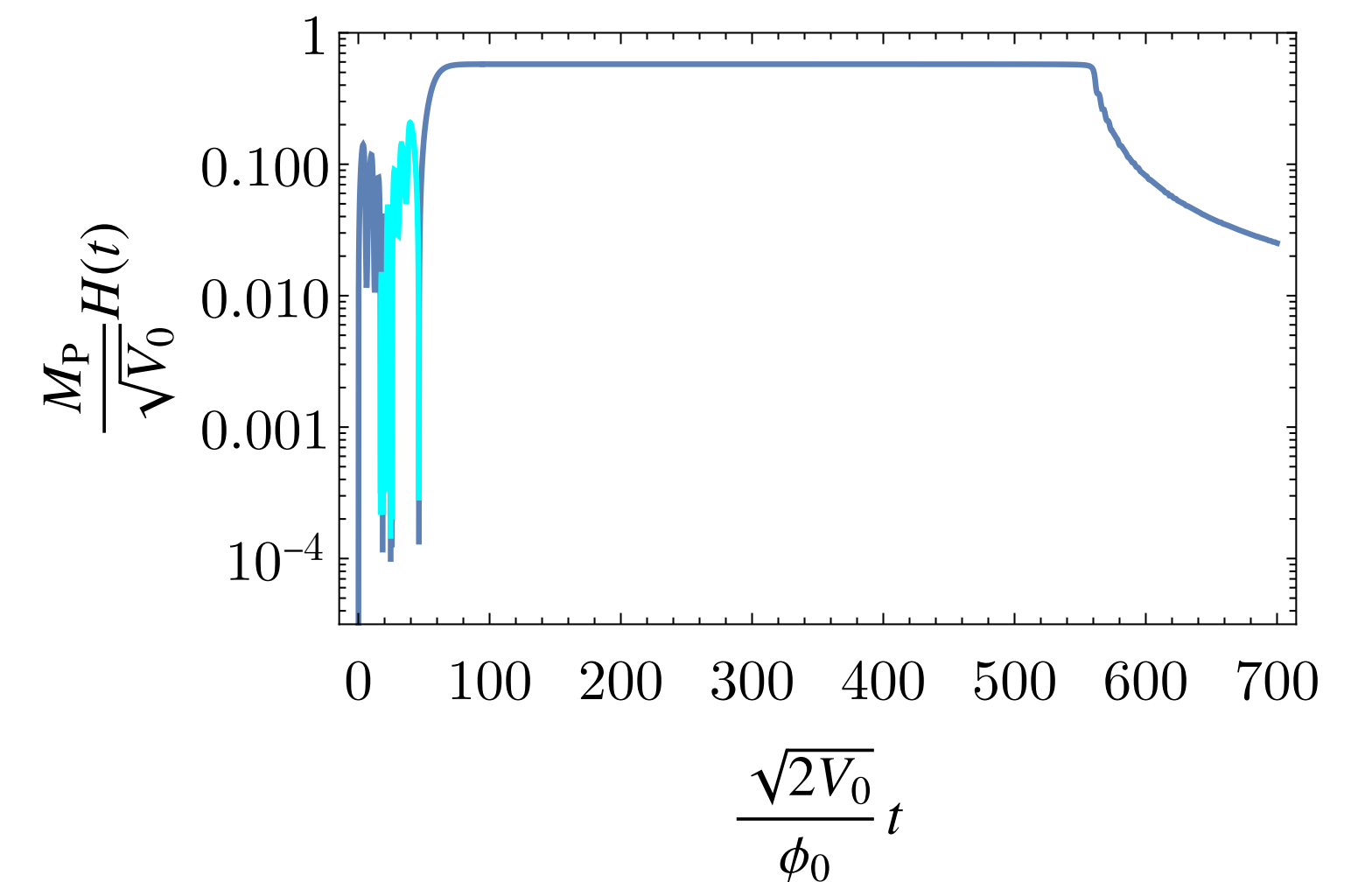
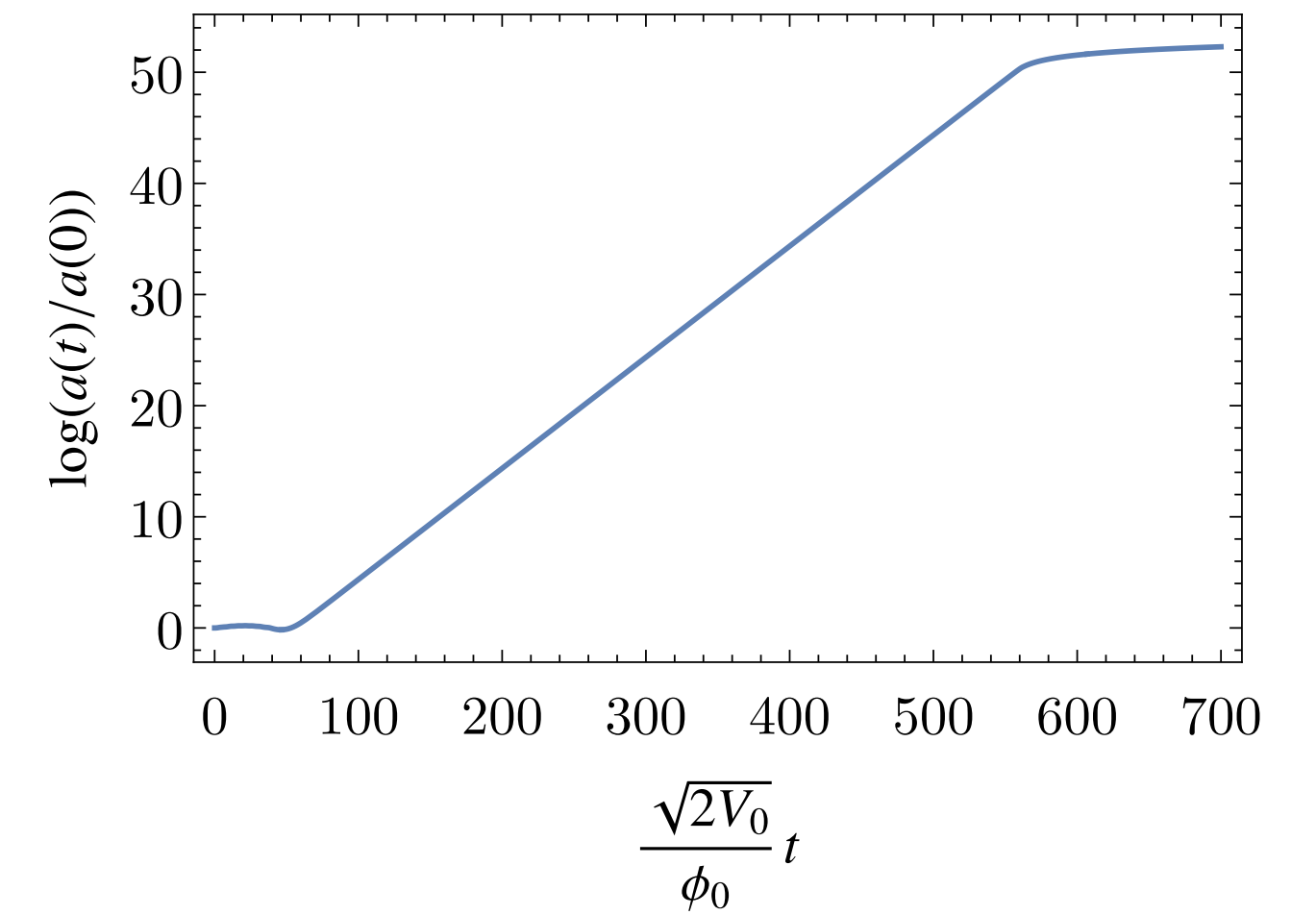
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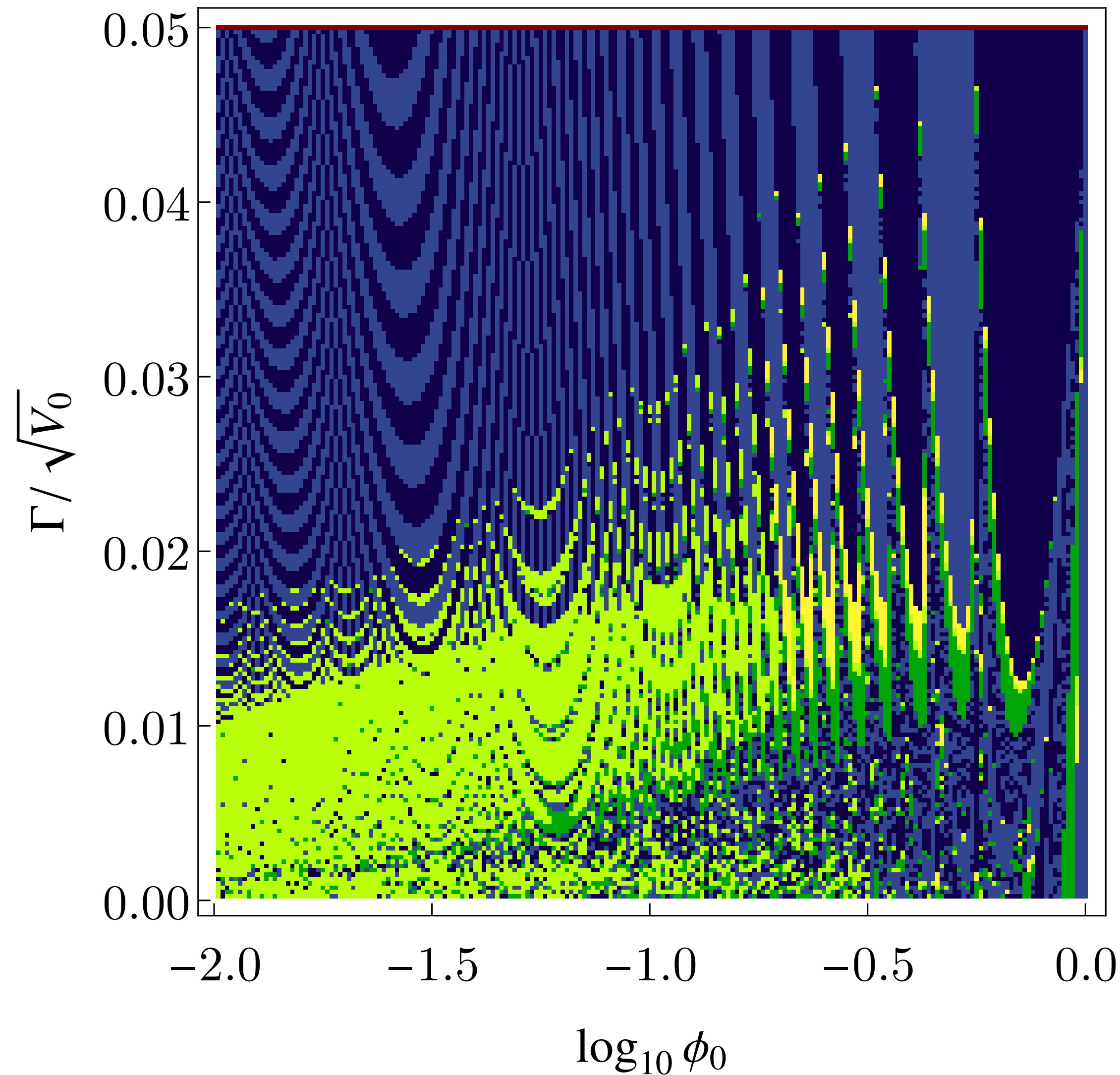
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Generality of the Mechanism



How generic is the beginning of inflation?

We set $\phi_0 / \sqrt{V_0} = 2.1 \times 10^5$ and $\phi_{\text{ini}} / \phi_0 = 1 - 1.3 \log_{10} \phi_0$.

- Big Crunch with $\dot{\phi} > 0$
- Big Crunch with $\dot{\phi} < 0$
- Cyclic Universe
- Short inflation ($N < 50$)
- Long inflation ($N \geq 50$) if fragmentation is neglected
- Long inflation ($N \geq 50$) before the fragmentation time

Probability of Inflation

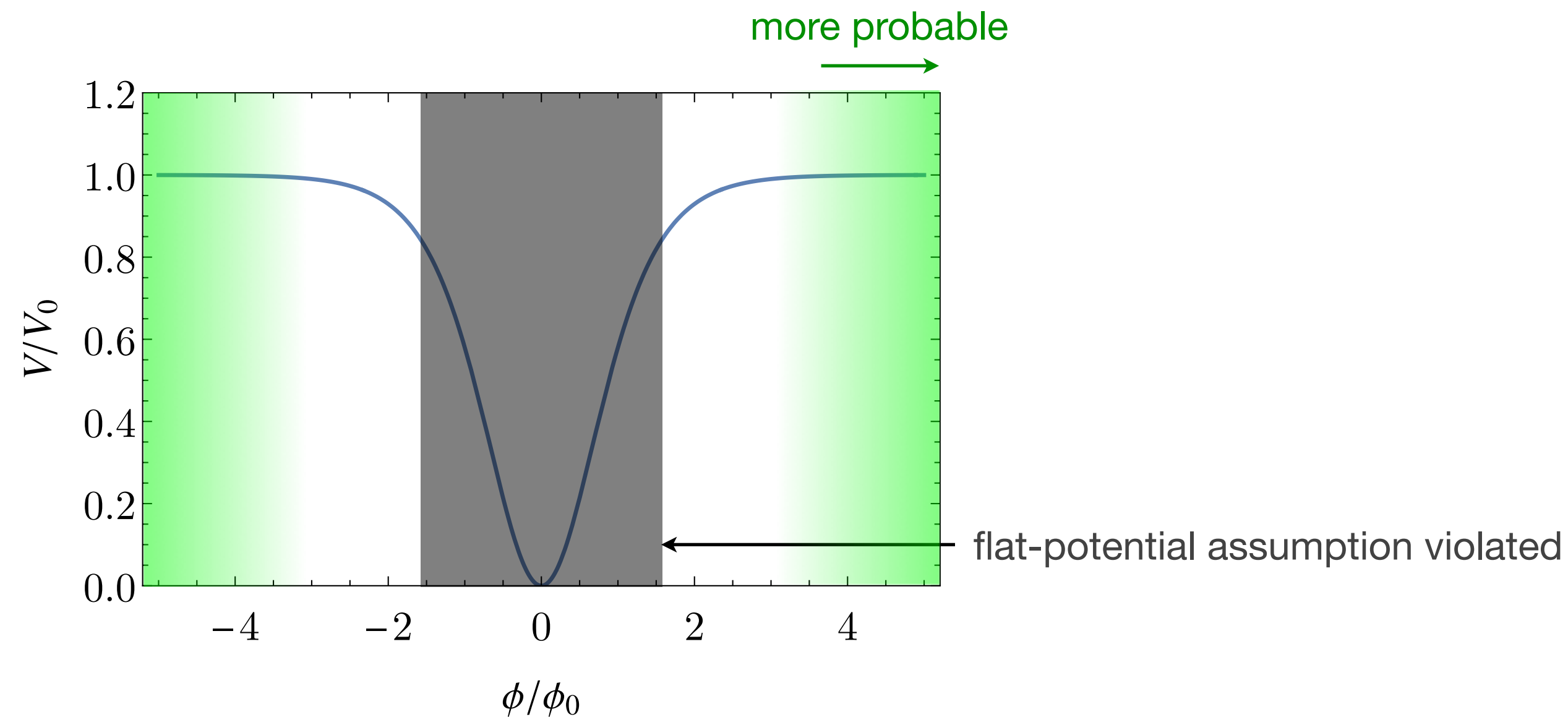
Probability distribution function of the homogeneous initial field value $\phi(0)$ in quantum cosmology

$$P[\phi(0)] \sim \exp\left(\pm \frac{24\pi^2}{V(\phi(0))}\right)$$

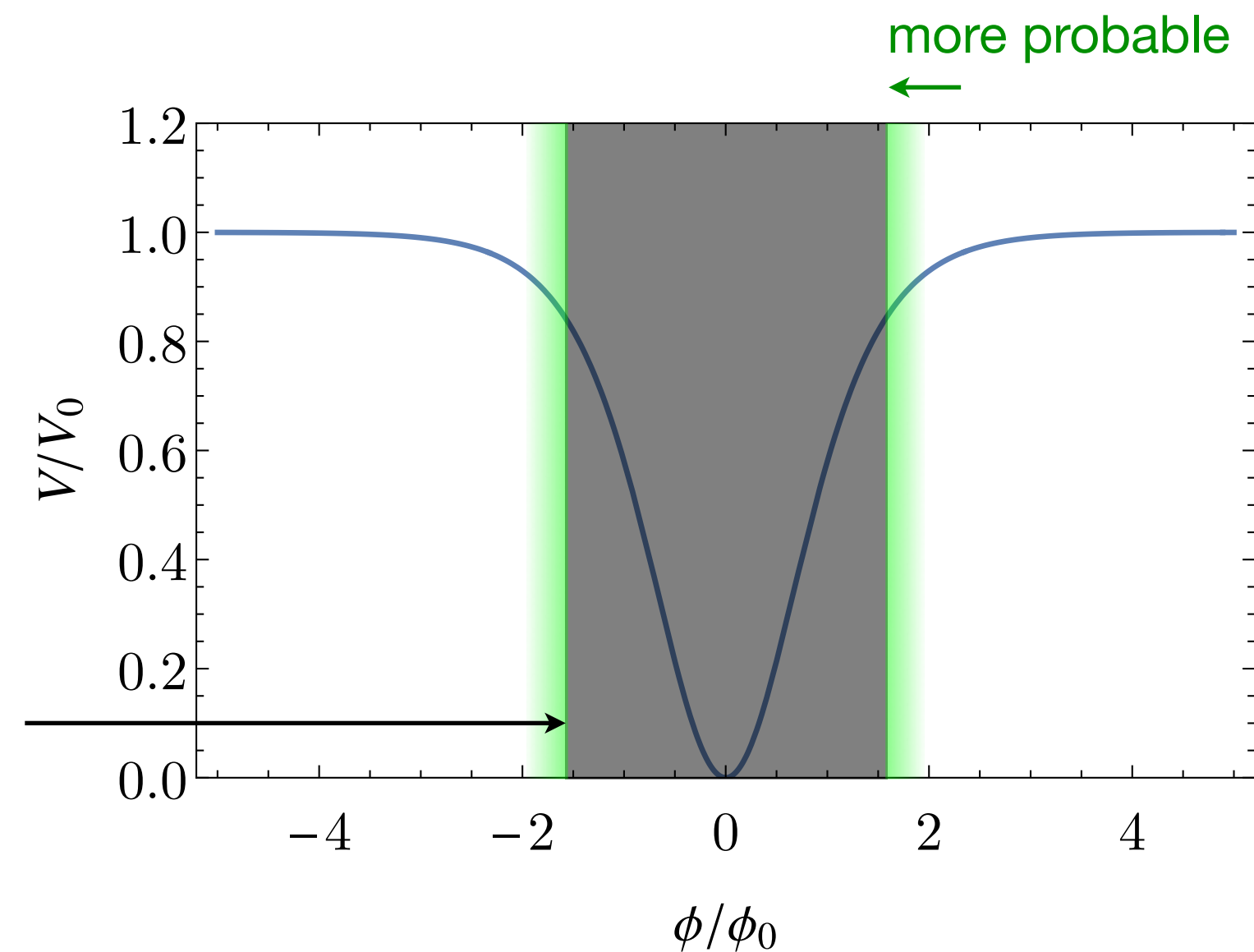
+: no-boundary proposal
 -: tunneling proposal

Note: the exponent is $\mathcal{O}(10^{14})$ for the CMB-compatible case.

tunneling proposal



no-boundary proposal



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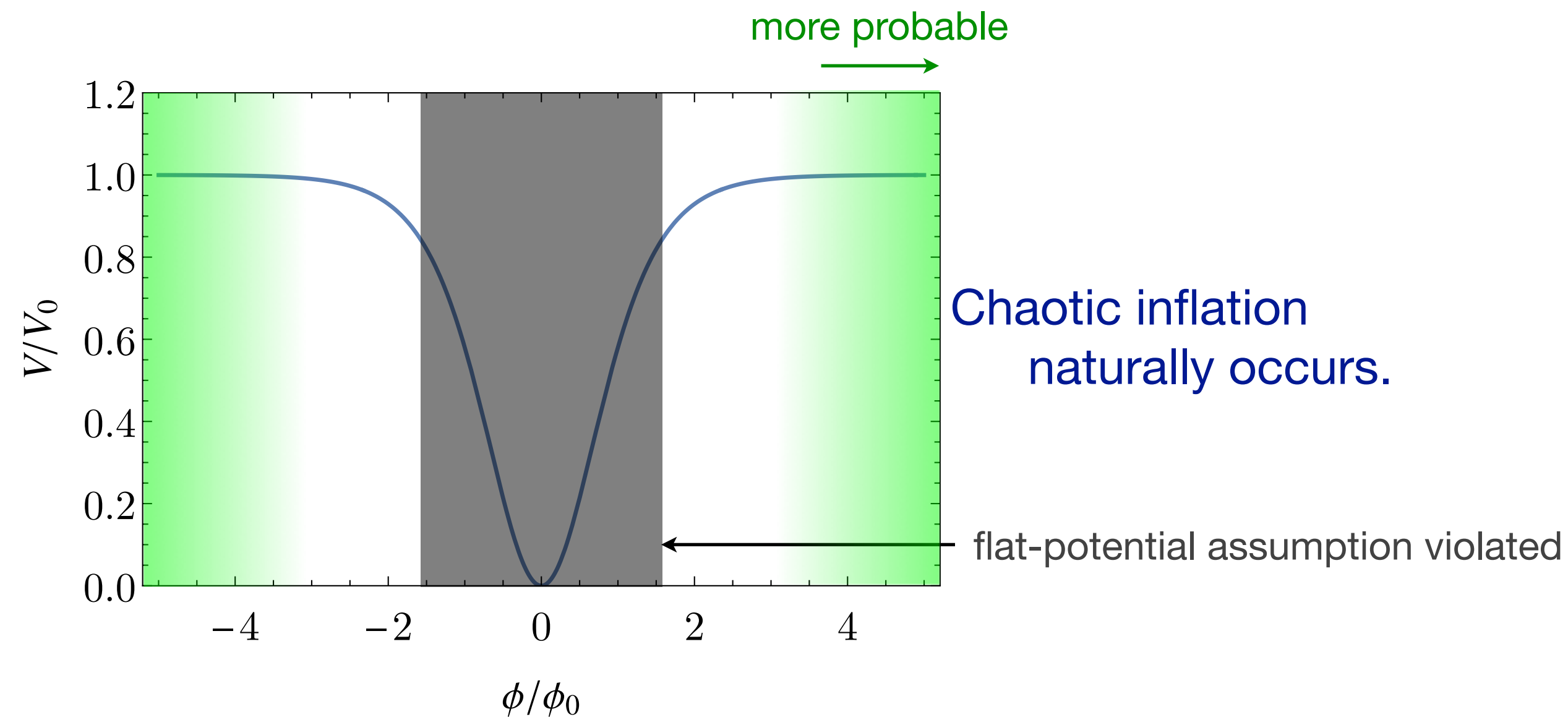
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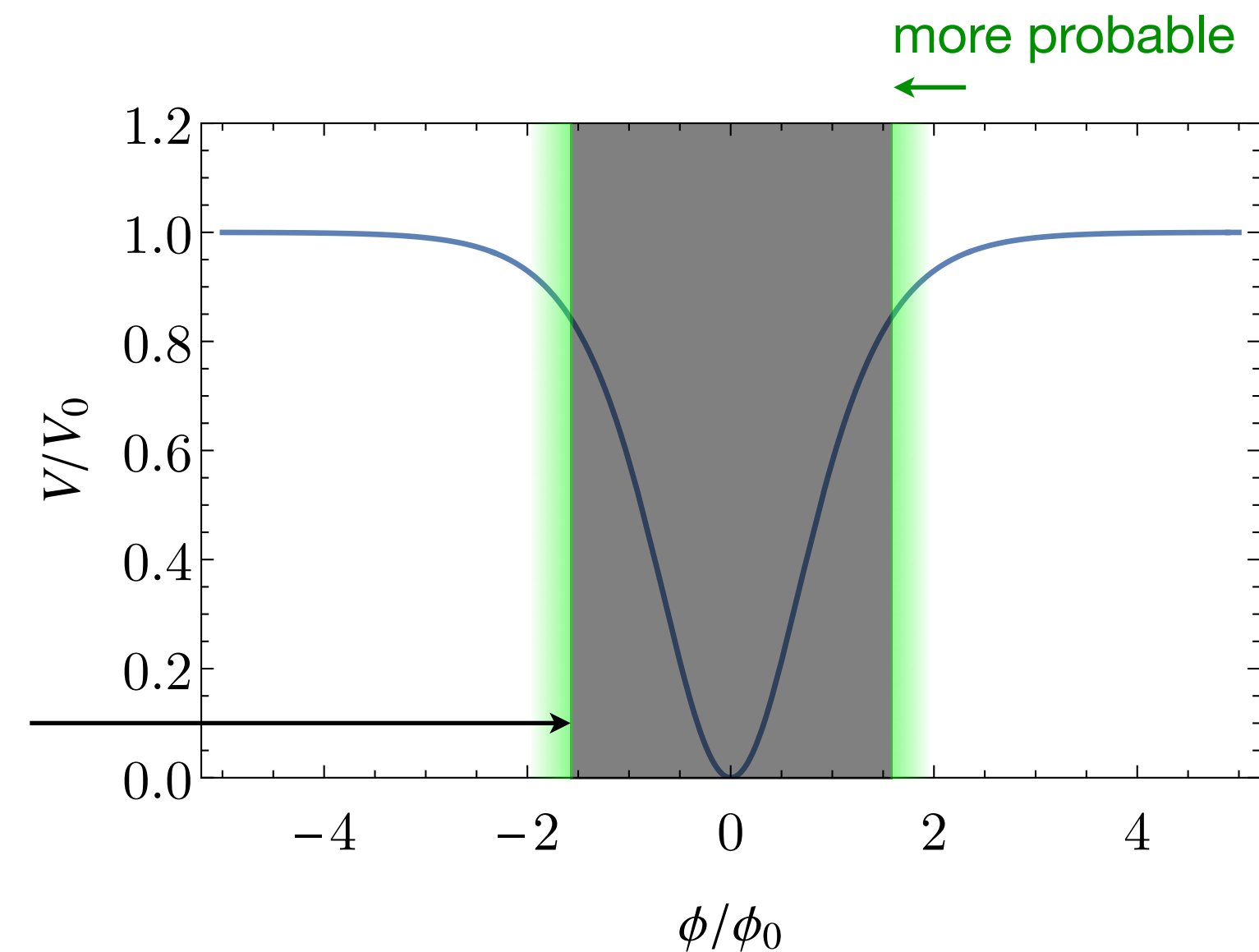
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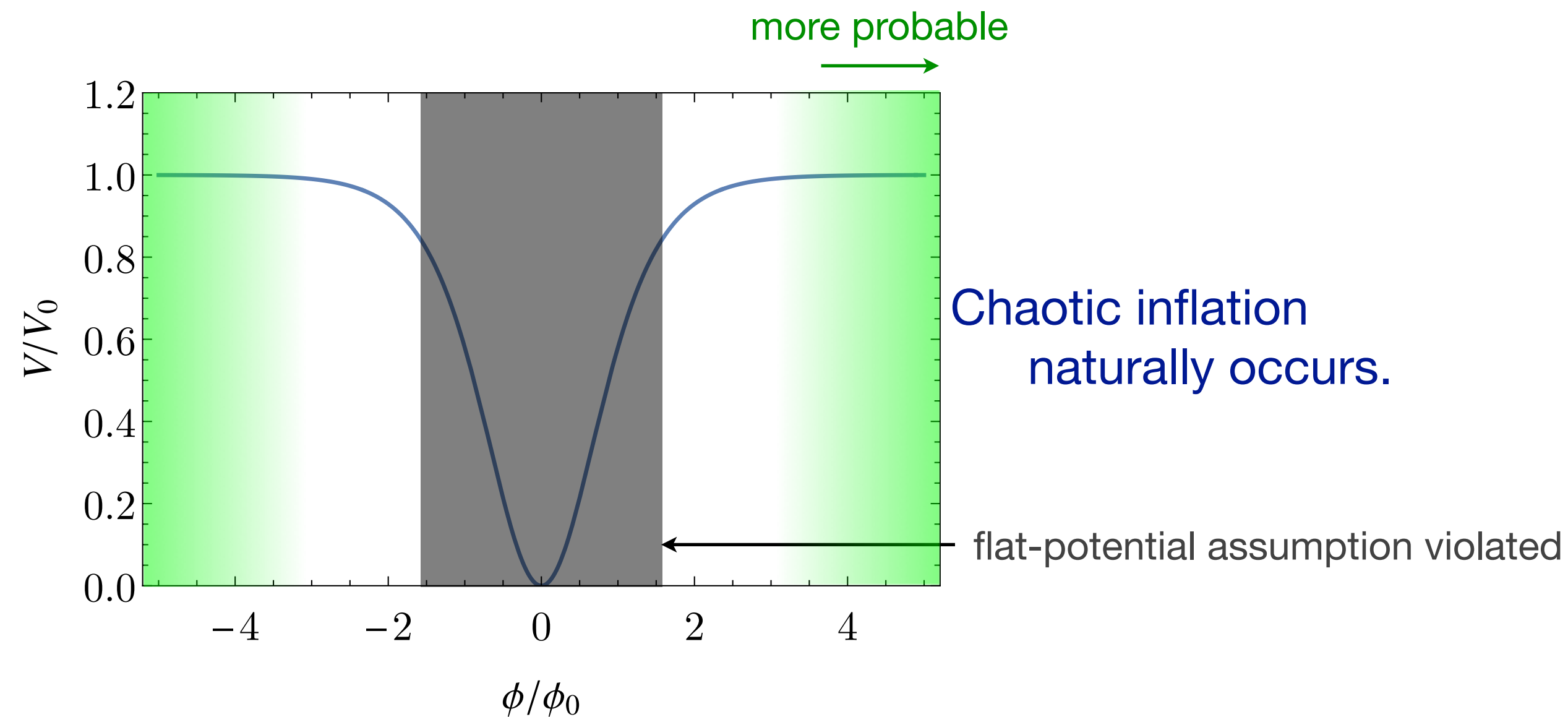
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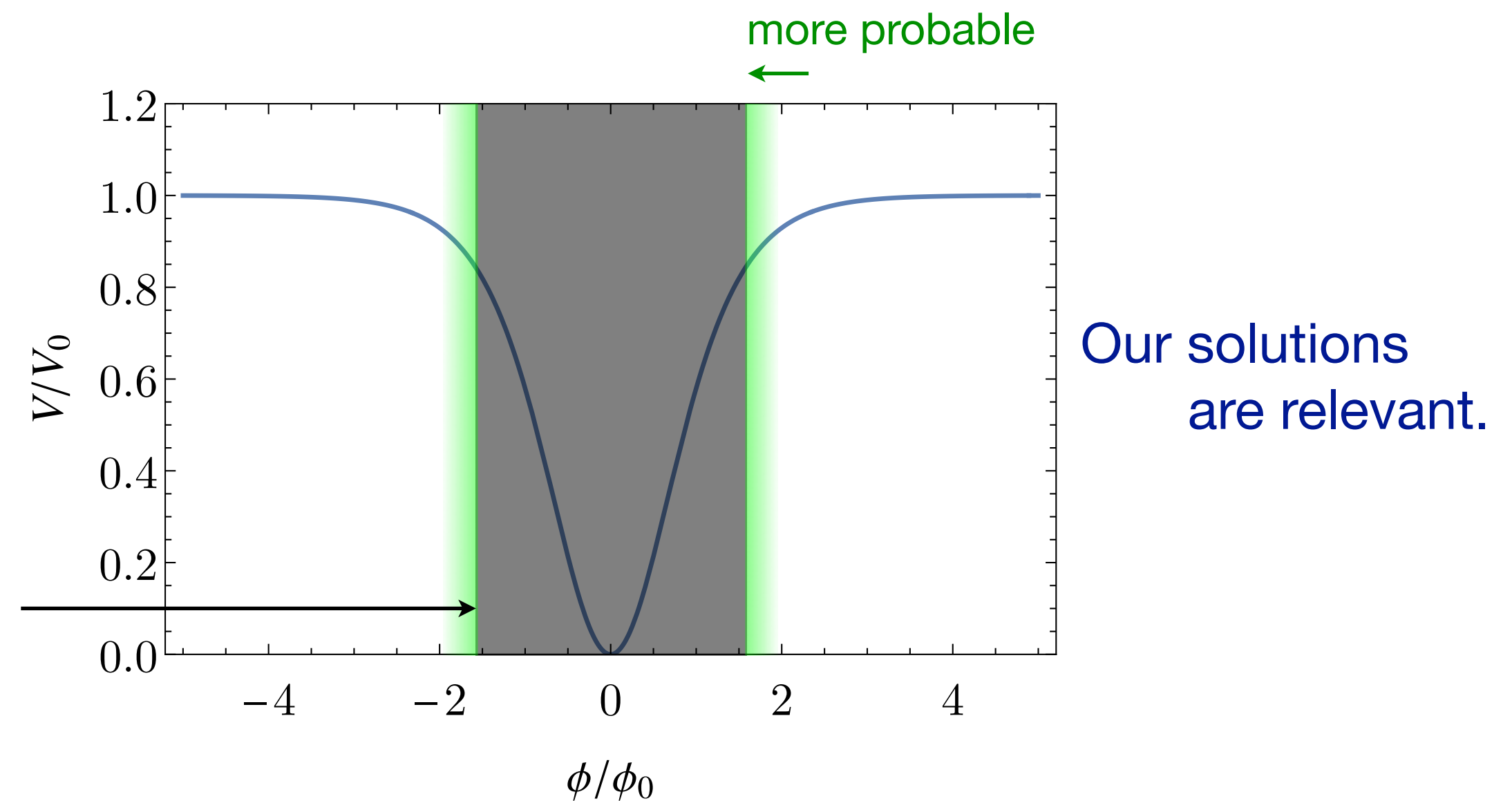
+: no-boundary proposal
 -: tunneling proposal

Note: the exponent is $\mathcal{O}(10^{14})$ for the CMB-compatible case.

tunneling proposal



no-boundary proposal



Probability of Inflation

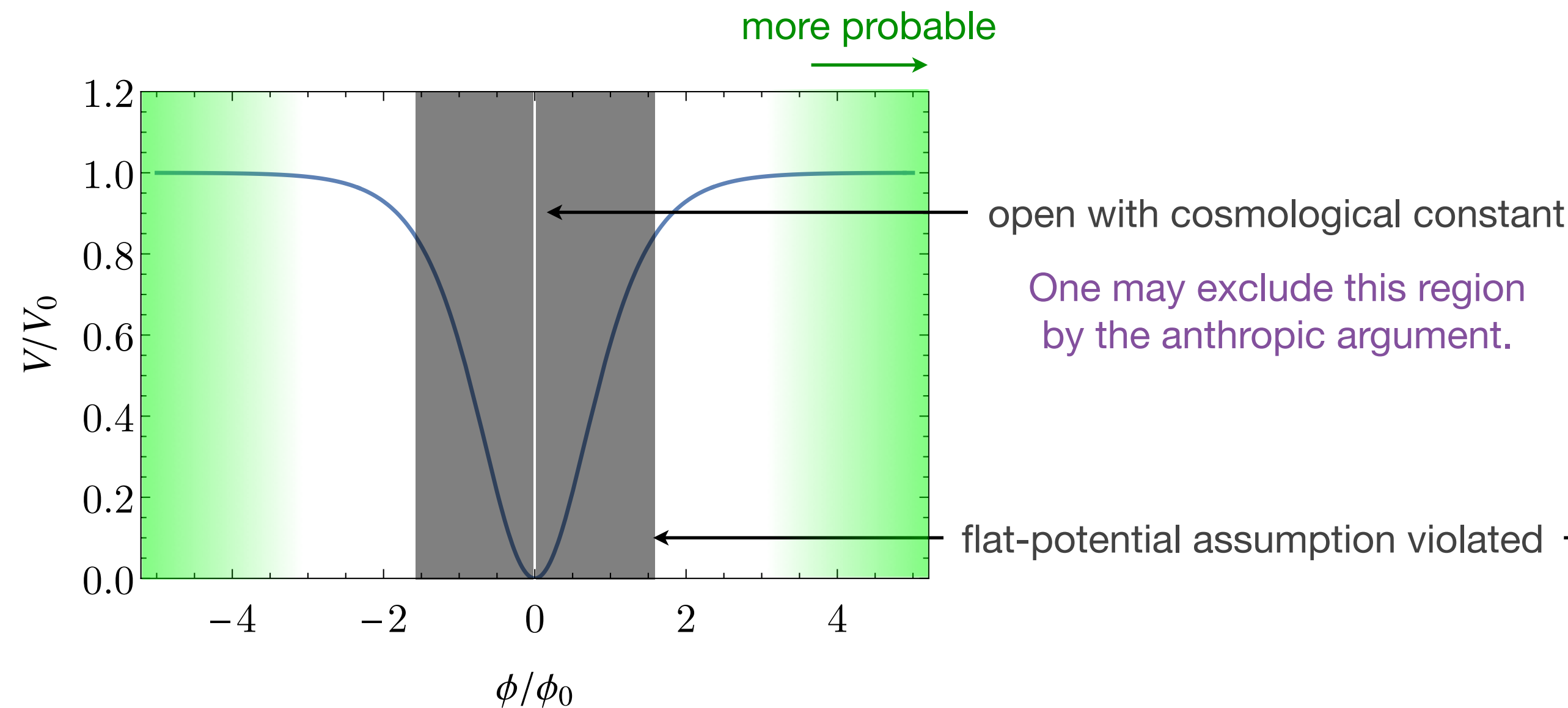
Probability distribution function of the homogeneous initial field value $\phi(0)$ in quantum cosmology

$$P[\phi(0)] \sim \exp\left(\pm \frac{24\pi^2}{V(\phi(0))}\right)$$

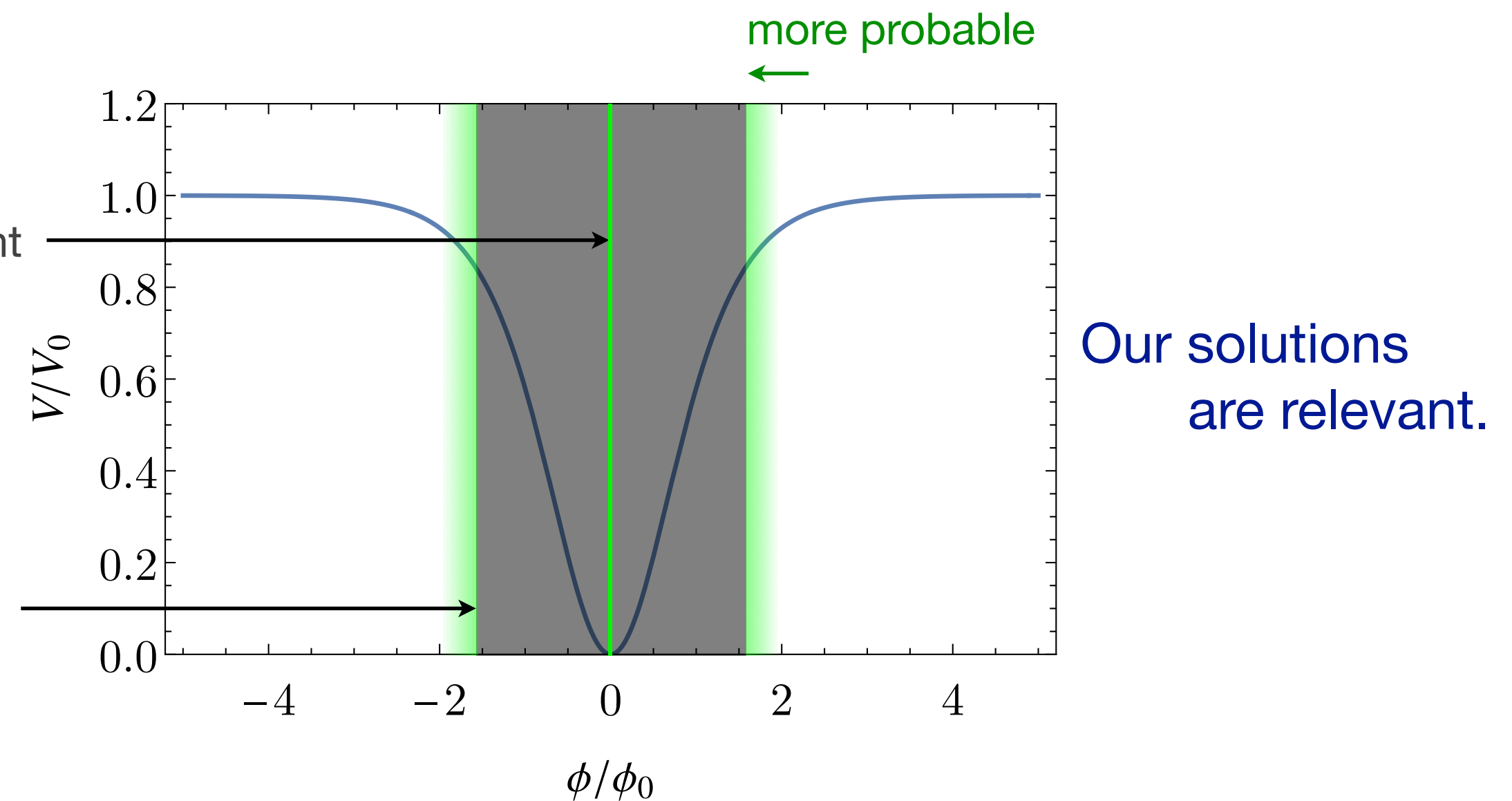
+: no-boundary proposal
 -: tunneling proposal

Note: the exponent is $\mathcal{O}(10^{14})$ for the CMB-compatible case.

tunneling proposal



no-boundary proposal

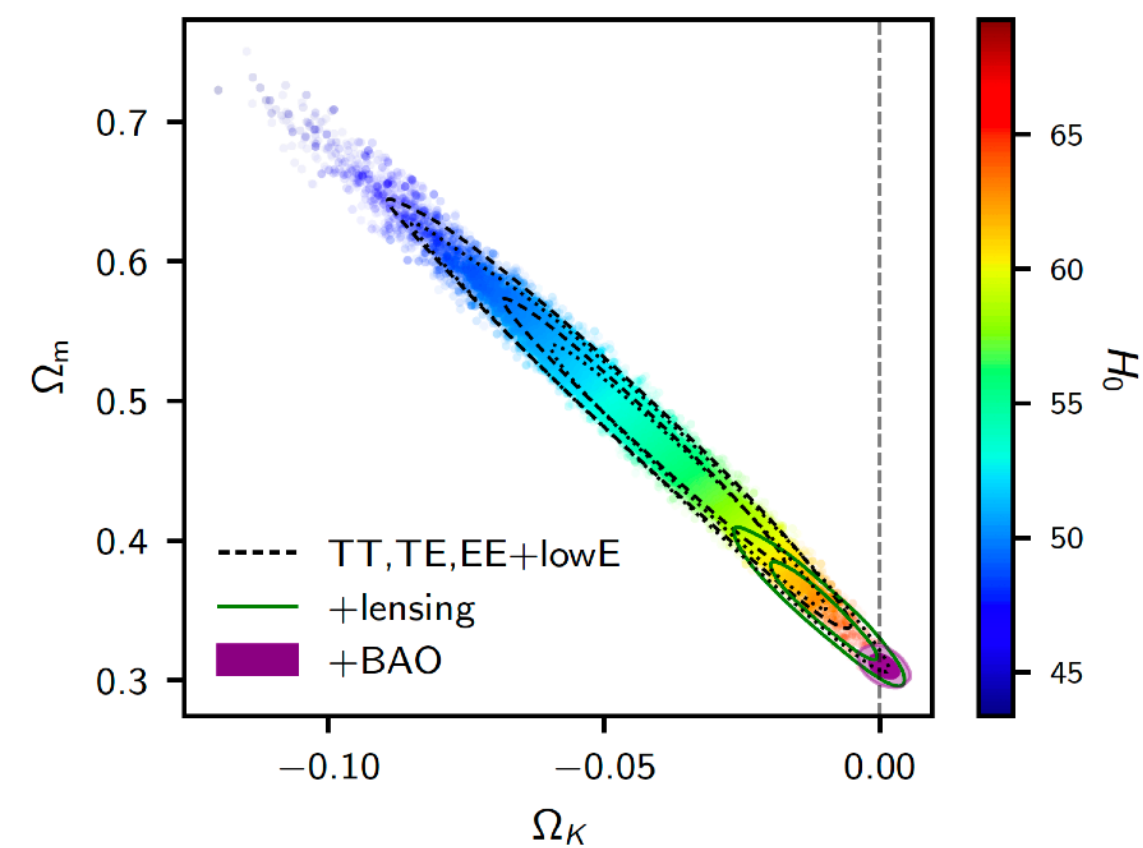


Observational Tests?

Spatial curvature constraints and the Hubble tension

The Planck data prefer positive spatial curvature.

Positive spatial curvature reduces the Hubble tension.



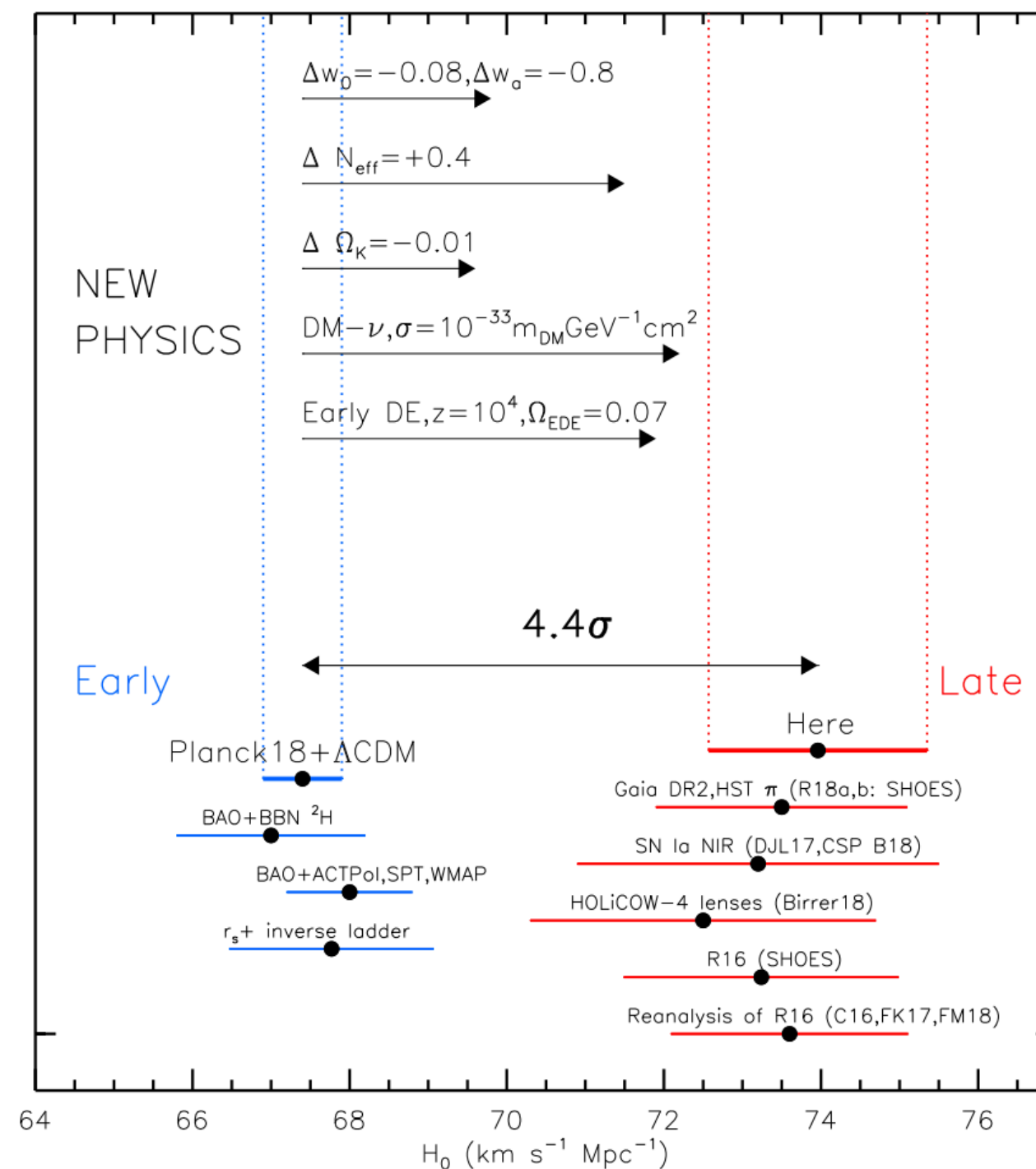
[Aghanim et al. (Planck 2018), 1807.06209]

$$\Omega_K = -0.044^{+0.018}_{-0.015}$$

(68%, Planck TT,TE,EE+lowE)

$$\Omega_K = 0.0007 \pm 0.0019$$

(68%, Planck TT,TE,EE+lowE+lensing+BAO)



[Riess, Casertano, Yuan, Macri, Scolnic, 1903.07603]

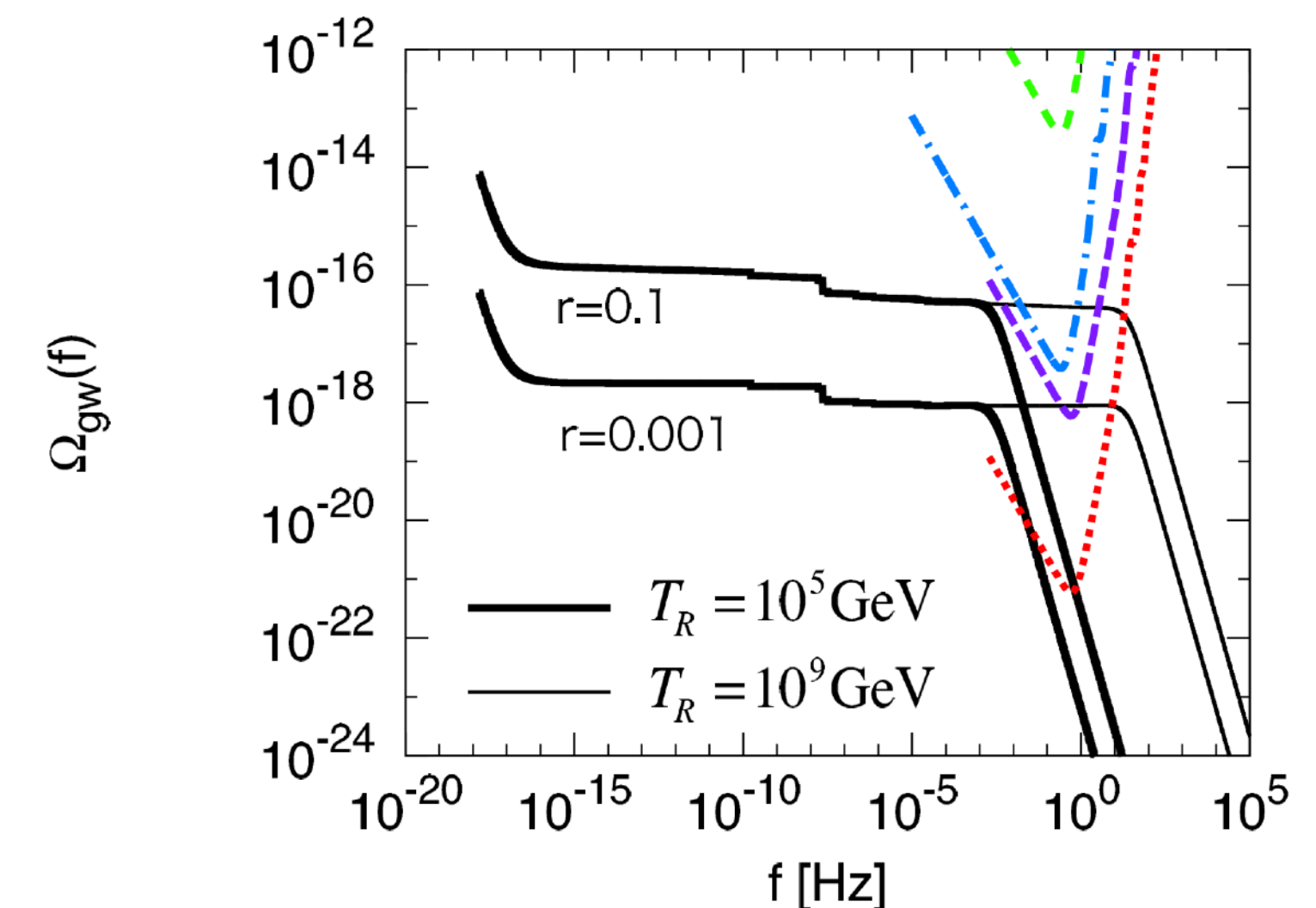
CMB low-multipole suppression

[Sloan, Dimopoulos, Karamitsos, 1912.00090] studied suppression of the power spectrum on large scales in a setup similar to ours.

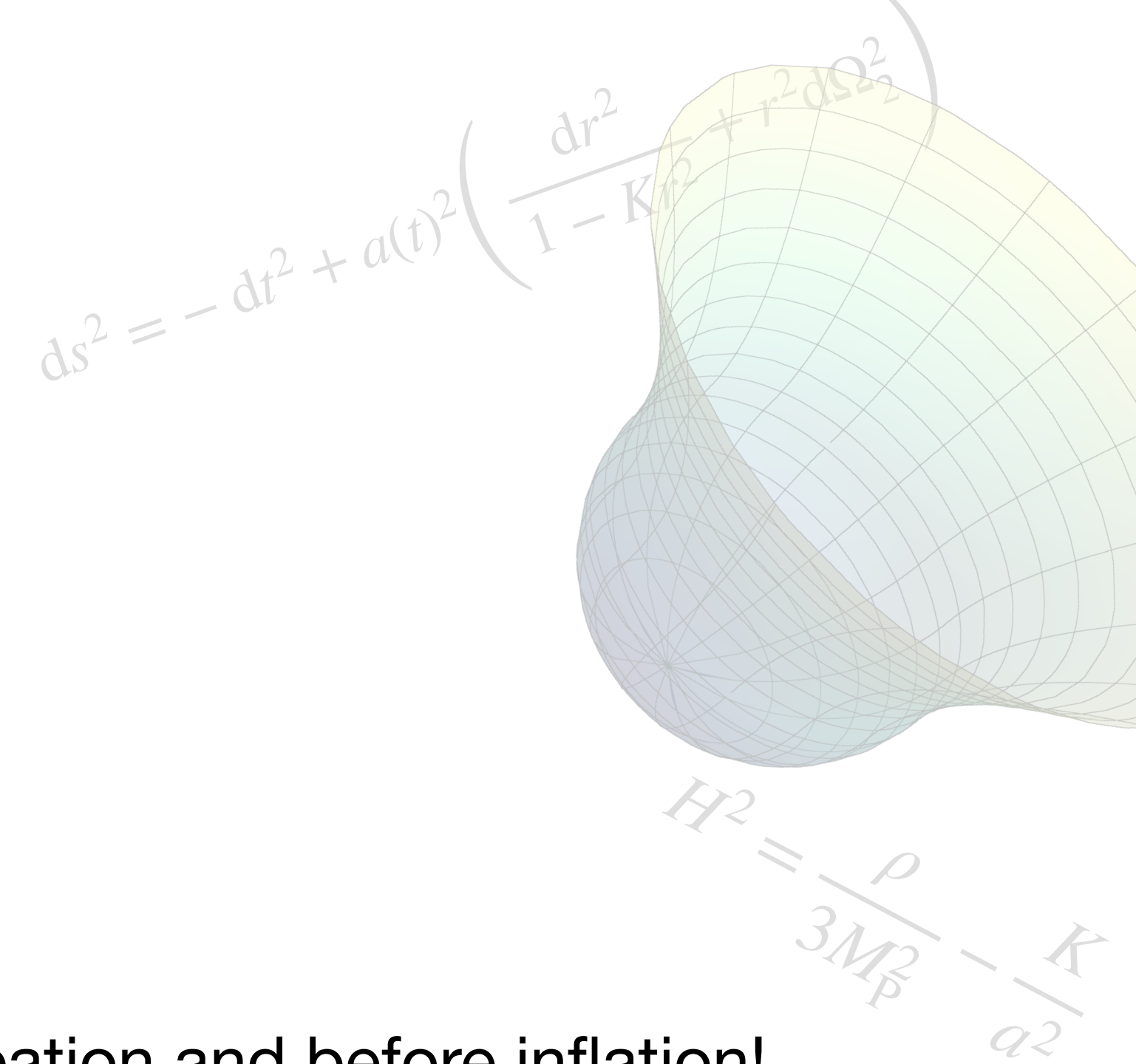
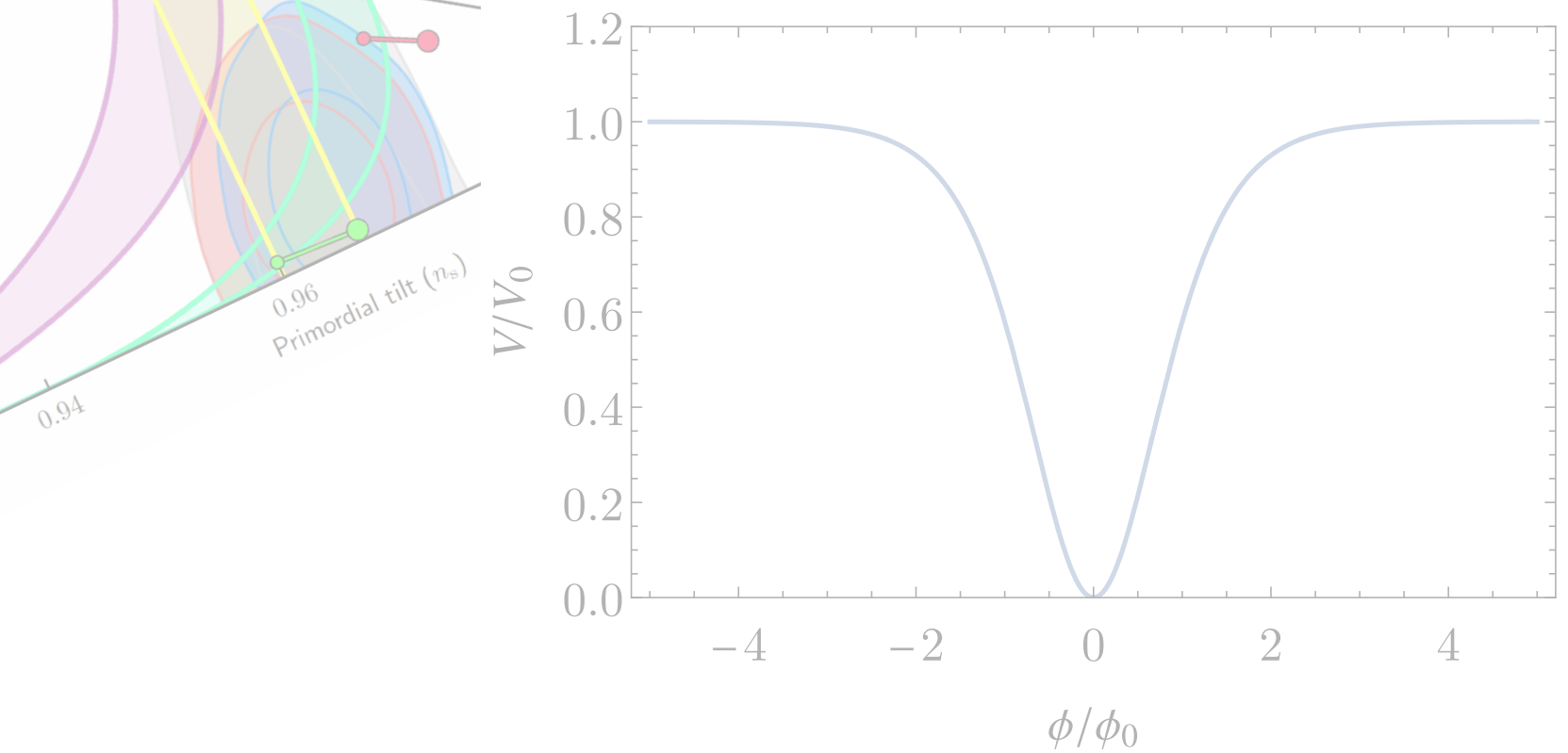
Ideas to probe the reheating of the Universe

e.g.) Measuring the reheating temperature through the spectral break of the gravitational waves.

[Nakayama, Saito, Suwa, Yokoyama, 0804.1827]

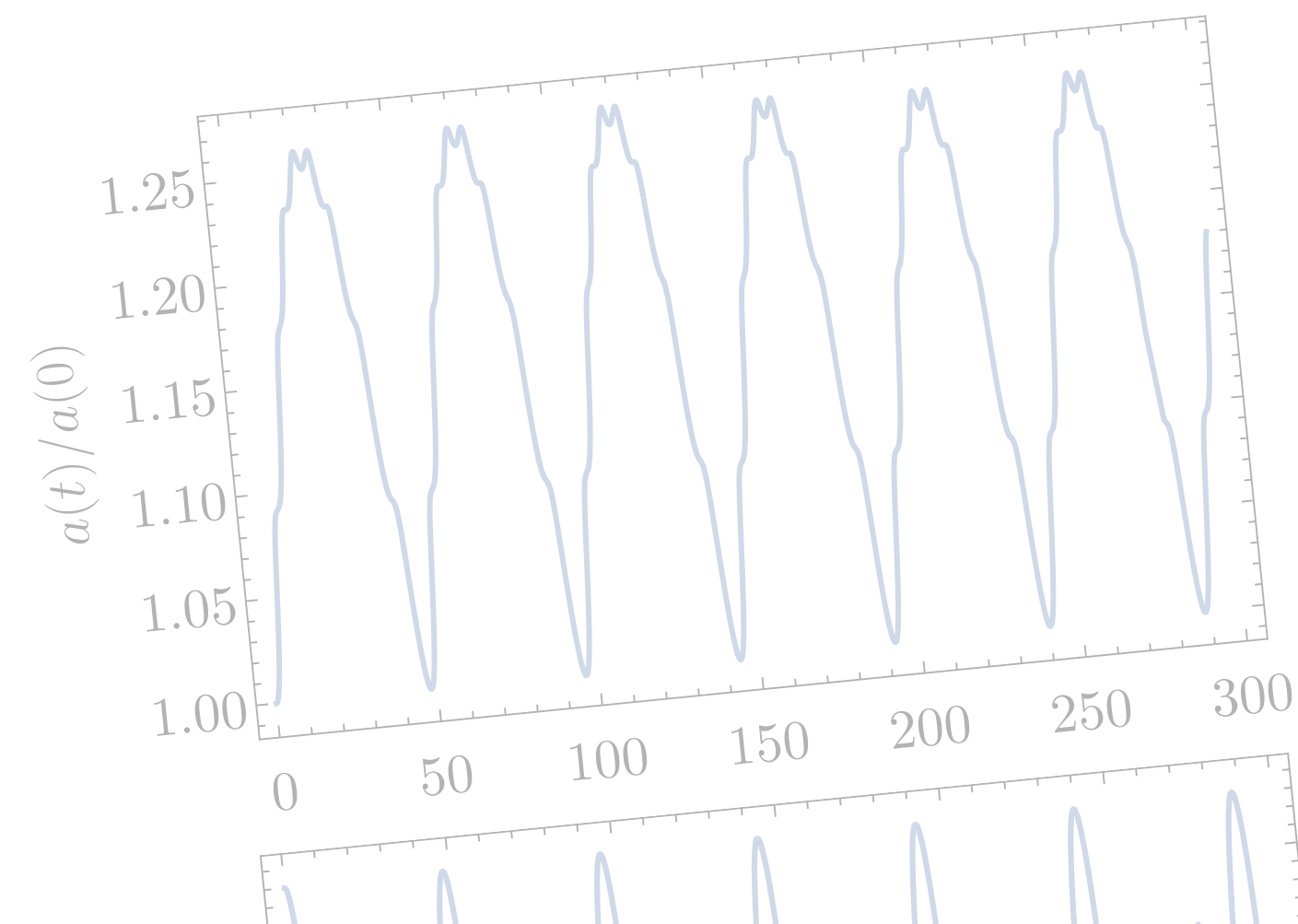


Conclusion



Our Universe may have experienced the quasi-cyclic period just after the creation and before inflation!

We hope to study the mechanism of starting inflation through observations related to the reheating of the Universe.



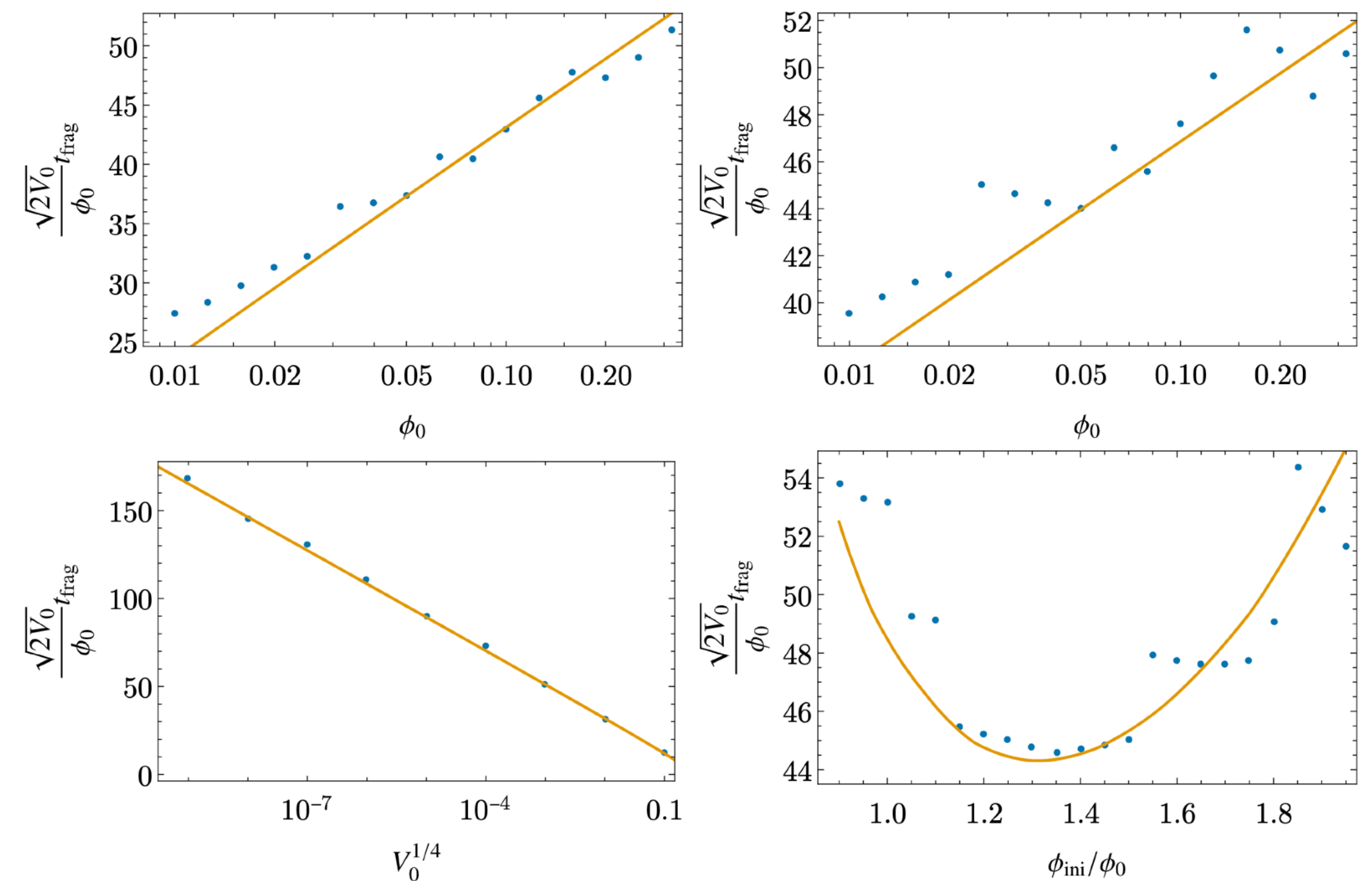
$$w = \frac{\dot{\phi}^2/2 - V}{\dot{\phi}^2/2 + V}$$

Fragmentation Time

Validation of the approximated formula with CosmoLattice calculation

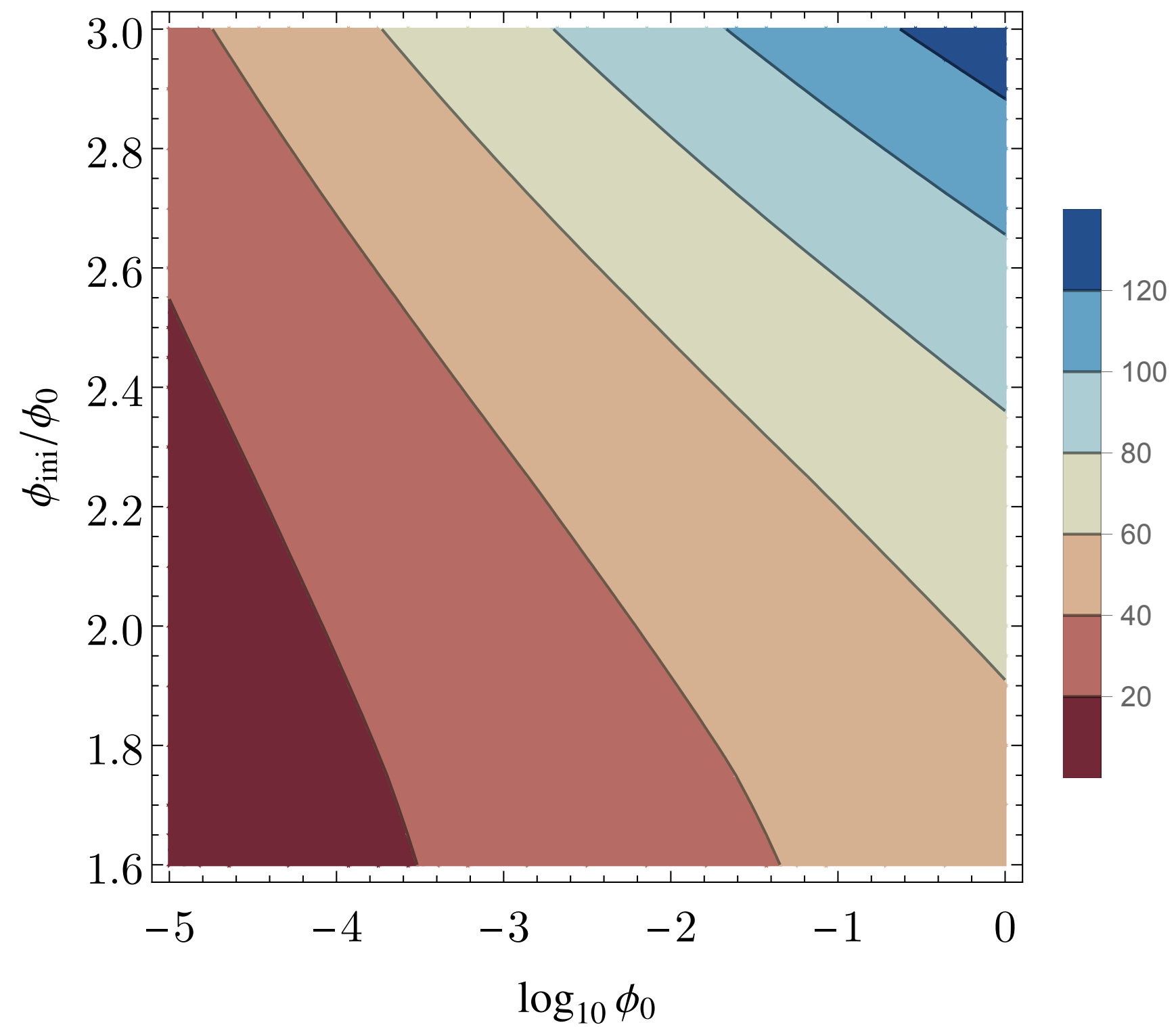
$$t_{\text{frag}} \approx \frac{1}{2\mu_{\text{peak}}} \left(\ln \left(\frac{4\pi^{3/2}c\rho}{(k_{\text{peak}}/a)^4} \right) + \frac{1}{2} \ln \ln \left(\frac{4\pi^{3/2}c\rho}{(k_{\text{peak}}/a)^4} \right) \right)$$

We set $c = 0.01$ and used numerical values of μ_{peak} and k_{peak} from [Tomberg, Veermäe, 2108.10767].

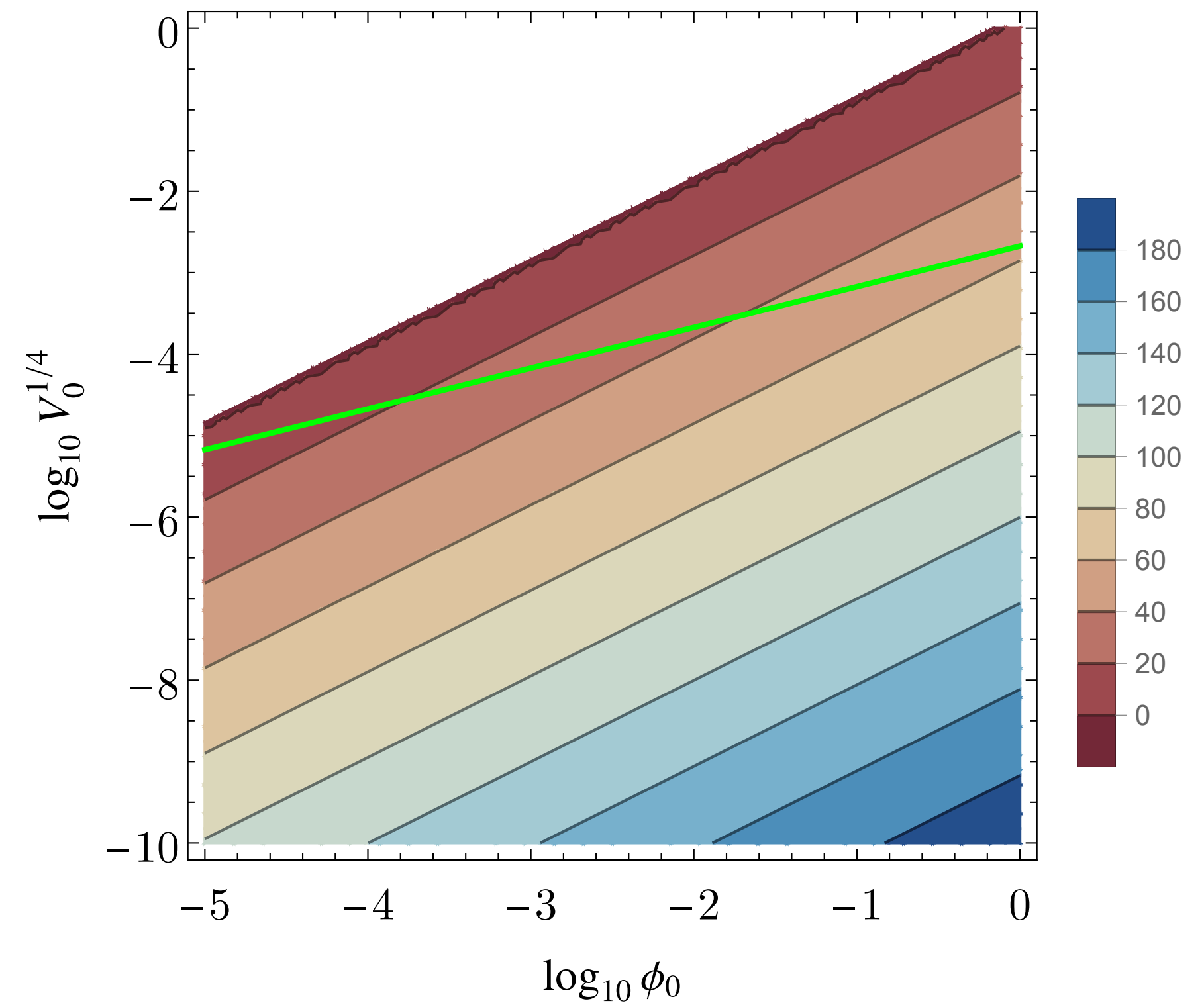


Fragmentation Time

Contour plot of $(\sqrt{2V_0}/\phi_0) t_{\text{frag}}$ with $\phi_0/\sqrt{V_0} = 2.1 \times 10^5$



Contour plot of $(\sqrt{2V_0}/\phi_0) t_{\text{frag}}$ with $\phi_{\text{ini}}/\phi_0 = 1.8$

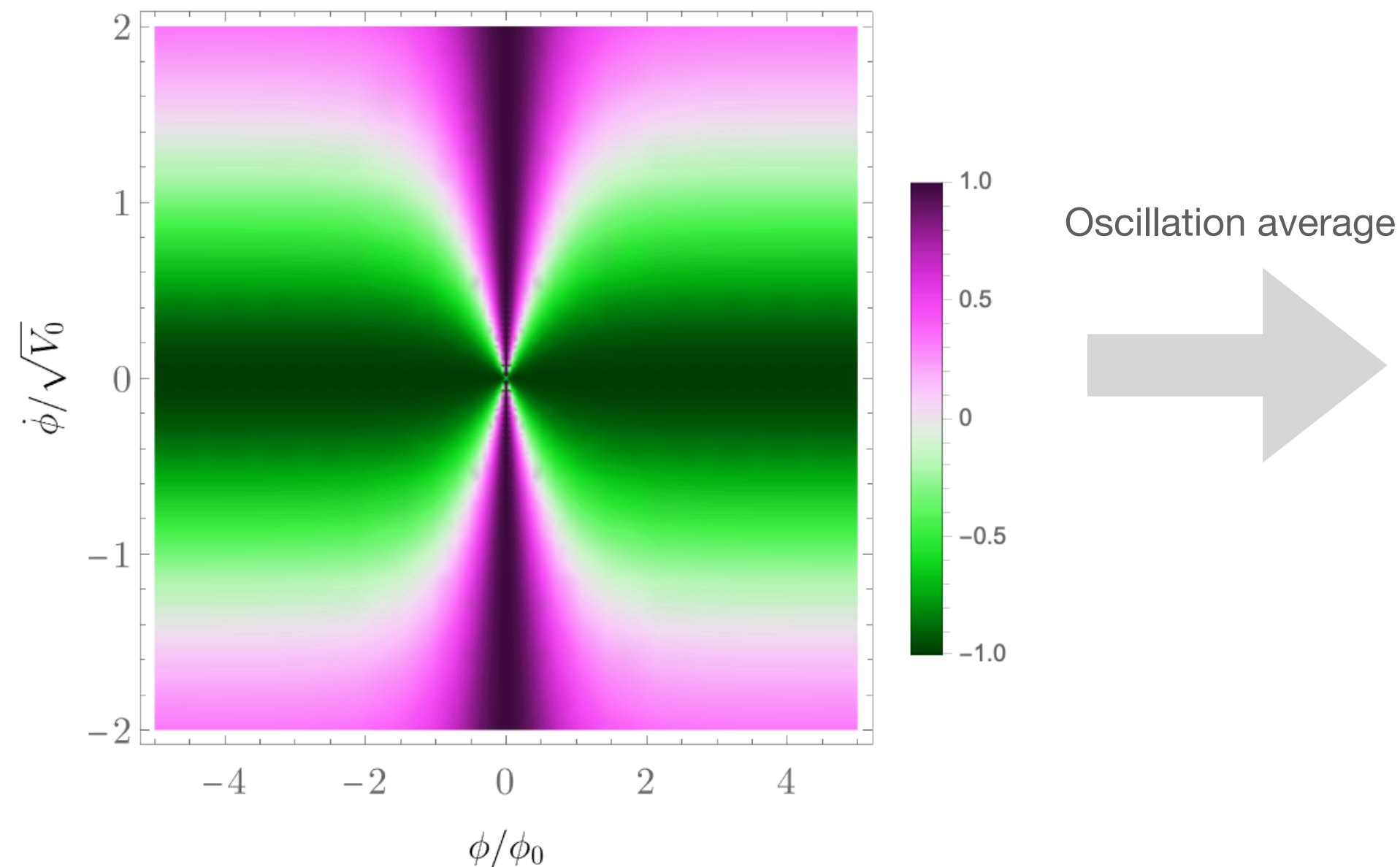


Analytic Understanding of Cyclic Solutions

Approximation based on the **average over oscillations of ϕ** (Note that this breaks down when the amplitude becomes too large.)

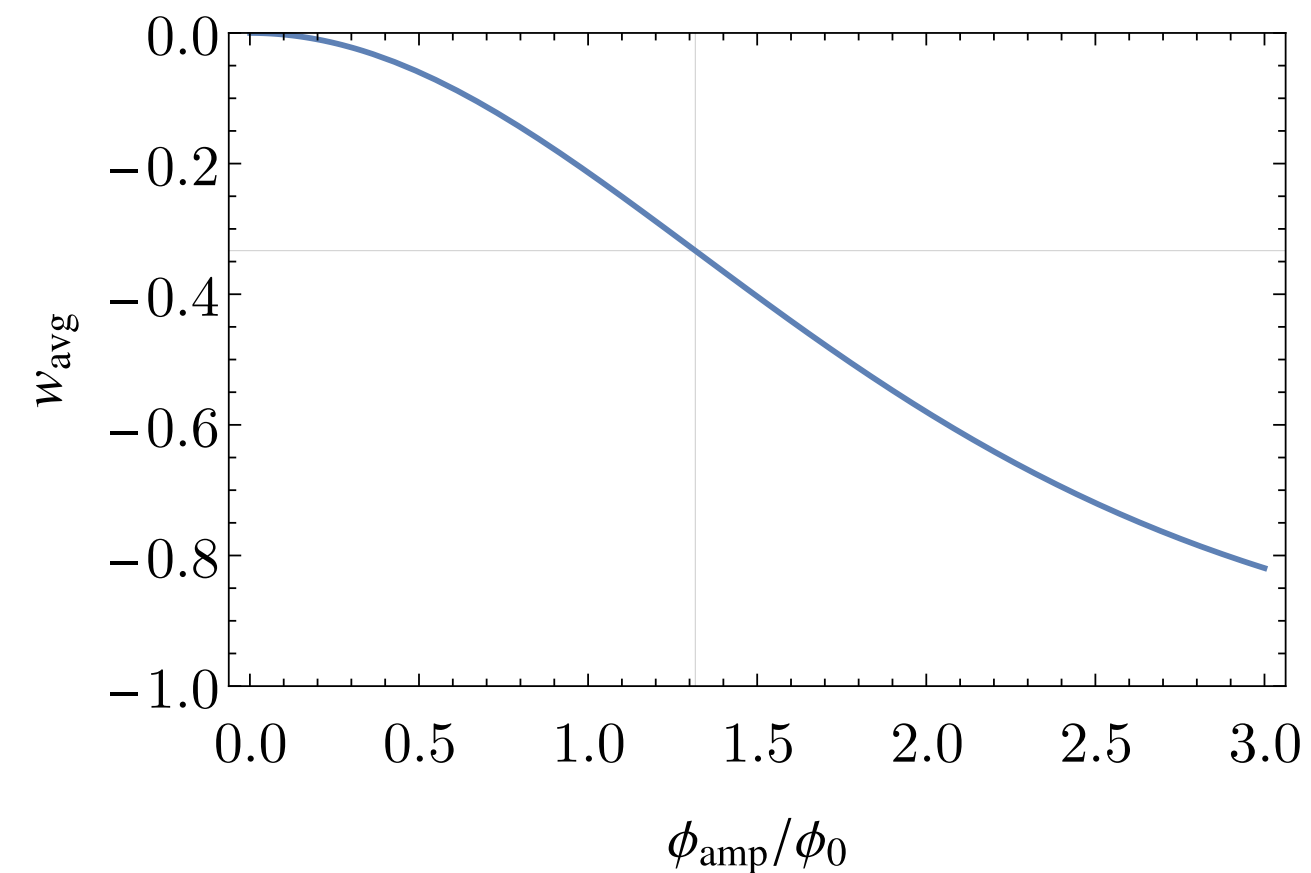
(The technique is based on [Tomberg, Veermäe, 2108.10767; Karam, Tomberg, Veermäe, 2102.02712])

The instantaneous equation-of-state parameter



The averaged equation-of-state parameter

$$w_{\text{avg}} = \frac{-2 + 2\sqrt{1 - \rho/V_0} + \rho/V_0}{\rho/V_0}$$



Energy density as a function of the scale factor

$$\frac{\rho_{\text{avg}}}{V_0} = 2 \left(1 - \sqrt{1 - \frac{\rho_{\text{avg},*}}{V_0}} \right) \left(\frac{a_*}{a} \right)^3 - \left(1 - \sqrt{1 - \frac{\rho_{\text{avg},*}}{V_0}} \right)^2 \left(\frac{a_*}{a} \right)^6$$

Friedmann equation

$$\left(\frac{\dot{a}}{a} \right)^2 = \frac{2V_0 X_*}{3} \left(\left(\frac{a_*}{a} \right)^3 - \frac{X_*}{2} \left(\frac{a_*}{a} \right)^6 - \left(1 - \frac{X_*}{2} \right) \left(\frac{a_*}{a} \right)^2 \right) \quad \text{with} \quad X_* \equiv 1 - \sqrt{1 - \rho_{\text{avg},*}/V_0}$$

Analytic Understanding of Cyclic Solutions

Approximation based on the **average over oscillations of ϕ** (Note that this breaks down when the amplitude becomes too large.)

(The technique is based on [Tomberg, Veermäe, 2108.10767; Karam, Tomberg, Veermäe, 2102.02712])

The averaged equation-of-state parameter

$$w_{\text{avg}} = \frac{-2 + 2\sqrt{1 - \rho/V_0} + \rho/V_0}{\rho/V_0}$$

The critical amplitude (bounce vs turn-around)

$$\phi_{\text{amp}, w_{\text{avg}}=-1/3} / \phi_0 = \log(2 + \sqrt{3}) \approx 1.31696$$

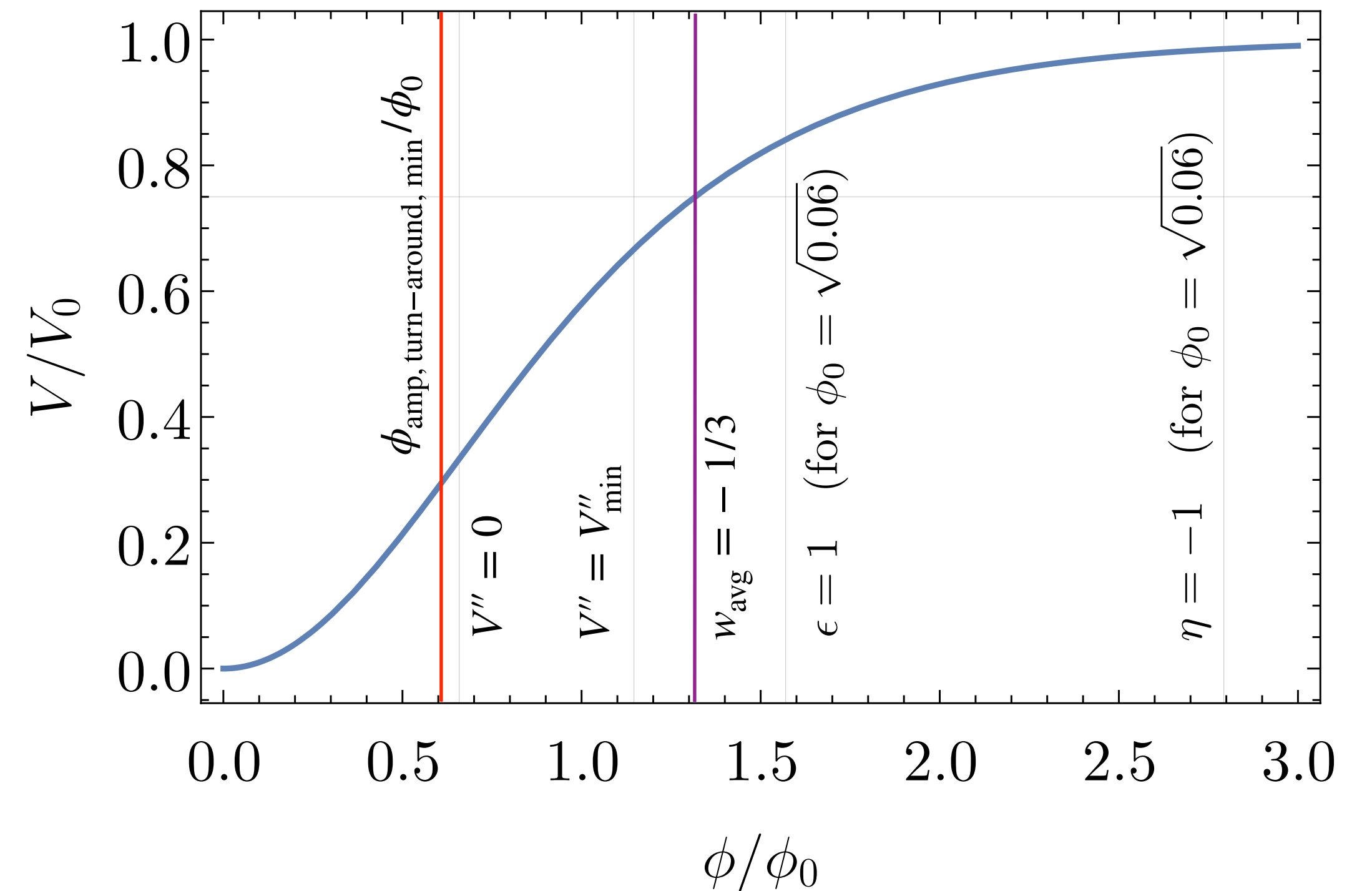
The critical amplitude (below which no cyclic solution)

$$\min \phi_{\text{amp}, \text{turn-around}} / \phi_0 = \text{artanh} \left(\frac{3}{1 + \sqrt[3]{19 - 3\sqrt{33}} + \sqrt[3]{19 + 3\sqrt{33}}} \right) \approx 0.609378$$

The maximal ratio of the max/min of the scale factor

$$\lim_{\rho_{\text{avg}, \text{bounce}} \rightarrow V_0} \frac{a_{\text{max}}}{a_{\text{min}}} = \frac{1 + \sqrt[3]{19 - 3\sqrt{33}} + \sqrt[3]{19 + 3\sqrt{33}}}{3} \approx 1.83929$$

$$V(\phi) = V_0 \tanh^2(\phi/\phi_0)$$



Characteristic field values.

Lattice Simulation

Lattice topology

For simplicity, we use \mathbb{T}^3 rather than \mathbb{S}^3 .

The IR cutoff due to \mathbb{S}^3 should be $\frac{k_{\text{IR},\mathbb{S}^3}}{a} = \frac{2\sqrt{K}}{a} \simeq 2\sqrt{\frac{\rho_{\text{avg}}}{3M_{\text{P}}^2}}$.

Spatial curvature

We modify the codes of *CosmoLattice* to include the spatial curvature.

Dimensionless variables

Quantities with a tilde are measured in units of $f_* = \phi_0$ and $\omega_* = \sqrt{2V_0}/\phi_0$.

IR cutoffs

The minimum wave number in *CosmoLattice* is defined to be $k_{\text{IR,lattice}} = \frac{2\pi}{L} = \frac{2\pi}{N_{\mathcal{E}\mathcal{L}}\delta x}$.

This has to be smaller than the most tachyonic mode $\frac{k_{\text{peak}}}{a} \approx \frac{3}{\Delta t_\phi}$.

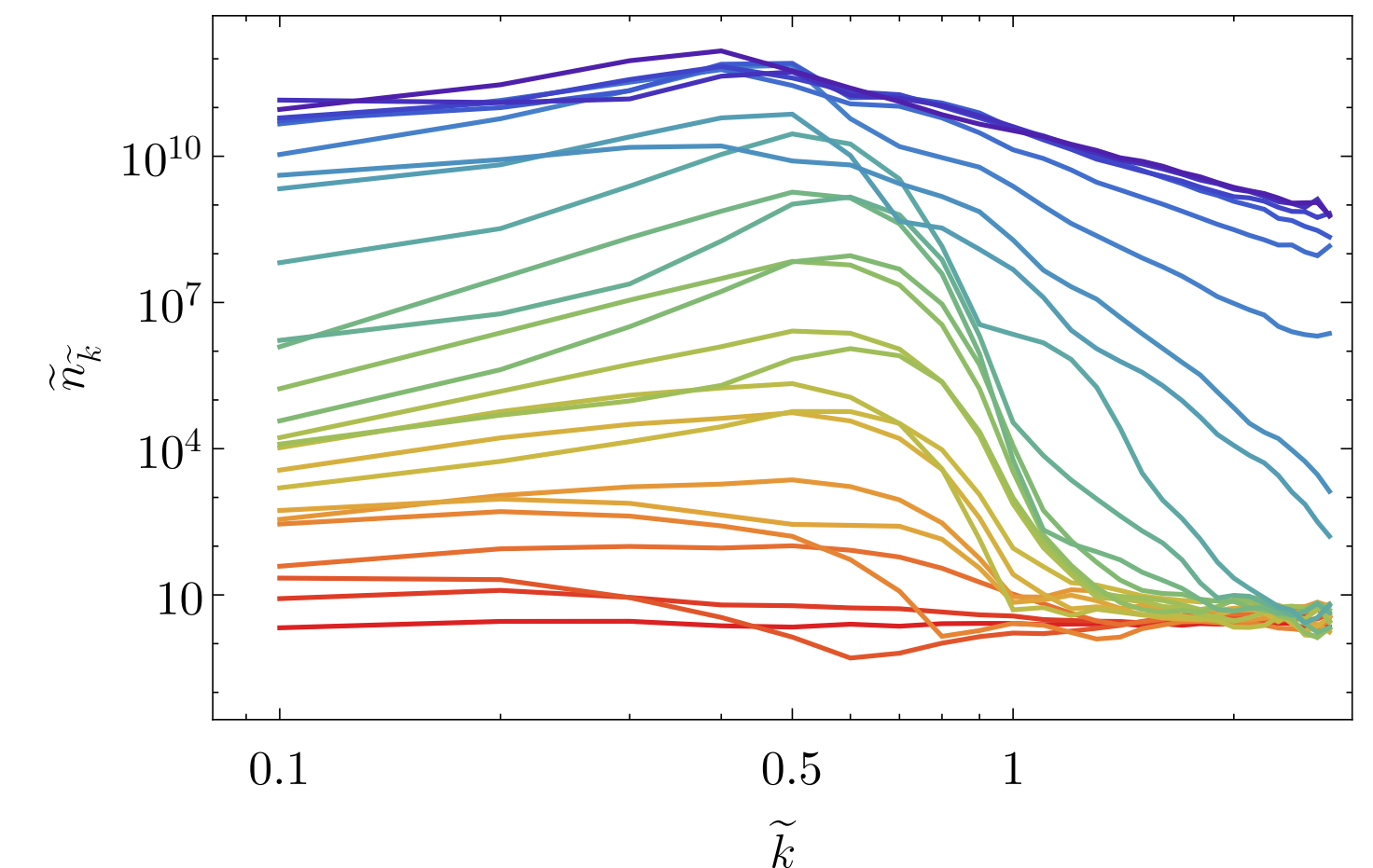
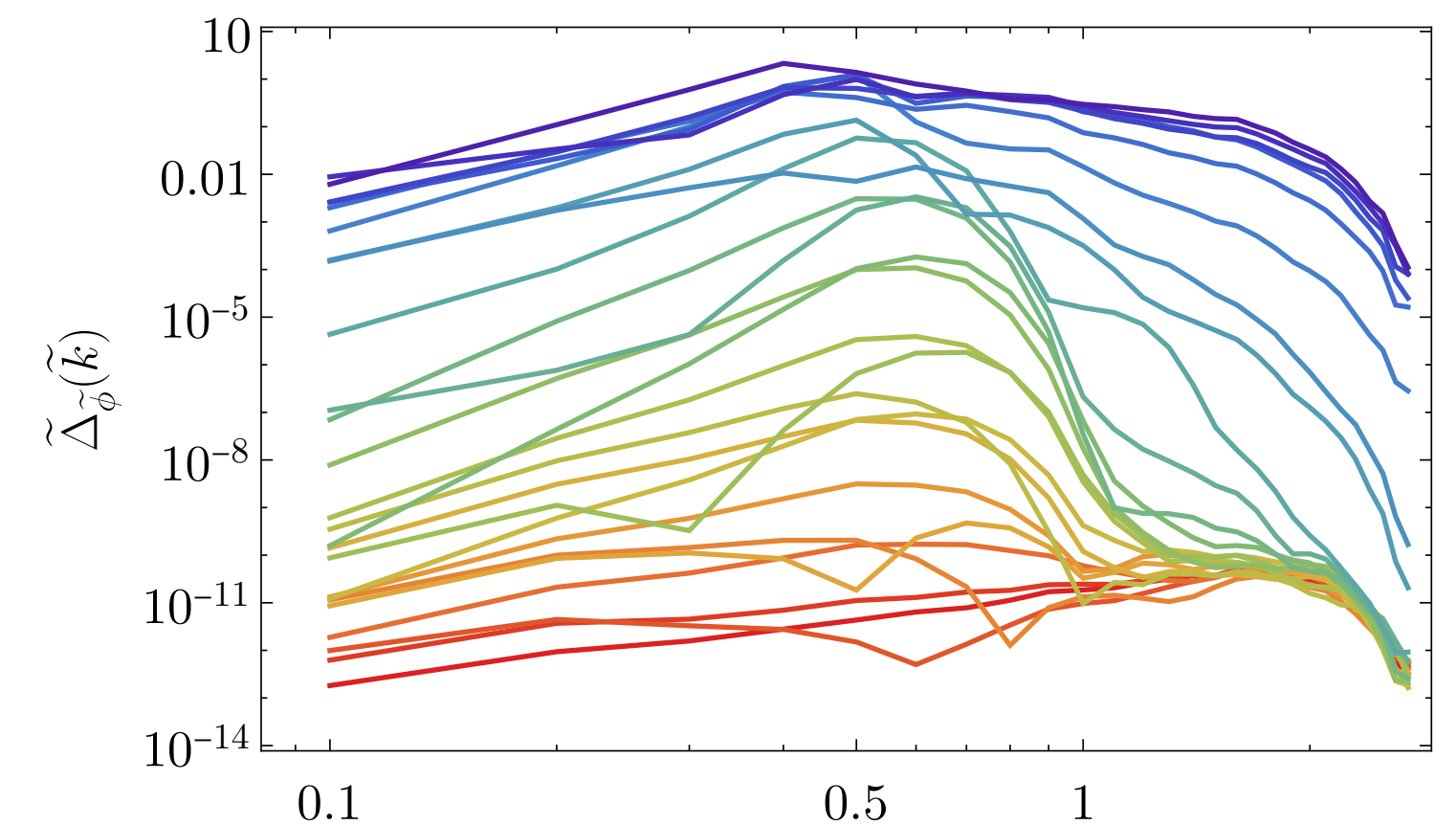
Evolver and input parameters

The default evolver (velocity Verlet algorithm: VV2).

$N_{\mathcal{E}\mathcal{L}} = 32$, $\tilde{k}_{\text{IR,lattice}} = 0.1$, and $\Delta\tilde{t} = 0.01$.

Spectrum of perturbations

The tachyonic peak structure is well resolved.



Explicit Model

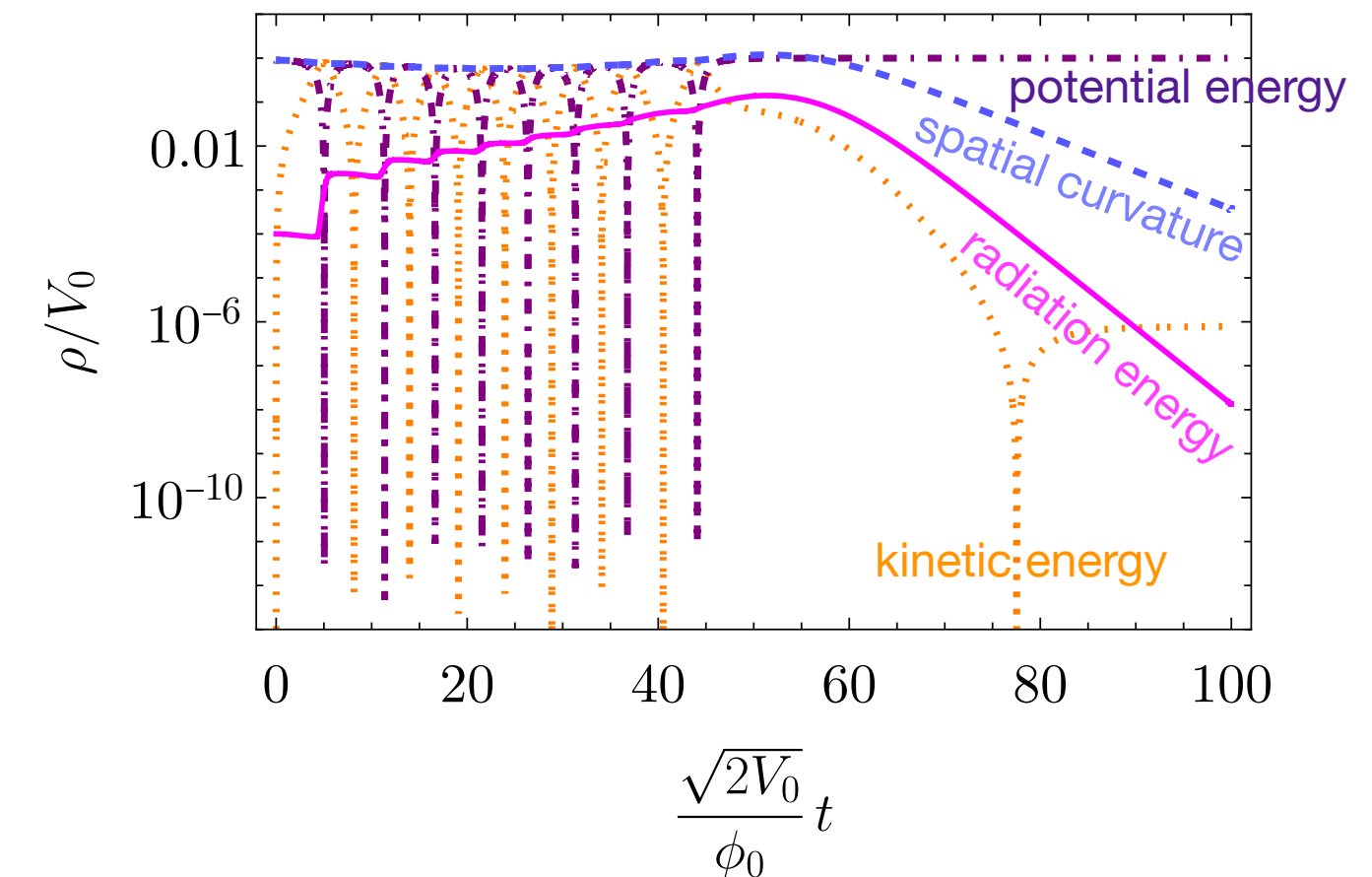
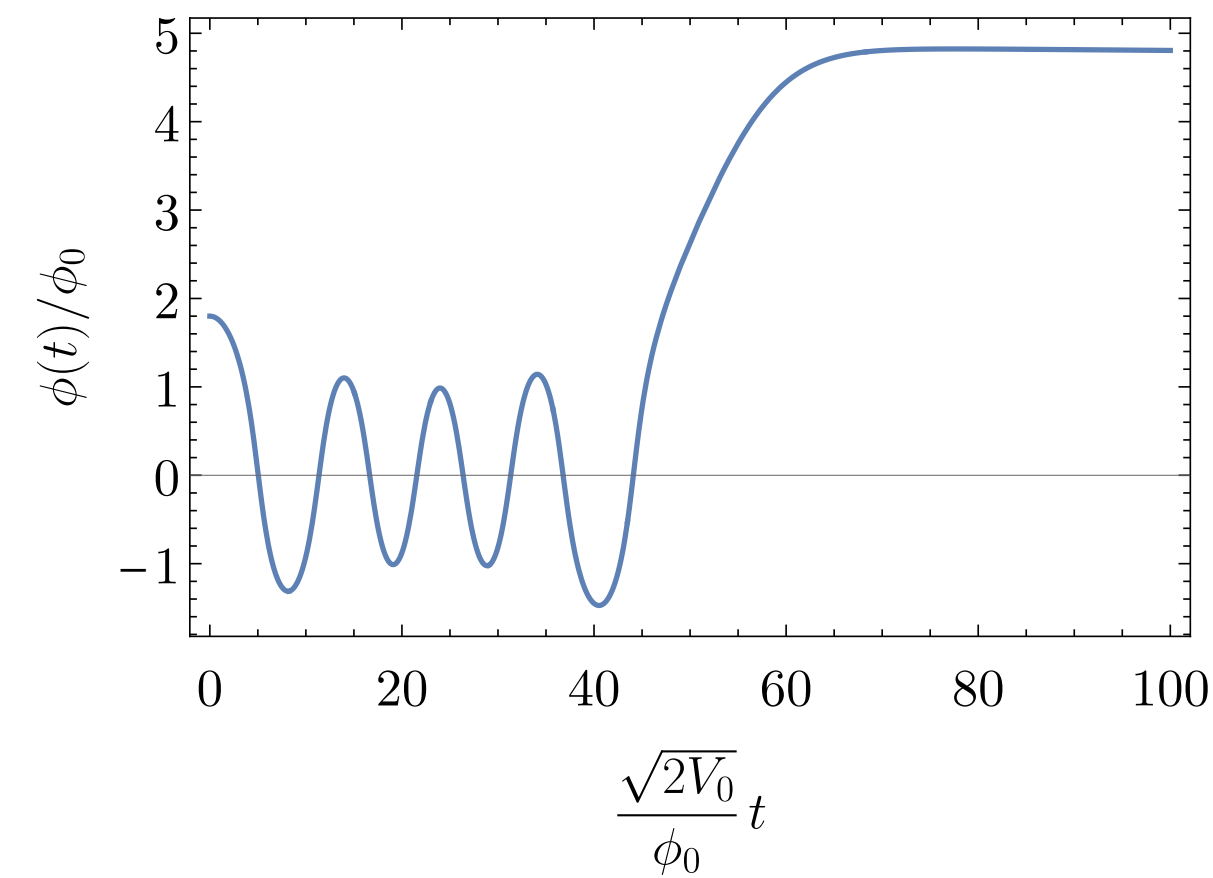
SU(N_c) gauge fields with $\mathcal{L}_{\text{int}} = -\frac{\alpha}{4\pi f} \phi \tilde{F}^{\mu\nu} F_{\mu\nu}$, $\mathcal{L}_{\text{matter}} = -\frac{1}{4} g^{\mu\rho} g^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma}$

Dissipation rate $\Gamma(T) = \Upsilon(T) + \Gamma_{\text{sct}}(T) + \Gamma_{\text{dec}}$

sphaleron-induced friction $\Upsilon(T) \sim \frac{(N_c \alpha)^5 T^3}{f^2}$

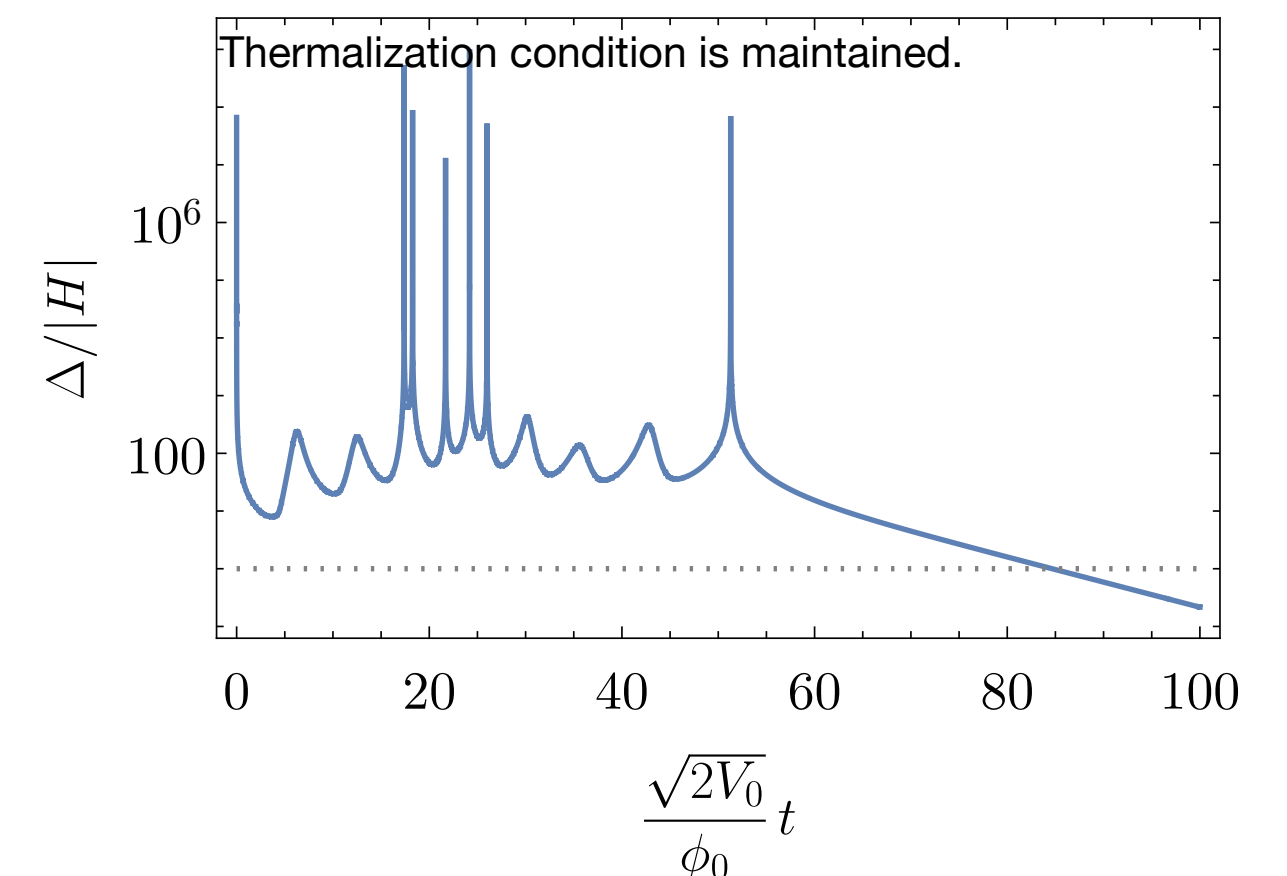
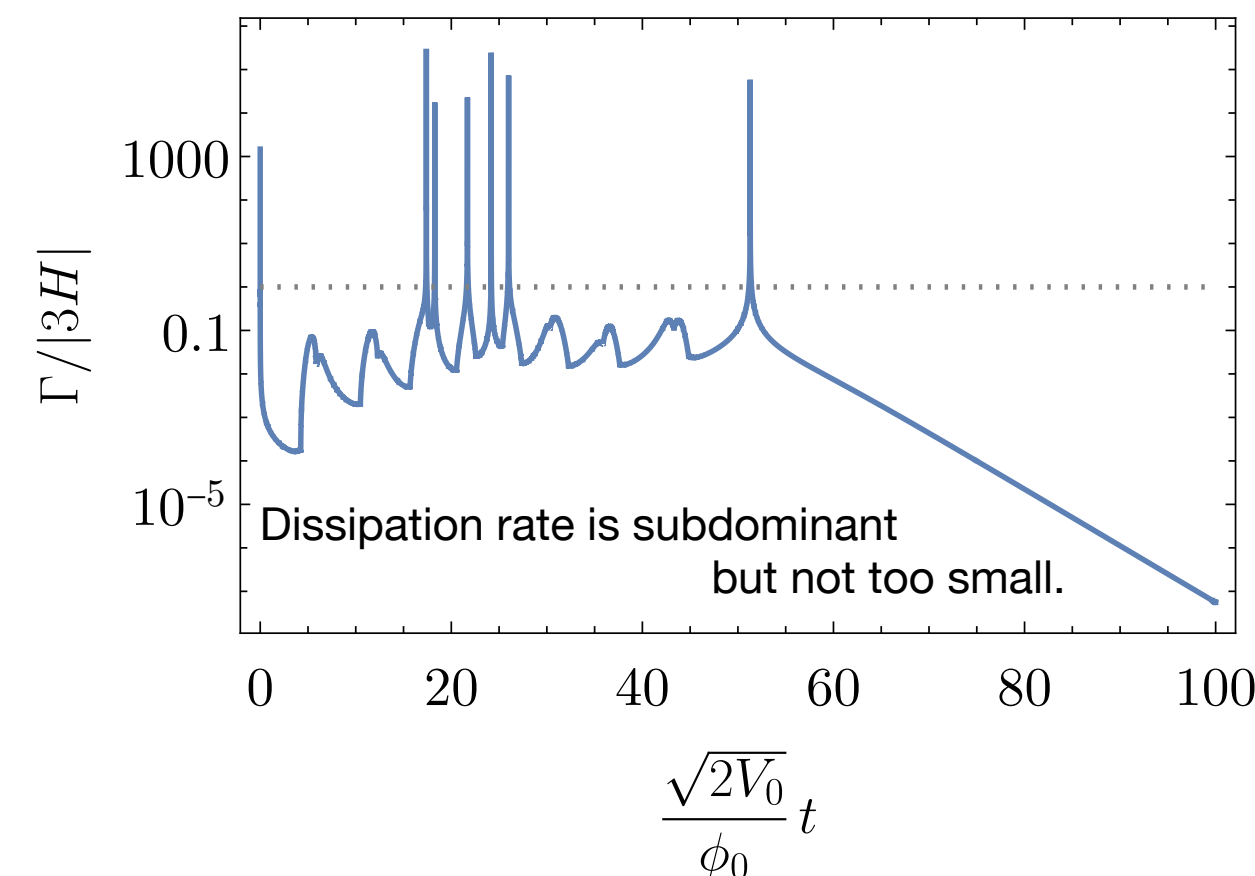
scattering rate $\Gamma_c \simeq \frac{C(N_c^2 - 1) T (p^0)^2}{64\pi^4 f^2}$

decay rate $\Gamma_{\text{dec}} = \frac{(N_c^2 - 1) \alpha^2 m_\phi^3}{64\pi^3 f^2}$



We follow [DeRocco, Graham, Kalia, 2107.0757] for the thermalization criteria.

- Nonlinearity develops in the relevant time scale.
- Backreaction is negligible until it becomes nonlinear.
- Thermalization rate is greater than the Hubble rate.



Explicit Model

SU(N_c) gauge fields with $\mathcal{L}_{\text{int}} = -\frac{\alpha}{4\pi f} \phi \tilde{F}^{\mu\nu} F_{\mu\nu}$, $\mathcal{L}_{\text{matter}} = -\frac{1}{4} g^{\mu\rho} g^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma}$.

Reheating temperature in the dark sector $T_{\text{R}}^{\text{dark}} = \max \left[T_{\text{R, dec}}, \min \left[T_{\text{R, sct}}, T_{\text{max}} \right] \right]$

where $T_{\text{R, sct}} = 1.5 \times 10^{14} \text{ GeV} \left(\frac{C}{1} \right) \left(\frac{g_*}{22} \right)^{-1/2} \left(\frac{N_c^2 - 1}{3^2 - 1} \right)$,

$$T_{\text{R, dec}} = 1.3 \times 10^{12} \text{ GeV} \left(\frac{g_*}{22} \right)^{-1/4} \left(\frac{N_c^2 - 1}{3^2 - 1} \right)^{1/2} \left(\frac{\alpha}{0.015} \right) \left(\frac{m_\phi}{1.6 \times 10^{13} \text{ GeV}} \right)^{3/2} \left(\frac{f}{5.8 \times 10^{13} \text{ GeV}} \right)^{-1},$$

and the maximal attainable temperature after inflation $T_{\text{max}} \leq T_{\text{inst}} \simeq 1.6 \times 10^{15} \text{ GeV} \left(\frac{g_*}{22} \right)^{-1/4} \left(\frac{V_0^{1/4}}{2.6 \times 10^{15} \text{ GeV}} \right)$.

Some options to heat the Standard Model sector

- Dark glueball decay via dimension-6 operators.
- Dark Higgs portal.

Gallery of Phase Space Trajectory

$$\phi_0 = \sqrt{0.06} \quad \text{and} \quad \Gamma/\sqrt{V_0} = 0.019$$

