Gravitational wave signals from the inflationary era

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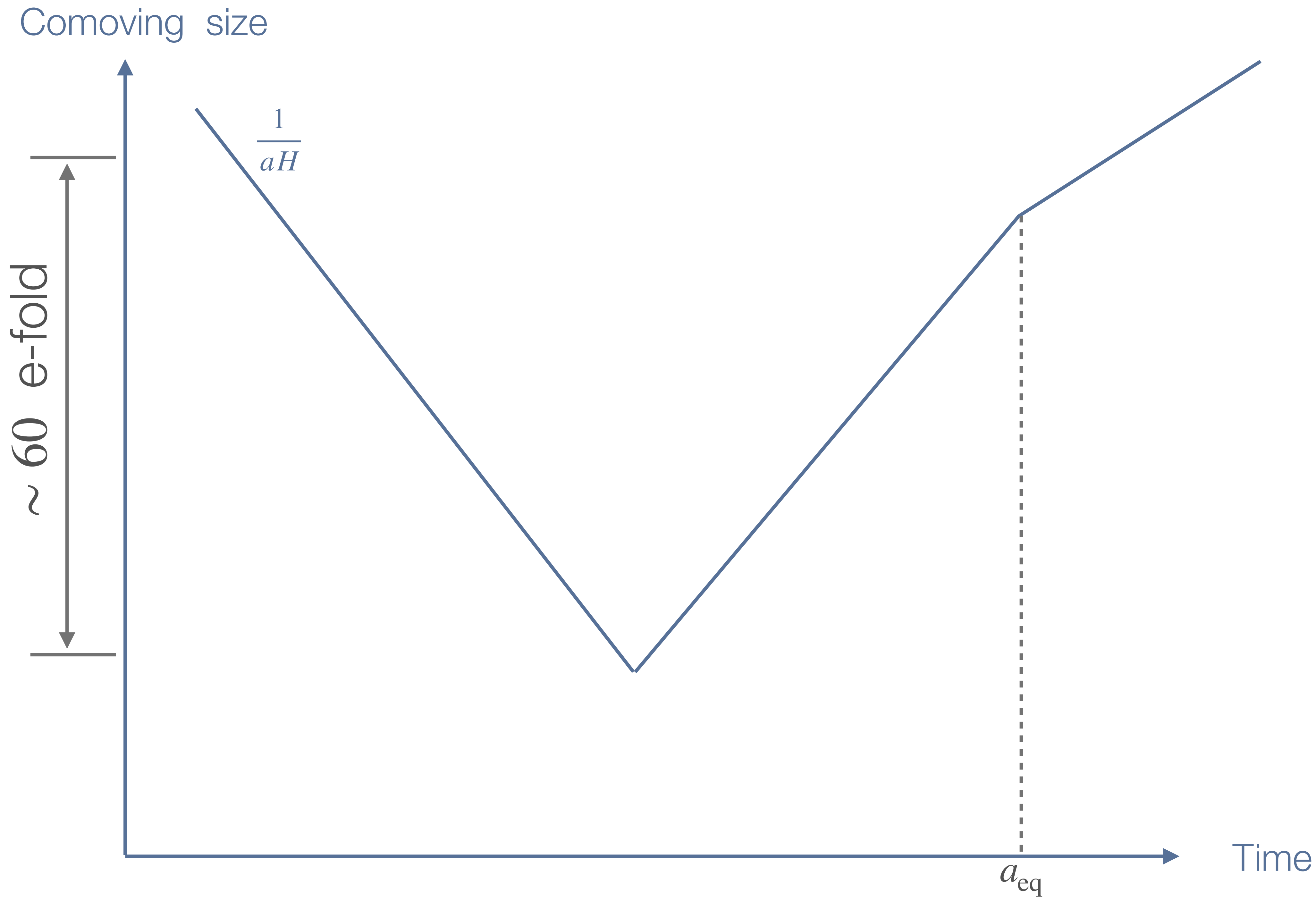
Univ. of Chicago

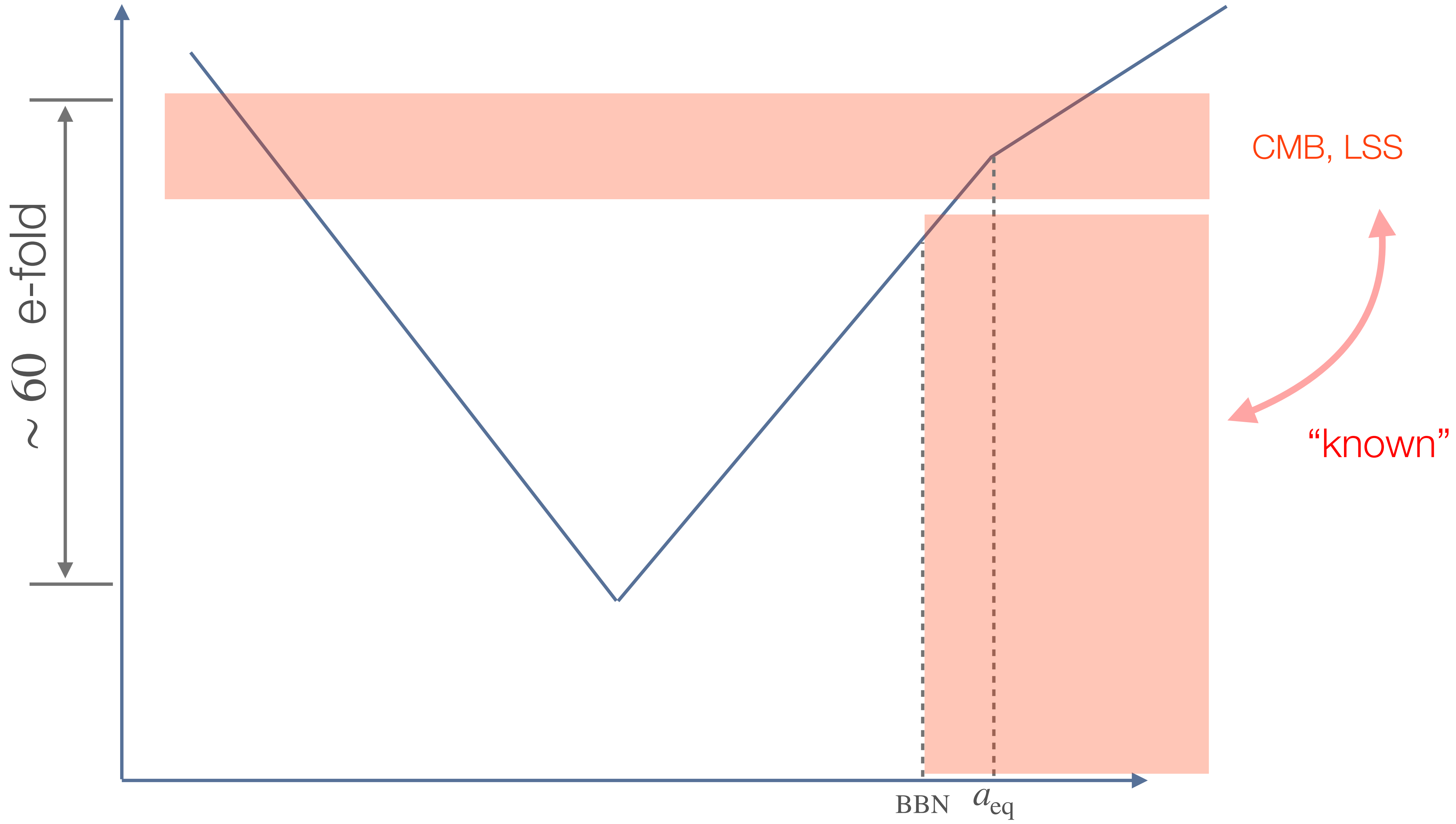
Work in collaboration with Reza Ebadi, Soubhik Kumar, Amara McCune, Hanwen Tai, 2307.12048
and Yunjia Bao, Keisuke Harigaya in progress

Rencontres du Vietnam, ICISE, Qui Nhon, Vietnam. Jan 12. 2024

Early universe



Early universe



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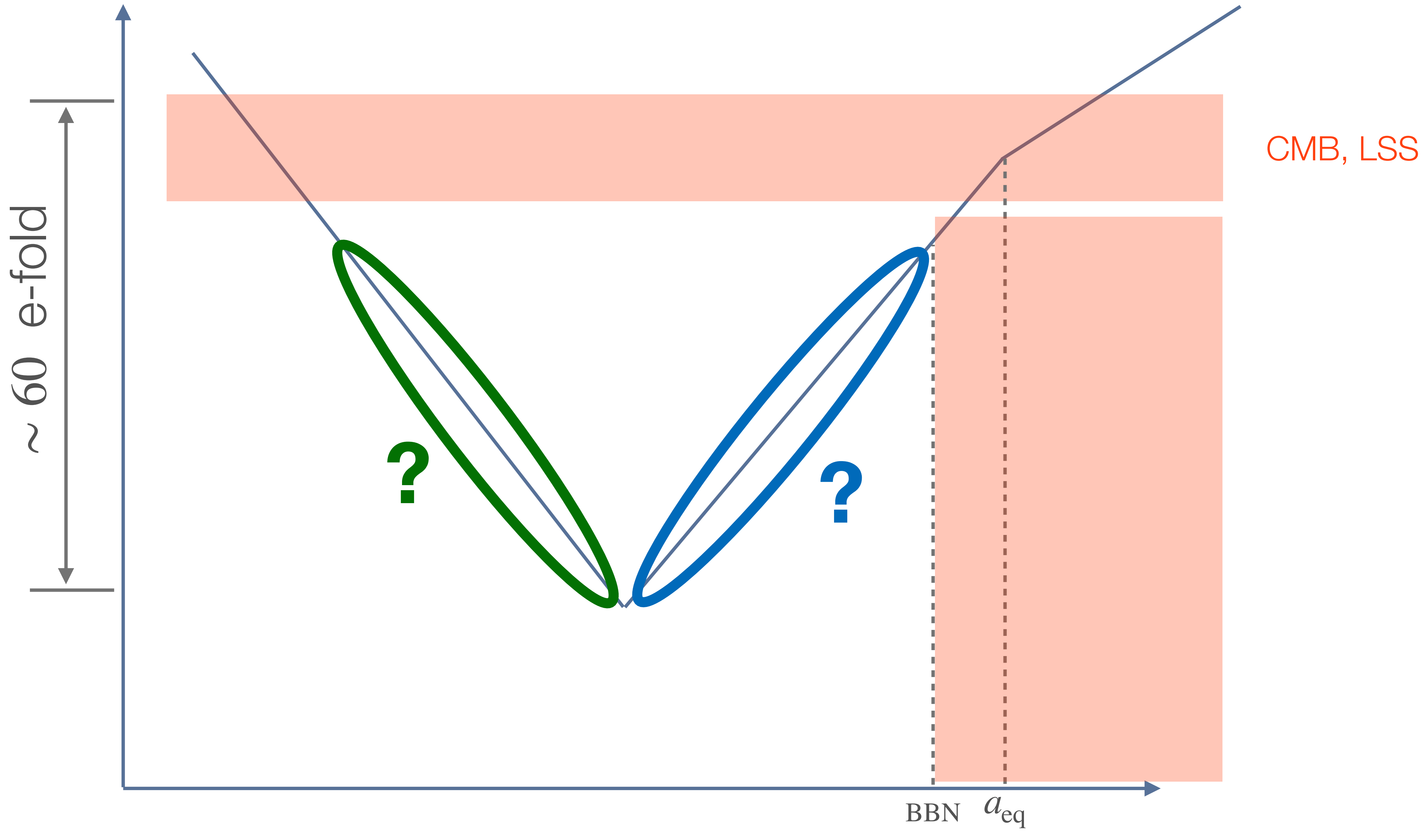

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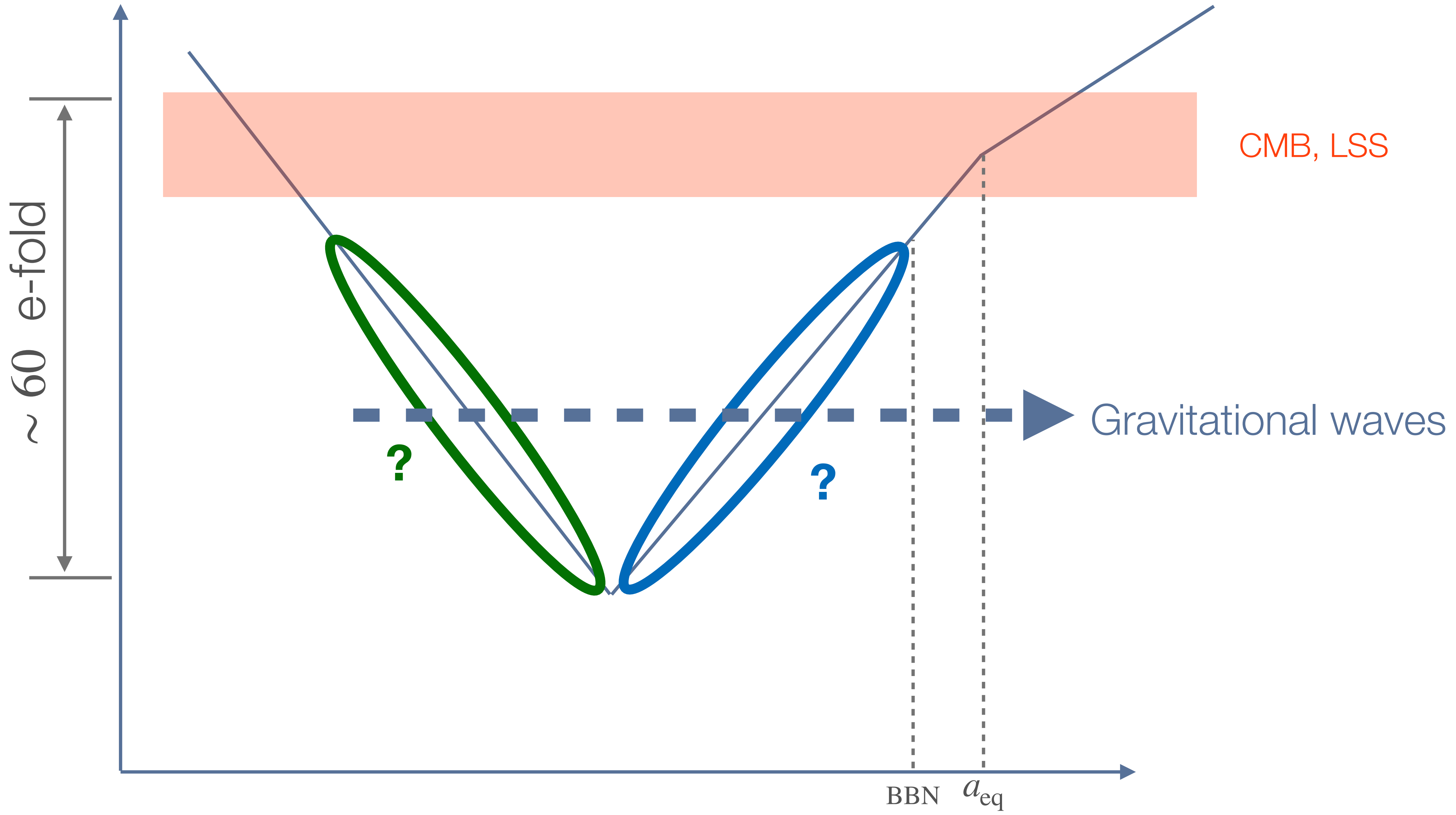
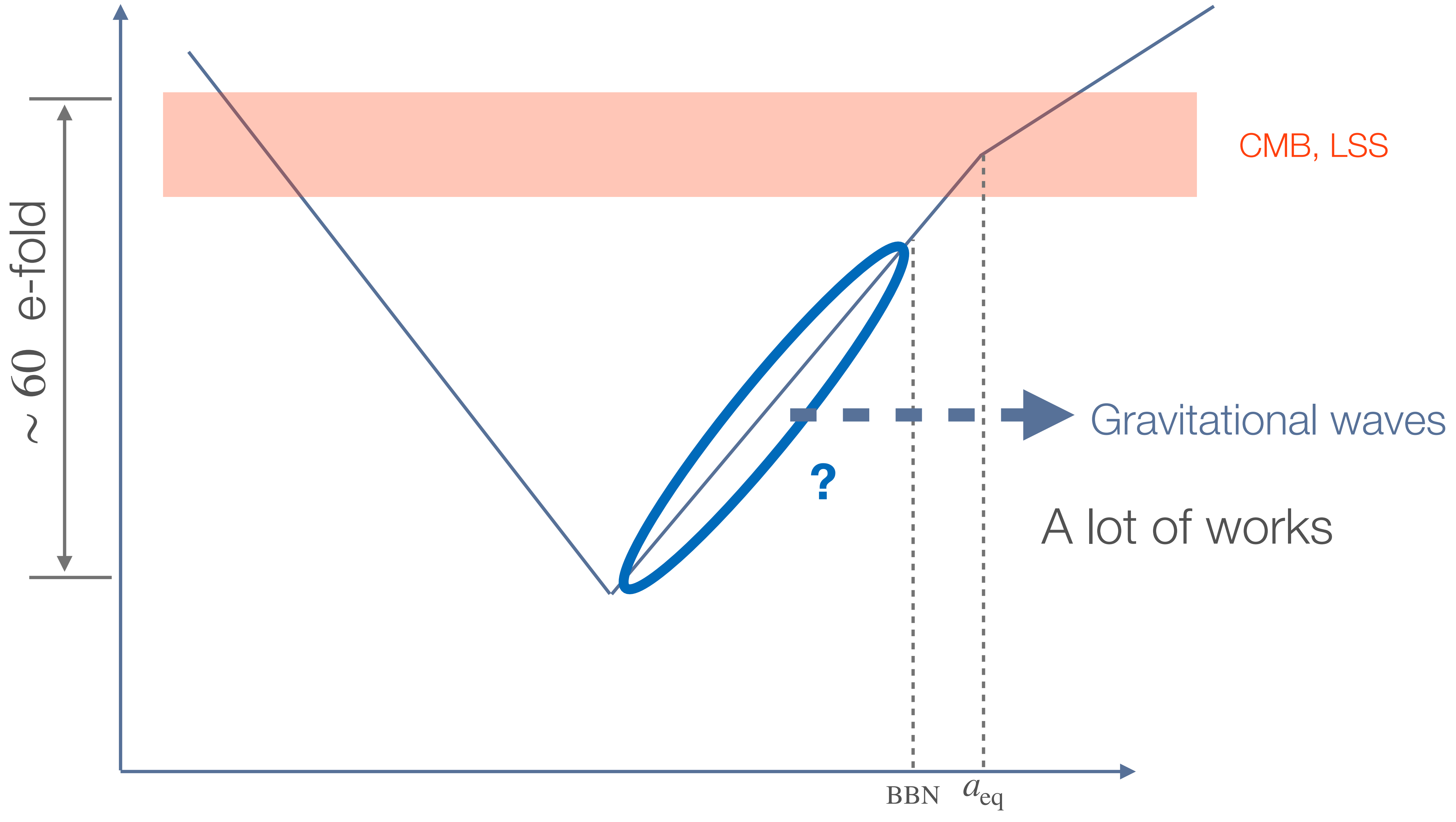
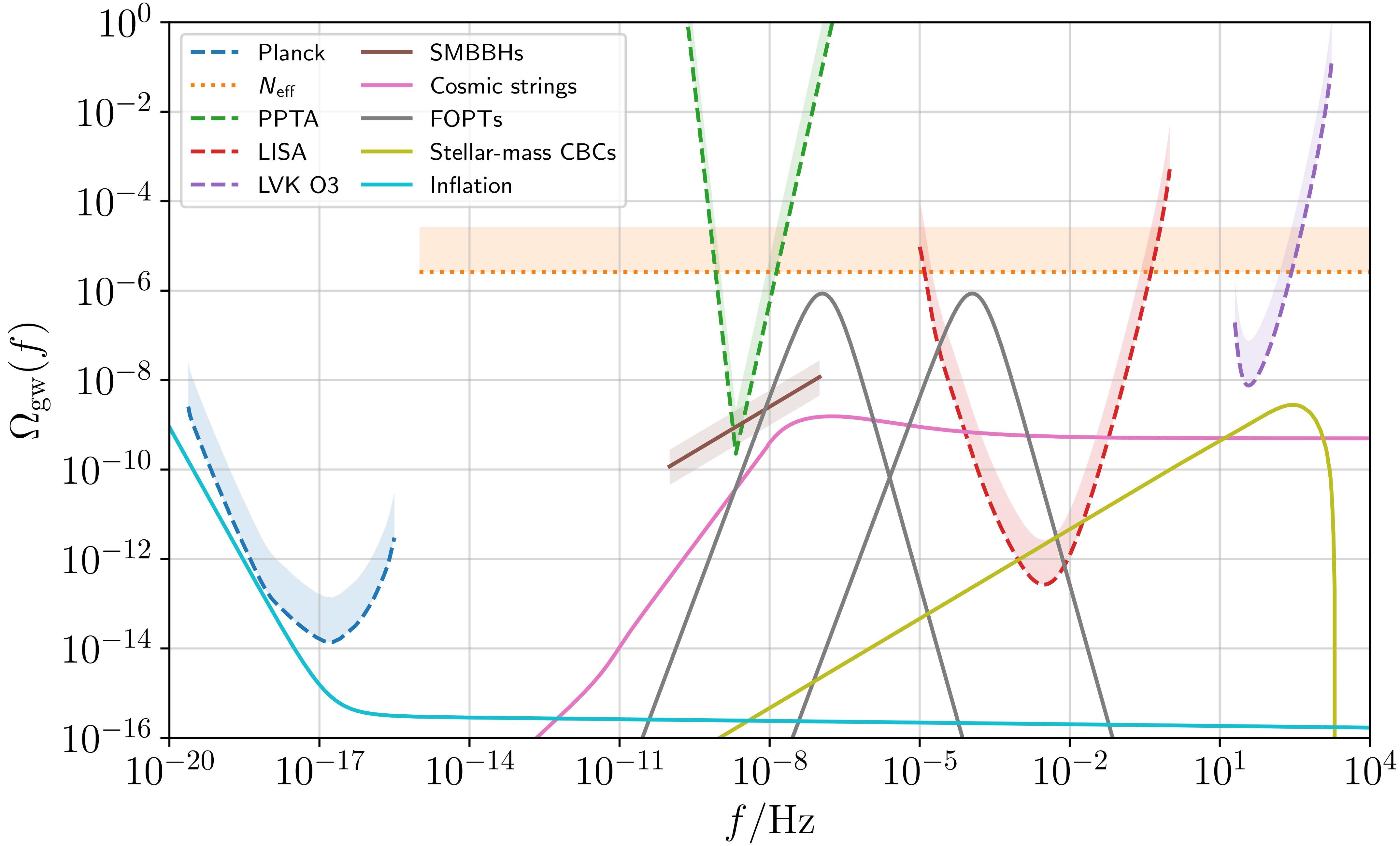


Early universe

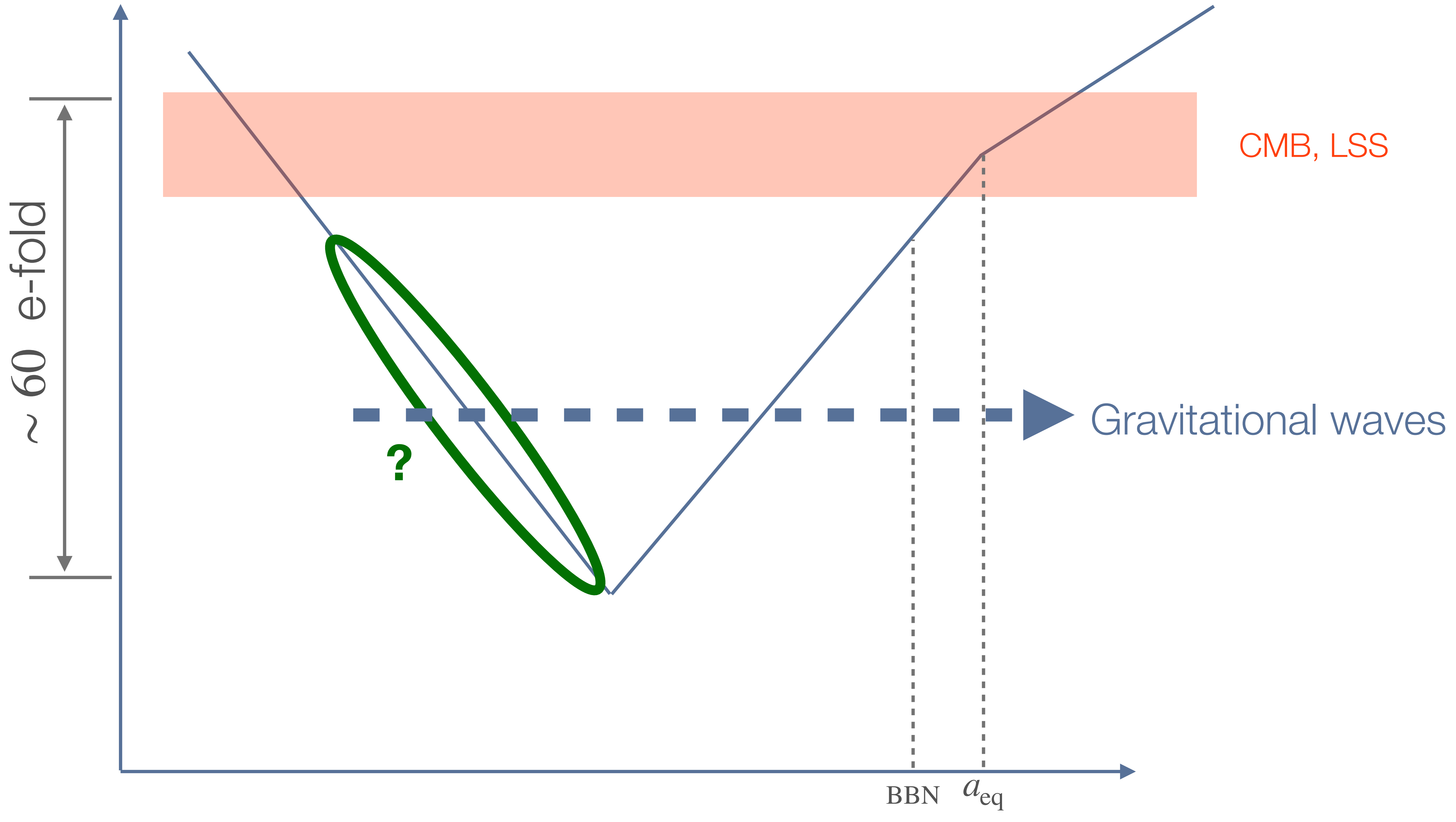


Review by Renzini et al, 2202.00178

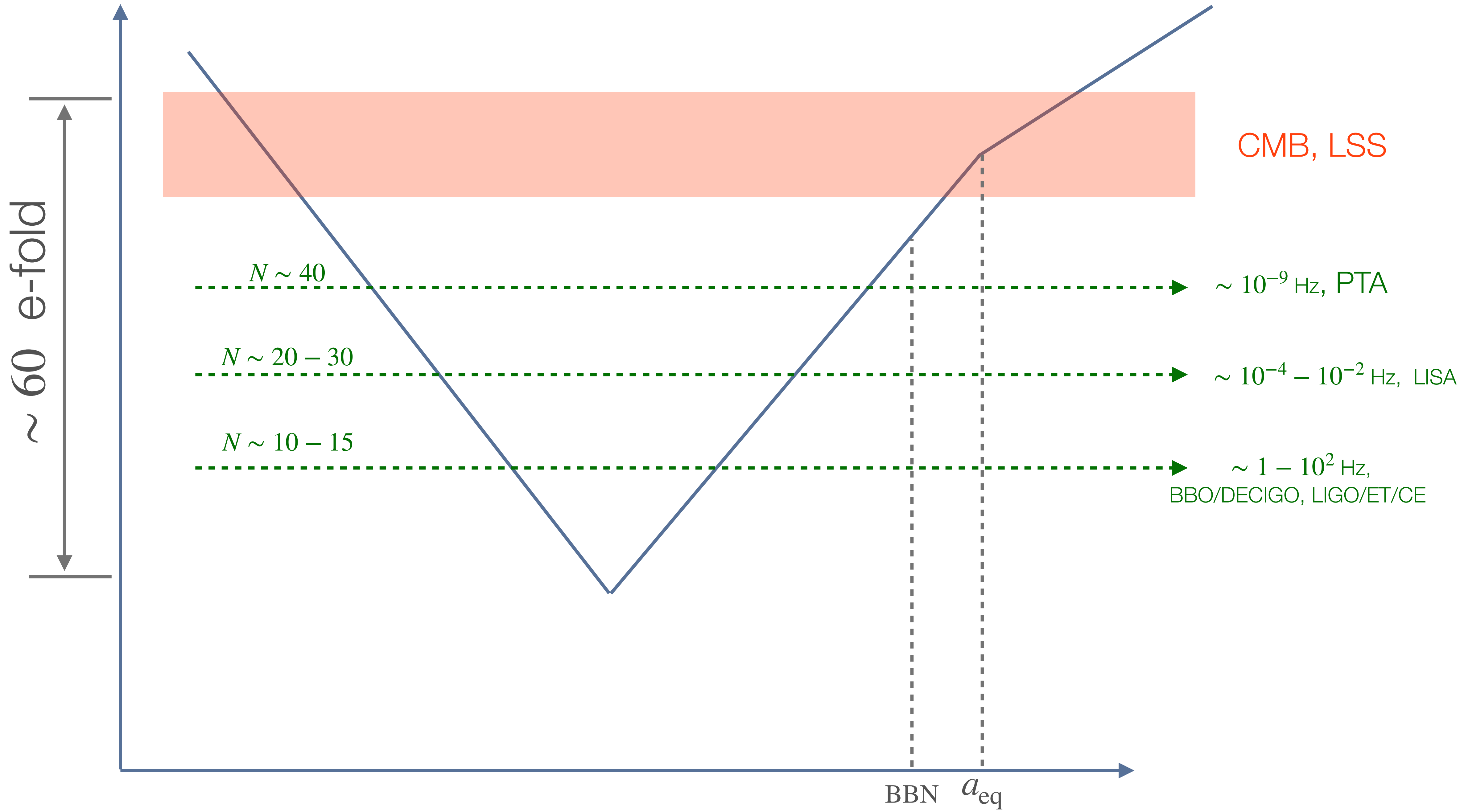


Typically, need something quite dramatic.

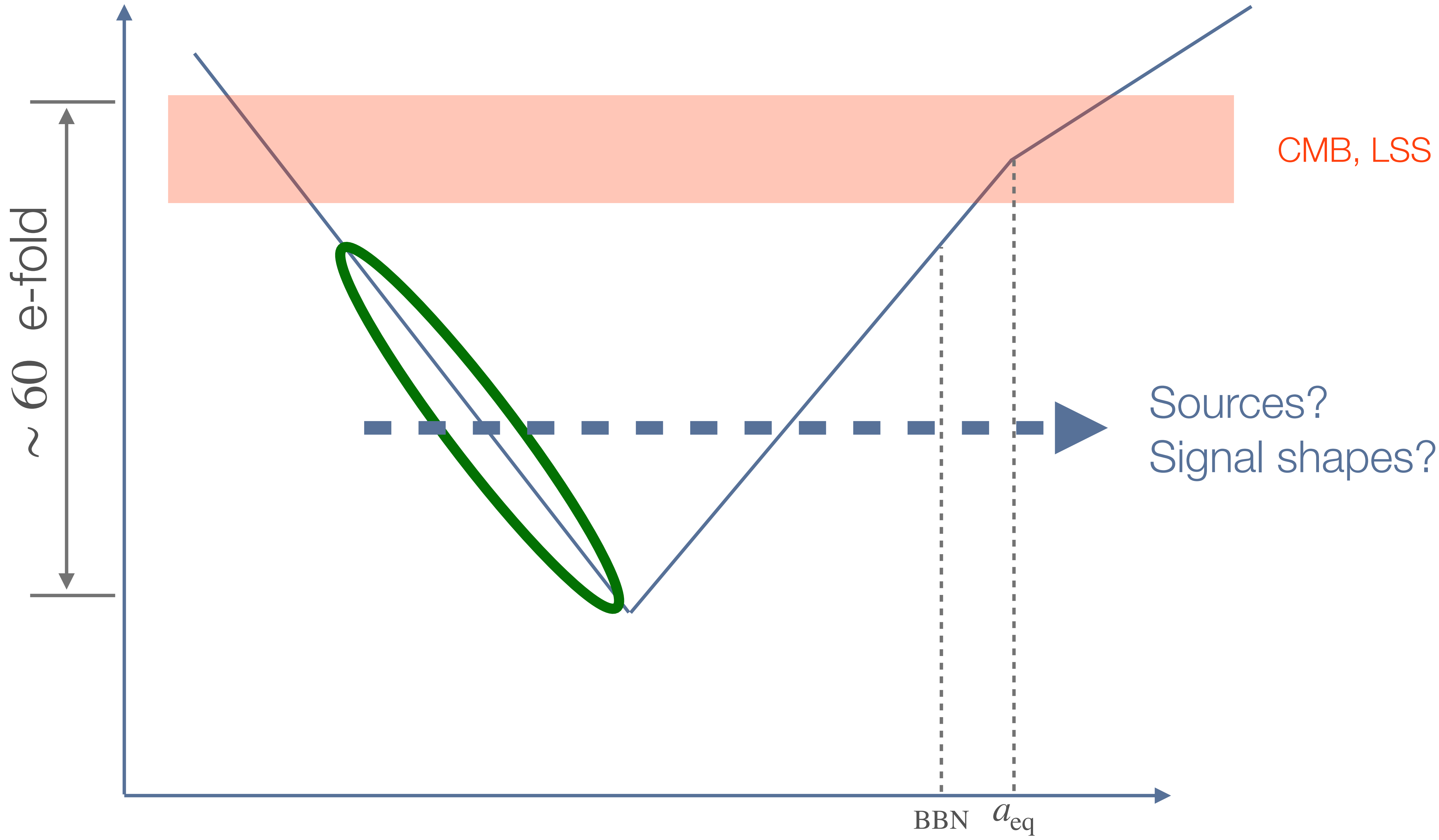
Early universe



Early universe

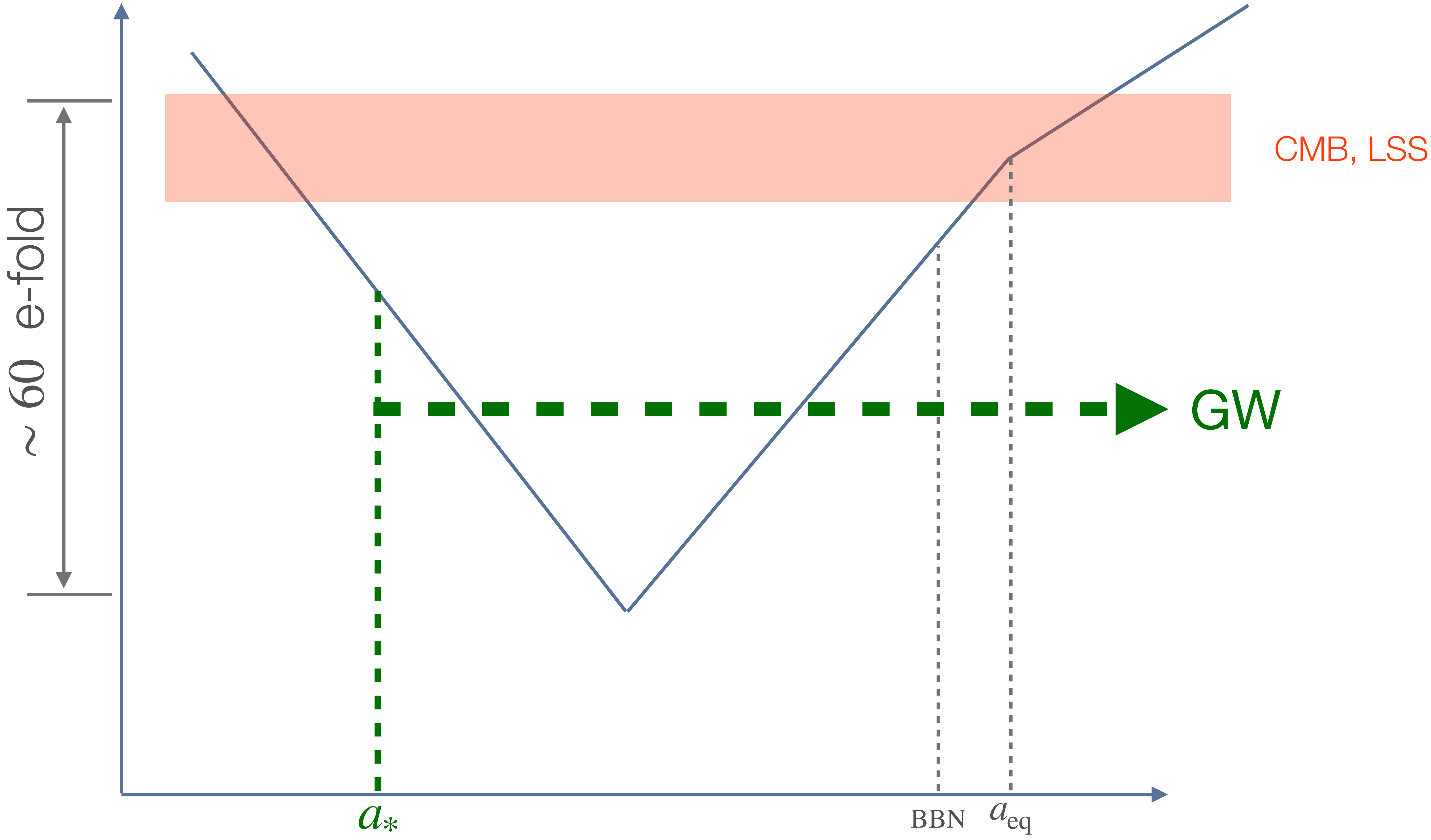


Early universe



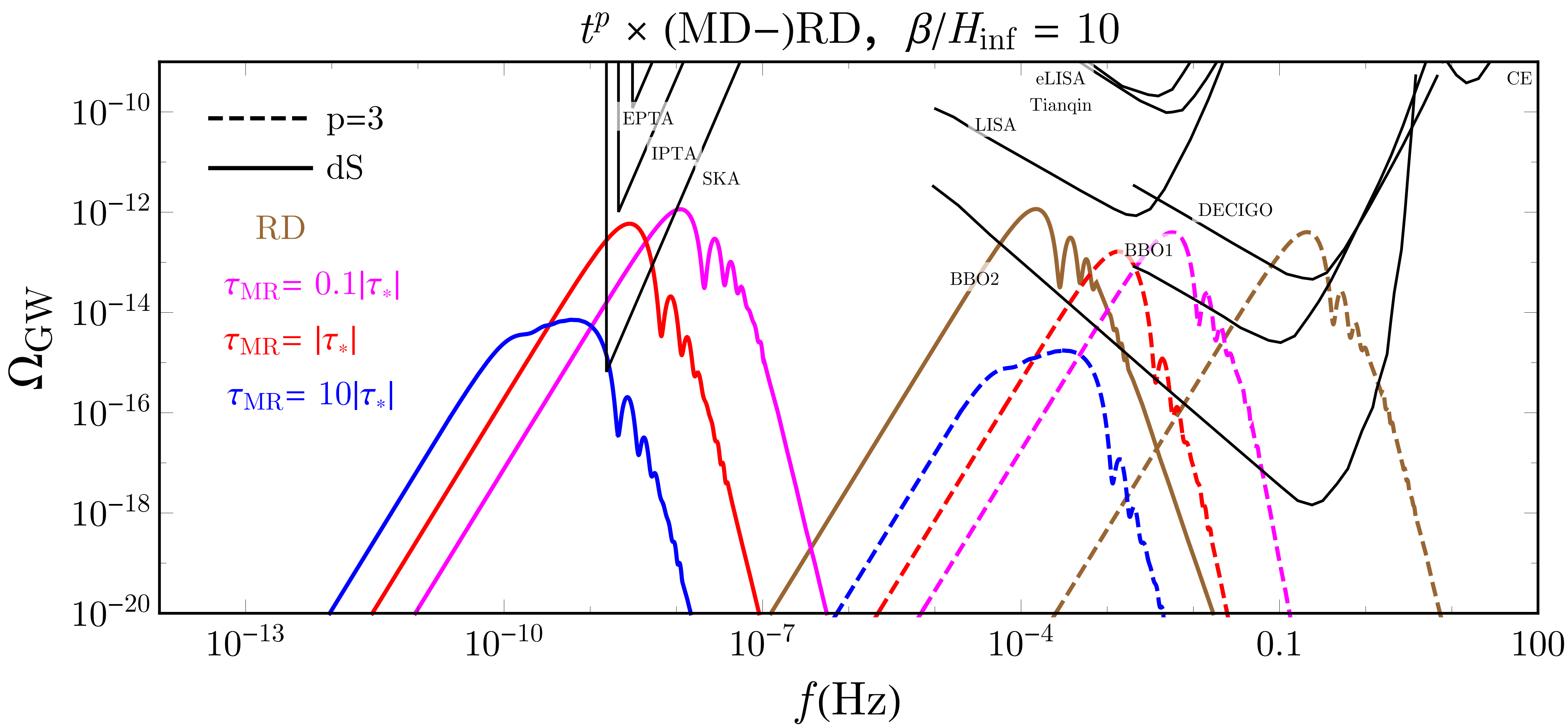
Primary GW

Early universe



Something dramatic generates GW
Example: 1st order PT.

Haipeng An, KunFeng Lyu, Siyi Zhou, and LTW 2009.12381, 2201.05171



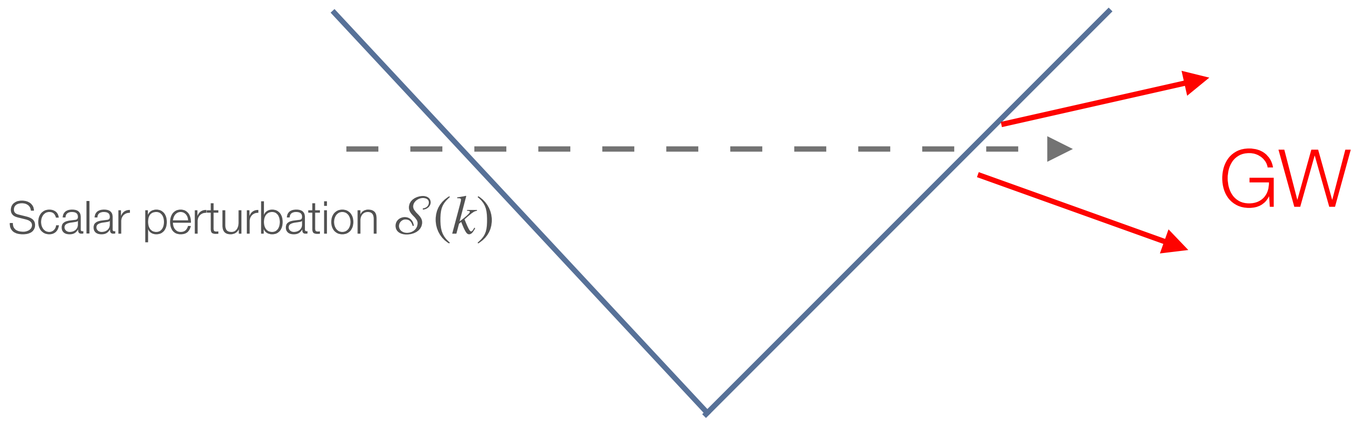
$$\Omega_{\rm GW}^{\rm max} \sim \Omega_R \times \left(\frac{\Delta\rho_{\rm vac}}{\rho_{\rm inf\star}}\right)^2 \times \left(\frac{H_\star}{\beta}\right)^5 \tilde{\Delta} \times F(H_\star/H_r, a_\star/a_r, \cdots)$$

$$\approx 10^{-13} \times \left(\frac{\Delta\rho_{\rm vac}/\rho_{\rm inf\star}}{0.1}\right)^2 \times \left(\frac{H_\star/\beta}{0.1}\right)^5$$

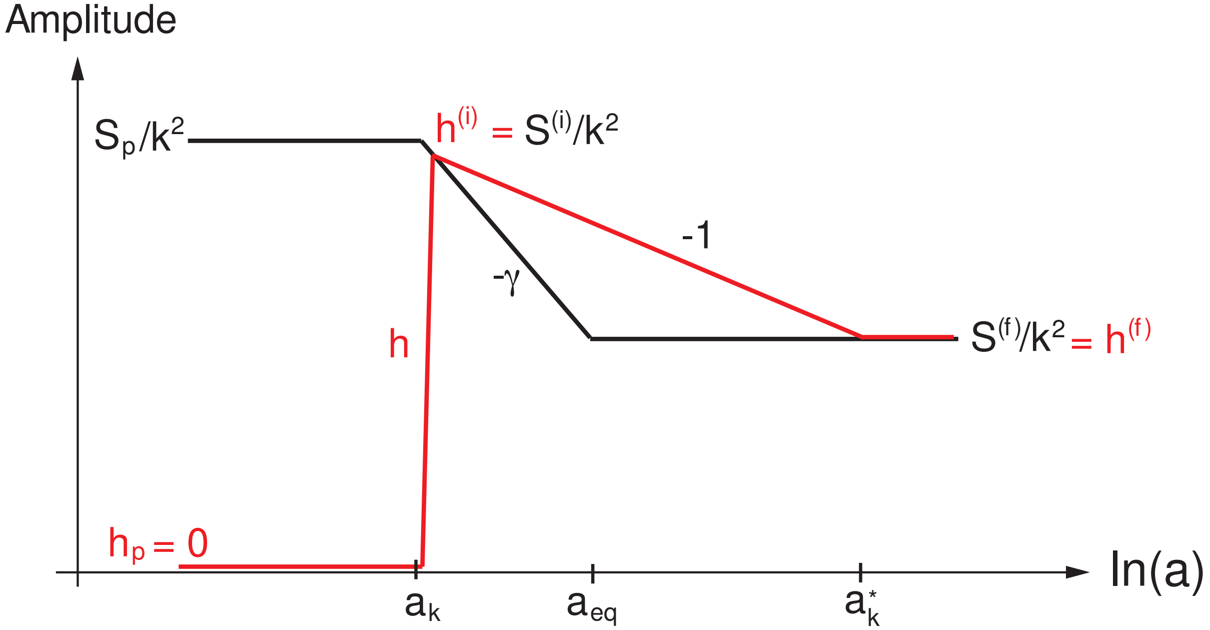
See talk by Haipeng An earlier in this workshop.

Secondary GW

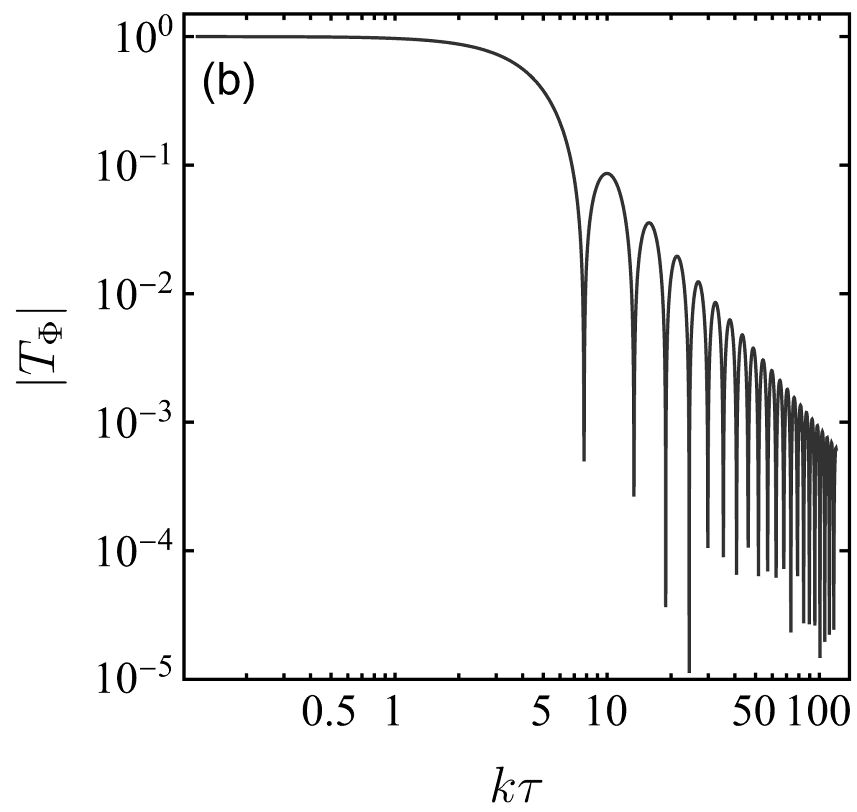
Secondary GW



Secondary GW

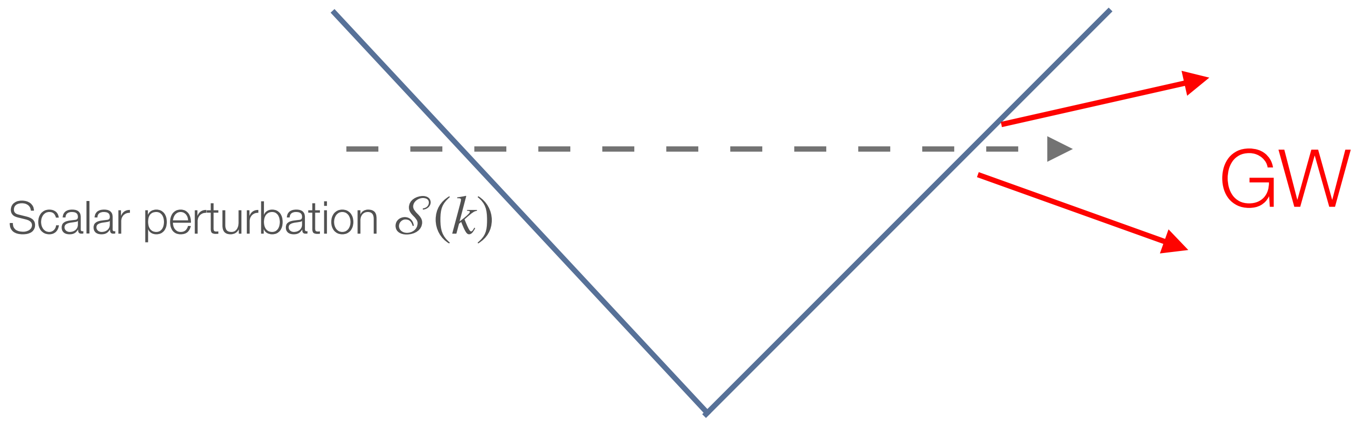


Baumann, Steinhardt, Takahashi, hep-th/0703290



Modes enter horizon during RD, starts oscillate, and generates GW

Secondary GW



$$\mathcal{S}(k) = ?$$

Example: A spectator light scalar

R. Ebadi, S. Kumar, A. McCune, H. Tai, LTW, 2307.12048

$$\mathcal{L} = \frac{1}{2}(\partial\sigma)^2 - \frac{1}{2}m^2\sigma^2 - \frac{\lambda}{4}\sigma^4 \qquad \text{with } m < H$$

The spectrum of its fluctuation can be studied by stochastic method

Starobinsky and Yokoyama, 1994; Markkanen, Rajantie, Stopyra, Tenkanen, 1904.11917

Stochastic method

The spectrum of its fluctuation can be studied by stochastic method

Starobinsky and Yokoyama, 1994; Markkanen, Rajantie, Stopyra, Tenkanen, 1904.11917

Fokker-Planck $$\frac{\partial P_{\mathrm{FP}}(t,\sigma)}{\partial t} = \left(\frac{V''(\sigma)}{3H} + \frac{V'(\sigma)}{3H}\frac{\partial}{\partial \sigma} + \frac{H^3}{8\pi^2}\frac{\partial^2}{\partial^2 \sigma} \right) P_{\mathrm{FP}}(t,\sigma)$$

$P_{\mathrm{FP}}(t,\sigma)$: 1-pt PDF

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Starobinsky and Yokoyama, 1994; Markkanen, Rajantie, Stopyra, Tenkanen, 1904.11917

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Classical evolution, drift

$P_{\mathrm{FP}}(t,\sigma)$: 1-pt PDF

Stochastic method

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Classical evolution, drift   Stochastic, diffusion

$P_{\rm FP}(t,\sigma)$: 1-pt PDF

Light field during inflation

$$m_\sigma^2 < H^2$$

1. Massless. “Stuck” at large field value.
   - Example: misaligned axion.
2. Massive but light.

Light field during inflation

- Massive but light. (Free field for simplicity)

$$\dot{\sigma} = -\frac{m_\sigma^2 \sigma}{3H} \rightarrow \sigma = \exp\left(-m_\sigma^2 \int^t \frac{dt'}{3H(t')}\right) \cdot \sigma_i$$

Initial field value

Light field during inflation

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Initial field value

- Roughly, $-\int^t \frac{dt'}{3H(t')} \sim \frac{1}{\dot{H}}$

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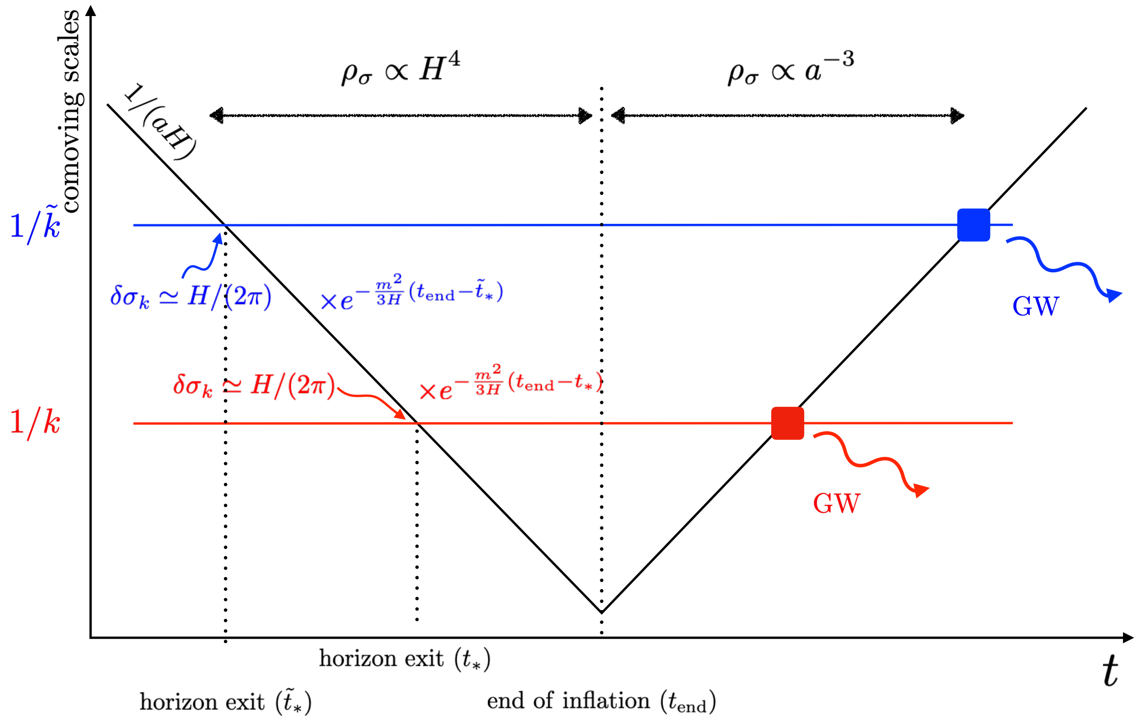
Initial field value

- Roughly, $-\int^t \frac{dt'}{3H(t')} \sim \frac{1}{\dot{H}}$

- If $m_\sigma^2 > \epsilon H^2$ $(\epsilon = \dot{H}/H^2)$,
  - Initial value of field does not matter. Amplitude of field dominated by stochastic fluctuation around origin

Blue tilt

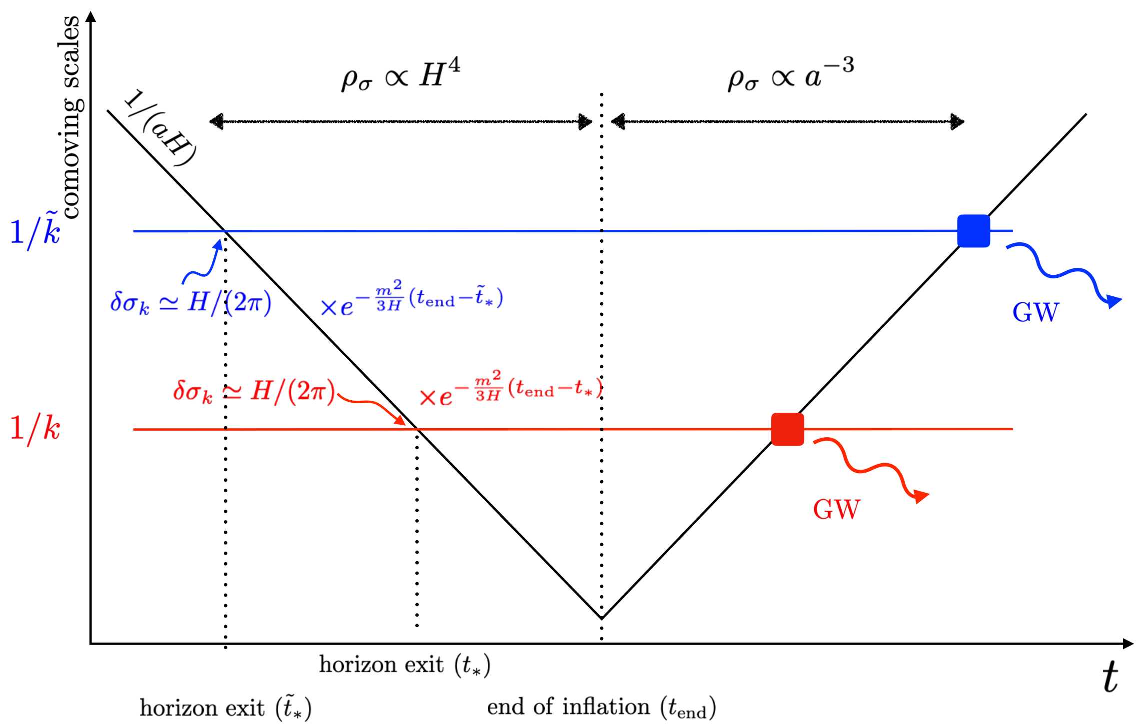


At horizon exit:
Amplitude ≈ H

After exit, damping

$$\dot{\sigma} = -\frac{m_\sigma^2 \sigma}{3H}$$

Blue tilt



At horizon exit:
Amplitude ≈ H

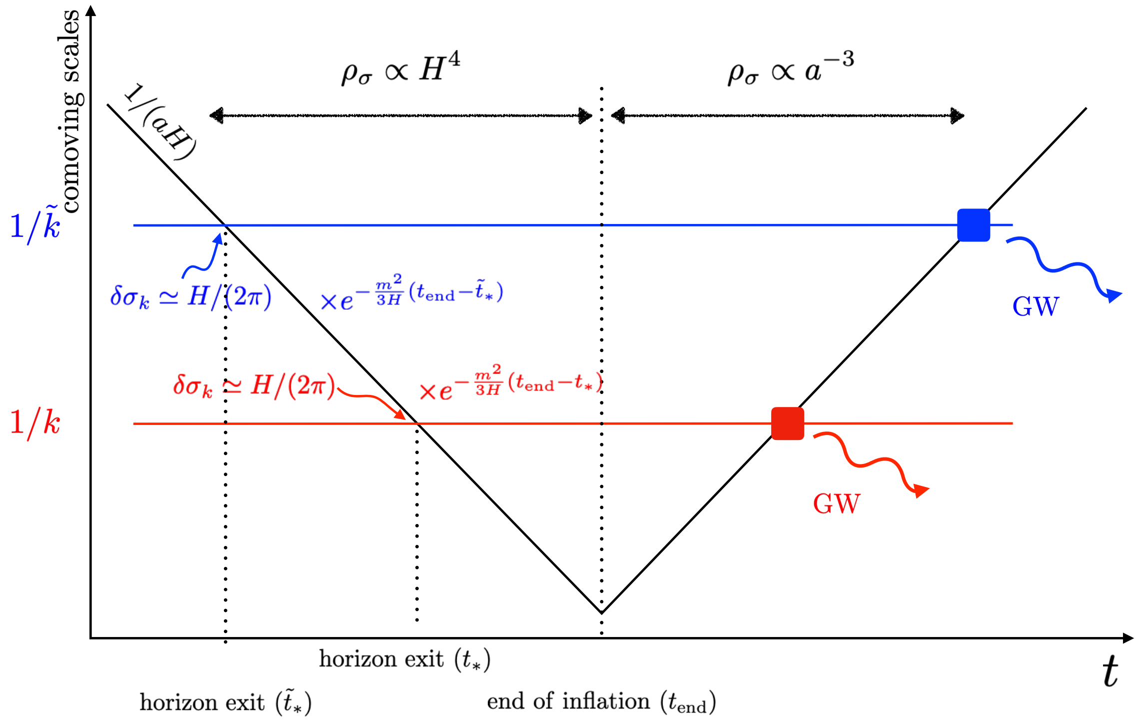
After exit, damping

$$\dot{\sigma} = -\frac{m_\sigma^2 \sigma}{3H}$$

$$\sigma_k(t) = \sigma(t_*) \exp\left(-\frac{m_\sigma^2}{3H}(t-t_*)\right) = \sigma(t_*)\left[\exp\left(-H(t-t_*)\right)\right]^{\frac{m_\sigma^2}{3H^2}} = \sigma(t_*)\left[\frac{k(t)}{H}\right]^{\frac{m_\sigma^2}{3H^2}}$$

More damping for longer wave-length (earlier exit) ⇒ blue tilt

Blue tilt



At horizon exit:
Amplitude ≈ H

After exit, damping

$$\dot{\sigma} = -\frac{m_\sigma^2 \sigma}{3H}$$

For more general scalar theory

$$\mathcal{P}_f(k) = \sum_n \frac{2}{\pi} f_n^2 \Gamma\left(2 - 2\frac{\Lambda_n}{H}\right) \sin\left(\frac{\Lambda_n \pi}{H}\right) \left(\frac{k}{H}\right)^{2\Lambda_n/H} \quad \to \mathscr{A} \left(\frac{k}{H}\right)^{\frac{2\Lambda_{\rm lowest}}{H}} \quad \text{for } k \ll H$$

Blue tilt

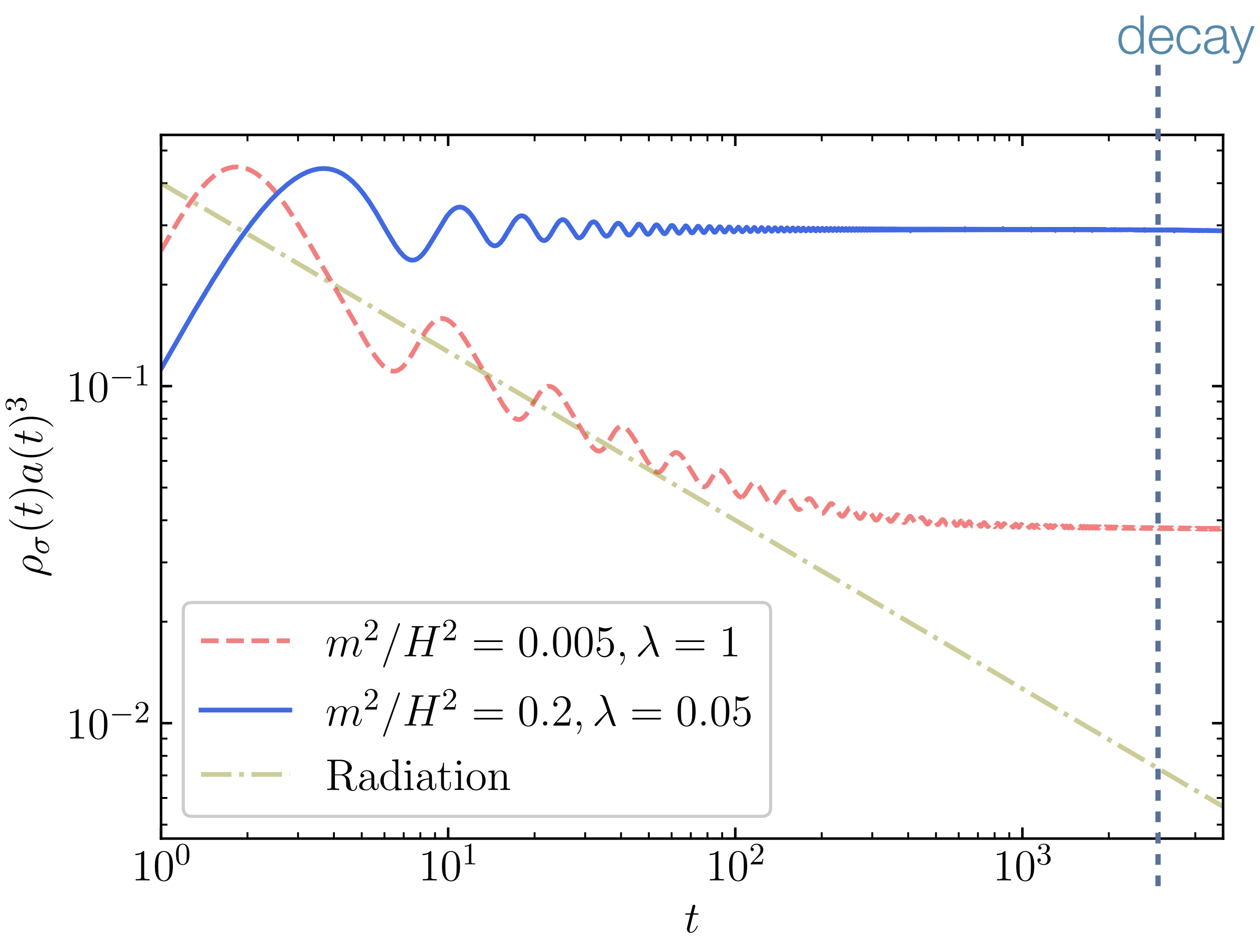
m^2/H^2	λ	Λ_2/H	g_2^2	Λ_4/H	g_4^2
0.2	0.05	0.16	1.99	0.37	0.03
0.2	0.07	0.17	1.98	0.40	0.05
0.2	0.1	0.18	1.98	0.44	0.07
0.25	0.05	0.19	1.99	0.42	0.02
0.25	0.07	0.20	1.99	0.45	0.03
0.25	0.1	0.21	1.98	0.49	0.05
0.3	0.05	0.22	1.99	0.48	0.01
0.3	0.07	0.23	1.99	0.51	0.02
0.3	0.1	0.24	1.99	0.54	0.03

Generic to have sizable blue tilt

$$\mathcal{P}_f(k) = \sum_n \frac{2}{\pi} f_n^2 \Gamma\left(2 - 2\frac{\Lambda_n}{H}\right) \sin\left(\frac{\Lambda_n \pi}{H}\right) \left(\frac{k}{H}\right)^{2\Lambda_n/H} \quad \to \mathscr{A} \left(\frac{k}{H}\right)^{\frac{2\Lambda_{\text{lowest}}}{H}}$$

for $k \ll H$

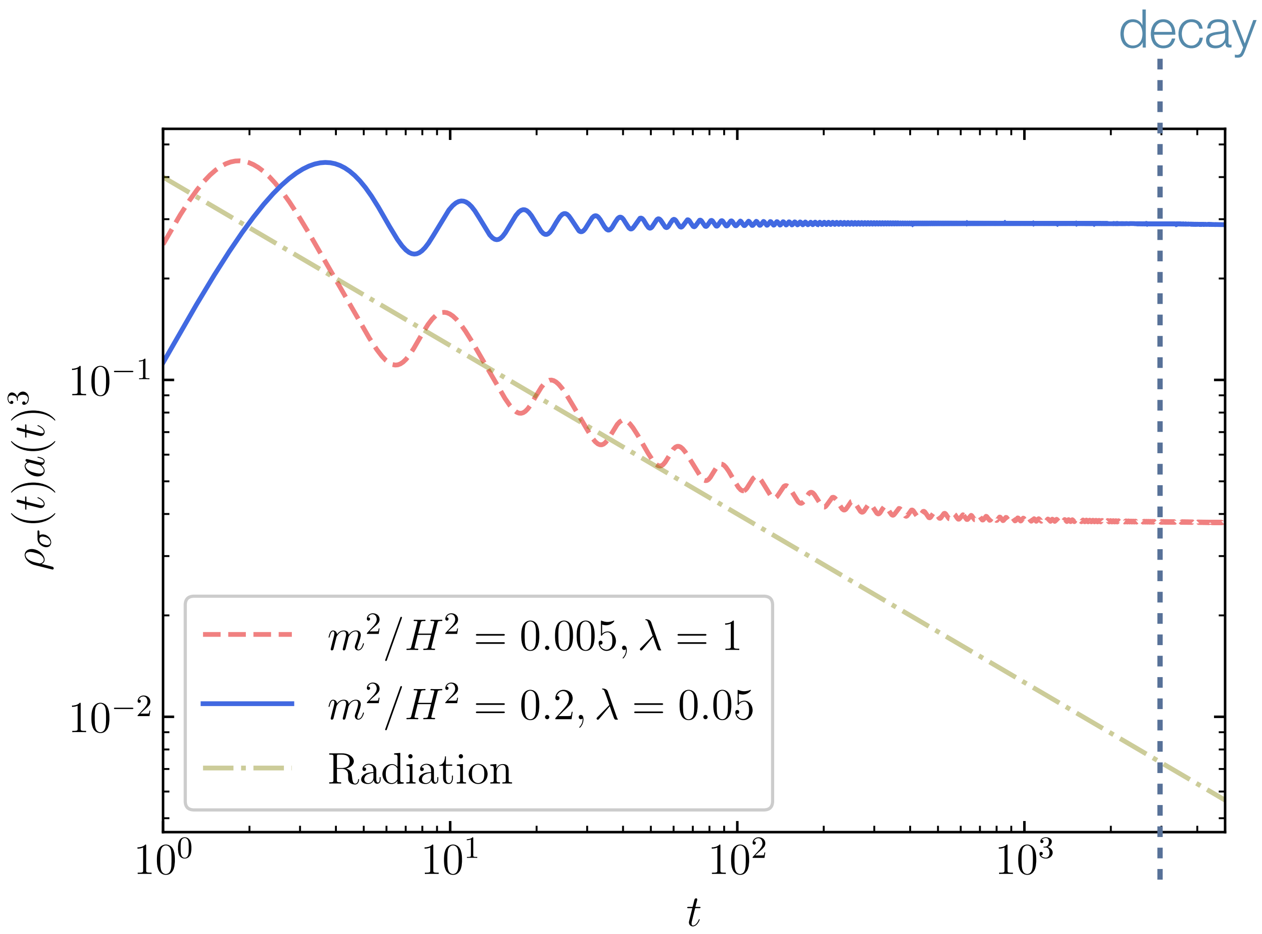
After inflation



Eventually,
evolve like matter

Can become important

After inflation



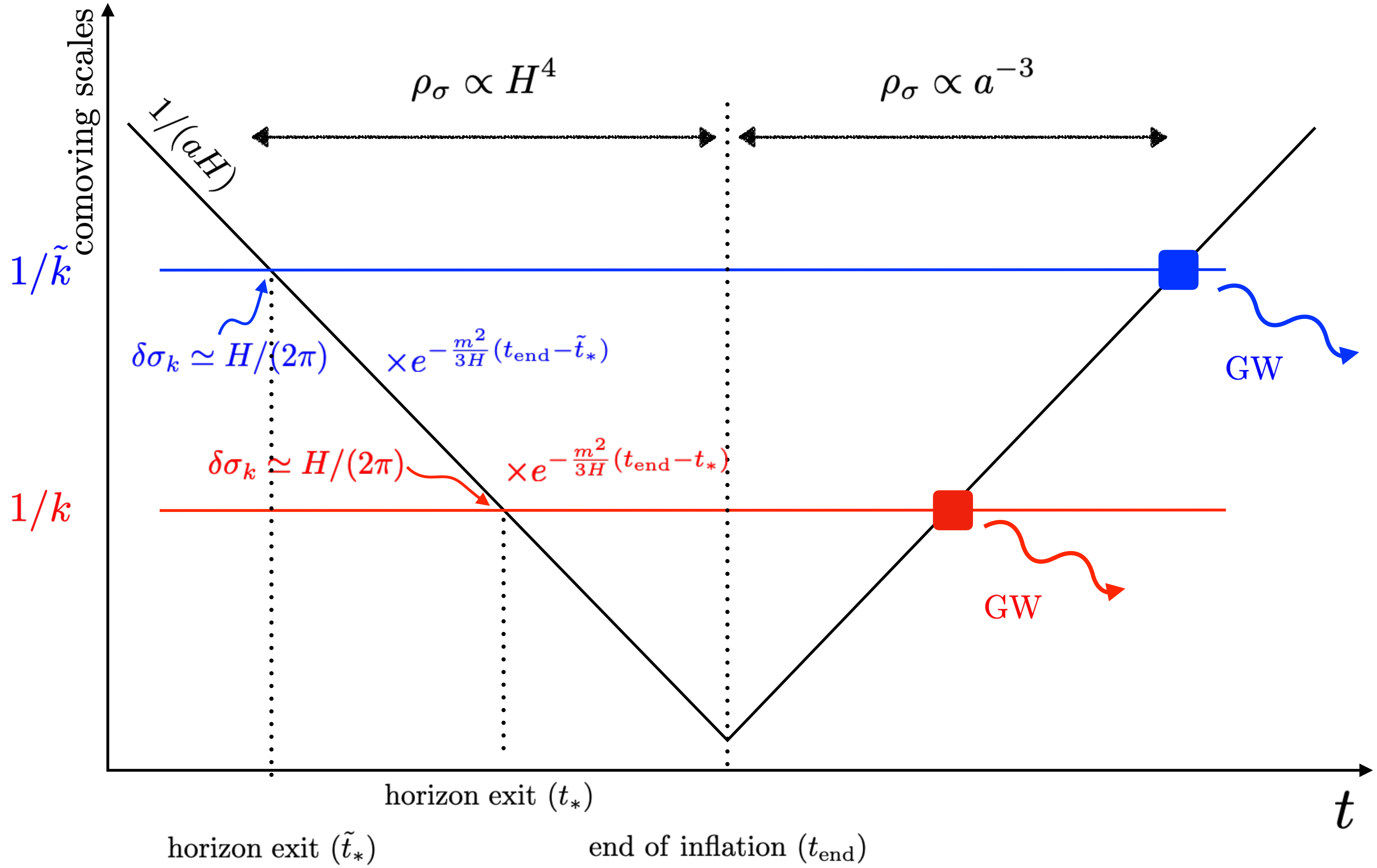
Eventually,
evolve like matter

Can become important

$$\Delta^2_\zeta(k) = \begin{cases} \Delta^2_{\zeta_r}(k) + \left(\frac{f_\sigma(t_d)}{4+3f_\sigma(t_d)}\right)^2 \Delta^2_{S_\sigma}(k), \ k < k_d, \\ \Delta^2_{\zeta_r}(k) + \left(\frac{f_\sigma(t_d)(k_d/k)}{4+3f_\sigma(t_d)(k_d/k)}\right)^2 \Delta^2_{S_\sigma}(k), \ k > k_d \end{cases}$$

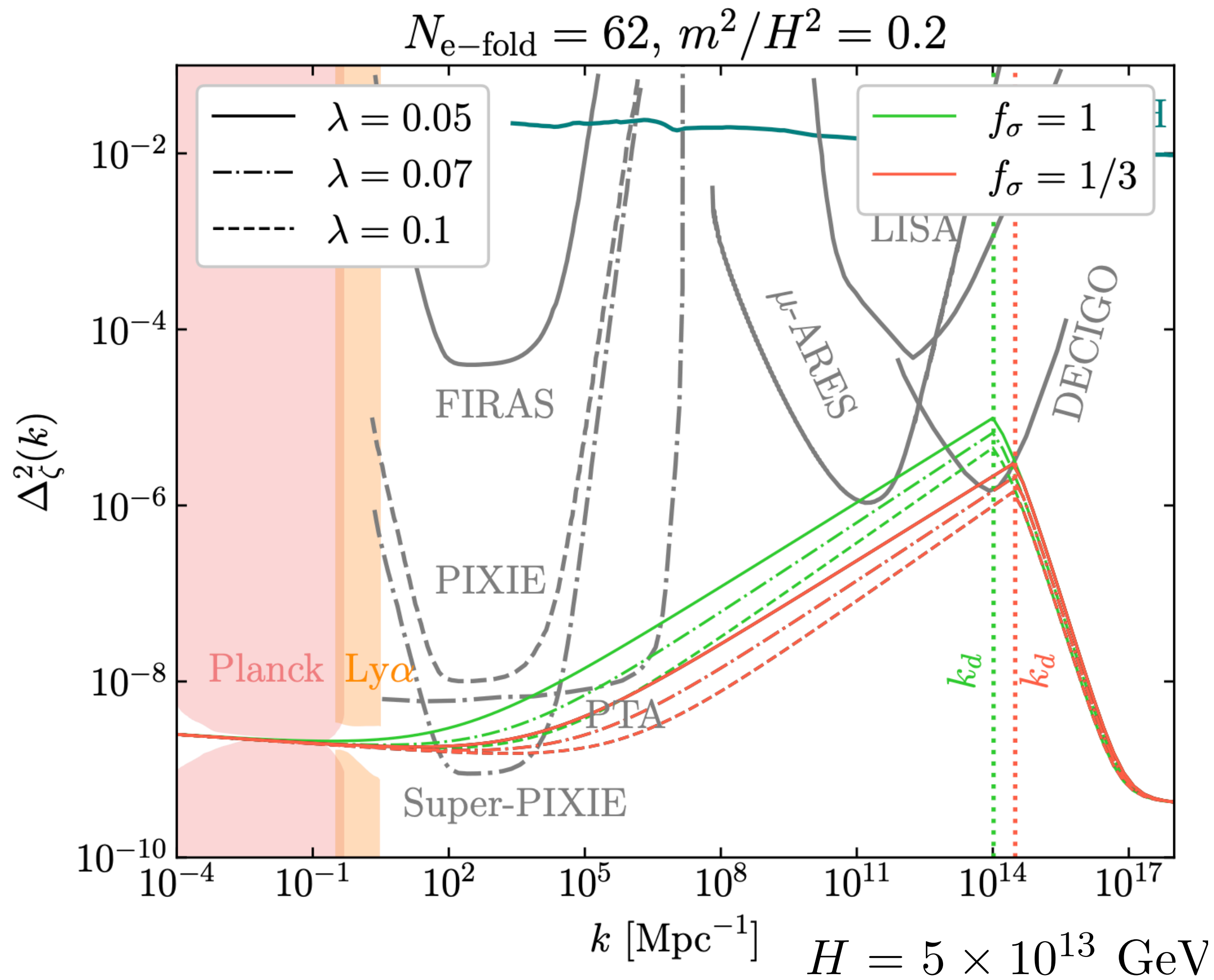
k_d, mode entering the horizon when the scalar decays.

2nd GW


comoving scales
$1/(aH)$
$\rho_\sigma \propto H^4$
$\rho_\sigma \propto a^{-3}$
$1/\tilde{k}$
$1/k$
$\delta\sigma_k \simeq H/(2\pi)$
$\times e^{-\frac{m^2}{3H}(t_{\rm end}-\tilde{t}_*)}$
$\delta\sigma_k \simeq H/(2\pi)$
$\times e^{-\frac{m^2}{3H}(t_{\rm end}-t_*)}$
GW
GW
t
horizon exit (t_*)
horizon exit $(\tilde{t}_*)$
end of inflation $(t_{\rm end})$

Power spectrum

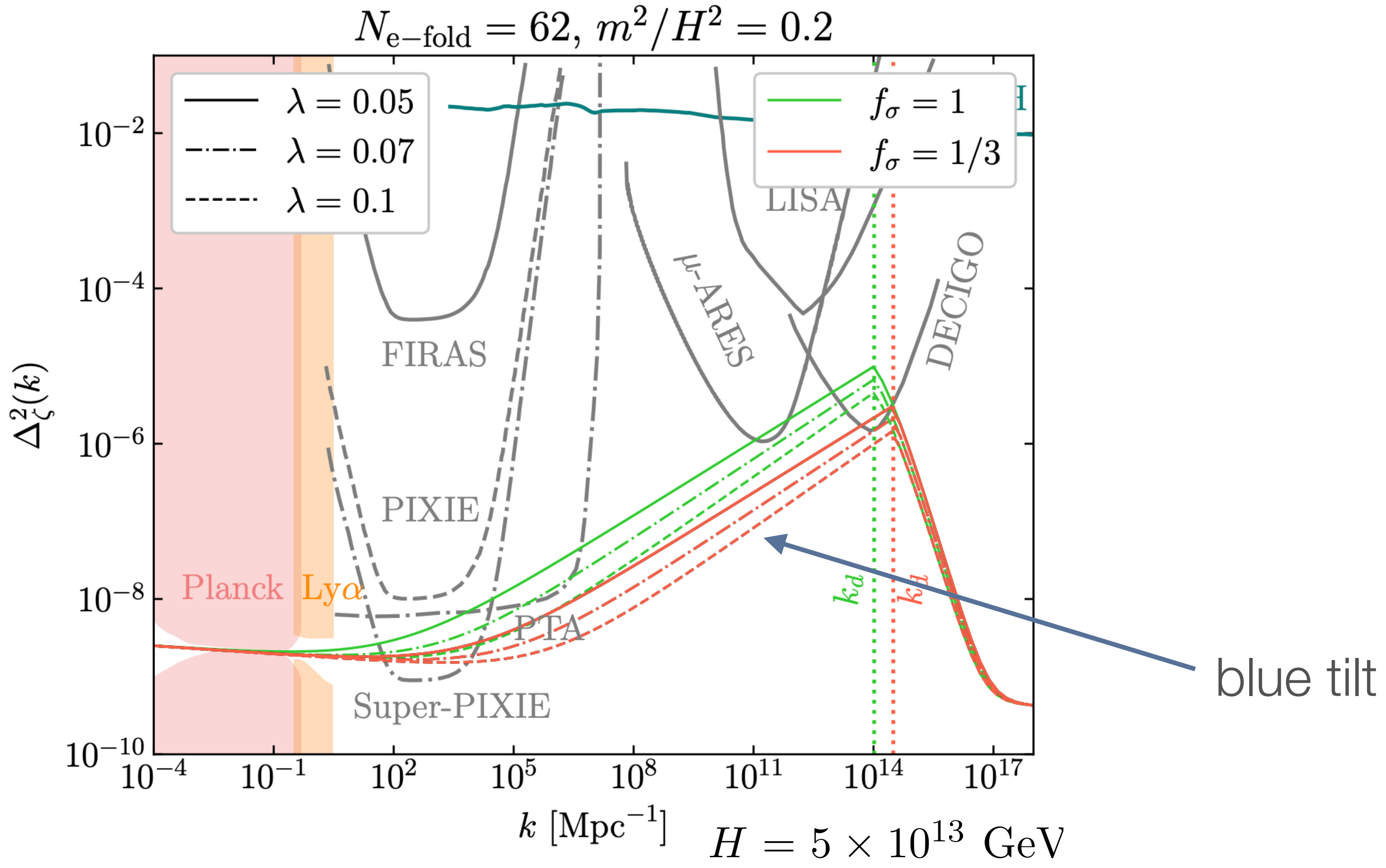
Reza Ebadi, Soubhik Kumar, Amara McCune, Hanwen Tai, LTW, 2307.12048



Assuming the scalar behave similar to curvaton.
Becoming important before decay.

Power spectrum

Reza Ebadi, Soubhik Kumar, Amara McCune, Hanwen Tai, LTW, 2307.12048



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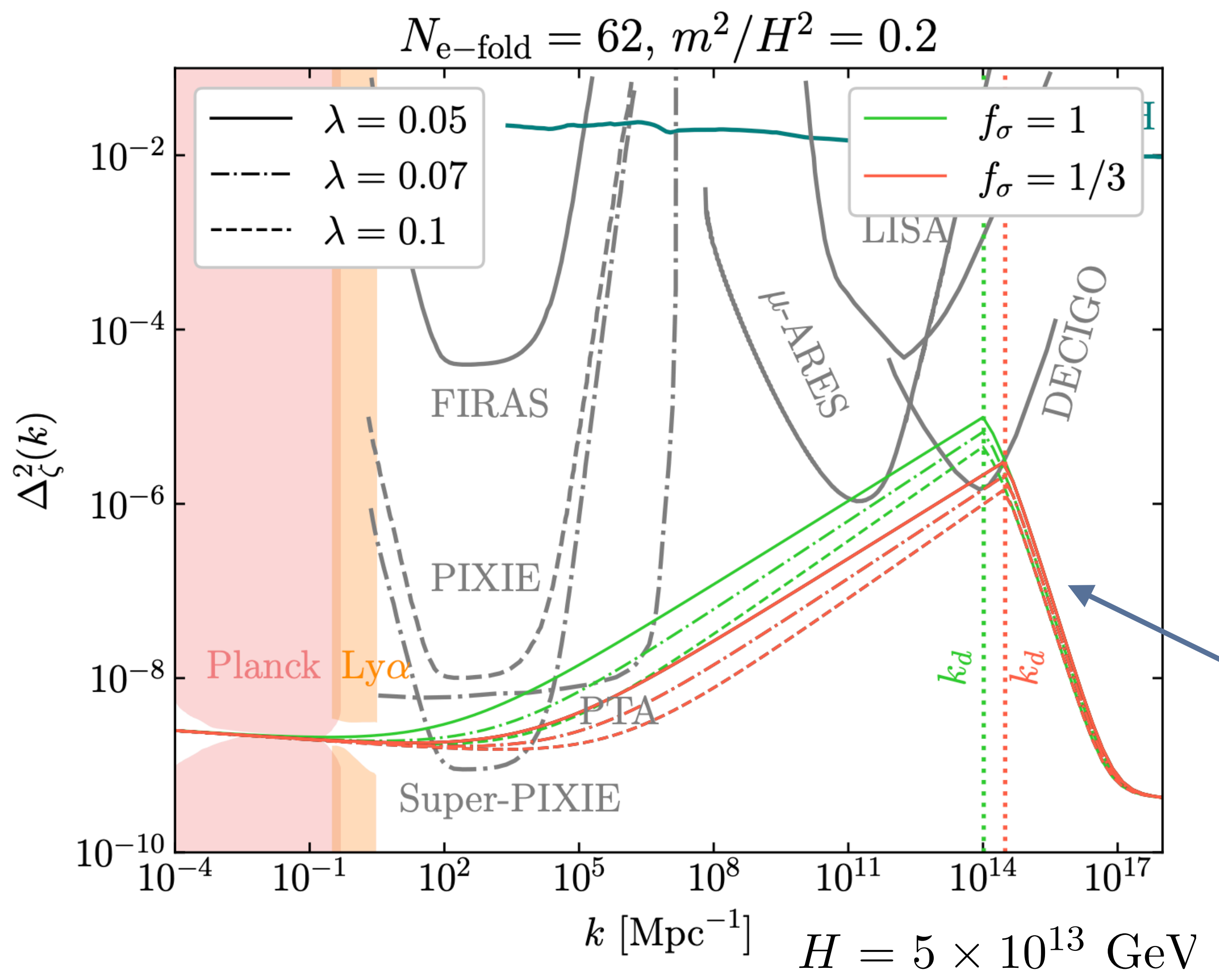

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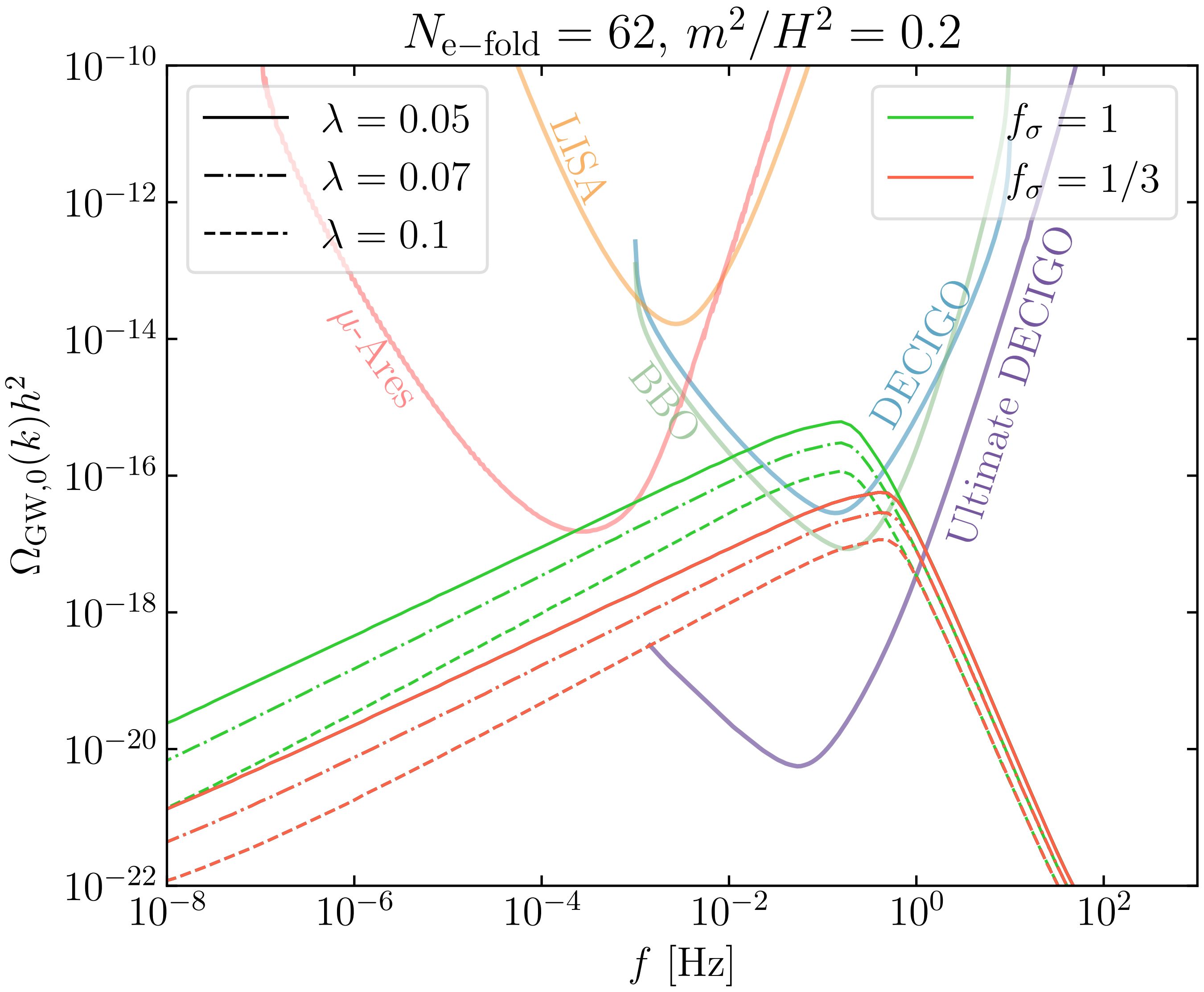
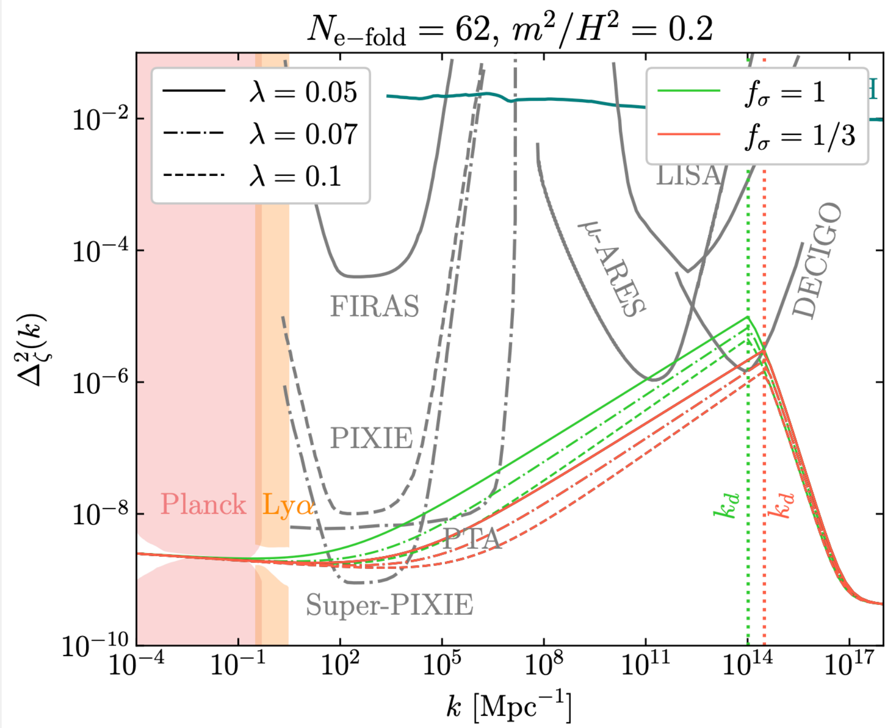
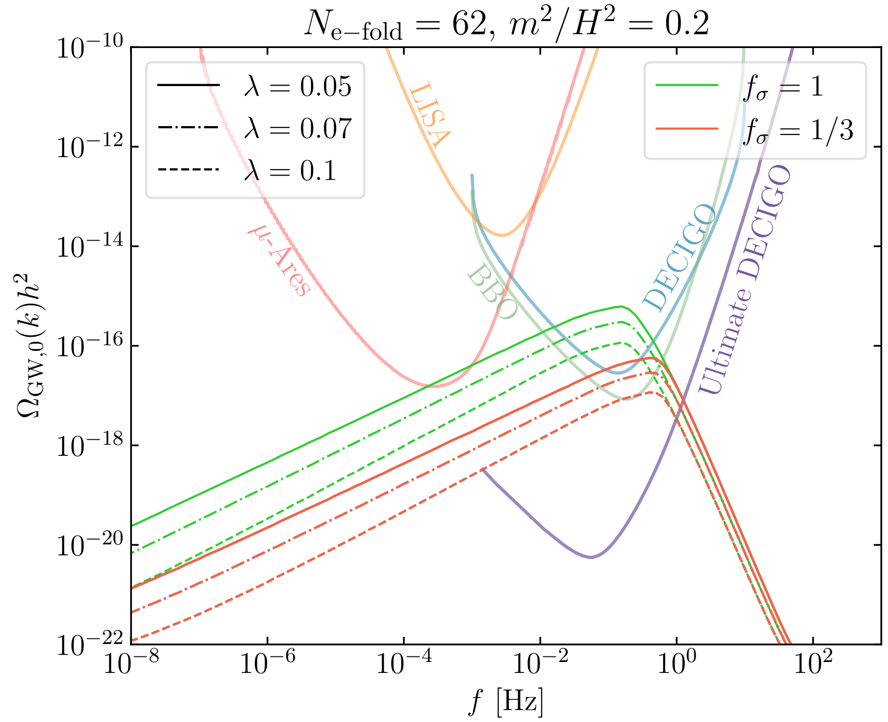
Gravitational wave

Reza Ebadi, Soubhik Kumar, Amara McCune, Hanwen Tai, LTW, 2307.12048



NanoGrav? No.





Blue tilt in the case not large enough to give rise to the signal.

Larger tilt?

$$\mathcal{P}_f(k) = \sum_n \frac{2}{\pi} f_n^2 \Gamma\left(2 - 2\frac{\Lambda_n}{H}\right) \sin\left(\frac{\Lambda_n \pi}{H}\right) \left(\frac{k}{H}\right)^{2\Lambda_n/H} \quad \rightarrow \mathscr{A} \left(\frac{k}{H}\right)^{\frac{2\Lambda_{\text{lowest}}}{H}} \quad \text{for } k \ll H$$

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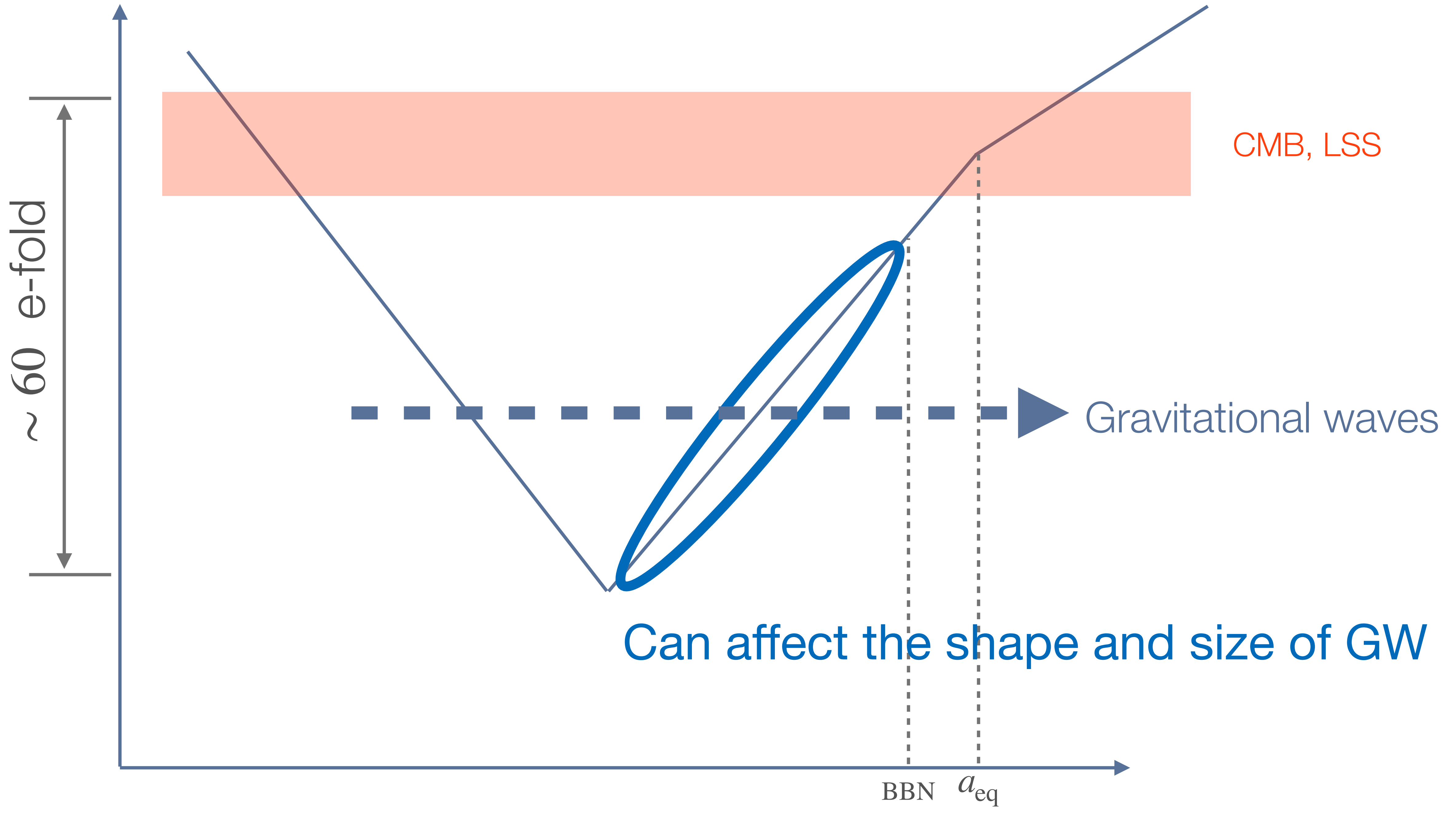
Where $$\frac{\Lambda}{H} \sim \frac{m^2}{H^2}$$

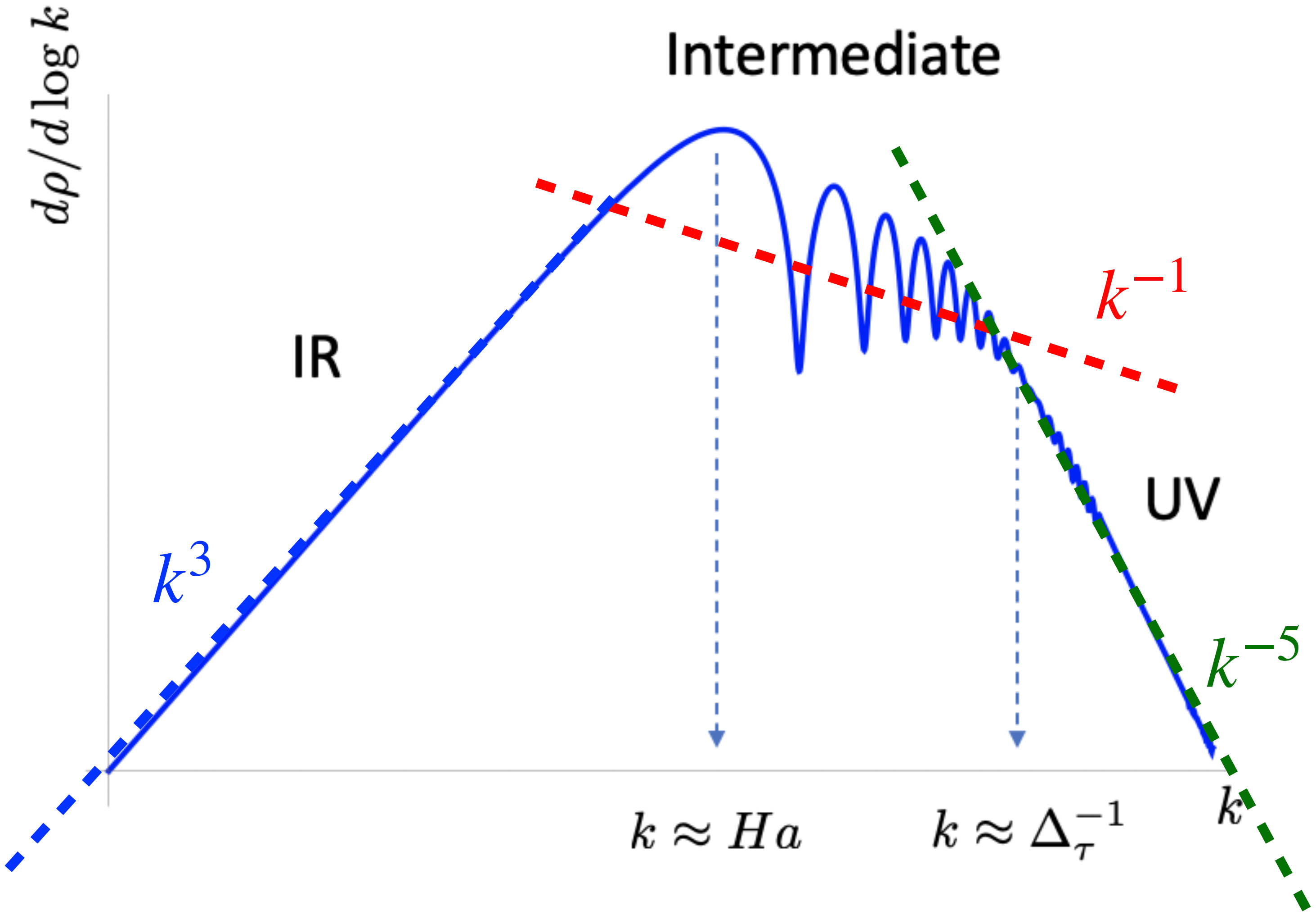
Larger tilt needs $m > H$, not a light field, fluctuation suppressed.

Post-inflationary evolution.

Early universe



Example 1: phase transition

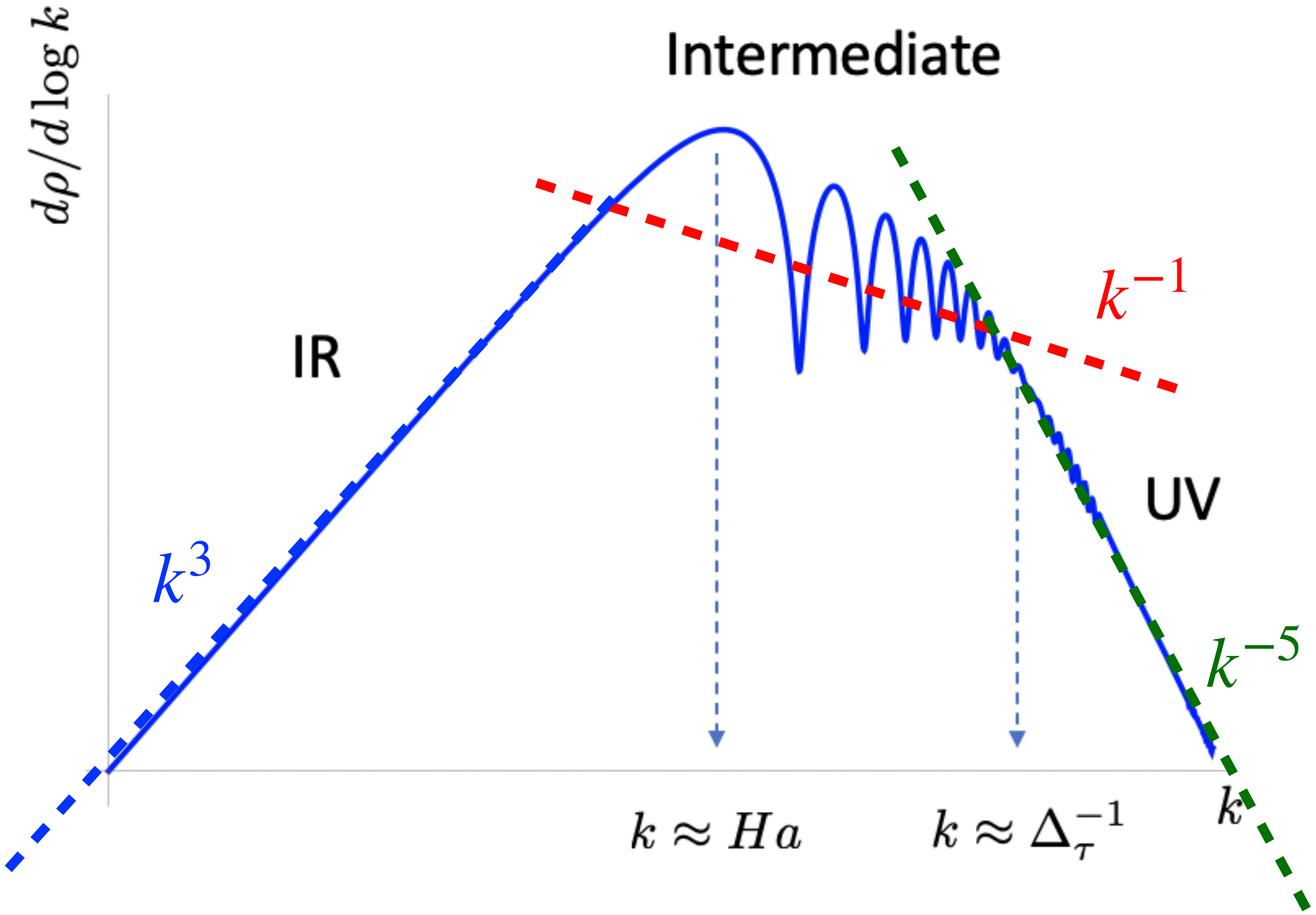


$$\frac{d\rho_{\rm GW}}{d\log k} \propto k^3 \langle (h')^2 \rangle$$

Assumption: de Sitter - instant reheating, RD

$$\frac{d\rho_{\rm GW}^{\rm osc}}{d\log k} = \frac{2G_N|\tilde{T}_{ij}(0,0)|^2}{\pi V a^4(\tau) a^2(\tau_\star)} \left\{ \left[\tilde{\mathcal{E}}_0^i(k)\tilde{\mathcal{G}}_0^f(k)\right]^2 k^3 \left[1 + \mathcal{S}(k\Delta_\tau)\cos 2k(\tau_\star - \tau_0)\right] \right\}$$

Example 1: phase transition



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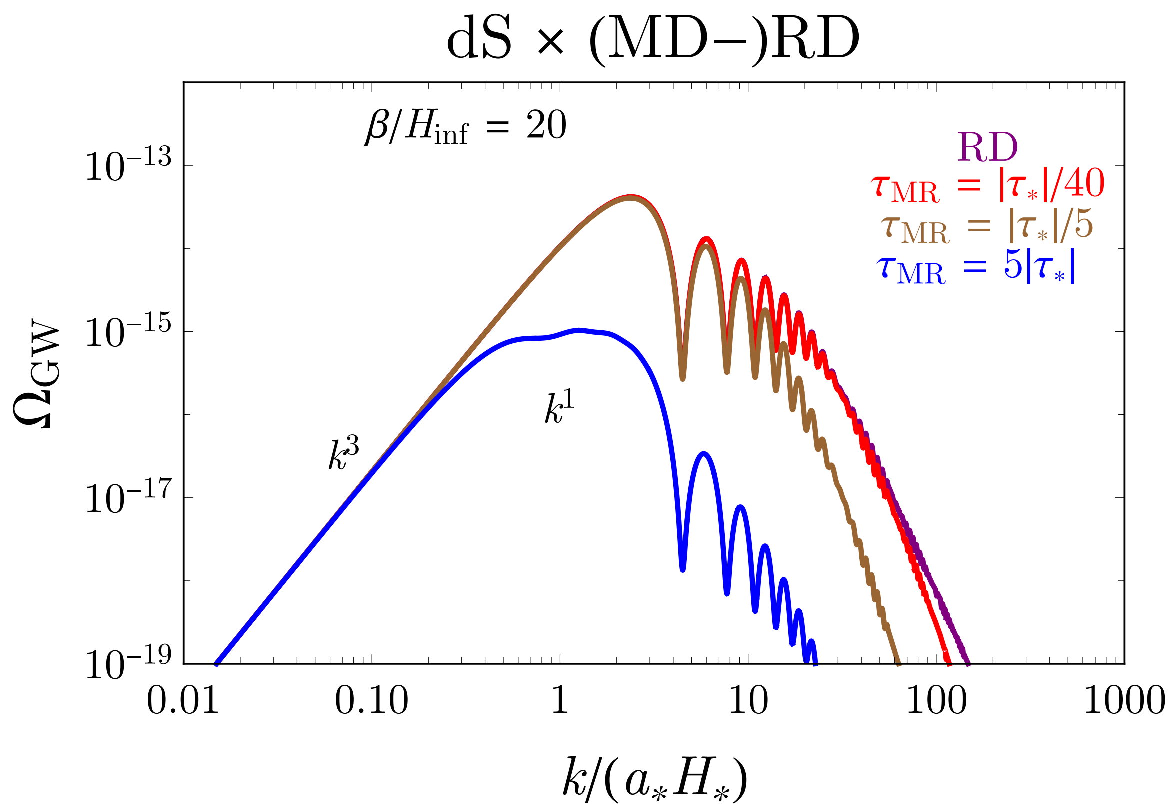
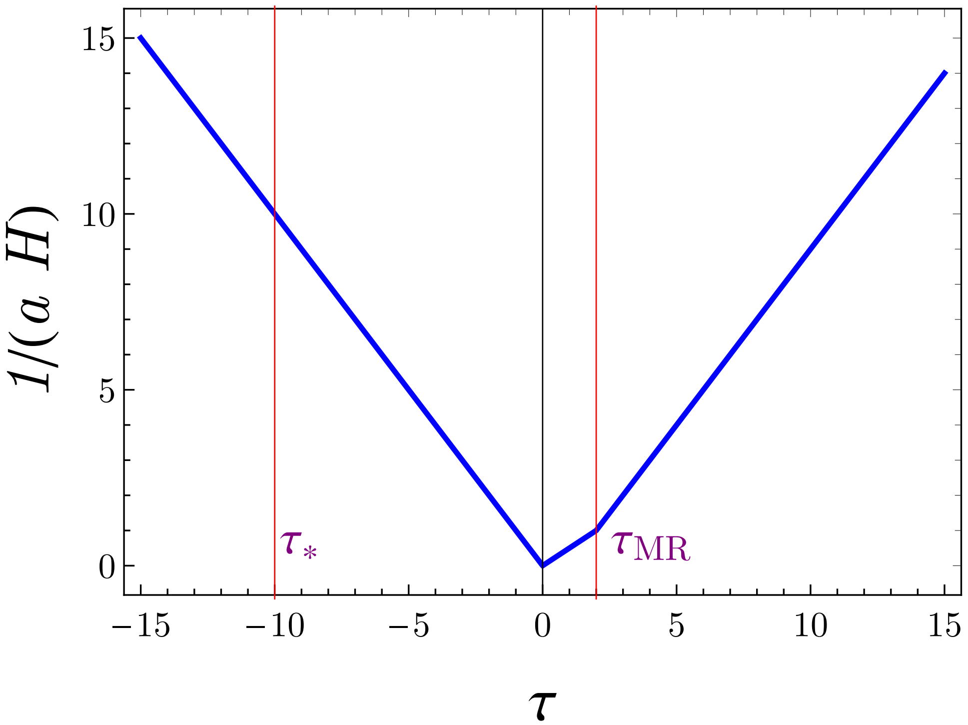
Depending on
Time evolution

Comparing scenarios



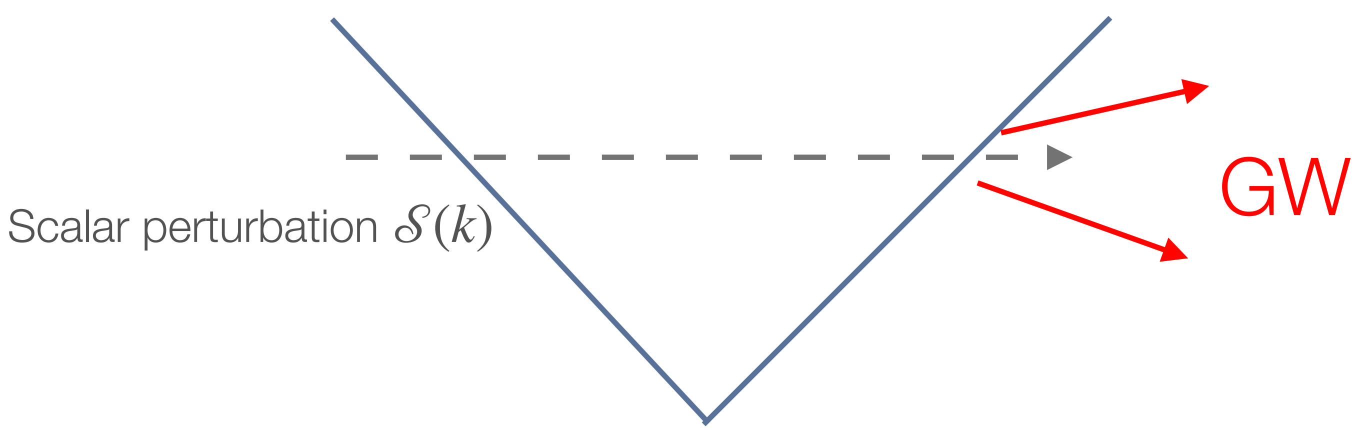


Scenarios after reheating.

τ_{MR} = MD-RD transition

See talk by Haipeng An earlier in this workshop.

Example 2: Secondary GW



Sensitive to evolution after re-entry

The result presented earlier assumes radiation domination.
If there is a MD⇒RD transition, answer can be different.

Gravitational wave

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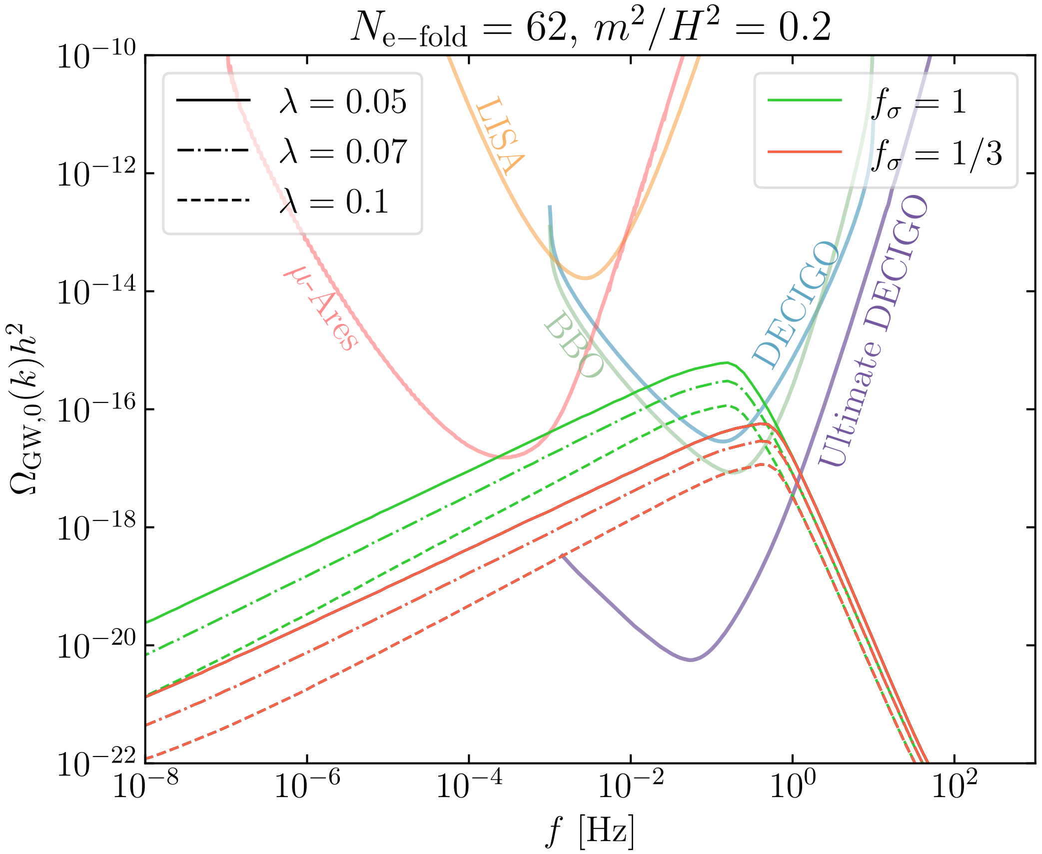

Assumption: Gravitational wave generated during RD.
Spectator decays right before it dominates.
Otherwise, there is an RD ⇒ MD ⇒ RD transition.

Gravitational wave

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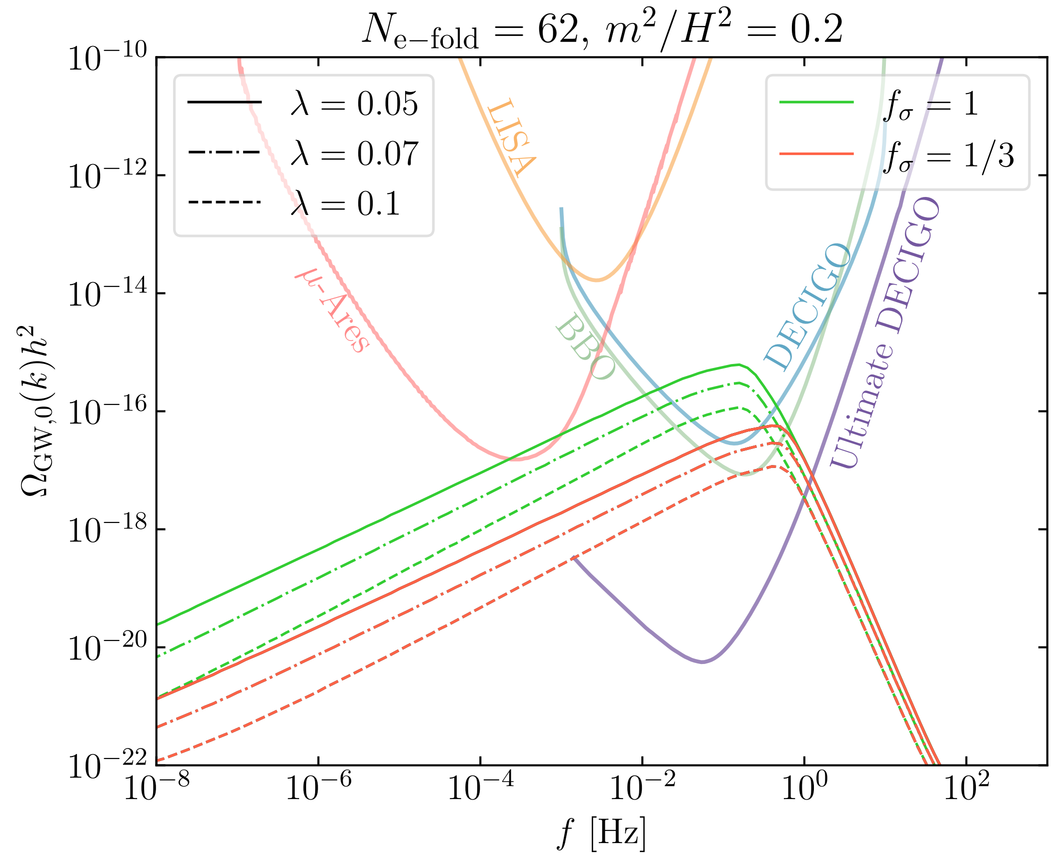


Assumption: Gravitational wave generated during RD.
Spectator decays right before it dominates.
Otherwise, there is an RD $\Rightarrow$ MD $\Rightarrow$ RD transition.

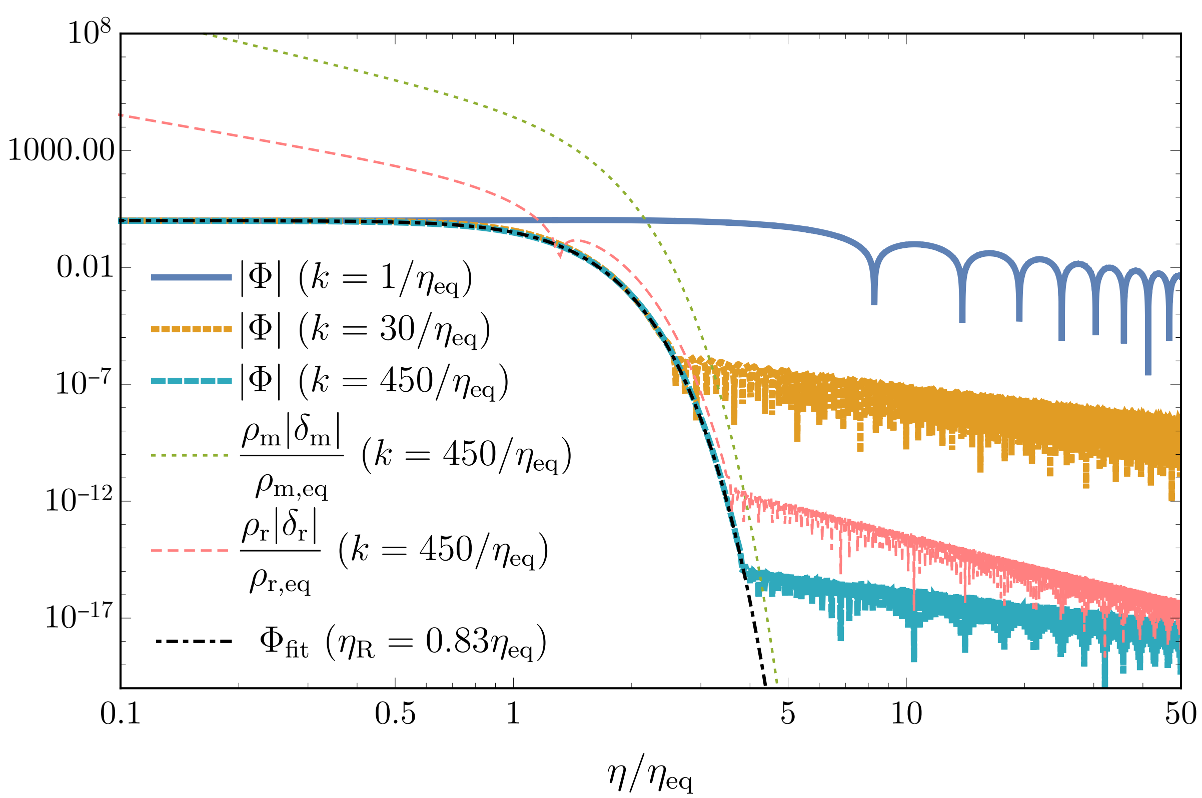
More generic!

Secondary GW

Early matter domination ⇒ Radiation

Inomata, Kohri, Nakama, Terada 1904.12878, 1904.12879

Gradual transition, transition time: $H \sim \Gamma_\sigma$

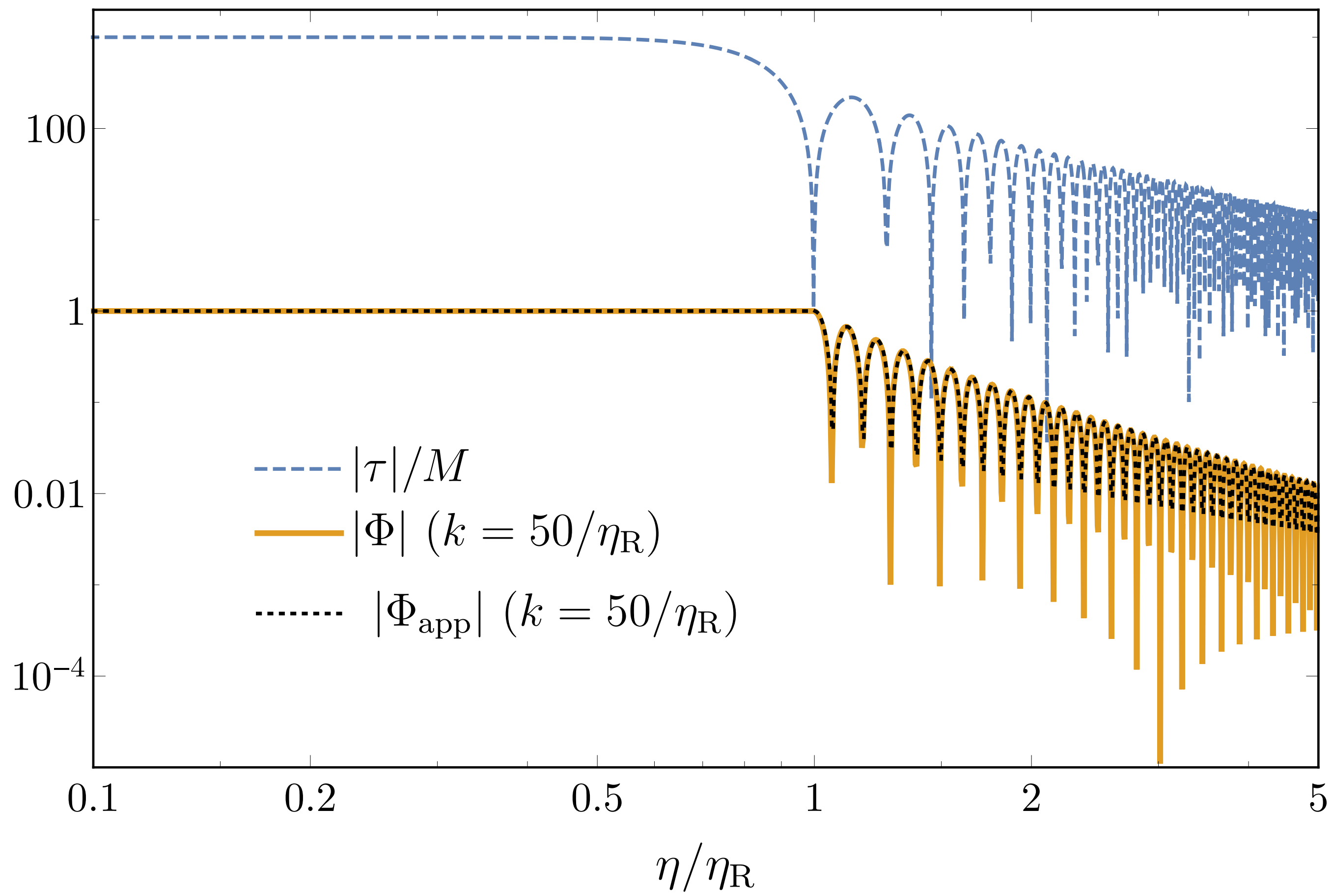


For modes entering in MD, no oscillation until RD

Grav. potential approx constant in MD. Decay in transition.

Gravitational potential

Sudden transition time: $H \gg \Gamma_\sigma$



Sudden transition

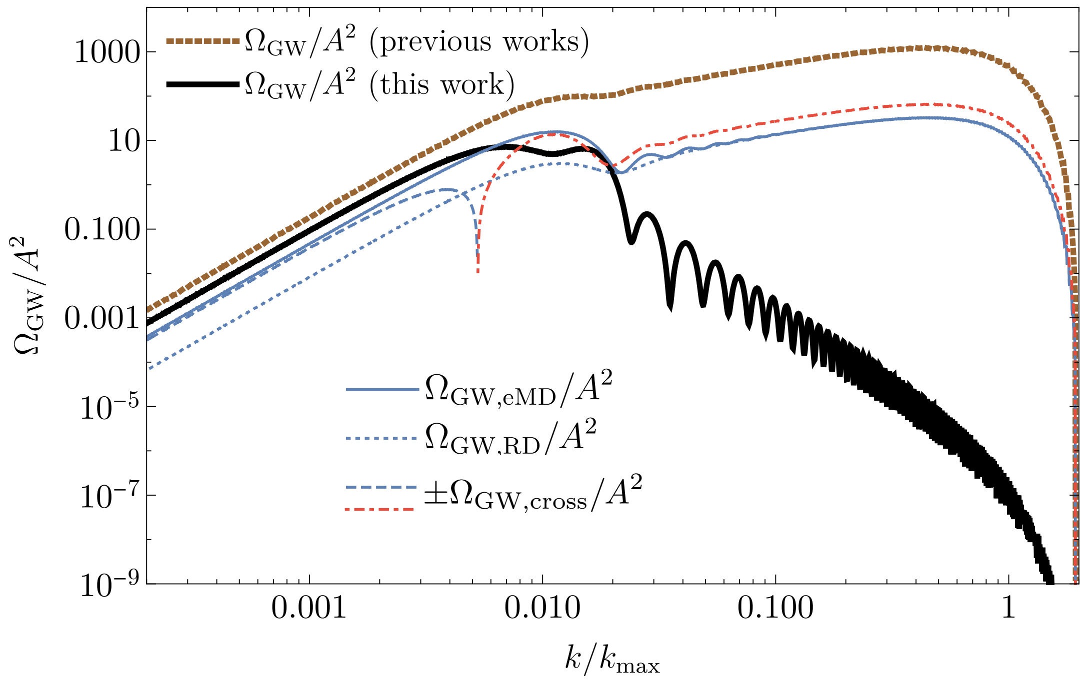
Enhanced in comparison with the gradual transition case

Inomata, Kohri, Nakama, Terada 1904.12878, 1904.12879

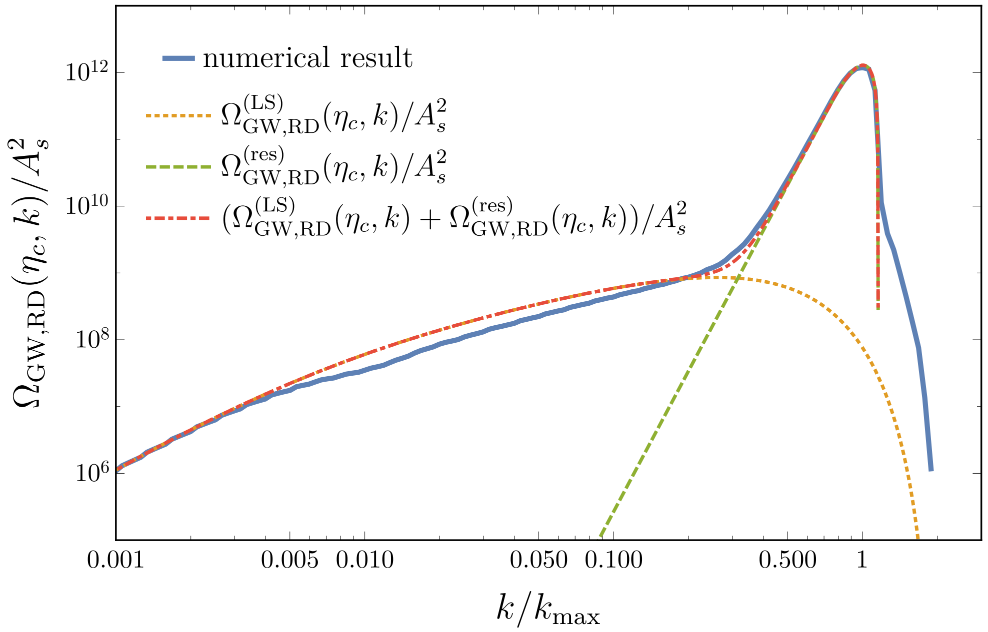
Secondary GW

Early matter domination ⇒ Radiation

Inomata, Kohri, Nakama, Terada 1904.12878, 1904.12879



Gradual transition

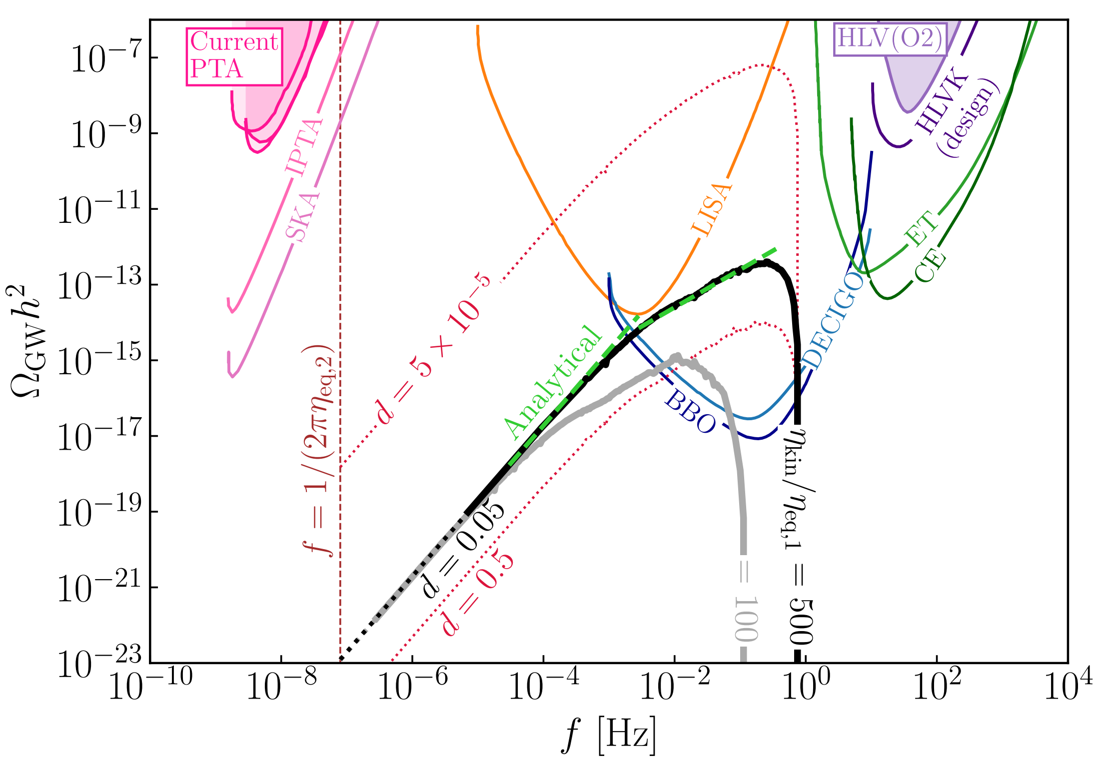


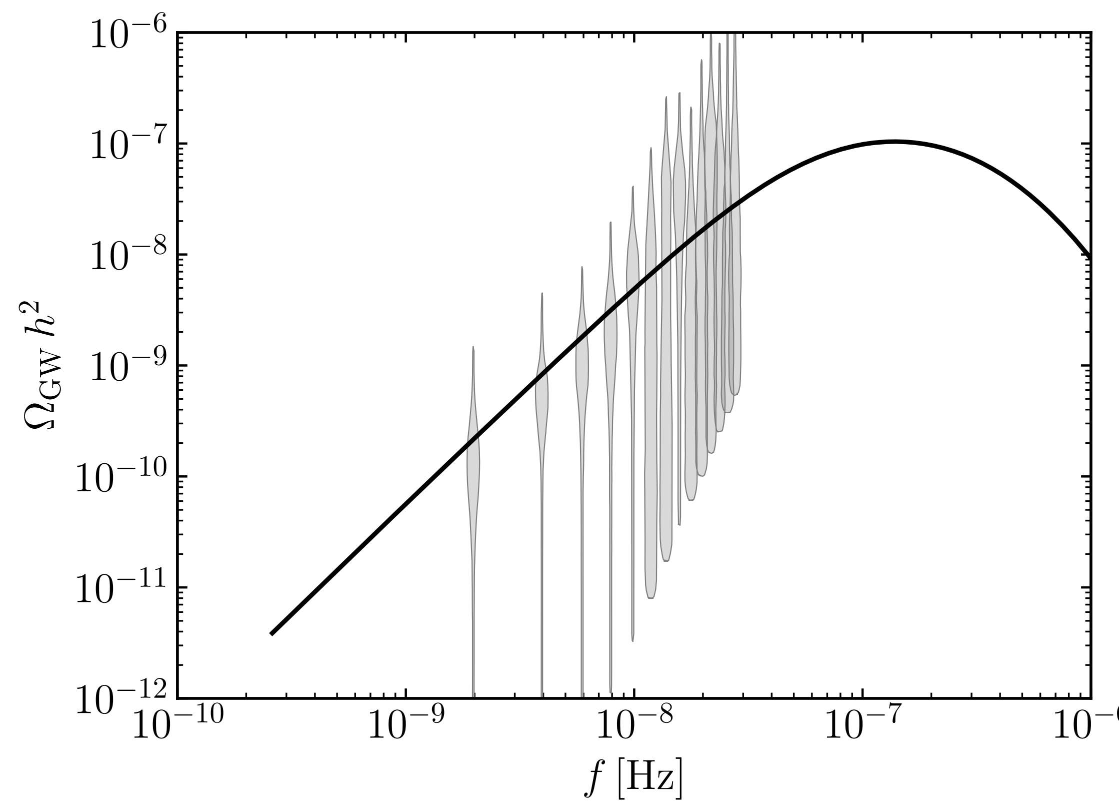
Sudden transition

Open question: better approximation in the gradual transition case needed

Another example: Matter → kination

Harigaya, Inomata, Terada 2305.14242, 2309.00228





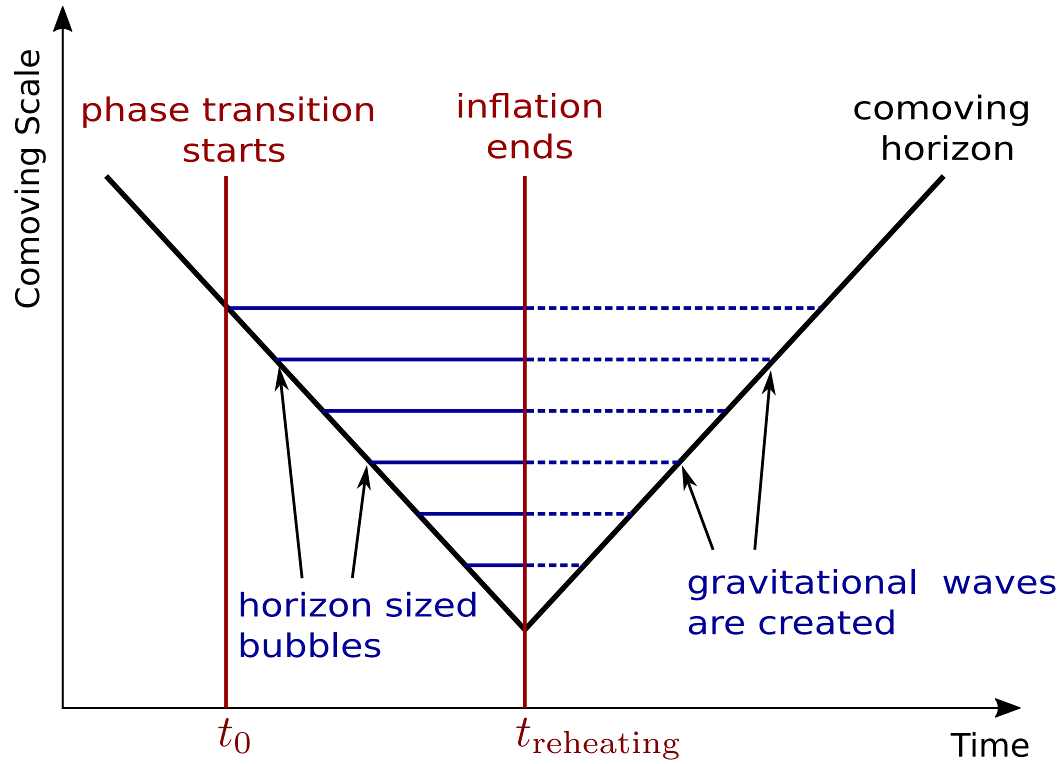
Conclusions

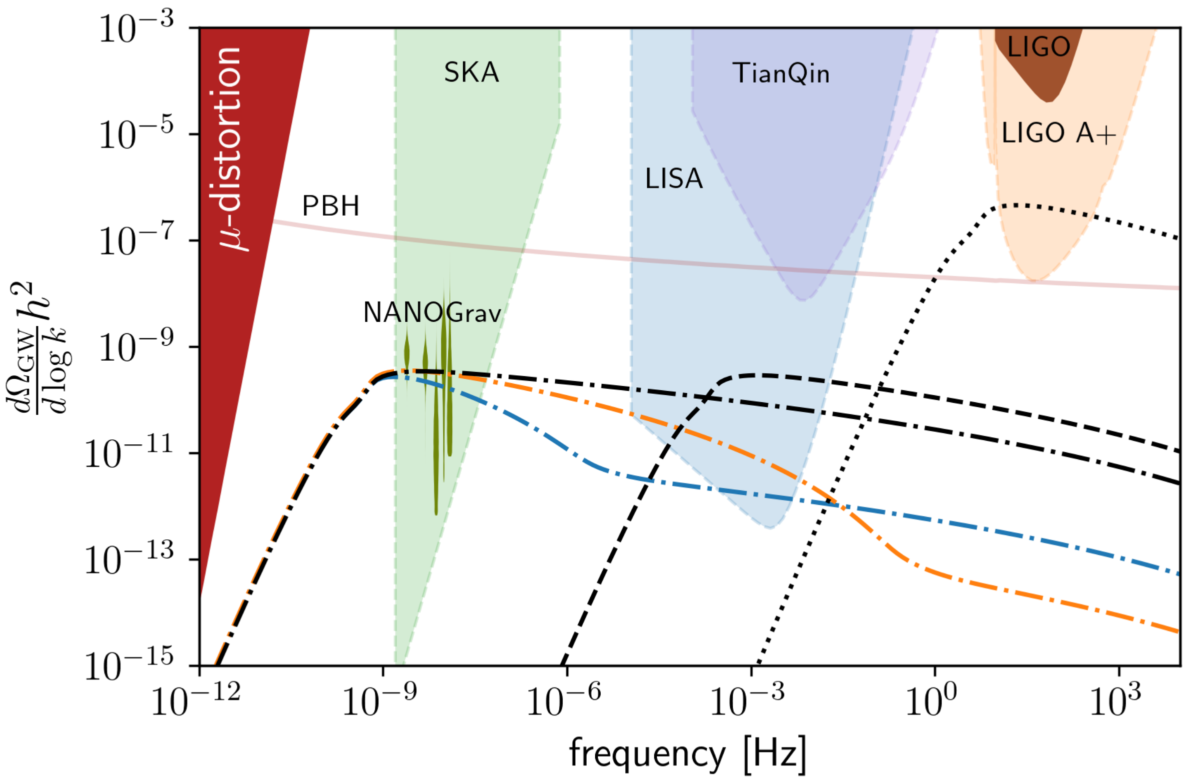
- GW will be a great tool in probing early universe, especially for epochs “invisible” through other means.
  - Long term prospect. Probably the only way to get these information.
- Inflation stage is a plausible place for interesting and observable GW signal can be generated.
  - Both primary and secondary GW.
  - Discovery and study its shape very informative.

extra

Another interesting limit

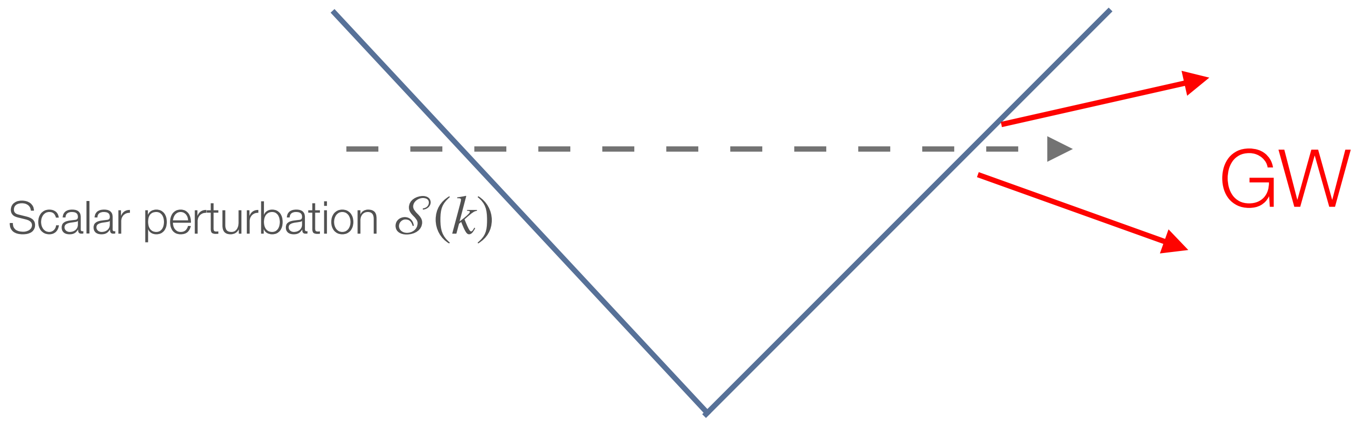
Barir, Geller, Sun, Volansky, 2203.00693



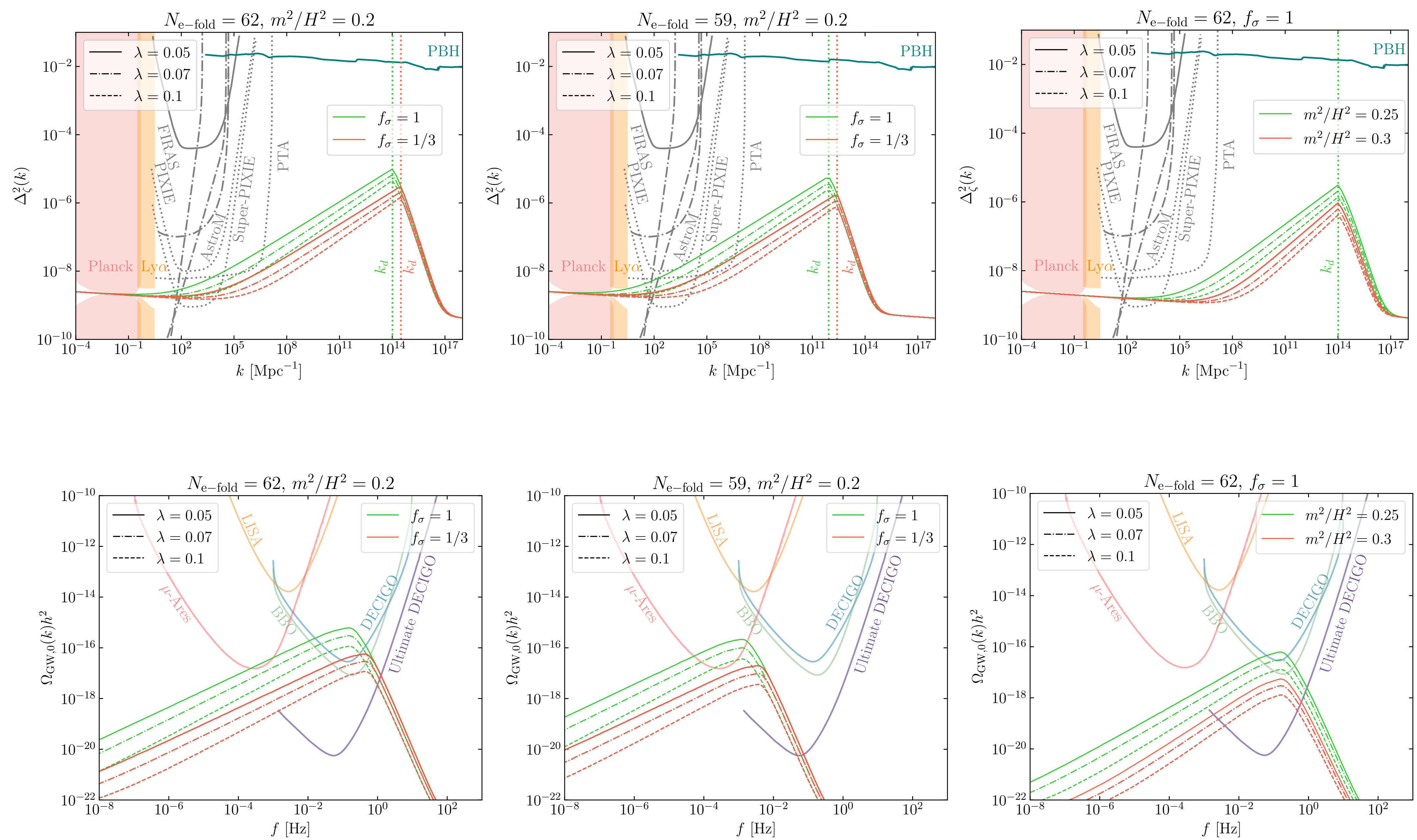


Large bubble does not percolate, generate large curvature perturbations
⇒ secondary gravitational wave at re-entry

Secondary GW



$$h''_{\mathbf{k}} + 2\mathcal{H}h'_{\mathbf{k}} + k^2 h_{\mathbf{k}} = \mathcal{S}(\mathbf{k}, \eta)$$

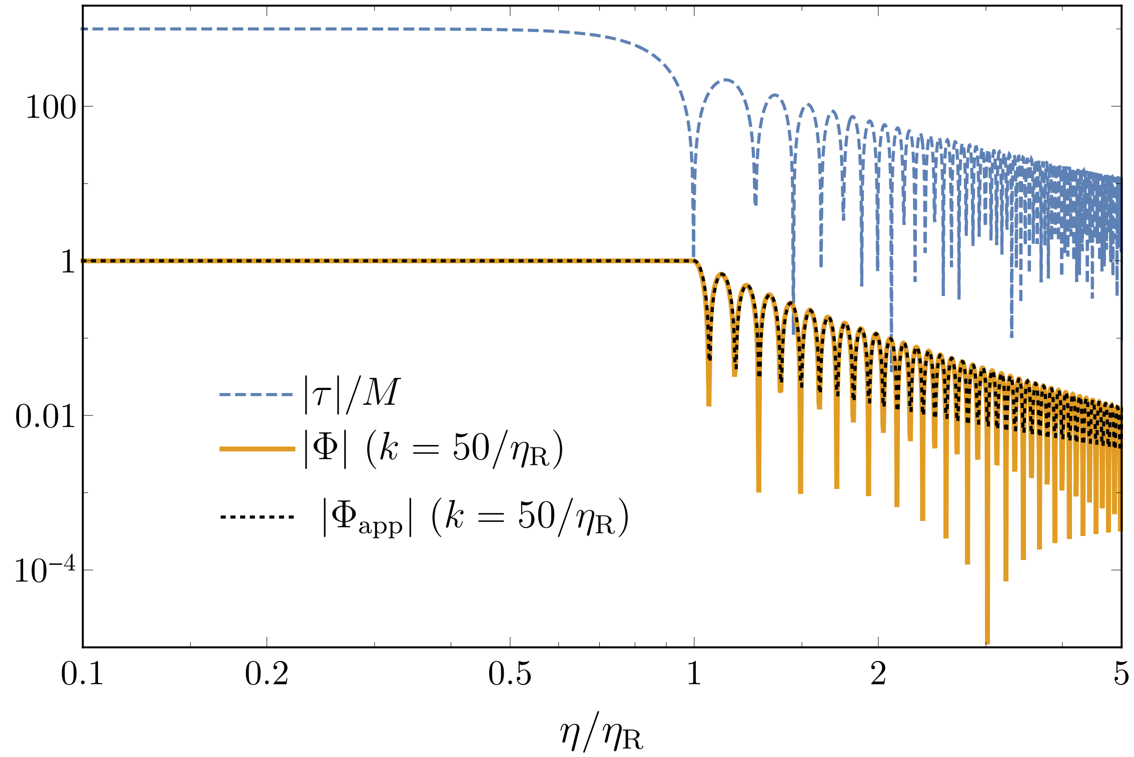


$N_{\rm e-fold} = 62,\ m^2/H^2 = 0.2$
$N_{\rm e-fold} = 59,\ m^2/H^2 = 0.2$
$N_{\rm e-fold} = 62,\ f_\sigma = 1$
$\lambda = 0.05$
$\lambda = 0.07$
$\lambda = 0.1$
$f_\sigma = 1$
$f_\sigma = 1/3$
$m^2/H^2 = 0.25$
$m^2/H^2 = 0.3$
PBH
FIRAS
PIXIE
AstroM
Super-PIXIE
PTA
Planck
Lyα
k_d
$\Delta_\zeta^2(k)$
k [Mpc^{-1}]
LISA
μ-Ares
BBO
DECIGO
Ultimate DECIGO
$\Omega_{\rm GW,0}(k)h^2$
f [Hz]

Gravitational potential

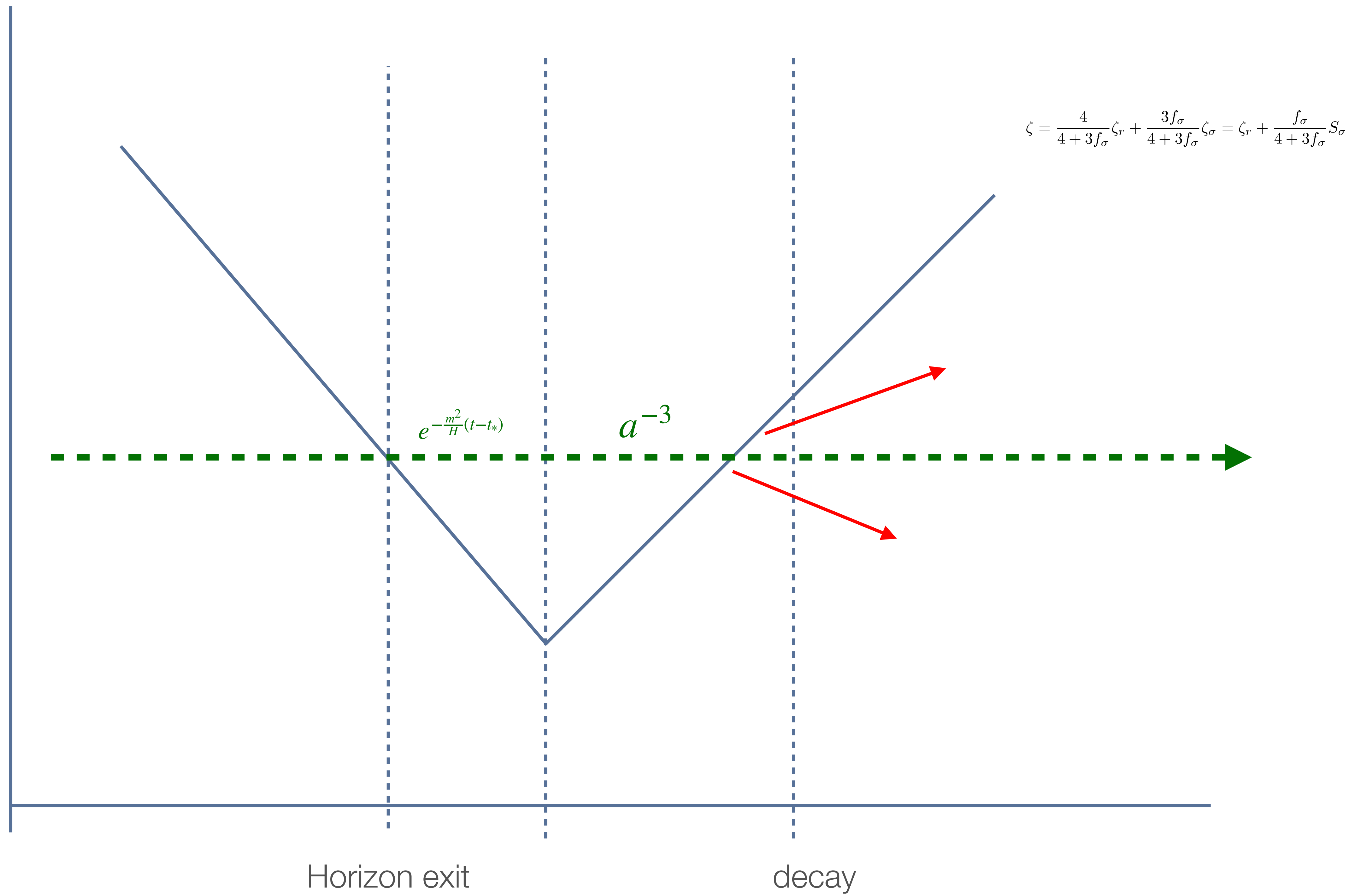
$$\Phi'' + 3(1+w)\mathcal{H}\Phi' + wk^2\Phi = 0$$

$$\Phi' = -\frac{k^2\Phi + 3\mathcal{H}^2\Phi + \frac{3}{2}\mathcal{H}^2\left(\frac{\rho_\mathrm{m}}{\rho_\mathrm{tot}}\delta_\mathrm{m} + \frac{\rho_\mathrm{r}}{\rho_\mathrm{tot}}\delta_\mathrm{r}\right)}{3\mathcal{H}}$$



Sudden transition

Inomata, Kohri, Nakama, Terada 1904.12878, 1904.12879


$\zeta = \frac{4}{4+3f_\sigma}\zeta_r + \frac{3f_\sigma}{4+3f_\sigma}\zeta_\sigma = \zeta_r + \frac{f_\sigma}{4+3f_\sigma}S_\sigma$
$e^{-\frac{m^2}{H}(t-t_*)}$
a^{-3}
Horizon exit
decay

$$\zeta = -\Psi - H\frac{\delta\rho}{\dot{\rho}} = (1-f_\sigma)\zeta_{\rm r} + f_\sigma \zeta_\sigma = \zeta_{\rm r} + \frac{f_\sigma}{3} S_\sigma$$

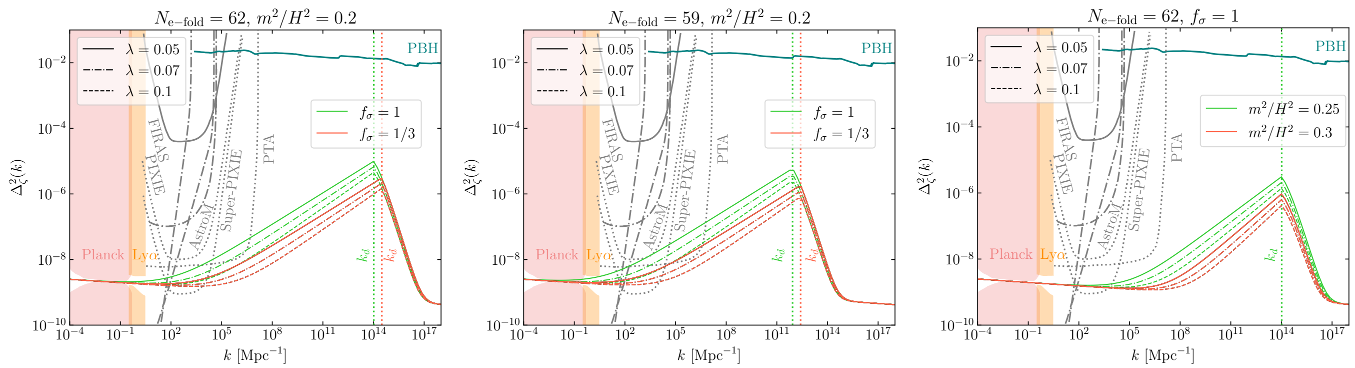
$$\zeta_{\rm r} = -\Psi + \frac{1}{4}\frac{\delta\rho_{\rm r}}{\rho_{\rm r}}\,, \quad \zeta_\sigma = -\Psi + \frac{1}{3}\frac{\delta\rho_\sigma}{\rho_\sigma}\,, \quad f_\sigma = \frac{3\rho_\sigma}{4\rho_{\rm r} + 3\rho_\sigma}\,, \quad S_\sigma = \frac{\delta\rho_\sigma}{\rho_\sigma} - \frac{3}{4}\frac{\delta\rho_{\rm r}}{\rho_{\rm r}} = 3(\zeta_\sigma - \zeta_{\rm r})$$

$$P_\zeta = P_{\zeta_{\rm r}} + \left(\frac{f_\sigma}{3}\right)^2 P_{S_\sigma}$$

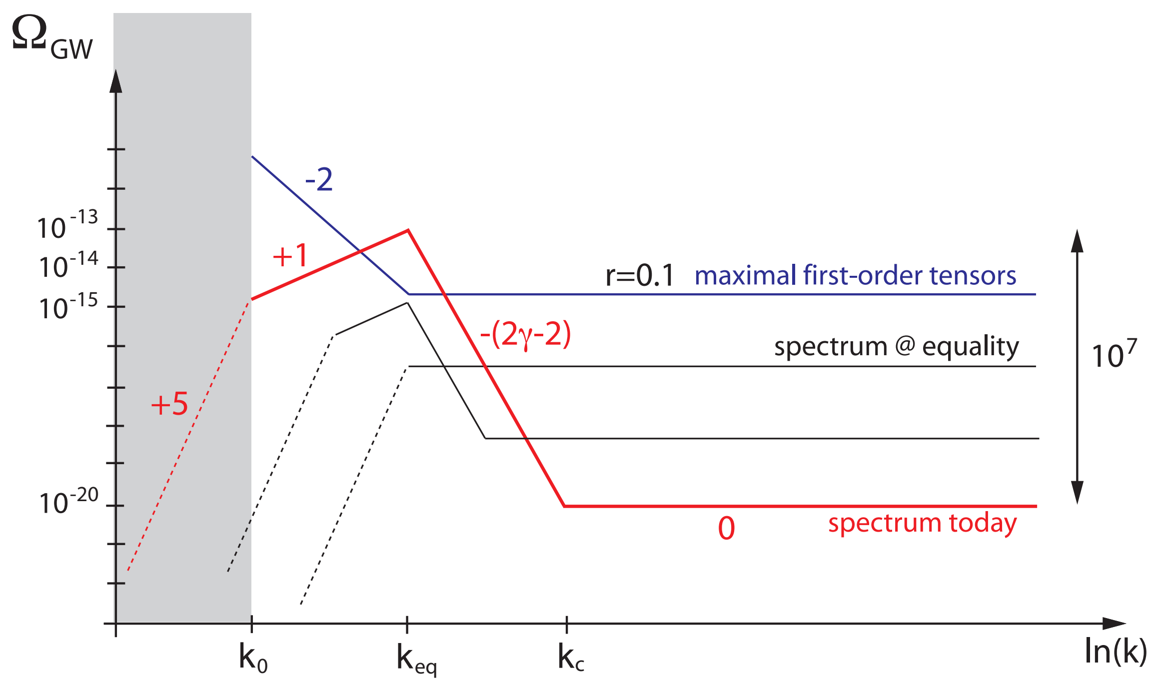
a. Benchmark 1. We focus on the first benchmark in eq. (55). For $m^2 = 0.2H^2$ and $\lambda \simeq 0.05 - 0.1$, we get $\langle V(\sigma)\rangle \approx 0.02H^4$ from eq. (41), implying $\langle V(\sigma)\rangle / V_k \approx 3\times 10^{-12}$ for $H = 5\times 10^{13}$ GeV. Assuming instantaneous reheating, and $\rho_{\rm end} \simeq V_k/100$, we see $f_\sigma \simeq 1$ for $a \simeq (1/3)\times 10^{10} a_{\rm end}$. As benchmarks, we assume σ decays when $f_\sigma = 1$ and $1/3$. Using $k_{\rm end} \approx 4\times 10^{23}$ Mpc^{-1}, we can then evaluate $k_d \approx 10^{14}$ Mpc^{-1} and $k_d \approx 3\times 10^{14}$ Mpc^{-1}, respectively. The result for the curvature power spectrum with these choices is shown in Fig. 3 (left).

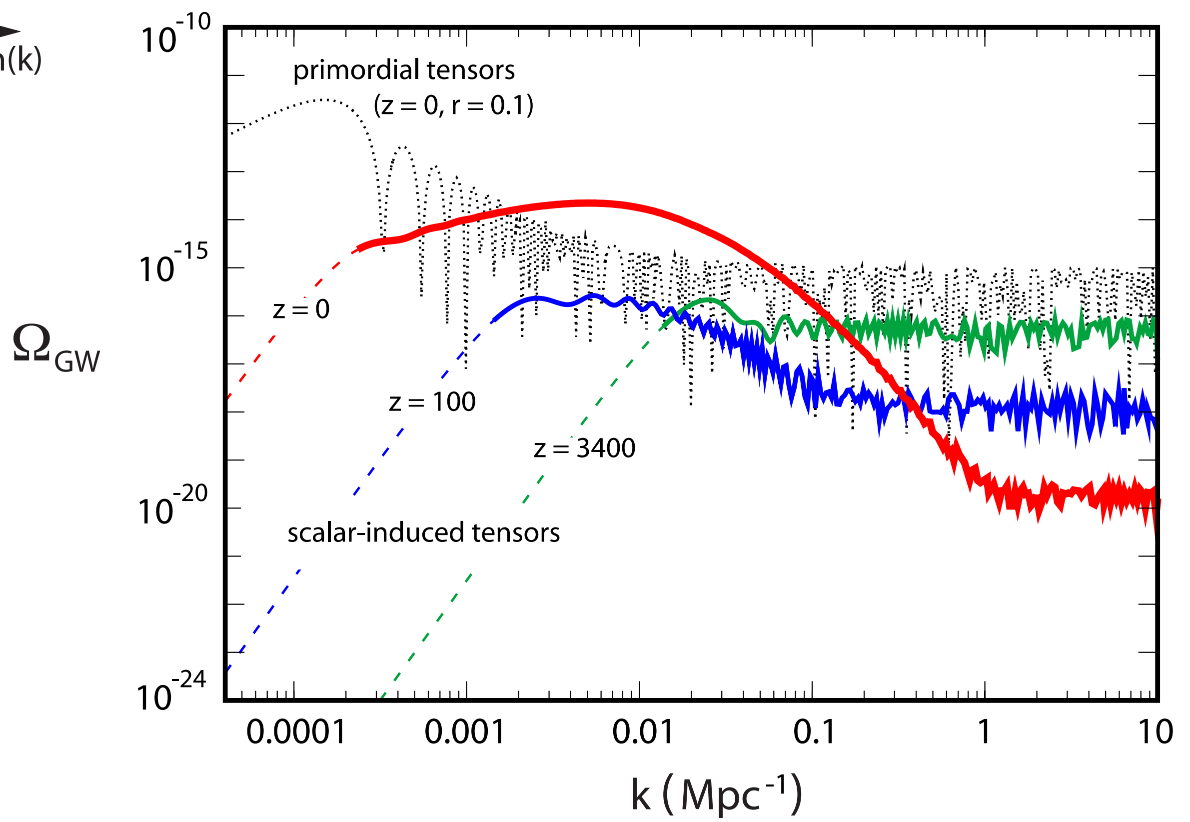
b. Benchmark 2. We now discuss the second benchmark in eq. (55). We again choose $m^2 = 0.2H^2$ and $\lambda \simeq 0.05 - 0.1$, for which we get $\langle V(\sigma)\rangle \approx 0.02H^4$ from eq. (41). This implies $\langle V(\sigma)\rangle / V_k \approx 3\times 10^{-12}$ for $H = 5\times 10^{13}$ GeV, as before. The rest of the parameters can be derived in an analogous way, with one difference. During the reheating epoch, with our assumption $w \approx 0$, f_σ does not grow with the scale factor since the dominant energy density of the Universe is also diluting as matter. Accounting for this gives $k_d \approx 8\times 10^{11}$ Mpc^{-1} and $k_d \approx 3\times 10^{12}$ Mpc^{-1}, for $f_\sigma = 1$ and $1/3$, respectively, with the resulting curvature power spectrum shown in Fig. 3 (center).

c. Benchmark 3. This is same as the first benchmark discussed above, except we focus on $m^2 = 0.25H^2$ and $0.3H^2$ along with $f_\sigma = 1$. The result is shown in Fig. 3 (right).



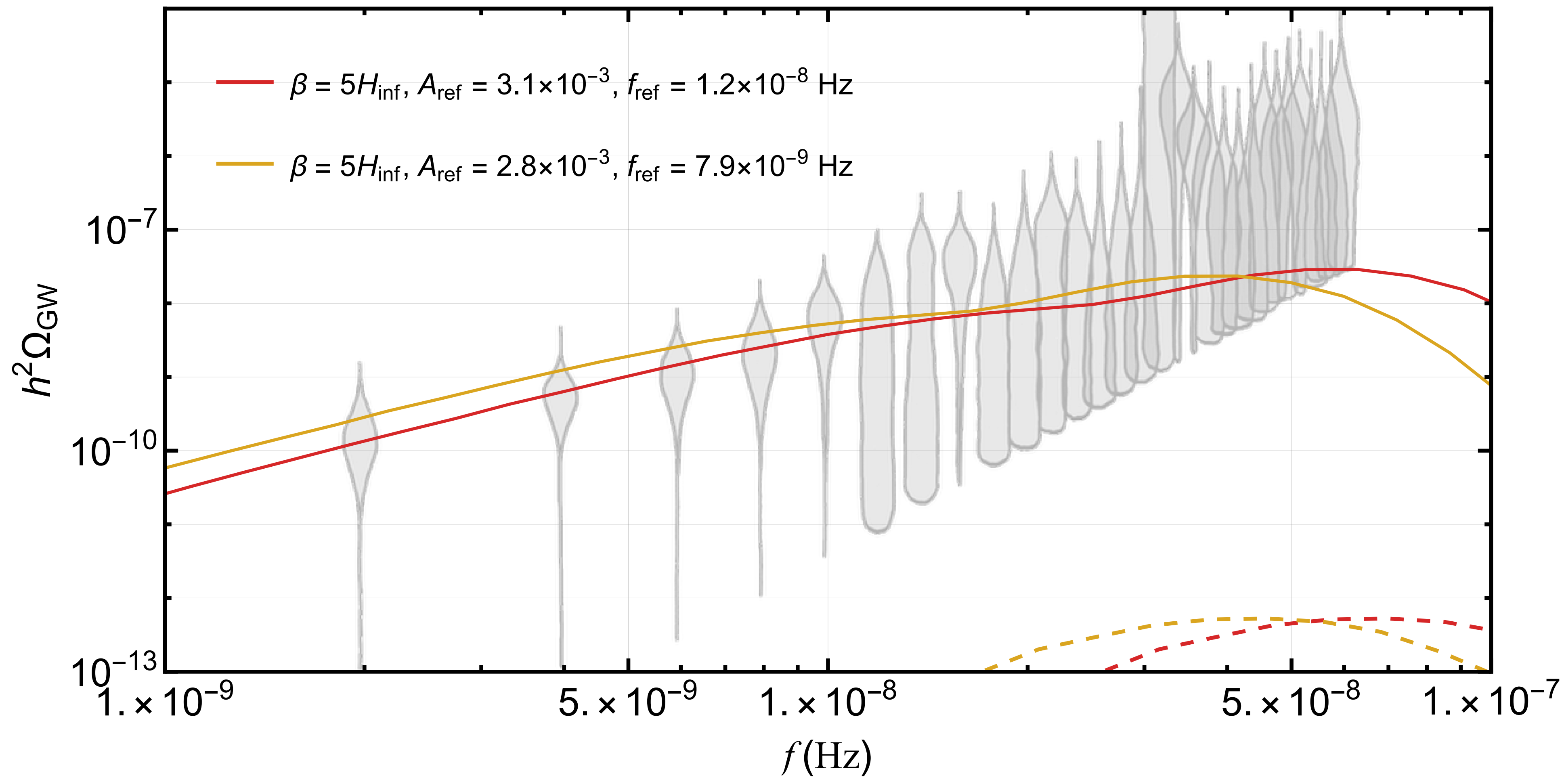
Secondary from $\Delta^2_{\mathcal{R}}$



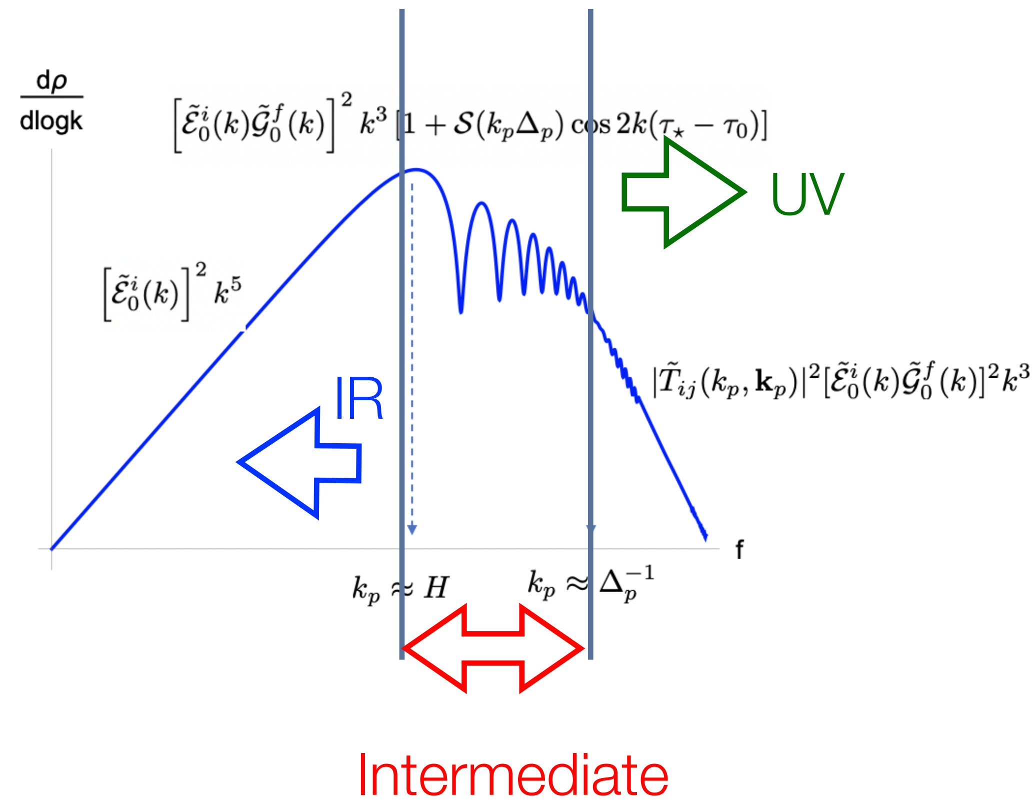


Could be interesting.

Haipeng An, Chen Yang, Boye Su, Hanwen Tai, LTW, 2308.00070



Impact on spectrum



UV

Scenarios after inflation →

Inflationary scenarios ↓

	RD	MD	$t^{\tilde{p}}$
dS	k^{-5}	k^{-7}	$k^{-3+2\frac{\tilde{p}}{\tilde{p}-1}}$
t^p	$k^{-3+2\frac{p}{1-p}}$	$k^{-5+2\frac{p}{1-p}}$	$k^{-1+2\left(\frac{p}{1-p}+\frac{\tilde{p}}{\tilde{p}-1}\right)}$

Intermediate

Scenarios after inflation →

Inflationary scenarios ↓

	RD	MD	$t^{\tilde{p}}$
dS	k^{-1}	k^{-3}	$k^{1+2\frac{\tilde{p}}{\tilde{p}-1}}$
t^p	$k^{1+2\frac{p}{1-p}}$	$k^{-1+2\frac{p}{1-p}}$	$k^{3+2\left(\frac{p}{1-p}+\frac{\tilde{p}}{\tilde{p}-1}\right)}$

IR

Scenarios after inflation →

Inflationary scenarios ↓

	RD	MD	$t^{\tilde{p}}$
dS	k^3	k^1	$k^{5+2\frac{\tilde{p}}{\tilde{p}-1}}$
t^p	k^3	k^1	$k^{5+2\frac{\tilde{p}}{\tilde{p}-1}}$

Slopes sensitive to the evolution.