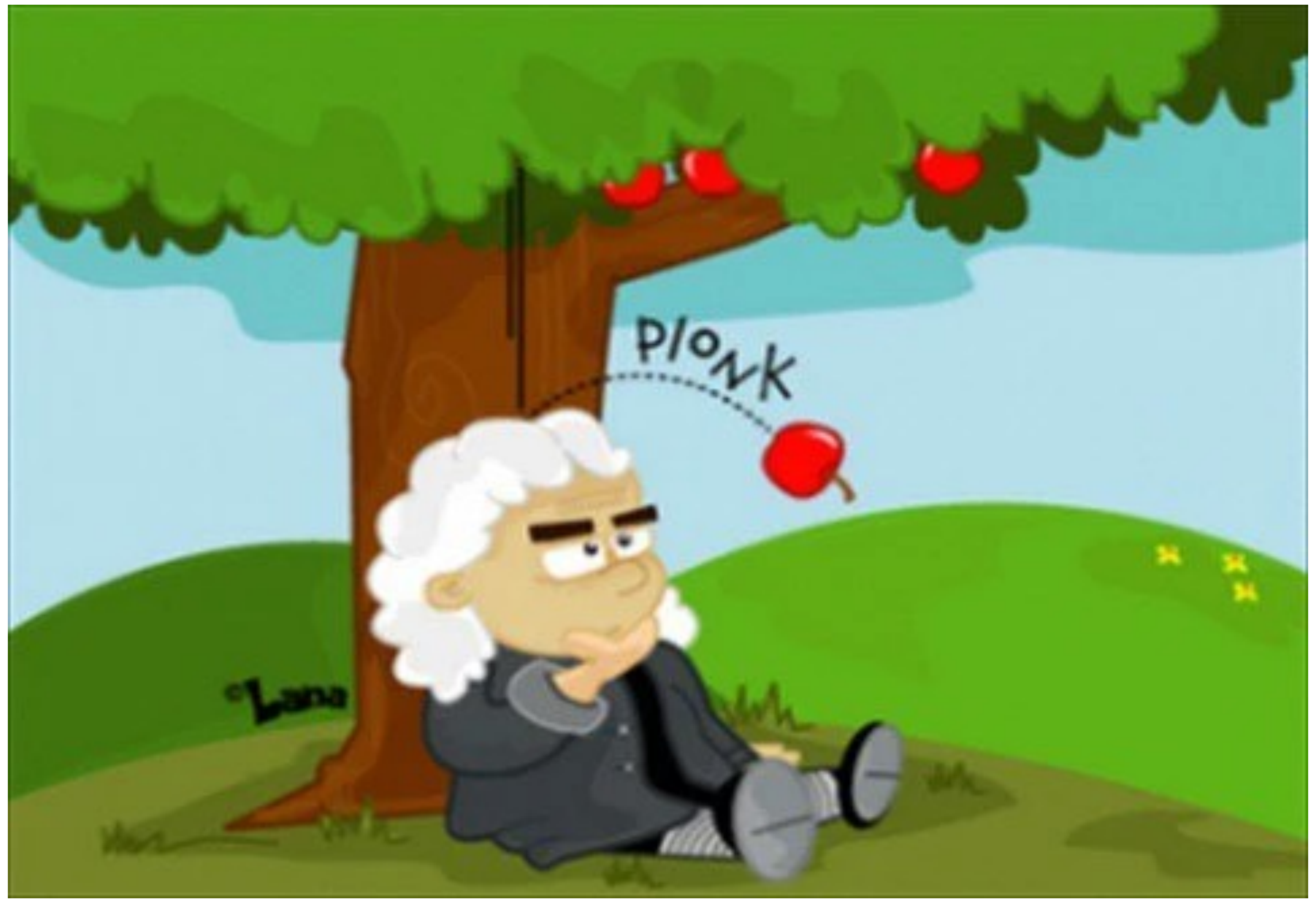
The Classical Equations of Quantized Gauge Theories

Surjeet Rajendran

With Anne-Katherine Burns, David E. Kaplan and Tom Melia

Classical and Quantum Particle Mechanics



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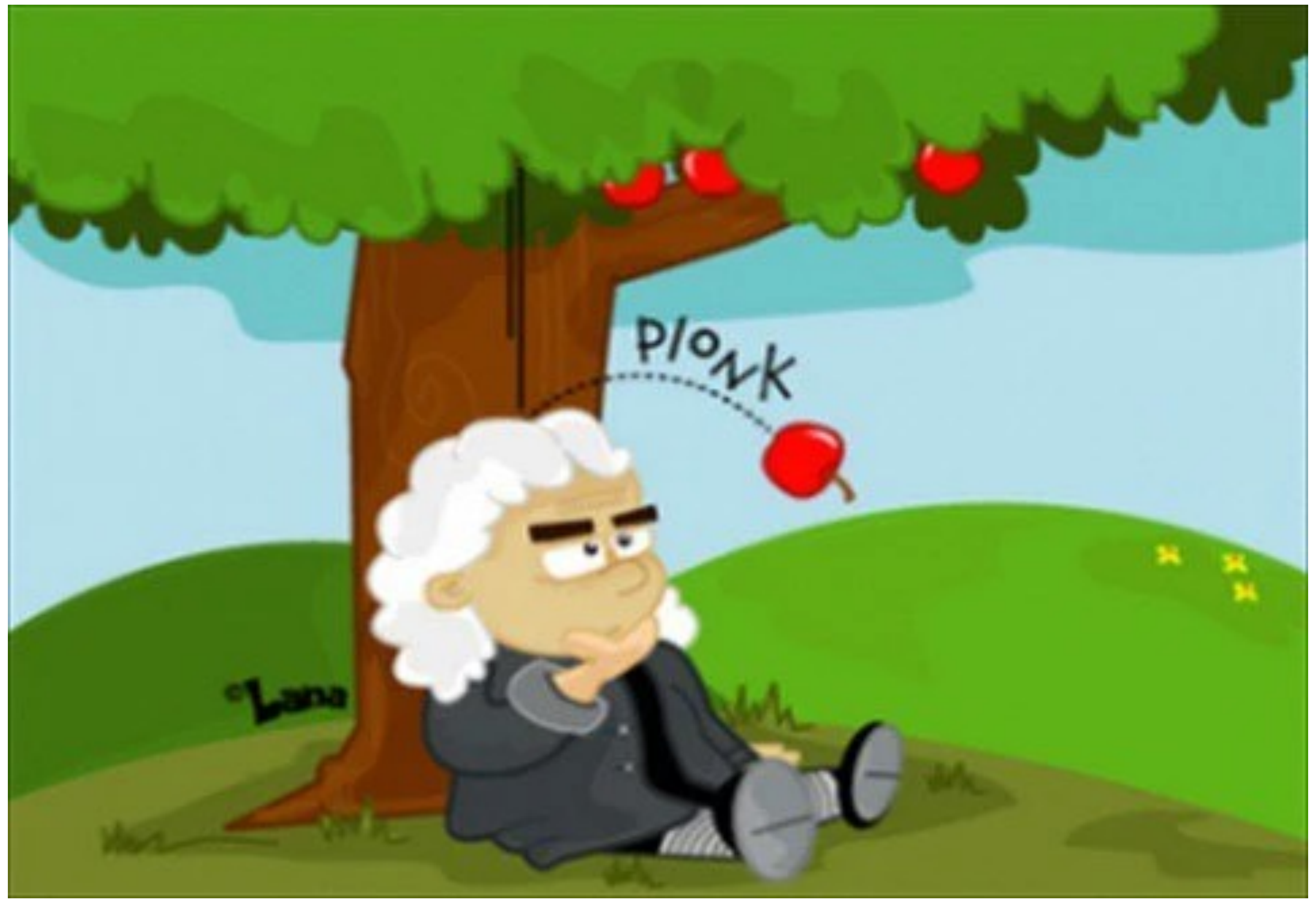

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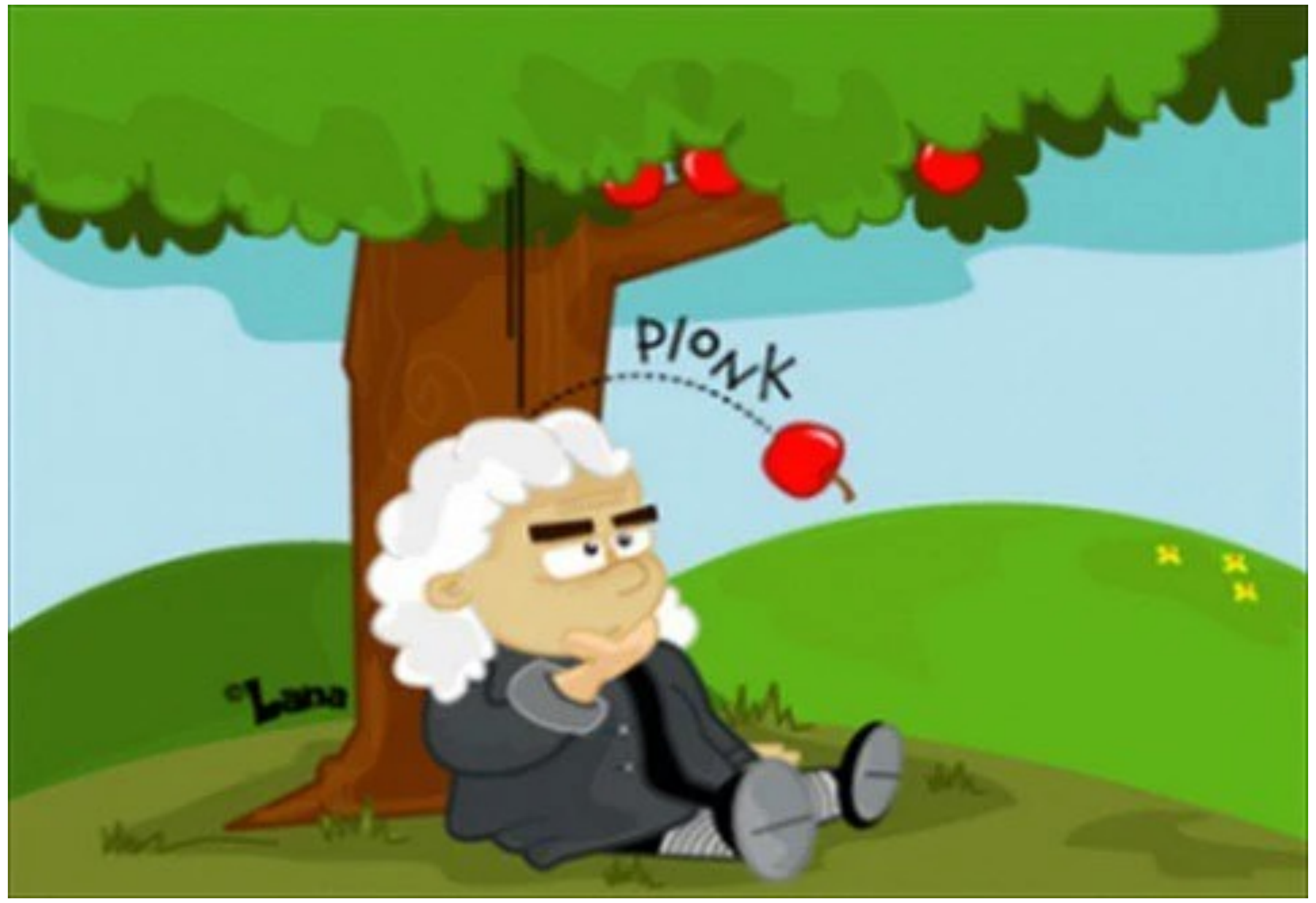

$$\vec{F} = m\vec{a}$$

$$\frac{d\langle\Psi|\hat{p}|\Psi\rangle}{dt} = -\langle\Psi|\frac{dV}{dx}|\Psi\rangle$$

$\Uparrow$

Ehrenfest's Theorem

$$i\frac{\partial|\Psi\rangle}{\partial t} = H|\Psi\rangle$$

✓

Classical and Quantum Field Theory

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(e.g. Klein Gordon)

$$\frac{\partial S}{\partial \phi} = 0$$

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$|\chi(t)\rangle$ Quantum State of Fields (e.g. in Fock states)

$\phi(x)$ Time Independent Operators

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$\phi(x)$ **Time Independent Operators**

$$H = \int d^3x \, \mathcal{H}\left(\phi(x), \pi(x)\right)$$

Time Evolution

$$i\frac{\partial|\chi(t)\rangle}{\partial t} = H|\chi(t)\rangle$$

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Schwinger Dyson

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Gauge theories?

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Dynamical Equations $$\frac{\delta S}{\delta A^i} = 0 \implies \frac{\partial \vec{E}}{\partial t} = \nabla \times \vec{B} + \vec{J}$$

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What about quantum mechanics?

Quantum Gauge Theories

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$$Z = \int DA^\mu e^{iS[A]} \implies \delta Z = \int DA^\mu \left(\frac{\delta S}{\delta A^\mu} \right) \delta A^\mu = 0$$

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Need to Gauge Fix to define Path Integral

$$Z = \int DA^\mu e^{iS_{gf}[A]} \implies \delta Z = \int DA^\mu \left(\frac{\delta S_{gf}}{\delta A^\mu}\right) \delta A^\mu = 0$$

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Not the Same ????????

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Imposed by hand - does not follow from Schrodinger

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First order ODE - can time evolve states that violate constraint

Does anything go wrong? i.e. massless photon?

Outline

1. Classical Mini Superspace Cosmology

2. Path Integral

3. Hamiltonian

4. Wheeler deWitt

5. Conclusions

Mini Superspace Cosmology

Homogeneous Universe

Study homogeneous space-times in General Relativity

$$g_{\mu\nu} = -N\left(t\right)^2 dt^2 + a\left(t\right)^2\left(dx^2 + dy^2 + dz^2\right)$$

Couple this to homogenous sources of matter, for e.g. rolling scalar field φ

$$S = \int dt\mathcal{L}\left(a\left(t\right), \dot{a}\left(t\right), N\left(t\right), \phi\left(t\right), \dot{\phi}\left(t\right)\right)$$

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Classical Equations?

2 FRW + 1 Scalar Field

Classical Equations

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$$\frac{\partial S}{\partial a} = 0 \implies \partial_t \left(\frac{\partial \mathcal{L}}{\partial \dot{a}}\right) - \frac{\partial \mathcal{L}}{\partial a} = 0 \text{ i.e. } \left(\frac{\ddot{a}}{a} + \cdots = 0\right)$$

2nd FRW Equation

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Klein Gordon

$$\frac{\partial S}{\partial N} = 0 \implies \frac{\partial \mathcal{L}}{\partial N} = 0 \text{ i.e. } \left(\left(\frac{\dot{a}}{a}\right)^2 + \cdots = 0\right)$$

1st FRW Equation (Hubble)

Classical Equations

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1st FRW Equation (Hubble)

Classical Solutions are over-constrained (3 equations for 2 variables)

Solve: Klein Gordon + 2nd FRW with boundary condition from 1st FRW

Path Integral

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$$\dot{N} = 0 \implies N\left(t_2\right) = N_0, N\left(t_1\right) = N_0$$

Path Integral

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Can show that path integral over a(t), ϕ(t) are finite

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Path Integral explicitly depends upon random gauge choice N_0

Path integral obviously not gauge invariant

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Is this a problem?

Gauge Invariant Physics

$$\langle \phi_f , a_f | T \left(t_2 ; t_1 \right) | \phi_i , a_i \rangle$$

Path integral defines the time evolution operator

$$g_{\mu\nu} = -N \left(t \right)^2 dt^2 + a \left(t \right)^2 \left(dx^2 + dy^2 + dz^2 \right)$$

Time coordinate is a gauge choice in General Relativity

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Perfectly Reasonable for the time evolution operator to be gauge dependent

We need gauge invariant physics. What does this mean?

Gauge Invariant Physics

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What happens if $N_0 \to \tilde{N}_0$?

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where $\tilde{t} = \dfrac{N_0}{\tilde{N}_0} t$

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$$\text{where } \tilde{t} = \frac{N_0}{\tilde{N}_0} t$$

$$|\phi_i, a_i\rangle \to \sum_{\phi_f, a_f} c_{\phi_f, a_f}(t_2, t_1) |\phi_f, a_f\rangle = \sum_{\phi_f, a_f} c_{\phi_f, a_f}\left(\tilde{t}_2, \tilde{t}_1\right) |\phi_f, a_f\rangle$$

$$|\Psi(t_1)\rangle = |\tilde{\Psi}\left(\tilde{t}_1\right)\rangle = |\Sigma\rangle \to |\Psi(t_2)\rangle = |\tilde{\Psi}\left(\tilde{t}_2\right)\rangle = |\Omega\rangle$$

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Fix manifold (R^1).

Pick any choice of time co-ordinate on the manifold

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Moral: Gauge invariant physics from gauge dependent path integral
Reasonable since time is gauge choice!

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2nd FRW Equation

Heisenberg Picture

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$$\langle\Psi|\frac{\partial\lambda}{\partial t} - \frac{\partial\mathcal{L}}{\partial N}|\Psi\rangle = 0$$

Tells you how λ evolves in the path integral - not 1st FRW Equation?????

Hamiltonian

Hamiltonian Construction

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$$S = \int dt \mathcal{L}\left(a(t), \dot{a}(t), N(t), \phi(t), \dot{\phi}(t)\right)$$

$$\Pi_a = \frac{\partial \mathcal{L}}{\partial \dot{a}}, \Pi_\phi = \frac{\partial \mathcal{L}}{\partial \dot{\phi}}$$

Quantize These

Hamiltonian Construction

$$g_{\mu\nu} = -N(t)^2 \, dt^2 + a(t)^2 \left(dx^2 + dy^2 + dz^2\right)$$

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Quantize These

$$\Pi_N = \frac{\partial \mathcal{L}}{\partial \dot{N}} = 0$$

What to do with N?

Hamiltonian Construction

$$S = \int dt \mathcal{L} \left(a\,(t)\,, \dot{a}\,(t)\,, N\,(t)\,, \phi\,(t)\,, \dot{\phi}\,(t) \right)$$

Construct Canonical Hamiltonian from this Lagrangian

$$H_N = N\, H_0\,(a, \Pi_a, \phi, \Pi_\phi)$$

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What is N? Different values of N yield different Hamiltonians
Different physics? Gauge Invariance?

If N is a non-trivial operator on the Fock space, no way to make physics gauge invariant
Possible choice: N is a c-number
But still, different choices of N yield different Hamiltonians!

Schrodinger Equation

$$i\frac{\partial|\chi(t)\rangle}{\partial t} = NH_0|\chi(t)\rangle$$

N is a c-number, different choices of N yield different Hamiltonians

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$$d\tilde{t} = N dt$$

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Different choices of N correspond to different choices of time co-ordinate.
Gauge invariant physics!

$$\langle\phi_f, a_f|T(t_2;t_1)|\phi_i, a_i\rangle = \int_{\phi(t_1)=\phi_i, a(t_1)=a_i}^{\phi(t_2)=\phi_f, a(t_2)=a_f} D\phi D a e^{i\int_{t=t_1}^{t=t_2}\left(\tilde{\mathcal{L}}(N_0)\right)}$$

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Consistent with Path Integral - you just pick N

Consequence of Schrodinger Equation

$$i\frac{\partial|\chi(t)\rangle}{\partial t} = NH_0|\chi(t)\rangle \implies \frac{d\langle\chi(t)|H_0|\chi(t)\rangle}{dt} = 0$$

Identity, similar to Ehrenfest and Schwinger-Dyson

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Can Show: $\langle\chi(t)|H_0|\chi(t)\rangle = \langle\chi(t)|a^3\frac{\partial\mathcal{L}}{\partial N}|\chi(t)\rangle$

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Can Show: $\langle\chi(t)|H_0|\chi(t)\rangle = \langle\chi(t)|a^3\frac{\partial\mathcal{L}}{\partial N}|\chi(t)\rangle$

Thus: $\dfrac{d\langle\chi|a^3\frac{\partial\mathcal{L}}{\partial N}|\chi\rangle}{dt} = 0$

This is almost the 1st FRW equation - but not quite.
1st FRW equation only satisfied up to overall constant

Initial State

$$a = A + A^\dagger, \Pi_a = i\left(A - A^\dagger\right) \qquad \phi = B + B^\dagger, \Pi_\phi = i\left(B - B^\dagger\right)$$

Create quantum states of a, ϕ

$$|\chi\rangle = f\left(A, A^\dagger, B, B^\dagger\right)|0>$$

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In classical physics, we demand 2 FRW + 1 KG equation to hold - so we restrict initial conditions to obey 1st FRW

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$$i\frac{\partial|\chi(t)\rangle}{\partial t} = NH_0|\chi(t)\rangle$$

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Quantum Dynamics

$$i\frac{\partial|\chi(t)\rangle}{\partial t} = NH_0|\chi(t)\rangle$$

First order ODE - no issue with time evolving

$$\langle\chi|a^3\frac{\partial\mathcal{L}}{\partial N}|\chi\rangle \neq 0$$

Violating 1st FRW

$$\langle \chi | a^3 \frac{\partial \mathcal{L}}{\partial N} | \chi \rangle = c$$

Quantum Dynamics: $$i \frac{\partial |\chi(t)\rangle}{\partial t} = N H_0 |\chi(t)\rangle$$

Violating 1st FRW

$$\langle\chi|a^3\frac{\partial\mathcal{L}}{\partial N}|\chi\rangle = c$$

Quantum Dynamics: $$i\frac{\partial|\chi(t)\rangle}{\partial t} = NH_0|\chi(t)\rangle$$

Implied Classical Dynamics

$$\langle\chi|\partial_t\left(\frac{\partial\mathcal{L}}{\partial\dot{\phi}}\right) - \frac{\partial\mathcal{L}}{\partial\phi}|\chi\rangle = 0$$

Klein Gordon

$$\langle\chi|\partial_t\left(\frac{\partial\mathcal{L}}{\partial\dot{a}}\right) - \frac{\partial\mathcal{L}}{\partial a}|\chi\rangle = 0$$

2nd FRW Equation

$$\langle\chi|\frac{\partial\mathcal{L}}{\partial N}|\chi\rangle = \langle\chi|\frac{c}{a^3}|\chi\rangle$$

1st FRW but with "Dark" Matter

Quantum "Dark" Matter

$$\langle \chi | a^3 \frac{\partial \mathcal{L}}{\partial N} | \chi \rangle = c$$

$$a = A + A^\dagger, \Pi_a = i\left(A - A^\dagger\right) \qquad \phi = B + B^\dagger, \Pi_\phi = i\left(B - B^\dagger\right)$$

Create quantum states of a, ϕ
Quantum Dynamics: Just 1 first order ODE (Schrodinger)
No reason to constrain initial state!

Failure manifests classically as "dark" matter - though no real particle excitation there. Conservation implies super-selection sector.

Can be positive or negative!

Wheeler deWitt

Can we get all of Einstein's Equations?

$$i\frac{\partial|\chi(t)\rangle}{\partial t} = NH_0|\chi(t)\rangle$$

$$\text{Want } \langle\chi|a^3\frac{\partial\mathcal{L}}{\partial N}|\chi\rangle = 0$$

Hamiltonian changes with N

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Universe is in an energy eigenstate

But... Time???

Path Integral Version

$$\langle \phi_f, a_f | T(t_2; t_1) | \phi_i, a_i \rangle = e^{iP(\phi_f, a_f, t_2; \phi_i, a_i, t_1; N_0)}$$

Gauge fixed path integral implies we lose 1st FRW equation

Path Integral explicitly depends upon random gauge choice N_0

Path integral obviously not gauge invariant

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Integrate over all N_0

Unsurprisingly, this yields infinity

Demanding gauge invariant path integral implies only possible states are static (Wheeler deWitt)

Conclusions

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Opinion A

Opinion B

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Gauge Dependent Time Evolution With Gauge Invariant Physics

1st FRW Equation only true up to constant

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All Equations Hold...but....

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Full General Relativity? Electromagnetism?

Quantizing Gauge Theories

Yukawa Theory

$$S \supset \int d^4x \, \lambda(x) \, \phi \bar{\Psi} \Psi$$

$$\text{Do Not } \frac{\partial S}{\partial \lambda} = 0$$

$$\text{Do Not } \phi \bar{\Psi} \Psi |\chi\rangle = 0$$

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$$\pi_\lambda = \frac{\partial \mathcal{L}}{\partial \dot{\lambda}} = 0$$

λ not d.o.f.

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Analogy with Yukawa: Treat N, A_0, $g_{0\mu}$ as c-number coupling constants

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Can show that for these states, still get massless photon and graviton - the only physical aspect of gauge invariance we actually care about

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Further, by suitable redefinitions, can show that the c-number coupling constants are not observable in linear quantum mechanics

General Relativity

$$g_{\mu\nu} = g_{0\mu} dt dx^{\mu} + g_{ij} dx^{i} dx^{j}$$

$g_{0\mu}$ do not have conjugate momenta - fixed c-number functions

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Classically:

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With: $H_{\mu\nu} = H_{0\mu}$

$$\nabla^{\mu} H_{\mu\nu} = 0$$

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New cosmological observables

Electromagnetism

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$$\partial_\mu F^{\mu\nu} = J^\nu + J_d^\mu$$

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New cosmological observables

Backup

Classical and Quantum Gauge Theories

Classical Field Theory
(e.g. General Relativity)

$$\frac{\delta S}{\delta g^{\mu\nu}} = 0$$

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Quantum Field Theory

$$Z = \int Dg^{\mu\nu} e^{iS[g]} \implies \delta Z = \int Dg^{\mu\nu} \left(\frac{\delta S}{\delta g^{\mu\nu}} \right) \delta g^{\mu\nu} = 0$$

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Not the Same
????????

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Quantum Supremacy

$$Z = \int Dg^{\mu\nu} e^{iS_{gf}[g]} \implies \delta Z = \int Dg^{\mu\nu} \left(\frac{\delta S_{gf}}{\delta g^{\mu\nu}}\right) \delta g^{\mu\nu} = 0$$

✓

Hamiltonian Construction

$$\Pi_a = \frac{\partial \mathcal{L}}{\partial \dot{a}}, \Pi_\phi = \frac{\partial \mathcal{L}}{\partial \dot{\phi}}$$

Quantize These

$$[a, \Pi_a] = [\phi, \Pi_\phi] = i$$

$$a = A + A^\dagger, \Pi_a = i\left(A - A^\dagger\right) \qquad \phi = B + B^\dagger, \Pi_\phi = i\left(B - B^\dagger\right)$$

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Quantize These

$$[a, \Pi_a] = [\phi, \Pi_\phi] = i$$

$$a = A + A^\dagger, \Pi_a = i\left(A - A^\dagger\right) \qquad \phi = B + B^\dagger, \Pi_\phi = i\left(B - B^\dagger\right)$$

Fully non-linear General Relativity - no "free" theory with "free" kinetic term

But, can still construct Hilbert space with Fock states of A, B - these are operator level statements independent of kinetic terms of the theory

$$A|0\rangle = 0, A^\dagger|0\rangle = |1\rangle \text{ etc.}$$