

ABSTRACT

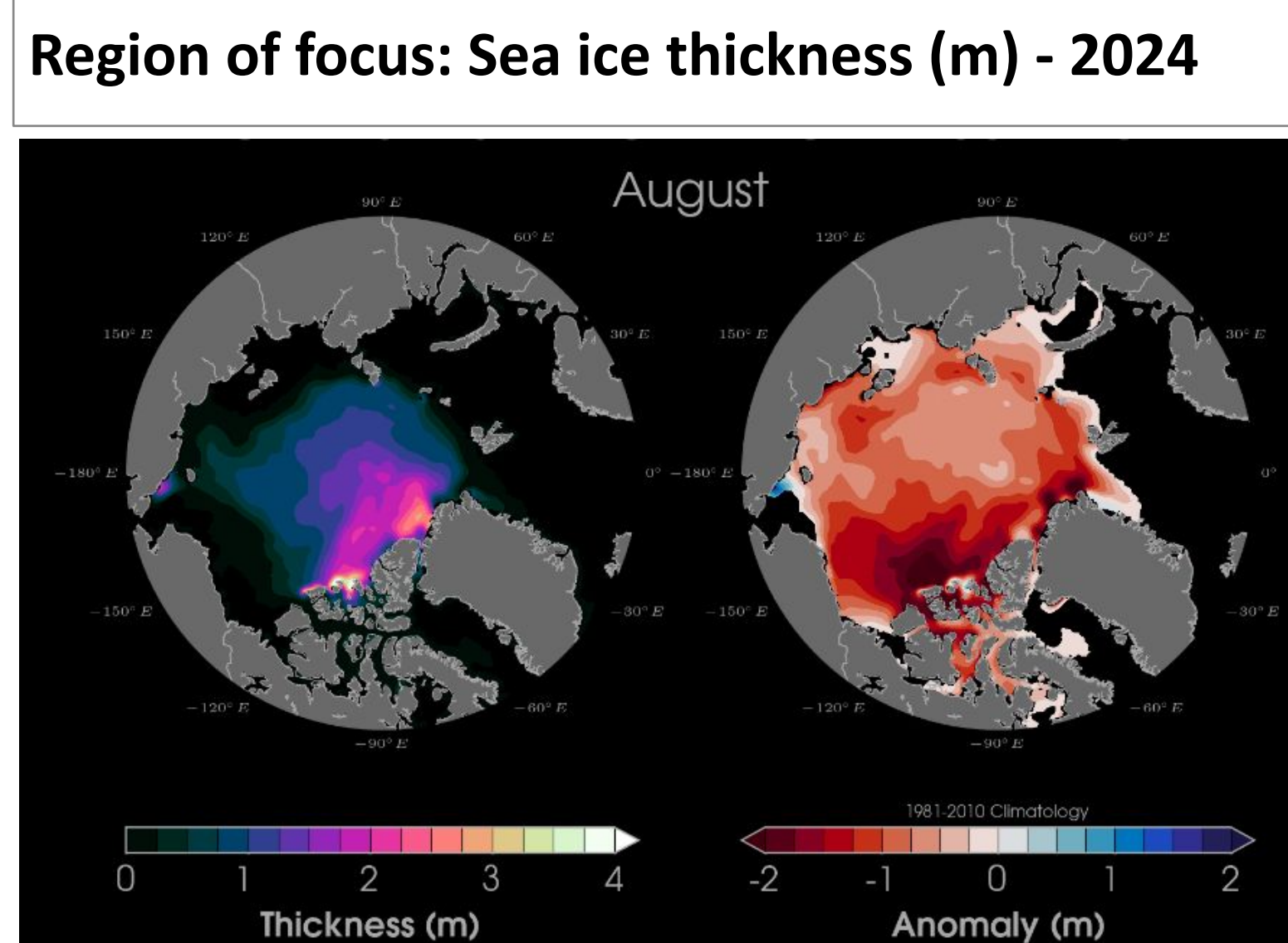
The Physics-Informed Sea Ice Thickness Model (PSTM) introduced in this study combines the strengths of physics-based models and machine learning techniques. By integrating recurrent neural networks (RNNs) with a Kolmogorov-Arnold Networks (KAN)-inspired spline module, PSTM effectively captures long-range dependencies and nonlinear relationships within sea ice data. The integration of optimal transport cost into backpropagation further enhances the model's ability to represent physical relationships.

PSTM is designed to predict sea ice thickness by focusing on the fundamental processes of ice growth, melt, and dynamics. By concentrating on these key factors, the model provides a more accurate and interpretable understanding of sea ice evolution.

Experimental results using Arctic sea ice data show that PSTM consistently outperforms traditional models, demonstrating the advantages of incorporating physical constraints into machine learning frameworks for spatio-temporal prediction tasks.

INTRODUCTION

Sea ice thickness refers to the vertical distance between the top surface of the sea ice (which may be covered by snow) and the bottom where it is in contact with the ocean water. It is a key measure used to understand the overall volume and structure of sea ice in polar regions.



[1]

Understanding the dynamics of sea ice thickness, particularly focusing on thermodynamic and dynamic processes, is key to comprehending freshwater discharge in the Arctic. Sea ice melt releases freshwater into the ocean, affecting salinity and influencing ocean circulation and heat transport. Changes in ocean currents, driven by the formation and melting of sea ice, further impact the distribution and timing of freshwater discharge.

Physics-informed machine learning (PIML) incorporates physical laws and constraints into the loss function of a machine learning model, ensuring that the model's predictions align with underlying physical principles.

DATASET

- The study uses the NEMO3.6-LIM3 global ocean-sea ice model simulations from [2], focusing on the Arctic Ocean (60-90 degrees N) during the period 1995-2014.
- Monthly gridded sea ice thickness data is obtained, where the Earth's surface is divided into grid cells, each representing sea ice thickness for that area.
- The grid has dimensions of 362 cells in the x-coordinate, 292 cells in the y-coordinate, and 480 time steps (t-coordinate) corresponding to monthly data points.
- The sea ice thickness variable is used as the target label for prediction tasks in the study.
- A total of 2,000,000 samples are utilized, which are split into training, validation, and testing datasets for evaluating prediction performance on historical data.

METHODS

We propose a neural network approach to solve the sea ice thickness distribution equation presented in [2]. The neural network is trained to approximate the solution, $h(x,y,t)$. During training, a physics loss function is minimized.

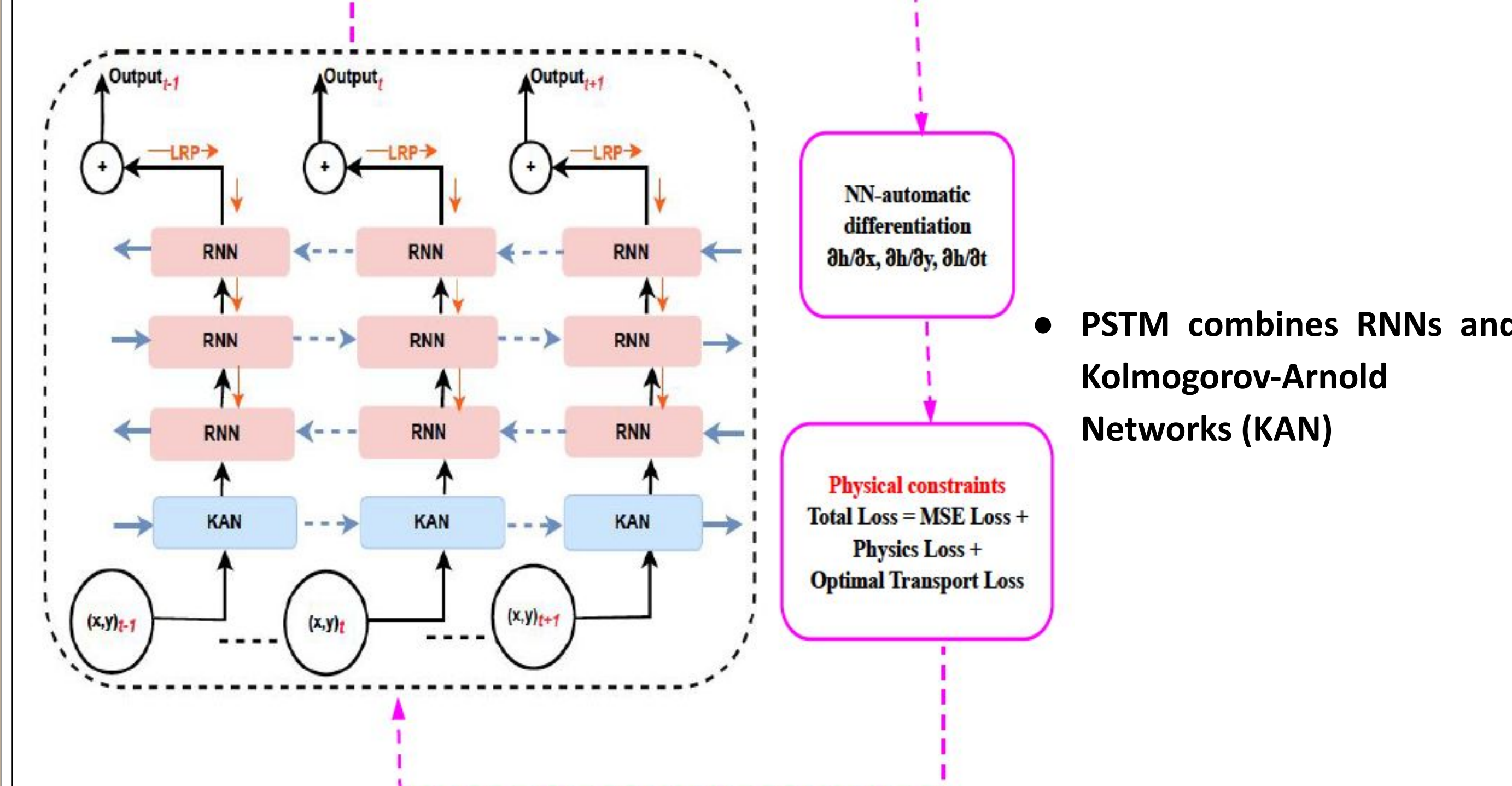
$$\frac{\partial g(h)}{\partial t} = -\nabla \cdot (\mathbf{u}g(h)) - \frac{\partial}{\partial h}(f * g(h)) + \psi(h) + L$$

$g(h)$ represents the ice thickness distribution, \mathbf{u} is the velocity of the ice pack, $f = dh/dt$ is the growth or melting rate of the ice, $\Psi(h)$ is the mechanical redistribution function. t is time. The equation describes the rate of change of the ice thickness distribution as the sum of two terms:

- First term:** represents the advection of ice thickness by the ice pack velocity.
- Second term:** represents the redistribution of ice thickness due to processes like rafting, ridging, and melting.

The **redistribution function $\Psi(h)$** is non-negative and dictates how ice thickness is redistributed. The term L corresponds to lateral melt

Fig.2. RNN/LSTM with KAN-Inspired Architecture for Physics-Informed Model for sea ice thickness (PSTM)



RESULTS

Table 1. Evaluation sea ice thickness prediction with and without physics.

Type	Model	MSE	SMAPE	MAPE	Theil
NoPhysics	Transformer_NP	1.9523	0.9822	6.9114	0.1805
NoPhysics	PSTM_NP (GRU)	1.9236	0.9803	6.8104	<u>0.1825</u>
NoPhysics	PSTM_NP (LSTM)	1.8986	0.9795	6.7029	0.1837
Physics	PGNN	18.9592	3.1262	8.6318	0.2021
Physics	PcudnnLSTM	2.2397	1.1905	3.5606	0.3004
Physics	PSTM (GRU)	<u>1.8564</u>	<u>0.9781</u>	6.5139	0.1859
Physics	PSTM (LSTM)	1.8394	0.9780	<u>6.4236</u>	0.1876

- Among the physics-based models, PSTM (LSTM) appears to have the best overall performance based on the metrics.

Table 2. Evaluation sea ice thickness prediction with and without physics.

Optimizer	MSE		SMAPE		MAPE		Theil	
	KAN	NO KAN	KAN	NO KAN	KAN	NO KAN	KAN	NO KAN
Adam	1.9362	2.0280	0.9812	1.1119	6.8533	4.3639	0.1813	0.3100
Adagrad	1.8641	3.2285	0.9765	1.6596	6.5936	2.7689	0.1886	0.4089
RMSprop	1.9324	1.7600	0.9810	0.9888	6.8397	5.7456	0.1814	0.2290

- NO KAN:** Traditional activation function ReLU is used
- KAN:** Cubic spline activation function is used. These spline-based activation function provides smoother transitions and more flexibility in modeling non-linear relationships
 - KAN generally leads to better MSE, SMAPE, and Theil metrics. KAN may increase MAPE, suggesting potential sensitivity to outliers.
 - Adagrad: Shows the most significant improvement in MSE and Theil metrics when using KAN.

CONCLUSION

- Our study suggests that physics-informed models show reasonable improvements in sea ice thickness prediction.
- Results demonstrate that PSTM, which combines RNNs and KAN, effectively captures complex spatiotemporal patterns in sea ice thickness data.
- The RNN models, with physics-informed loss functions, better represent physical relationships and improve prediction accuracy.
- Future research** could explore applying PSTM to other geophysical phenomena and assessing the impact of various physical constraints on

REFERENCES

- [1] <https://zacklabe.com/arctic-sea-ice-volumethickness/>
 [2] Massonnet, F. Climate Models as Guidance for the Design of Observing Systems: the Case of Polar Climate and Sea Ice Prediction. (Em Current Climate Change Reports). (textbf(5) pp. 334-344 (2019)