Dark matter candidate emerging from 3-form gauge theory

7th Sydney CPPC Meeting

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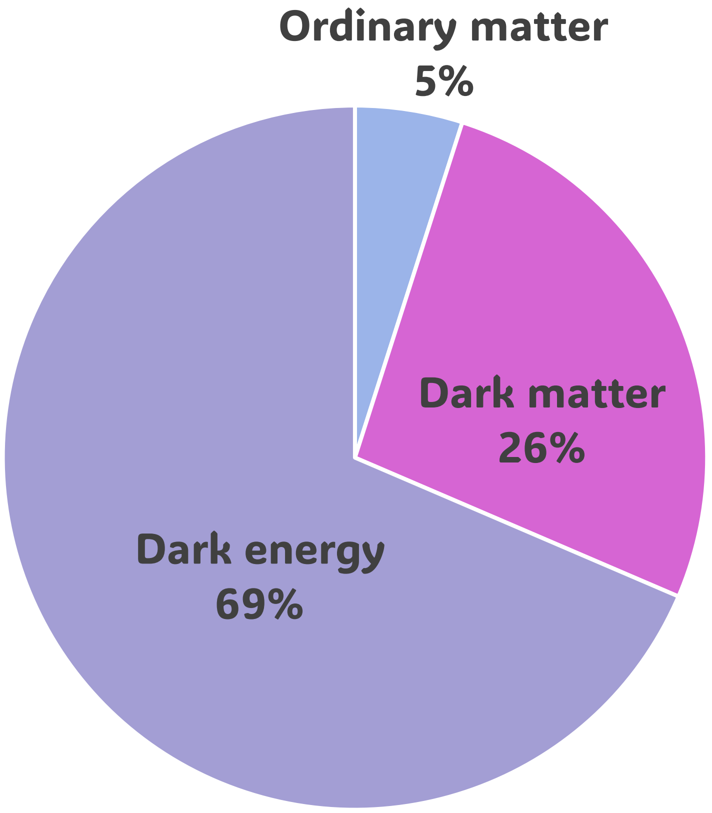
Dark matter and dark energy



Dark matter (DM)	Dark energy (DE)
Properties	
• *Matter*: clumps under gravity • *Dark*: Does not emit/absorb light • *Cold*: Most non-relativistic today	• Causes accelerated expansion (negative pressure) • Doesn't clump under gravity
Evidence	
• Rotational curves of galaxies • Gravitational lensing • Galaxy formation • CMB	• SN Ia • CMB
Candidates	
• New particles (e.g. WIMPs, ALPs) • Primordial black holes • Modified gravity (e.g. MOND)	• Cosmological constant Λ • Quintessence

Are dark matter and dark energy related?

- Most DM theories related to new particles.
- DE traditionally a constant vacuum energy Λ permeating whole Universe – no particles.

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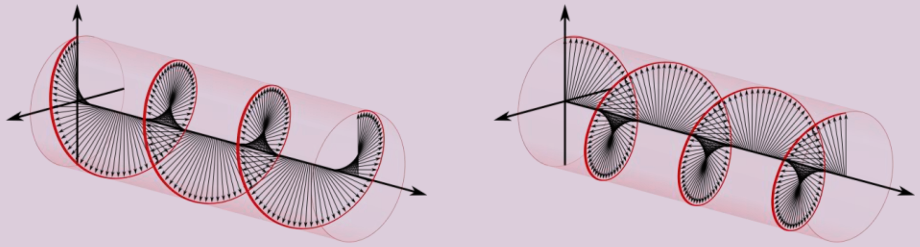
- This work proposes a model that can explain DM and DE using **3-form gauge fields**.

3-form gauge theory

Electromagnetism	3-form gauge theory
• Photon A_μ with gauge redundancy: $$\delta A_\mu = \partial_\mu \theta$$	• Tensor field $A_{\nu\rho\sigma}$ with gauge redundancy: $$\delta A_{\nu\rho\sigma} = \partial_{[\nu}\Omega_{\rho\sigma]} \propto \epsilon_{\mu\nu\rho\sigma}\partial^\mu \theta$$
• Field strength tensor: $$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = \partial_{[\mu}A_{\nu]}$$	• Field strength tensor: $$F_{\mu\nu\rho\sigma} = \partial_{[\mu}A_{\nu\rho\sigma]}$$
• 2 propagating d.o.f. (2 polarisation states)  Source: Wikimedia Commons	• $F_{\mu\nu\rho\sigma}$ is dual to a scalar F $$F_{\mu\nu\rho\sigma} = \epsilon_{\mu\nu\rho\sigma}F$$
	• 0 propagating d.o.f. (no particles)

Dynamics of 3-forms in vacuum

$$\mathcal{L}_{gauge} = \sqrt{-g}\left[-\frac{1}{2\cdot 4!}F_{\mu\nu\rho\sigma}F^{\mu\nu\rho\sigma}\right]$$

- Equations of motion: $\partial_\mu F = 0 \implies F = \text{constant} \equiv \lambda$

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No new particles!

- Energy-momentum tensor:

$$T_{\mu\nu} = g_{\mu\nu}\cdot\frac{1}{2}\lambda^2 \implies \begin{cases} \rho = T_{00} = \frac{1}{2}\lambda^2 \\ p = T_{ii} = -\frac{1}{2}a^2\lambda^2 \end{cases}$$

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- Negative pressure $\implies$ dark energy!

3-form in vacuum behaves like a cosmological constant!

Generating a mass for 3-forms

- Need to get DM candidate – apply **Anderson-Higgs mechanism** to 3-form.

- Analogous to photons propagating in plasma:
  - $+1$ d.o.f. due to collective plasma oscillations.
  - Photon now has 3 d.o.f. $\Longrightarrow$ **effective mass**.

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- 3-form permeates Universe containing ordinary matter.

Anderson-Higgs mechanism on 3-form $\Longrightarrow$ $+1$ d.o.f. for 3-form $\Longrightarrow$ Effective mass arises (DM candidate)

Modelling the Universe

$$g_{\mu\nu} = \mathrm{diag}\big(1, -a(t)^2, -a(t)^2, -a(t)^2\big)$$

- *Cosmological principle* $\Longrightarrow$ model ordinary matter as a **perfect cosmic fluid**.
  - Described by real scalar field $\phi(x)$ with shift symmetry $\delta\phi = c$.

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  - Described by real scalar field $\phi(x)$ with shift symmetry $\delta\phi = c$.

- Construct Lagrangian (using first-order formalism):

$$\mathcal{L}_{fluid} = \sqrt{-g}\left[\partial_\mu \phi V^\mu - \underbrace{\mu^4}_{\text{Energy scale of theory}} P(X)\right] \quad \text{where} \quad X = \frac{1}{2\mu^4} V_\mu V^\mu$$

Ordinary matter as a perfect fluid

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- Energy-momentum tensor: $T_{\mu\nu} = 2V_\mu \partial_\nu \phi - V_\mu V_\nu - g_{\mu\nu}(V^\alpha \partial_\alpha \phi - \mu^4 P)$

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  - Equations of motion: $P_X V^\mu = \partial^\mu \phi$

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- Rewrite as: $T_{\mu\nu} = \underbrace{2\mu^4 X P_X}_{\rho + p} u_\mu u_\nu - g_{\mu\nu} \underbrace{(2\mu^4 X P_X - \mu^4 P)}_{p}$

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- Fix equation of state $w = p/\rho$ and solve for ρ in terms of w:

$$\boxed{\rho = \mu^4 P(X) = \rho_\Lambda X^{(1+w)/2}}$$

Gauging the theory with a 3-form

Coupling strength of 3-form

Energy scale of theory

$$\mathcal{L} = \sqrt{-g}\left[-\frac{1}{2\cdot 4!}F_{\mu\nu\rho\sigma}F^{\mu\nu\rho\sigma} + \left(\partial_\mu\phi - \frac{g_A\mu}{3!}\epsilon_{\mu\nu\rho\sigma}A^{\nu\rho\sigma}\right)V^\mu - \mu^4 P(X)\right]$$

Gauge coupling between 3-form and cosmic fluid

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- Energy-momentum tensor:

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- Again, parameterise energy density by equation of state $w = p/\rho$:

$$\boxed{\rho = \mu^4 P + \frac{1}{2}F^2 = \rho_\Lambda X^{(1+w)/2}}$$

Energy density at different epochs

Find background energy densities – assume only **time-dependent solutions**.

- DE-dominated era $(w = -1)$:

$$\bar{\rho}_\Lambda = \mu^4 P_0 + \frac{1}{2}\lambda^2$$

Energy density at different epochs

Find background energy densities – assume only **time-dependent solutions**.

- DE-dominated era ($w = -1$):

$$\bar{\rho}_\Lambda = \mu^4 P_0 + \frac{1}{2}\lambda^2$$

- Matter-dominated era ($w = 0$):

$$\bar{\rho}_M = \bar{\rho}_B + \left(\frac{4}{H_0}\frac{\Omega_{M,0}}{\Omega_{\Lambda,0}}\right)^2 \frac{g_A^2 \mu^6}{a^3}$$

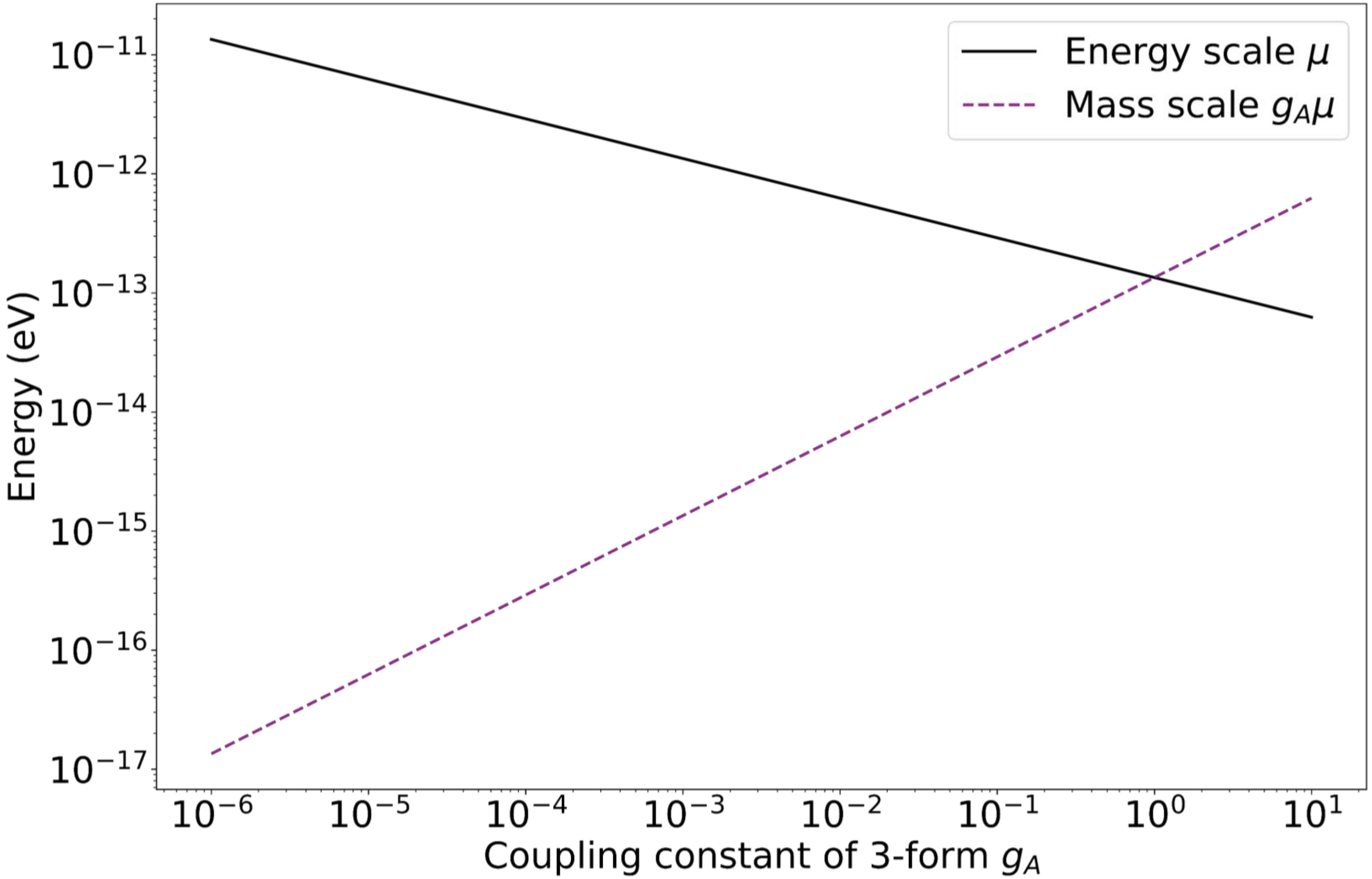
New contribution from 3-form's effective mass!

Relating DM density to model parameters

- Assume all DM consists of the 3-form's effective mass.

$$\bar{\rho}_{DM,0} = \left(\frac{4}{H_0}\frac{\Omega_{M,0}}{\Omega_{\Lambda,0}}\right)^2 g_A^2 \mu^6$$

- $\bar{\rho}_{DM,0} = 9.75 \times 10^{-48}$ GeV^4
- $H_0 = 1.44 \times 10^{-42}$ GeV
- $\frac{\Omega_{M,0}}{\Omega_{\Lambda,0}} = 0.459$



Conclusion

- Introduced novel theory of DM involving a 3-form gauge field that also contributes to DE.
- By considering energy density of background fields, one can find a relationship between μ and g_A of the model.
- To make a viable DM candidate, need to **study perturbed fields of the theory** to verify large scale structure formation.

BACKUP SLIDES

Levi-Civita tensor

Normalisation:

- $\epsilon^{\mu\nu\rho\sigma}\epsilon_{\mu\nu\rho\sigma} = -4!$
- $\epsilon_{0123} = \sqrt{-g}$
- $\epsilon^{0123} = 1/\sqrt{-g}$

Alternatively, can write in terms of Levi-Civita *symbol* $\tilde{\epsilon}_{\mu\nu\rho\sigma}$:

- $\epsilon_{\mu\nu\rho\sigma} = \sqrt{-g}\tilde{\epsilon}_{\mu\nu\rho\sigma}$
- $\epsilon^{\mu\nu\rho\sigma} = (1/\sqrt{-g})\tilde{\epsilon}^{\mu\nu\rho\sigma}$

Energy-momentum tensors

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\partial \mathcal{L}}{\partial g^{\mu\nu}} = 2 \frac{\partial(\mathcal{L}/\sqrt{-g})}{\partial g^{\mu\nu}} - g_{\mu\nu}(\mathcal{L}/\sqrt{-g})$$

$$T_{\mu\nu}^{fluid} = (\rho + p)u_\mu u_\nu - g_{\mu\nu} p$$

Cosmic fluid through first-order formalism

$$\mathcal{L}_{fluid} = \sqrt{-g}\left[\partial_\mu \phi V^\mu - \mu^4 P(X)\right] \quad \text{where} \quad X = \frac{1}{2\mu^4} V_\mu V^\mu$$

Equations of motion:

- $\partial_\mu\left(\sqrt{-g}V^\mu\right) = 0$
- $V^\mu = \frac{1}{P_X}\partial^\mu \phi$

4-velocity of fluid: $V^\mu = \mu^2\sqrt{2X}u^\mu$

$$T_{\mu\nu} = 2V_\mu \partial_\nu \phi - V_\mu V_\nu - g_{\mu\nu}(V^\alpha \partial_\alpha \phi - \mu^4 P) = 2\mu^4 X P_X u_\mu u_\nu - g_{\mu\nu}(2\mu^4 X P_X - \mu^4 P)$$

$$\rho(w) = \mu^4 P = \alpha \mu^4 X^{(1+w)/2} \quad \text{where} \quad w = p/\rho$$

3-form gauge theory in vacua

Gauge invariance:

- $\delta A_{\nu\rho\sigma} = \frac{1}{g_A \mu} \epsilon_{\mu\nu\rho\sigma} \nabla^\mu \theta$
- $\delta F_{\mu\nu\rho\sigma} = \frac{1}{g_A \mu} \nabla_{[\mu} \epsilon_{\nu\rho\sigma]\alpha} \nabla^\alpha \theta$
- $\delta\left(F_{\mu\nu\rho\sigma} F^{\mu\nu\rho\sigma}\right) = 0 \Longrightarrow \nabla_\mu \nabla^\mu \theta = 0$
- $\delta F \propto \nabla_\mu \nabla^\mu \theta = 0$

Dual 1-form:

- $B_\mu = \frac{1}{3!} \epsilon_{\mu\nu\rho\sigma} A^{\nu\rho\sigma}$ where $\delta B_\mu = \frac{1}{g_A \mu} \nabla_\mu \theta$
- $F = -\frac{1}{4} \nabla_\mu B^\mu$

3-form gauge theory in vacua

Equations of motion:

$$\partial_\mu(\sqrt{-g}F^{\mu\nu\rho\sigma}) = 0 \implies \partial_\mu\left(\sqrt{-g}(\frac{1}{\sqrt{-g}}\tilde{\epsilon}^{\mu\nu\rho\sigma}F)\right) = 0 \implies \tilde{\epsilon}^{\mu\nu\rho\sigma}\partial_\mu F = 0 \implies \partial_\mu F = 0$$

Lagrangian requires a boundary term $+\frac{1}{4!}\partial_\mu\left(\sqrt{-g}F^{\mu\nu\rho\sigma}A_{\nu\rho\sigma}\right)$ to ensure:

1. The variation of the fields vanishes at the boundary.
2. The energy-momentum tensor derived from the *on-shell* Lagrangian reproduces the correct sign.

3-form gauge theory + cosmic fluid

Gauge invariance:

- $\delta A_{\nu\rho\sigma} = \frac{1}{g_A \mu} \epsilon_{\mu\nu\rho\sigma} \nabla^{\mu} \theta$
- $\delta B_{\mu} = \frac{1}{g_A \mu} \nabla_{\mu} \theta$
- $\delta F_{\mu\nu\rho\sigma} = \frac{1}{g_A \mu} \nabla_{[\mu} \epsilon_{\nu\rho\sigma]\alpha} \nabla^{\alpha} \theta$
- $\delta F = \nabla_{\mu} \nabla^{\mu} \theta = 0$
- $\delta \phi = \theta$

Equations of motion:

- $\partial_{\mu}\left(\sqrt{-g}\, V^{\mu}\right) = 0$
- $V^{\mu} = \frac{1}{P_X} (\partial^{\mu} \phi - g_A \mu B^{\mu})$
- $\partial_{\mu}\left(\sqrt{-g}\, F^{\mu\nu\rho\sigma}\right) = 4 g_A \mu \sqrt{-g}\, \epsilon^{\mu\nu\rho\sigma} V_{\mu} \Longrightarrow \partial_{\mu} F = 4 g_A \mu V_{\mu}$

3-form gauge theory + cosmic fluid

Energy-momentum tensor:

$$T_{\mu\nu} = 2\mu^4 X P_X u_\mu u_\nu - g_{\mu\nu}\left(2\mu^4 X P_X - \mu^4 P - \frac{1}{2}F^2\right)$$

Energy density:

$$\rho = \mu^4 P + \frac{1}{2}F^2 = \rho_\Lambda X^{(1+w)/2}$$

Dark energy-dominated era ($w = -1$)

$$\bar{\rho}_\Lambda = \mu^4 \bar{P} + \frac{1}{2}\bar{F}^2 = \mu^4 P_0 + \frac{1}{2}\lambda^2 = \text{constant}$$

On-shell background fields:

- $\bar{V}^0 = \frac{\mu^2 c_\Lambda}{a^3} = \frac{1}{\bar{P}_{\bar{X}}}\left(\dot{\bar{\phi}} - g_A \mu \bar{B}^0\right)$
- $\bar{X} = \frac{1}{2\mu^4}\left(\bar{V}^0\right)^2$
- $\bar{F} = -\frac{4 g_A \mu^3 c_\Lambda}{3 H_0}\frac{1}{a^3} + \lambda$

If you plug $\bar{F}$ into $\bar{\rho}_\Lambda$, you find terms $\sim a^{-6}$ and $\sim a^{-3}$. But since $\bar{\rho}_\Lambda$ is constant, the function $\bar{P}(\bar{X})$ must contain terms that exactly cancel out these terms, leaving behind only constant contributions P_0 as well as λ^2 from $\bar{F}$.

Matter-dominated era ($w = 0$)

$$\bar{\rho}_M = \mu^4 \bar{P} + \frac{1}{2}\bar{F}^2 = \bar{\rho}_\Lambda \bar{X}^{1/2} = \frac{c_M}{\sqrt{2}}\frac{\bar{\rho}_\Lambda}{a^3}$$

On-shell background fields:

- $\bar{V}^0 = \frac{\mu^2 c_M}{a^3} = \frac{1}{\bar{P}_{\bar{X}}}\left(\dot{\bar{\phi}} - g_A \mu \bar{B}^0\right)$
- $\bar{X} = \frac{1}{2\mu^4}\left(\bar{V}^0\right)^2$
- $\bar{F} = -\frac{4 g_A \mu^3 c_M}{H_0}\frac{1}{a^{3/2}} + \lambda$

If you plug $\bar{F}$ into $\bar{\rho}_M$, you find terms $\sim a^{-3/2}$ and a constant $\sim \lambda^2$. But since $\bar{\rho}_M \sim a^{-3}$, the function $\bar{P}(\bar{X})$ must contain terms that exactly cancel out the terms $\sim a^{-3/2}$ and the constants, leaving behind $\sim a^{-3}$ terms from $\bar{P}(\bar{X})$ and $\bar{F}^2$. We assume that whatever remaining term from $\bar{P}(\bar{X})$ corresponds to ordinary matter ρ_B.

Radiation-dominated era ($w = 1/3$)

$$\bar{\rho}_R = \mu^4 \bar{P} + \frac{1}{2}\bar{F}^2 = \bar{\rho}_\Lambda \bar{X}^{2/3} = \left(\frac{c_R^2}{2}\right)^{2/3} \frac{\bar{\rho}_\Lambda}{a^4}$$

On-shell background fields:

- $\bar{V}^0 = \frac{\mu^2 c_R}{a^3} = \frac{1}{\bar{P}_{\bar{X}}}\left(\dot{\bar{\phi}} - g_A \mu \bar{B}^0\right)$
- $\bar{X} = \frac{1}{2\mu^4}\left(\bar{V}^0\right)^2$
- $\bar{F} = -\frac{8 g_A \mu^3 c_R}{a H_0} + \lambda$

If you plug $\bar{F}$ into $\bar{\rho}_R$, you find terms $\sim a^{-2}$, $\sim a^{-1}$ and a constant $\sim \lambda^2$. But since $\rho_R \sim a^{-4}$, the function $\bar{P}(\bar{X})$ must contain terms that exactly cancel outs the entire contribution of $\bar{F}^2$. Thus, the 3-form doesn’t contribute to the radiation-dominated era.