Finite Nuclear Contributions to Atomic Hyperfine Structure

AIP Summer Meeting 2023

James Vandeleur

Supervised by Dr. Jacinda Ginges
Thanks to George Sanamyan, Ben Roberts

The University of Queensland

Table-top Physics

- Table top searches for new physics are gaining popularity.
- Parity Violation, testing QED, EDMs, Isotope Shift - all improved with precise nuclear models.

Table-top Physics

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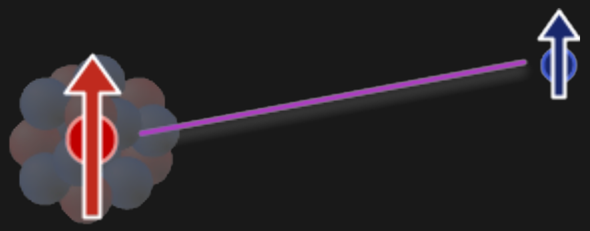
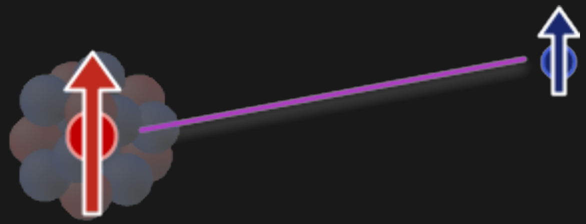
- Parity Violation, testing QED, EDMs, Isotope Shift - all improved with precise nuclear models.
- Atomic Hyperfine structure is the playground for finite nuclear models.

Hyperfine Interaction



$$h_{\mathrm{hfs}} \propto \boldsymbol{\mu} \cdot \mathbf{B}$$

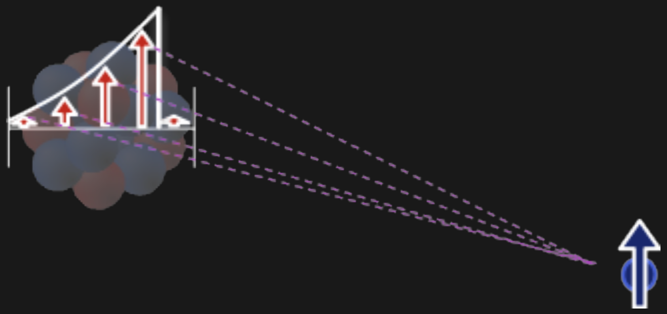
Hyperfine Interaction



$$h_{\mathrm{hfs}} \propto \boldsymbol{\mu} \cdot \mathbf{B}$$

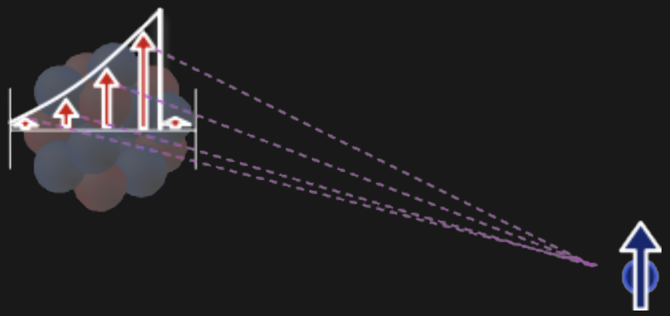
$$\mathcal{A} \propto \langle \mathrm{atom} | \, h_{\mathrm{hfs}} \, | \mathrm{atom} \rangle$$

Finite Nuclear Effects



$$\mu \to F(r)\mu$$

Finite Nuclear Effects

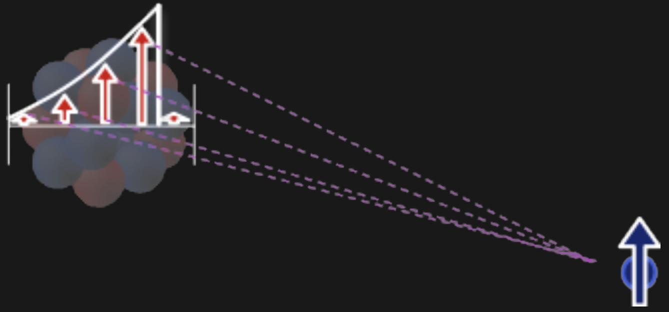


$$\mu \rightarrow F(r)\mu$$

$$\downarrow$$

$$\mathcal{A} = \mathcal{A}_{\mathrm{point}} + \mathcal{A}_{\mathrm{charge}} + \mathcal{A}_{\mathrm{BW}} + \mathcal{A}_{\mathrm{QED}}$$

Finite Nuclear Effects



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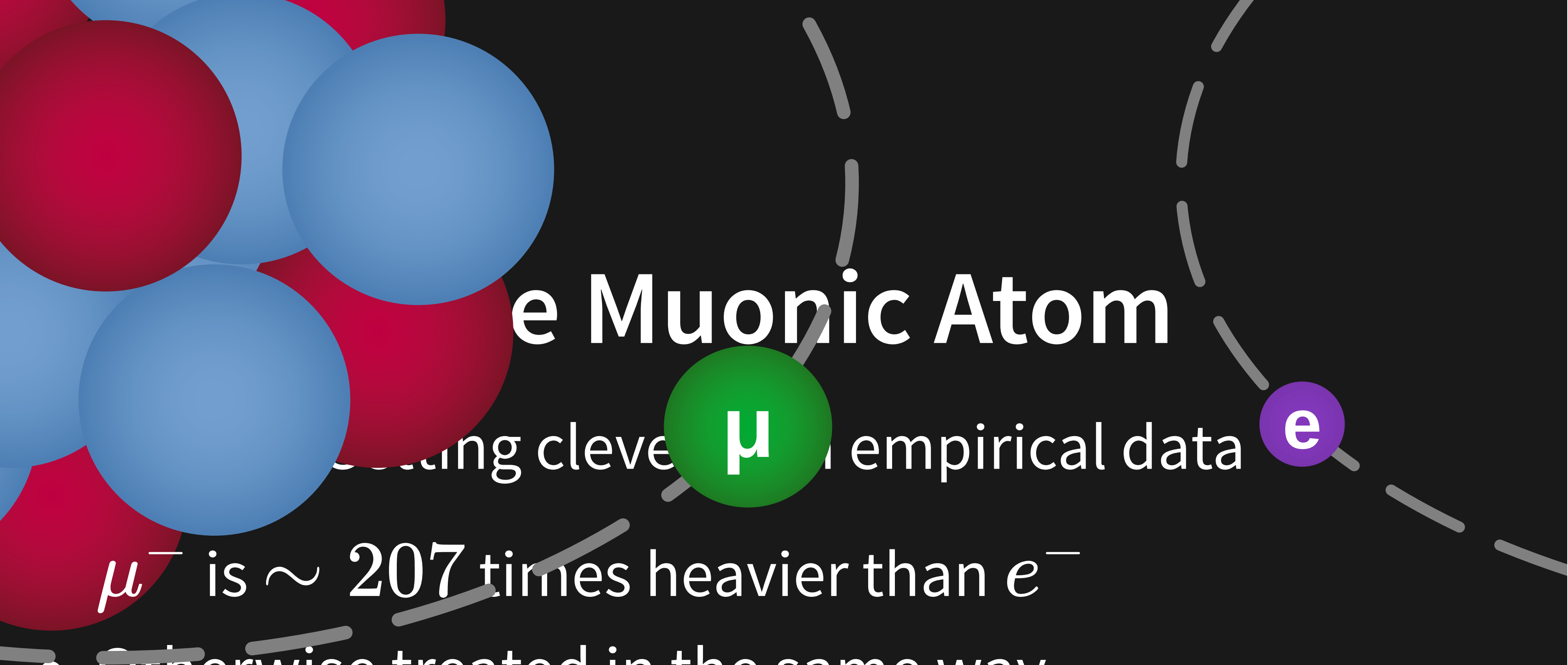
Need to improve nuclear models!



e Muonic Atom

- tting cleve empirical data
- μ^- is ~ 207 times heavier than e^-
- Otherwise treated in the same way
- Muon is much closer to nucleus and more sensitive to nuclear effects

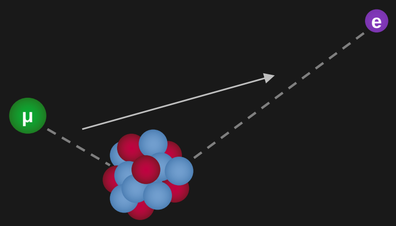
Outline

Method

Fitting nucleon distribution.

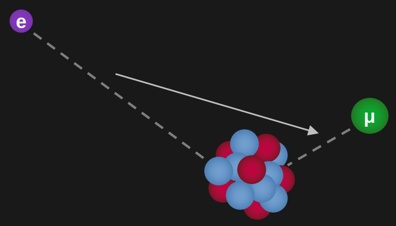
Bohr-Weisskopf (BW) Effect in Hg

(Finite Magnetisation Distribution)



Nuclear Polarisability (NP) Effect

(Electron-Nucleon Excitations)



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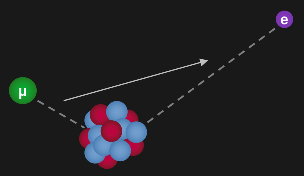

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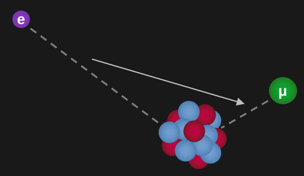

* A. A. Elizarov, et al. , Optics and Spectroscopy **100**, 361 (2006).

Calculate $\mathcal{A}_{\mathrm{BW}}$?

- Need to know the magnetisation distribution.
- Single particle model - requires odd nucleon wavefunction.
- Extra (muonic) system provides one piece of information.

Total hyperfine constant

$$\mathcal{A} = \mathcal{A}_{\mathrm{point}} + \mathcal{A}_{\mathrm{charge}} + \mathcal{A}_{\mathrm{BW}} + \mathcal{A}_{\mathrm{QED}}$$

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Solutions to Dirac equation.

Fermi charge distribution with radius from scattering experiment.

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$$\mathcal{A}_{\mathrm{QED}}$$

Vacuum Polarisation
(dominant in muonic atoms)

Self Energy

Total hyperfine constant

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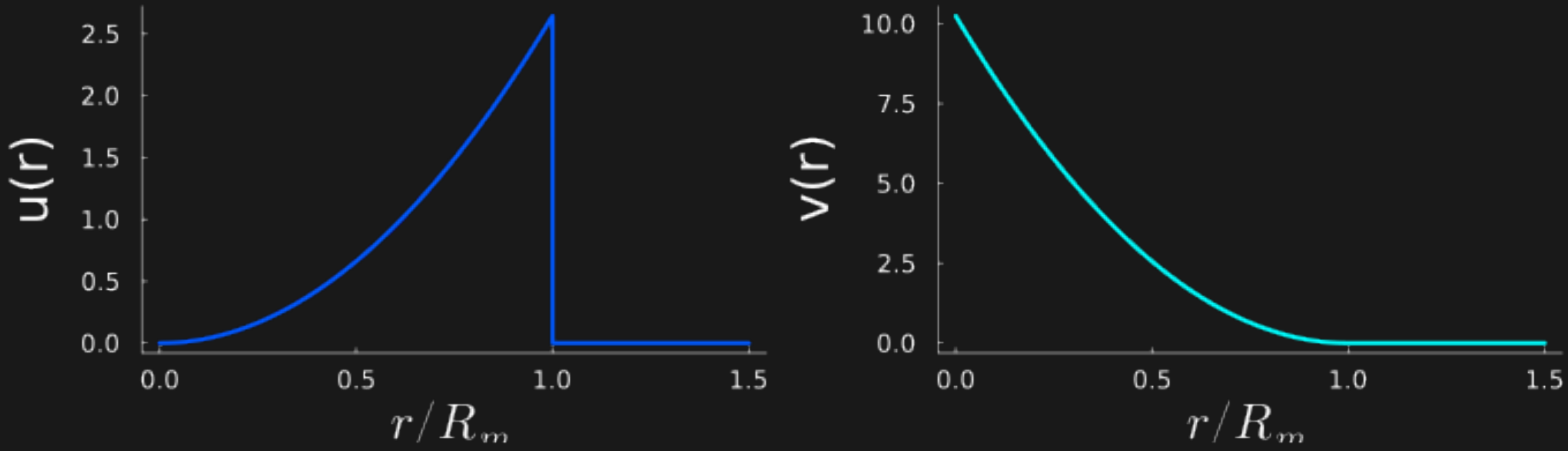
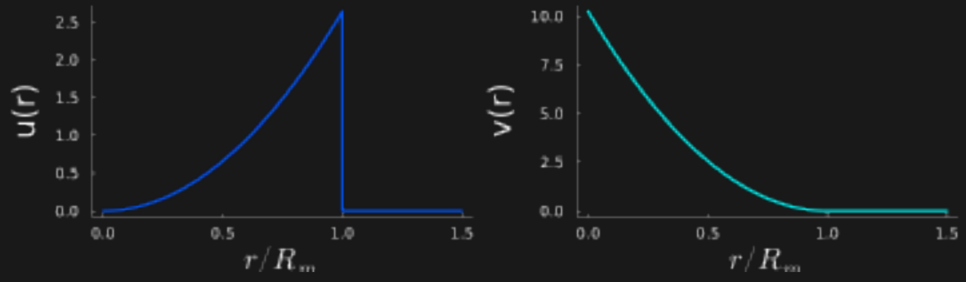
Nucleon Wavefunctions

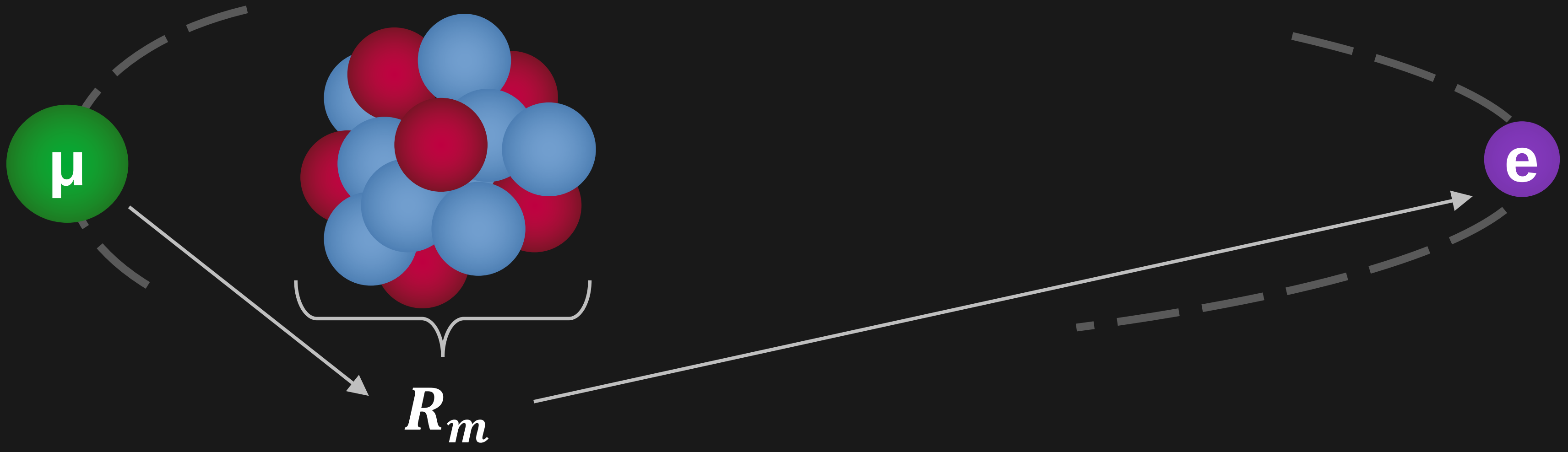
Experiment gives us **1** piece of information - R_m.



Nucleon Wavefunctions



Fitting Nuclear Magnetic Radius - R_m



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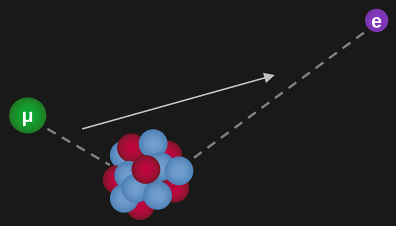

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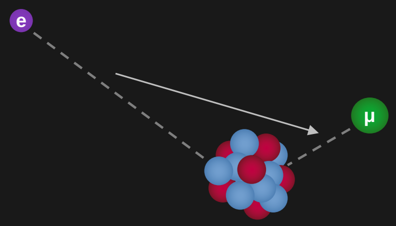


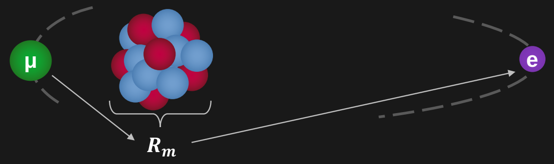
BW Effect in Mercury

- Sanamyan et al. recently applied this method in Cs.
- Also data available for ^{199}Hg.
- Hg isotopes used by Moskowitz and Lombardi.

G. Sanamyan, B. M. Roberts, and J. S. M. Ginges , Physical Review Letters **130**, 053001 (2023).

Results

muonic $\mathrm{Hg} \longrightarrow$ H-like Hg



Stand-alone value for BW effect in $^{199}\mathrm{Hg}$.

Model	u			v			
n	0	1	2	1	2	Variance	Final Value
R_m (fm)	5.76(56)	5.16(47)	4.91(44)	8.81(96)	11.9(1.3)		
ϵ_BW (%)	-2.33(40)	-2.26(37)	-2.24(36)	-2.50(46)	-2.61(50)	0.16	-2.39(45)

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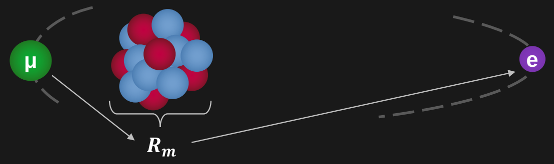

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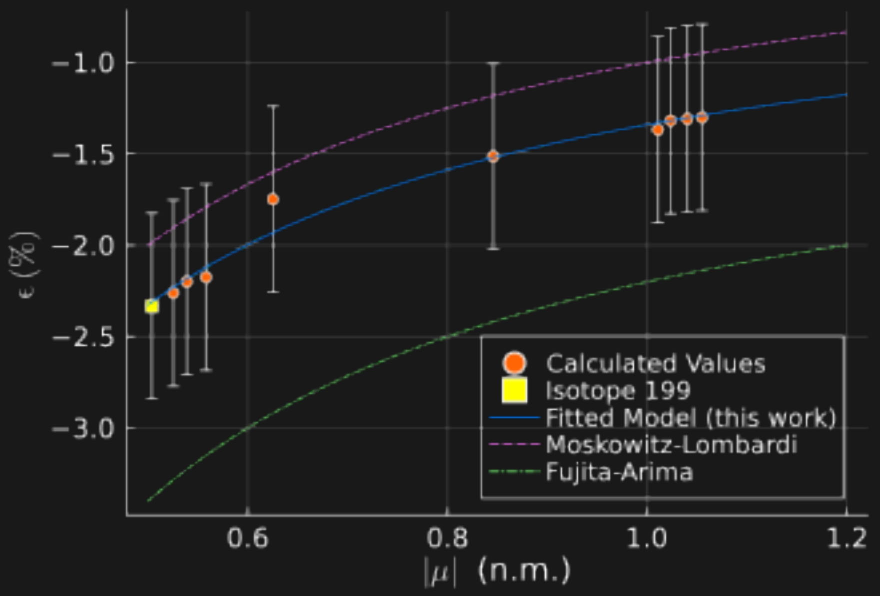
screening ($\approx \times 1$) $\rightarrow \mathrm{Hg}^+, \mathrm{Hg}$

Differential BW Effect

$$\epsilon^{A} \approx \epsilon^{199} + {}^{A}\Delta^{199}$$

- ${}^{A}\Delta^{199}$ from experiment.
- Finding ϵ^{199} immediately gives all isotopes.

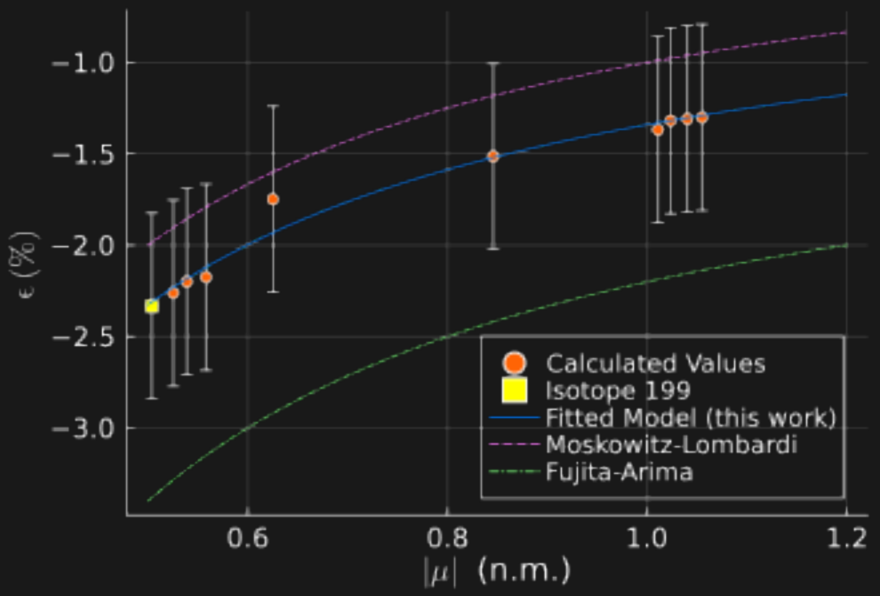
BW effect in Mercury Isotopes



$$\epsilon_{\mathrm{BW}} = \alpha \frac{\mu_N}{|\mu|} + c$$

P. A. Moskowitz and M. Lombardi , Physics Letters B **46**, 334 (1973).

BW effect in Mercury Isotopes



$$\epsilon_{\mathrm{BW}} = \alpha \frac{\mu_N}{|\mu|} + c$$

$$\alpha = \ 0.01$$ (all agree)

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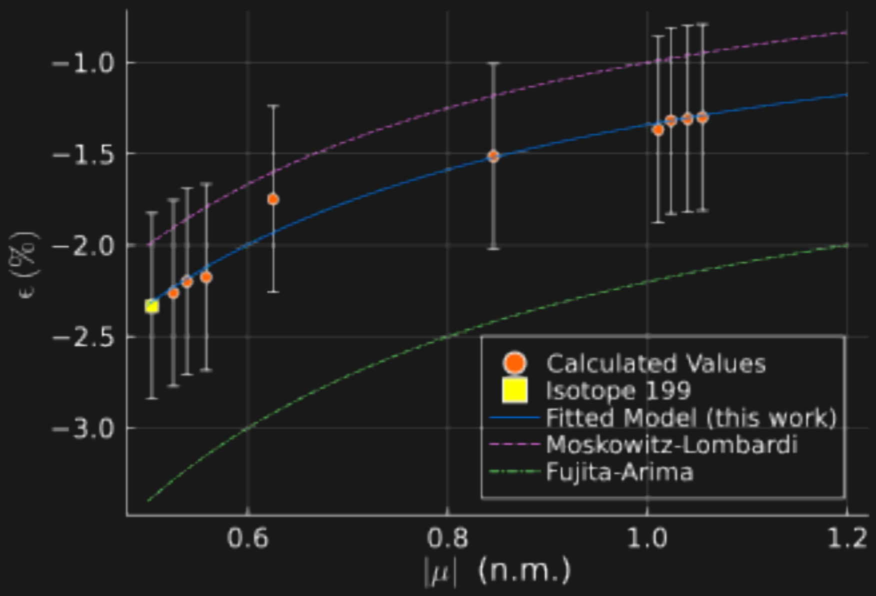

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$\alpha = \ 0.01$ (all agree)

$c \sim -0.01$ (Fujita & Arima)

$c \sim -0.0036(8)$ (our result)

P. A. Moskowitz and M. Lombardi , Physics Letters B **46**, 334 (1973).

T. Fujita and A. Arima , Nuclear Physics A **254**, 513 (1975).

BW Summary

- Now have BW in all Hg isotopes.
- Empirical shift in ML rule gives insight into nuclear models.
- Method relies on available muonic data - more coming!

- (In Preparation) J. Vandeleur, G. Sanamyan, B. J. Roberts and J. S. M. Ginges, Empirical determination of the Bohr-Weisskopf effect in Hg Isotopes and comparison with the Moskowitz Lombardi rule.

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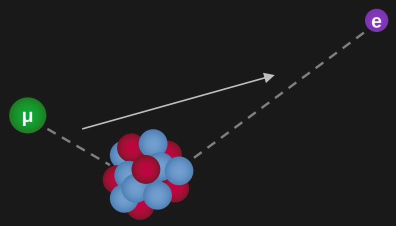

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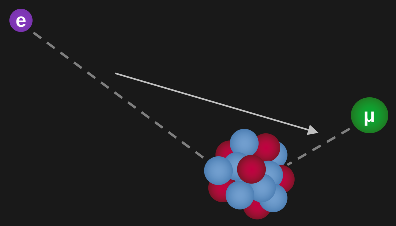


The muonic orbital has a strong overlap with the nucleus.

Nuclear degrees of freedom are excited - Nuclear Polarisability (NP)

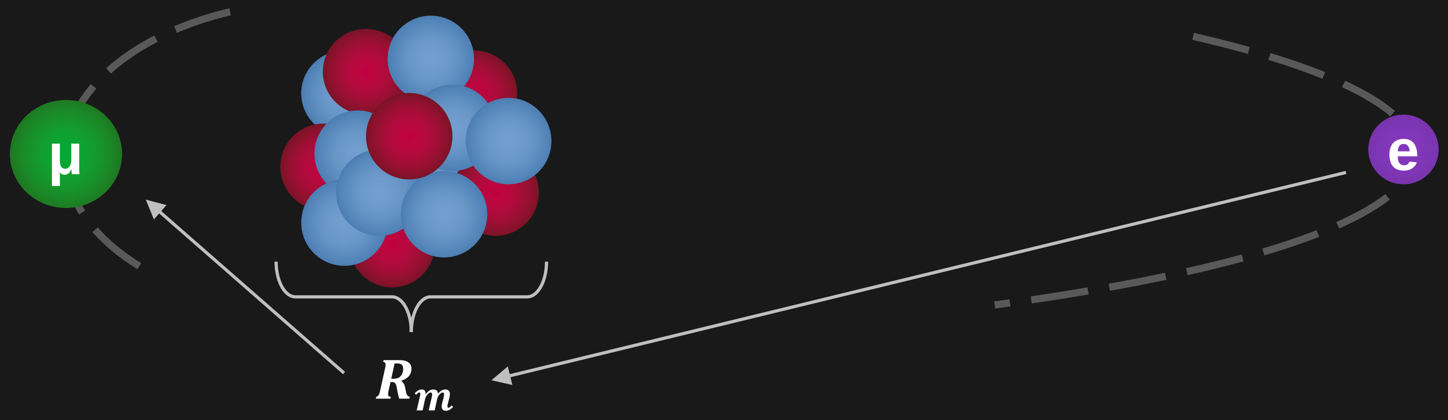
$$\mathcal{A} = \mathcal{A}_0 + \mathcal{A}_{\text{BW}} + \mathcal{A}_{\text{QED}} + \mathcal{A}_{\text{NP}}$$

The NP contribution is preventing progress with muonic atom experiments.

- Nuclear structure tests.
- Radius of a proton (muonic Hydrogen).
- Previous method...

Finding $\mathcal{A}_{\mathrm{NP}}$?

Can do previous method in reverse.



$$\mathcal{A}_{\mathrm{NP}}^{\mathrm{exp}} = \mathcal{A}_{\mathrm{exp}} - \mathcal{A}_0 - \mathcal{A}_{\mathrm{BW}} - \mathcal{A}_{\mathrm{QED}}$$

Results

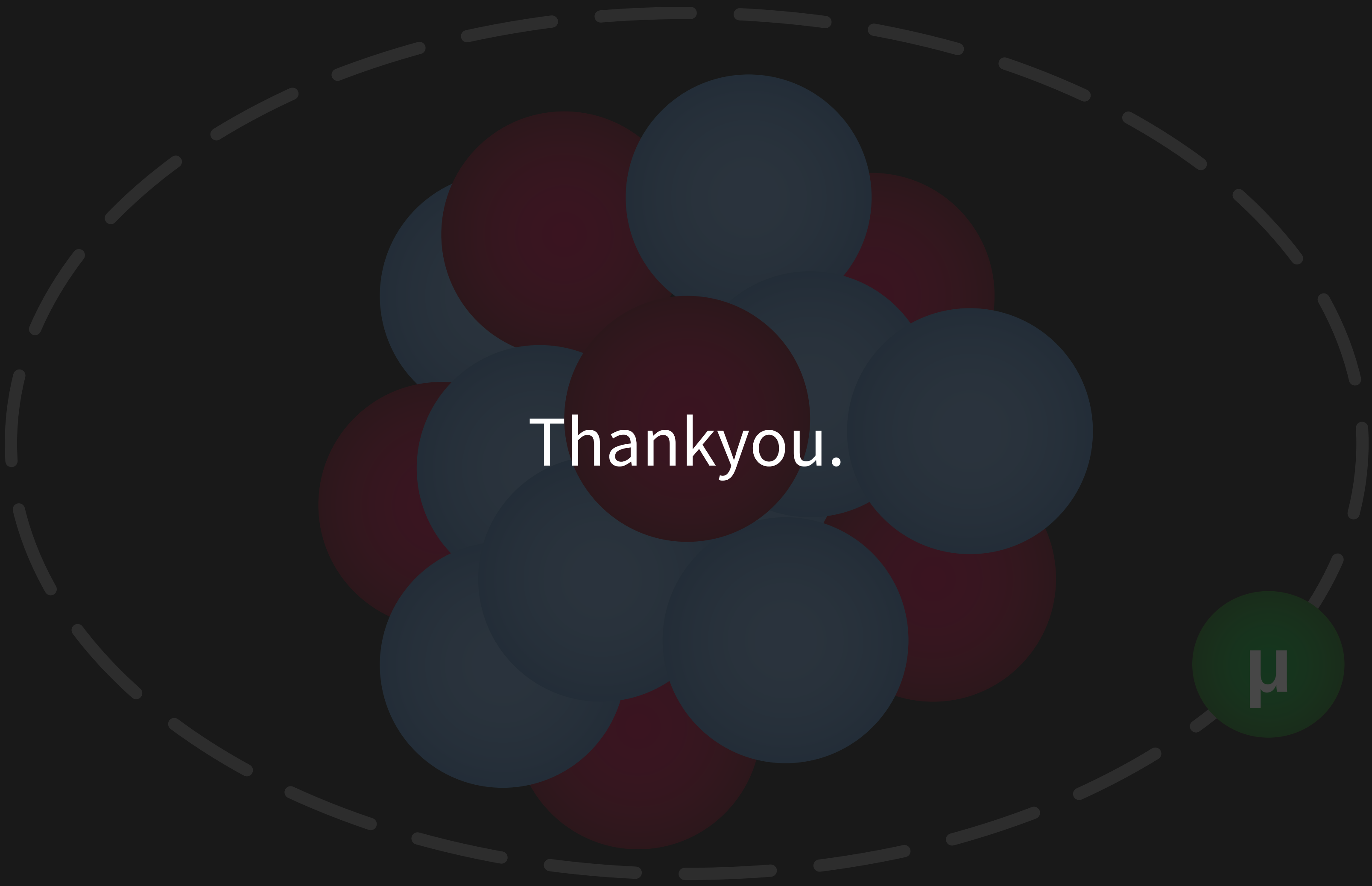
(keV)	$\mathcal{A}_{\mathrm{exp}}$	$\mathcal{A}_0$	$\mathcal{A}_{\mathrm{BW}}$	$\mathcal{A}_{\mathrm{QED}}$	$\boldsymbol{\mathcal{A}_{\mathrm{NP}}}$
^{203}Tl	2.340(80)	4.695(6)	-2.355(148)	0.023(12)	**-0.023(169)**
^{205}Tl	2.309(35)	4.726(6)	-2.719(149)	0.023(12)	**-0.068(154)**
^{209}Bi	0.959(52)	1.3394(7)	-0.4115(199)	0.006(3)	**0.0251(557)**

- NP contributions are very small.
- Uncertainties dominated by model variance through BW.

NP Summary

- NP contributions $\sim$ BW uncertainty.
- Muonic atom experiments can progress.
- Previous results in mercury still valid.

- (In Preparation) J. Vandeleur, G. Sanamyan, N.S. Oreshkina and J. S. M. Ginges, Determination of nuclear polarizability effects in the hyperfine structure of muonic atoms from a combination of H-Like and muonic atom experiments.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