Imprints of Dark Photons on Gravitational Waves

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Outline

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3. Dark Photon Dark Matter
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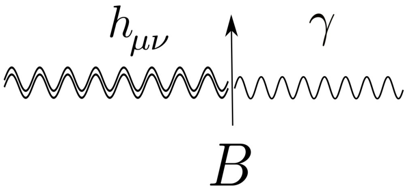
The Gertsenshtein effect: Graviton-Photon Conversion

In a background magnetic field, an incident graviton has a non-zero probability of conversion into a photon.

Derive through the coupling to the energy-momentum tensor $h_{\mu\nu} T^{\mu\nu}$.

Dependent on the magnetic field strength and the length of propagation.



Sufficiently large B and distances $\rightarrow$ graviton-photon oscillations.

Conversion in Cosmological Magnetic Fields

Large scale magnetic fields allow for large propagation lengths.

Possible route for detecting high frequency gravitational waves.

- Upper bound on size of intergalactic magnetic fields: $B_0 < 10^{-9}$ G.
- Effective photon mass induced by plasma effects, suppresses the conversion probability.
- Strongly suppressed in the early universe despite stronger B.

Dark Magnetic Fields

Consider instead a dark $U(1)$, that will undergo graviton-dark photon conversion

- Weakened constraints on dark magnetic fields today, by 3 orders of magnitude: $B_0 < 10^{-6}$ G from ΔN_{eff}.
- There may be no dark matter plasma to suppress the conversion probability, or the coupling and density can be small.
- Possibly generated by inflation or first order phase transition, providing information about the early universe.
- Dark matter candidate with imprints on the gravitational wave spectrum.

Dark Photon Dark Matter

The dark matter energy density of the universe today,

$$\rho_{\mathrm{DM}} \simeq 9.6 \cdot 10^{-48} \ \mathrm{GeV}^4 \ .$$

- No convincing experimental evidence for proposed dark matter candidates.
- Go beyond the usual WIMP paradigm.
- Novel production methods, such as couplings to the inflaton.
- Differentiable observational and experimental signatures.
- Kinetic mixing to photon.
- Stuekelberg mass or from spontaneous symmetry breaking.

Initial Set-up

Assume that, the universe is radiation dominated with,

- A stochastic dark magnetic field with energy density $\rho_{DM} = \frac{B^2}{2}$ and $\Omega^r_{DM} < 0.01$, with characteristic k_*,
- There is gravitational waves with ω propagating in this background with $\Omega^r_{GW} < 0.01$,
- The dark magnetic field is slowly varying relative to the GW, $k_* \ll \omega$.
- It has a small mass m_{DM}, and becomes non-relativistic before matter-radiation equality, $k_*(T_m) < m_{DM}$, ($T_m \simeq 1$ eV)
- It is the dark matter today - components with $k_*(T_0)$ and $\omega(T_0)$.

Calculating the Conversion Probability

Initial Set-up

Consider the following action,

$$S = \int d^4x \sqrt{-g}\, \frac{M_p^2}{2} R + \int d^4x \sqrt{-g} \left[-\frac{1}{4} g^{\mu\rho} g^{\nu\sigma} X_{\mu\nu} X_{\rho\sigma} + m_{DM}^2 g^{\mu\nu} X_\mu X_\nu \right],$$

with linear perturbations:

$$ds^2 = a(\tau)^2 (-d\tau^2 + (\delta_{ij} + h_{ij}) dx_i dx_j) \ ,$$

and

$$X_\mu = \bar{X}_\mu + \delta X_\mu$$

where $\bar{X}_\mu$ is the approximate background with characteristic scale k_*, with $E_i = -X_{0i}$, and $B_i = \frac{1}{2a^2} \epsilon_{ijk} X^{jk}$.

Conversion Probability

Using the transverse traceless gauge for the graviton, consider a gravitational wave propagating in the z direction,

$$h_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+(\tau, z) & h_\times(\tau, z) & 0 \\ 0 & h_\times(\tau, z) & -h_+(\tau, z) & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Take the radiation gauge for the dark photon, and consider a stochastic $\bar{B} = \frac{1}{a^2}(B_0/\sqrt{3}, B_0/\sqrt{3}, B_0/\sqrt{3})$ that only evolves adiabatically, with $\bar{E} = 0$.

Obtain the EoM, in the radiation dominated epoch and $\omega \gg k_*, H$,

$$(\omega + i\partial_z)h_\lambda \simeq -\delta X_\lambda \frac{B_0}{\sqrt{3}M_p a^2} \ ,$$

$$(\omega + i\partial_z)\delta X_\lambda + m_D(a)\delta X_\lambda \simeq -h_\lambda \frac{B_0}{\sqrt{3}M_p a^2} \ .$$

Conversion Probability

Solve EOMs of photon and gravitational wave in the dark magnetic field.

In Minkowski spacetime, with propagation distance L and small m_{DM}, the probability is approximately,

$$P_{g\to\gamma} \simeq \sin^2\left(\frac{B_0}{\sqrt{3}M_p}L\right)$$

In FRW spacetime, must include the scale factor and solve numerically:

$$i\frac{d}{da}\begin{pmatrix} h_\lambda(a) \\ \delta X_\lambda(a) \end{pmatrix} = \frac{1}{aH}\begin{pmatrix} 0 & m_{gD}(a) \\ m_{gD}(a) & m_D(a) \end{pmatrix}\begin{pmatrix} h_\lambda(a) \\ \delta X_\lambda(a) \end{pmatrix},$$

where $m_{gD}(a) = \frac{B_0}{\sqrt{3}M_p a^2}$ and $m_D(a) = \frac{m_{DM}^2}{2\omega}$.

For $m_D \to 0$, the probability is given by approximately,

$$P_{g\to\gamma} \simeq \sin^2(\sqrt{\Omega^r_{\rm DM}}\ln(a))$$

where $\Omega^r_{\rm DM} = B^2/(6M_p^2H^2)$.

Conversion Probability

Starting from gravitational waves incident on a background dark magnetic field, the conversion probability of a graviton to a dark photons is ρ_{DD} .

We must solve the following (density matrix description):

$$\begin{aligned} \rho'_{DD} &= \frac{-2m_{gD}I - \Gamma_D \rho_{DD}}{Ha} \ , \quad R' = \frac{m_D I - \Gamma_D R/2}{Ha} \ , \\ \rho'_{gg} &= \frac{2m_{gD}I}{Ha} \ , \quad I' = \frac{-m_D R - \Gamma_D I/2 - m_{gD}(\rho_{gg} - \rho_{DD})}{Ha}, \end{aligned}$$

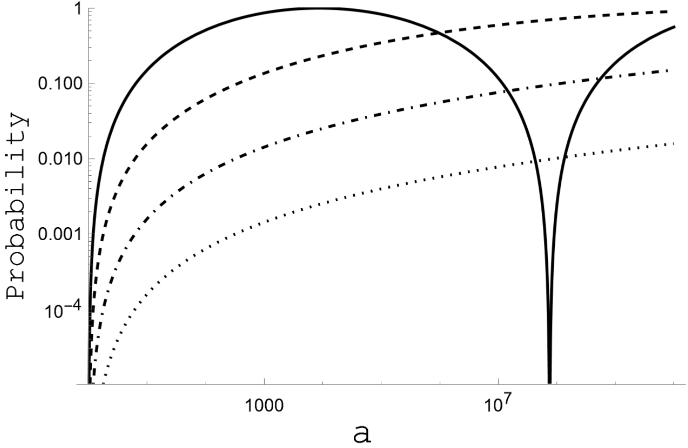
where $\rho_{gD} = \rho^*_{Dg} = R + iI$.

Consider that $\Gamma_D = 0$ (no dark electrons), and we take the initial conditions $\rho_{gg}(a_i) = 1$, $\rho_{DD}(a_i) = 0$, $I(a_i) = 0$, and $R(a_i) = 0$.

Solve this set of equations numerically.

Conversion Probability

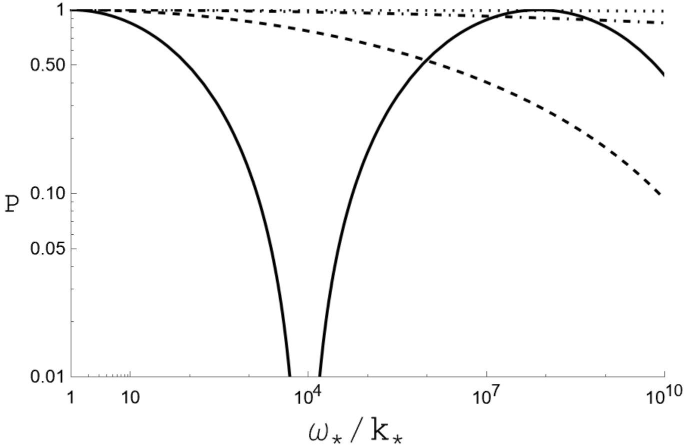
For a small mass and large scalar dark magnetic field,



for $\Omega^r_{\mathrm{DM}} = B^2/(6M_p^2H^2) = 0.01, 0.001, 0.0001, 0.00001$

Conversion Probability

Imprint on gravitational wave spectrum observed today,



for $\Omega^r_{\rm DM} = 0.01, 0.001, 0.0001, 0.00001.$

Conversion Probability

The graviton to photon conversion probability is approximately,

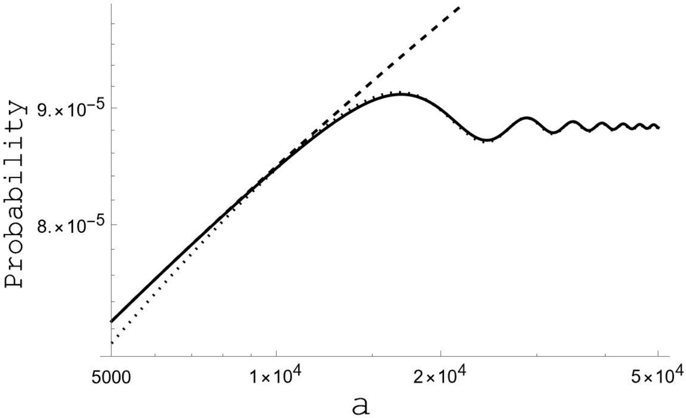
$$P_{g\to\gamma} \simeq \sin^2(\sqrt{\Omega_{\rm DM}}\ln(a))$$

Dependent upon the validity of the assumption $k_* \ll \omega$

This simple behaviour will be cut off when:

- After sufficiently long GW propagation $a_* \simeq k_*/\omega$,
- The mass term of the dark photon becomes important,
- Presence of dark electron density or significant kinetic mixing.

Conversion Probability with Mass Term



Once the mass becomes important, the conversion quickly slows.

Conversion Probability

There exists two distinct regimes, that are well approximated by:

- for $m_{gD} \gg m_D$

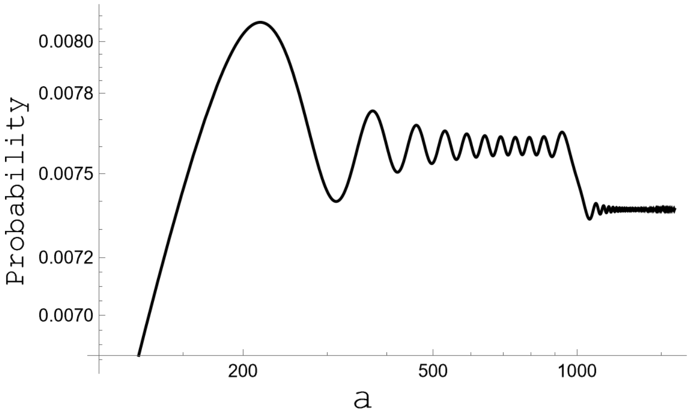
$$P \sim \left[\sin\left(\frac{m_{g\gamma,i}}{H_i}\ln(a)\right)\right]^2 \sim \Omega^r_{\rm DM}\ln(a)^2$$

- for $m_{gD} \ll m_D$,

$$P \sim P_* + \mathrm{Ci}\left(\frac{m^2_{DM}}{3H_i\omega_i}a^3\right)$$

where $P_* \sim \Omega^r_{\rm DM}\ln(\frac{3H_i}{m_D})^2$ and $\mathrm{Ci}(x)$ is the Cosine Integral.

Conversion Probability



In special cases, it is possible to have cancellation between terms in m_D - giving resonance behaviour.

Dark Photon Dark Matter

Dark Photon Dark Matter

If we want the dark photon to also be dark matter:

- ρ_{DM} dilutes as radiation until the characteristic momenta becomes comparable to its mass m_{DM}.
- Then it will become non-relativistic and will instead redshift in a matter-like way.
- The temperature at which this occurs is given by,

$$T_m = m\frac{T_{\rm reh}}{k_*} = m\left(\frac{90}{\pi^2 g_*}\right)^{1/4}\frac{\sqrt{M_p H_{\rm inf}}}{k_*}$$

where we have taken $k_{\rm phys} = k_*\frac{T}{T_{\rm reh}}$ in this case.

Dark Photon Dark Matter

Thus, the required dark photon density at the end of inflation is:

$$\rho_{\rm DM}^{\rm req} = 7 \cdot 10^{57}\ {\rm GeV}^4 \left(\frac{10^{-10}\ {\rm GeV}}{m} \right) \left(\frac{H_{\rm inf}}{10^{12}\ {\rm GeV}} \right)^{5/2}$$

which can be converted to a ratio of the total energy density of the universe ($\rho = 3M_p^2 H_{\rm inf}^2$),

$$\Omega_{\rm DM}^{\rm req} = 0.01 \left(\frac{4 \cdot 10^{-12}\ {\rm GeV}}{m} \right) \left(\frac{H_{\rm inf}}{10^{12}\ {\rm GeV}} \right)^{1/2}$$

if the original dark photons dominate the dark matter today.

Possible to calculate the maximum contribution to the DM from GWs: $\sim 0.5\%$.

Gravitational Wave Sources

This effect could leave an imprint on the gravitational wave spectrum of various sources.

- Evaporation of Primordial Black Holes - high frequency and energy density,
- First order phase transitions,
- Inflationary gravitational waves and Preheating,
- Topological defects,
- Cosmic gravitational wave background from early universe plasma.

Other Phenomenological Implications

Additional Dark Sector Components

- Effects of possible dark fermions are dependent on size of coupling g_D and mass scales.
- Plasma effects strongly suppressed relative to the magnetic field case, preserving the efficient conversion. Possible resonant effects.
- Kinetic mixing to SM photon.
- Additional $U(1)$ fields.
- Inclusion of E_0 component - ratio to B_0 important to determining possible chiral effects and enhancement/suppression.
- The dark $U(1)$ could have Stuekelberg mass or be generated by spontaneous symmetry breaking.

Presence of Dark Fermions

There may exist a dark electron with m_{eD} and dark photon coupling g_D.

Initially ignored them, assuming diluted by inflation or heavy.

- The induced plasma mass is $m_{\rm plasma} = -\frac{\omega_D^2}{\omega} = -\frac{g_D^2 n_{eD}}{m_{eD}\omega}$,
- Can make gauge coupling small, and take a large dark electron mass to minimise the suppression of the oscillation probability.
- Also, contribution from vacuum polarisation: $m_{DQED} = \frac{\alpha_D}{45\pi}\left(\frac{B}{B_c}\right)^2\omega$
- The opposing signs of these two terms (and the dark photon mass term) means a resonance can occur, where efficient oscillations are induced even for small dark magnetic fields.

Kinetic Mixing

There may be a non-zero kinetic mixing with the SM photon $\epsilon F_{\mu\nu} X^{\mu\nu}$.

- If ϵ is large, we can't ignore its interactions with the SM plasma.
- For relevant dark photon masses, we have $\epsilon < 10^{-9}$.
- Thus, $\omega^2_{\mathrm{plasma}} \sim e^2 \epsilon^2 T^2 < 3 \cdot 10^{-19} T^2$, with $\frac{\omega^2_{\mathrm{plasma}}}{\omega} < 0.2$ GeV
- This term may dominate over the dark matter mass, causing an early suppression of the probability - probing ϵ.
- Provides an extra contribution to m_D - possible resonance behaviour.
- Kinetic mixing could also generate a primordial SM magnetic field.

Inclusion of E component

So far, we have considered only the dark magnetic field.

- There may be dark electric fields, depending on the production mechanism and dark sector components.
- Alters the probabilities and features imprinted on the gravitational wave spectrum.
- Example: Three Orthogonal $U(1)$'s in flavour-locked configuration E_0 and B_0, where

$$F^{(i)}_{j0} = E_0 \delta^i_j, \qquad F^{(i)}_{kj} = B_0 \epsilon^{ijk}$$

  for isotropy and homogeneity (2105.08073).

Example Case: Triplet of $U(1)$'s

The equations of motion are,

$$u_\lambda'' + \left(k^2 + 2\frac{B_0^2 - E_0^2}{a^2 M_p^2}\right) u_\lambda = \frac{2}{aM_p}(E_0 v_\lambda' - \lambda k B_0 v_\lambda)$$

$$v_\lambda'' + k^2 v_\lambda = \frac{2}{aM_p}\left(E_0\left(\frac{a'}{a}u_\lambda - u_\lambda'\right) - \lambda k B_0 u_\lambda\right)$$

where $u_\lambda = ah_\pm$ and $v_\lambda = \delta X_\lambda$.

- Oscillatory imprints on gravitational wave spectrums,
- Dependence on relative amplitude of E_0 and B_0 ,
- Induces a tilt for superhorizon modes depending on E_0/B_0 ratio,
- Chiral effects increase with larger E_0.

Conclusion

Dark magnetic fields lead to efficient graviton-photon conversion in the early universe.

- Imprints of gravitational wave sources on the dark matter spectrum.
- Correlated imprints in the gravitational wave spectrum.
- Avoids effects that suppress the conversion probability for ordinary magnetic fields.
- Kinetic mixing between dark photon and SM photon provides experimental tests.
- Signatures of other dark sector components also imprinted in gravitational waves.

Thank You! :)