Resonant emittance mixing of flat beams in plasma accelerators

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Carl B. Schroeder,[3,4] and Maxence Thévenet[1]

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3: Lawrence Berkeley National Laboratory, USA
4: University of California, Berkeley, USA

ALEGRO Workshop
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Linear colliders use flat beams to avoid beamstrahlung

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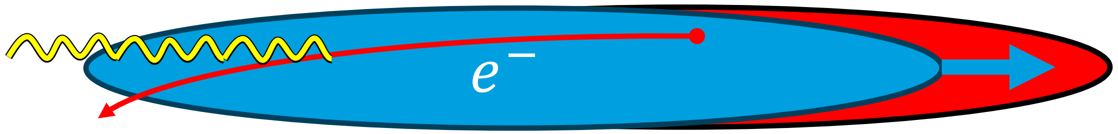

Beams focus each other at IP
Particles are deflected → emit synchrotron radiation, thereby loosing energy
significant energy spread → luminosity spectrum

Beamstrahlung scales with[1] $\sim 1/(\sigma_x + \sigma_y)$

[1] Schroeder et al, JINST 2022

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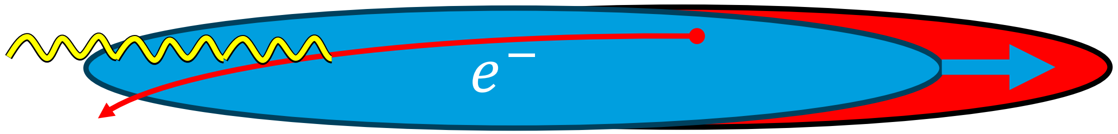

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→ Flat beams with $\sigma_x \gg \sigma_y$ minimize beamstrahlung while allowing for high luminosity

[1] Schroeder et al, JINST 2022
[2] Schulte, RAST 2016

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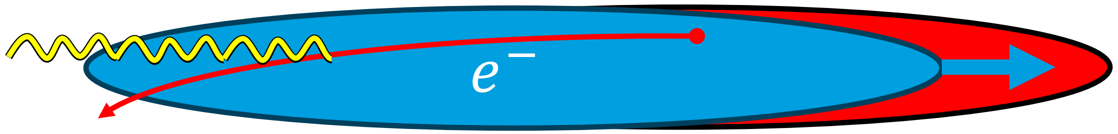

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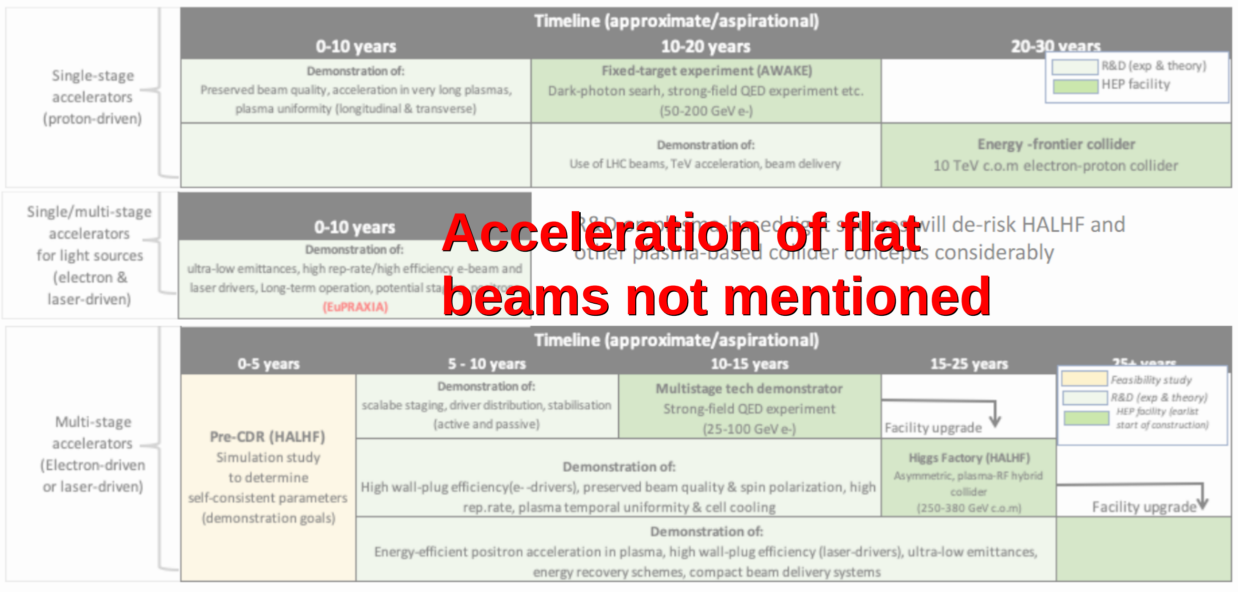
Damping rings produce flat beams automatically
Due to asymmetric focusing of quadrupoles, it is easier to focus flat beams at IP[3]

[1] Schroeder et al, JINST 2022
[2] Schulte, RAST 2016
[3] Raubenheimer, SLAC PUB 1993

Acceleration of flat beams in plasma accel. largely unknown

Key R&D challenges according to ESPP



	Timeline (approximate/aspirational)		
	0-10 years	10-20 years	20-30 years
Single-stage accelerators (proton-driven)	Demonstration of: Preserved beam quality, acceleration in very long plasmas, plasma uniformity (longitudinal & transverse)	Fixed-target experiment (AWAKE) Dark-photon searh, strong-field QED experiment etc. (50-200 GeV e-)	
		Demonstration of: Use of LHC beams, TeV acceleration, beam delivery	Energy -frontier collider 10 TeV c.o.m electron-proton collider

- ▭ R&D (exp & theory)
- 🟩 HEP facility

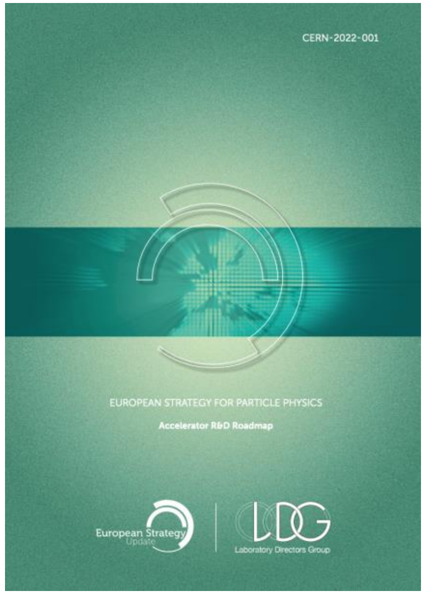
	0-10 years
Single/multi-stage accelerators for light sources (electron & laser-driven)	Demonstration of: ultra-low emittances, high rep-rate/high efficiency e-beam and laser drivers, Long-term operation, potential sta… positron… (EuPRAXIA)

R&D on plasma-based light sources will de-risk HALHF and other plasma-based collider concepts considerably

Acceleration of flat beams not mentioned

	Timeline (approximate/aspirational)				
	0-5 years	5 - 10 years	10-15 years	15-25 years	25+ years
Multi-stage accelerators (Electron-driven or laser-driven)	Pre-CDR (HALHF) Simulation study to determine self-consistent parameters (demonstration goals)	Demonstration of: scalabe staging, driver distribution, stabilisation (active and passive)	Multistage tech demonstrator Strong-field QED experiment (25-100 GeV e-)	Facility upgrade	
		Demonstration of: High wall-plug efficiency(e- -drivers), preserved beam quality & spin polarization, high rep.rate, plasma temporal uniformity & cell cooling		Higgs Factory (HALHF) Asymmetric, plasma-RF hybrid collider (250-380 GeV c.o.m)	Facility upgrade
		Demonstration of: Energy-efficient positron acceleration in plasma, high wall-plug efficiency (laser-drivers), ultra-low emittances, energy recovery schemes, compact beam delivery systems			

- 🟨 Feasibility study
- ▭ R&D (exp & theory)
- 🟩 HEP facility (earlist start of construction)



CERN-2022-001

EUROPEAN STRATEGY FOR PARTICLE PHYSICS

Accelerator R&D Roadmap

Acceleration of flat beams in plasma accel. largely unknown

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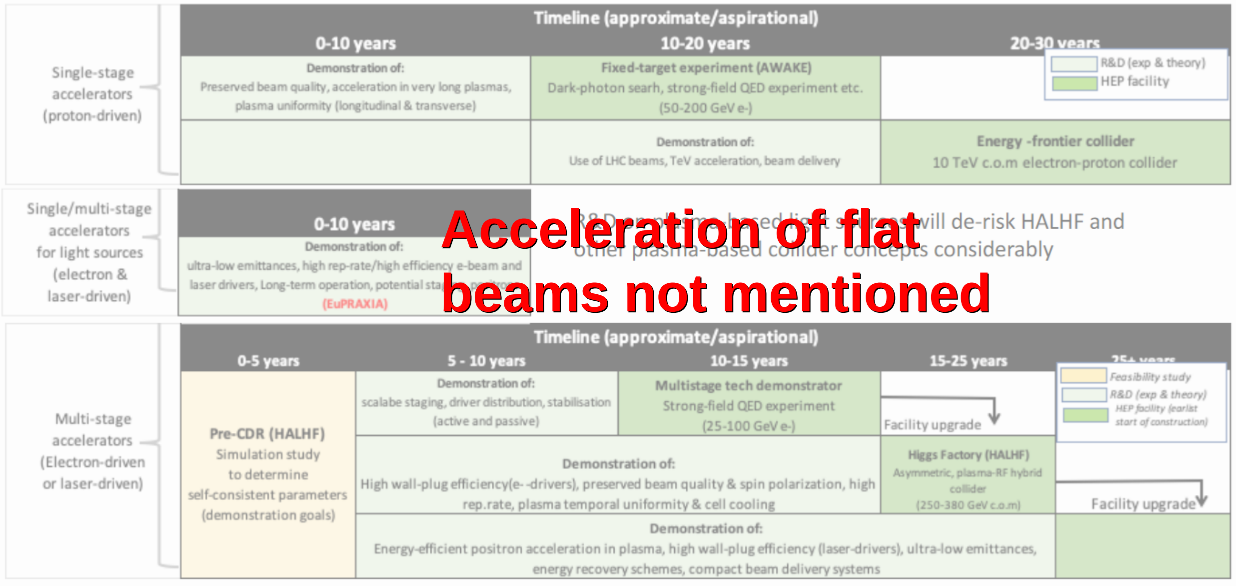

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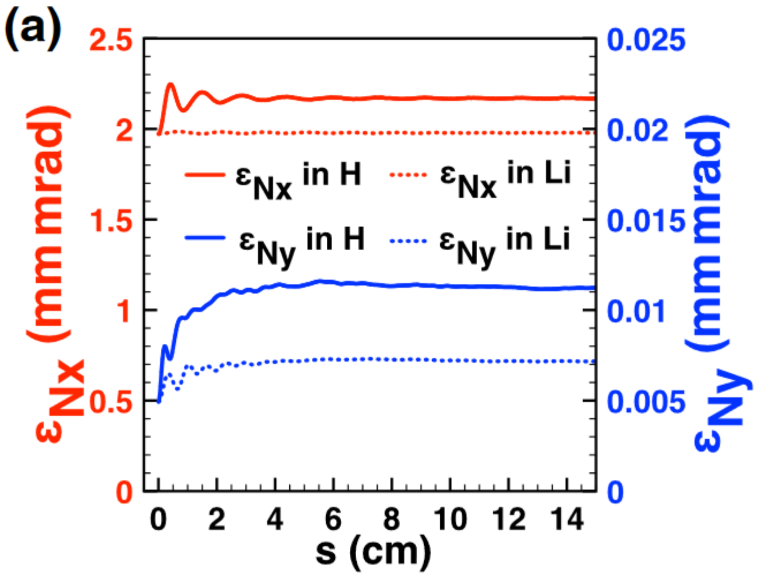
Acceleration of flat beams not mentioned

Acceleration of a flat electron beam in the blowout regime with ion motion[1]



Flat beams are generally not considered a problem in plasma accelerators

[1] An et al, PRL 2017

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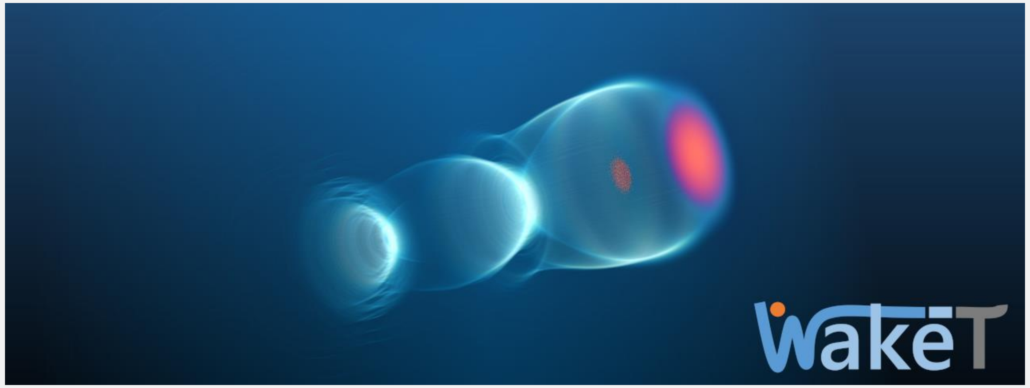
Open-source tools allow for modeling collider-relevant beams

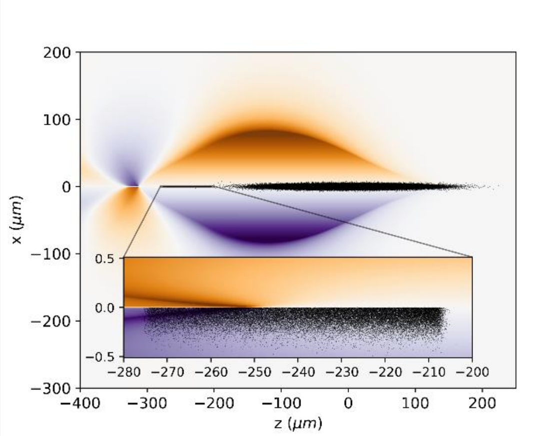
- 2D (axisymmetric) quasistatic code for laser/beam-driven plasma acceleration, incl. ion motion



A. Ferran Pousa et al., J. Phys.: Conf. Ser. (2019)
A. Ferran Pousa, et al. Proc. IPAC23, 1533
https://github.com/AngelFP/Wake-T

HiPACE++

- GPU-capable multi-physics 3D quasistatic particle-in-cell code (laser-driven or beam-driven)
- Collaboration DESY. BERKELEY LAB



S. Diederichs et al., *Comput. Phys. Comm.* 278: 108421 (2022)
https://agenda.infn.it/event/35577/contributions/208606
https://github.com/Hi-PACE/hipace

Let’s model a collider: HALHF

Drive bunch:
Charge: 4.28 nC
Length: (rms): 42 µm
Emittance (x, y): free parameter

Witness bunch:
Charge: 1.6 nC,
Length (rms): 18 µm
Emittance (x, y): 160 µm, 0.54 µm

Plasma:
Argon, ionized to first level
$\mathrm{n}_0 = 7 \times 10^{15}$ cm^{-3}
Length: 5 m (first stage 2.5)
Stages: 16

Foster et al, NJP 2023

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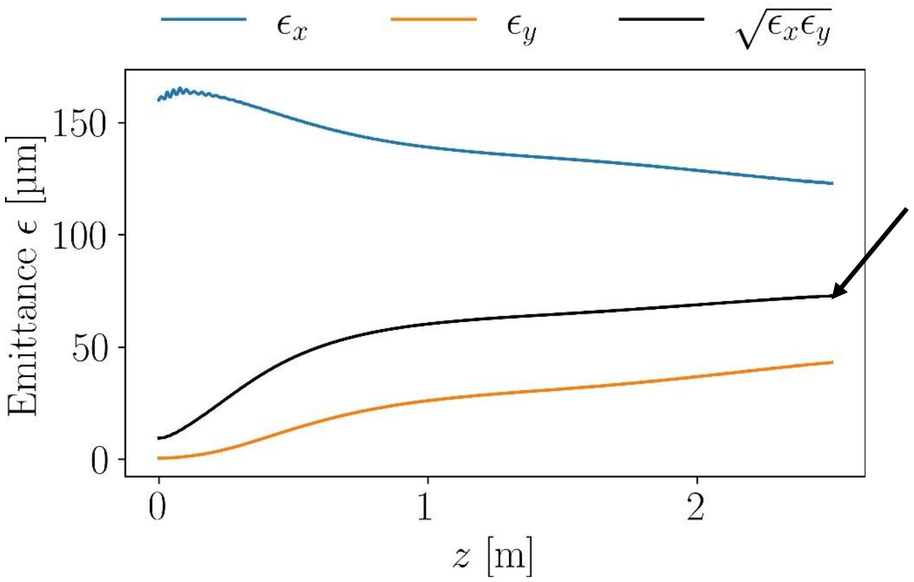
Large emittance: → significant head erosion over 5m

Small emittance: → significant ion motion

trade-off: $\epsilon_{[x,y]} = [60, 60]$ µm

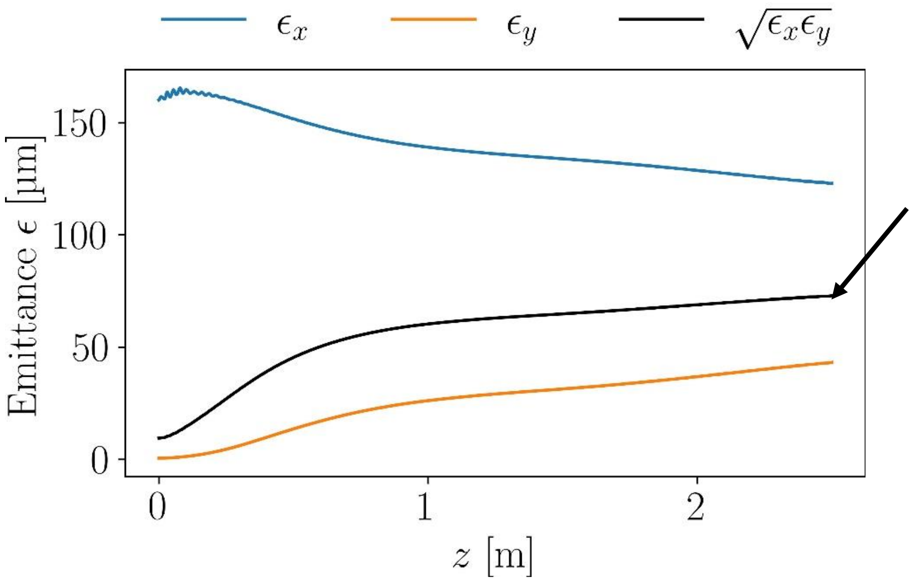
Foster et al, NJP 2023

Drastic emittance growth in argon



Emittance grows
by factor of **7.7!**

Drastic emittance growth in argon due to ionization

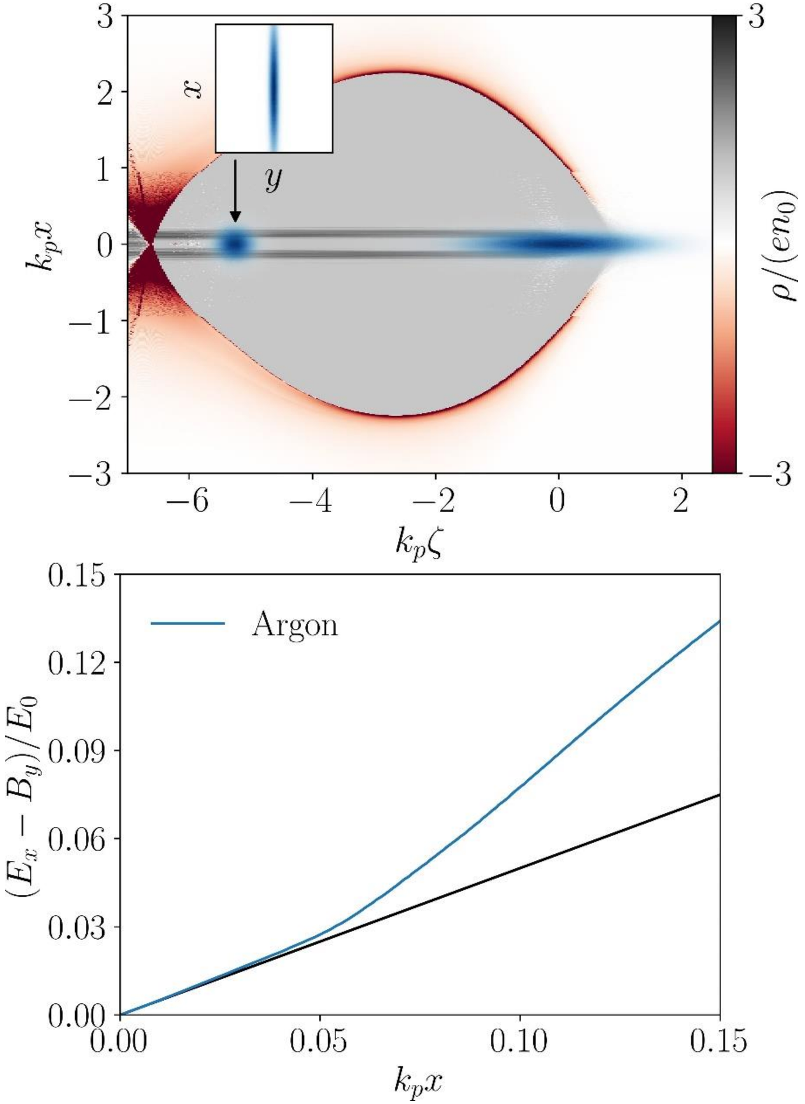


Emittance grows by factor of **7.7!**

Intense space charge fields ionize Argon to higher levels

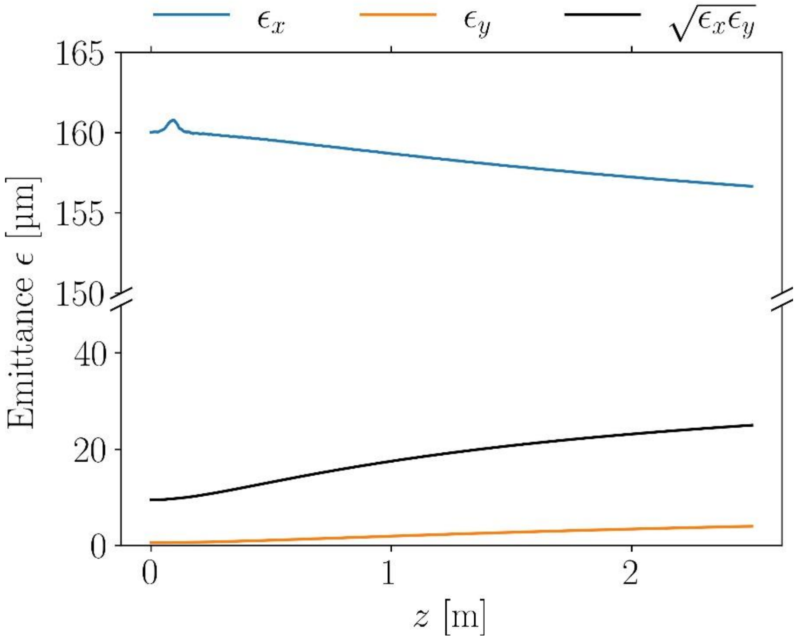
Final witness beam (500 GeV) ionizes Argon to **5th level**

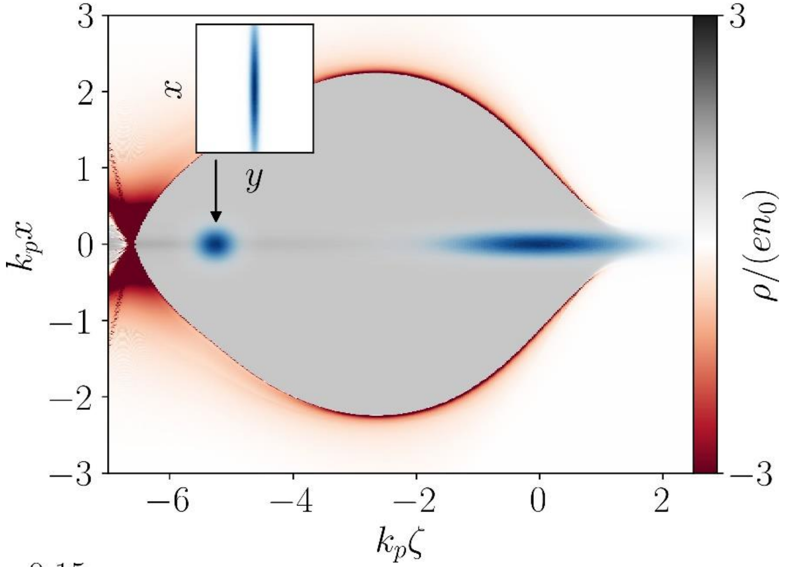
Witness beam must not ionize plasma further, high ionization states also increase ion motion (charge-mass-ratio)

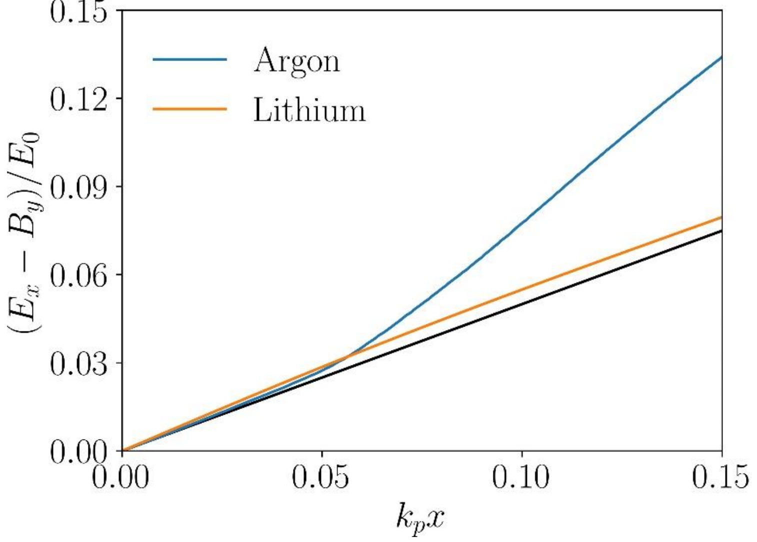


Choice of plasma species: ion motion vs ionization

Using lithium to avoid ionization

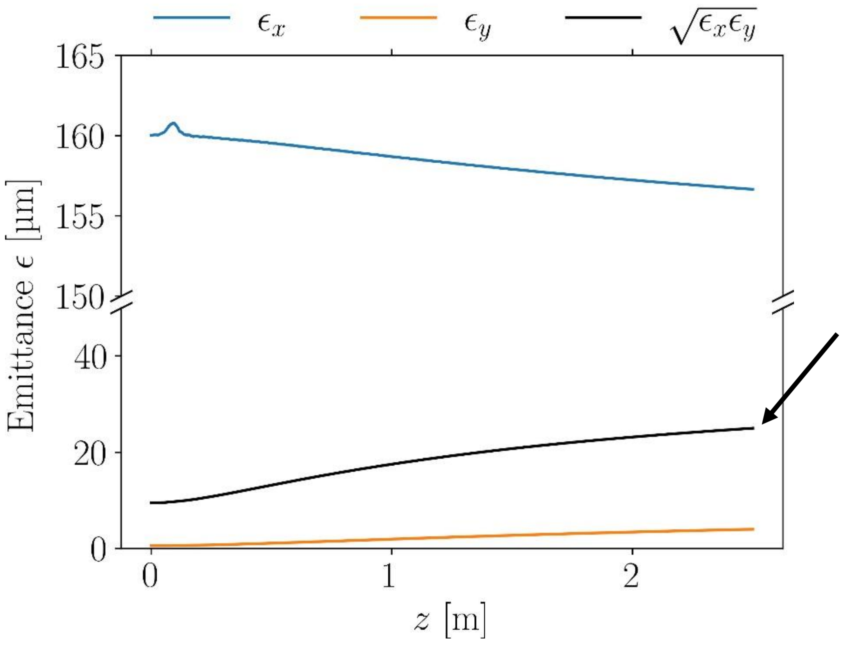






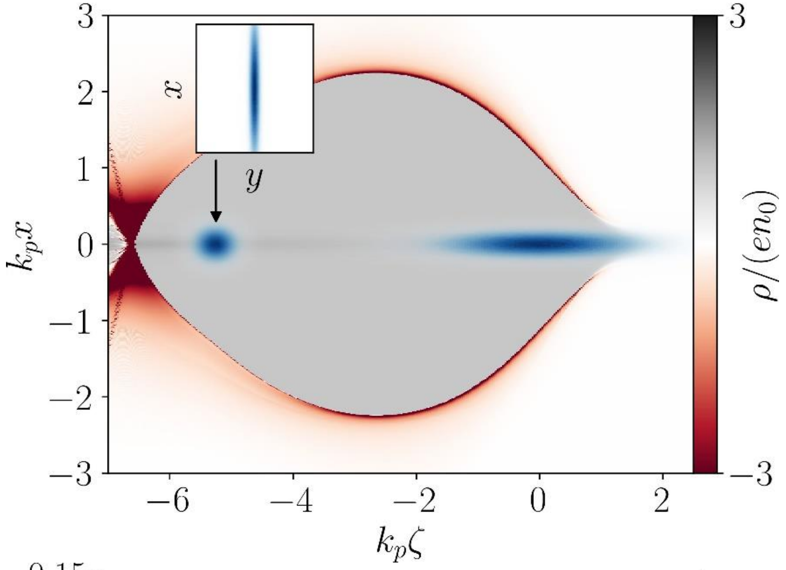
Still significant emittance growth for mild ion motion

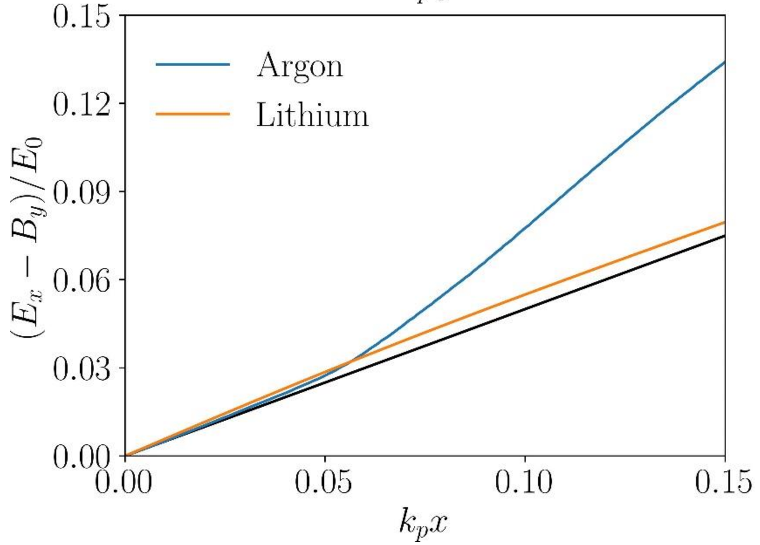
Using lithium to avoid ionization



Emittance grows by factor of **2.6!**

Only mild **drive beam ion motion** present





Ion motion not a problem per se

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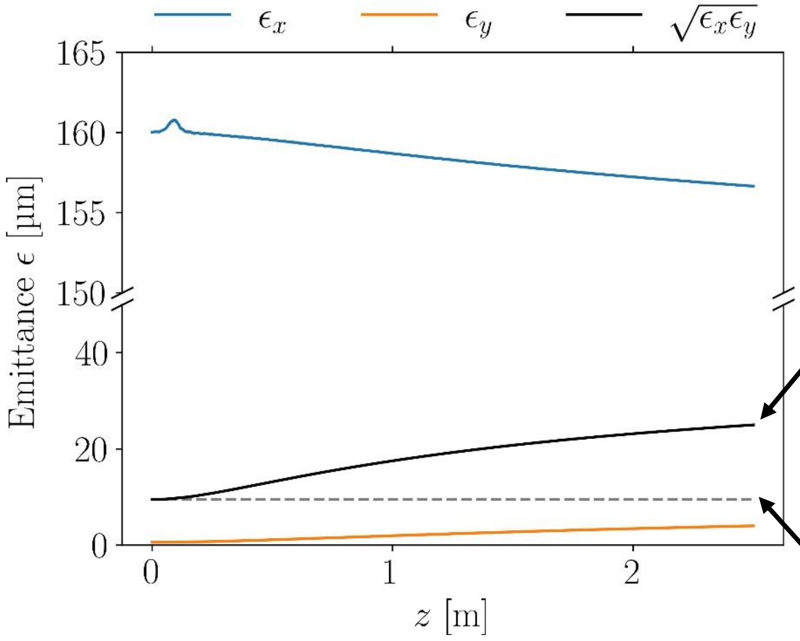
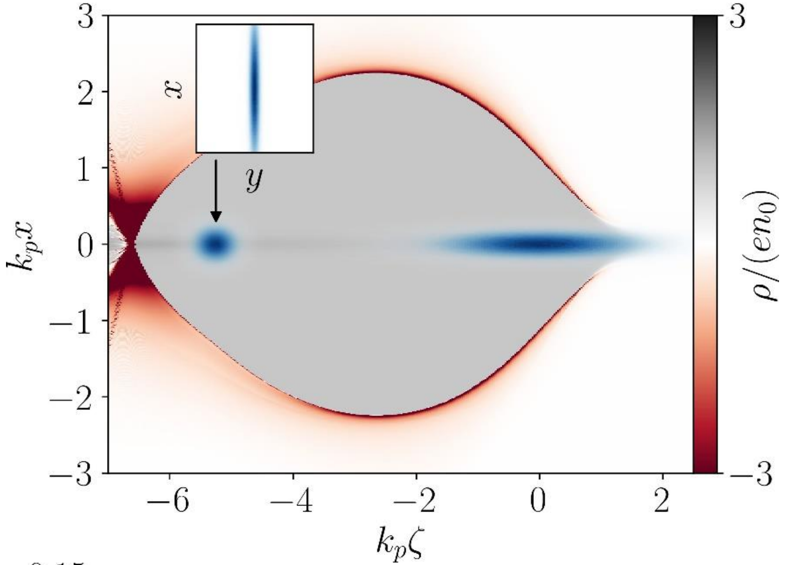
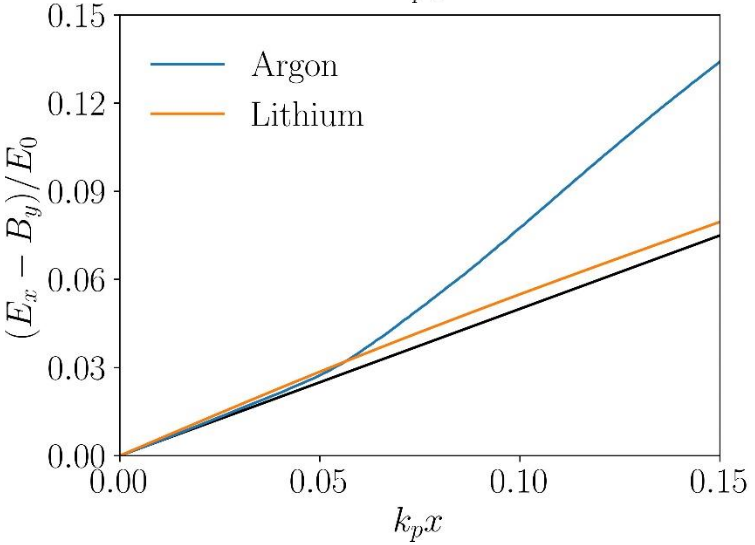


Emittance grows by factor of **2.6!**

Emittance of round witness beam grows by only **0.3%**

Only mild **drive beam ion motion** present





Ion motion leads to *coupled*, nonlinear transverse wakefields

Transverse wakefields for nonrelativistic ion motion

$$\frac{W_r}{E_0} = \frac{k_p r}{2}\left[1 + Z_i \frac{m}{M_i}\frac{n_{b,0}}{n_0}\frac{(k_p\zeta)^2}{2} H\left(\frac{r^2}{2\sigma_x^2}\right)\right]$$

with $H(q) = (1 - e^{-q})/q$

Benedetti et al. PRAB 2017

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Nonlinearity coefficient α

Length scale L

Benedetti et al. PRAB 2017

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approximated to

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Coupled, **nonlinear** wakefields

$r = \sqrt{x^2 + y^2}$

Benedetti et al. PRAB 2017

Emittance mixing depends on nonlinearity strength

3D particle tracking using Wake-T

Drive beam ion motion regime

Initial HALHF witness beam

$\alpha_{[x,y]} = (0, 2.5) \times (0, 2.5)$

$L_{[x,y]} = \sigma_{d,x} = 6\ \mu m$

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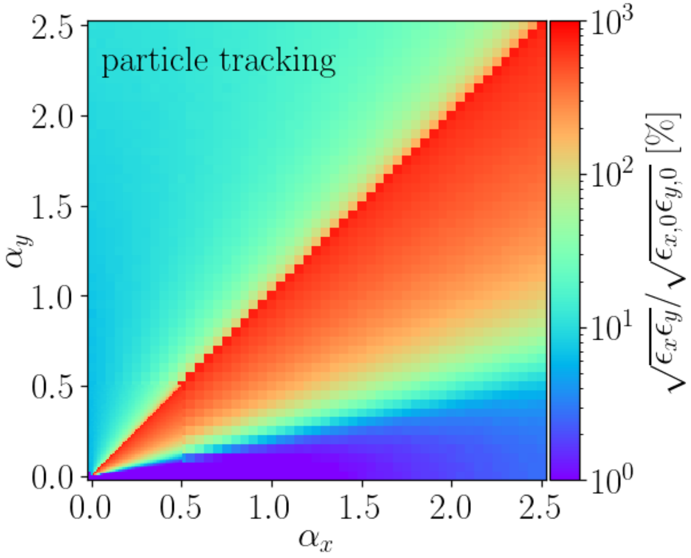

Diederichs et al.
https://arxiv.org/abs/2403.05871

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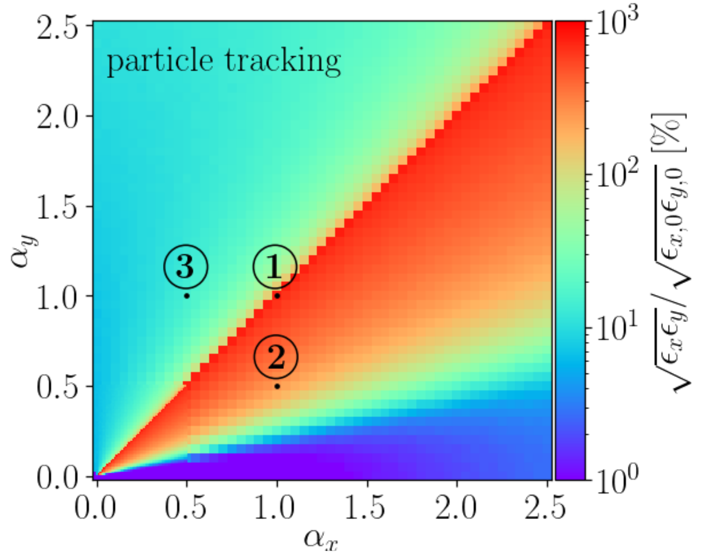

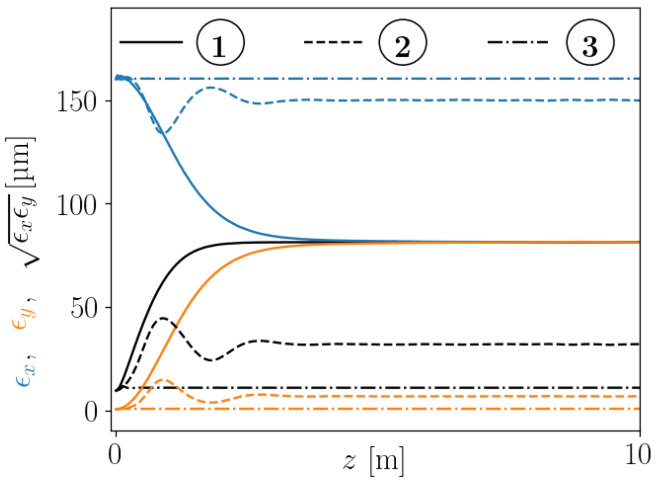

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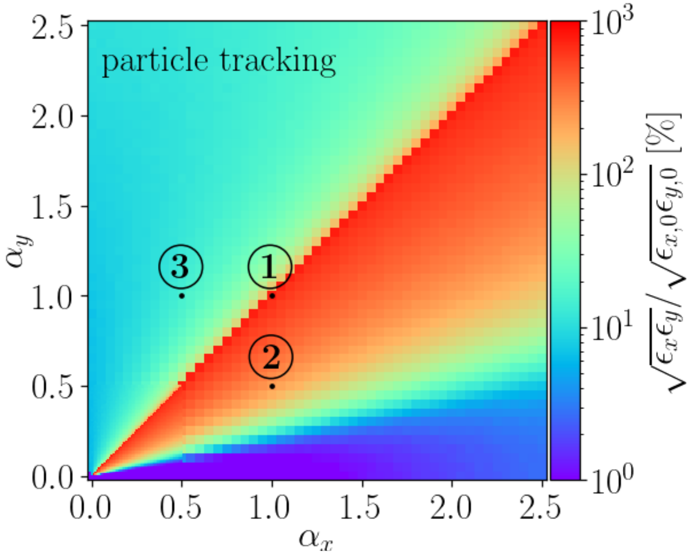

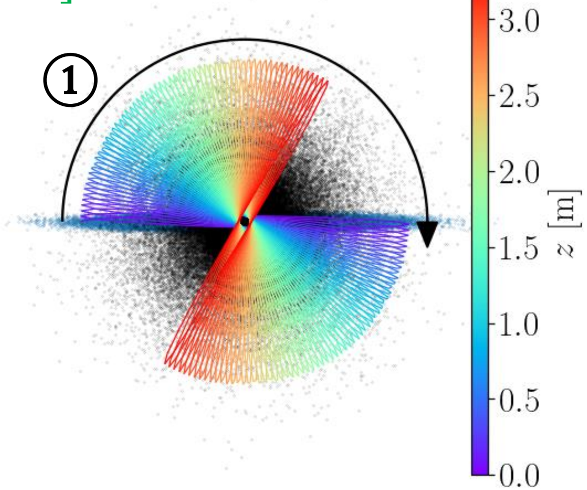

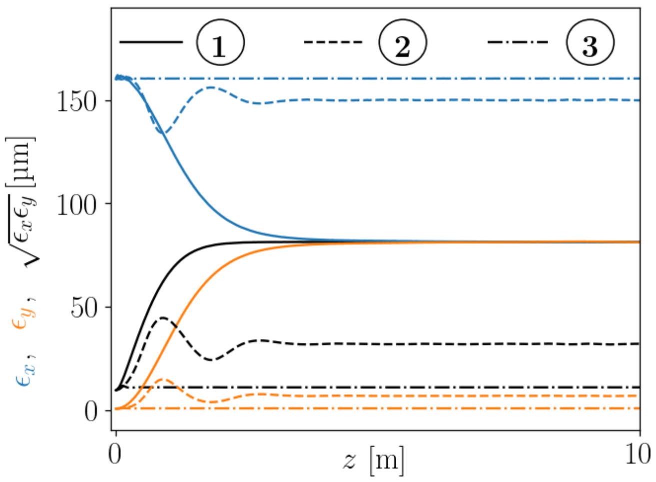

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Emittance mixing occurs when there are resonant particles with (almost) the same betatron period in x and y

Theory based on **action-angle variables**

$$\frac{E_x - B_y}{E_0} = \frac{k_p x}{2}\left[1 + \alpha_x H\left(\frac{r^2}{2L_x^2}\right)\right]$$

$$\frac{E_y + B_x}{E_0} = \frac{k_p y}{2}\left[1 + \alpha_y H\left(\frac{r^2}{2L_y^2}\right)\right]$$

Resonant particles: $k_{\beta,x} \simeq k_{\beta,y}$

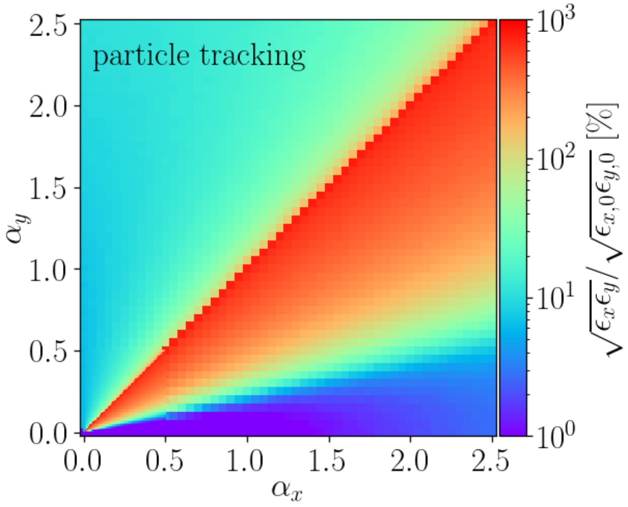
Emittance at saturation:

$$\epsilon^*_{w,x} \simeq (1-\eta_r)\epsilon_{w,x} + \frac{1}{2}\eta_r j^{(r)}_{x,0} \longleftarrow$$

Average action of resonant particles

$$\epsilon^*_{w,y} \simeq (1-\eta_r)\epsilon_{w,y} + \frac{1}{2}\eta_r \frac{\alpha_y}{\alpha_x}\frac{L_x^2}{L_y^2} j^{(r)}_{x,0}$$

Fraction of resonant particles



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$\uparrow$

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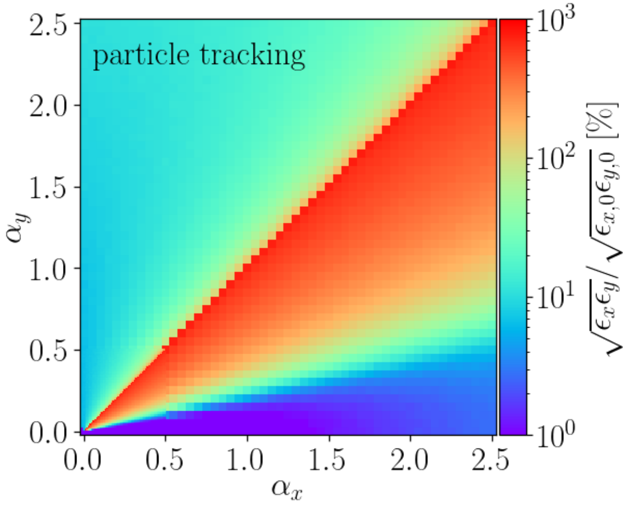

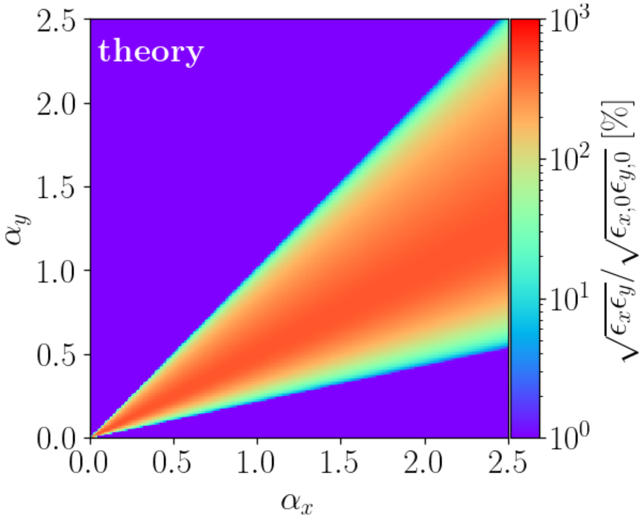

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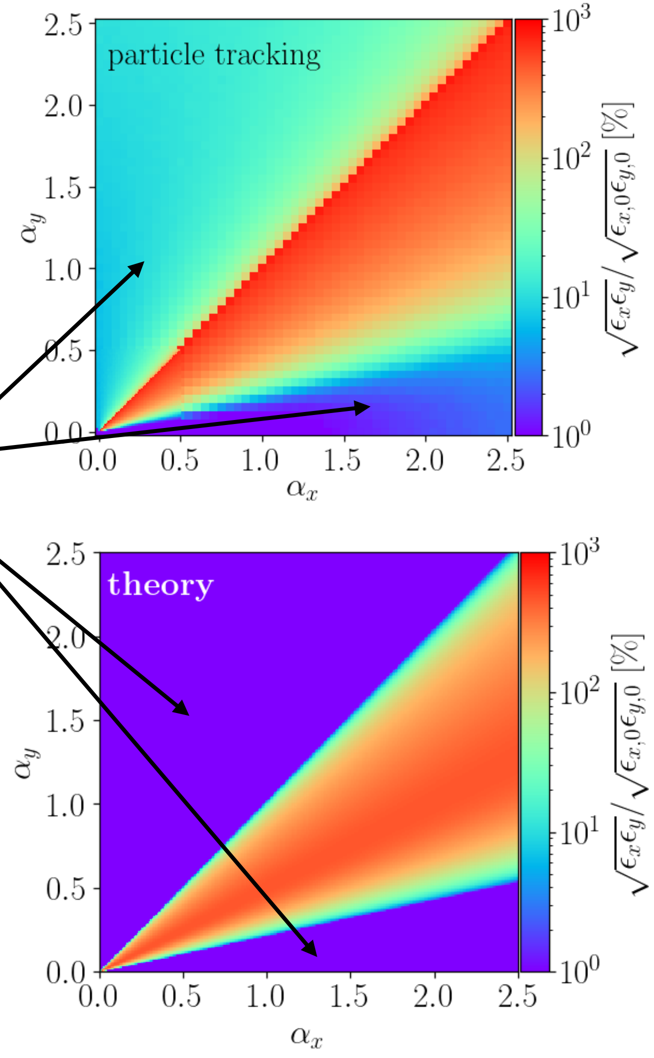
↑

Fraction of resonant particles

Safe parameter space without mixing



Flat drive beams break the resonance and prevent mixing in the first 6 stages

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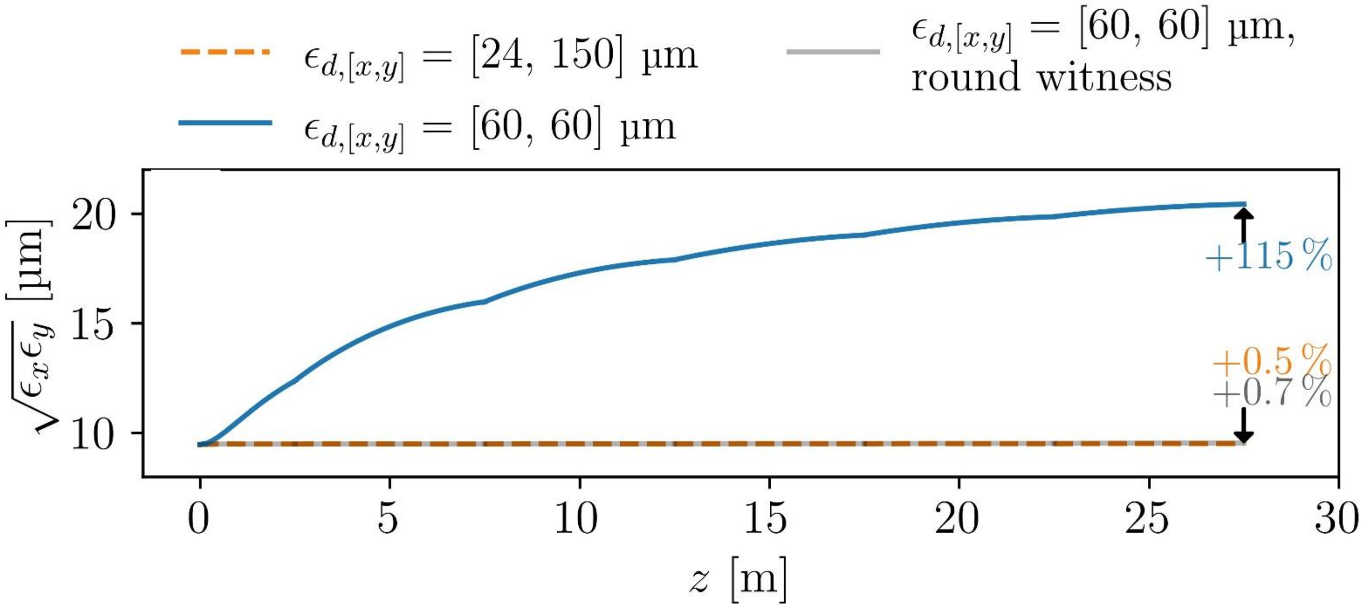
Length: (rms): 42 µm
Emittance (x, y): 60 µm 60 µm
24 µm 150 µm

Witness bunch:
Charge: 1.6 nC,
Length (rms): 18 µm
Emittance (x, y): 160 µm, 0.54 µm

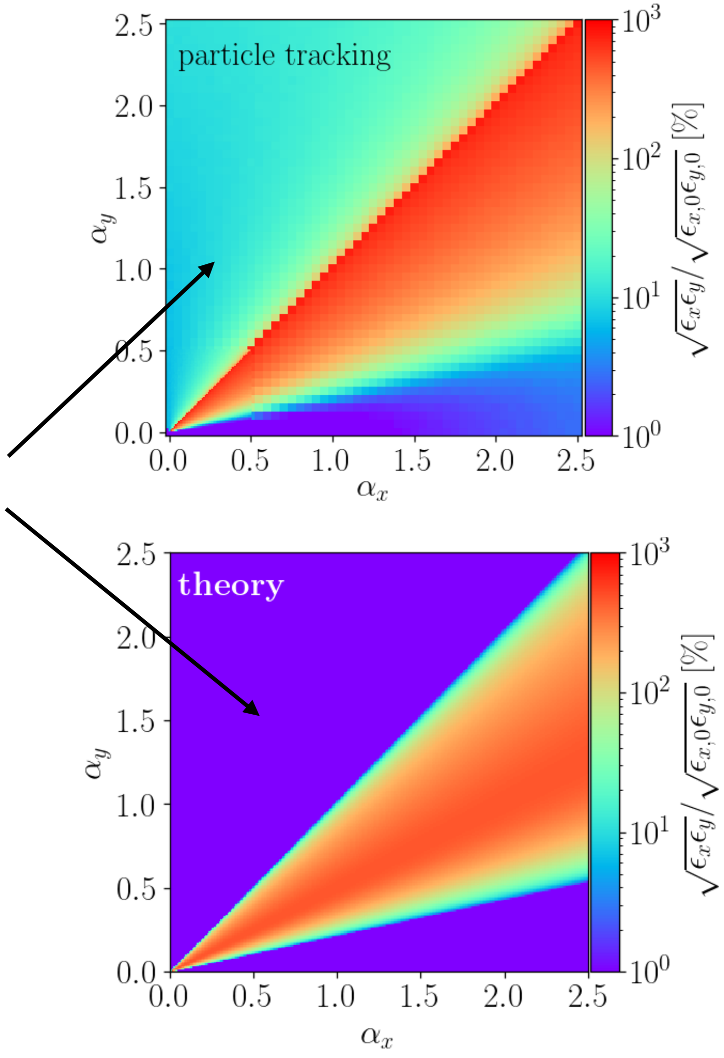
Plasma:
Sodium, ionized to first level
$n_0 = 7 \times 10^{15}$ cm^{-3}
Length: 5 m (first stage 2.5)
Stages: 6, no interstage modeling



Are flat beams always in the safe parameter space?

Flat beams with $\sigma_x \gg \sigma_y$ should cause ion motion with $\alpha_x \ll \alpha_y$ and $L_x > L_y$

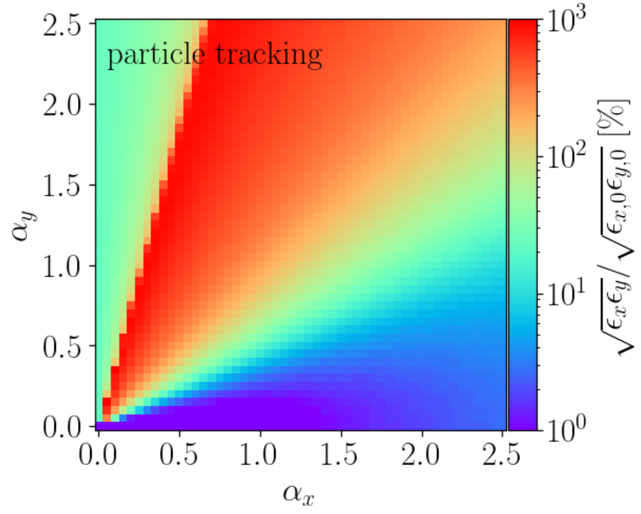
Safe parameter space without mixing



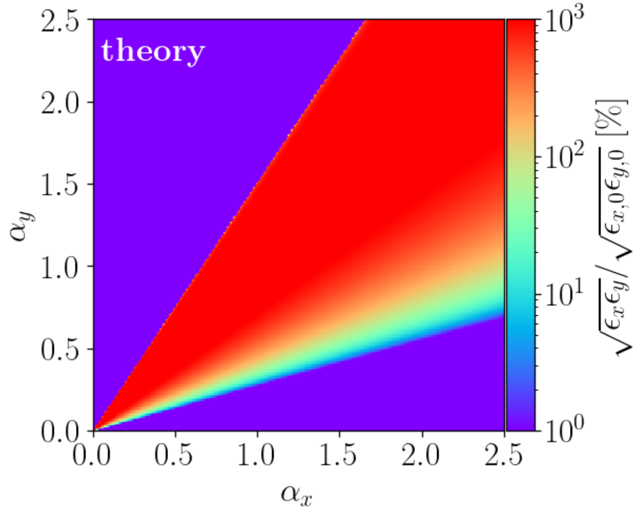
Are flat beams always in the safe parameter space?

Particle tracking using Wake-T

Witness beam ion motion regime
Initial HALHF witness beam
$\alpha_{[x,y]} = (0, 2.5) \times (0, 2.5)$
$L_x = \sigma_{d,x} = 6\ \mu m$
$L_y = 0.5\ L_x = 3\ \mu m$



$\boldsymbol{\alpha_{[x,y]}}$ **and** $\boldsymbol{L_{[x,y]}}$ **cannot be choosen freely** in a real accelerator



Also, feedback of witness beam on ions neglected

Self-consistent PIC simulations required

Witness beam ion motion regime exhibits some emittance mixing

Drive bunch:
Charge: 4.28 nC
Length: (rms): 42 µm
Emittance (x, y): 2.8 mm
rigid

Witness bunch:
Charge: 1.6 nC,
Length (rms): 18 µm
Emittance (x, y): 5 µm, 35 nm

Plasma:
Hydrogen
$\mathrm{n}_0 = 7 \times 10^{15}$ cm^{-3}
Length: 77.5 m

Witness beam ion motion regime exhibits some emittance mixing

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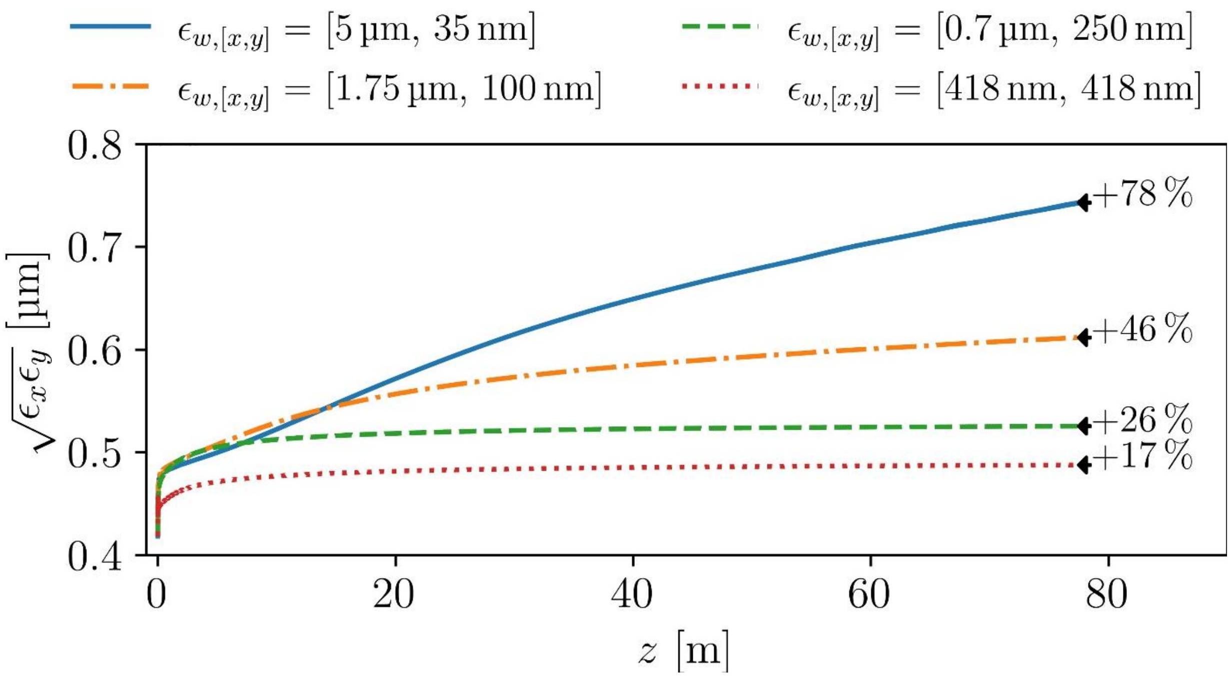


Recap: two different relevant regimes

1. Axisymmetric, coupled, nonlinear wakefields can cause severe emittance mixing:

e.g., due to drive beam ion motion, ionization, ~~parabolic plasma channels~~

This symmetry must be avoided! Flat drivers, laser drivers

2. **Non-axisymmetric, coupled,** nonlinear wakefields still cause some degree of emittance mixing

e.g., due to witness beam ion motion

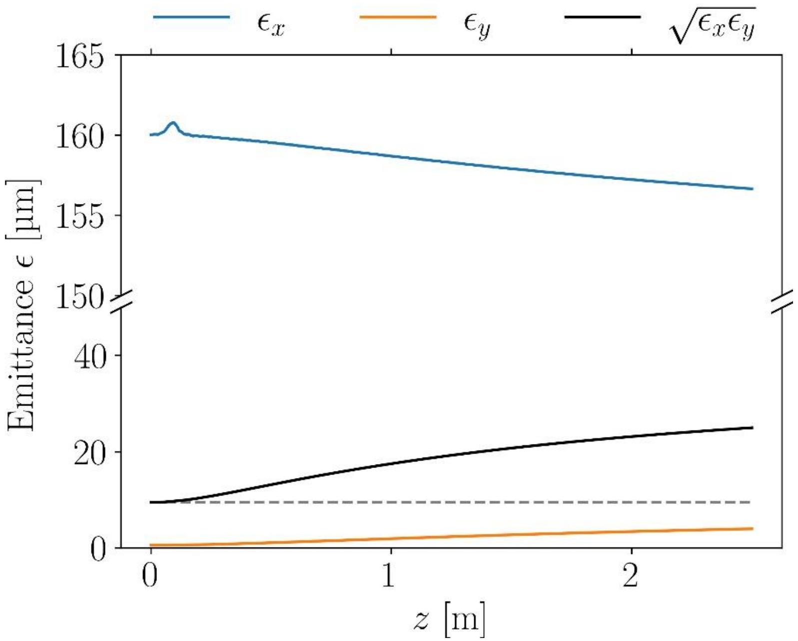
Effect must be considered in design

Identification of optimal parameter space needs to be addressed

Hosing needs to be re-evaluated for flat beams

Preliminary results

Drive beam ion motion regime:
First stage of HALHF, lithium, round driver

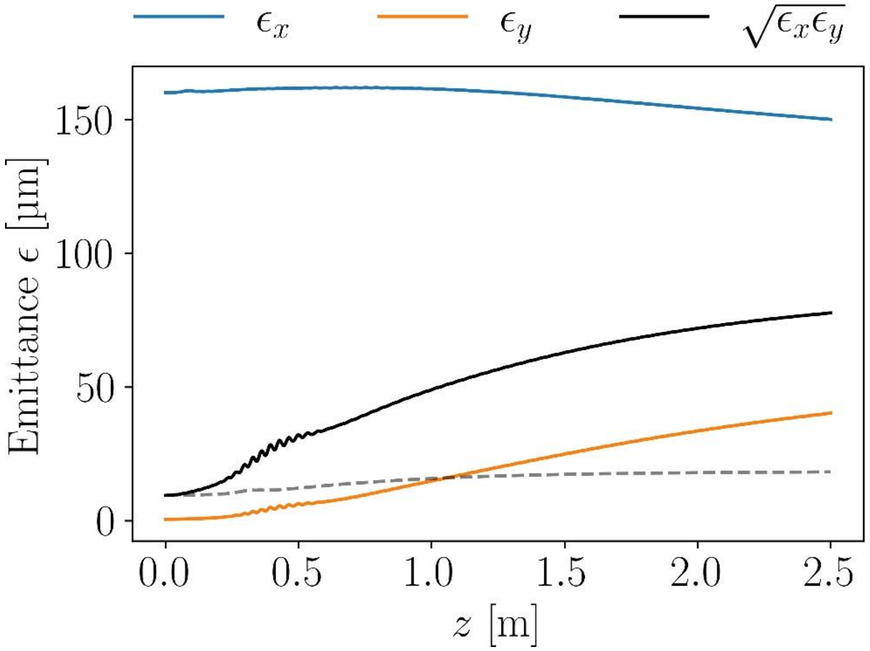


Hosing needs to be re-evaluated for flat beams

Preliminary results

Drive beam ion motion regime:
First stage of HALHF, lithium, round driver

Offset in x and y by 1µm



Hosing needs to be re-evaluated for flat beams

Preliminary results

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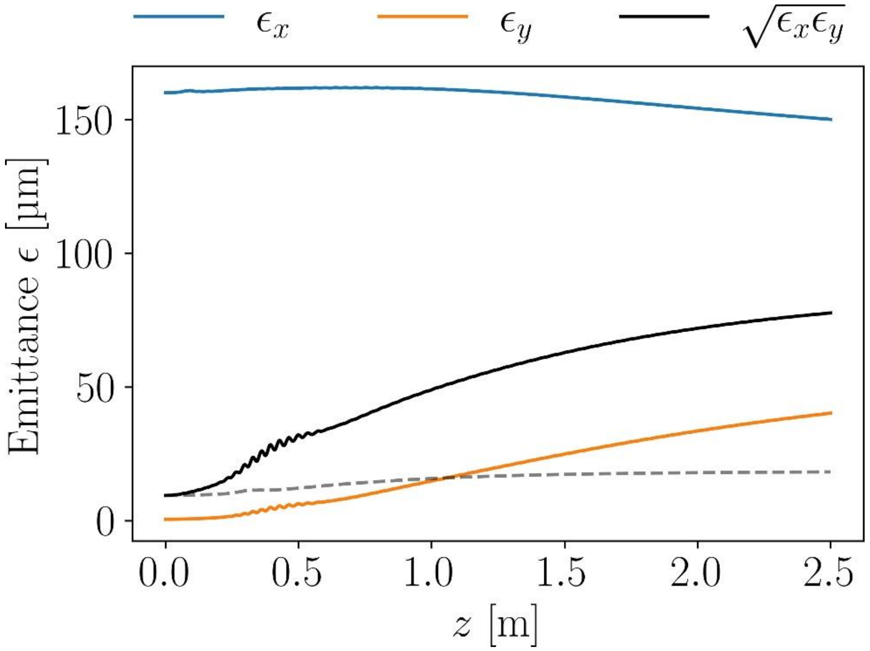
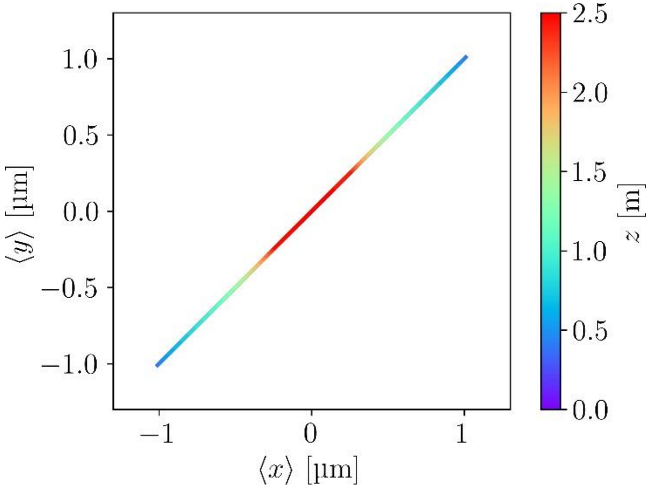
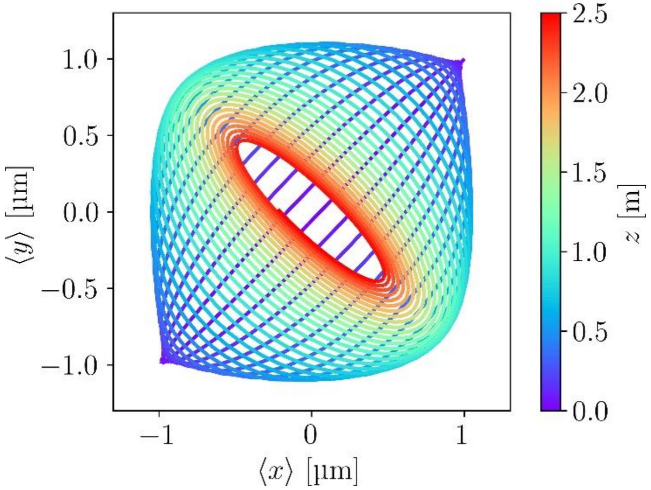


Misaligned flat beams exhibit collective **angular momentum**!

Round witness beam

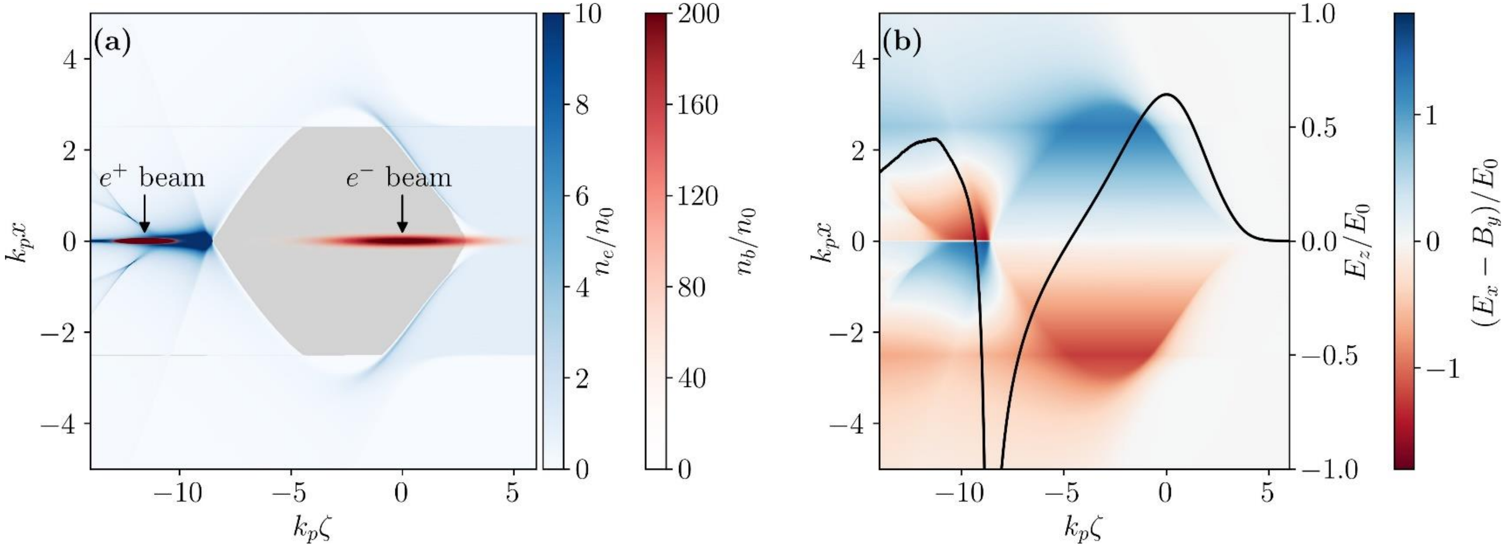


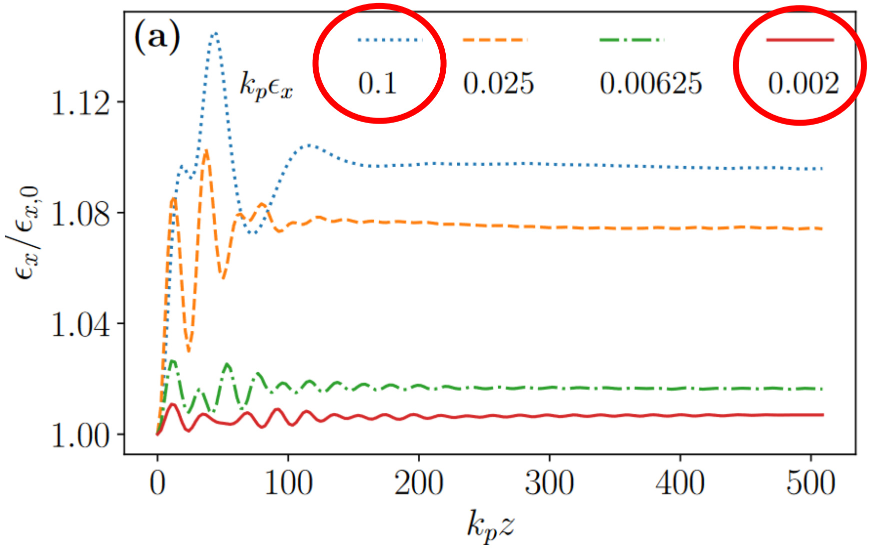
Flat witness beam



Almost all plasma-based positron acceleration schemes rely on coupled, nonlinear transverse wakefields

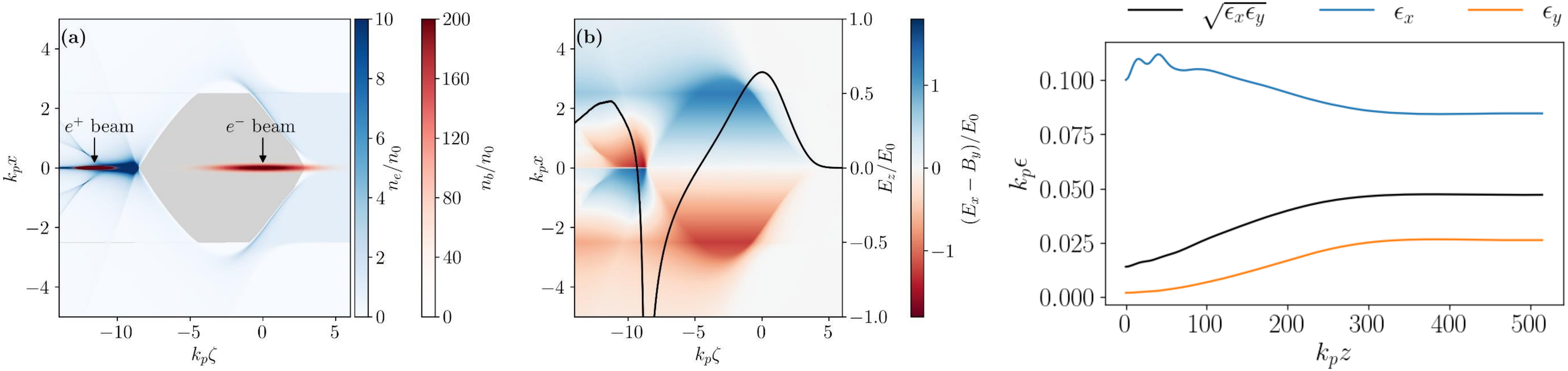
Positron acceleration in a plasma column enables emittance preservating acceleration of round beams





Almost all plasma-based positron acceleration schemes rely on coupled, nonlinear transverse wakefields

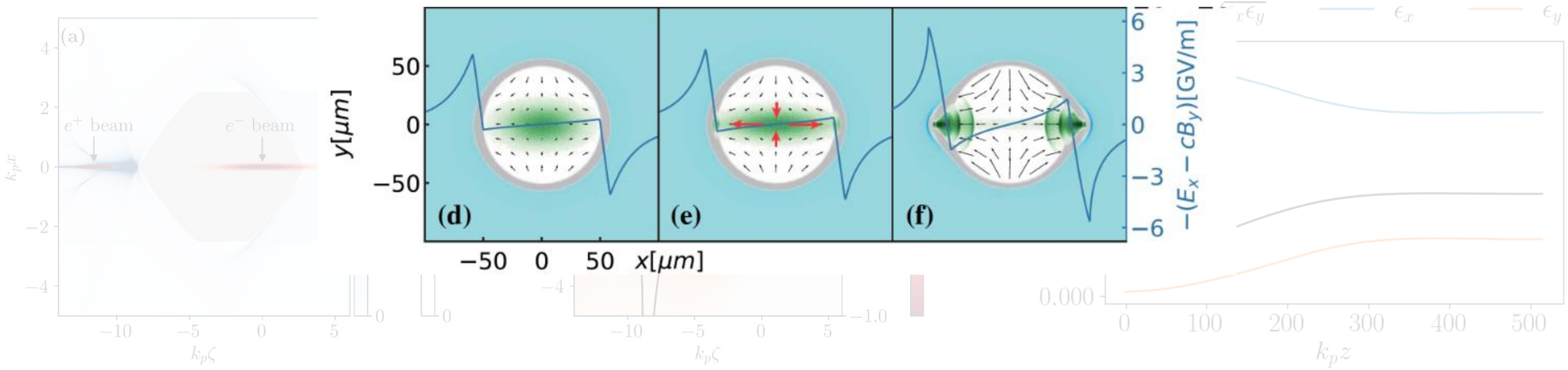
Positron acceleration in a plasma column enables ~~emittance preservating acceleration~~ of ***flat beams***



Only **hollow core plasma channels** do not exhitib coupled nonlinear transverse wakefields,
all other positron acceleration schemes do!

Almost all plasma-based positron acceleration schemes rely on coupled, nonlinear transverse wakefields

Positron acceleration in a plasma column enables emittance preservating acceleration of round beams



Flat beams in hollow core plasma channels induce **quadrupole field**[1]

Acceleration of flat beams needs to be evaluated with detailed modeling

[1] Zhou et al, PRL 2021

Acceleration of flat beams in plasmas is nontrivial

***Coupled*, nonlinear transverse wakefields lead to emittance exchange** for resonant particles with the same betatron frequency in x and y

Resonance can be mitigated by flat drivers or laser drivers

Coupled, nonlinearities are must be considered for collider relevant parameters:

→ possible better parameter spaces?

→ plasma element important: *ion motion* vs *ionization*

Hosing needs to be reconsidered. Coupling of transverse planes increases emittance growth

Positron acceleration of flat beams even more challenging in plasmas

Flat beams in **hollow core plasma channels** induce **quadrupole moment**

Round beams fully avoid emittance mixing

Diederichs et al.
https://arxiv.org/abs/2403.05871

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Jean-Luc Vay

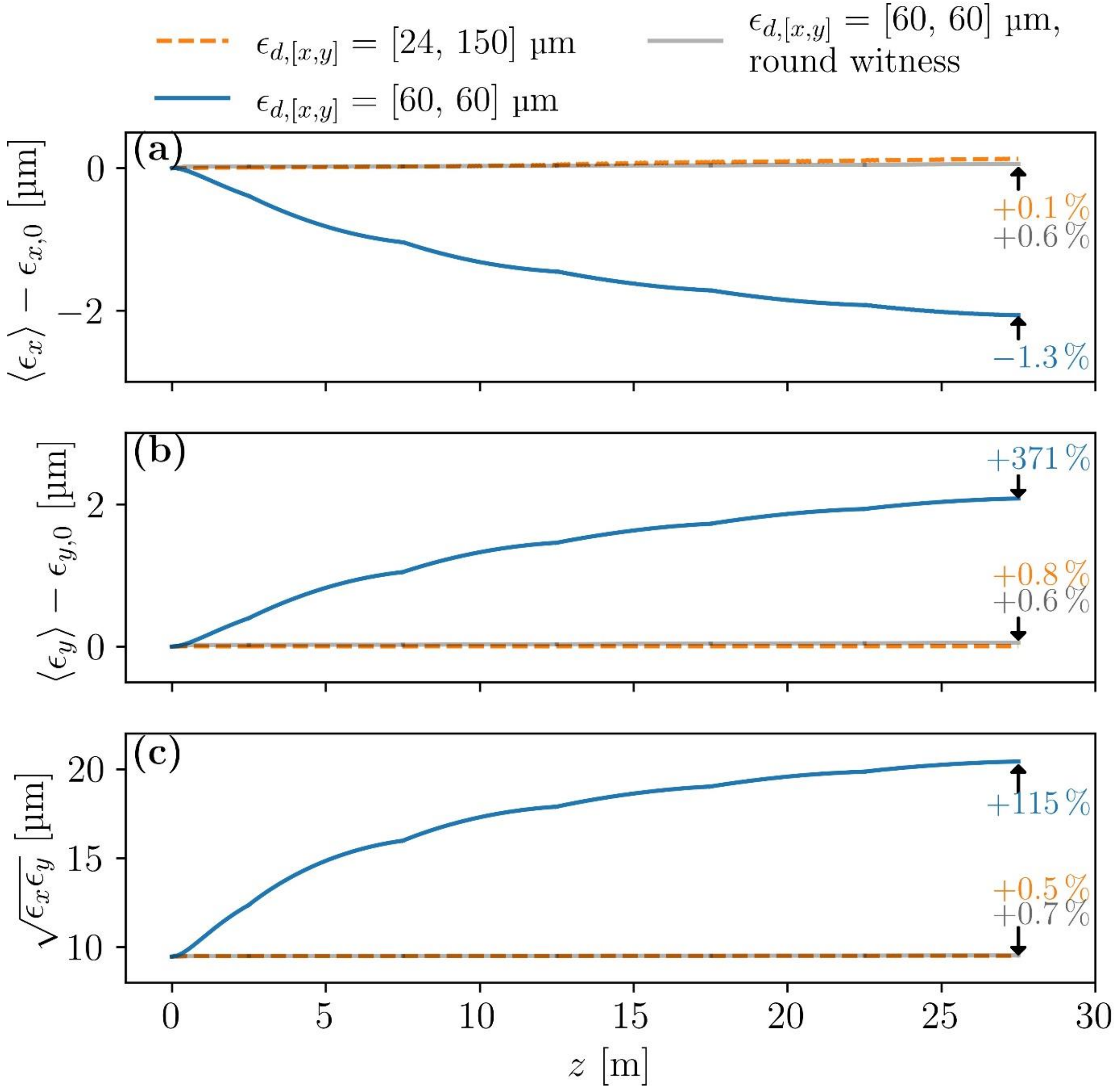
Backup

Flat drive beams break the resonance and prevent mixing in the first 6 stages

Drive bunch:
Charge: 4.28 nC
Length: (rms): 42 µm
Emittance (x, y): 60 µm 60 µm
24 µm 150 µm

Witness bunch:
Charge: 1.6 nC,
Length (rms): 18 µm
Emittance (x, y): 160 µm, 0.54 µm

Plasma:
Sodium, ionized to first level
$n_0 = 7 \times 10^{15}$ cm^{-3}
Length: 5 m (first stage 2.5)
Stages: 6, no interstage modeling



Witness beam ion motion regime exhibits some emittance mixing

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Plasma:
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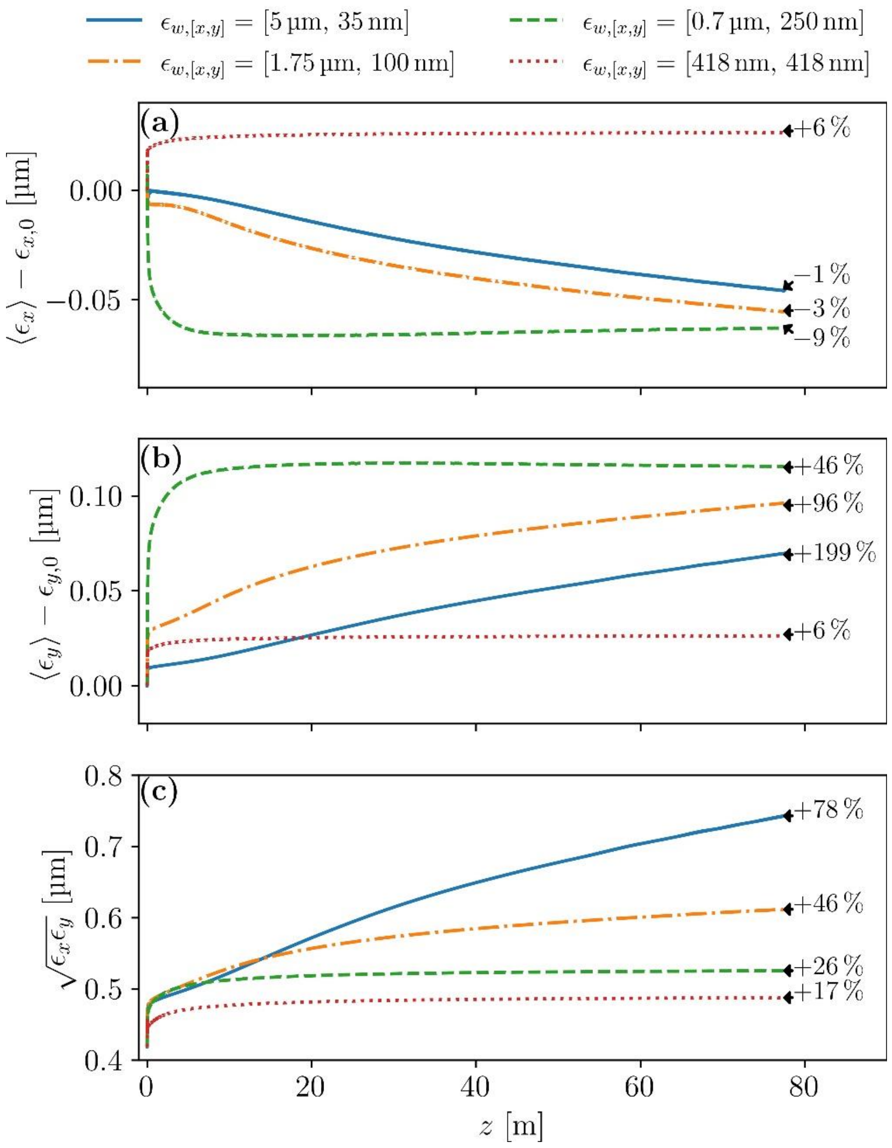
Length: 77.5 m

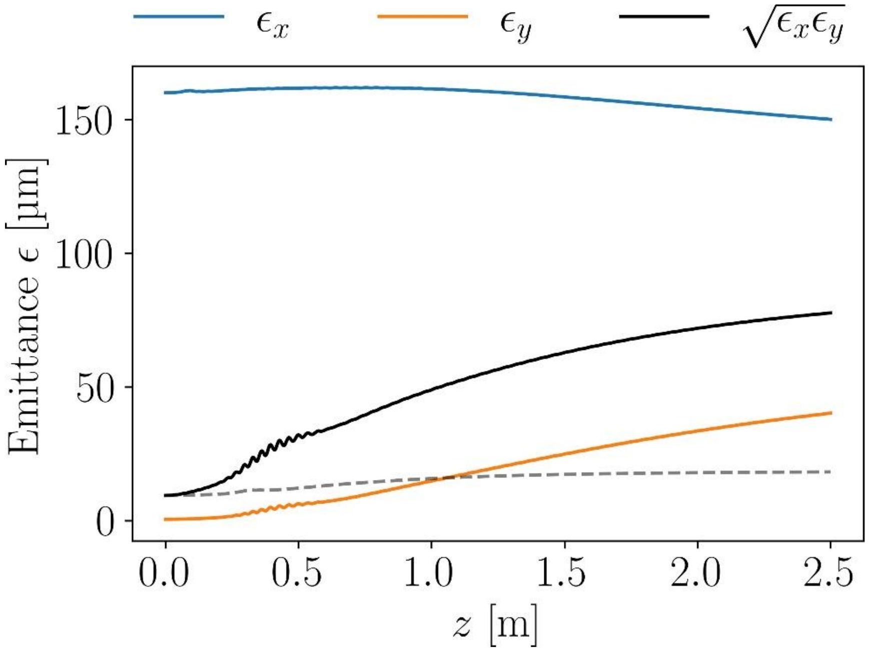


Hosing needs to be re-evaluated for flat beams

Preliminary results

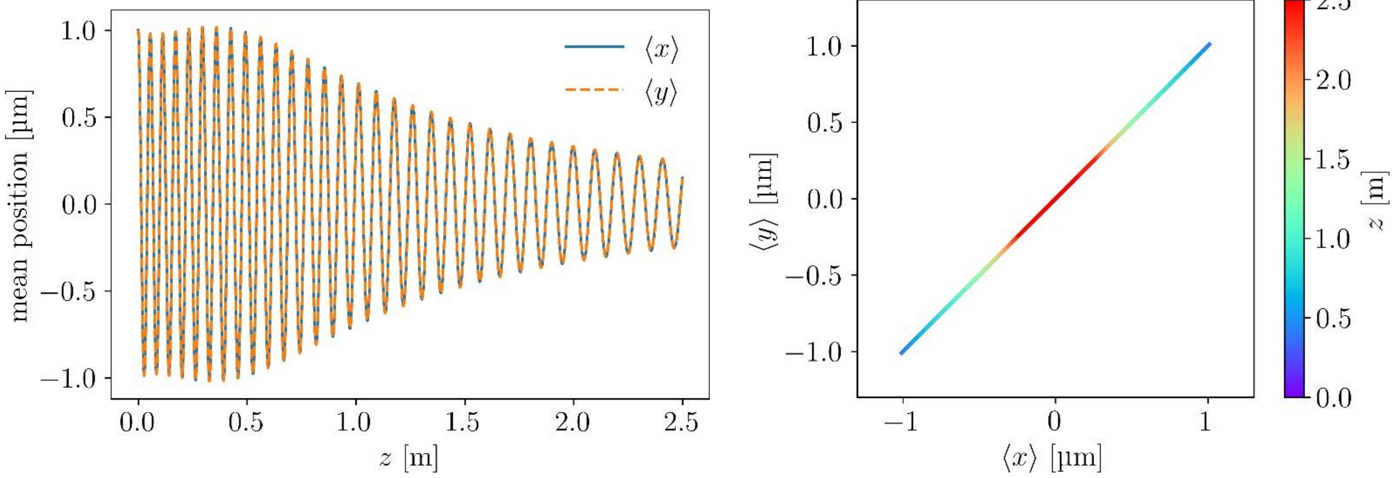
Drive beam ion motion regime:
First stage of HALHF, lithium, round driver

Offset in x and y by 1µm

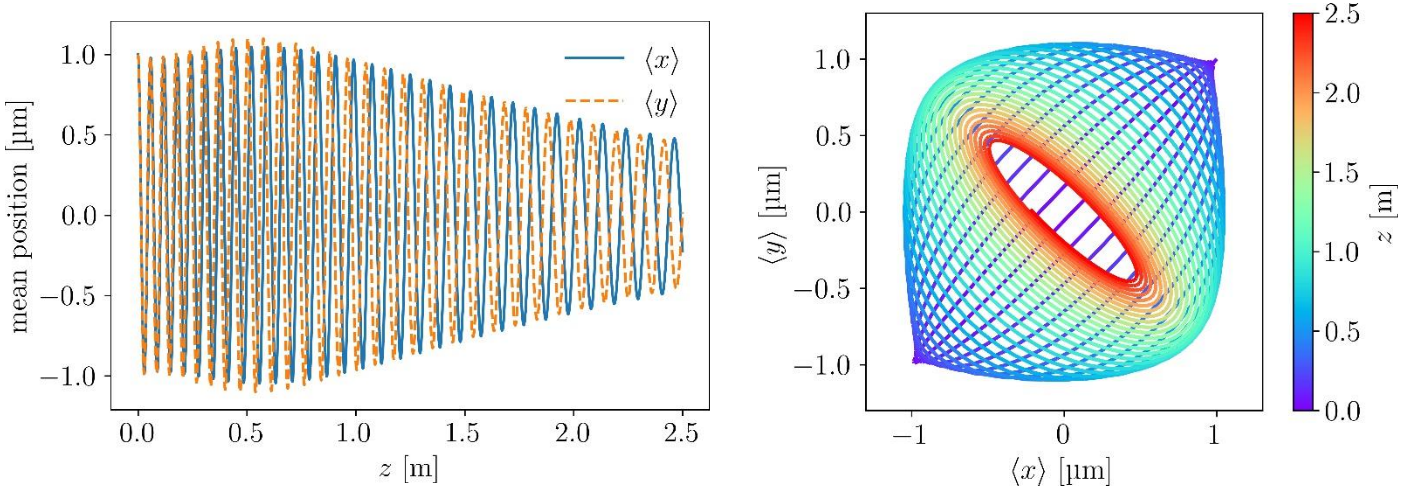


Misaligned flat beams exhibit collective **angular momentum**!

Round witness beam



Flat witness beam

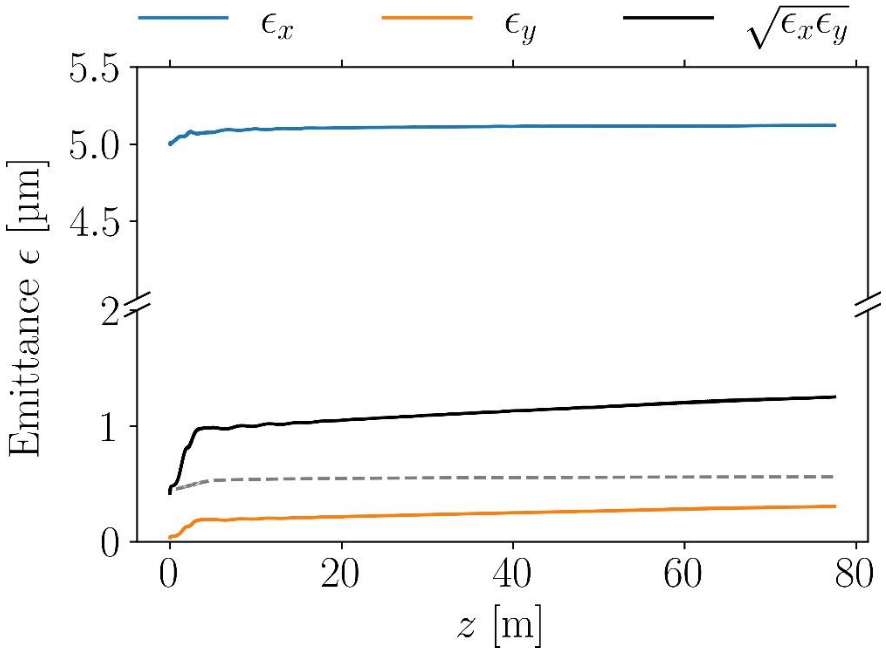


Hosing needs to be re-evaluated for flat beams

Preliminary results

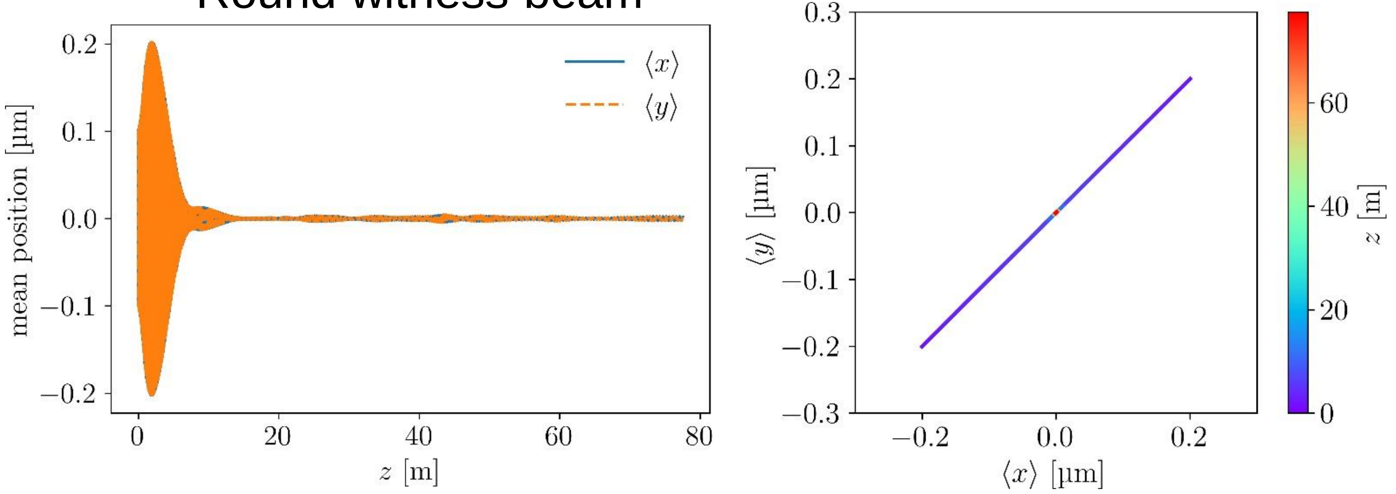
Witness beam ion motion regime:
77.5 m stage, hydrogen, rigid driver
$\epsilon_{[x,y]} = [5\ \mu m,\ 35\ nm]$

Offset in x and y by 0.1µm

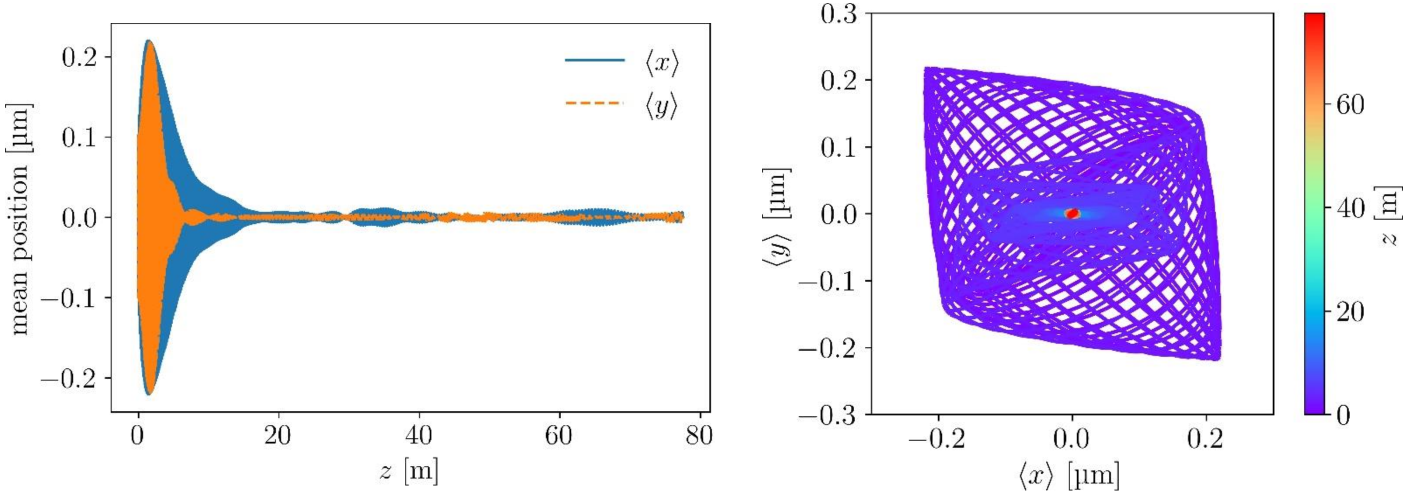


Hosing is strongly damped by strong ion motion[1]

Round witness beam



Flat witness beam



[1] Mehrling et al, PRL 2018