Needs and challenges in electroweak physics

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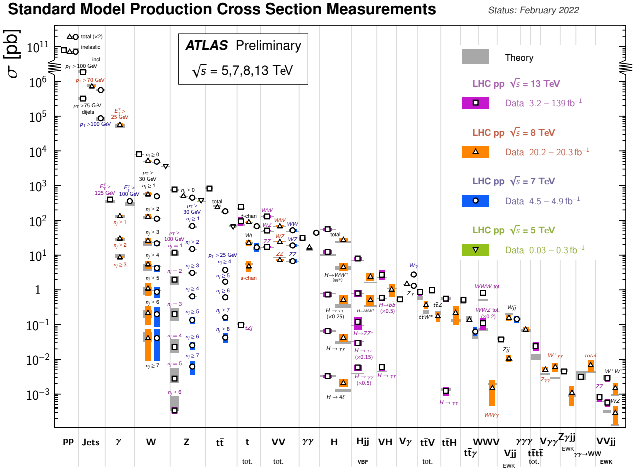
Precision physics at the LHC – role of electroweak (EW) corrections



- excellent agreement between SM predictions and LHC data,
  $\hookrightarrow$ SM can only be challenged with highest possible precision!
- NNLO QCD $\oplus$ NLO EW corrections meanwhile standard in most $2 \to 2$ key processes

Relevance of EW corrections at the LHC

Precision measurements at the LHC

- cross-section uncertainties for single-W/Z production:
  Δ(luminosity) $\sim 4\%$, Δ(PDF) ~ 2–3%
- often 1% precision on shapes of distributions or ratios of cross sections
- high-precision measurements of M_{W}, $\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$:
  $\Delta M_{\mathrm{W}}/M_{\mathrm{W}} \lesssim 2\cdot 10^{-4}$, $\Delta\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}/\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}} \lesssim 4\cdot 10^{-4}$
- energy reach deep into the TeV range with several-% precision

Size of EW corrections

generic size $\mathcal{O}(\alpha) \sim \mathcal{O}(\alpha_{\mathrm{s}}^2) \sim 1\%$ suggests NLO EW $\sim$ NNLO QCD

but systematic enhancements possible, e.g.

- by photon emission
  $\hookrightarrow$ kinematical effects, mass-singular logs $\propto \alpha\ln(m_\mu/Q)$ for muons, etc., often several-10% effects near shoulders of distributions
- at high energies
  $\hookrightarrow$ EW Sudakov logs $\propto (\alpha/s_{\mathrm{W}}^2)\ln^2(M_{\mathrm{W}}/Q)$ and subleading logs, typically several-10% effects in the TeV range

Further peculiarities of EW corrections

Large universal corrections

- induced by photonic vacuum polarization
  and corrections to the ρ-parameter
- can often be absorbed into leading-order predictions
  by appropriate choice of EW input parameter scheme

Instability of W and Z bosons

- realistic observables have to be defined via decay products (leptons, γs, jets)
- off-shell effects $\sim \mathcal{O}(\Gamma/M) \sim \mathcal{O}(\alpha)$ are part of the NLO EW corrections

Photon–jet separation

- non-trivial due to $q \to q + \gamma$ splitting

  $\hookrightarrow$ separation, e.g., by quark-to-photon "fragmentation function"
- complication by photon-induced jets via $\gamma^* \to q\bar{q}$

  $\hookrightarrow$ description by "fragmentation" or "conversion function"

State of the art in the calculation of EW corrections:

- NLO machinery worked out in recent decades
  - on-shell / $\overline{\mathrm{MS}}$ renormalization
  - all multi-leg, multi-scale 1-loop integrals known with complex masses
  - NLO treatment of W/Z resonances
    (pole expansions, complex-mass scheme)
  - IR slicing and subtractions
- Numerous NLO EW calculations for specific processes,
  including multi-leg calculations up to $2 \to 8$ particle processes
- QED parton showers
  (PHOTOS, showers in HERWIG, MADGRAPH, PYTHIS, SHERPA)
- NLO EW automation accomplished
  (MADGRAPH5_AMC@NLO, OPENLOOPS, RECOLA/COLLIER, etc.)
- few mixed NNLO QCD×EW corrections exist
  (several decays, Drell–Yan processes, first results for $\mathrm{e}^+\mathrm{e}^- \to \mathrm{WW}$)
- NNLO EW results still extremely rare
  (μ decay and M_W predictions, $\mathrm{Z}\bar{f}f$ formfactors, partial results for $\mathrm{e}^+\mathrm{e}^- \to \mathrm{ZH}$)

Plan for this talk:

- highlight role and significance of EW corrections
- review key features of EW corrections
  $\hookrightarrow$ exemplified via Drell–Yan + multi-boson processes at the LHC
- consider combination of QCD and EW corrections
  (including results on NNLO$\times$EW corrections)
- emphasize challenges for high-precision physics at future $\mathrm{e}^+\mathrm{e}^-$ colliders

Note: Selection of topics by far not exhaustive (and personally biased)

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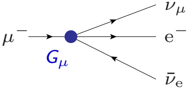
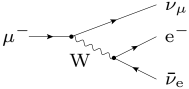
Features of EW corrections

Universal EW corrections, muon decay, and input parameter schemes

μ decay including higher-order corrections



+ QED corrections $\Longleftrightarrow$



+ EW corrections

$\hookrightarrow$ Relation between G_μ, $\alpha(0)$, M_W, and M_Z including corrections:

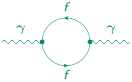
$$\alpha_{G_\mu} \equiv \frac{\sqrt{2}}{\pi} G_\mu M_\mathrm{W}^2 \left(1 - \frac{M_\mathrm{W}^2}{M_\mathrm{Z}^2}\right) = \alpha(0)(1 + \Delta r)$$

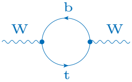
Δr comprises quantum corrections to μ decay
(beyond electromagnetic corrections in Fermi model)

Sirlin '80, Marciano, Sirlin '80

$$\Delta r_{1-\mathrm{loop}} = \Delta\alpha(M_\mathrm{Z}^2) - \frac{c_\mathrm{W}^2}{s_\mathrm{W}^2}\Delta\rho_\mathrm{top} + \Delta r_\mathrm{rem}(M_\mathrm{H})$$

$\sim 6\%$	$\sim 3\%$	$\sim 1\%$
$\alpha \ln(m_f/M_\mathrm{Z})$	$G_\mu m_\mathrm{t}^2$	$\alpha \ln(M_\mathrm{H}/M_\mathrm{Z})$





Predicting M_W from muon decay

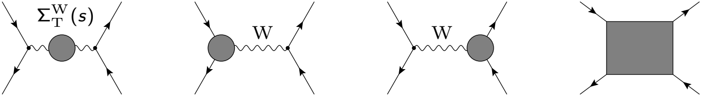
Measure G_μ in μ decay and trade M_W for G_μ as input in

$$\frac{\sqrt{2}}{\pi}\, G_\mu\, M_\mathrm{W}^2 \left(1 - \frac{M_\mathrm{W}^2}{M_\mathrm{Z}^2}\right) = \alpha(0)(1+\Delta r) \qquad \to \text{ solve for } M_\mathrm{W}$$

Δr depends on all input parameters $\quad \to$ sensitivity to m_t, M_H in SM fit

Contributions to Δr:

$+$ virtual corrections:



W self-energy $\qquad$ $\mathrm{W}l\nu_l$ vertex correction $\qquad$ box diagrams

$+$ photonic bremsstrahlung in the SM

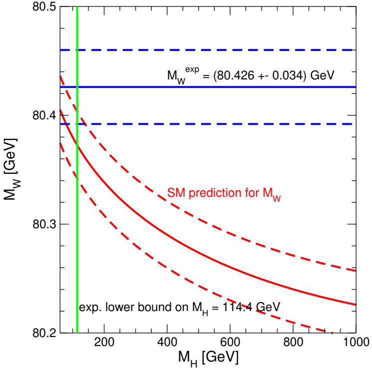
$-$ photonic bremsstrahlung in the Fermi model

$+$ full two-loop contributions $+$ higher-order corrections to ρ-parameter

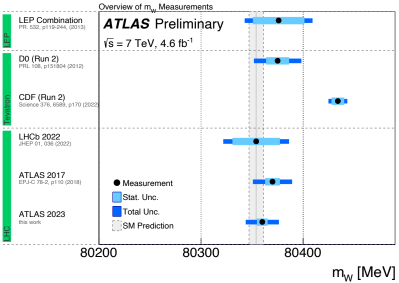
v.Ritbergen,Stuart '98; Seidensticker,Steinhauser '99; Freitas et al. '00-'02; Awramik,Czakon '02/'03; Onishchenko,Veretin '02

Confronting predicted and measured values of M_W

Hollik et al. '03



ATLAS '23



- Current theoretical precision: $\Delta M_\mathrm{W} \sim 0.003\,\mathrm{GeV}$
- Most precise measurements:

CDF '22: $(80.4335 \pm 0.0094)\,\mathrm{GeV}$ (controversial analysis)

ATLAS '23: $(80.360 \pm 0.016)\,\mathrm{GeV}$

EW input parameter schemes for cross-section predictions

Aim: absorb universal corrections from $\Delta\alpha$ and $\Delta\rho$
into leading-order (LO) predictions as much as possible

$$\sigma_{\mathrm{NLO}} = \alpha^N A_{\mathrm{LO}} \, (1 + \delta_{\mathrm{EW}}), \qquad \delta_{\mathrm{EW}} = \mathcal{O}(\alpha)$$

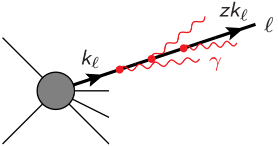
$\hookrightarrow$ minimize missing higher-order corrections!

- $\Delta\alpha^n$ terms can be absorbed to all orders
- $\Delta\rho^n$ terms can be absorbed at least to two-loop order
- factor α in δ_{EW} can still be adjusted appropriately
  (e.g. $\alpha \to \alpha(0)$ if γ radiation dominates, $\alpha \to \alpha_{G_\mu}$ if weak corrections dominate)
- Typical scheme choices: EW input quantities:
  - $\alpha(0)$ scheme: $\alpha(0)$, M_{W}, M_{Z}
  - $\alpha(M_{\mathrm{Z}})$ scheme: $\alpha(M_{\mathrm{Z}})$, M_{W}, M_{Z}
  - G_μ scheme: G_μ, M_{W}, M_{Z}
  - hybrid schemes: e.g. $|\mathcal{M}|^2 \propto \alpha(0)^n \, \alpha_{G_\mu}^m$

  $\hookrightarrow$ optimal choice depends on #(external photons), energy, etc.

Collinear final-state radiation (FSR) off leptons

Leading logarithmic effect is universal:



$$\sigma_{\mathrm{LL,FSR}} = \int \underbrace{\mathrm{d}\sigma^{\mathrm{LO}}(k_l)}_{\text{hard scattering}} \int_0^1 \mathrm{d}z \, \underbrace{\Gamma_{\ell\ell}^{\mathrm{LL}}(z, Q^2)}_{\substack{\text{leading-log structure} \\ \text{function, } Q = \text{typ. scale}}} \Theta_{\mathrm{cut}}(zk_l)$$

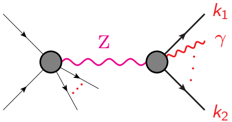
- $\Gamma_{\ell\ell}^{\mathrm{LL}}(z, Q^2)$ known to $\mathcal{O}(\alpha^5)$ + soft exponentiation, equivalent description by QED parton showers
- $\mathcal{O}(\alpha)$ approximation: $\Gamma_{\ell\ell}^{\mathrm{LL},1}(z, Q^2) = \dfrac{\alpha(0)}{2\pi}\left[\ln\left(\dfrac{Q^2}{m_\ell^2}\right) - 1\right]\left(\dfrac{1+z^2}{1-z}\right)_+$
- Alternative approach: QED parton shower
  $\hookrightarrow$ advantage: photons described with finite p_{T} and definite multiplicity

Impact on predictions:

- log-enhanced corrections for "bare" leptons (muons) $\rightarrow$ large radiative tails
- KLN theorem:
  mass-singular FSR effects cancel if $(\ell\gamma)$ system is inclusive
  (full integration over z)
- full FSR not universal,
  in general not even separable from other EW corrections
  (possible only if LO amplitudes do not include W bosons)

Radiative tail from final-state radiation

occurs if resonances reconstructed from decay products



Typical situations: $\quad \mathrm{e^+e^-} \to \mathrm{WW/ZZ} \to 4f,$

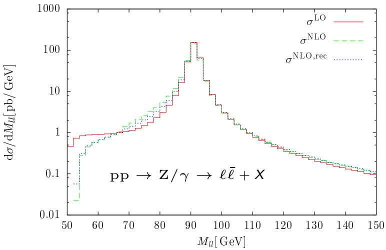
$\qquad\qquad\qquad\qquad \mathrm{pp} \to \mathrm{Z}/\gamma \to \ell\bar{\ell} + X$

Final-state radiation:

resonance for

$$M^2 = (k_1+k_2)^2 < (k_1+k_2+k_\gamma)^2 \sim M_\mathrm{Z}^2$$

$\hookrightarrow$ radiative tail in distribution $\frac{\mathrm{d}\sigma}{\mathrm{d}M}$
of reconstructed invariant mass M
for $M < M_\mathrm{Z}$

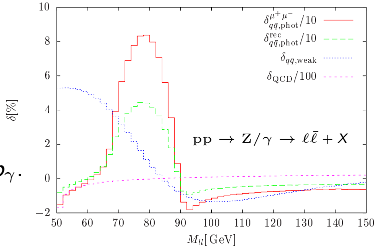


S.D., Huber '09

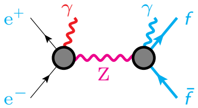
Example: Single-Z production

- radiative tail with corrections up to $\sim 80\%$
- FSR effect drastically reduced
  by photon recombination ("rec"):

  If $R_{l\gamma} < 0.1$ then $(l\gamma) \to \tilde{l}$ with $p_{\tilde{l}} = p_l + p_\gamma$.



Comparison with radiative tail from initial-state radiation



occurs if initial state is fixed

Typical situations: $\quad e^+e^- \to Z/\gamma \to f\bar{f},$
$\mu^+\mu^- \to Z, H, ? \to f\bar{f}$

$\hookrightarrow$ scan over s-channel resonance in $\sigma_{\mathrm{tot}}(s)$ by changing CM energy $\sqrt{s}$

Initial-state radiation:

Z can become resonant for $s = (p_+ + p_-)^2 \; > \; (p_+ + p_- - k_\gamma)^2 \sim M_Z^2$

$\hookrightarrow$ radiative tail for $s > M_Z^2$ due to "radiative return"

Final-state radiation:

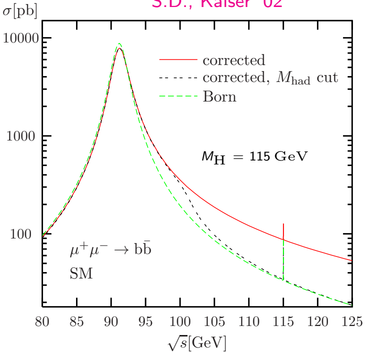
$s = k_Z^2 \sim M_Z^2$ for FSR

$\hookrightarrow$ only rescaling of resonance

Example:

cross section for $\mu^-\mu^+ \to b\bar{b}$ in lowest order,
including photonic and QCD corrections,
with and without invariant-mass cut
$\sqrt{s} - M(b\bar{b}) < 10\,\mathrm{GeV}$

S.D., Kaiser '02



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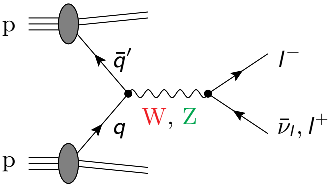
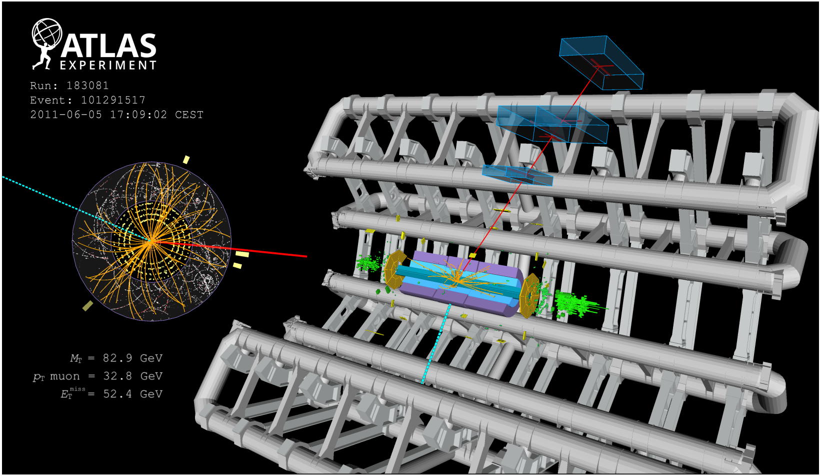
Single-W/Z production



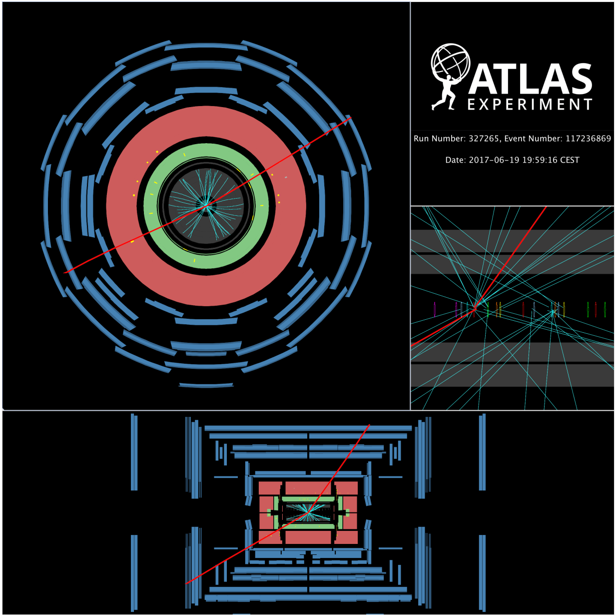
Physics goals:

- M_{Z} $\to$ detector calibration by comparing with LEP1 result
- $\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$ $\to$ comparable precision with LEP1 and SLC
- M_{W} $\to$ exceeds LEP2 precision by factor of 2–3,
  most recent $\Delta M_{\mathrm{W}}^{\mathrm{ATLAS}} = 16\,\mathrm{MeV}$
  (tension with $\Delta M_{\mathrm{W}}^{\mathrm{CDF}} = 9\,\mathrm{MeV}$)
- σ, $\mathrm{d}\sigma$ $\to$ precision SM studies
- decay widths Γ_{Z} and Γ_{W} from M_{ll} or $M_{\mathrm{T},l\nu_l}$ tails
- search for Z' and W' at high M_{ll} or $M_{\mathrm{T},l\nu_l}$
- information on PDFs

A $\mathrm{W} \to \mu\nu_\mu$ event from ATLAS



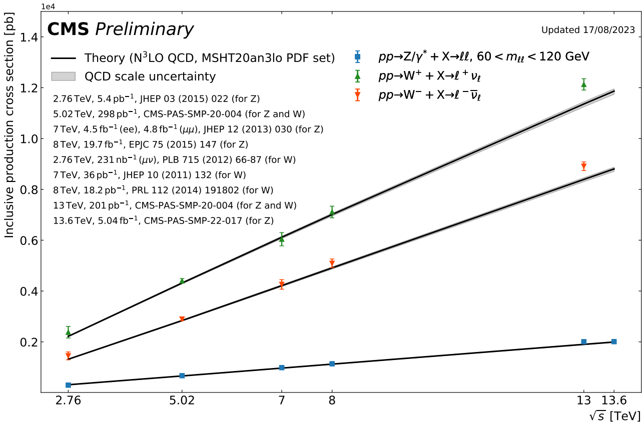
A $Z \to \mu^+\mu^-$ event from ATLAS



Comments on the theory status

- fixed-order QCD corrections known to N^{3}LO for cross sections, Duhr et al. '20
  to NNLO for differential distributions
  Hamberg et al '90; . . . Melnikov et al. '06; Catani et al. '09, . . .
- EW corrections known to NLO Baur et al. '97; Zykunov '01; S.D. et al. '01; . . .
  $+$ higher-order improvements (universal corrections, multi-γ)
- fixed-order mixed $\mathcal{O}(\alpha_\mathrm{s}\alpha)$ corrections
  (pole approximation for W/Z, for Z even fully off-shell)
  S.D. et al. '14;'15;'20; Behring et al. '20; Bonciani et al. '21;
  Armadillo et al. '22; Buccioni et al. '22; . . .
- QCD resummations (q_T resummation, SCET, etc.),
  QCD/QED parton showers, etc.
  $\hookrightarrow$ essential to describe p_T spectra of W/Z bosons

$\mathrm{W/Z}$ cross-section measurements at the LHC:

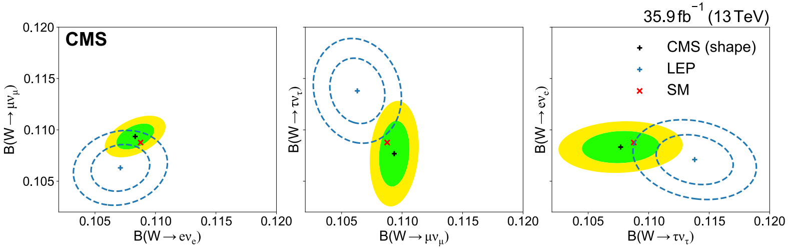


Good agreement between LHC data and N^{3}LO QCD + NLO EW predictions

(tension for 13 TeV W-boson cross sections to be clarified, PDFs?)

Further recent results from the LHC

Test of lepton universality in W decays: (mostly from $\mathrm{t\bar{t}}$ events)

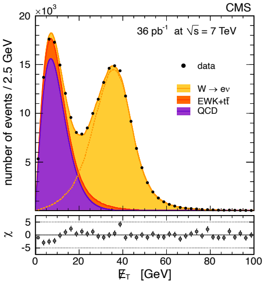


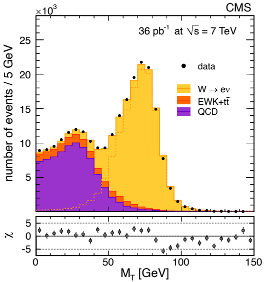
$\hookrightarrow$ tension in LEP results not confirmed

Differential W/Z cross sections

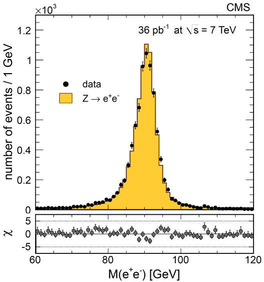
$\hookrightarrow$ information on M_{W}, $\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$, etc.

W bosons:





Z bosons:

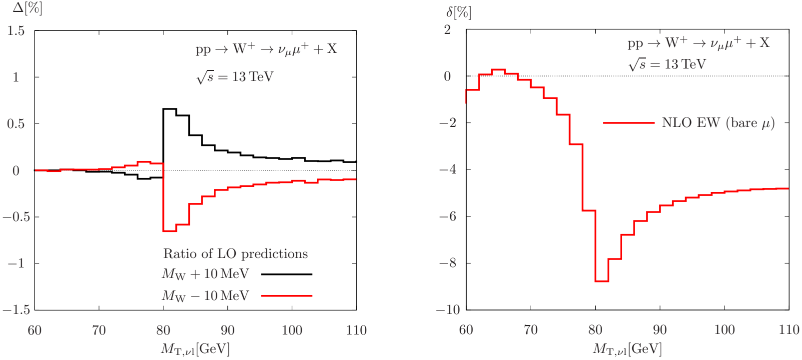


Note locations of Jacobian peaks / resonance at

$$\not{E}_{\mathrm{T}} \sim M_{\mathrm{W}}/2,$$
$$M_{\mathrm{T}} \sim M_{\mathrm{W}},$$
$$M_{\mathrm{e^+e^-}} \sim M_{\mathrm{Z}}$$

Sensitivity of distributions to M_{W} versus NLO EW corrections:

(based on S.D., Krämer '01)



Shape prediction at the level of few 0.1% required!

$\hookrightarrow$ Proper inclusion of EW corrections at NLO + beyond crucial!

$\hookrightarrow$ In particular, check resonance treatment!

Exercise: Compare two different resonance treatments!

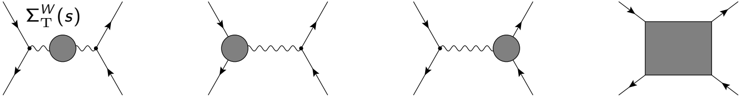
Complex-mass scheme (CMS) Denner et al. '99,'05; see also Denner, S.D. 1912.06823

$\hookrightarrow$ Complex on-shell renormalization with complex EW couplings

$\hookrightarrow$ Gauge invariance and NLO accuracy in resonance and off-shell regions!

Treatment of W productionv ia some "factorization scheme (FS)":
SD, Krämer '01

Virtual corrections:



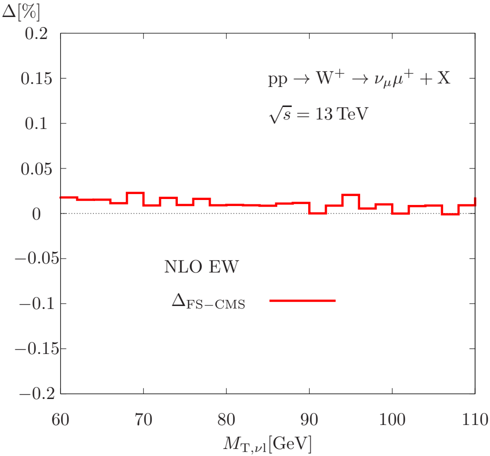
W self-energy | $Wq\bar{q}'$ and $W\nu_l l$ vertex corrections | box diagrams

$$\mathrm{d}\sigma_{\mathrm{virt}}^{\mathrm{FS}}(\hat{s},\hat{t}) = \underbrace{\mathrm{d}\sigma_{\mathrm{LO}}}_{\propto \frac{1}{|\hat{s}-M_{\mathrm{W}}^2+\mathrm{i}M_{\mathrm{W}}\Gamma_{\mathrm{W}}|^2}} \times \underbrace{\Big[\delta_{WW}(\hat{s})+\delta_{Wdu}(\hat{s})+\delta_{W\nu_l l}(\hat{s})+\delta_{\mathrm{box}}(\hat{s},\hat{t})\Big]}_{\Gamma_{\mathrm{W}}\neq 0 \text{ only in } \log(\hat{s}-M_{\mathrm{W}}^2+\mathrm{i}M_{\mathrm{W}}\Gamma_{\mathrm{W}})}$$

Real photonic corrections:

- amplitude gauge invariant for complex W-boson mass μ_{W} and real s_{W}
- IR divergences exactly match between $\mathrm{d}\sigma_{\mathrm{virt}}^{\mathrm{FS}}$ and $\mathrm{d}\sigma_{\mathrm{real}}^{\mathrm{FS}}$

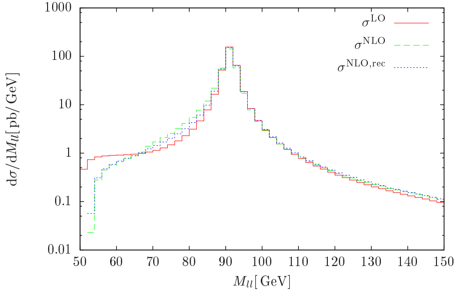
Comparison of width schemes for W production at NLO EW

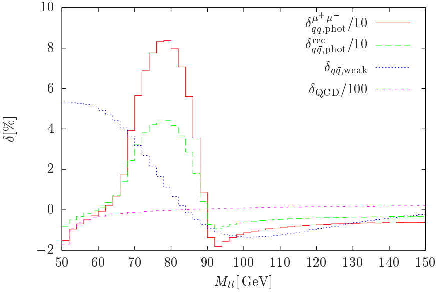


Consistency between the FS and CMS at the level of
$\Delta_{\mathrm{FS-CMS}} = \frac{\mathrm{d}\sigma_{\mathrm{FS}}}{\mathrm{d}\sigma_{\mathrm{CMS}}} - 1 \sim 0.02\%$!

Survey of EW corrections to Z production

SD, Huber '09

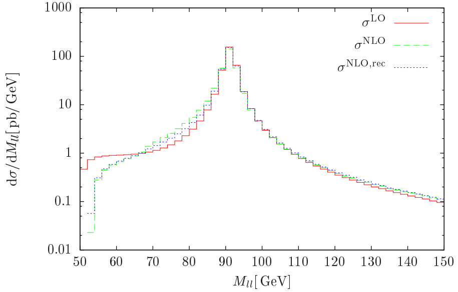


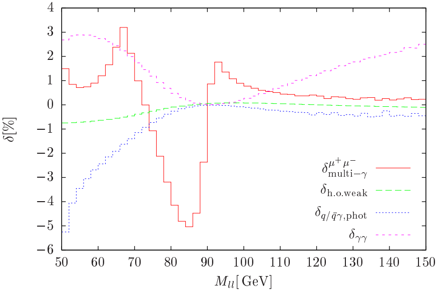


- NLO QED corrections (mostly FSR) several 10%
  [maximally $\sim$ 40%(80%) for dressed leptons (bare muons)]
- Mulit-γ effects still at the few-% level
- Weak NLO corrections at the few-% level
  $\hookrightarrow$ most sensitive to width scheme

Survey of EW corrections to Z production

SD, Huber '09





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- Mulit-γ effects still at the few-% level
- Weak NLO corrections at the few-% level
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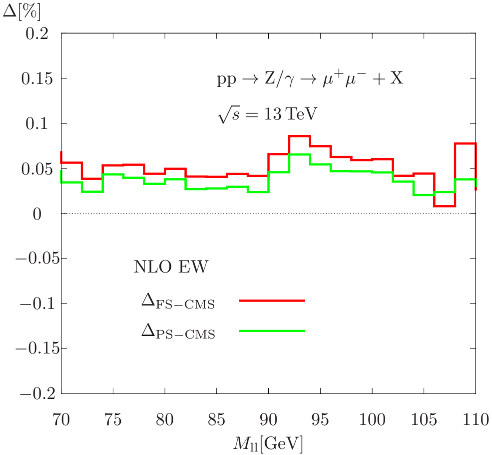
Comparison of width schemes for Z production at NLO EW

(based on S.D., Huber 0911.2329)



Resonance schemes:

(see also 1912.06823)

CMS = complex-mass scheme

PS = pole scheme

FS = factorization scheme
(less solid, more tricky due to γ/Z interference)

Consistency between the PS, FS, and CMS at the level of
$\Delta_{\mathrm{FS/PS-CMS}} = \frac{\mathrm{d}\sigma_{\mathrm{FS/PS}}}{\mathrm{d}\sigma_{\mathrm{CMS}}} - 1 \lesssim 0.1\%$!

Forward–backward asymmetry $A_{\mathrm{FB}}(M_{\ell\ell})$ in neutral-current Drell–Yan production

Issue: symmetric pp initial state at the LHC, i.e. no preferred forward direction!

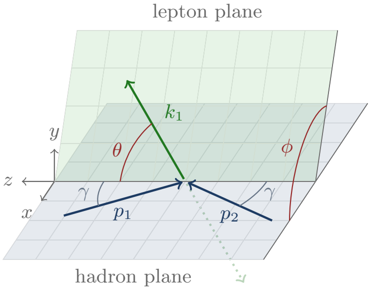
Solution: exploit PDF difference between (valence) q and (sea) $\bar{q}$

$\hookrightarrow$ on average, q carries more momentum than $\bar{q}$!

$\hookrightarrow$ on average, CM($q\bar{q}$) $\approx$ CM(Z) $\approx$ CM($\ell^+\ell^-$) $\rightarrow$ q direction!

$\Rightarrow$ Collins–Soper angle θ, ϕ:

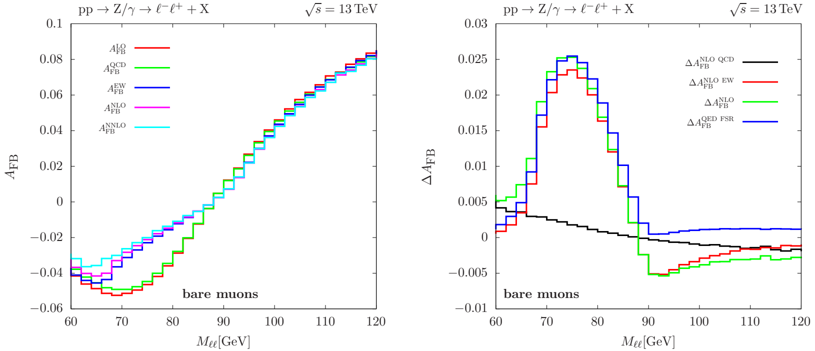
- go into centre-of-mass frame CM(Z) of the Z boson
- z axis $=$ line of intersection of leptonic and hadronic planes
- $+z$ direction inherited from Z direction in LAB frame
- $+x$ direction from beams
- $+y$ direction completes right-handed coordinate system
- $\theta, \phi =$ polar angles of ℓ^- momentum $\vec{k}_1$



FB asymmetry A_{FB} in Z production – weak corrections and width schemes

A_{FB} defined via Collins–Soper angles $\quad\rightarrow\quad$ sensitivity to $\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$

S.D., Huss, Schwarz '24



Large EW corrections!

Experimental uncertainties and precision targets:

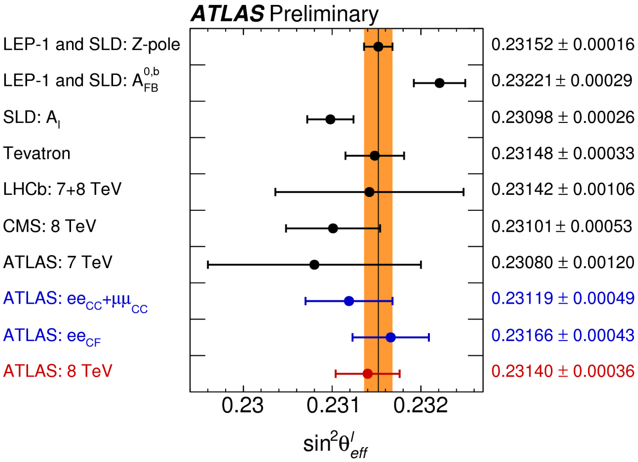
- Z resonance at LEP: $\quad\Delta A_{\mathrm{FB}}^{\mathrm{b}} = 0.0016, \quad \Delta A_{\mathrm{FB}}^{\ell} = 0.0010$

  $\hookrightarrow\ \Delta\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}} = 0.00029$ from $\Delta A_{\mathrm{FB}}^{\mathrm{b}}$

- LHC precision target for predictions: $\quad \Delta A_{\mathrm{FB}}(M_{\ell\ell}) \lesssim 10^{-4}$

  $\hookrightarrow$ great challenge (not yet completely reached)

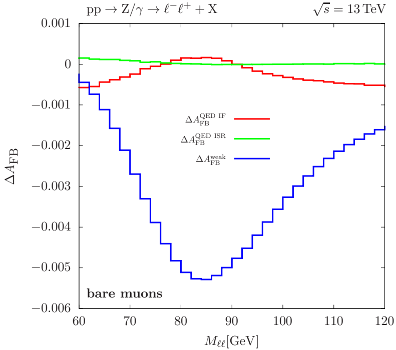
Measurements of the effective weak mixing angle – current status

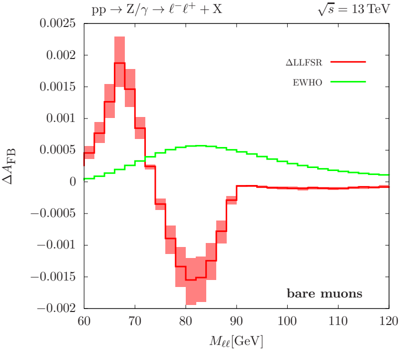


$\hookrightarrow$ LHC closes in on LEP precision!

FB asymmetry A_{FB} – different sources of EW corrections

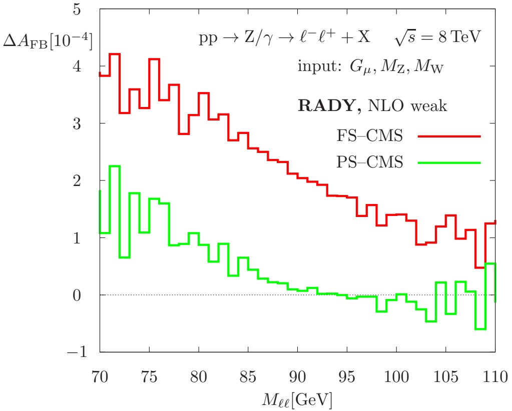
S.D., Huss, Schwarz '24





- NLO weak corrections very important
- large QED corrections due to FSR (previous plot)
- little impact from QED ISR and IF interference
- multi-photon FSR effects significant
  $\hookrightarrow$ leading-log treatment (ΔLLFSR) not sufficient!
- universal EW higher-order effects (EWHO) due to $\Delta\alpha$, $\Delta\rho$ relevant

FB asymmetry A_{FB} – differences of width schemes differentially



$\hookrightarrow$ |PS-CMS| $\lesssim 10^{-4}$

FS less accurate (theoretically not as solid as PS/CMS)

$\hookrightarrow$ theoretical improvements beyond NLO EW very desirable!

NNLO QCD$\times$EW corrections

Calculation in pole approximation (PA)

S.D., Huss, Schwinn '14,'15; S.D., Huss, Schwarz '24

- leading term of resonance expansion
  - $\hookrightarrow$ valid in vicinity of W/Z resonance
  - $\hookrightarrow$ relevant for M_{W}, $\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}}$ analyses
- on-shell production/decay as building blocks
  - $\hookrightarrow$ reduced 2-loop complexity

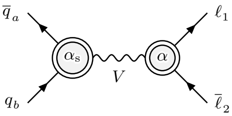
De Florian ey al. '18; Delto et al. '19; Bonciani et al. '19–'21; Behring et al. '20; Buccioni et al. '20

Full off-shell calculation

- important for off-shell tails of $M_{\ell\ell}$, $M_{\mathrm{T},\nu\ell}$, $k_{\mathrm{T},\ell}$ distributions
- full 2-loop complexity (e.g. boxes with internal masses)
- $\mathcal{O}(N_f\alpha_{\mathrm{s}}\alpha)$ parts, complex renormalization — S.D., Schmidt, Schwarz '20
- neutral-current process fully known — Bonciani et al. '21; Armadillo et al. '22; Buccioni et al. '22
- charged-current process approximately known (2-loop part approximated) — Buonocore et al. '21

NNLO QCD×EW corrections in pole approximation

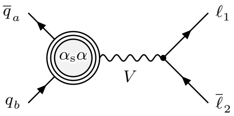
Factorizable initial–final (IF) corrs.:



S.D., Huss, Schwinn '15

- large corrections due to collinear FSR

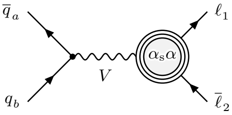
Factorizable initial–initial (II) corrs.:



De Florian ey al. '18; Delto et al. '19; Bonciani et al. '19–'21; Behring et al. '20; Buccioni et al. '20; S.D., Huss, Schwarz '24

- moderate/small corrections, widely absorption into PDF redefinition

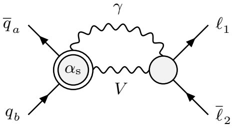
Factorizable final–final (FF) corrs.:



S.D., Huss, Schwinn '14

- only $V\bar{\ell}\ell$ counterterms (small)

Non-factorizable (NF) corrs.:



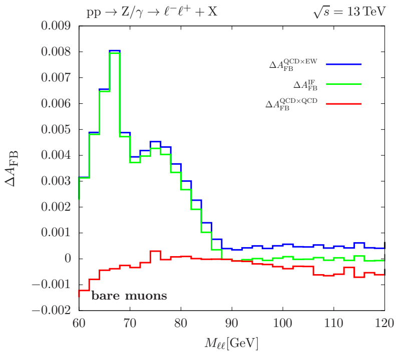
S.D., Huss, Schwinn '14

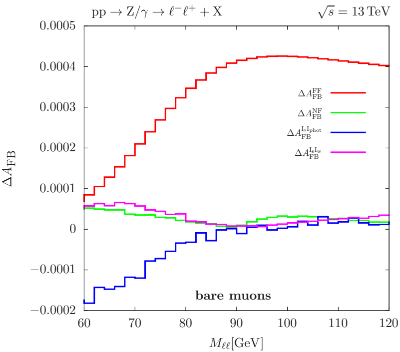
- corrections negligible

New: Evaluation of $\mathcal{O}(\alpha_s\alpha)$ corrections to FB asymmetry! S.D., Huss, Schwarz '24

FB asymmetry A_{FB} – NNLO corrections (QCD×EW in pole aproximation)

S.D., Huss, Schwarz '24

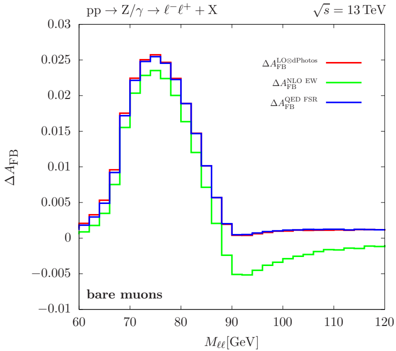




- NNLO QCD × FSR QED (IF) by far dominating NNLO effect!
- NNLO QCD × weak final-state (FF) corrections still relevant
- other NNLO QCD × EW corrections (initial state, non-factorizable) negligible

Fixed-order $\mathcal{O}(\alpha_s\alpha)$ corrections verses QCD $\times$ QED parton shower

S.D., Huss, Schwarz '24

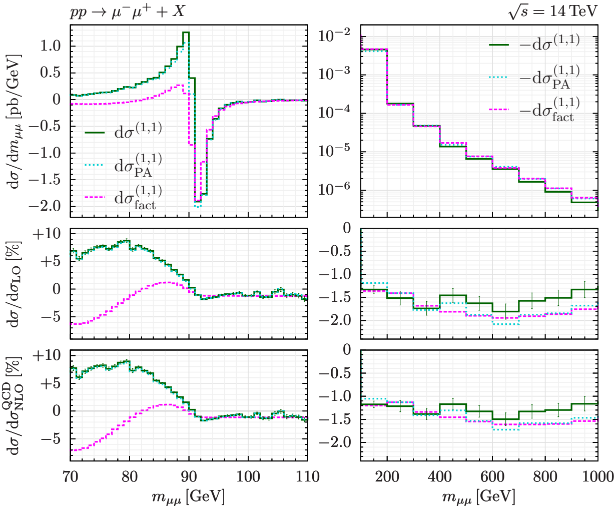


- Z production:
  QED parton showers (like PHOTOS) capture FSR effects well
  But:
  Approximative quality only known by comparison to full MS-based results
- Note: Concept of FSR not well defined for charged-current processes!

$\mathcal{O}(\alpha_s\alpha)$ corrections to high-energy tails in Drell–Yan processes

NNLO QCD×EW corrections to $M_{\mu\mu}$ distribution (bare muons)

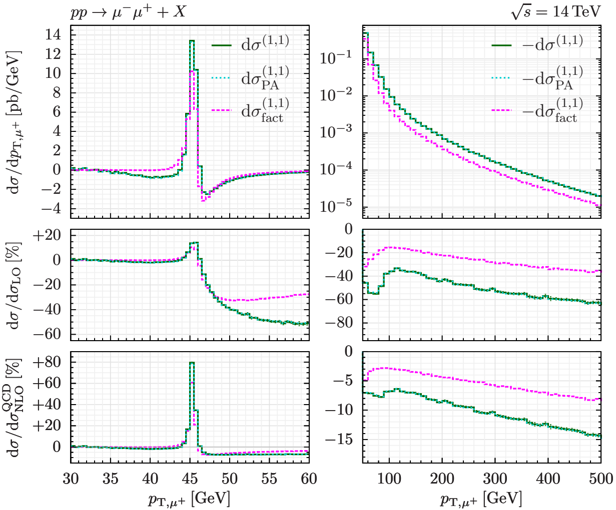
Bonciani et al. '21



$\delta \sim 1$–2% in TeV range

NNLO QCD×EW corrections $p_{\mathrm{T},\mu}$ distribution (bare muons)

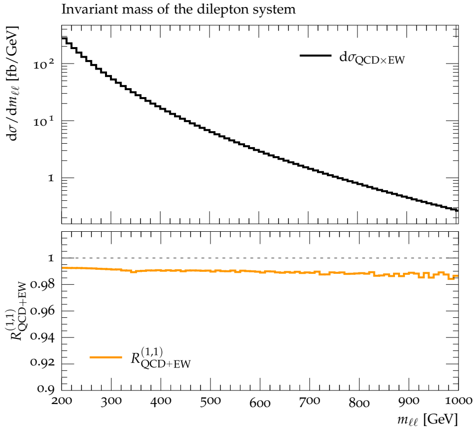
Bonciani et al. '21



$\delta \sim 10$–15% for $p_{\mathrm{T},\mu} \sim 500\,\mathrm{GeV}$

NNLO QCD$\times$EW corrections to $M_{\ell\ell}$ distribution (dressed leptons)

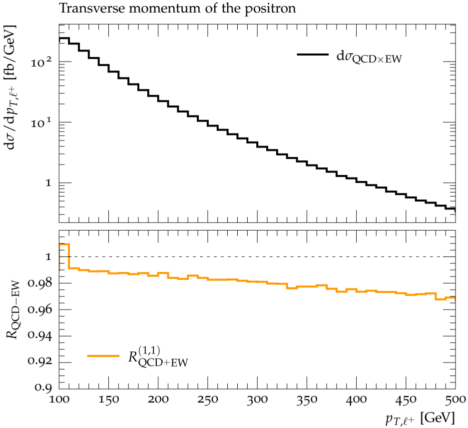
Buccioni et al. '22



Effect from γ recombination seems small?

NNLO QCD×EW corrections to $p_{\mathrm{T},\ell}$ distribution (dressed leptons)

Buccioni et al. '22



Effect from γ recombination very significant?

Upshot:

Great progress on NNLO QCD×EW frontier!

But more flexibility / comparability of results wrt. γ recombination desirable ...

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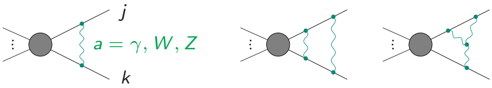
Electroweak corrections at high energies

Sudakov logarithms induced by soft gauge-boson exchange



etc.

$+$ sub-leading logarithms from collinear singularities

Typical impact on $2 \to 2$ reactions at $\sqrt{s} \sim 1\,\mathrm{TeV}$:

$$\delta_{\mathrm{LL}}^{1-\mathrm{loop}} \sim -\frac{\alpha}{\pi s_{\mathrm{W}}^2} \ln^2\left(\frac{s}{M_{\mathrm{W}}^2}\right) \simeq -26\%, \qquad \delta_{\mathrm{NLL}}^{1-\mathrm{loop}} \sim +\frac{3\alpha}{\pi s_{\mathrm{W}}^2} \ln\left(\frac{s}{M_{\mathrm{W}}^2}\right) \simeq 16\%$$

$$\delta_{\mathrm{LL}}^{2-\mathrm{loop}} \sim +\frac{\alpha^2}{2\pi^2 s_{\mathrm{W}}^4} \ln^4\left(\frac{s}{M_{\mathrm{W}}^2}\right) \simeq 3.5\%, \qquad \delta_{\mathrm{NLL}}^{2-\mathrm{loop}} \sim -\frac{3\alpha^2}{\pi^2 s_{\mathrm{W}}^4} \ln^3\left(\frac{s}{M_{\mathrm{W}}^2}\right) \simeq -4.2\%$$

$\Rightarrow$ Corrections still relevant at 2-loop level

Note: differences to QED/QCD where Sudakov logs cancel

- massive gauge bosons W, Z can be reconstructed
  $\hookrightarrow$ no need to add "real W, Z radiation"
- non-Abelian charges of W, Z are "open" $\rightarrow$ Bloch–Nordsieck theorem not applicable

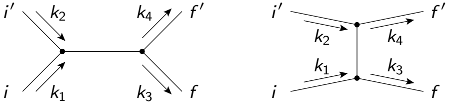
Extensive theoretical studies at fixed perturbative (1-/2-loop) order and suggested resummations via evolution equations

Beccaria et al.; Beenakker, Werthenbach; Ciafaloni, Comelli; Denner, Pozzorini; Fadin et al.; Hori et al.; Melles; Kühn et al., Denner et al.; Manohar et al. '00–

High-energy limit – Sudakov versus Regge regime

Sudakov regime: all invariants $k_i \cdot k_j \gg M_{\mathrm{W}}^2$!

Example:
$2 \to 2$ particle process



Kinematic variables in centre-of-mass frame in high-energy limit ($k_j^2 \to 0$):

$$
\begin{aligned}
s &= (k_1+k_2)^2 \sim 4E^2, && E = \text{beam energy},\\
t &= (k_1-k_3)^2 \sim -4E^2\sin^2(\theta/2), && \theta = \text{scattering angle},\\
M_{34} &= \sqrt{s} \sim 2E, &&\\
k_{\mathrm{T}} &= k_{3,\mathrm{T}} \sim E\sin\theta &&
\end{aligned}
$$

High-energy limits in distributions:

- $\frac{\mathrm{d}\sigma}{\mathrm{d}k_{\mathrm{T}}}$: $k_{\mathrm{T}} \gg M_{\mathrm{W}} \quad \Rightarrow \quad s, |t| \gg M_{\mathrm{W}}^2 \quad \Rightarrow$ Sudakov domination
- $\frac{\mathrm{d}\sigma}{\mathrm{d}M_{34}}$: $M_{34} \gg M_{\mathrm{W}} \;\Rightarrow$ small $|t|$ possible $\;\Rightarrow$ in general no Sudakov domination
  (i.e. typically smaller corrections)

Example: Drell–Yan production

Neutral current: $\mathrm{pp} \to \ell^+\ell^-$ at $\sqrt{s} = 14\,\mathrm{TeV}$ (based on S.D./Huber arXiv:0911.2329)

$M_{\ell\ell}/\mathrm{GeV}$	$50-\infty$	$100-\infty$	$200-\infty$	$500-\infty$	$1000-\infty$	$2000-\infty$
σ_0/pb	738.733(6)	32.7236(3)	1.48479(1)	0.0809420(6)	0.00679953(3)	0.000303744(1)
$\delta^{\mathrm{rec}}_{\mathrm{q\bar{q},phot}}/\%$	-1.81	-4.71	-2.92	-3.36	-4.24	-5.66
$\delta_{\mathrm{q\bar{q},weak}}/\%$	-0.71	-1.02	-0.14	-2.38	-5.87	-11.12
$\delta^{(1)}_{\mathrm{Sudakov}}/\%$	0.27	0.54	-1.43	-7.93	-15.52	-25.50
$\delta^{(2)}_{\mathrm{Sudakov}}/\%$	-0.00046	-0.0067	-0.035	0.23	1.14	3.38

no Sudakov domination!

Charged current: $\mathrm{pp} \to \ell^+\nu_\ell$ at $\sqrt{s} = 14\,\mathrm{TeV}$ (based on Brensing et al. arXiv:0710.3309)

$M_{\mathrm{T},\nu_\ell\ell}/\mathrm{GeV}$	$50-\infty$	$100-\infty$	$200-\infty$	$500-\infty$	$1000-\infty$	$2000-\infty$
σ_0/pb	4495.7(2)	27.589(2)	1.7906(1)	0.084697(4)	0.0065222(4)	0.00027322(1)
$\delta^{\mu^+\nu_\mu}_{\mathrm{q\bar{q}}}/\%$	$-2.9(1)$	$-5.2(1)$	$-8.1(1)$	$-14.8(1)$	$-22.6(1)$	$-33.2(1)$
$\delta^{\mathrm{rec}}_{\mathrm{q\bar{q}}}/\%$	$-1.8(1)$	$-3.5(1)$	$-6.5(1)$	$-12.7(1)$	$-20.0(1)$	$-29.6(1)$
$\delta^{(1)}_{\mathrm{Sudakov}}/\%$	0.0005	0.5	-1.9	-9.5	-18.5	-29.7
$\delta^{(1)}_{\mathrm{EWslog}}/\%$	0.008	0.9	2.3	3.8	4.8	5.9
$\delta^{(2)}_{\mathrm{Sudakov}}/\%$	-0.0002	-0.023	-0.082	0.21	1.3	3.8

Sudakov domination!

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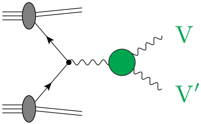
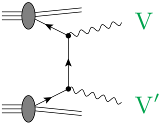
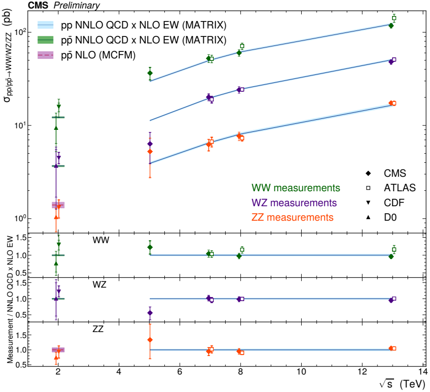
Multi-boson production / scattering at the LHC

Massive di-boson production

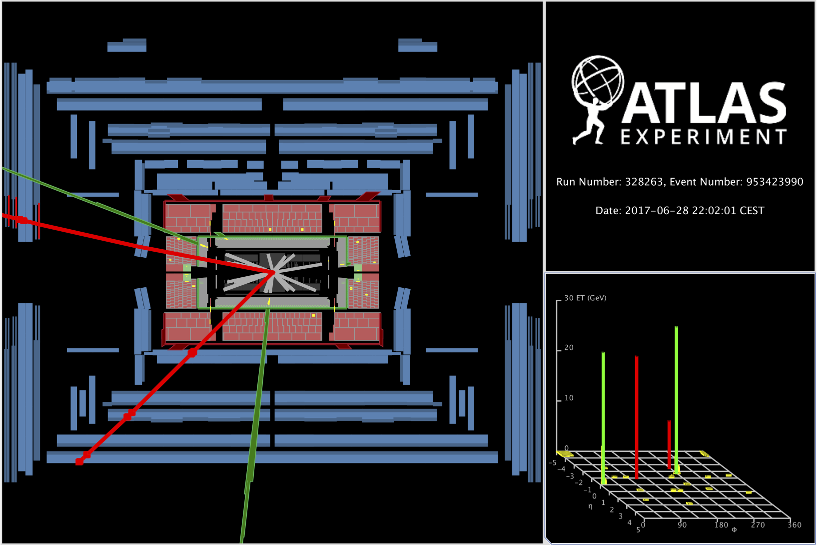






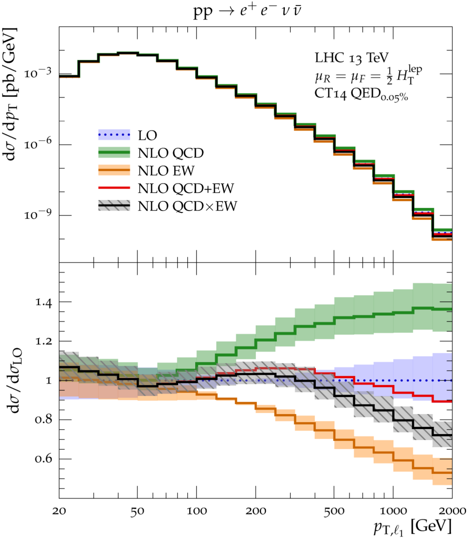
- overall good agreement between data and SM
- NNLO QCD corrections essential for proper descritpion of data
- NLO EW corrections important in differential distributions
- data constrain anomalous *VVV* couplings

A $\mu^+\mu^-e^+e^-$ event from ATLAS



$\mathrm{pp} \to \mathrm{WW}/\mathrm{ZZ} \to \mathrm{e}^+\mathrm{e}^-\nu\bar{\nu} + X$: survey of different NLO contributions

Kallweit et al. '17

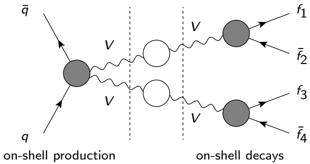


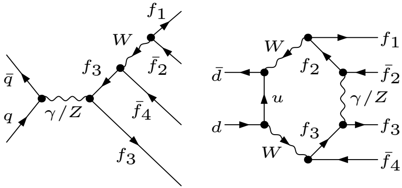
- XS contributions:
  $\mathrm{WW} + \mathrm{ZZ} +$ interferences
- Jet veto:
  $$H_{\mathrm{T}}^{\mathrm{jet}} = \sum_{i \in \mathrm{jets}} p_{\mathrm{T},i} \;>\; H_{\mathrm{T}}^{\mathrm{lep}}$$
  $\hookrightarrow$ K_{QCD} moderate
- EW corrections
  $\sim -40\%$ in TeV range
  (EW Sudakov logarithms)
- Combination of QCD and EW corrections:
  $|\,\mathrm{QCD{+}EW} - \mathrm{QCD{\times}EW}\,|$
  $\sim \delta_{\mathrm{QCD}} \times \delta_{\mathrm{EW}}$
  $\sim$ 10–20% for $p_{\mathrm{T},\ell_1} \gtrsim 1\,\mathrm{TeV}$

  Note: product better motivated!

EW corrections – full NLO versus pole approximation

Double-pole approximation (DPA) calculation vs. Full off-shell $q\bar{q} \to 4f$





- ▶ expansion about resonance poles
  $\hookrightarrow$ factorizable & non-fact. corrs.
- ▶ not many diagrams (2→2 production)
- \+ numerically fast
- – validity only for $\sqrt{\hat{s}} > 2M_V + \mathcal{O}(\Gamma_V)$

- ▶ off-shell calculation with complex-mass scheme
- ▶ many off-shell diagrams ($\sim 10^3$/channel)
- – CPU intensive
- \+ NLO accuracy everywhere

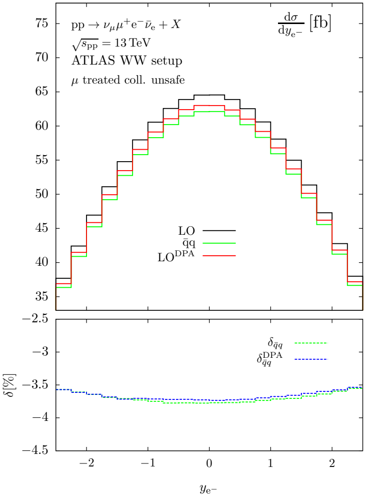
Approaches compared for $\mathrm{e^+e^-/pp \to WW} \to 4f$, etc.

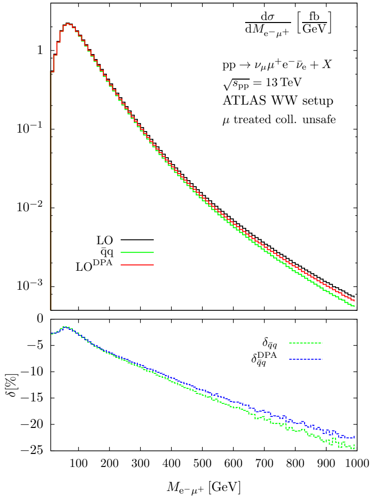
(similarly for $\mathrm{pp \to WWW} \to 6\ell$, $\mathrm{pp(WW \to WW)} \to 4\ell 2\mathrm{j}$, etc.)

DPA versus full off-shell EW correction in $\mathrm{pp} \to \nu_\mu \mu^+ \mathrm{e}^- \bar{\nu}_\mathrm{e} + X$

Biedermann et al. '16

Rapidity and invariant-mass distributions





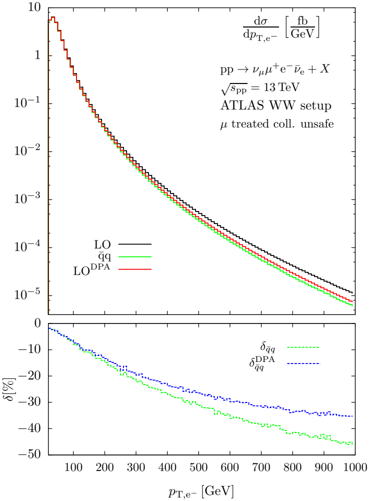
Level of agreement as expected (dominance of doubly-resonant diagrams)

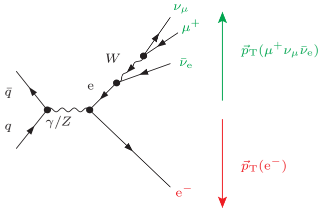
$\hookrightarrow$ difference $\lesssim 0.5\%$ whenever cross section sizable

DPA versus full off-shell EW correction in $\mathrm{pp} \to \nu_\mu \mu^+ \mathrm{e}^- \bar{\nu}_\mathrm{e} + X$

Biedermann et al. '16

Transverse-momentum distribution of a single lepton



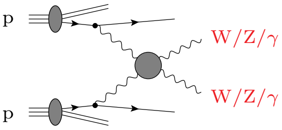


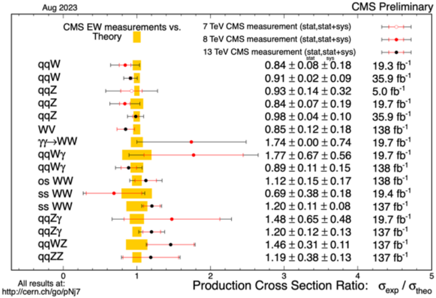
Impact of singly-resonant diagrams where e^- takes recoil from $(\mu^+\nu_\mu\bar{\nu}_\mathrm{e})$

(W bremsstrahlung to Drell–Yan production of $\mathrm{e}^+\mathrm{e}^-$)

Agreement degrades for $p_\mathrm{T} \gtrsim 300\,\mathrm{GeV}$, since off-shell diagrams get enhanced

Electroweak gauge-boson scattering





Physics interest:

- strong sensitivity to EW gauge-boson self-interaction
- window to EW symmetry breaking (EWSB) via off-shell Higgs exchange, complementary to direct analyses of (on-shell) Higgs bosons

Analysis framework:

- "SM Effective Theory (SMEFT)" based on SM particle content

$$\mathcal{L}_{\mathrm{eff}} = \mathcal{L}_{\mathrm{SM}} + \sum_i \frac{c_i}{\Lambda^2} \mathcal{L}_i^{(\mathrm{dim}-6)},$$

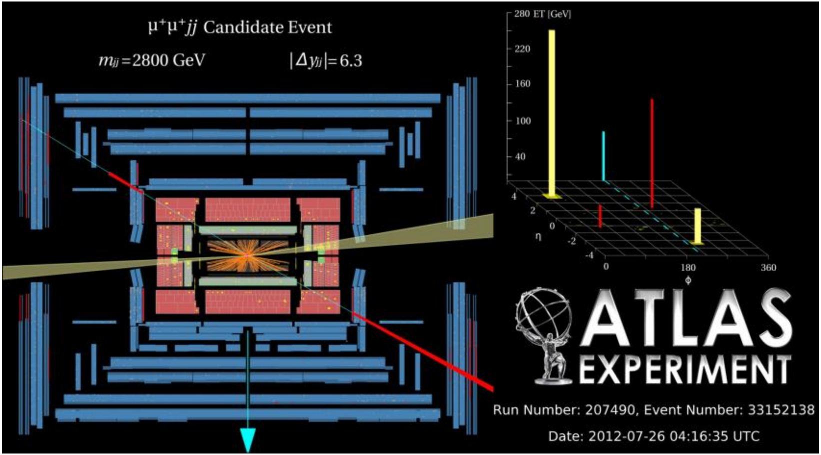
effective dim-6 operators
Buchmüller, Wyler '85; Grzadkowski et al. '10

- Specific SM extensions (extended Higgs sectors, modified EWSB, etc.)

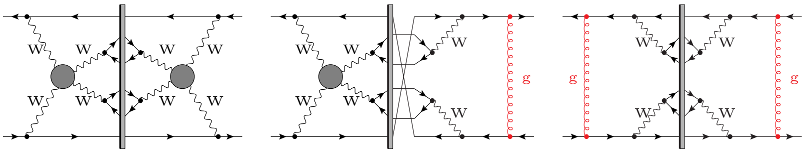
All channels measured by ATLAS & CMS $\rightarrow$ compatibility with SM

$\Rightarrow$ BSM effects (if accessible) subtle and small $\rightarrow$ highest precision required !

A typical W^+W^+ scattering event at the LHC



Schematic view of perturbative orders at LO and NLO



$\mathcal{M}^{\mathrm{LO}}_{\mathrm{EW}}\,\mathcal{M}^{\mathrm{LO}*}_{\mathrm{EW}}$ $\mathcal{M}^{\mathrm{LO}}_{\mathrm{EW}}\,\mathcal{M}^{\mathrm{LO}*}_{\mathrm{QCD}}+\mathrm{c.c.}$ $\mathcal{M}^{\mathrm{LO}}_{\mathrm{QCD}}\,\mathcal{M}^{\mathrm{LO}*}_{\mathrm{QCD}}$

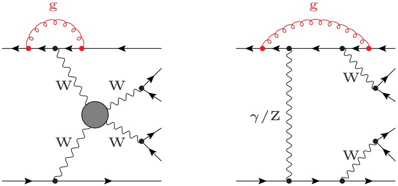
$$\begin{array}{lccccccc}
\textbf{LO} & & \mathcal{O}(\alpha^6) & & \mathcal{O}(\alpha_s\alpha^5) & & \mathcal{O}(\alpha_s^2\alpha^4) & \\
& \swarrow^{\mathrm{EW}} & & \searrow^{\mathrm{QCD}} \swarrow^{\mathrm{EW}} & & \searrow^{\mathrm{QCD}} \swarrow^{\mathrm{EW}} & & \searrow^{\mathrm{QCD}} \\
\textbf{NLO} & \mathcal{O}(\alpha^7) & & \mathcal{O}(\alpha_s\alpha^6) & & \mathcal{O}(\alpha_s^2\alpha^5) & & \mathcal{O}(\alpha_s^3\alpha^4)
\end{array}$$

EW mixed EW–QCD QCD

$\Rightarrow$ Tower of mixed EW–QCD corrections at NLO

Survey of NLO contributions of QCD type

QCD corrections to EW channels



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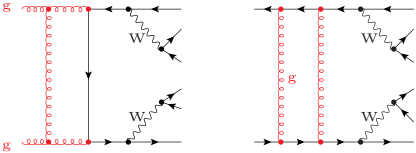

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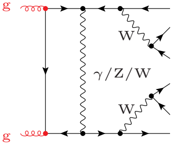
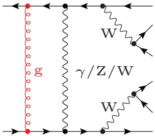
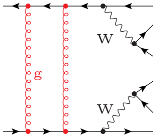




$\times \mathcal{M}_{\mathrm{QCD}}^{\mathrm{LO}*}$



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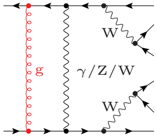

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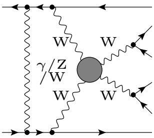

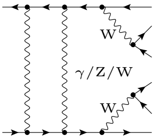

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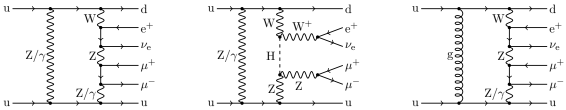

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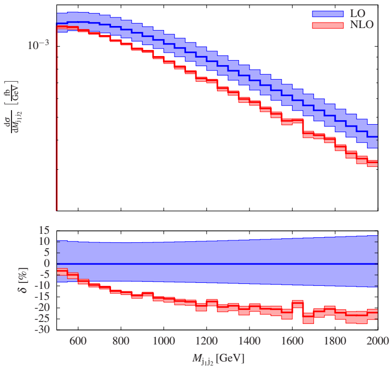

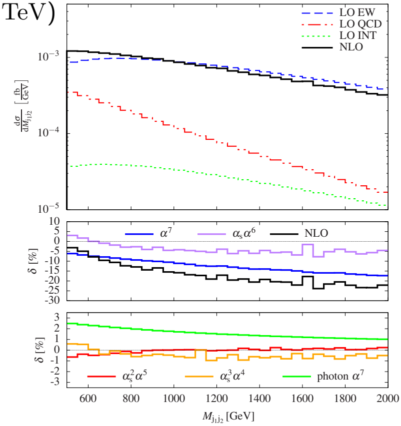

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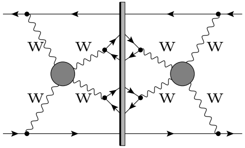
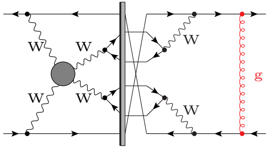
(due to soft/collinear W/Z exchange)

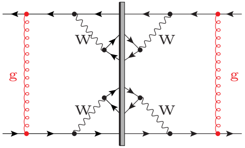
Schematic view of perturbative orders at LO and NLO



$\mathcal{M}^{\mathrm{LO}}_{\mathrm{EW}}\,\mathcal{M}^{\mathrm{LO}*}_{\mathrm{EW}}$



$\mathcal{M}^{\mathrm{LO}}_{\mathrm{EW}}\,\mathcal{M}^{\mathrm{LO}*}_{\color{red}{\mathrm{QCD}}} + \mathrm{c.c.}$



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EW QCD EW QCD EW QCD

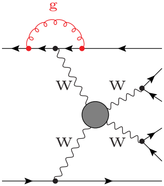
NLO $\mathcal{O}(\alpha^7)$ $\mathcal{O}(\alpha_{\mathrm{s}}\alpha^6)$ $\mathcal{O}(\alpha_{\mathrm{s}}^2\alpha^5)$ $\mathcal{O}(\alpha_{\mathrm{s}}^3\alpha^4)$

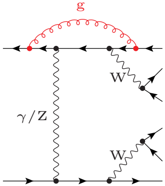
EW mixed EW–QCD QCD

$\Rightarrow$ Tower of mixed EW–QCD corrections at NLO

Survey of NLO contributions of QCD type

QCD corrections to EW channels





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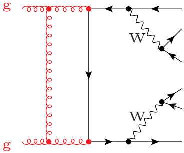

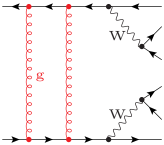

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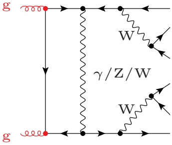

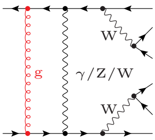

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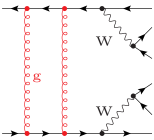

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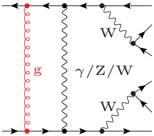

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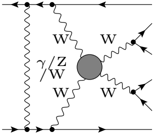

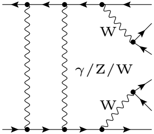

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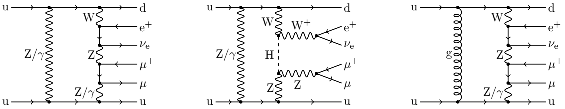

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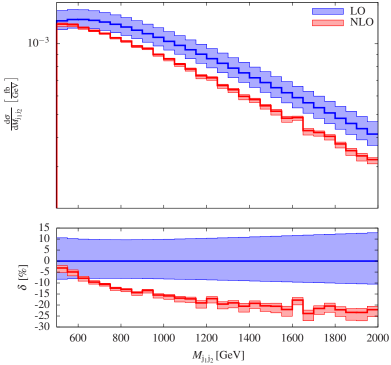

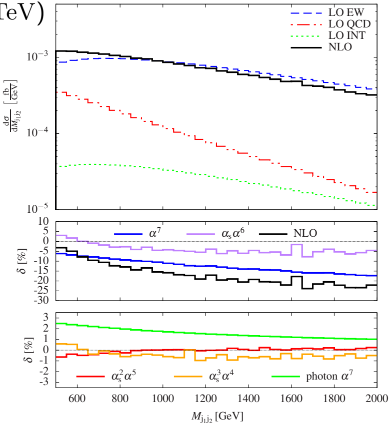

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Recent recalculation of NLO corrections to QCD $\mathrm{W^+W^+} + 2\mathrm{j}$ channel

S.D., Maierhöfer, Schwan, Winterhalter '23

Results with VBS cuts:

	Order	Result [fb]	δ [%]	Scale uncertainty	
LO	$\mathcal{O}(\alpha^6\alpha_s^0)$	1.24597(5)		-7.7%	9.9%
	$\mathcal{O}(\alpha^5\alpha_s^1)$	0.051133(3)		-14.0%	17.7%
	$\mathcal{O}(\alpha^4\alpha_s^2)$	0.18649(2)		-22.2%	31.6%
	sum	1.48359(5)		-9.8%	12.1%
NLO	$\mathcal{O}(\alpha^7\alpha_s^0)$	$-0.1747(5)$	-11.8%		
	$\mathcal{O}(\alpha^6\alpha_s^1)$	$-0.0902(8)$	-6.1%		
	$\mathcal{O}(\alpha^5\alpha_s^2)$	$-0.00017(19)$*	0.0%		
	$\mathcal{O}(\alpha^4\alpha_s^3)$	$-0.0033(7)$	-0.2%		
	sum	$-0.268(1)$	-18.1%		
LO+NLO	sum	1.215(1)		-4.0%	1.5%

- interesting interplay of QCD and EW corrections
- large EW corrections from high-energy domain
  $\hookrightarrow$ inclusion of leading effects beyond NLO?
- approximations for complex $2 \to 6$ process non-trivial, but possible

* Error in earlier calculation (Biedermann et al. '16,'17) corrected

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Challenges in electroweak corrections beyond NLO

EW corrections at NLO

- problem conceptually solved, corrections widely automated
- dedicated calculations for high-multiplicity processes $(2 \to 6, 7, 8, \dots)$ certainly still welcome
  $\hookrightarrow$ non-trivial cross-checks, ansatz for approximations, improvements beyond NLO, ...

EW renormalization at NNLO

- concept widely straightforward for on-shell and $\overline{\mathrm{MS}}$ schemes
- few applications for decays exist
- subtleties expected (unstable-particles effects, imaginary parts, etc.)
- major challenge: complex-mass scheme for unstable particles at NNLO

Massive 2-loop integrals (and beyond)

- majority of graphs involve triple-massive cuts $\quad \to$ elliptic integrals
- numerical methods unavoidable
  $\hookrightarrow$ try out and compare different approaches
- often analytical expansions provide an alternative

Challenges in electroweak corrections beyond NLO (continued)

IR singularities / QED radiation

- borrow subtraction methods from QCD
- small masses of fermions often desirable
  $\hookrightarrow$ massification of massless limits
- control QED radiation way beyond NNLO (large effects on tails)
  $\hookrightarrow$ factorization into (perturbative!) QED lepton/photon PDFs

Approximations

- Important, but validate/check carefully!
- Don't oversimplify! E.g. include W/Z decays in processes
- Resonance expansions for W/Z/H production often good approximations!
- Effective vector-boson approximations not appropriate for precision physics

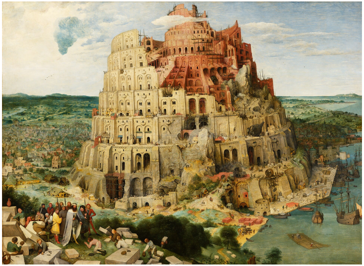
Challenges in electroweak corrections beyond NLO (continued)

Important calculations $\rightarrow$ required for successful phenomenology

- LHC:
  - NNLO QCD$\times$EW corrections and/or QCD/QED PS matching for $2 \rightarrow 2$ key processes
  - Drell–Yan: NNLO EW in pole approximation for M_W, $\sin^2\theta_\mathrm{eff}^\mathrm{lept}$
  - leading EW corrections beyond NLO at high energies
  - ...
- Future $\mathrm{e}^+\mathrm{e}^-$ colliders:
  - N^{3}LO EW for $\mu \rightarrow \mathrm{e}\bar{\nu}_\mathrm{e}\nu_\mu$ for M_W
  - Multi-loop corrections to EWPOs (e.g. ρ-parameter)
  - NNLO EW for $\mathrm{e}^+\mathrm{e}^- \rightarrow \mathrm{Z}/\gamma^* \rightarrow f\bar{f}$

    $\hookrightarrow$ check validity of pseudo-observable approach
  - NNLO EW for WW production at threshold
  - NNLO EW for ZH production, multi-loop calculations for H decays
  - ...

Challenges in electroweak corrections beyond NLO (continued)



Extremely huge effort,
highly specialized concepts/techniques,
long-lasting projects, ...

$\hookrightarrow$ Don't build a new Babel tower!

Validation, sustainability, legacy

- Proper documentation of methods/results
  $\hookrightarrow$ benchmark results, ancillary files for analytical results, public programs
- Libraries for integrals of even amplitudes?
- Tuned comparisons of independent results $\quad\to\quad$ working groups / reports
- Excite, engage and support young talents!

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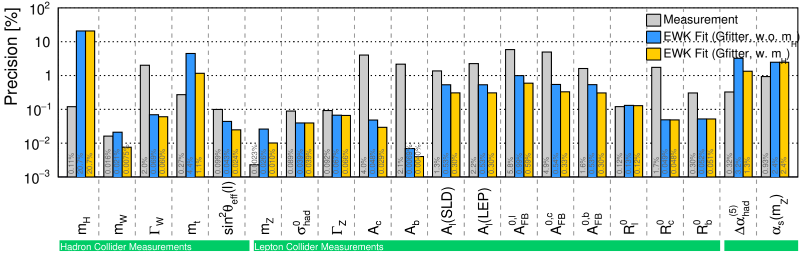
Outlook: electroweak precision physics at future e^+e^- colliders

Status of (not only) EW precision physics in the (pre HL-)LHC era

Erler, Schott '19



Current precision: typically $\lesssim 1\%$, even $\sim 0.01{-}0.1\%$ in some cases

Future projections: promise improvements by 1–2 orders of magnitude

$\hookrightarrow$ ultimate challenge of the SM at future e^+e^- colliders

But: Can theory provide adequate predictions?

Experimental errors and theory uncertainties

Experimental errors:

$\left.\begin{array}{l}\text{systematic errors}\\ \text{statistical errors}\end{array}\right\} \rightarrow$ LHC status + projections to HL/HE-LHC, ILC, FCC-ee
= input in the following

Theory uncertainties in predictions:

- Intrinsic uncertainties due to missing higher-order corrections, estimated from
  - generic scaling of higher order via coupling factors
  - renormalization and factorization scale variations
  - tower of known corrections, e.g. $\Delta_{\mathrm{NNLO}} \sim \delta_{\mathrm{NLO}}^2$ if δ_{NLO} known
  - different variants to include/resum leading higher-order effects
- Parametric uncertainties due to errors in input parameters, induced by
  - experimental errors in measurements
  - theory uncertainties in analyses

Note:
Estimates of theory uncertainties often (too) optimistic in projections of exp. results...

Physics at the Z pole – central EW precision (pseudo-)observables

FCC-ee: Freitas et al., 1906.05379; ILC: Moortgat-Pick et al., 1504.01726

	experimental accuracy			intrinsic theory uncertainty		
	current	ILC	FCC-ee	current	current source	prospect
$\Delta M_{\mathrm{Z}}[\mathrm{MeV}]$	2.1	–	0.1			
$\Delta \Gamma_{\mathrm{Z}}[\mathrm{MeV}]$	2.3	1	0.1	0.4	$\alpha^3, \alpha^2\alpha_{\mathrm{s}}, \alpha\alpha_{\mathrm{s}}^2$	0.15
$\Delta \sin^2\theta_{\mathrm{eff}}^{\ell}[10^{-5}]$	23	1.3	0.6	4.5	$\alpha^3, \alpha^2\alpha_{\mathrm{s}}$	1.5
$\Delta R_{\mathrm{b}}[10^{-5}]$	66	14	6	11	$\alpha^3, \alpha^2\alpha_{\mathrm{s}}$	5
$\Delta R_{\ell}[10^{-3}]$	25	3	1	6	$\alpha^3, \alpha^2\alpha_{\mathrm{s}}$	1.5

Theory requirements for Z-pole pseudo-observables:

- needed:
  - ◇ EW and QCD–EW 3-loop calculations
  - ◇ $1 \to 2$ decays, fully inclusive
- problems:
  - ◇ technical: massive multi-loop integrals, γ_5
  - ◇ conceptual: pseudo-obs. on the complex Z-pole

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$\Delta R_\ell[10^{-3}]$	25	3	1	6	1.5	1.3	α_s

Parametric uncertainties of EW pseudo-observables:

- QCD:
  - most important: $\delta\alpha_\mathrm{s} \sim 0.00015$ @ FCC-ee?
    $\hookrightarrow$ α_s from EW POs competitive $\Rightarrow$ cross-check with other results!
  - quark masses m_t, m_b, m_c
- $\Delta\alpha_\mathrm{had}$: $\delta(\Delta\alpha_\mathrm{had}) \sim 5(3)\times 10^{-5}$ for/from FCC-ee?
  - new exp. results from BES III / Belle II on $\mathrm{e^+e^-} \to \mathrm{hadrons}$
  - $\Delta\alpha_\mathrm{had}$ from fit to radiative return $\mathrm{e^+e^-} \to \gamma + \mathrm{hadrons}$
- other EW parameters: M_Z, M_W, M_H less critical (improved at ILC/FCC-ee)

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Parametric

- QCD:
  - mos
    $\hookrightarrow \alpha$ … s!
  - quark
- $\Delta\alpha_{\mathrm{had}}$: … for/from FCC-ee?
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Theory challenges

- Z Lineshape at NNLO EW precision (massive 2-loop, concept of unstable particles)
- higher-order ISR corrections way beyond NNLO
- concept of pseudo-observables at much higher level

W-boson mass measurements vs. prediction from μ decay

ILC: Baak et al., 1310.6708 FCC-ee: Freitas et al., 1906.05379

	experimental accuracy				theory uncertainty				
		σ_{WW} @ threshold			intrinsic			parametric	
	current	LEP2	ILC	FCC-ee	current	source	prospect	prospect	source
$\Delta M_{\mathrm{W}}[\mathrm{MeV}]$	13	200	3–6	0.5–1	3	$\alpha^3, \alpha^2\alpha_{\mathrm{s}}$	1	1(0.6)	$\Delta\alpha_{\mathrm{had}}$

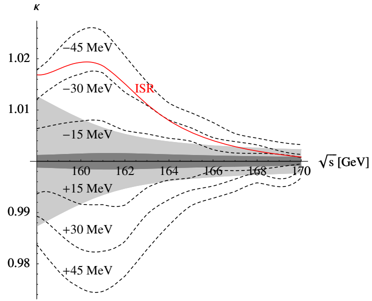
complicated reconstructions
Amoroso et al., 2308.09417

basically counting experiments

M_{W} calculated from μ decay

Sensitivity of σ_{WW} to M_{W}:

Beneke et al. '07



$$\kappa = \frac{\sigma_{\mathrm{WW}}(s, M_{\mathrm{W}} + \delta M_{\mathrm{W}})}{\sigma_{\mathrm{WW}}(s, M_{\mathrm{W}})}$$

$\Delta\kappa = 0.1\%\,(0.02\%) \leftrightarrow \delta M_{\mathrm{W}} = 1.5\,(0.3)\,\mathrm{MeV}$ for $\sqrt{s} = 161\,\mathrm{GeV}$

$\Rightarrow$ FCC-ee requires $\Delta_{\mathrm{TH}} \sim 0.01$–$0.04\%$ in σ_{WW}

Shaded areas / ISR curve:
some uncertainties of NLO(EFT) calculation,
improveable via full NLO($\mathrm{ee}\to 4f$) and NNLO(EFT)

W-boson mass measurements vs. prediction from μ decay

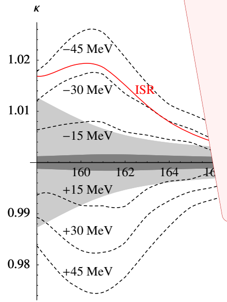
ILC: Baak et al., 1310.6708 FCC-ee: Freitas et al., 1906.05379

	experimental accuracy				theory uncertainty				
		σ_{WW} @ threshold			intrinsic			parametric	
	current	LEP2	ILC	FCC-ee	current	source	prospect	prospect	source
ΔM_W[MeV]	13	200	3–6	0.5–1	3	$\alpha^3, \alpha^2\alpha_s$ …		…0.6)	$\Delta\alpha_{had}$

complicated reconstruct…

Amoroso et al., 2308…

Sensitivity of σ_{WW}…



Theory challenges

- Full NLO $e^+e^- \to 4f$ prediction for each $4f$ type
- ISR to very high orders
- full NNLO calculation in threshold EFT + improvements
- for M_W analysis: M_W prediction from μ decay at 3 loops

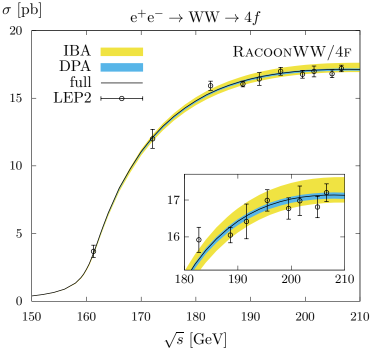
…(0.3) MeV

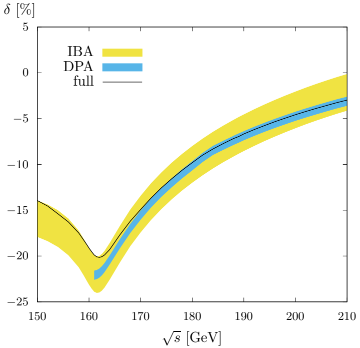
$\Rightarrow$ FCC-ee requires $\Delta_{TH} \sim 0.01$–0.04% in σ_{WW}

Shaded areas / ISR curve:
some uncertainties of NLO(EFT) calculation,
improveable via full NLO($ee\to4f$) and NNLO(EFT)

State-of-the-art prediction of σ_{WW} in LEP2 energy range

Denner, S.D., 1912.06823





- IBA = based on leading-log ISR and universal EW corrections ($\Delta \sim 2\%$)
  $\hookrightarrow$ shows large ISR impact near threshold (also by GENTLE)
- DPA = "Double-Pole Approximation" (leading term of resonance expansion)
  $\hookrightarrow$ $\Delta \sim 0.5\%$ above threshold, not applicable at threshold RacoonWW, YFSWW
- "full" = full NLO prediction for $e^+e^- \to 4f$ via charged current Denner et al. '05
  + leading-log improvements for ISR beyond NLO
  $\hookrightarrow$ $\Delta \sim 0.5\%$ everywhere

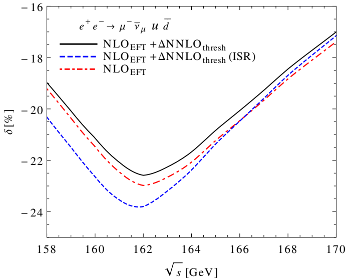
Improvements for σ_{WW} @ threshold via EFT

Beneke et al. '07; Actis et al. '08

EFT provides expansion of σ_{WW} for $\beta = \sqrt{1 - 4M_{\mathrm{W}}^2/s} \sim \sqrt{\Gamma_{\mathrm{W}}/M_{\mathrm{W}}} \sim \sqrt{\alpha}$:

$$
\begin{aligned}
\sigma_{\mathrm{WW}} \;=\; C\alpha^2\beta\Big[\; & 1 + c^{(0)}\beta && \text{LO} \\
& + \alpha\Big(\frac{c_1^{(1)}}{\beta} + c_2^{(1)}\ln\beta L_{\mathrm{e}} + c_3^{(1)} L_{\mathrm{e}} + c_4^{(1)} + c_5^{(1)}\beta\Big) && \text{NLO} \\
& + \alpha^2 \underbrace{\Big(\frac{c_1^{(2)}}{\beta^2} + \frac{c_2^{(2)}}{\beta} + c_3^{(2)}\ln^2\beta L_{\mathrm{e}}^2 + c_4^{(2)}\ln\beta L_{\mathrm{e}}^2 + \dots\Big)}_{\text{leading NNLO parts known}} + \dots\Big] && \text{NNLO}
\end{aligned}
$$

NNLO $\downarrow$ required for FCC-ee



ISR enhancement factor $L_{\mathrm{e}} = \ln(m_{\mathrm{e}}/M_{\mathrm{W}})$

Resummation of leading $(\alpha L_{\mathrm{e}})^n$ and subleading $\alpha(\alpha L_{\mathrm{e}})^{n-1}$ ISR necessary!

Theory issues in scan of $\sigma_{\mathrm{WW}}(s)$ over WW threshold

- Definition of σ_{WW} via $4f$ final states
  - $\mathrm{e}^{\pm}$ final states: separation or inclusion of single-W channels?
    $\hookrightarrow$ TH precision versus EXP accuracy
  - Hadronic final states: separation of multi-jet events (2j,3j,4j,...)
    $\hookrightarrow$ TH precision versus EXP accuracy
- Required for the best achievable theory prediction for σ_{WW}:
  - Full NLO $\mathrm{e}^+\mathrm{e}^- \to 4f$ prediction for each $4f$ type
    (interferences with ZZ and forward-$\mathrm{e}^{\pm}$ channels)
  - full NNLO EFT calculation (only leading terms available)
  - leading 3-loop Coulomb-enhanced EFT corrections
  - matching of all fixed-order $\mathrm{e}^+\mathrm{e}^- \to 4f$ and threshold-EFT ingredients
  - convolution of matched and corrected XS with higher-order ISR

  $\hookrightarrow$ Estimate of theory uncertainty:
  $\Delta \sim 0.01-0.04\%$ for σ_{WW} @ threshold   Freitas et al., 1906.05379

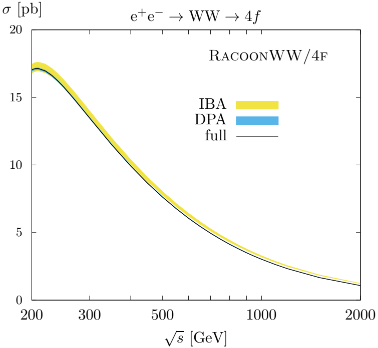
Improved M_{W} prediction from μ decay

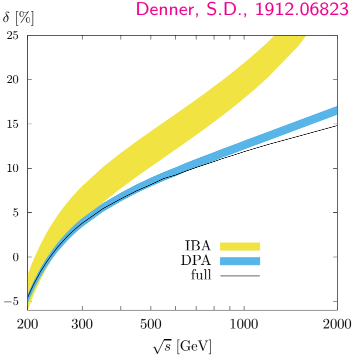
- Massive 3-loop computations (vacuum graphs, self-energies)

WW production beyond LEP2 energy range

Fixed-order NLO + leading-log ISR prediction:

Denner, S.D., 1912.06823





Note: large non-universal weak corrections + sizeable off-shell effects

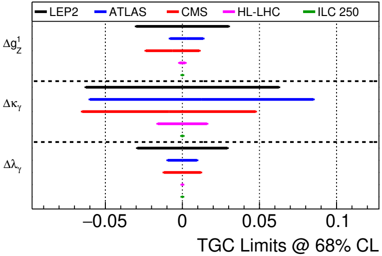
Achievable precision:

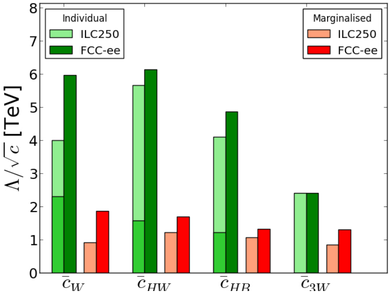
- by full NLO for $e^+e^- \to 4f$ + leading NNLO corrections + ISR resummation
- estimate: $\Delta \sim 0.5\%$ in distributions ($\sim 1\%$ in tails) up to $\sqrt{s} \sim 1\,\mathrm{TeV}$

Triple-gauge couplings (TGC) analyses in $\mathrm{e^+e^-} \to \mathrm{WW}$

- $\mathrm{e^+e^-}$ is ideal framework: no formfactors for damping required!
- SMEFT framework:
  sensitivity to dim-6 operators complementary to Higgs analyses Ellis, You '15

Bambade et al. '19





- Impact of $\Delta\kappa_\gamma$ on $\mathrm{d}\sigma_{\mathrm{WW}}$:

$\sqrt{s}/\mathrm{GeV}$	200	250	500
$\Delta\kappa_\gamma$	0.05	0.004	0.001
$\mathrm{d}\sigma_{\mathrm{WW}}(\kappa_\gamma)/\mathrm{d}\sigma^{\mathrm{SM}}_{\mathrm{WW}} - 1$	3%	$\sim$ 0.5%	$\sim$ 0.5%

$\hookrightarrow$ SM precision limits reach in TGCs for moderate $\sqrt{s}$!

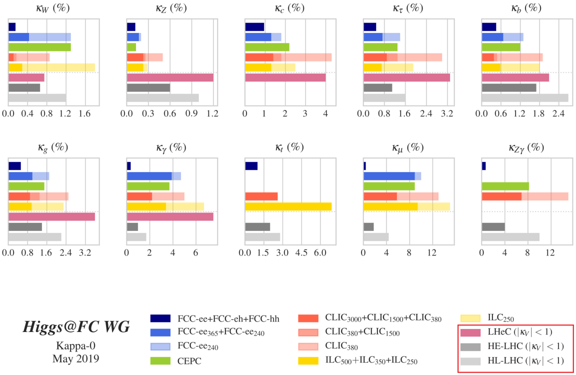
Theory homework for high-precision W-boson physics

- Exclusive analyses & predictions for $\mathrm{e^+e^-} \to 4f$:
  - $\mathrm{e}^{\pm}$ final states: proper treatment / separation of single-W channels
  - Hadronic final states: separation of multi-jet events (2j,3j,4j,...)
  - Full NLO $\mathrm{e^+e^-} \to 4f$ prediction for each $4f$ type (interferences with ZZ and forward-$\mathrm{e}^{\pm}$ channels)
  - more leading corrections beyond NLO
- σ_{WW} in threshold region:
  - full NNLO EFT calculation (only leading terms available)
  - leading 3-loop Coulomb-enhanced EFT corrections
  - matching of all fixed-order $\mathrm{e^+e^-} \to 4f$ and threshold-EFT ingredients

  $\hookrightarrow$ Estimate of theory uncertainty:
  $\Delta \sim 0.01 - 0.04\%$ for σ_{WW} @ threshold Freitas et al., 1906.05379
- For M_{W} analysis: Improved M_{W} prediction from μ decay
  - massive 3-loop computations (vacuum graphs, self-energies)

Higgs couplings analyses at present and future colliders

de Blas et al., 1905.03764



- Many different assumptions in different analyses! Read fine-print!
  Important details: $\Gamma_{\mathrm{H,BSM}} = 0$? $|\kappa_{\mathrm{W}}|, |\kappa_{\mathrm{Z}}| \le 1$? $\kappa_\gamma, \kappa_{\mathrm{g}}$ independent?
- Theory limitations!
  H couplings $\neq$ free parameters, rescaled model $\neq$ consistent field theory
  $\hookrightarrow$ QCD corrections often ok, but EW corrections ($\sim 5\%$) inconsistent!
  $\hookrightarrow$ Coupling rescalings (e.g. κ framework) uncertain to $\sim 5\%$!
  $\Rightarrow$ Use EFT like SMEFT (with corrections)!

Higgs decay widths and Higgs couplings at ILC and FCC-ee

LHC HXS WG; de Blas et al., 1905.03764; HL-LHC: Cepeda et al., 1902.00134; ILC: Bambade et al., 1903.01629 FCC-ee: Freitas et al., 1906.05379

	experimental accuracy			theory uncertainty			param. unc.	
	HL-LHC	ILC250	FCC-ee	current	source	prospect	prospect	source
$\mathrm{H} \to \mathrm{b}\bar{\mathrm{b}}$	4.4%	2%	0.8%	0.4%	α_{s}^5	0.2%	0.6%	m_{b}
$\mathrm{H} \to \tau\tau$	2.9%	2.4%	1.1%	0.3%	α^2	0.1%	negligible	
$\mathrm{H} \to \mu\mu$	8.2%	8%	12%	0.3%	α^2	0.1%	negligible	
$\mathrm{H} \to \mathrm{gg}$	1.6% (prod.)	3.2%	1.6%	3.2%	α_{s}^4	1%	0.5%	α_{s}
$\mathrm{H} \to \gamma\gamma$	2.6%	2.2%	3.0%	1%	α^2	1%	negligible	
$\mathrm{H} \to \gamma\mathrm{Z}$	19%			5%	α	1%	0.1%	M_{H}
$\mathrm{H} \to \mathrm{WW}$	2.8%	1.1%	0.4%	0.5%	$\alpha_{\mathrm{s}}^2, \alpha_{\mathrm{s}}\alpha, \alpha^2$	0.3%	0.1%	M_{H}
$\mathrm{H} \to \mathrm{ZZ}$	2.9%	1.1%	0.3%	0.5%	$\alpha_{\mathrm{s}}^2, \alpha_{\mathrm{s}}\alpha, \alpha^2$	0.3%	0.1%	M_{H}

Note: $\mathrm{e}^+\mathrm{e}^-$ colliders from $\sigma_{\mathrm{e}^+\mathrm{e}^-\to\mathrm{ZH}}$ with *inclusive* Higgs decays!

$\Rightarrow$ Absolute normalization of Higgs BRs

Higgs decay widths and Higgs couplings at ILC and FCC-ee

LHC HXS WG; de Blas et al., 1905.03764; HL-LHC: Cepeda et al., 1902.00134; ILC: Bambade et al., 1903.01629 FCC-ee: Freitas et al., 1906.05379

	experimental accuracy			th...	param. unc.	
	HL-LHC	ILC250	FC...		prospect	source
$H \to b\bar{b}$	4...				0.6%	m_b
$H \to \tau\tau$					negligible	
$H \to \mu\mu$	8...				negligible	
$H \to gg$	1.6...				...5%	α_s
$H \to \gamma\gamma$	2...				negligible	
$H \to \gamma Z$	19...				1%	M_H
$H \to WW$	2.8...				1%	M_H
$H \to ZZ$	2.9...			0.3%	0.1%	M_H

Theory challenges

- massive EW 2-loop caslculations for $e^+e^- \to ZH$, ...
- 4-/5-loop QCD calculations for $H \to b\bar{b}, gg$, ...
- off-shell NLO calculations if Higgs boson not fully reconstructible
- EFT calculations with radiative corrections

Note: e^+e^- colli... $\sigma_{e^+e^- \to ZH}$ with *inclusive* Higgs decays!

$\Rightarrow$ Absolute normalization of Higgs BRs

Enormous challenges for theory!

Can theory provide adequate predictions?

My expectation: Yes.

... anticipating progress + support for young theorists

Backup Slides

Electroweak input parameter schemes

SM input parameters: (natural choice)

α_{s}, α, M_{W}, M_{Z}, M_{H}, m_f, V_{CKM}

Issues:

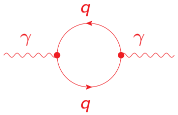
- Setting of α: process-specific choice to
  - avoid sensitivity to non-perturbative light-quark masses
  - minimize universal EW corrections

  Schemes: fix M_{W}, M_{Z}, and α
  - $\alpha(0)$-scheme: $\alpha = \alpha(0) = 1/137.0\ldots$
  - $\alpha(M_{\mathrm{Z}})$-scheme: $\alpha = \alpha(M_{\mathrm{Z}}^2) \approx 1/129$
  - G_μ-scheme: $\alpha = \alpha_{G_\mu} = \sqrt{2}G_\mu M_{\mathrm{W}}^2(1 - M_{\mathrm{W}}^2/M_{\mathrm{Z}}^2)/\pi \approx 1/132$

  $\hookrightarrow$ Some arbitrariness of $\sim 3{-}6\%$ per factor of α in LO prediction

- Warnings / pitfalls:
  - α must not be set diagram by diagram, but global factors like $\alpha(0)^m\alpha_{G_\mu}^n$ in gauge-invariant contributions mandatory !
  - weak mixing angle: $s_{\mathrm{W}} \neq$ free parameter if M_{W} and M_{Z} are fixed !
  - Yukawa couplings are uniquely fixed by fermion masses !

Running electromagnetic coupling $\alpha(s)$:



becomes sensitive to unphysical quark masses m_q
for $|s|$ in GeV range and below (non-perturbative regime)
$\hookrightarrow$ δZ_e and δZ_{AA} involve $\ln m_f$ with $f = q, \ell$

Solution: fit hadronic part of $\Delta\alpha(s) \;=\; -\mathrm{Re}\{\Sigma^{AA}_{\mathrm{T,R}}(s)/s\}$ and thus of δZ_e

via dispersion relation to $R(s) \;=\; \dfrac{\sigma(\mathrm{e}^+\mathrm{e}^- \to \text{hadrons})}{\sigma(\mathrm{e}^+\mathrm{e}^- \to \mu^+\mu^-)}$ Jegerlehner et al.

$\Rightarrow$ Running elmg. coupling: $\quad \alpha(s) \;=\; \dfrac{\alpha(0)}{1 - \Delta\alpha_{\mathrm{ferm}\neq\mathrm{top}}(s)}$

Universal contribution of $\Delta\alpha(M_\mathrm{Z}^2)$ to renormalization constants:

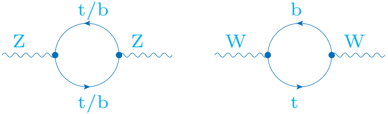
$$\delta Z_e = \tfrac{1}{2}\,\Delta\alpha(M_\mathrm{Z}^2) + \ldots, \qquad \delta Z_{AA} = -\Delta\alpha(M_\mathrm{Z}^2) + \ldots$$

Leading correction to the ρ-parameter:

mass differences in fermion doublets break custodial SU(2) symmetry

$\hookrightarrow$ large effects from bottom–top loops in W/Z self-energies   Veltman '77

- large corrections $\propto m_\mathrm{t}^2$ in $\Sigma_\mathrm{T}^{VV}(s)$, $V = W, Z$



$$\frac{\Sigma_\mathrm{T}^{ZZ}(s)}{M_\mathrm{Z}^2} - \frac{\Sigma_\mathrm{T}^{WW}(s)}{M_\mathrm{W}^2} \underset{|s|\ll m_\mathrm{t}^2}{\widetilde{\quad}} \frac{3G_\mu m_\mathrm{t}^2}{8\sqrt{2}\pi^2} \equiv \Delta\rho_\mathrm{top}$$

- m_t^2-enhanced terms show up in δs_W, δc_W, but cancel in $\Sigma_{\mathrm{T,R}}^{VV}(s)$
- leading terms to $\Delta\rho$ known beyond NLO

Universal contribution of $\Delta\rho$ to renormalization constants:

$$\frac{\delta c_\mathrm{W}^2}{c_\mathrm{W}^2} = -\Delta\rho_\mathrm{top} + \ldots, \qquad \frac{\delta s_\mathrm{W}^2}{s_\mathrm{W}^2} = \underbrace{\frac{c_\mathrm{W}^2}{s_\mathrm{W}^2}}_{\text{major effect due to } 1/s_\mathrm{W}^2 \text{ enhancement}} \Delta\rho_\mathrm{top} + \ldots$$

Adaption of input parameter schemes for cross-section predictions

Aim: absorb universal corrections from $\Delta\alpha$ and $\Delta\rho$
into leading-order (LO) predictions as much as possible

- $\Delta\alpha^n$ terms can be absorbed to all orders
- $\Delta\rho^n$ terms can be absorbed at least to two-loop order
- factor α in δ_{EW} can still be adjusted appropriately
  (e.g. $\alpha \to \alpha(0)$ if γ radiation dominates, $\alpha \to \alpha_{G_\mu}$ if weak corrections dominate)

Consider NLO cross section:

$$\sigma_{\mathrm{NLO}} = \alpha^N A_{\mathrm{LO}} \, (1 + \delta_{\mathrm{EW}}), \qquad \delta_{\mathrm{EW}} = \mathcal{O}(\alpha)$$

- for process at some generic energy scale $Q \gtrsim M_{\mathrm{W}}$
- with N_γ external photons (separable from $\gamma^* \to f\bar{f}$)
- with N_W couplings of $\mathrm{W/Z}$ in dominating LO diagrams
  ($\Delta\rho$ effects from c_{W} from difference between $\mathrm{W/Z}$ ignored)
  $\hookrightarrow$ N_W factors of $g_2^2 \propto 1/s_{\mathrm{W}}^2$ in LO cross section

$\alpha(0)$-scheme: $\sigma_{\mathrm{LO}} = \alpha(0)^N A_{\mathrm{LO}}$

$$\delta_{\mathrm{EW}}^{\alpha(0)} = 2N\,\delta Z_e + N_\gamma\,\delta Z_{AA} - N_W \frac{\delta s_{\mathrm{W}}^2}{s_{\mathrm{W}}^2} + \dots$$

$\alpha(0)$-scheme: $\quad \sigma_{\mathrm{LO}} = \alpha(0)^N A_{\mathrm{LO}}$

$$\delta_{\mathrm{EW}}^{\alpha(0)} = (N - N_\gamma)\,\Delta\alpha(M_Z^2) - N_W \frac{c_W^2}{s_W^2} \Delta\rho_{\mathrm{top}} + \dots$$

$\Rightarrow$ cancellation of $\Delta\alpha$, $\Delta\rho$ for $N_\gamma = N$, $N_W = 0$,

i.e. for processes such as $\gamma\gamma \to \ell^+\ell^-, \mathrm{W^+W^-}$, $\mathrm{e}\gamma \to \mathrm{e}\gamma$, etc.

$\alpha(M_Z)$-scheme: $\quad \sigma_{\mathrm{LO}} = \alpha(M_Z^2)^N A_{\mathrm{LO}}$

$$\delta_{\mathrm{EW}}^{\alpha(M_Z)} = \delta_{\mathrm{EW}}^{\alpha(0)} - N\Delta\alpha(M_Z) + \dots = -N_\gamma\,\Delta\alpha(M_Z^2) - N_W \frac{c_W^2}{s_W^2} \Delta\rho_{\mathrm{top}} + \dots$$

$\Rightarrow$ cancellation of $\Delta\alpha$, $\Delta\rho$ for $N_\gamma = 0$, $N_W = 0$,

which is not possible, since there is at least one Z exchange for $N_\gamma = 0$.

But: γ exchange dominates over Z exchange for $Q \ll M_W$ $(N_W \to 0)$

$\Rightarrow$ "$\alpha(Q)$ scheme" for neutral-current processes appropriate, $\mathrm{e^+e^-}/q\bar{q} \to \ell^+\ell^-$, etc.

G_μ-scheme: $\quad \sigma_{\mathrm{LO}} = \alpha_{G_\mu}^N A_{\mathrm{LO}}$

$$\delta_{\mathrm{EW}}^{G_\mu} = \delta_{\mathrm{EW}}^{\alpha(0)} - N\Delta r + \dots = -N_\gamma\,\Delta\alpha(M_Z^2) + (N - N_W) \frac{c_W^2}{s_W^2} \Delta\rho_{\mathrm{top}} + \dots$$

$\Rightarrow$ cancellation of $\Delta\alpha$, $\Delta\rho$ for $N_\gamma = 0$, $N_W = N$,

i.e. for $\mathrm{W/Z}$ decays, all EW processes without external γ at $Q \gtrsim M_W$

Mixed scheme: $\sigma_{\rm LO} = \alpha(G_\mu)^n \alpha(0)^m A_{\rm LO}$

$$\delta_{\rm EW}^{\rm mix} = \delta_{\rm EW}^{\alpha(0)} - n\,\Delta r + \ldots = (m - N_\gamma)\,\Delta\alpha(M_{\rm Z}^2) + (n - N_W)\,\frac{c_{\rm W}^2}{s_{\rm W}^2}\Delta\rho_{\rm top} + \ldots$$

$\Rightarrow$ cancellation of $\Delta\alpha$, $\Delta\rho$ for $N_\gamma = m$, $N_W = n$,

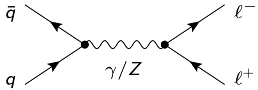
i.e. for all EW processes with m external γ at $Q \gtrsim M_{\rm W}$

Note: m does not include γ as parton from $\mathrm{p}/\bar{\mathrm{p}}$, because processes induced by $\gamma \to q\bar{q}, \ell\bar{\ell}$ cannot be separated form pure γ processes

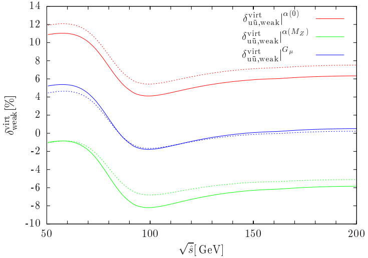
Harland-Lang et al. '16

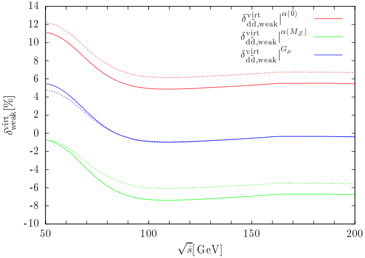
Example: weak corrections to Z production

(partonic cross sections, no photonic corrections)



S.D., Huber '09





- expected off-sets between NLO EW corrections in different schemes
- most suited EW input parameter schemes:

  $\sqrt{\hat{s}} \gtrsim M_{\rm Z}$: G_μ scheme

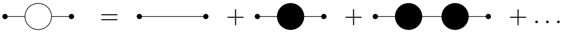
  $\sqrt{\hat{s}} \lesssim 70\,{\rm GeV}$: $\alpha(M_{\rm Z})$ scheme scheme ($\alpha(Q)$ scheme for $Q=\sqrt{\hat{s}} \ll M_{\rm Z}$)
- dashed lines include leading 2-loop effects from $\Delta\alpha$ and $\Delta\rho$
  $\hookrightarrow$ highest stability against h.o. corrections in recommended schemes

Unstable particles in Quantum Field Theory

description of resonances requires resummation of propagator corrections

$\hookrightarrow$ mixing of perturbative orders potentially violates gauge invariance

Dyson series and propagator poles (scalar example)



$$G_{\mathrm{R}}^{\phi\phi}(p) = \frac{\mathrm{i}}{p^2 - m^2} + \frac{\mathrm{i}}{p^2 - m^2}\,\mathrm{i}\Sigma_{\mathrm{R}}(p^2)\,\frac{\mathrm{i}}{p^2 - m^2} + \ldots = \frac{\mathrm{i}}{p^2 - m^2 + \Sigma_{\mathrm{R}}(p^2)}$$

$\Sigma_{\mathrm{R}}(p^2) =$ renormalized self-energy, $m =$ ren. mass

stable particle: $\mathrm{Im}\{\Sigma_{\mathrm{R}}(p^2)\} = 0$ at $p^2 \sim m^2$

$\hookrightarrow$ propagator pole for real value of p^2,
renormalization condition for physical mass m: $\Sigma_{\mathrm{R}}(m^2) = 0$

unstable particle: $\mathrm{Im}\{\Sigma_{\mathrm{R}}(p^2)\} \neq 0$ at $p^2 \sim m^2$

$\hookrightarrow$ location μ^2 of propagator pole is complex,
possible definition of mass M and width Γ: $\mu^2 = M^2 - \mathrm{i}M\Gamma$

Commonly used mass/width definitions:

- "on-shell mass/width" $M_{\mathrm{OS}}/\Gamma_{\mathrm{OS}}$: $\quad M_{\mathrm{OS}}^2 - M_0^2 + \mathrm{Re}\{\Sigma(M_{\mathrm{OS}}^2)\} \stackrel{!}{=} 0$

$$\hookrightarrow \quad G^{\phi\phi}(p) \underset{p^2\to M_{\mathrm{OS}}^2}{\widetilde{\quad\quad}} \frac{1}{(p^2 - M_{\mathrm{OS}}^2)(1+\mathrm{Re}\{\Sigma'(M_{\mathrm{OS}}^2)\}) + \mathrm{i}\,\mathrm{Im}\{\Sigma(p^2)\}}$$

comparison with form of Breit–Wigner resonance $\quad \dfrac{R_{\mathrm{OS}}}{p^2 - m^2 + \mathrm{i}m\Gamma}$

yields: $\quad M_{\mathrm{OS}}\Gamma_{\mathrm{OS}} \equiv \mathrm{Im}\{\Sigma(M_{\mathrm{OS}}^2)\} \,/\, (1+\mathrm{Re}\{\Sigma'(M_{\mathrm{OS}}^2)\}), \quad \Sigma'(p^2) \equiv \dfrac{\partial\Sigma(p^2)}{\partial p^2}$

- "pole mass/width" M/Γ: $\quad \mu^2 - M_0^2 + \Sigma(\mu^2) \stackrel{!}{=} 0$

complex pole position: $\quad \mu^2 \equiv M^2 - \mathrm{i}M\Gamma$

$$\hookrightarrow \quad G^{\phi\phi}(p) \underset{p^2\to \mu^2}{\widetilde{\quad\quad}} \frac{1}{(p^2-\mu^2)[1+\Sigma'(\mu^2)]} = \frac{R}{p^2 - M^2 + \mathrm{i}M\Gamma}$$

Note:

μ = gauge independent for any particle (pole location is property of S-matrix)

M_{OS} = gauge dependent at 2-loop order — Sirlin '91; Stuart '91; Gambino, Grassi '99; Grassi, Kniehl, Sirlin '01

Relation between "on-shell" and "pole" definitions:

Subtraction of defining equations yields:

$$M_{\mathrm{OS}}^2 + \mathrm{Re}\{\Sigma(M_{\mathrm{OS}}^2)\} \;=\; M^2 - \mathrm{i}M\Gamma + \Sigma(M^2 - \mathrm{i}M\Gamma)$$

Equation can be uniquely solved via recursion in powers of coupling α:

ansatz:
$$\begin{aligned} M_{\mathrm{OS}}^2 &= M^2 + c_1\alpha^1 + c_2\alpha^2 + \dots \\ M_{\mathrm{OS}}\Gamma_{\mathrm{OS}} &= M\Gamma + d_2\alpha^2 + d_3\alpha^3 + \dots, \qquad c_i, d_i = \text{real} \end{aligned}$$

counting in α: $\quad M_{\mathrm{OS}}, M = \mathcal{O}(\alpha^0), \quad \Gamma_{\mathrm{OS}}, \Gamma, \Sigma(p^2) = \mathcal{O}(\alpha^1)$

Result:

$$\begin{aligned} M_{\mathrm{OS}}^2 &= M^2 + \mathrm{Im}\{\Sigma(M^2)\}\,\mathrm{Im}\{\Sigma'(M^2)\} \;+\; \mathcal{O}(\alpha^3) \\ M_{\mathrm{OS}}\Gamma_{\mathrm{OS}} &= M\Gamma + \mathrm{Im}\{\Sigma(M^2)\}\,\mathrm{Im}\{\Sigma'(M^2)\}^2 \\ &\quad + \tfrac{1}{2}\,\mathrm{Im}\{\Sigma(M^2)\}^2\,\mathrm{Im}\{\Sigma''(M^2)\} \;+\; \mathcal{O}(\alpha^4) \end{aligned}$$

i.e. $\quad \{M_{\mathrm{OS}}, \Gamma_{\mathrm{OS}}\} \;=\; \{M, \Gamma\} \;+$ gauge-dependent 2-loop corrections

Important examples: W and Z bosons

In good approximation: $\mathrm{W} \to f\bar{f}'$, $\mathrm{Z} \to f\bar{f}$ with masses fermions f, f'

so that: $\mathrm{Im}\{\Sigma_\mathrm{T}^\mathrm{V}(p^2)\} = p^2 \times \frac{\Gamma_\mathrm{V}}{M_\mathrm{V}}\, \theta(p^2), \qquad \mathrm{V} = \mathrm{W}, \mathrm{Z}$

$$\hookrightarrow \quad M_\mathrm{OS}^2 = M^2 + \Gamma^2 + \mathcal{O}(\alpha^3) \qquad M_\mathrm{OS}\Gamma_\mathrm{OS} = M\Gamma + \frac{\Gamma^3}{M} + \mathcal{O}(\alpha^4)$$

In terms of measured numbers:

W boson: $M_\mathrm{W} \approx 80\,\mathrm{GeV}, \qquad \Gamma_\mathrm{W} \approx 2.1\,\mathrm{GeV}$

$\hookrightarrow \quad M_\mathrm{W,OS} - M_\mathrm{W,pole} \approx 28\,\mathrm{MeV}$

Z boson: $M_\mathrm{Z} \approx 91\,\mathrm{GeV}, \qquad \Gamma_\mathrm{Z} \approx 2.5\,\mathrm{GeV}$

$\hookrightarrow \quad M_\mathrm{Z,OS} - M_\mathrm{Z,pole} \approx 34\,\mathrm{MeV}$

Exp. accuracy: $\Delta M_\mathrm{W,exp}^\mathrm{ATLAS} = 16\,\mathrm{MeV}, \qquad \Delta M_\mathrm{Z,exp} = 2.1\,\mathrm{MeV}$

$\hookrightarrow$ Difference in definitions phenomenologically important !

Example of W and Z bosons continued:

Approximation of massless decay fermions:

$$\Gamma_{\mathrm{V,OS}}(p^2) \;=\; \Gamma_{\mathrm{V,OS}} \times \frac{p^2}{M^2_{\mathrm{V,OS}}}\,\theta(p^2), \qquad \mathrm{V} = \mathrm{W},\mathrm{Z}$$

Fit of W/Z resonance shapes to experimental data:

- ansatz $\left|\dfrac{R'}{p^2 - m'^2 + \mathrm{i}\gamma' p^2/m'}\right|^2$ yields: $m' = M_{\mathrm{V,OS}}, \quad \gamma' = \Gamma_{\mathrm{V,OS}}$
- ansatz $\left|\dfrac{R}{p^2 - m^2 + \mathrm{i}\gamma m}\right|^2$ yields: $m = M_{\mathrm{V,pole}}, \quad \gamma = \Gamma_{\mathrm{V,pole}}$

Note: The two forms are equivalent:

$$R = \frac{R'}{1+\mathrm{i}\gamma'/m'}, \quad m^2 = \frac{m'^2}{1+\gamma'^2/m'^2}, \quad m\gamma = \frac{m'\gamma'}{1+\gamma'^2/m'^2}$$

$\hookrightarrow$ consistent with relation between “on-shell” and “pole” definitions !

The issue of gauge invariance

Preliminary remarks:

The issue of gauge invariance goes

- beyond the definition of M and Γ and also
- beyond the question of parametrizing the resonance!

It is about the consistency of amplitudes everywhere in phase space, i.e.

- on resonance,
- in off-shell regions, and
- in the transition region between on-/off-shell domains.

Gauge-invariance requirements in amplitude calculations:

- proper cancellation of gauge-parameter dependences
  (relations between self-energies, vertex corrections, boxes, etc.)
- validity of (internal) Ward identities
  (e.g. ruling cancellations for forward scattering of $\mathrm{e}^{\pm}$ or at high energies)

$\Rightarrow$ Required: schemes to introduce width Γ

- without breaking gauge invariance
- maintaining (at least) NLO accuracy everywhere in phase space

Width schemes for LO calculations:

Naive propagator substitutions in full tree-level amplitudes:

$$\frac{1}{k^2 - m^2} \rightarrow \frac{1}{k^2 - m^2 + \mathrm{i} m\Gamma(k^2)}$$ for resonant or all propagators

- constant width $\Gamma(k^2) = \text{const.}$ $\rightarrow$ U(1) respected (if all propagators dressed), SU(2) "mildly" violated
- step width $\Gamma(k^2) \propto \theta(k^2)$ $\rightarrow$ U(1) and SU(2) violated
- running width $\Gamma(k^2) \propto \theta(k^2) \times k^2$ $\rightarrow$ U(1) and SU(2) violated
  $\hookrightarrow$ results can be totally wrong !

Complex-mass scheme

Denner et al. '99

Complex masses for $V = \mathrm{W}, \mathrm{Z}$ from

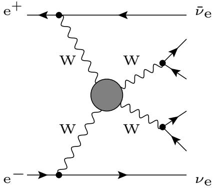
$$\mu_V^2 = M_V^2 - \mathrm{i} M_V \Gamma_V = \text{location of complex poles in } V \text{ propagators}$$

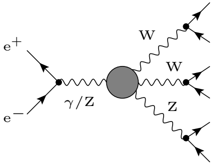
Complex (on-shell) weak mixing angle via $c_\mathrm{W} = \mu_\mathrm{W}/\mu_\mathrm{Z}$

$\Rightarrow$ All algebraic relations expressing gauge invariance hold exactly
(gauge-parameter cancellation, Ward identities).

Major benefit: Generalization to NLO Denner et al. '05; Denner, SD '19
provides NLO accuracy everywhere in phase space!

LO example from e^+e^- physics: $\sigma[\mathrm{fb}]$ for $e^+e^- \to \nu_e\bar{\nu}_e\mu^-\bar{\nu}_\mu u\bar{d}$ (with cuts)





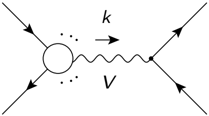
$+$ etc.

$\sqrt{s}$	500 GeV	800 GeV	2 TeV	10 TeV
constant width	1.633(1)	4.105(4)	11.74(2)	26.38(6)
running width	1.640(1)	4.132(4)	12.88(1)	12965(12)
complex mass	1.633(1)	4.104(3)	11.73(1)	26.39(6)

S.D., Roth '02

$\leftarrow$ totally wrong!

High-energy behaviour of longitudinal $V = \mathrm{W/Z}$ bosons:



$$\underset{k^0 \gg M_V}{\sim} \quad \frac{1}{k^2 - M_V^2}\, k^\mu\, T^V_\mu \;=\; \frac{1}{k^2 - M_V^2}\, c_V M_V T^S$$

(S = Goldstone partner of V)

SU(2) Ward identity $k^\mu T^V_\mu = c_V M_V T^S$ essential to cancel factor k^0,
otherwise gauge-invariance/unitarity-breaking terms enhanced by k^0/M_V

Width schemes for higher-order calculations:

- **Pole Scheme (PS)** Stuart '91; Aeppli et al. '93, '94; etc.
  Isolate resonance in a gauge-invariant way and introduce Γ only there:

$$\mathcal{M} = \frac{R(p^2)}{p^2 - M^2} + N(p^2) \; = \; \frac{R(M^2)}{p^2 - M^2} + \frac{R(p^2) - R(M^2)}{p^2 - M^2} + N(p^2)$$

$$\to \underbrace{\frac{\tilde{R}(M^2 - \mathrm{i}M\Gamma)}{p^2 - M^2 + \mathrm{i}M\Gamma}}_{\text{resonant}} + \underbrace{\frac{R(p^2) - R(M^2)}{p^2 - M^2}}_{\text{non-res./non-fact. corrs.}} + \underbrace{\tilde{N}(p^2)}_{\text{non-resonant}}$$

  $\hookrightarrow$ consistent, gauge invariant, NLO everywhere possible,
  but subtle and cumbersome in practice (complex kinematics, pole location is branch point rather than pole, IR structure of radiation)

- **Leading pole approximation (PA)**
  Take term with highest resonance enhancement of pole expansion
  = leading term of Pole Scheme

  $\hookrightarrow$ consistent, gauge invariant, straightforward,
  but valid only in resonance neighbourhood,
  rel. uncertainty for EW corrections $= \frac{\alpha}{\pi} \times \mathcal{O}(\Gamma/M)$

▶ Complex-mass scheme at NLO Denner et al. '05; Denner, S.D. '19

mass^2 = location of propagator pole in complex p^2 plane

↪ complex mass renormalization: $\underbrace{M^2_{\mathrm{W},0}}_{\text{bare mass}} = \mu^2_{\mathrm{W}} + \underbrace{\delta\mu^2_{\mathrm{W}}}_{\text{ren. constant}},$ etc.

Gauge invariance by complex weak mixing angle:

$$c_{\mathrm{W}} = \frac{\mu_{\mathrm{W}}}{\mu_{\mathrm{Z}}}, \qquad \frac{\delta c^2_{\mathrm{W}}}{c^2_{\mathrm{W}}} = \frac{\delta\mu^2_{\mathrm{W}}}{\mu^2_{\mathrm{W}}} - \frac{\delta\mu^2_{\mathrm{Z}}}{\mu^2_{\mathrm{Z}}}$$

Features of the complex-mass scheme:

- ⊕ perturbative calculations as usual (with complex masses and couplings)
- ⊕ no double counting of contributions (bare Lagrangian unchanged!)
- ⊕ gauge invariance (ST identities, gauge-parameter independence)
- ⊕ NLO accuracy everywhere in phase space
- ▶ spurios terms are beyond NLO, but spoil unitarity
- ▶ complex gauge-boson masses also in loop integrals (all known)
- ⊖ unstable particles only allowed as resonances (not as external states)
- ⊖ generalization to NNLO not yet known (but expected to work)

Technical details, exemplified for W bosons:

OS renormalization conditions for renormalized (transverse) self-energy

$$\Sigma^{W}_{\mathrm{T,R}}(\mu^2_{\mathrm{W}}) \;=\; 0, \quad \Sigma'^{W}_{\mathrm{T,R}}(\mu^2_{\mathrm{W}}) \;=\; 0$$

$\hookrightarrow$ μ^2_{W} is location of propagator pole, and residue $= 1$

Solution of renormalization conditions:

$$\delta\mu^2_{\mathrm{W}} \;=\; \Sigma^{W}_{\mathrm{T}}(\mu^2_{\mathrm{W}}), \quad \delta\mathcal{Z}_W \;=\; -\Sigma'^{W}_{\mathrm{T}}(\mu^2_{\mathrm{W}})$$

Note: Evaluation of $\Sigma^{W}_{\mathrm{T}}(p^2)$ at complex p^2 can be avoided

$$\Sigma^{W}_{\mathrm{T}}(\mu^2_{\mathrm{W}}) \;=\; \Sigma^{W}_{\mathrm{T}}(M^2_{\mathrm{W}}) + (\mu^2_{\mathrm{W}} - M^2_{\mathrm{W}})\Sigma'^{W}_{\mathrm{T}}(M^2_{\mathrm{W}}) + \underbrace{\tfrac{\alpha}{\pi}\mathrm{i}M_{\mathrm{W}}\Gamma_{\mathrm{W}}}_{\substack{\text{from non-analyticity}\\ \text{at } p^2 \,=\, M^2_{\mathrm{W}}}} + \underbrace{\mathcal{O}(\alpha^3)}_{\substack{\text{beyond one loop}\\ \text{and finite}}}$$

$\Rightarrow$ Renormalized W self-energy:

$$\Sigma^{W}_{\mathrm{T,R}}(p^2) \;=\; \Sigma^{W}_{\mathrm{T}}(p^2) - \delta M^2_{\mathrm{W}} + (p^2 - M^2_{\mathrm{W}})\delta Z_W$$

$$\text{with} \quad \delta M^2_{\mathrm{W}} \;=\; \Sigma^{W}_{\mathrm{T}}(M^2_{\mathrm{W}}) + \tfrac{\alpha}{\pi}\mathrm{i}M_{\mathrm{W}}\Gamma_{\mathrm{W}}, \quad \delta Z_W \;=\; -\Sigma'^{W}_{\mathrm{T}}(M^2_{\mathrm{W}})$$

Differences to the usual on-shell scheme:

- no real parts taken from Σ^{W}_{T}
- Σ^{W}_{T} evaluated with complex masses and couplings