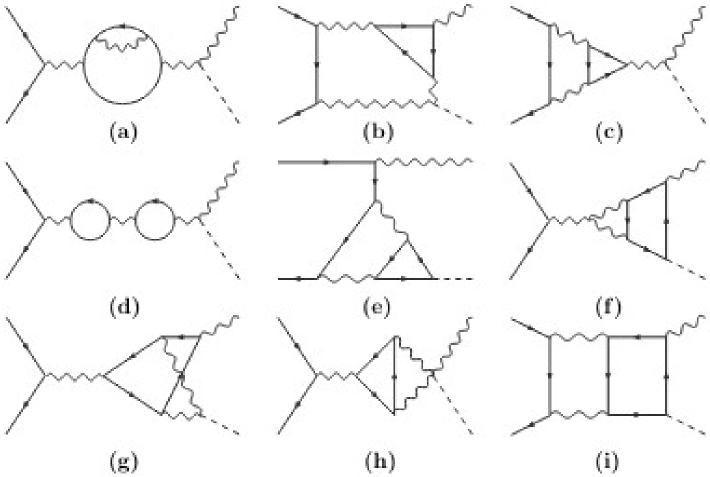
Techniques for 2-loop electroweak calculations

A. Freitas (U. Pittsburgh)

- Introduction
- Computational approach
- UV divergences
- Numerical results
- Conclusions



A. Freitas and Q. Song,
JHEP 04 (2021) 179 [arXiv:2101.00308]

A. Freitas and Q. Song,
PRL 130, 031801 [arXiv:2209.07612]

A. Freitas, Q. Song and K. Xie,
PRD 108, 053006 [arXiv:2305.16547]

Introduction

- Higher-order calculations in electroweak SM are challenging (many mass scales: m_Z, m_W, m_H, m_t)
- Analytic calculations:
  - IBP reduction to master integrals: large expressions and computing resources
  - Complete function space of master integrals unknown (harmonic polylogs, iterated elliptic integrals, …)
- Numerical calculations (e.g. in momentum or Feynman par. space)
  - Multi-dim. integration space, slowly converging
- New approaches using series solutions of diff. eqs.
  - still require IBP reduction

[Liu, Ma, Wang, 1711.09572] [Moriello, 1907.13234]
[Hidding, 2006.05510] [Liu, Ma, 2201.11669]
[Armadillo, Bonciani, Devoto, Rana, Vicini, 2205.03345]

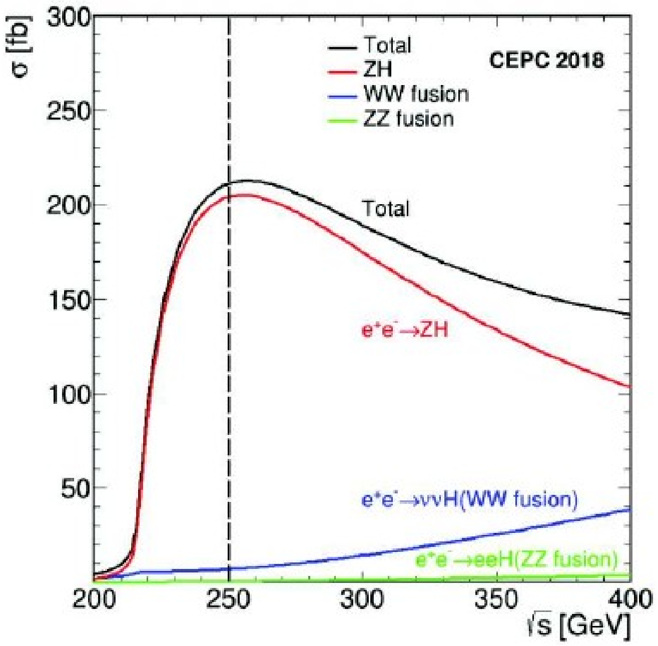
- **This work:** semi-numerical approach, tailored for EW 2-loop problems

Introduction

- **Application**: ee→HZ: dominant Higgs prod. process at e+e- colliders below 500 GeV

- Expected precision:

ILC	1.2%	[1903.01629]
CEPC	0.5%	[1811.10545]
FCC-ee	0.4%	[EPJ ST 228, 261]

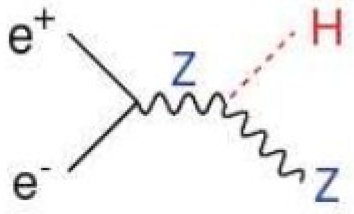


- Need higher-order corrections:

	$\alpha(0)$ scheme	G_μ scheme
σ^{LO} [fb]	223.14	239.64
σ^{NLO} [fb]	229.78	232.46
$\sigma^{\mathrm{NNLO,EW}\times\mathrm{QCD}}$ [fb]	232.21	233.29

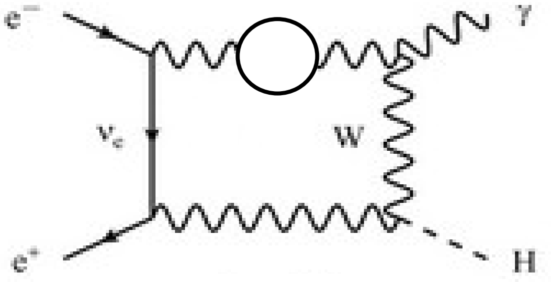
EW NNLO expected O(1%)

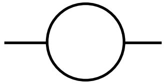
Gong et al. '16
Chen, Feng, Jia, Sang '18



Computational approach

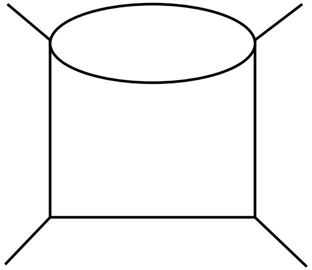
- Basic idea: use dispersion relation for sub-loop





$$= B_0(p^2, m_1^2, m_2^2) = \int_{(m_1+m_2)^2}^{\infty} \mathrm{d}\sigma \, \frac{\Delta B_0(\sigma, m_1^2, m_2^2)}{\sigma - p^2 - i\epsilon},$$

$$\Delta B_0(\sigma, m_1^2, m_2^2) \equiv \frac{1}{\pi} \mathrm{Im}\, B_0(\sigma, m_1^2, m_2^2)$$

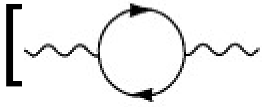


$$= -\int d\sigma \, \Delta B_0(\sigma, \ldots) \, D_0(\ldots, \sigma)$$

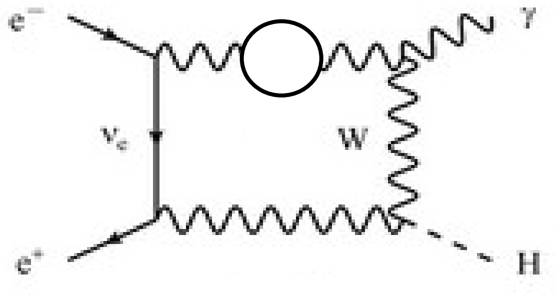
[Bauberger, Berends, Bohm, Buza, hep-ph/9409388]

Computational approach

- Basic idea: use dispersion relation for sub-loop
- including numerator terms:



$$\Big[\;\cdots\;\Big]_T = \int d\sigma \, \frac{c_0 \Delta B_0(\sigma, \ldots) + c_1 \Delta B_1(\sigma, \ldots) + c_{00} \Delta B_{00}(\sigma, \ldots) + \ldots}{\sigma - p^2 - i\epsilon}$$



$$= -\int d\sigma \, [c_0 \Delta B_0 + c_1 \Delta B_1 + c_{00} \Delta B_{00} + \ldots]$$
$$\times \, [a_1 D_0 + a_2 D_1 + \ldots + a_n C_0 + a_{n+1} C_1 + \ldots]$$

(coefficients depend on masses, external momenta and σ)

Computational approach: box diagrams

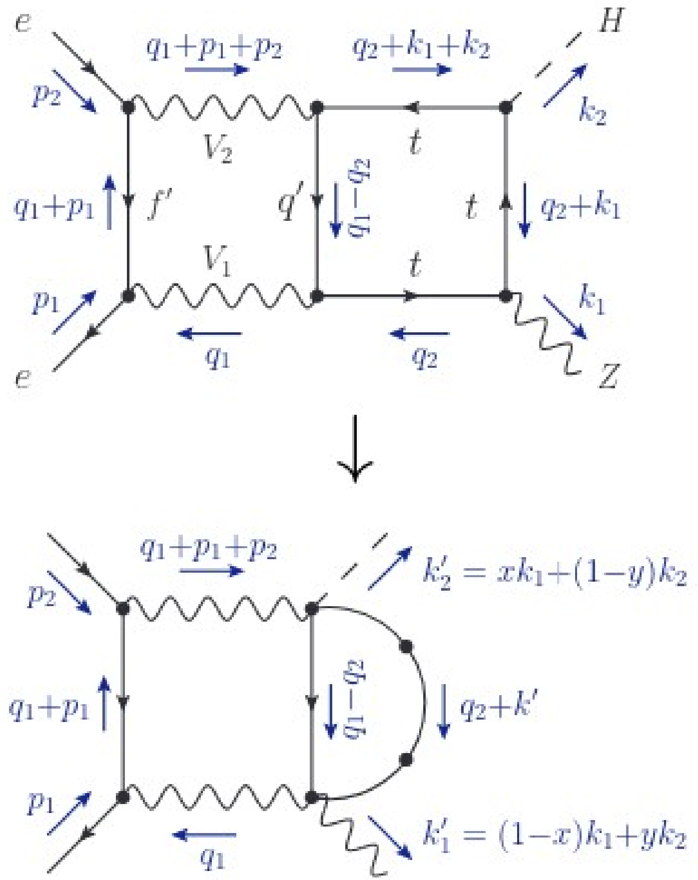
- Introduce Feynman parameters

- q_2 loop $= \int dx\, dy \, \frac{\partial^2}{\partial(m'^2)^2} \int_{\sigma_0}^{\infty} d\sigma \, \frac{\Delta B_0(\sigma, m'^2, m_{q'}^2)}{\sigma - \tilde{q}_1^2}$

$$\tilde{q}_1 = q_1 + k' + i\epsilon,$$

$$m'^2 = m_t^2 - xy(k_1 + k_2)^2 - (1 - x - y)(x k_1^2 + y k_2^2).$$

- Similarly use Feynman pars. for other box and vertex diagrams



Computational approach: box diagrams

- q_2 loop $= \int dx\, dy\, \frac{\partial^2}{\partial (m'^2)^2} \int_{\sigma_0}^{\infty} d\sigma\, \frac{\Delta B_0(\sigma, m'^2, m_{q'}^2)}{\sigma - \tilde{q}_1^2}$ $\quad \sigma_0 = (m' + m_{q'})^2$

$$= \int dx\, dy \left\{ \int_{\sigma_0}^{\infty} d\sigma\, \frac{\partial_{m'}^2 \Delta B_0(\sigma, m'^2, m_{q'}^2)}{\sigma - \tilde{q}_1^2} - \left[\frac{\partial_{m'} \Delta B_0(\sigma, m'^2, m_{q'}^2)}{\sigma - \tilde{q}_1^2} \right]_{\sigma \to \sigma_0} \right\}$$

Derivatives of ΔB_0 can be easily computed

- Problem: Each term blows up for $\sigma \to \sigma_0$
- Solution: Modify integrand to cancel boundary term

$$\int dx\, dy \left\{ \int_{\sigma_0}^{\infty} d\sigma\, \partial_{m'}^2 \Delta B_0(\sigma, m'^2, m_{q'}^2) \left(\frac{1}{\sigma - \tilde{q}_1^2} - \frac{\sigma_0}{\sigma(\sigma_0 - \tilde{q}_1^2)} \right) + \frac{\sigma_0}{\sigma_0 - \tilde{q}_1^2}\, \partial_{m'}^2 B_0(0, m'^2, m_{q'}^2) \right\}.$$

Computational approach: box diagrams

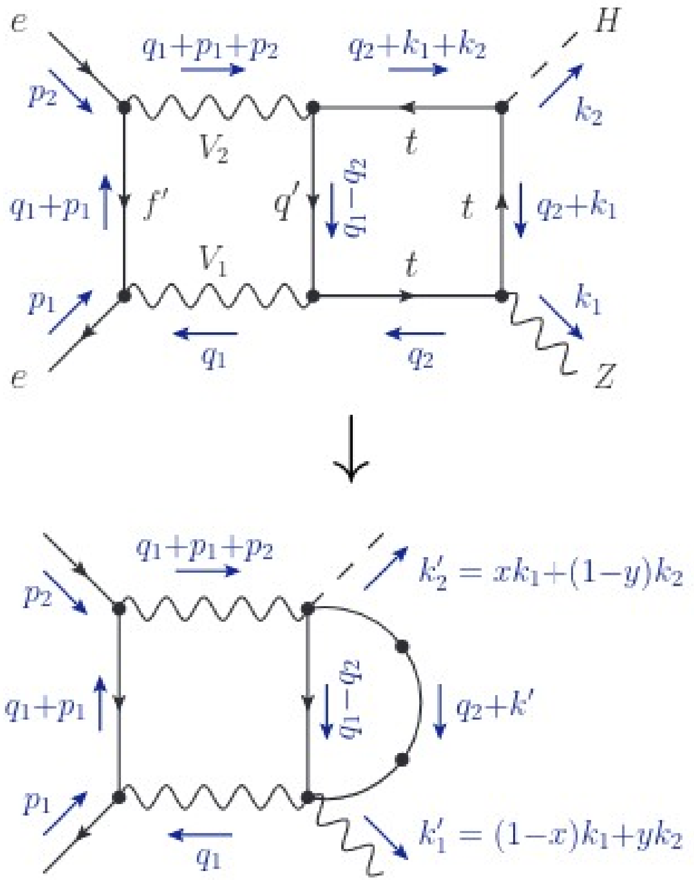
- Introduce Feynman parameters

- q_2 loop $= \int dx\, dy\, \frac{\partial^2}{\partial(m'^2)^2} \int_{\sigma_0}^{\infty} d\sigma\, \frac{\Delta B_0(\sigma, m'^2, m_{q'}^2)}{\sigma - \tilde{q}_1^2}$

$$\tilde{q}_1 = q_1 + k' + i\epsilon,$$

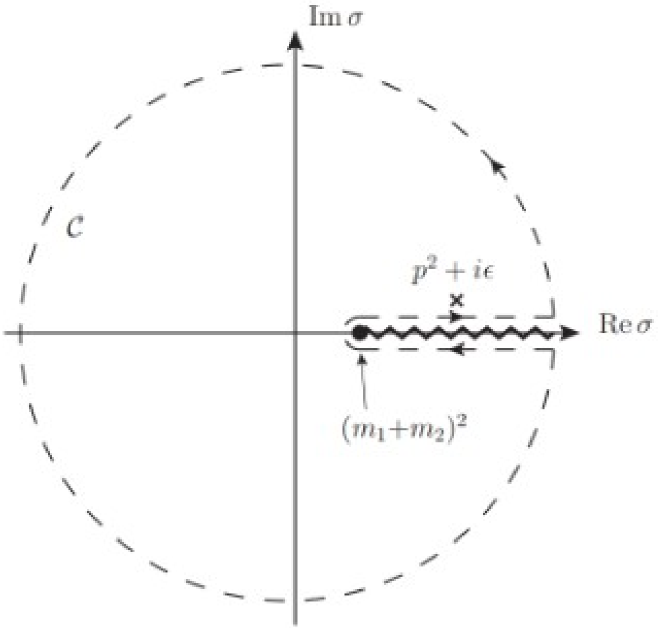
$$m'^2 = m_t^2 - xy(k_1 + k_2)^2 - (1 - x - y)(xk_1^2 + yk_2^2).$$

- Problem: m'^2 can in general become negative!



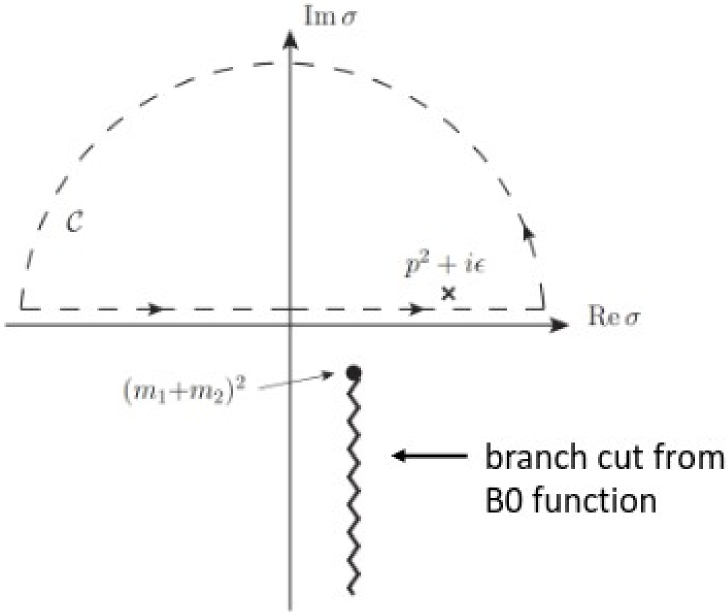
Computational approach: box diagrams

$m_1^2 \geq 0\ ,\ m_2^2 \geq 0$



integration
contours

$m_1^2 > 0\ ,\ m_2^2 < 0$



formulas

$$B_0(p^2, m_1^2, m_2^2) = \frac{1}{2\pi i} \oint d\sigma \frac{B_0(\sigma, m_1^2, m_2^2)}{\sigma - p^2 - i\epsilon}$$
$$= \int_{(m_1+m_2)^2}^{\infty} d\sigma \frac{\Delta B_0(\sigma, m_1^2, m_2^2)}{\sigma - p^2 - i\epsilon}$$

$$B_0(p^2, m_1^2, m_2^2) = \frac{1}{2\pi i} \oint_C d\sigma \frac{B_0(\sigma, m_1^2, m_2^2)}{\sigma - p^2 - i\epsilon}$$
$$= \frac{1}{2\pi i} \int_{-\infty}^{+\infty} d\sigma \frac{B_0(\sigma, m_1^2, m_2^2)}{\sigma - p^2 - i\epsilon}$$

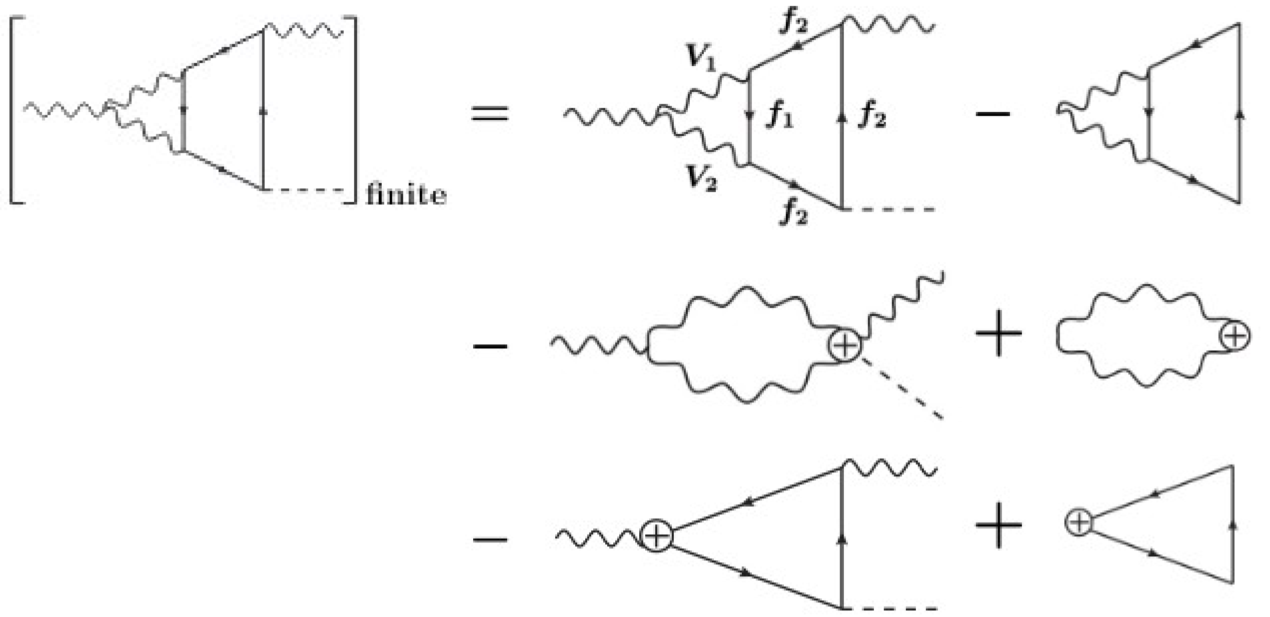
UV divergences

- UV divergences will cause the num. integral to diverge
- Need to subtract terms so make integral finite
- Subtraction terms simple enough to integrate analytically and add back

$$
\begin{aligned}
|M_0 M_2^*| &\sim \int dx \int dy \int d\sigma \times \underbrace{[\text{integrand}]}_{\text{UV div}} \\
&= \int dx \int dy \int d\sigma \times \underbrace{[\text{integrand} - I_{\text{subtra}}]}_{\text{UV finite, integrate numerically}} \\
&+ \int dx \int dy \int d\sigma \times \underbrace{[I_{\text{subtra}}]}_{\text{UV div, integrate analytically}}
\end{aligned}
$$

UV divergences

- Separate treatment for global divergence and two sub-loop divergences



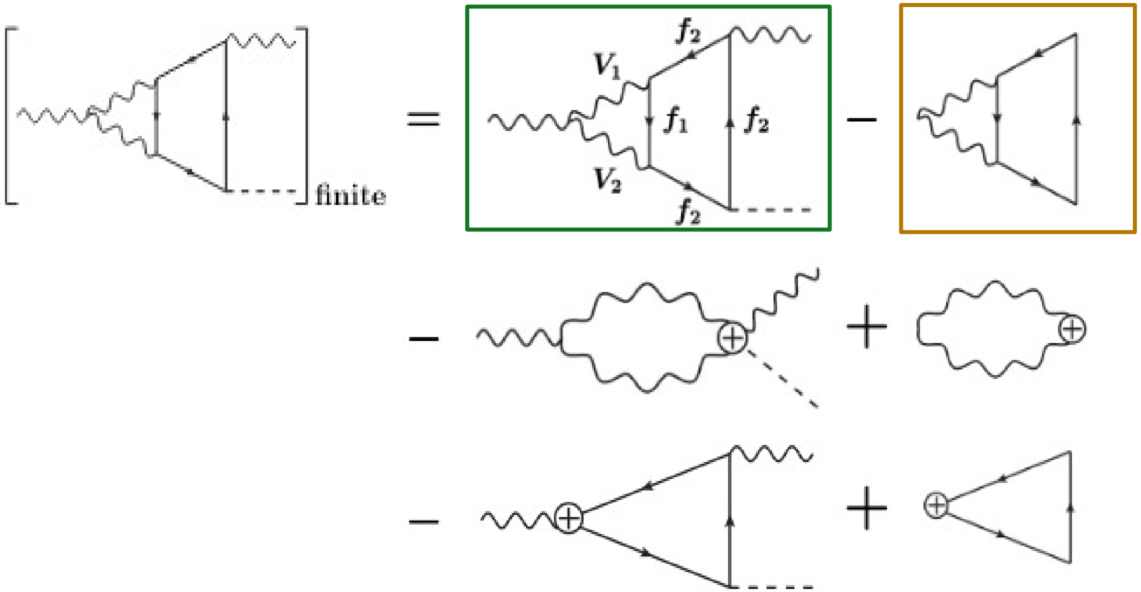
UV divergences: Example

$$\mathcal{I} = \int\!\!\int \frac{q_1^4}{(q_2^2 - m_{V_2}^2)((q_2+p)^2 - m_{V_1}^2)((q_2+q_1)^2 - m_{f_1}^2)(q_1^2 - m_{f_2}^2)((q_1-p_h)^2 - m_{f_2}^2)((q_1-p)^2 - m_{f_2}^2)}$$

$$\mathcal{I}_{\text{sub}}^{\text{glob}} = \int\!\!\int \frac{q_1^4}{(q_2^2 - m_{V_2}^2)(q_2^2 - m_{V_1}^2)((q_2+q_1)^2 - m_{f_1}^2)(q_1^2 - m_{f_2}^2)^3}$$

- Global divergence: Subtract integral with zero external momenta
- 2-loop vacuum integrals known analytically (here using **`FIRE`** and **`TVID`**)

  [A. Smirnov, 2020]
  [Bauberger and Freitas, 2017]

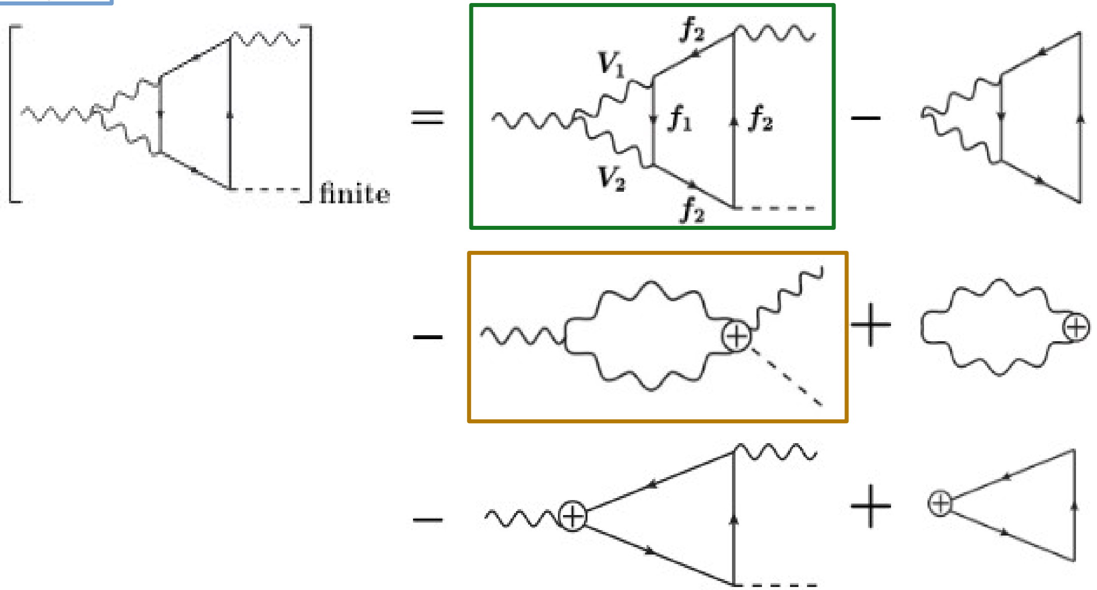


UV divergences: Example

$$\mathcal{I} = \int\int \frac{q_1^4}{(q_2^2 - m_{V_2}^2)((q_2+p)^2 - m_{V_1}^2)((q_2+q_1)^2 - m_{f_1}^2)(q_1^2 - m_{f_2}^2)((q_1-p_h)^2 - m_{f_2}^2)((q_1-p)^2 - m_{f_2}^2)}$$

$$\mathcal{I}_{\text{sub}}^{q_1} = \int\int \frac{q_1^4}{(q_2^2 - m_{V_2}^2)((q_2+p)^2 - m_{V_1}^2)(q_1^2 - m_{f_1}^2)(q_1^2 - m_{f_2}^2)^3}$$

$$= B_0(p^2, m_{V_1}^2, m_{V_2}^2) \times [c_1 A_0(m_{f_1}^2) + c_2 A_0(m_{f_2}^2)]$$



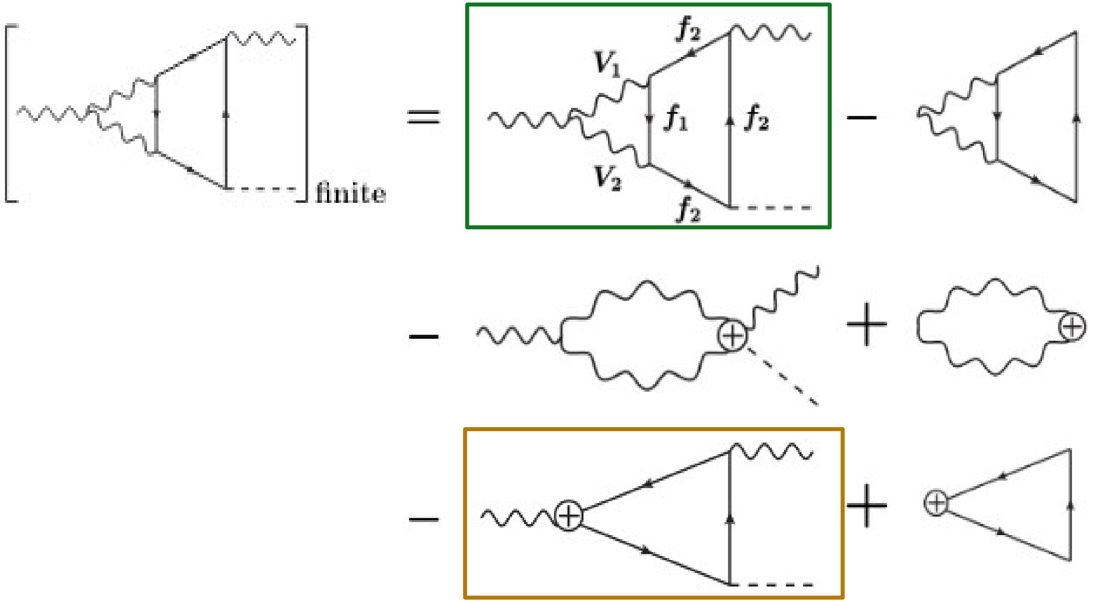
- Sub-loop divergence: Subtract sub-loop in large q_1 limit
- factorizes into product of 1-loop functions

UV divergences: Example

$$\mathcal{I} = \int\int \frac{q_2^2}{(q_2^2 - m_{V_2}^2)((q_2+p)^2 - m_{V_1}^2)((q_2+q_1)^2 - m_{f_1}^2)(q_1^2 - m_{f_2}^2)((q_1-p_h)^2 - m_{f_2}^2)((q_1-p)^2 - m_{f_2}^2)}$$

$$\mathcal{I}_{\rm sub}^{q_2} = \int\int \frac{q_2^2}{(q_2^2 - m_{V_2}^2)(q_2^2 - m_{V_1}^2)(q_2^2 - m_{f_1}^2)(q_1^2 - m_{f_2}^2)((q_1-p_h)^2 - m_{f_2}^2)((q_1-p)^2 - m_{f_2}^2)}$$

- Sub-loop divergence: Subtract sub-loop in large q_2 limit
- factorizes into product of 1-loop functions



Implementation

- Diagram generation with **`FeynArts`** [Hahn, 2001]
- Algebraic manipulations in **`Mathematica`**:
  - Construction of integrand (Feynman parameters & dispersion relation) for each diagram type
  - No IBP reduction
  - UV subtraction terms
  - Generate C++ code for subtracted integrand
- Numerical integration in C++
  - Passarino-Veltman functions (D_0, D_1, etc.) from **`LoopTools`** [Hahn, Perez-Victoria, 1999]
  - Adaptive Gauss integration (>3 digit accuracy in minutes in single core for one diagram type)

Implementation

- Disp. integral $\int_{\sigma_0}^{\infty} d\sigma$ … can become noisy for large σ

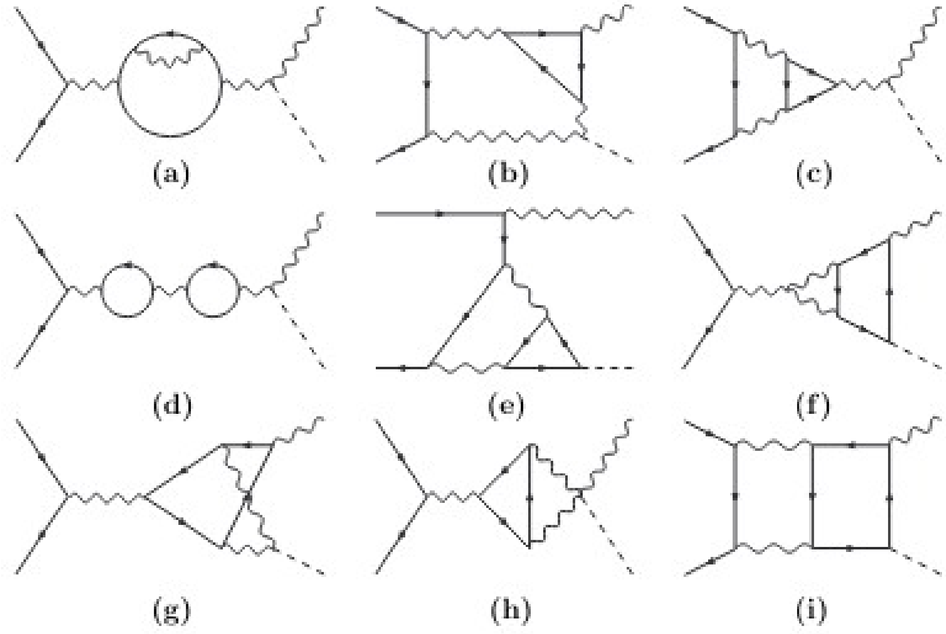
  → use cutoff (check cutoff independence within uncertainties)

- Small imag. part for negative masses ensures that **`LoopTools`** picks correct complex branch

- Overall precision limited by floating point numbers
  - ~3 digits with double precision
  - >6 digits with quadruple precision (but more running time)

Numerical results

- Computed full EW NNLO corrections with closed fermion loops (finite and gauge-invariant subset, typically dominant)
- Universal ISR QED effects factorized
- Final-state Z-boson defined as leading-pole term, final-state Higgs in narrow-width approx.



Numerical results

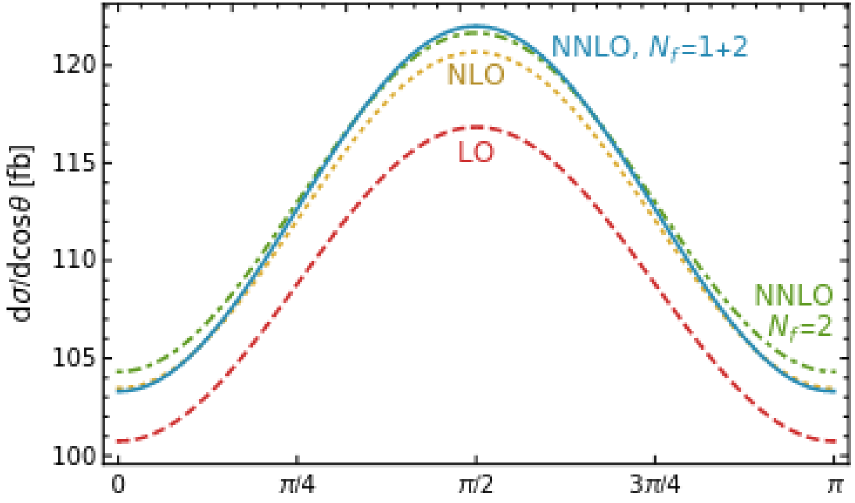
- Use complex pole mass scheme [e.g. Freitas, Hollik, Walter, Weiglein, hep-ph/0202131]

$$m_W^{\exp} = 80.379 \text{ GeV} \quad \Rightarrow \quad m_W = 80.352 \text{ GeV},$$
$$m_Z^{\exp} = 91.1876 \text{ GeV} \quad \Rightarrow \quad m_Z = 91.1535 \text{ GeV},$$

$$m_Z = m_Z^{\exp}\,[1 + (\Gamma_Z^{\exp}/m_Z^{\exp})^2]^{-1/2},$$
$$\Gamma_Z = \Gamma_Z^{\exp}\,[1 + (\Gamma_Z^{\exp}/m_Z^{\exp})^2]^{-1/2}.$$

$$m_H = 125.1 \text{ GeV}, \qquad m_t = 172.76 \text{ GeV},$$
$$\alpha^{-1} = 137.036, \qquad \Delta\alpha = 0.059,$$

	(fb)	Contribution	(fb)
σ^{LO}	222.958		
σ^{NLO}	229.893		
		$\mathcal{O}(\alpha_{N_f=1})$	21.130
		$\mathcal{O}(\alpha_{N_f=0})$	−14.195
σ^{NNLO}	231.546		
		$\mathcal{O}(\alpha^2_{N_f=2})$	1.881
		$\mathcal{O}(\alpha^2_{N_f=1})$	−0.226



Numerical results

- Scheme dependence:
  [EWxQCD and input pars from Sun, Feng, Jia, Sang, 1609.03995]

	$\alpha(0)$ scheme	G_μ scheme
$\sigma^{\rm LO}$ [fb]	223.14	239.64
$\sigma^{\rm NLO}$ [fb]	229.78	232.46
$\sigma^{\rm NNLO,EW\times QCD}$ [fb]	232.21	233.29
$\sigma^{\rm NNLO,EW}$ [fb]	233.86	233.98

$\alpha(0)$ scheme:

$$\alpha = e^2/(4\pi)$$

$$g = \frac{e}{\sin\theta_W} = \frac{e}{\sqrt{1 - m_W^2/m_Z^2}}$$

G_μ scheme:

$$\frac{G_\mu}{\sqrt{2}} = \frac{g^2}{8m_W^2}(1+\Delta r).$$

[Δr from Freitas, Hollik, Walter, Weiglein, hep-ph/0202131]

- Corrections smaller in G_μ scheme
- Very good agreement between two schemes

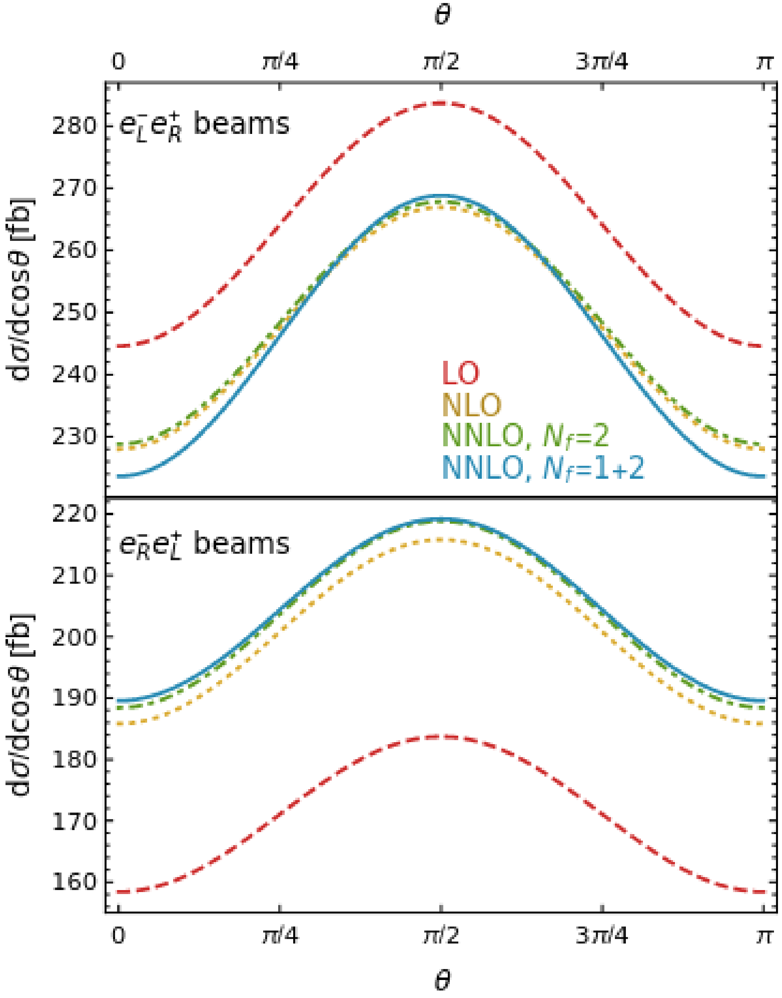
Conclusions

- EW NNLO corrections important for many scattering processes ($ee \to HZ$, $ee \to WW$, $pp \to \ell^+\ell^-$, ...)
- Semi-numerical technique based on dispersion relations and Feynman parameters
  - Minor resources needed for numerical evaluation
  - Avoids reduction to master integrals
  - Precision cannot be increased arbitrarily but sufficient for applications
- Fermionic EW NNLO corrections to $ee \to HZ$ found to be modest in size
  - Scheme dependence much reduced
- Bosonic EW NNLO expected to be numerically less important, but still desirable

Backup

Results for polarized beams

	$e^+_R e^-_L$	$e^+_L e^-_R$
σ^{LO} [fb]	541.28	350.55
σ^{NLO} [fb]	507.92	411.66
σ^{NNLO} [fb]	507.51	418.68
$\mathcal{O}(\alpha^2_{N_f=2})$	1.75	5.77
$\mathcal{O}(\alpha^2_{N_f=1})$	−2.15	1.25



Error estimate

- Main theory uncertainty:
  missing bosonic NNLO corrections
- Partial estimates:

Difference btw. $\alpha(0)$ and G_μ schemes	0.12 fb (0.05%)
$\lvert\mathcal{M}_{(1,\mathrm{bos})}\rvert^2$	0.65 fb (0.3%)