Semi-inclusive diffractive DIS (SIDDIS) at small-x

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with Feng Yuan and Bowen Xiao, PRD106 (2022) 094015

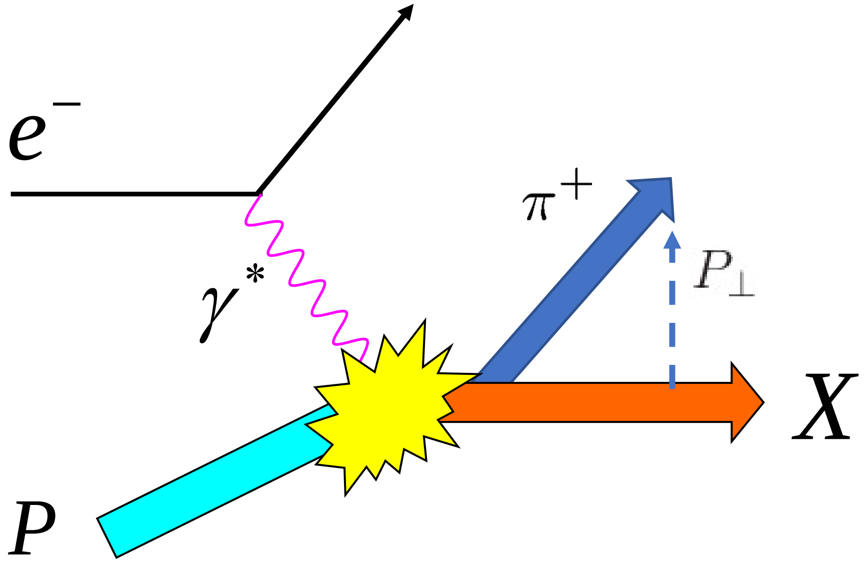
Semi-inclusive DIS (SIDIS)

Tag one hadron species
with fixed transverse momentum $P_\perp$

When $P_\perp$ is small, TMD factorization

$$\frac{d\sigma}{dP_\perp} = H \otimes f(x, k_\perp) \otimes D(z, q_\perp)$$

TMD PDF TMD FF



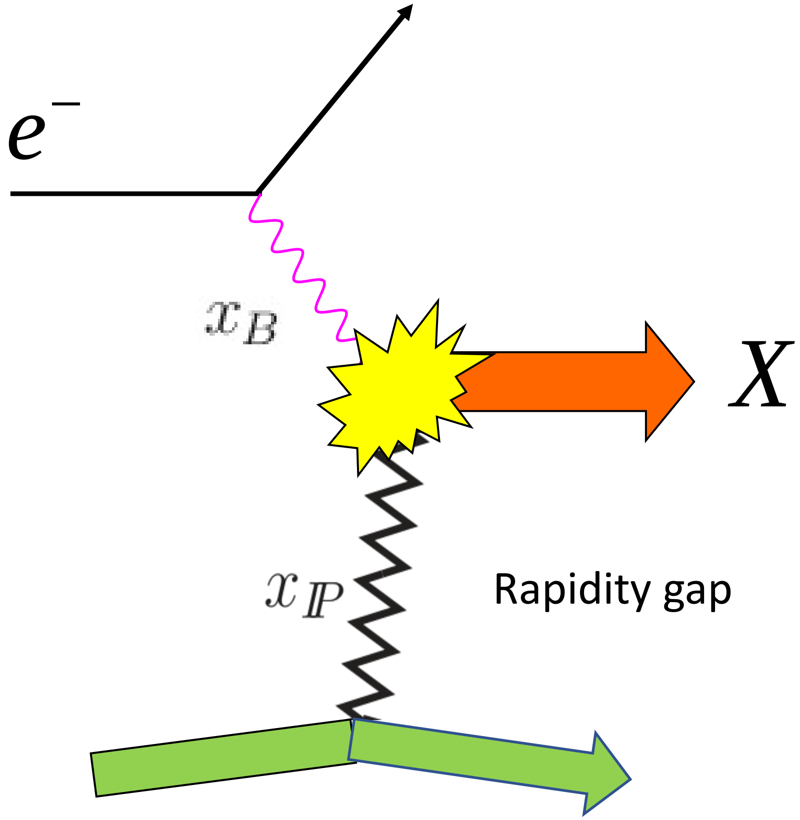
Open up a new class of observables where perturbative QCD is applicable.
Variety of novel phenomena due to intrinsic transverse momentum
3D imagining of partons in momentum space

Diffractive DIS

~10% of HERA events

Factorization in terms of diffractive PDF

At small-x, probe of BFKL/gluon saturation



$$\Delta Y_\beta = \ln \frac{1}{\beta}$$

$$\Delta Y_P = \ln \frac{1}{x_P}$$

$$F_{2/L}^{D(3)}(\beta, Q^2, x_{\mathbb{P}}) = \sum_i \int_\beta^1 \frac{dz}{z} C_{2/L,i}\left(\frac{\beta}{z}\right) f_i^D(z, x_{\mathbb{P}}; Q^2)$$

$$\beta = \frac{Q^2}{Q^2 + M_X^2} = \frac{x_B}{x_P}$$

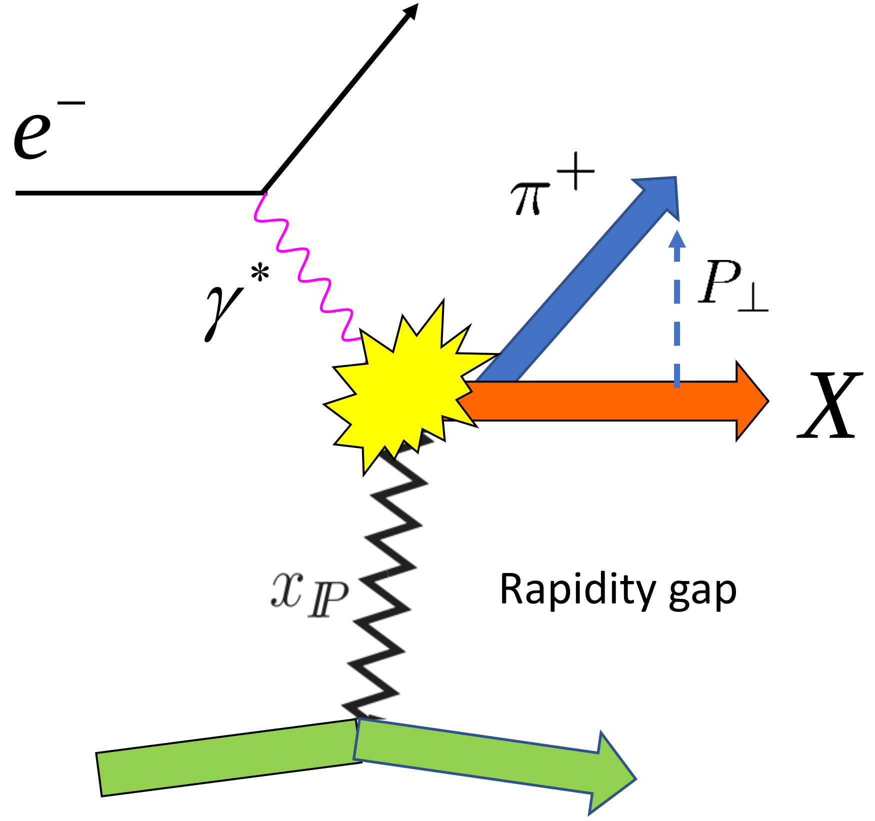
$$2E_{P'} \frac{df_q^D(x, x_P, t)}{d^3P'} = \int \frac{d\xi^-}{2(2\pi)^4} e^{-ix\xi^- P^+} \langle PS|\bar{\psi}(\xi)\gamma^+|P'X\rangle\langle P'X|\psi(0)|PS\rangle$$

Semi-inclusive diffractive DIS (SIDDIS)

$$\frac{d\sigma^{\text{SIDDIS}}(\ell p \to \ell' p' q X)}{dx_B dy d^2 k_\perp dY_{IP} dt} = \sigma_0 e_q^2 x_B \frac{df_q^D(\beta, k_\perp; x_{IP})}{dY_{IP} dt}$$

TMD version of diffractive PDF

$$2E_{P'} \frac{df_q^D(x, k_\perp; x_{IP}, t)}{d^3 P'}$$
$$= \int \frac{d\xi^- d^2\xi_\perp}{2(2\pi)^6} e^{-ix\xi^- P^+ + i\vec{\xi}_\perp \cdot \vec{k}_\perp}$$
$$\times \langle PS|\bar{\psi}(\xi)\mathcal{L}_n^\dagger(\xi)\gamma^+|P'X\rangle\langle P'X|\mathcal{L}_n(0)\psi(0)|PS\rangle$$



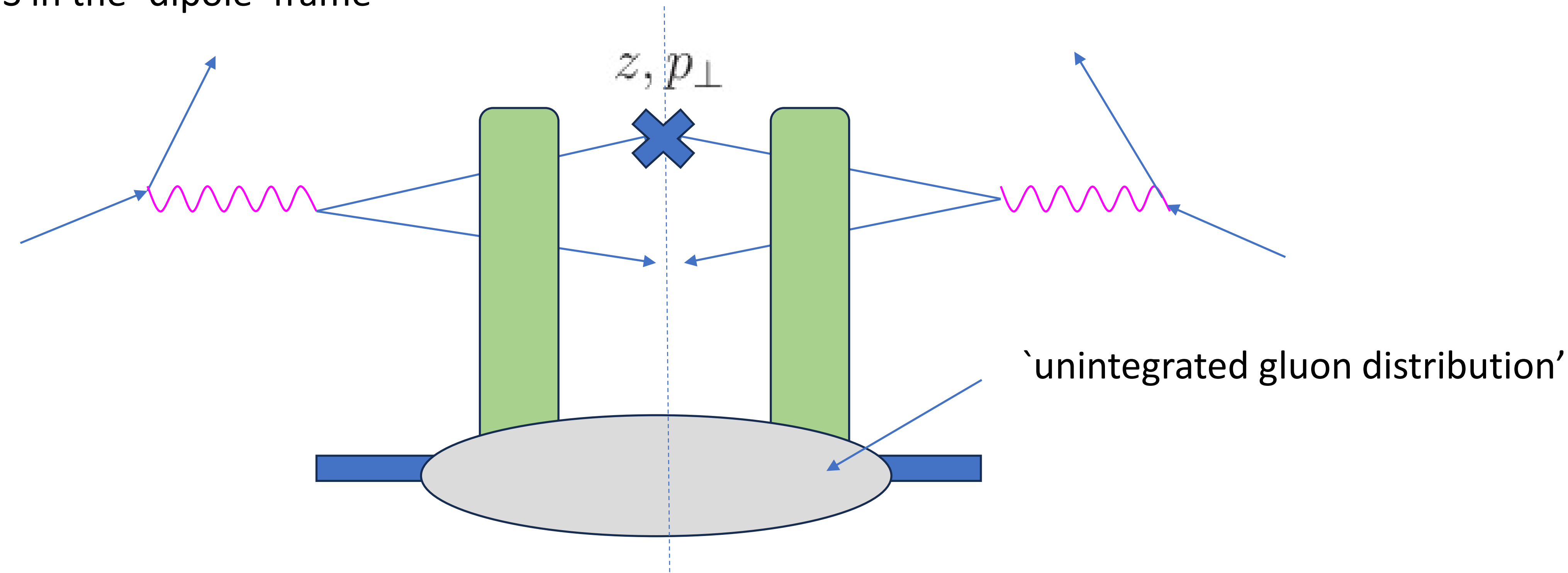
QCD factorization?

SIDIS at small-x

- TMD factorization well established at large to medium x
- Challenging to include small-x resummation & gluon saturation effects
- A variety of alternative approaches developed at small-x

  BFKL/kt factorization/color dipole/Color Glass Condensate/rapidity factorization

SIDIS in the `dipole' frame



Gluon TMD and color dipole

Start with the (dipole type) gluon TMD

$$F(x,k_\perp) \;=\; \frac{2}{p^+}\int\frac{dz^-d^2z_\perp}{(2\pi)^3}e^{ixp^+z^- -ik_\perp\cdot z_\perp}\langle p|\mathrm{Tr}[F^{+i}(0)WF^{+i}(z^-,z_\perp)W]|p\rangle$$

Take the formal Regge limit $x\to 0 \qquad e^{ixp^+z^-}\approx 1$

$$F(x,k_\perp)\approx\frac{2N_ck_\perp^2}{\alpha_s}\int\frac{d^2b_\perp}{(2\pi)^2}\int\frac{d^2r_\perp}{(2\pi)^2}e^{-ik_\perp\cdot r_\perp}\frac{\langle p|\frac{1}{N_c}\mathrm{Tr}[U(x_\perp)U^\dagger(y_\perp)]|p\rangle}{\langle p|p\rangle}$$

Dipole S-matrix

$$U(z_\perp)=P\exp\left(ig\int_{-\infty}^{\infty}dz^-A^+(z^-,z_\perp)\right)$$

x-dependence encoded in the evolution equation for the operator $UU^\dagger$ Balitsky (1996)

Quark TMD at small-x

McLerran, Venugopalan (1994)
Mueller (1999)
Marquet, Yuan, Xiao (2009)

$$f(x,k_\perp)=\int\frac{d^3\xi}{2(2\pi)^3}e^{-ixP^+\xi^-+ik_\perp\cdot\xi_\perp}\langle P|\bar\psi(\xi^-,\xi_\perp)\mathcal{L}\gamma^+\psi(0)|P\rangle$$

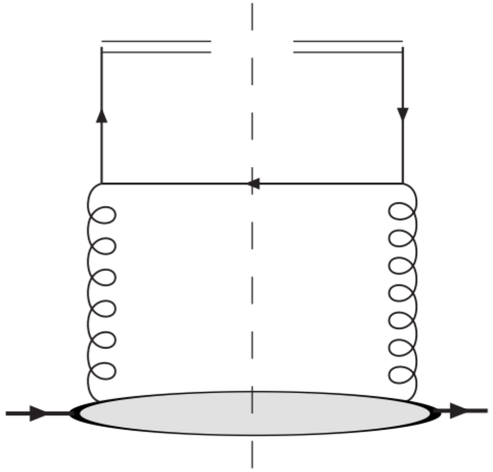
$$=\frac{T_R}{4\pi^4}S_\perp N_c\int d^2k_{g\perp}\int_x\frac{dx_g}{x_g^2}\left(\frac{\vec{k}_\perp|k_\perp-k_{g\perp}|}{\hat{x}(k_{g\perp}-k_\perp)^2+(1-\hat{x})k_\perp^2}-\frac{\vec{k}_\perp-\vec{k}_{g\perp}}{|k_\perp-k_{g\perp}|}\right)^2\frac{\langle P|\frac{1}{N_c}\mathrm{tr}UU^\dagger(k_{g\perp})|P\rangle}{\langle P|P\rangle}$$

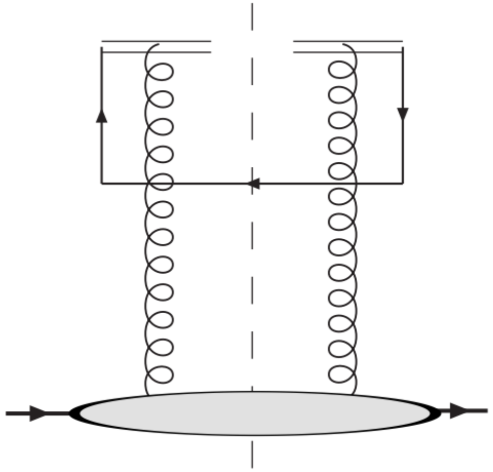
$$\sim\frac{1}{k_\perp^2}$$

Color dipole

SIDIS cross section

$$\frac{d\sigma}{dx_Bdyd^2P_\perp}=\sigma_0e_q^2x_Bf_q(x,k_\perp)\otimes D(z)$$





Geometric scaling

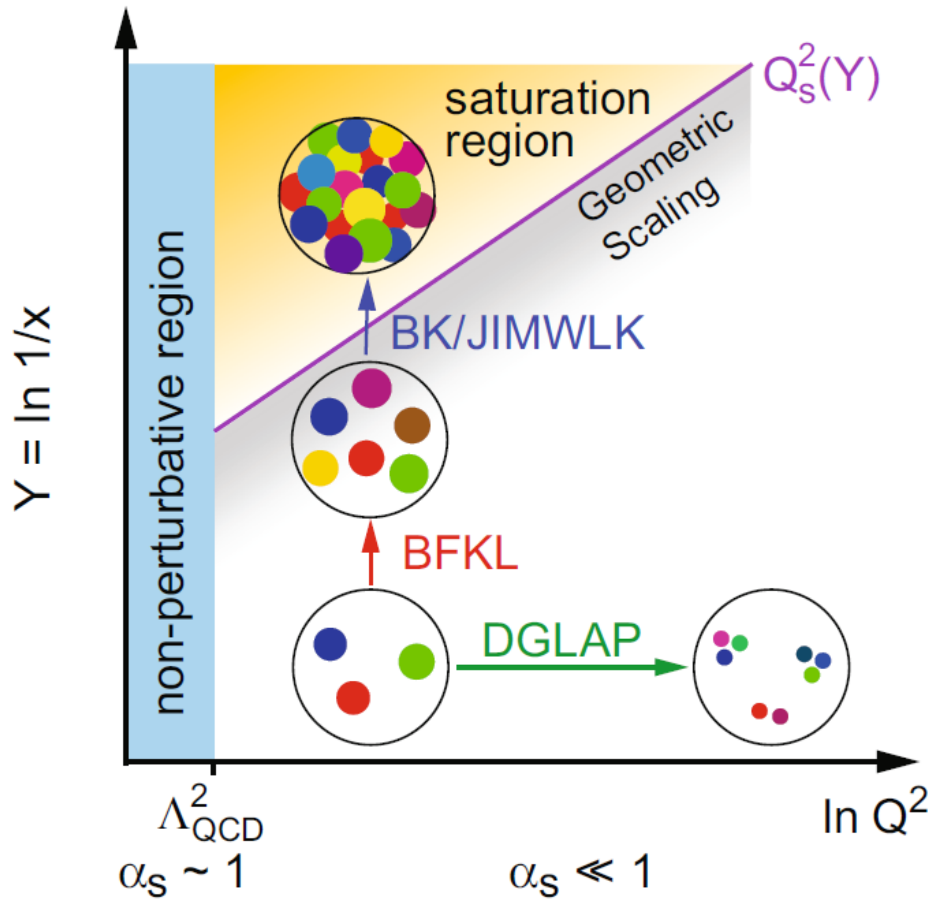
A priori, $F(x, k_\perp)$ depends separately on x and $k_\perp$

However, at small-x there is a dynamically generated scale called saturation momentum

$$Q_s(x) = \Lambda_{QCD} \left(\frac{1}{x}\right)^\gamma$$

$F(x, k_\perp)$ becomes a function only of the ratio in a certain kinematical window

$$F(x, Q_s) \sim \frac{1}{Q_s^2} f\left(\frac{k_\perp}{Q_s(x)}\right)$$



Off-forward color dipole = gluon Wigner distribution

$$W(x, q_\perp, \Delta_\perp) = \frac{2N_c}{\alpha_s}\left(q_\perp^2 - \frac{\Delta_\perp^2}{4}\right) \mathrm{F.T.}\langle P'|\mathrm{Tr}U_{x_\perp}U_{y_\perp}^\dagger|P\rangle$$

YH, Xiao, Yuan (2016)

off-forward

Unpol TMD, linearly polarized distribution,
gluon Sivers and other T-odd TMDs
gluon GPD H_g, E_g
transversity GPD

→ Application to diffractive PDF Cf. Hautmann, Kunszt, Soper (1999)

Quark diffractive TMD at small-x

YH, Xiao, Yuan (2022)

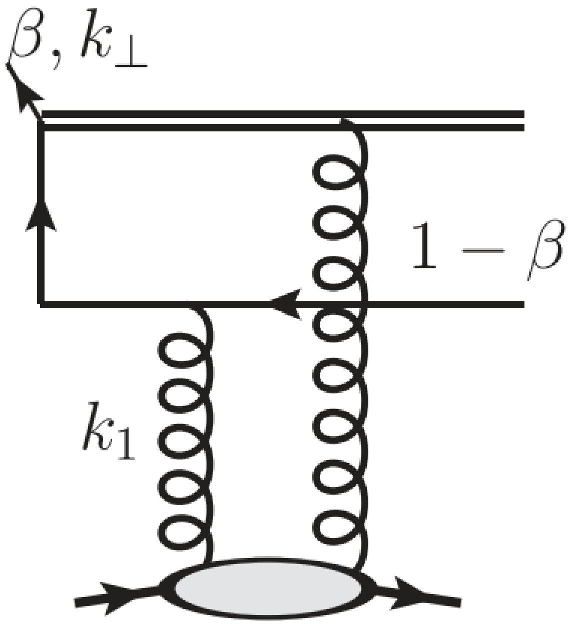
Start with the operator definition

$$x\frac{df_q^D(\beta,k_\perp;x_{IP})}{dY_{IP}dt} = \int d^2k_{1\perp}d^2k_{2\perp}\mathcal{F}_{x_{IP}}(k_{1\perp},\Delta_\perp)$$

$$\times\mathcal{F}_{x_{IP}}(k_{2\perp},\Delta_\perp)\frac{N_c\beta}{2\pi}\frac{k'_{1\perp}\cdot k'_{2\perp}k_\perp^2}{[\beta k_\perp^2+(1-\beta)k'^2_{1\perp}][\beta k_\perp^2+(1-\beta)k'^2_{2\perp}]}+\cdots$$

$$k'_{1\perp}=k_\perp-k_{1\perp}$$

color dipole

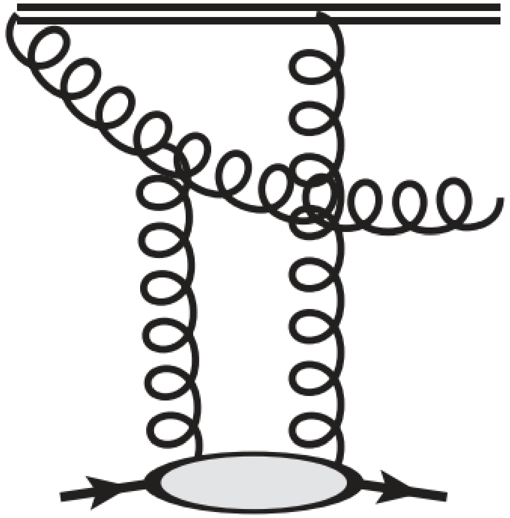


At large transverse momentum

$$\sim\frac{1}{k_\perp^4}(H_g(x_P,t))^2$$

Gluon GPD YH, Xiao, Yuan (2017)

Gluon diffractive TMD at small-x



$$
x\frac{df_g^D(\beta,k_\perp;x_{IP})}{dY_{IP}dt} = \int d^2k_{1\perp}d^2k_{2\perp}\mathcal{G}_{x_{IP}}(k_{1\perp},\Delta_\perp)\,\mathcal{G}_{x_{IP}}(k_{2\perp},\Delta_\perp)
$$

$$
\times\,\frac{N_c^2-1}{\pi(1-\beta)}\,\frac{1}{[\beta k_\perp^2+(1-\beta)k_{1\perp}'^2]}\,\frac{1}{[\beta k_\perp^2+(1-\beta)k_{2\perp}'^2]}
$$

$$
\times\left[\beta(1-\beta)k_\perp^2\,\frac{k_{1\perp}'^2+k_{2\perp}'^2}{2}+(1-\beta)^2(k_{1\perp}'\cdot k_{2\perp}')^2+\beta^2\,\frac{(k_\perp^2)^2}{2}\right]\;+\cdots
$$

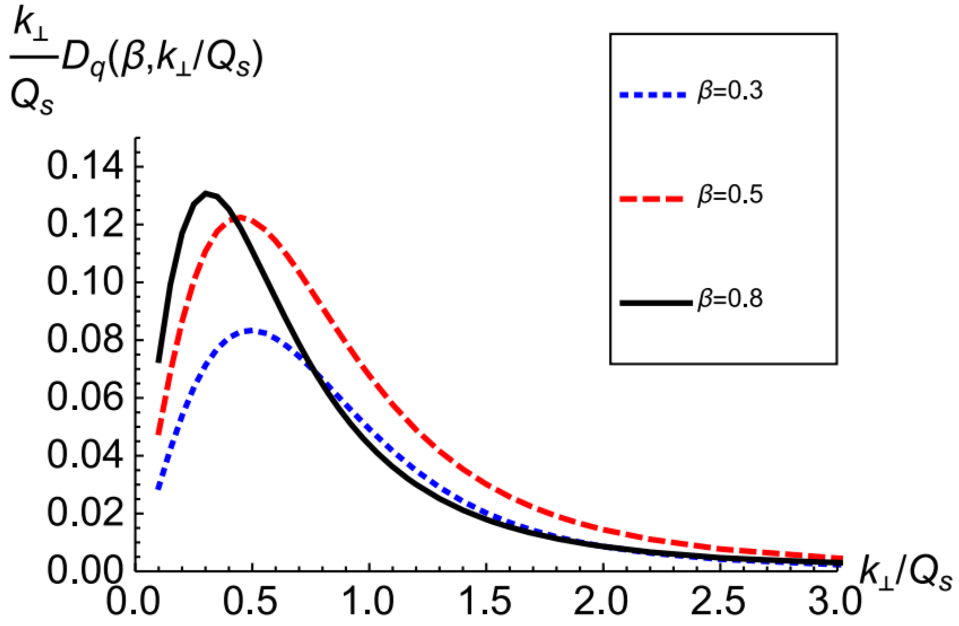
$$
\mathcal{G}_x(q_\perp,\Delta_\perp) = \int\frac{d^2b_\perp d^2r_\perp}{(2\pi)^4}\,e^{iq_\perp\cdot r_\perp+i\Delta_\perp\cdot b_\perp}\,\frac{1}{N_c^2-1}\left\langle \mathrm{Tr}\left[\tilde{U}\left(b_\perp+\frac{r_\perp}{2}\right)\tilde{U}^\dagger\left(b_\perp-\frac{r_\perp}{2}\right)\right]\right\rangle_x
$$

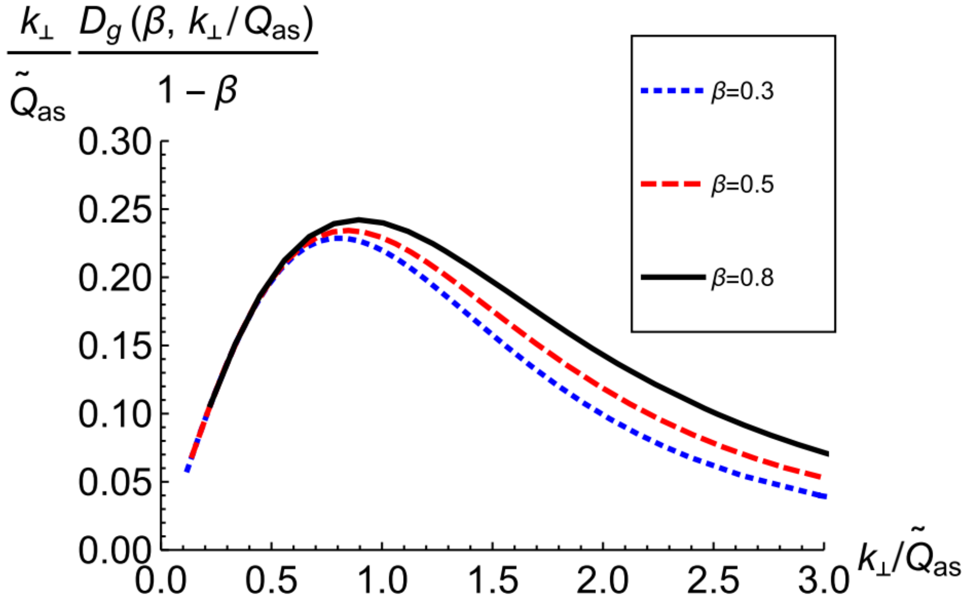
color dipole (adjoint rep.)

Modified geometric scaling

Iancu, Mueller, Triantafyllopoulos (2021)
YH, Xiao, Yuan (2022)

$$x\frac{df^D_{q,g}(\beta, k_\perp; x_{IP})}{dY_{IP}dt} = \mathcal{N}_{q,g} D_{q,g}\left(\beta, \frac{k_\perp}{Q_{s,as}}\right) \sim D_{q,g}\left(\frac{k_\perp}{\sqrt{1-\beta}Q_{s,as}(x_P)}\right)$$



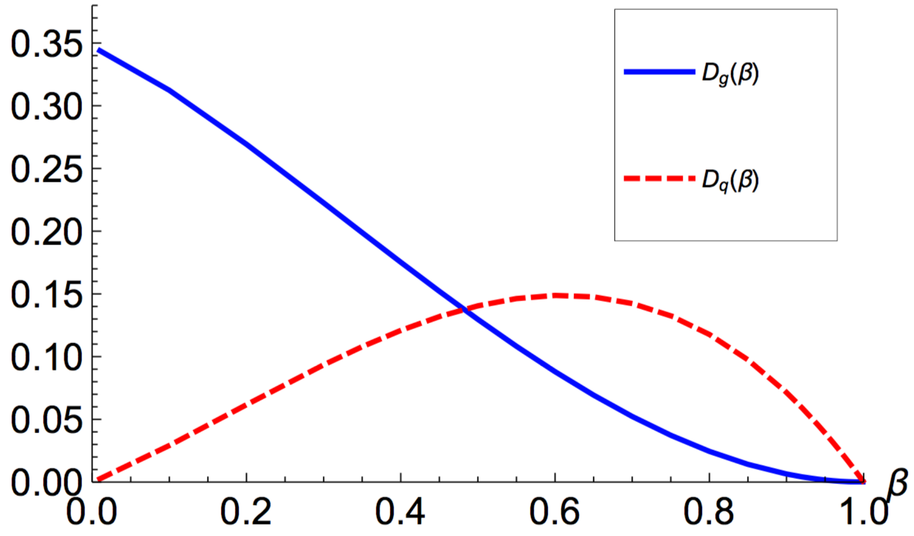


Collinear DPDF

Integrate over $k_\perp$

$$x\frac{df^D_{q,g}(\beta;x_{IP})}{dY_{IP}dt} = \mathcal{N}_{q,g}2\pi\mathcal{D}_{q,g}(\beta)Q^2_{s,as}$$

$$\mathcal{D}_q(\beta) = \beta\left(b_1(1-\beta)+b_2(1-\beta)^2\right) \qquad \mathcal{D}_g(\beta) = (a_0+a_1\beta)(1-\beta)^2$$



The end point behavior analytically computed for Gaussian models.

$$b_1 = \frac{3\pi^2}{16} - 1, \quad b_2 = \frac{20-3\pi^2}{16}$$

$$a_0 = \frac{\ln(2)}{2} \qquad a_1 = \frac{45\pi^2-272}{256} - \frac{\ln(2)}{2}$$

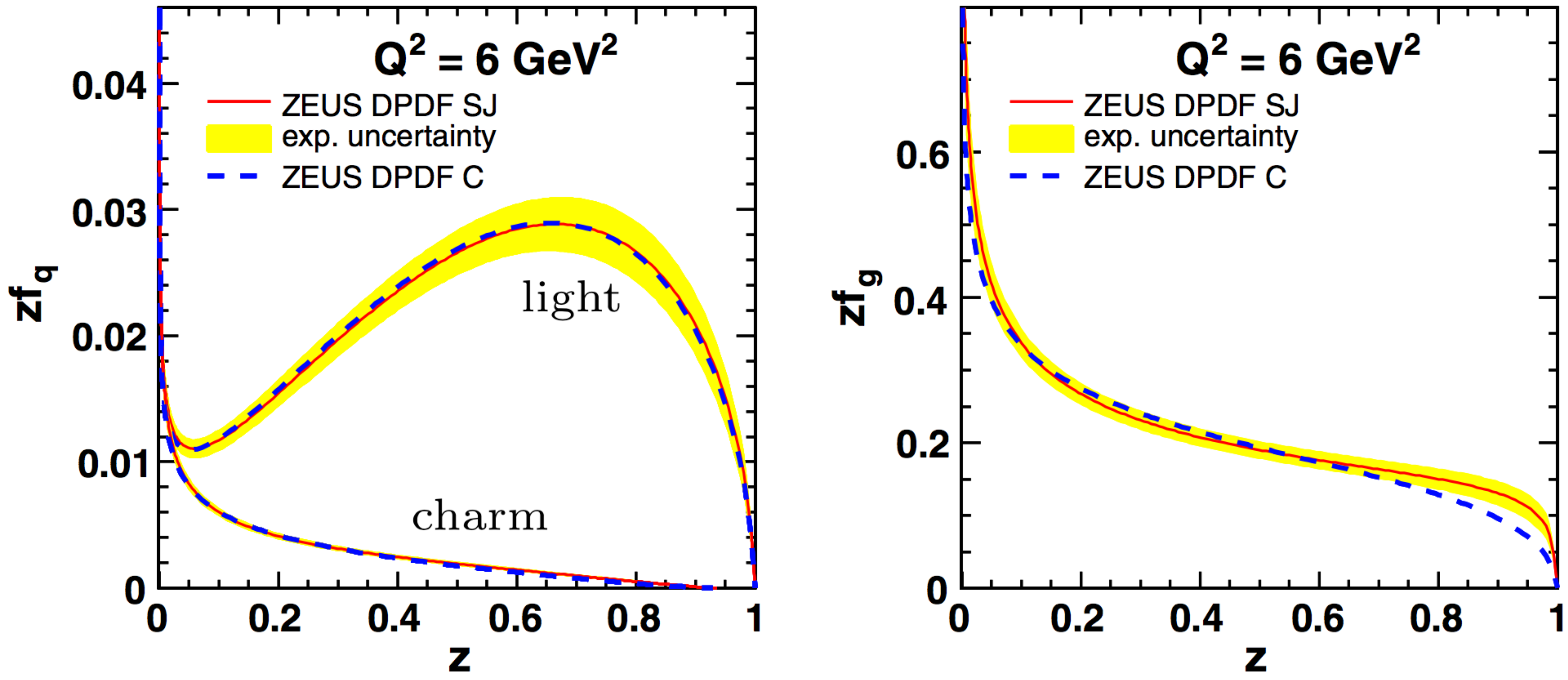
Buchmuller, Gehrmann, Hebecker (1999)

Initial condition for the DGLAP evolution

HERA data

ZEUS, NPB831, 1 (2010)

$$F_{2/L}^{D(3)}(\beta, Q^2, x_{\mathbb{P}}) = \sum_i \int_\beta^1 \frac{dz}{z} C_{2/L,i}\left(\frac{\beta}{z}\right) f_i^D(z, x_{\mathbb{P}}; Q^2)$$



Diffractive structure functions

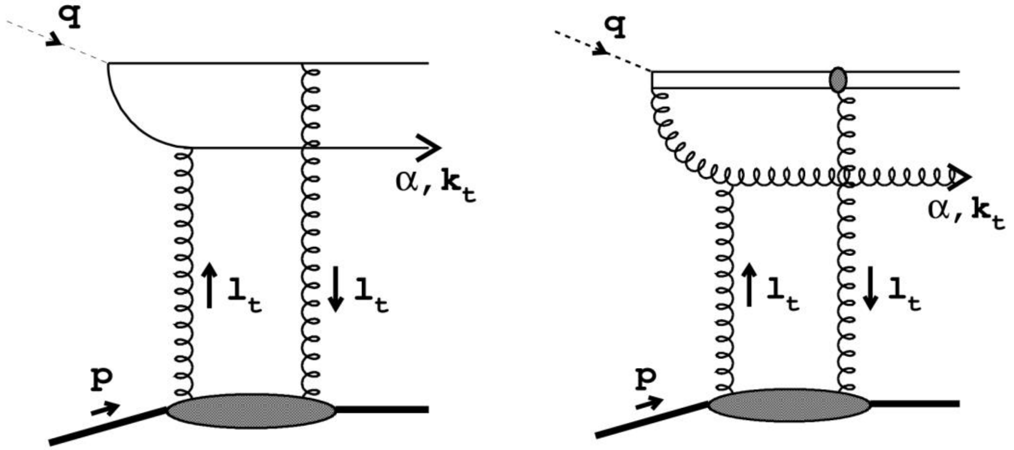
Wusthoff (1997)

Directly compute the cross section (diffractive structure functions)

At large- Q^2, one can identify TMD DPDF in the integrand

$$F^D_{\{t,q\bar{q}\}}(Q^2,\beta,x_{IP}) = Q^2\pi(1-\beta)\int_0^1 d\alpha(\alpha^2+(1-\alpha)^2)\,\frac{df_q^D(\beta,k_\perp;x_{IP})}{dY_{IP}}$$

$$x_{IP}F^D_{\{t,q\bar{q}g\}}(Q^2,\beta,x_{IP}) = \int_\beta^1 d\xi((1-\xi)^2+\xi^2)\int^{(1-\beta')Q^2}\frac{d^2k_\perp}{k_\perp^2}\frac{\alpha_s}{2\pi^2}\int^{k_\perp^2} d^2k'_\perp x'\,\frac{df_g(\beta',k'_\perp;x_{IP})}{dY_{IP}}$$



More complete calculation

Beuf, Hanninen, Lappi, Mulian, Mantysaari (2022)

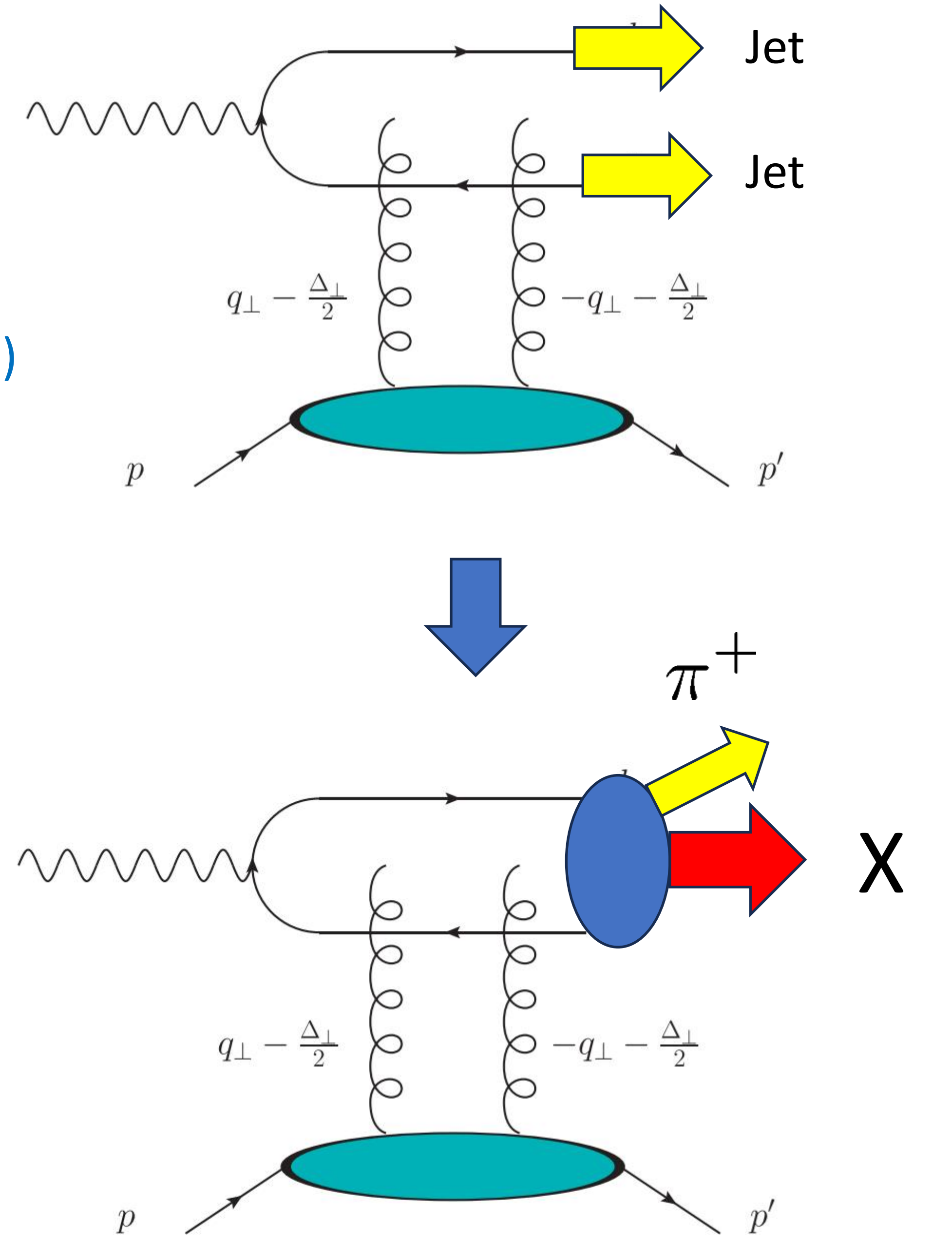
From dijet to SIDDIS

Start with the cross section for diffractive dijet production

Integrate over the antiquark phase space to get YH, Xiao, Yuan (2022)

$$\frac{d\sigma^{\rm SIDDIS}(\ell p \to \ell' p' q X)}{dx_B dy d^2 k_\perp dY_{IP} dt} = \sigma_0 e_q^2 x_B \frac{df_q^D(\beta, k_\perp; x_{IP})}{dY_{IP} dt}$$

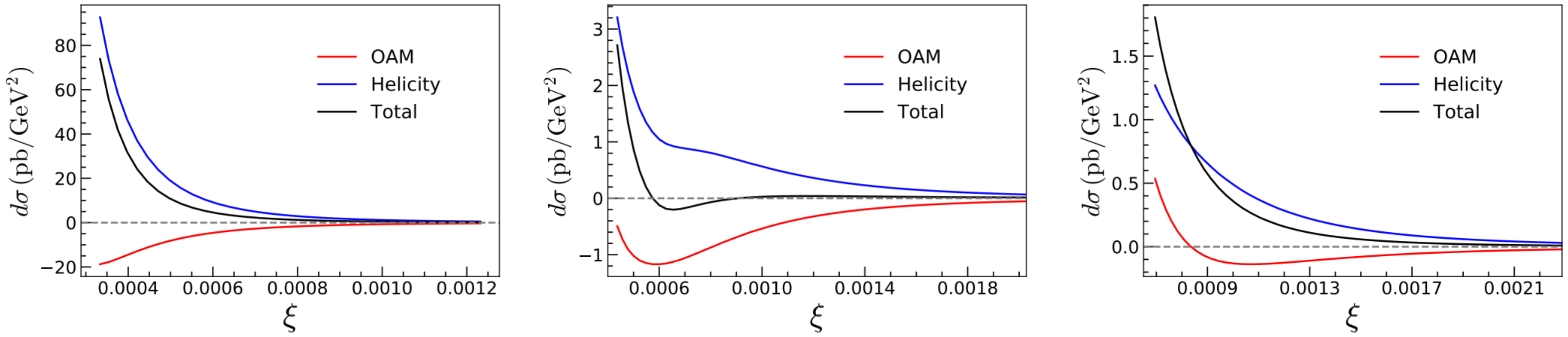
Additional vector $P'_\perp$ compared to SIDIS.
Rich pattern of angular correlations between $P'_\perp, S_\perp, k_\perp, \ell'_\perp$



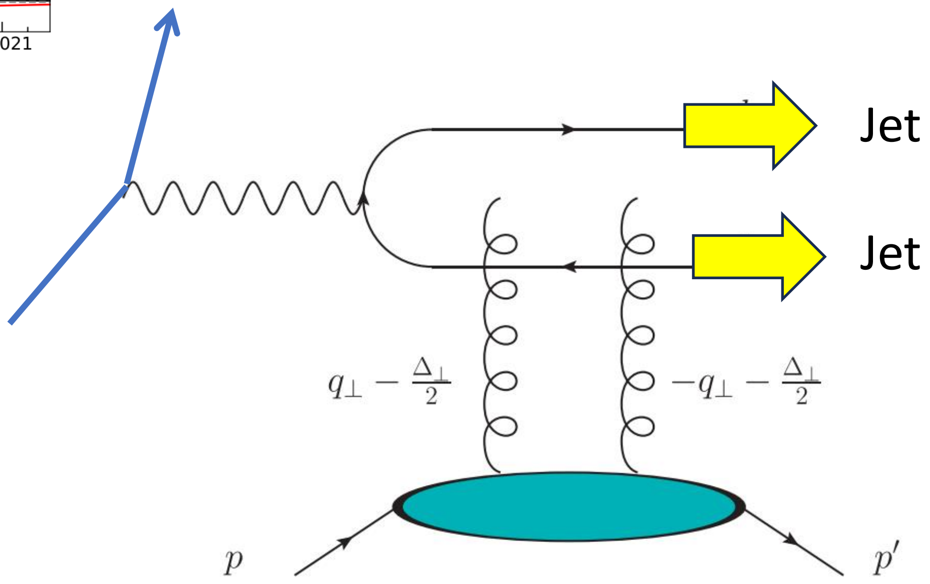
Application: Longitudinal double spin asymmetry at the EIC

Previously we proposed DSA in dijet production as a signal of gluon orbital angular momentum

Bhattacharya, Boussarie, YH (2022)



$$\cos(\phi_{l_\perp} - \phi_{\Delta_\perp})$$



Problem: dijet transverse momentum is too low to reconstruct as jets

Solution: Reinterpret as SIDDIS. Invariant mass is easy to measure.

Bhattacharya, Boussarie, YH, to appear

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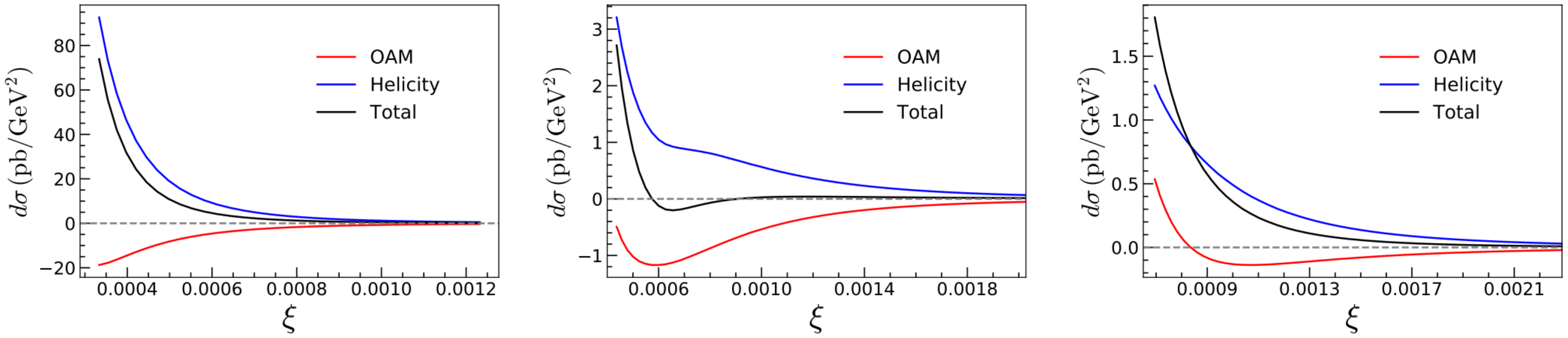

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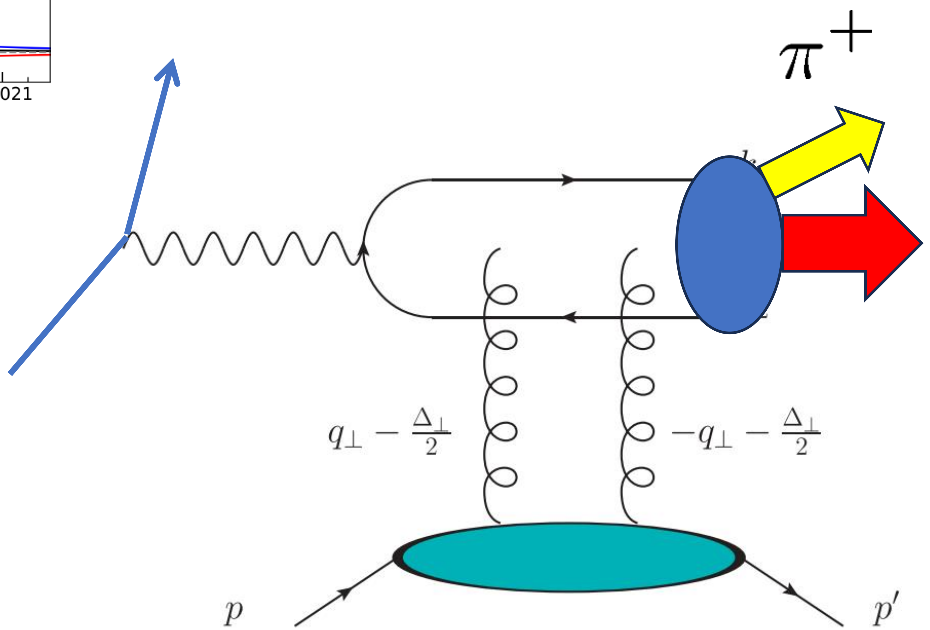

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Conclusions

- Small-x expression of diffractive quark/gluon TMD from the operator definition. Connection to gluon Wigner
- Modified geometric scaling in terms of $\tilde{Q}_s = \sqrt{1-\beta}Q_s$
- Semi-inclusive diffractive DIS (SIDDIS): new research avenue
- Additional vector $P'_\perp$ compared to SIDIS. Rich pattern of angular correlations between $P'_\perp, S_\perp, k_\perp, \ell'_\perp$