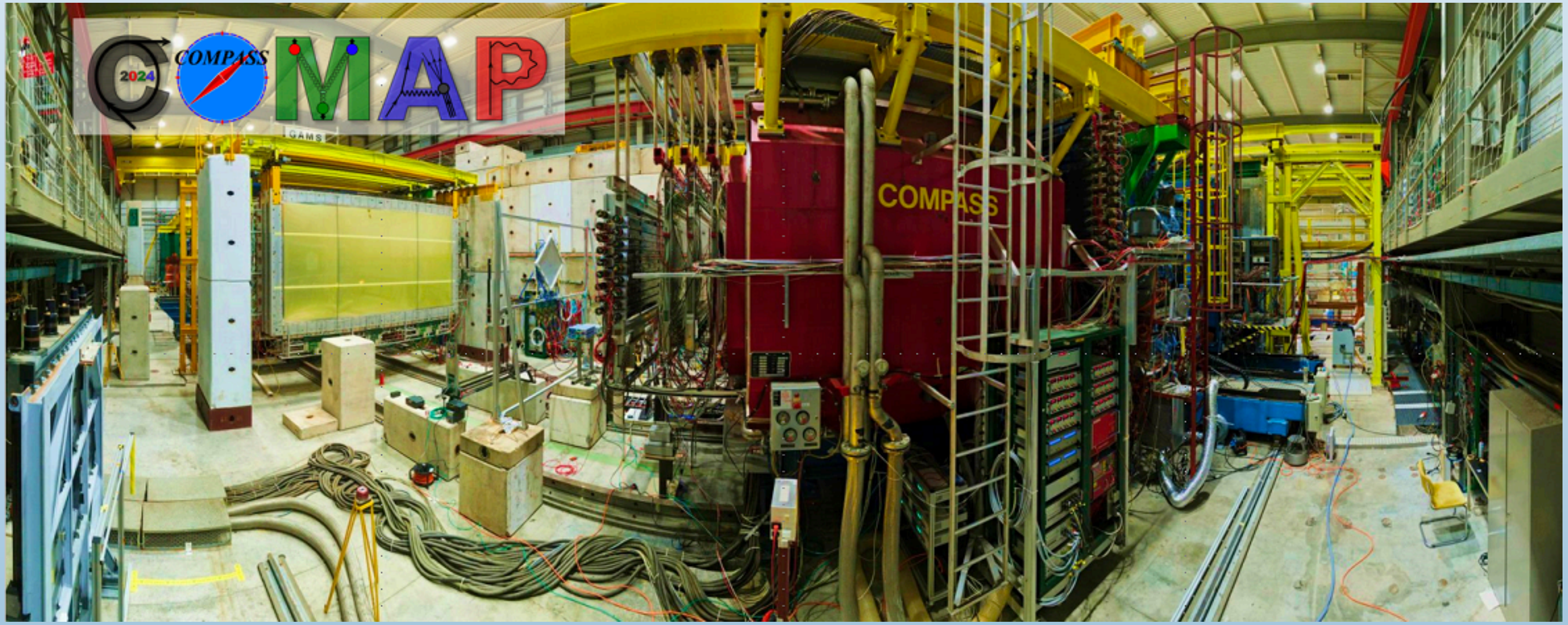
Recent Updates from the JAM Collaboration on Helicity PDFs

Christopher Cocuzza

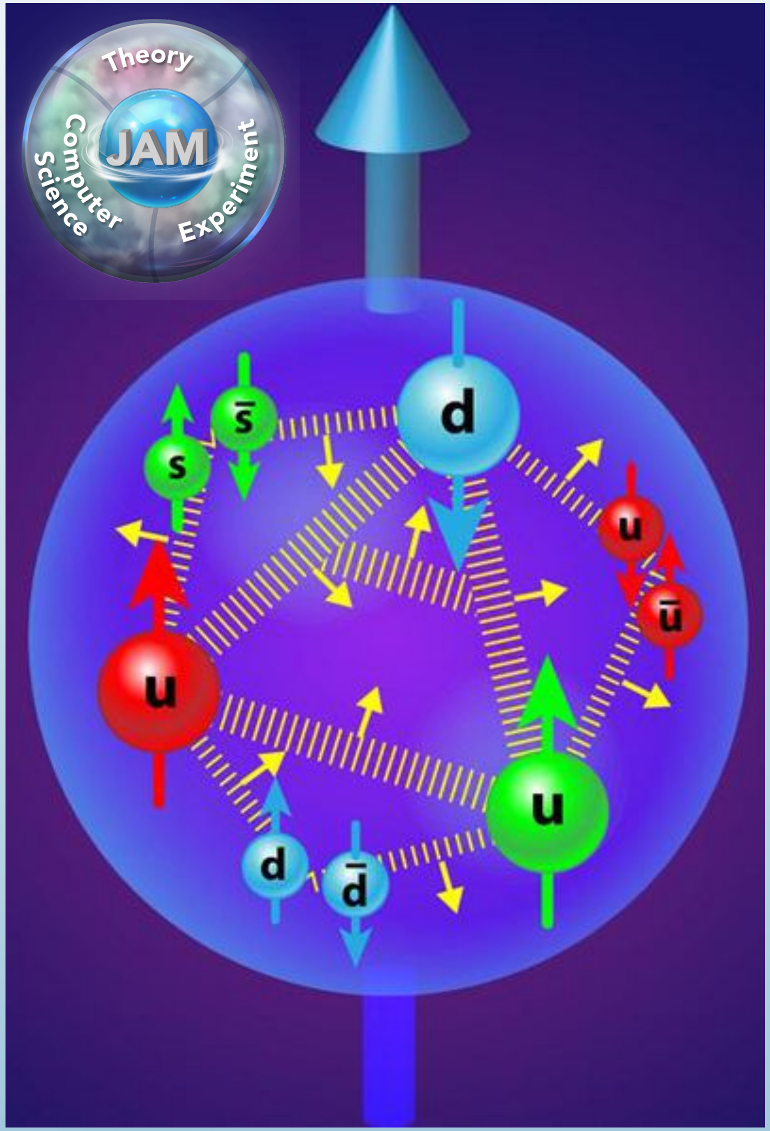
January 24, 2024



JAM Collaboration

3-dimensional structure of nucleons:

- Parton distribution functions (PDFs)
- Fragmentation functions (FFs)
- Transverse momentum dependent distributions (TMDs)
- Generalized parton distributions (GPDs)



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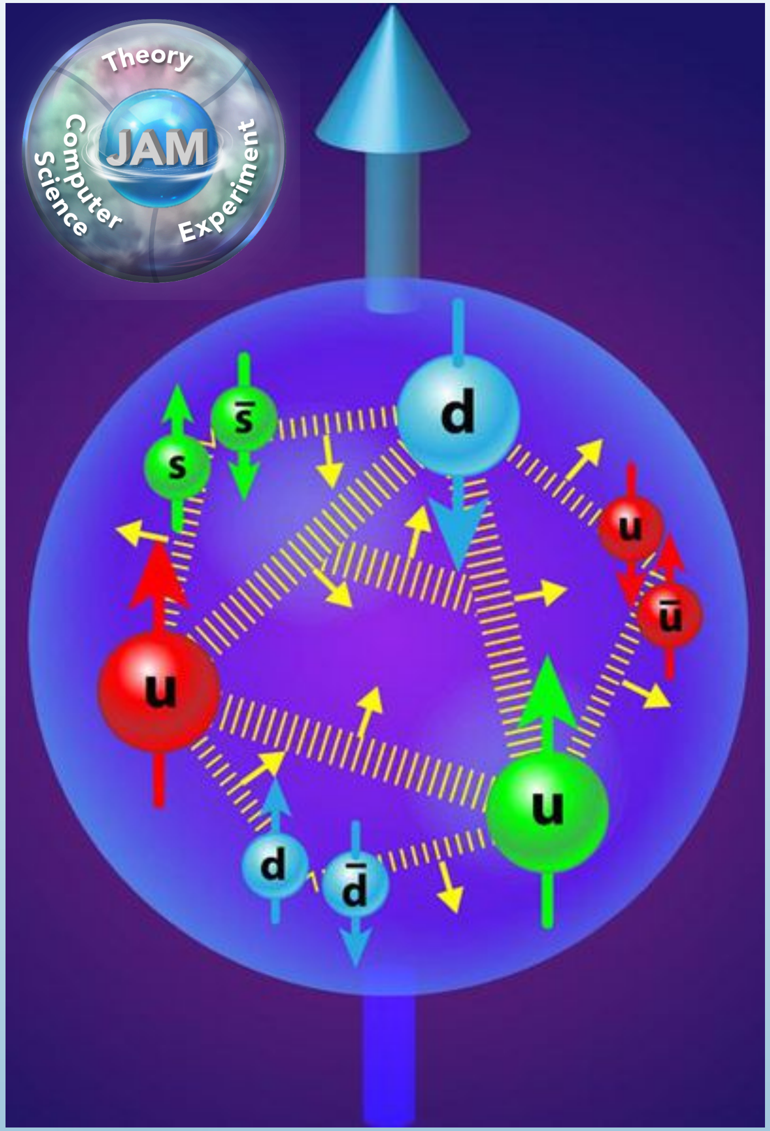
3-dimensional structure of nucleons:
- Parton distribution functions (PDFs)
- Fragmentation functions (FFs)
- Transverse momentum dependent distributions (TMDs)
- Generalized parton distributions (GPDs)

- Collinear factorization in perturbative QCD
- Simultaneous determinations of PDFs, FFs, etc.
- Monte Carlo methods for Bayesian inference



Hadron Structure

Global QCD Analysis

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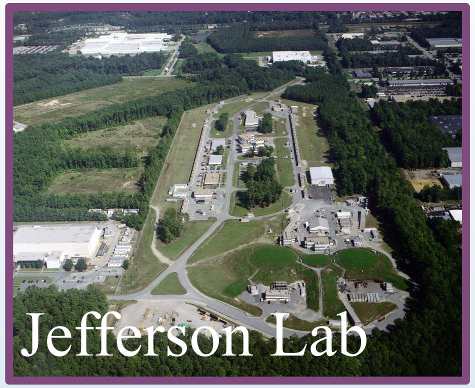
Introduction



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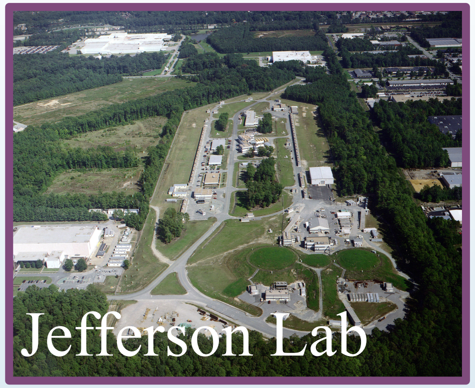

Global QCD Analysis

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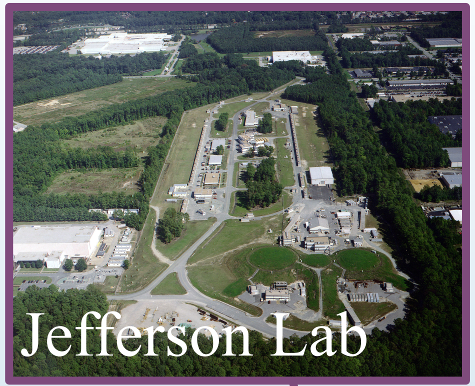
USQCD

Extended Twisted Mass Collaboration



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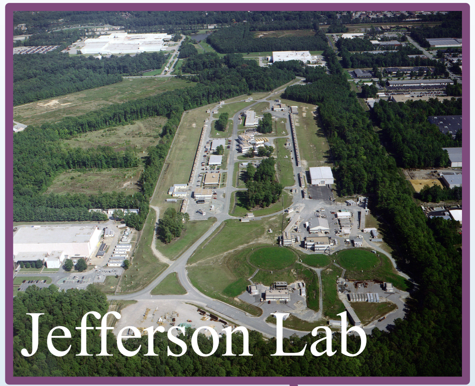

Hadron Structure

$$\frac{d}{d\ \ln(\mu^2)} f_i(x,\mu) = \sum_j \int_x^1 \frac{dz}{z} P_{ij}(z,\mu) f_j(\frac{x}{z},\mu)$$

Param. + Evolve + Factorization

$$\sigma = \sum_{i,j} H_{ij} \otimes f_i \otimes f_j$$

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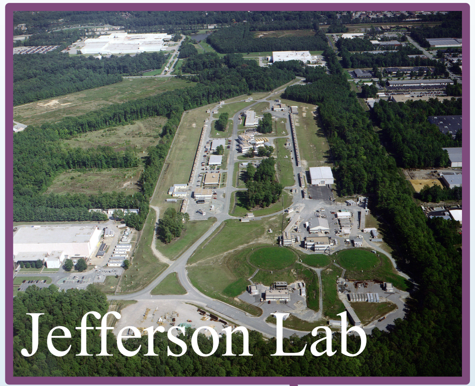
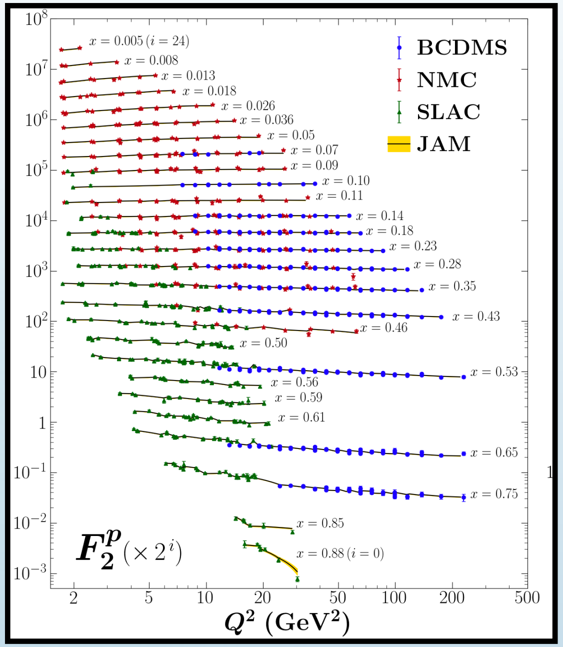


$$\chi^2(\boldsymbol{a}) = \sum_{i,e}\left(\frac{d_{i,e} - \sum_k r_e^k \beta_{i,e}^k - T_{i,e}(\boldsymbol{a})/N_e}{\alpha_{i,e}}\right)^2 + \sum_k \left(r_e^k\right)^2 + \left(\frac{1-N_e}{\delta N_e}\right)^2$$

χ^2 Minimization

$$\mathcal{L}(\boldsymbol{a}, \text{data}) = \exp\left(-\frac{1}{2}\chi^2(\boldsymbol{a}, \text{data})\right)$$

$$\mathcal{P}(\boldsymbol{a}|\text{data}) \sim \mathcal{L}(\boldsymbol{a}, \text{data})\, \pi(\boldsymbol{a})$$



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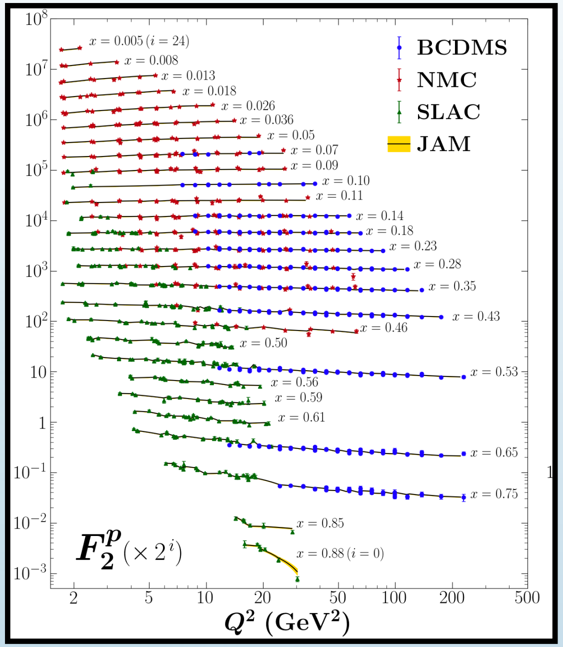

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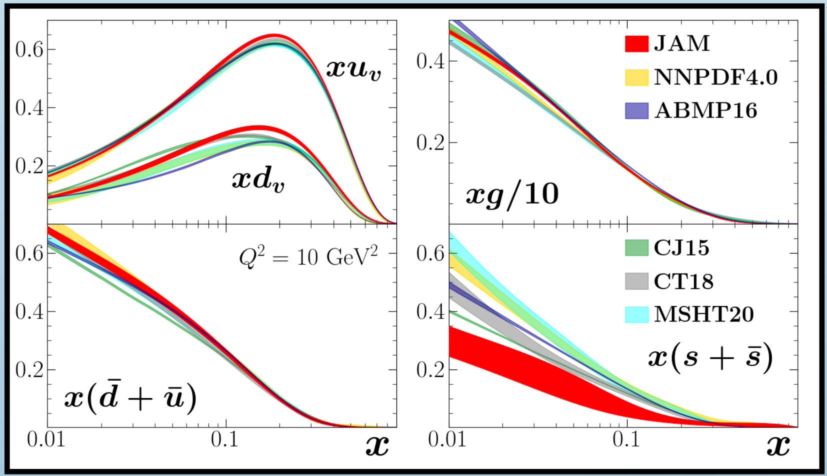

Data Resampling

$$\tilde{\sigma} = \sigma + N(0,1)\ \alpha$$

Current State of Helicity PDFs

Proton spin puzzle:

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g$$

$$\Delta\Sigma = \int_0^1 dx \sum_q \Delta q^+$$

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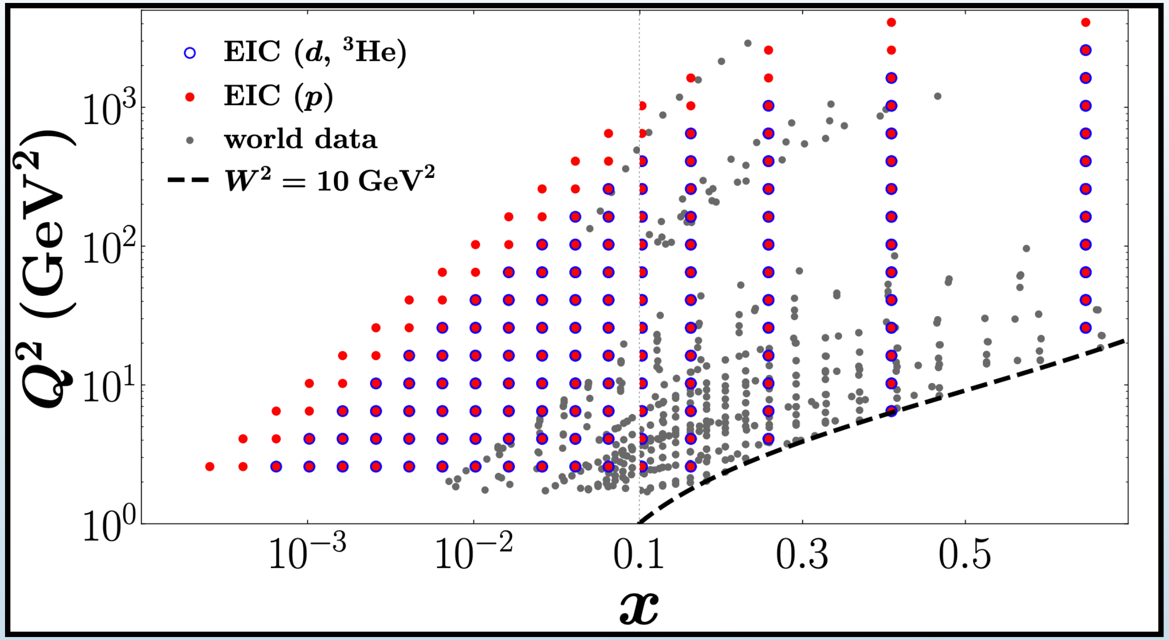

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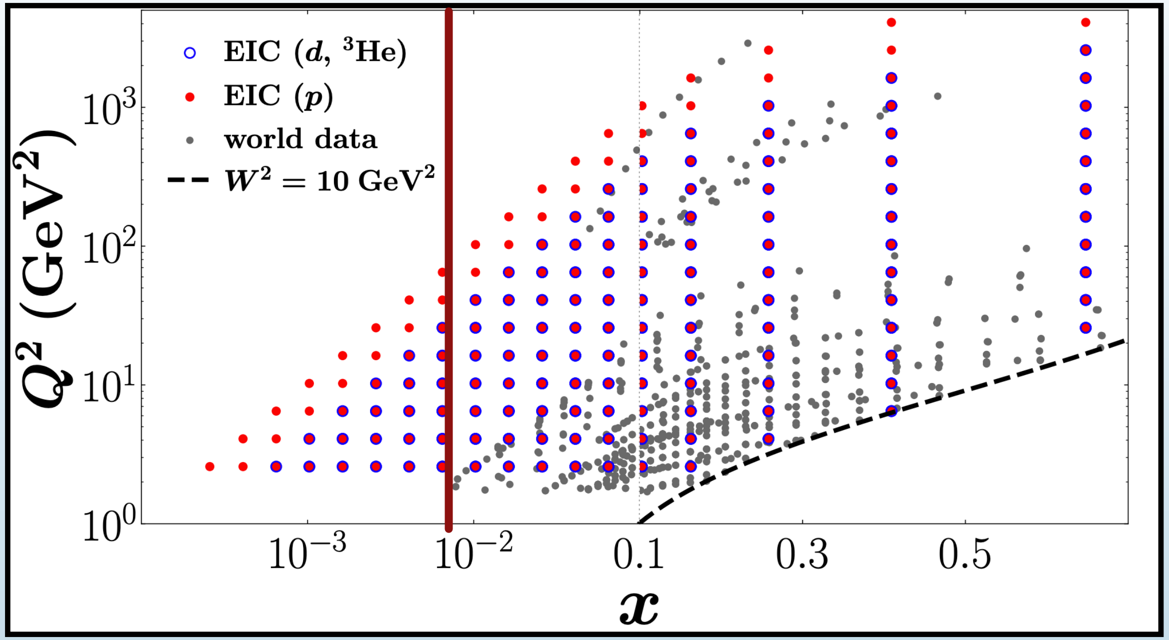

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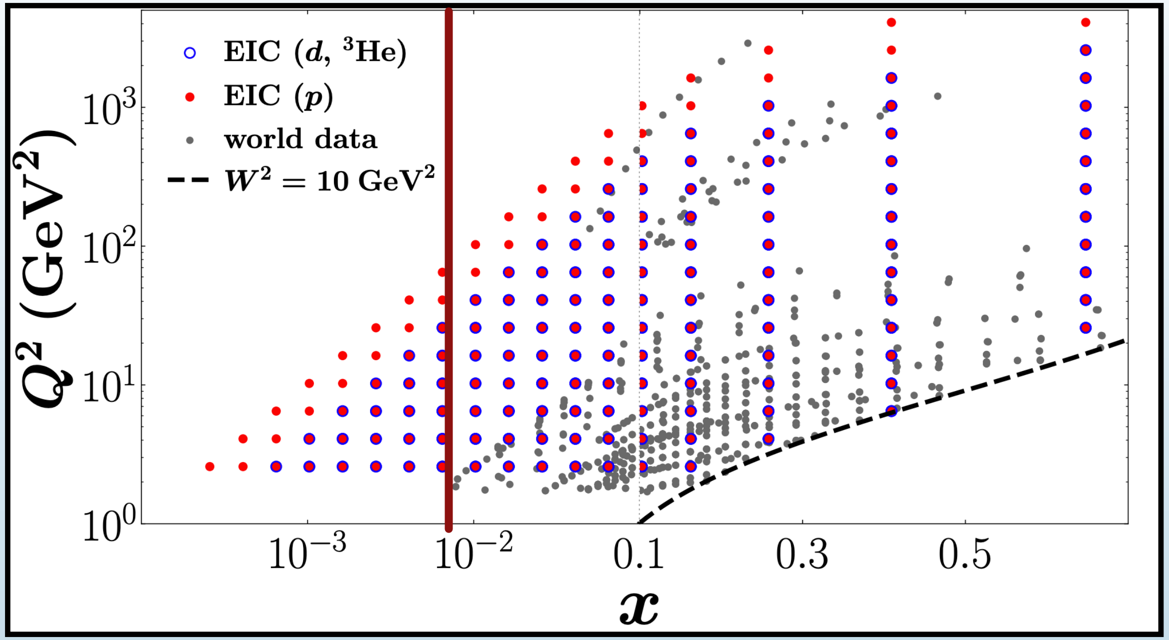

Still a lot to learn about helicity PDFs!
(antiquarks and gluon)

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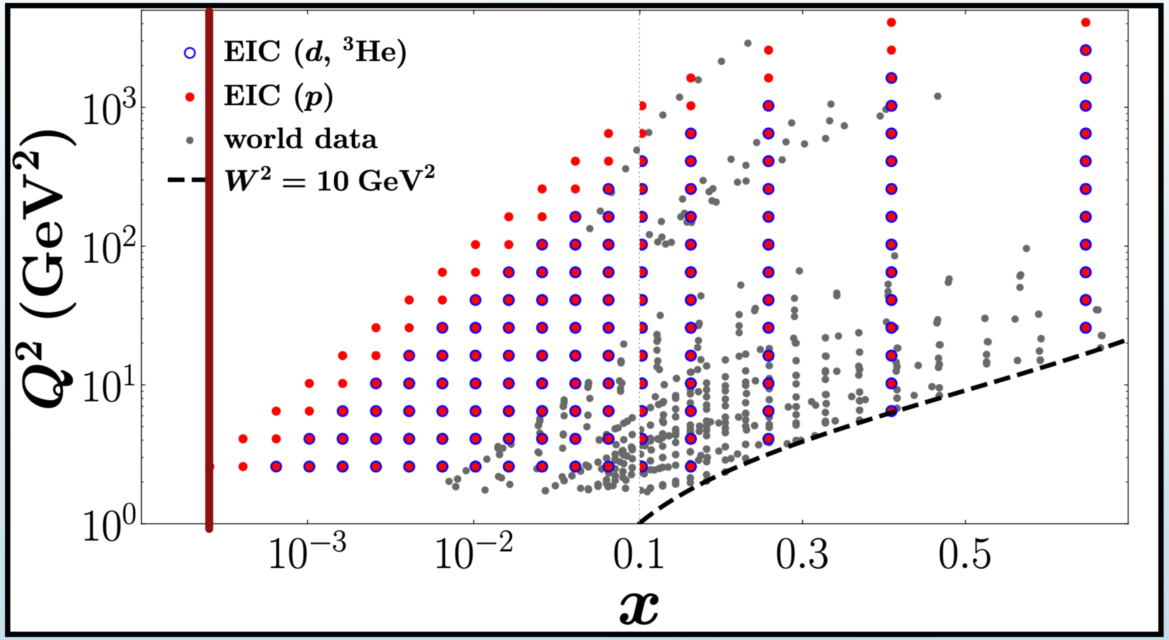

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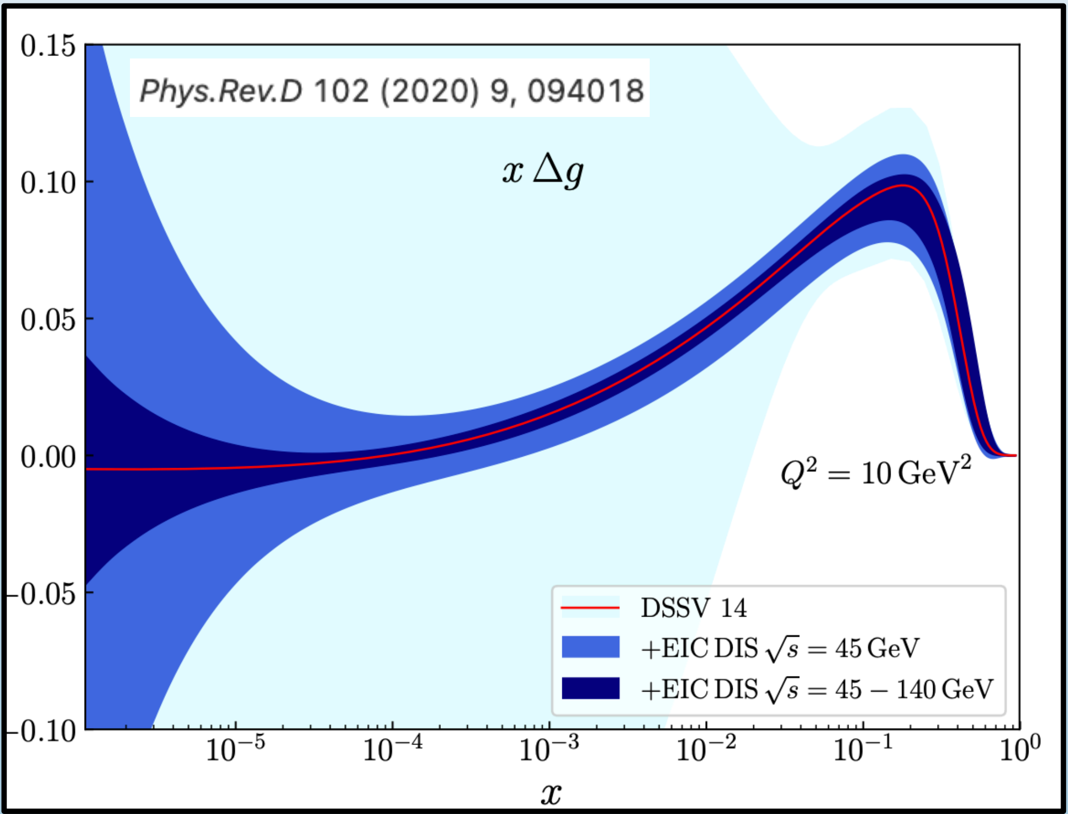

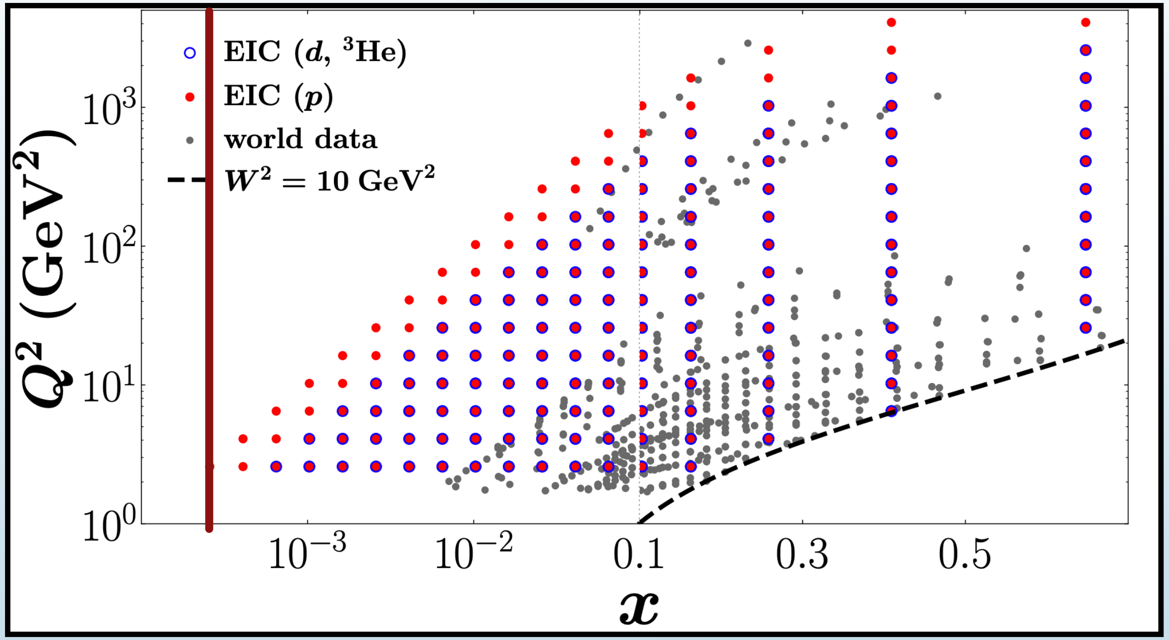

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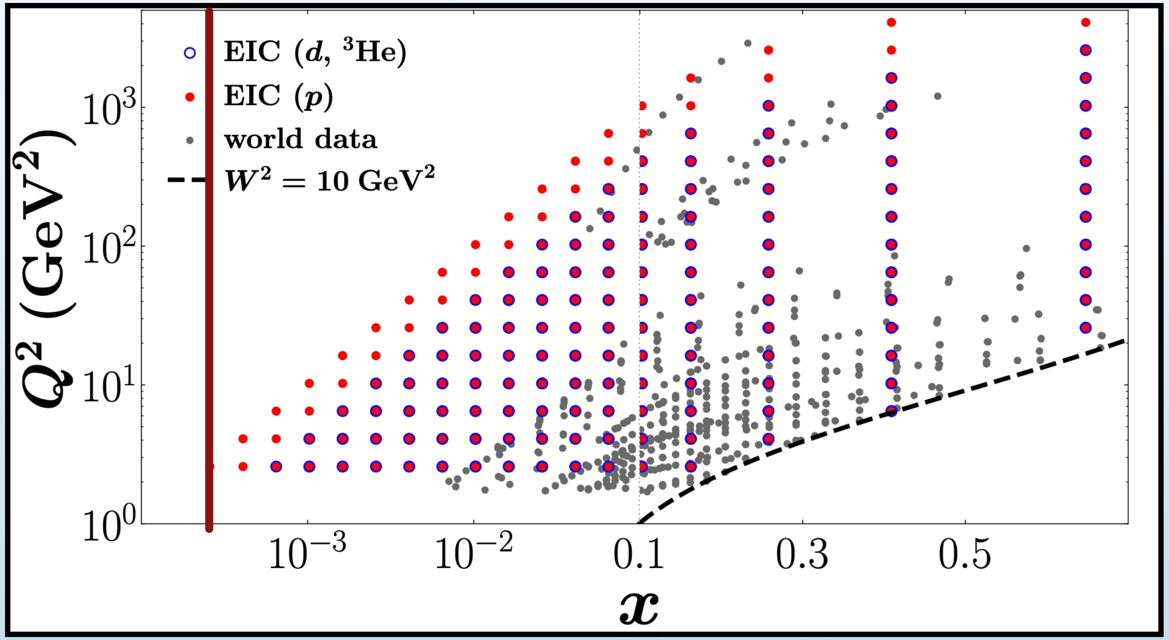

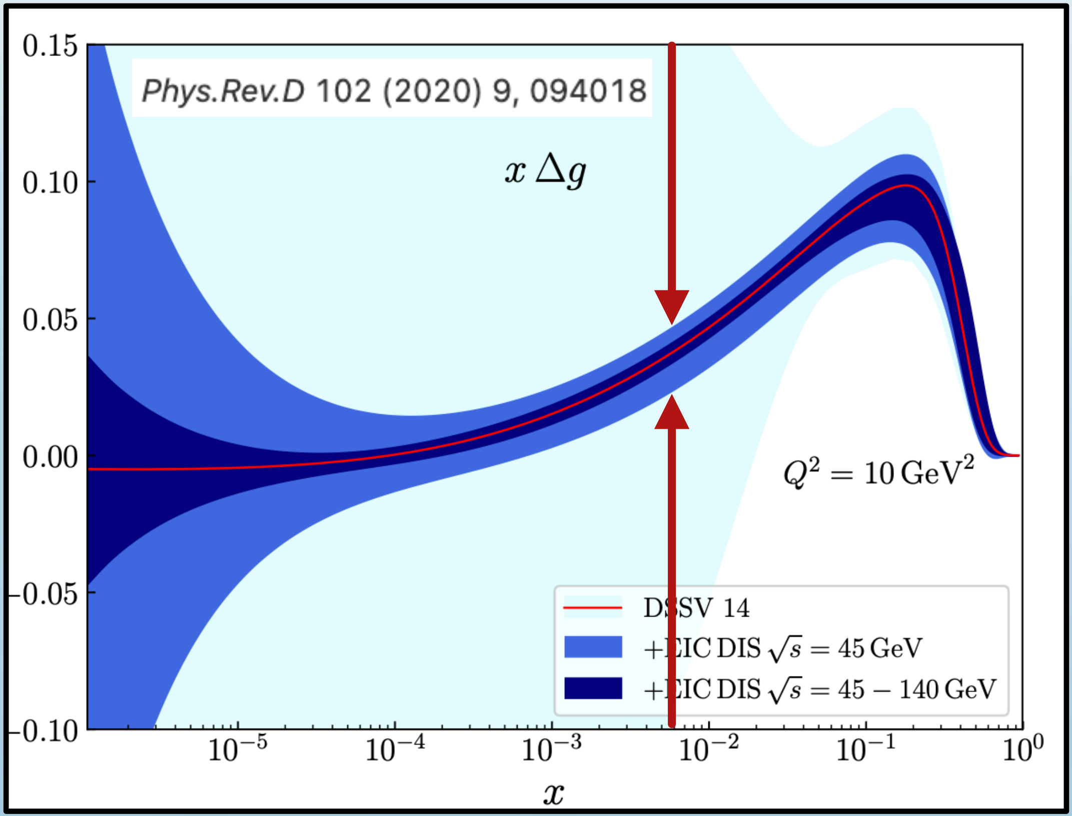

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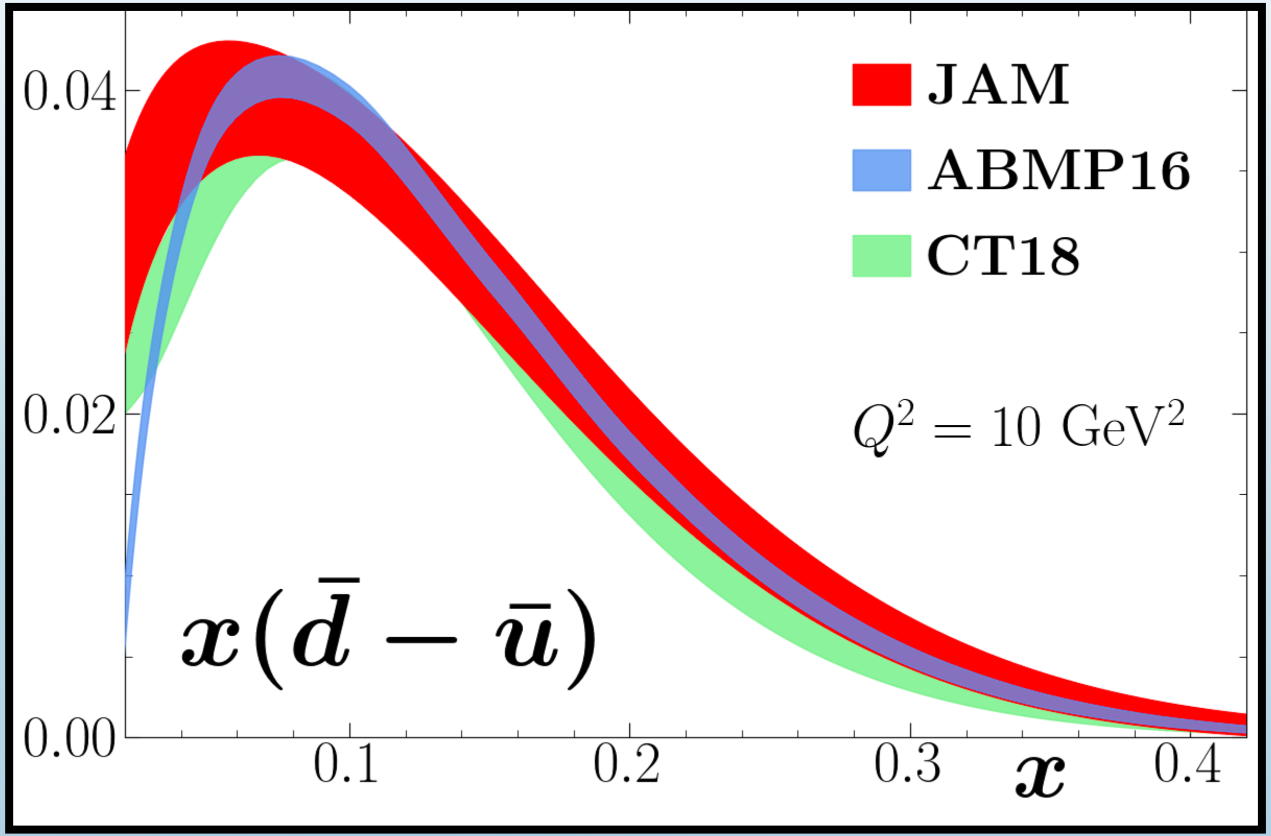
Introduction to Sea Asymmetry



Unpolarized

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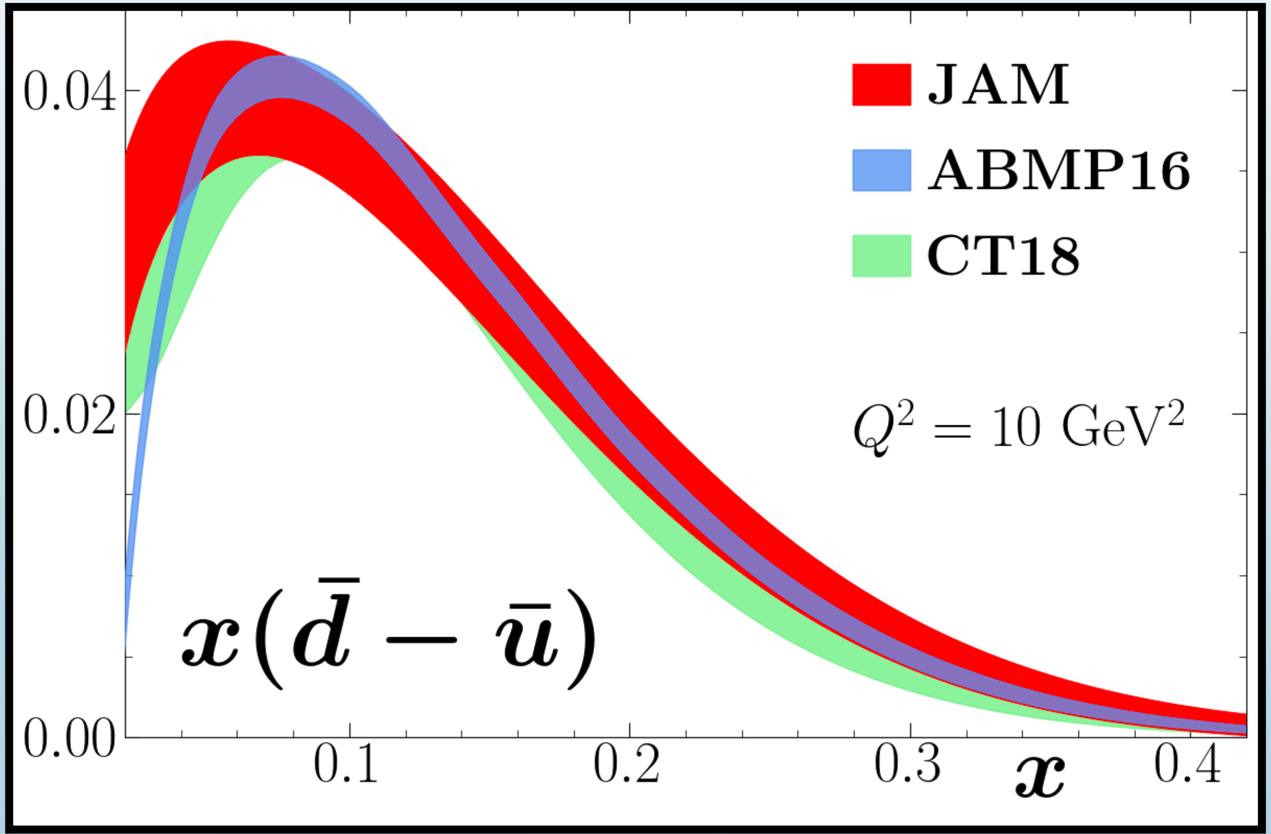

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Cannot be explained from gluons splitting into quark-antiquark pairs

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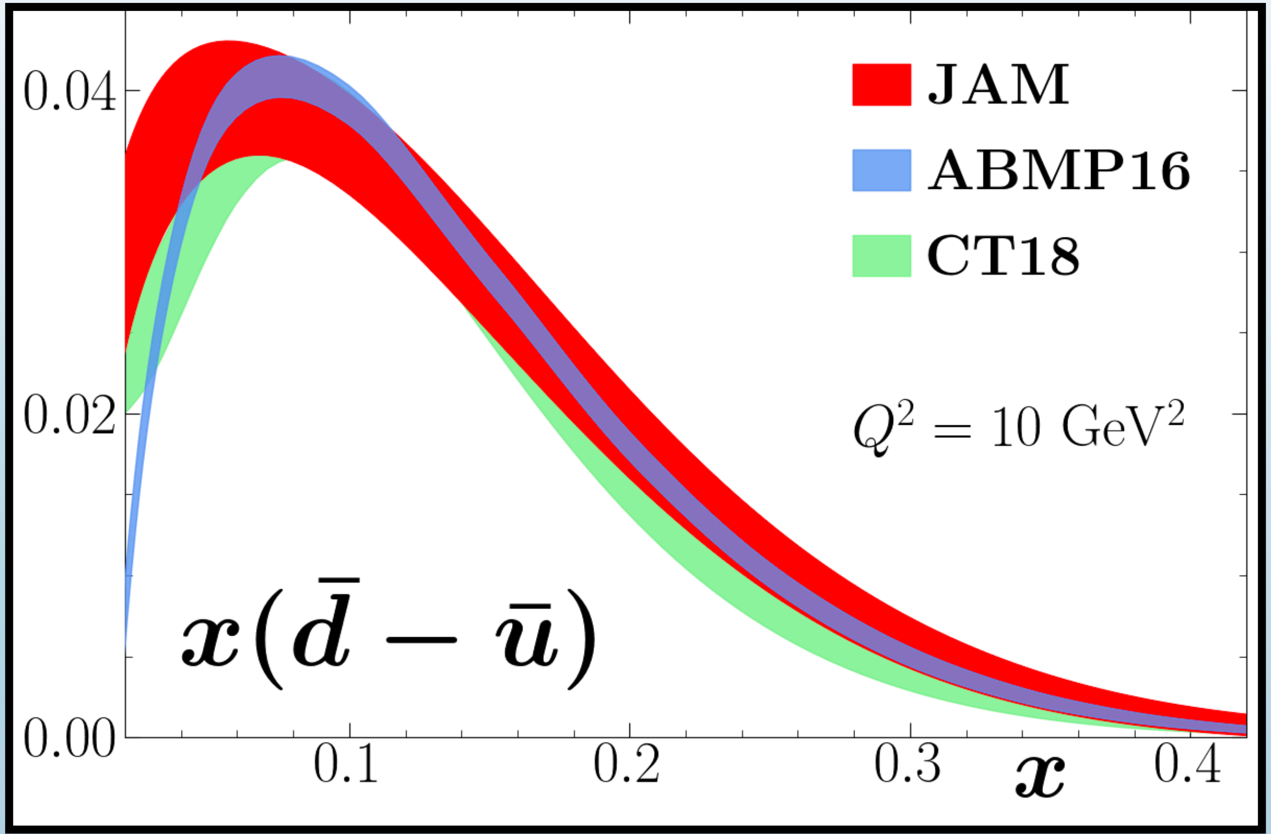

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Meson Cloud Models
Chiral Soliton Models
Statistical Models

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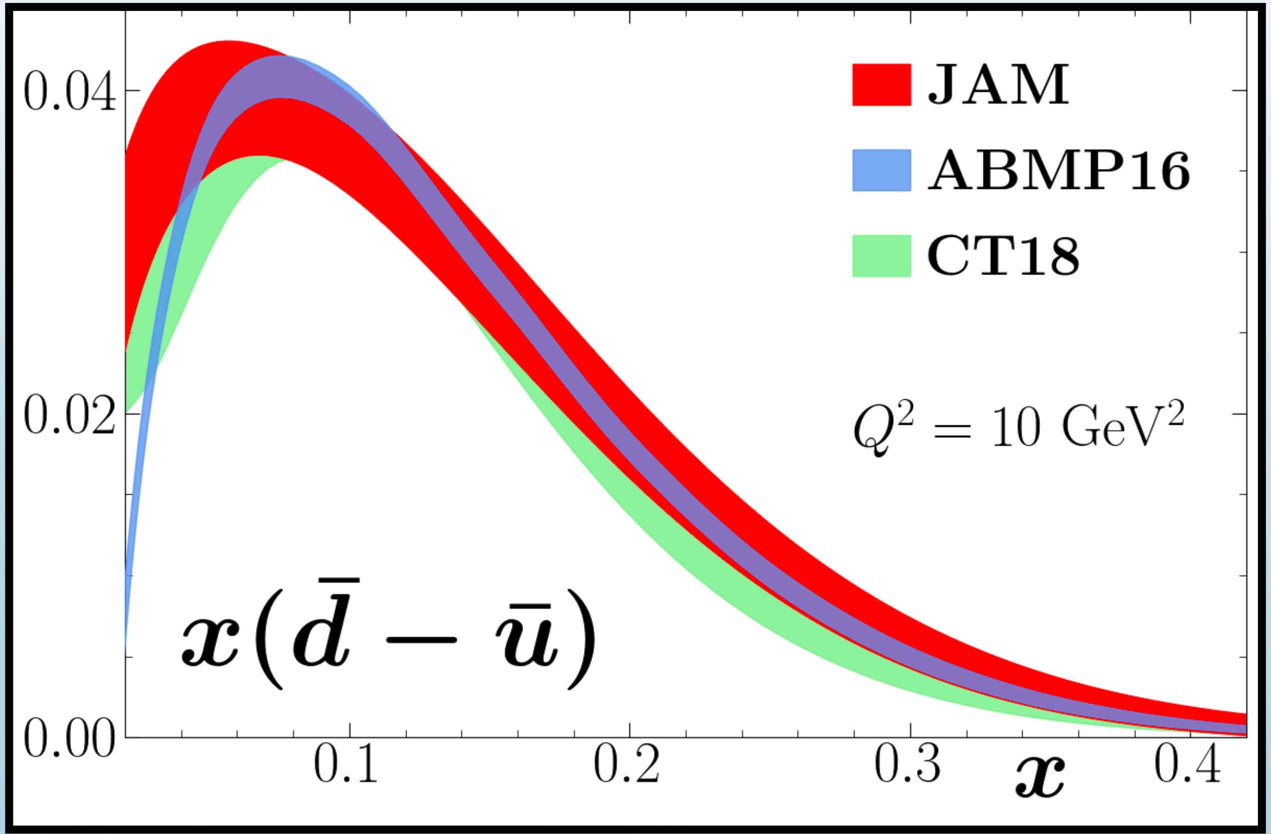

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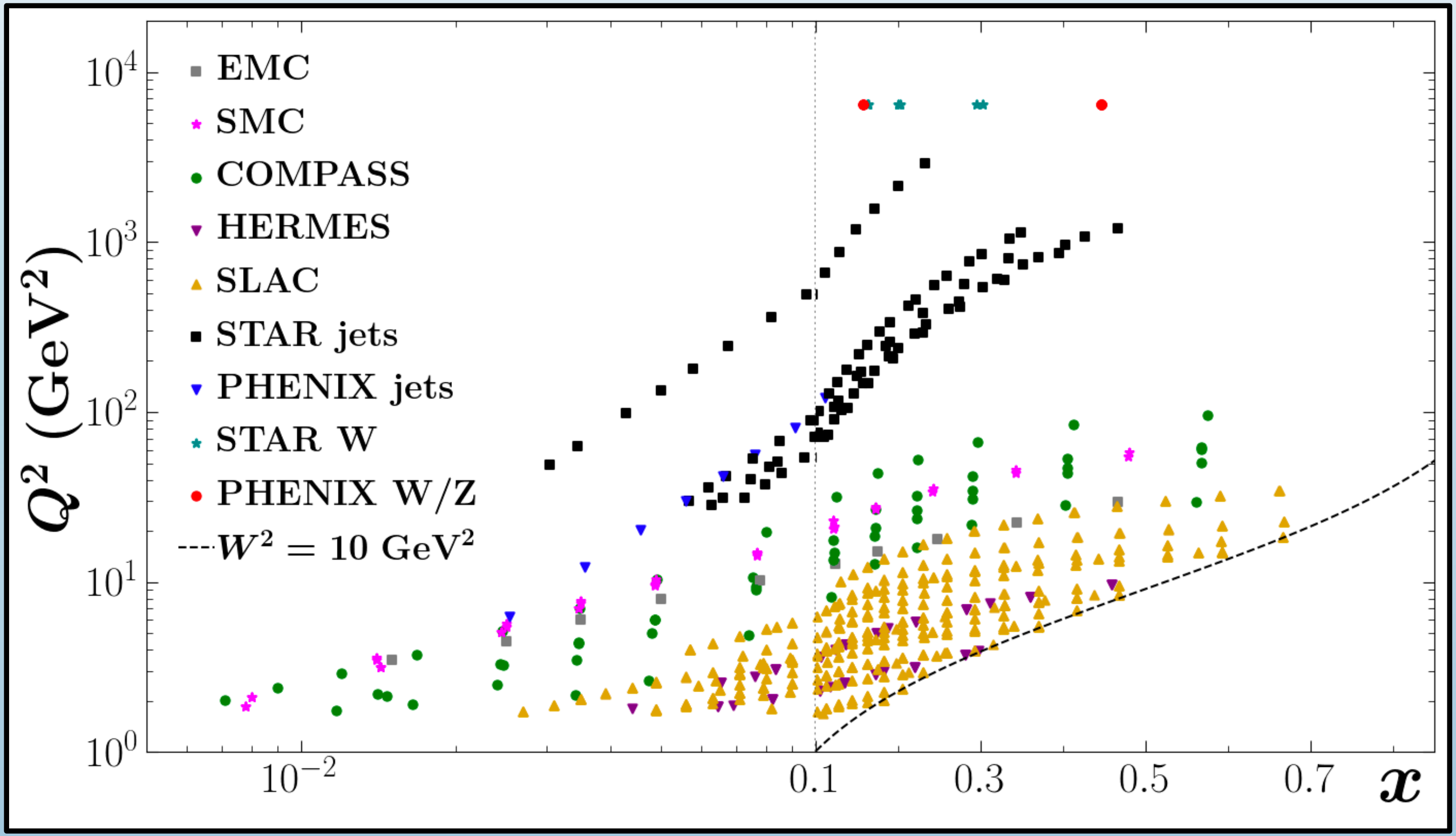
Meson Cloud Models
Chiral Soliton Models
Statistical Models

Still questions for helicity asymmetry

Kinematic Coverage (Helicity)

Deep Inelastic Scattering	COMPASS, EMC, HERMES, SLAC, SMC	365 points
Semi-Inclusive DIS	COMPASS, HERMES, SMC	231 points
W/Z Boson Production	STAR, PHENIX	18 points
Jets	STAR, PHENIX	61 points



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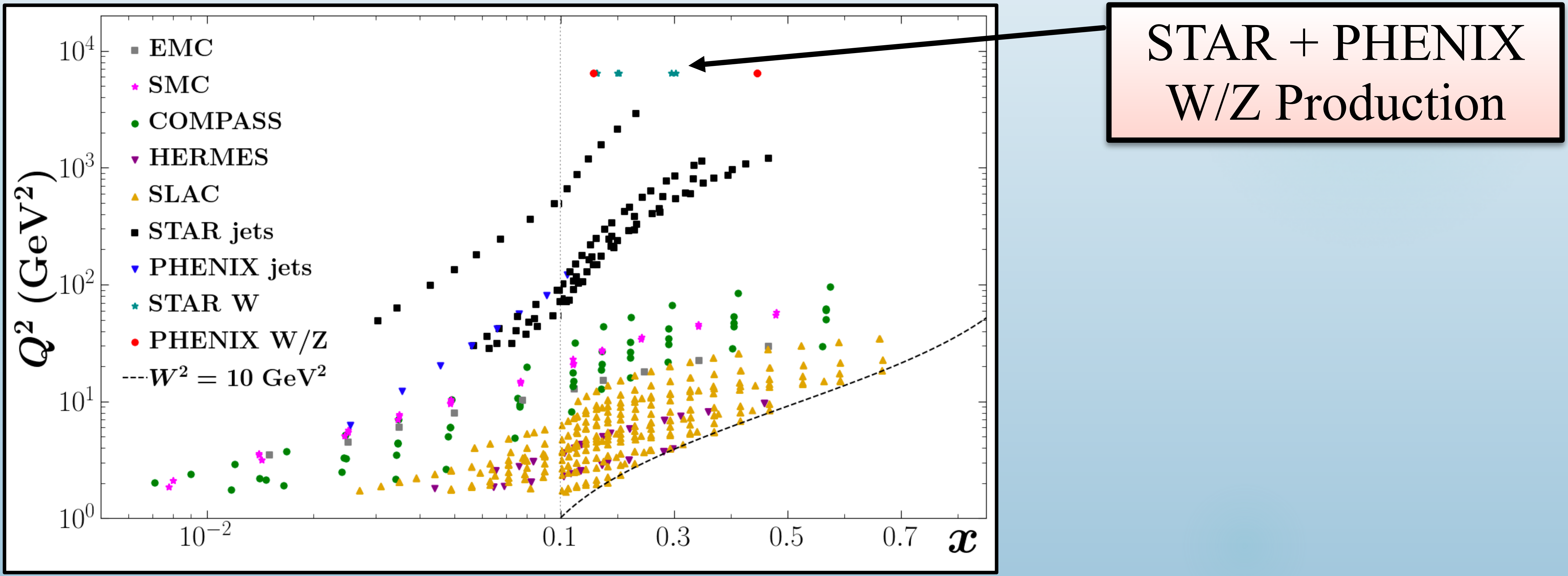

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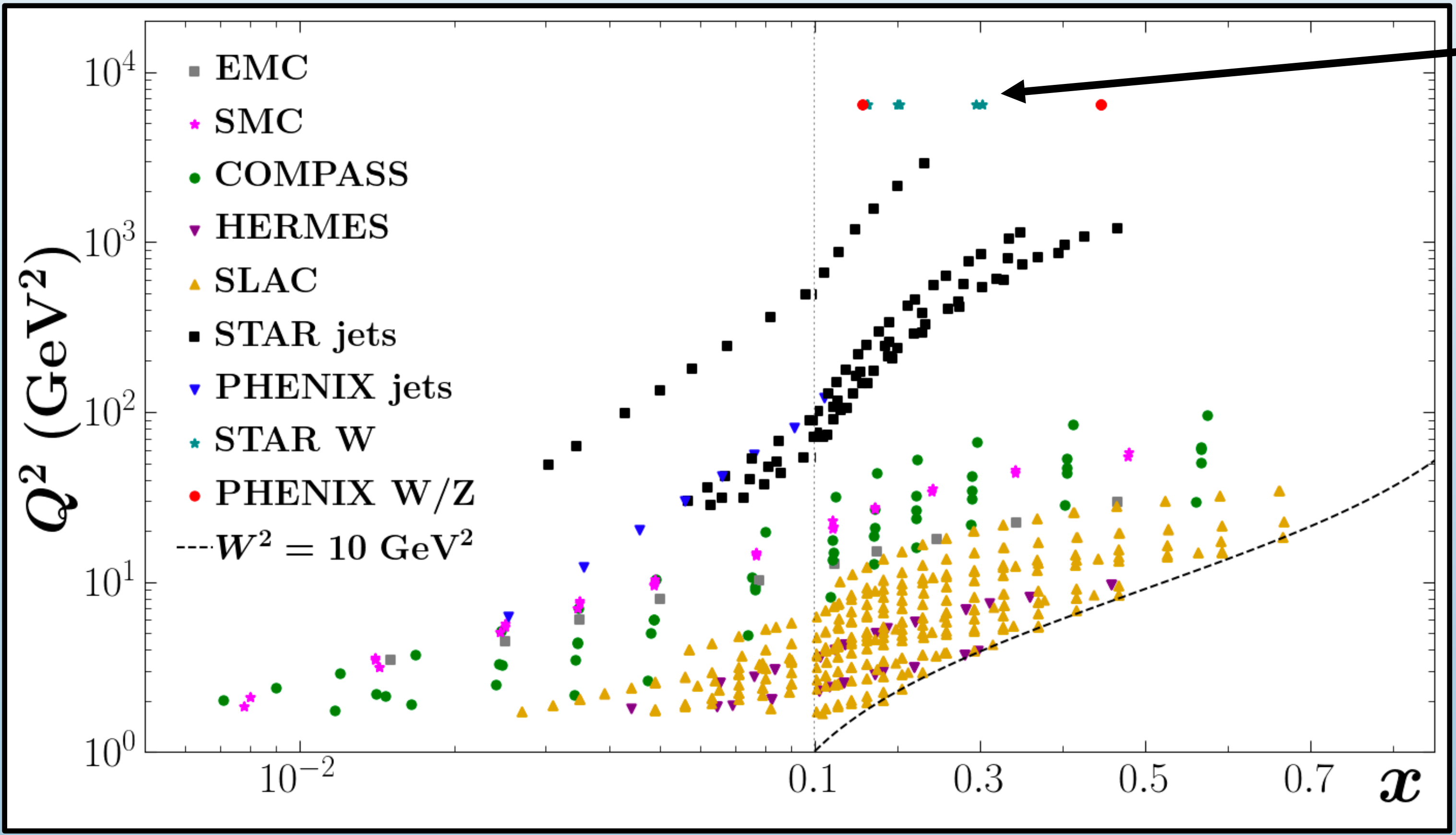
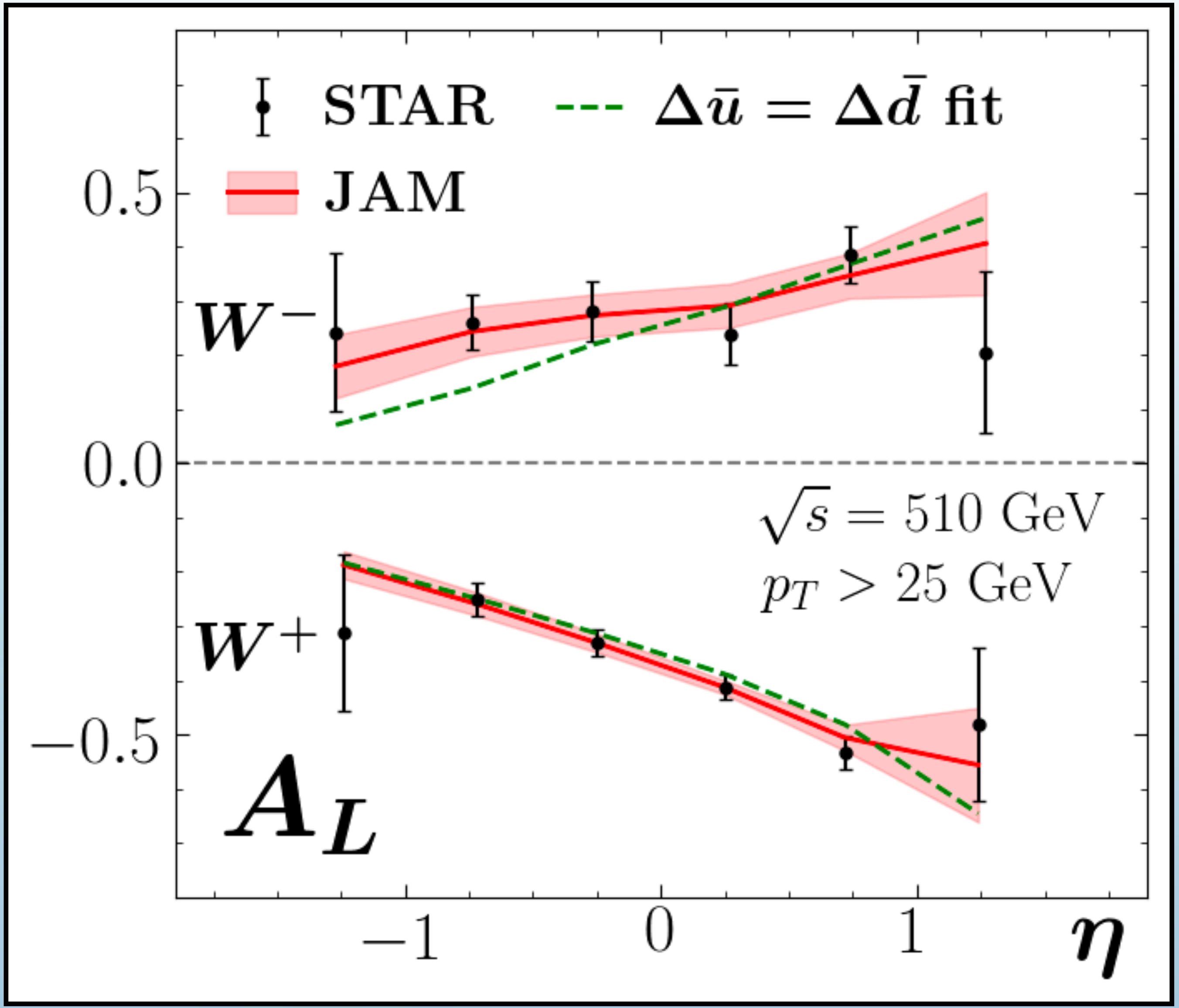


STAR + PHENIX
W/Z Production

process	$N_{\rm dat}$	$\chi^2/N_{\rm dat}$
polarized		
inclusive DIS	365	0.95
SIDIS (π^+, π^-)	64	1.05
SIDIS (K^+, K^-)	57	0.42
SIDIS (h^+, h^-)	110	0.95
inclusive jets	83	0.84
STAR $W^\pm$	12	0.65
PHENIX $W^\pm/Z$	6	0.50
total	**697**	**0.89**
unpolarized		
inclusive DIS	3908	1.17
SIDIS (π^+, π^-)	498	0.94
SIDIS (K^+, K^-)	494	1.31
SIDIS (h^+, h^-)	498	0.71
inclusive jets	198	1.28
Drell-Yan	205	1.21
W/Z production	153	1.01
total	**5954**	**1.12**
SIA ($\pi^\pm$)	231	0.91
SIA ($K^\pm$)	213	0.70
SIA ($h^\pm$)	120	1.07
total	**7215**	**1.08**

STAR Quality of Fit



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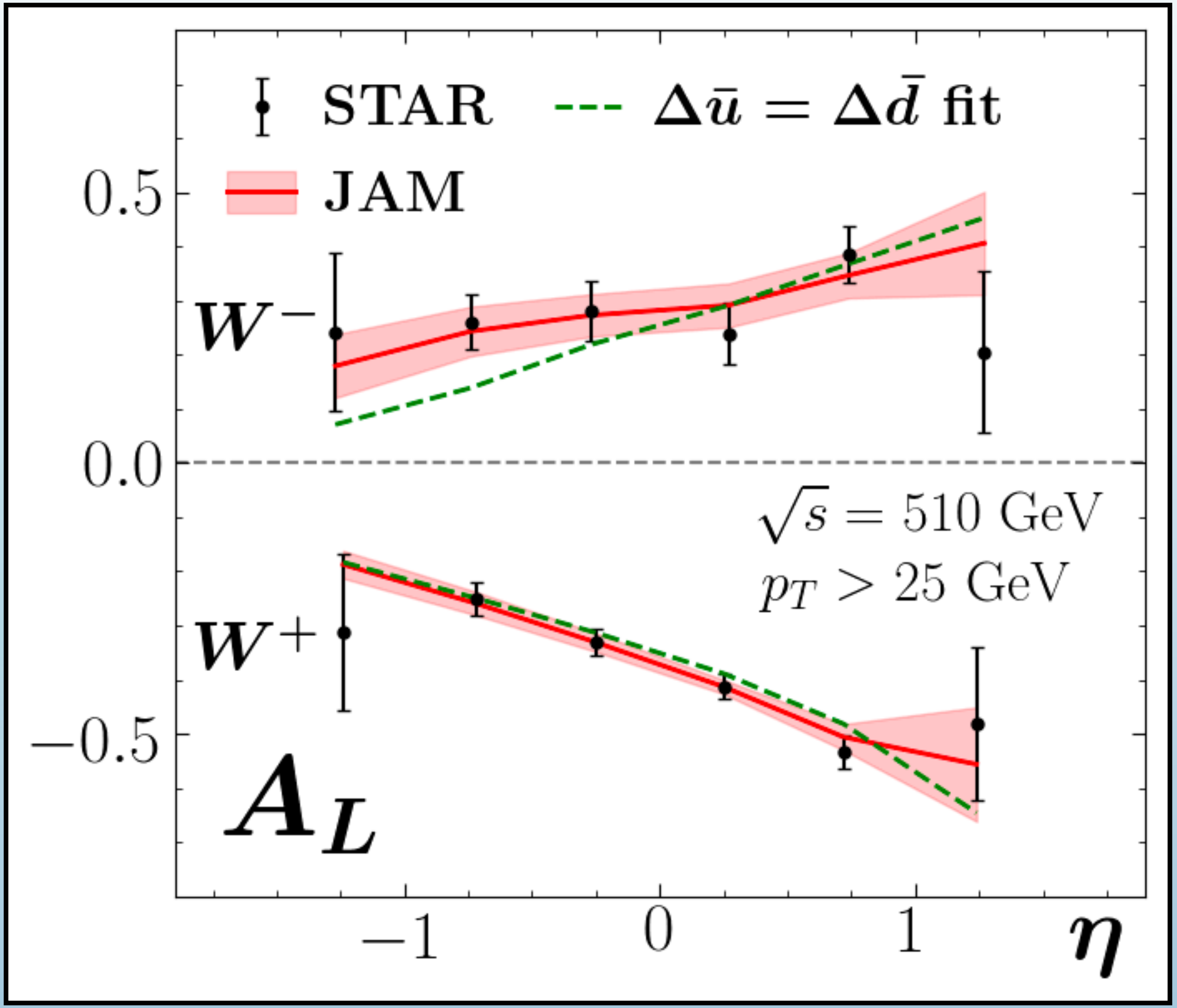
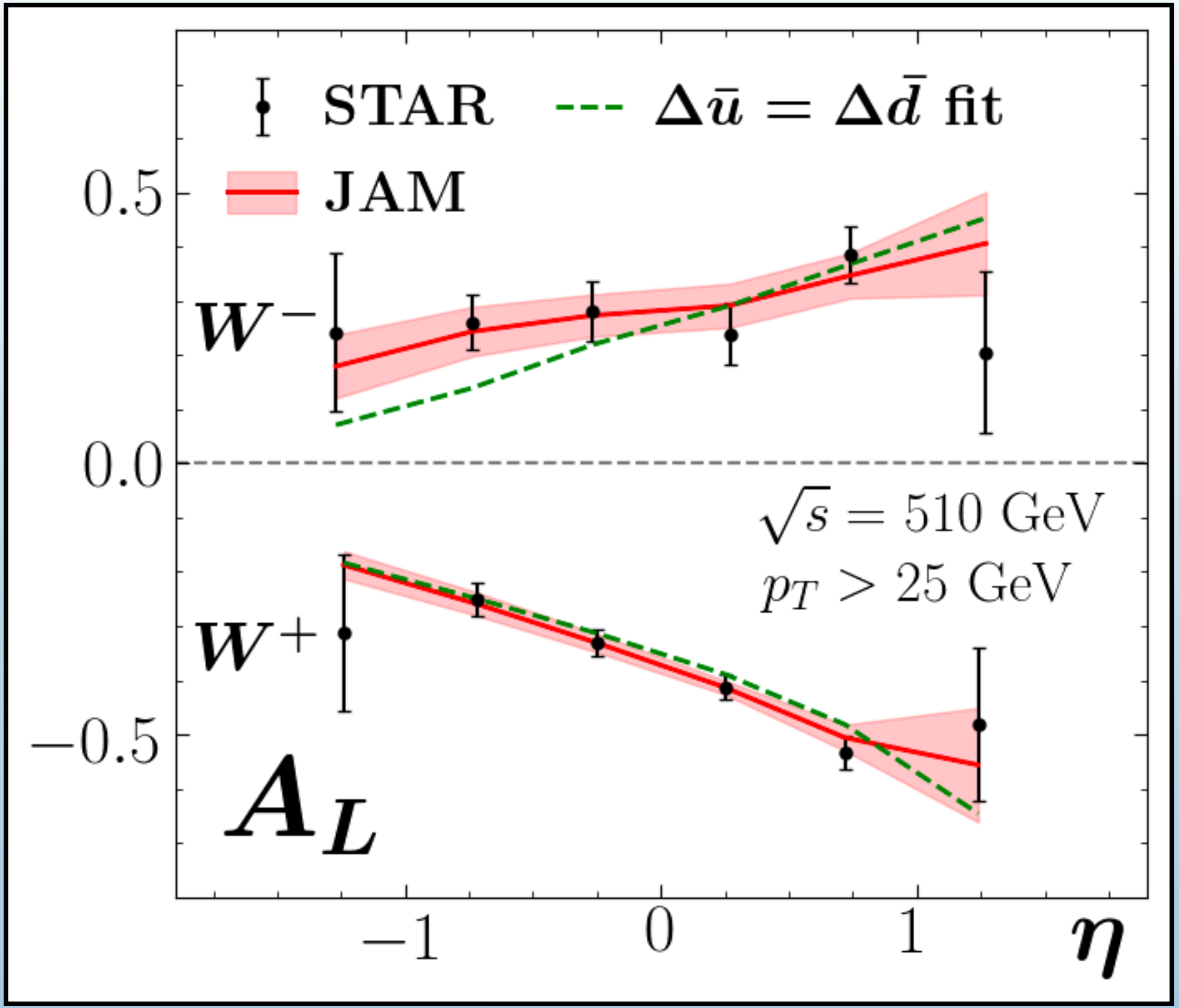


$$A_L^{W^+}(y_W) \propto \frac{\Delta\bar{d}(x_1)u(x_2) - \Delta u(x_1)\bar{d}(x_2)}{\bar{d}(x_1)u(x_2) + u(x_1)\bar{d}(x_2)}$$

$$A_L^{W^-}(y_W) \propto \frac{\Delta\bar{u}(x_1)d(x_2) - \Delta d(x_1)\bar{u}(x_2)}{\bar{u}(x_1)d(x_2) + d(x_1)\bar{u}(x_2)}$$

STAR Quality of Fit



process	$N_{\rm dat}$	$\chi^2/N_{\rm dat}$ JAM	+Pos.	$\Delta\bar{u} = \Delta\bar{d}$
STAR $W^{\pm}$	12	0.45	0.61	1.53
PHENIX $W^{\pm}/Z$	6	0.47	0.46	0.48
pol. DIS	365	0.93	0.93	0.93
pol. jet	61	1.00	1.03	1.00
total	**444**	**0.92**	**0.94**	**0.95**

$$A_L^{W^+}(y_W) \propto \frac{\Delta\bar{d}(x_1)u(x_2) - \Delta u(x_1)\bar{d}(x_2)}{\bar{d}(x_1)u(x_2) + u(x_1)\bar{d}(x_2)}$$

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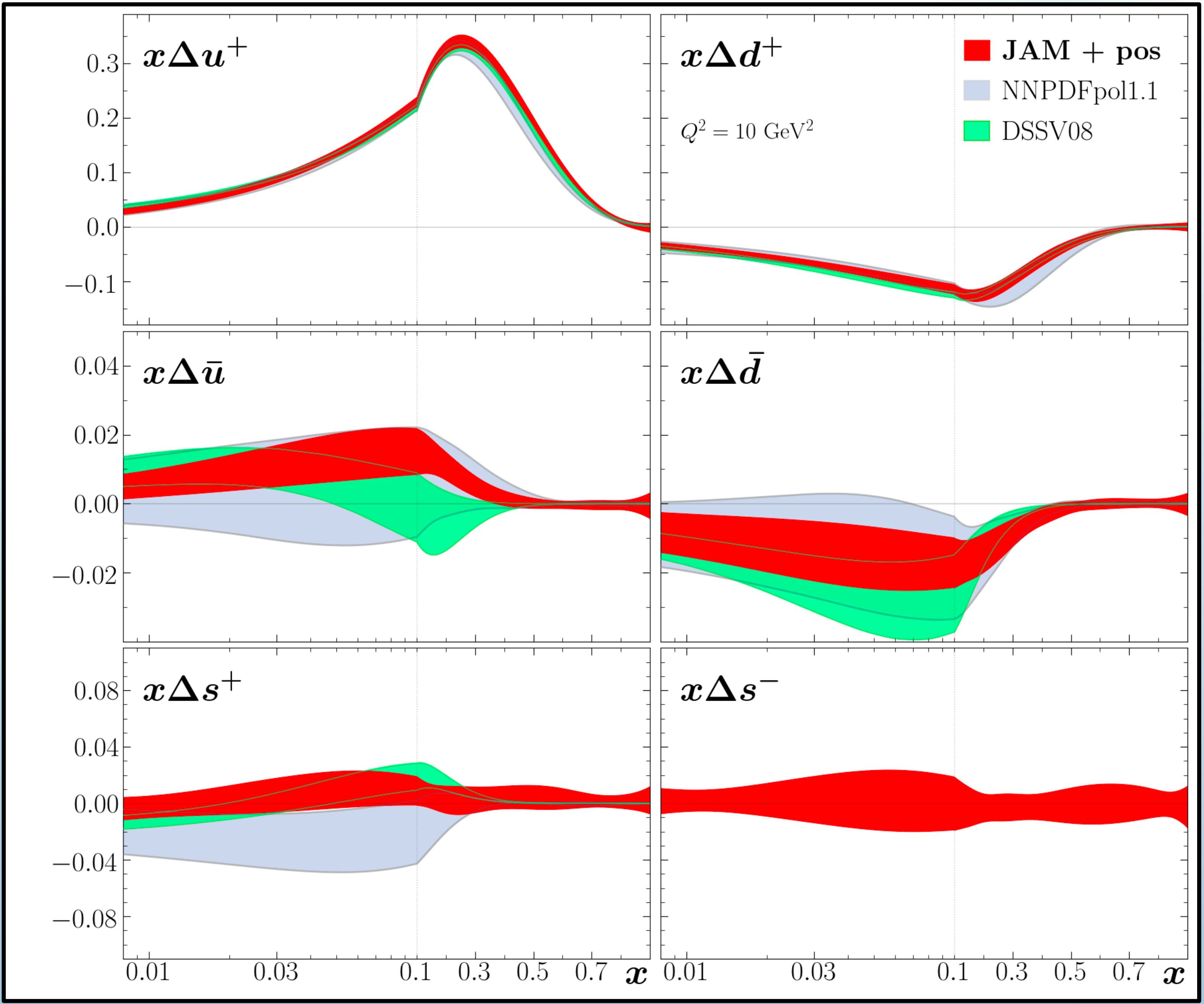
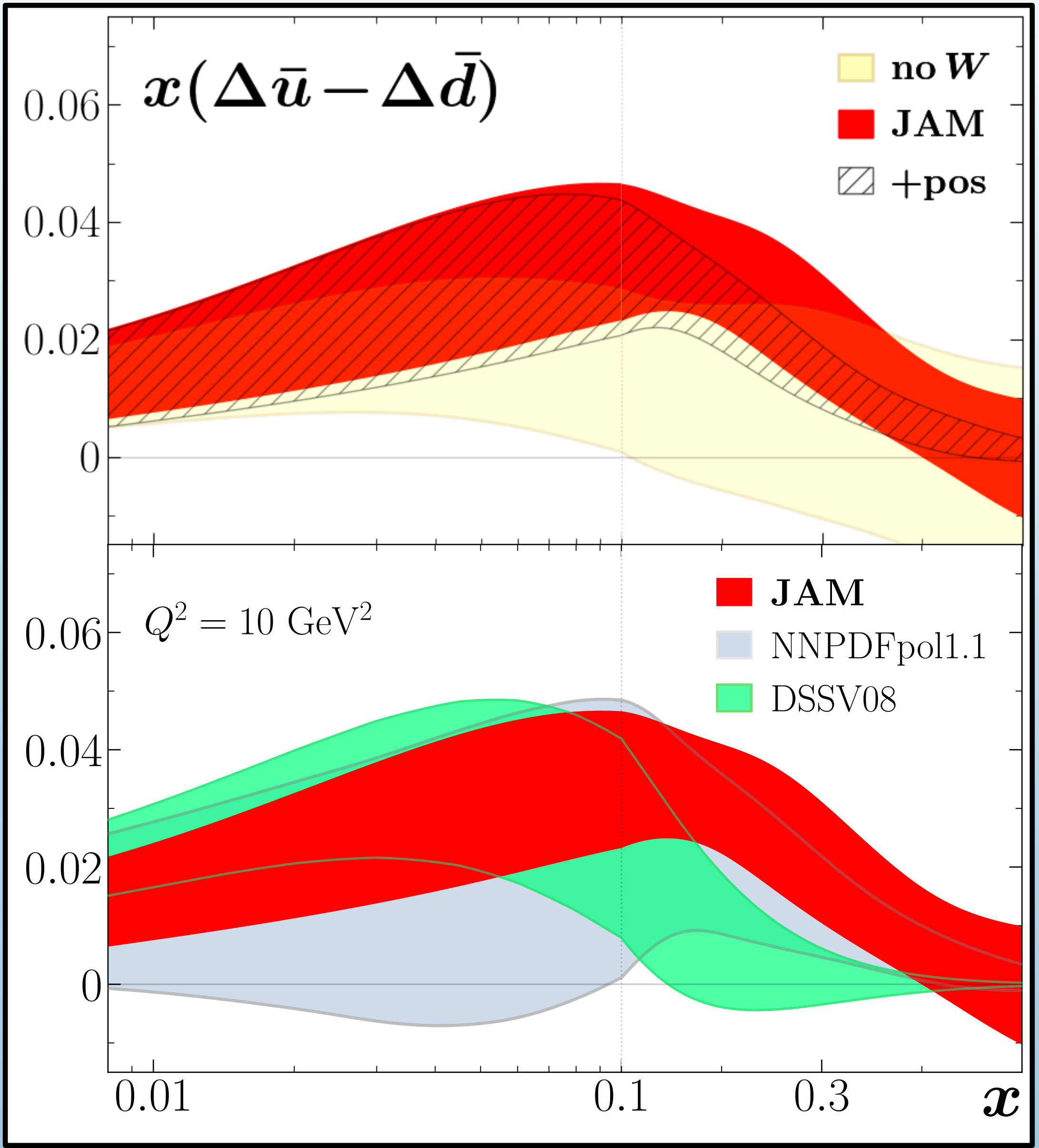
Sea Asymmetry



Resulting Asymmetry



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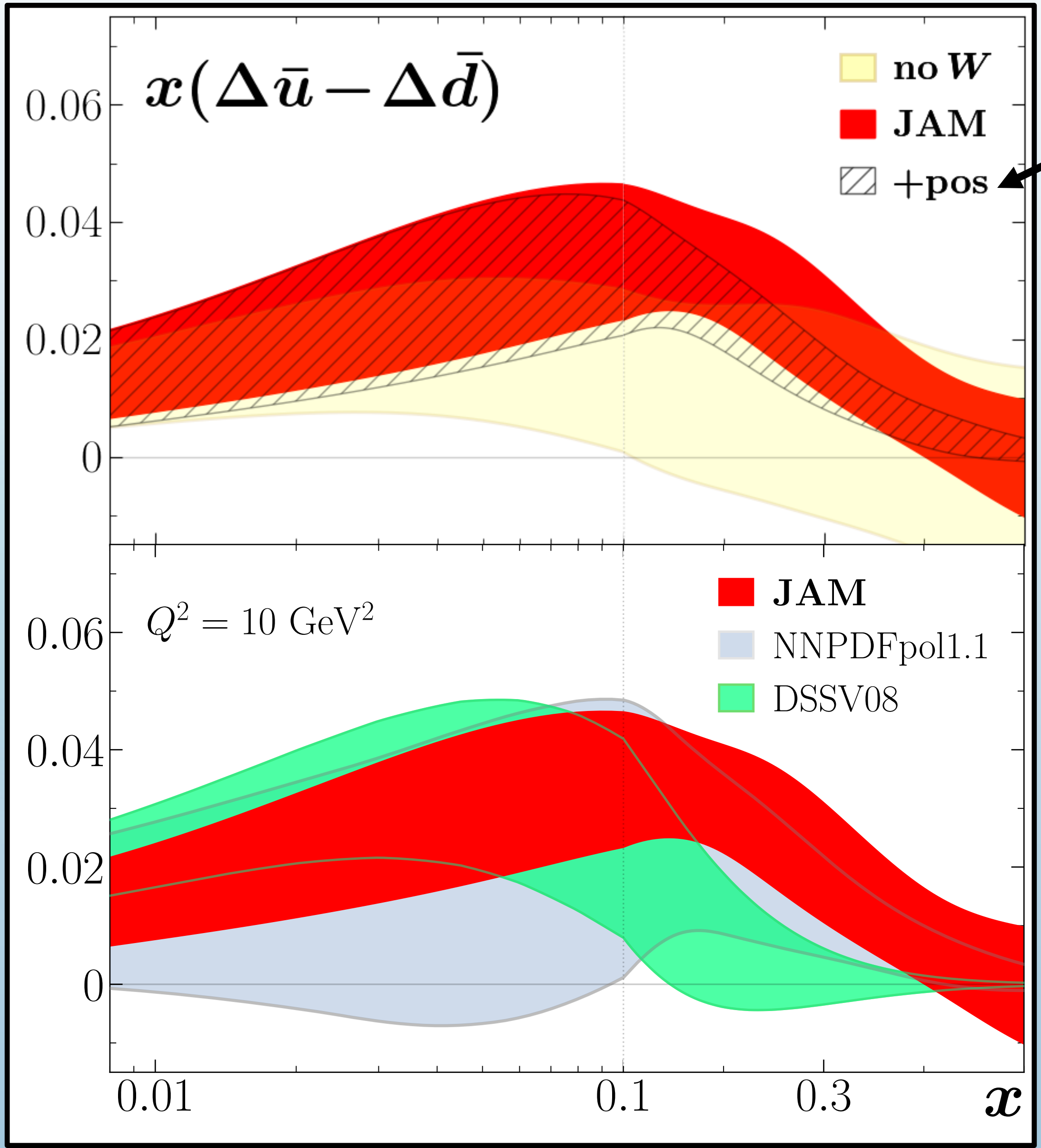

Positivity Constraints:
$|\Delta f(x, Q^2)| < f(x, Q^2)$

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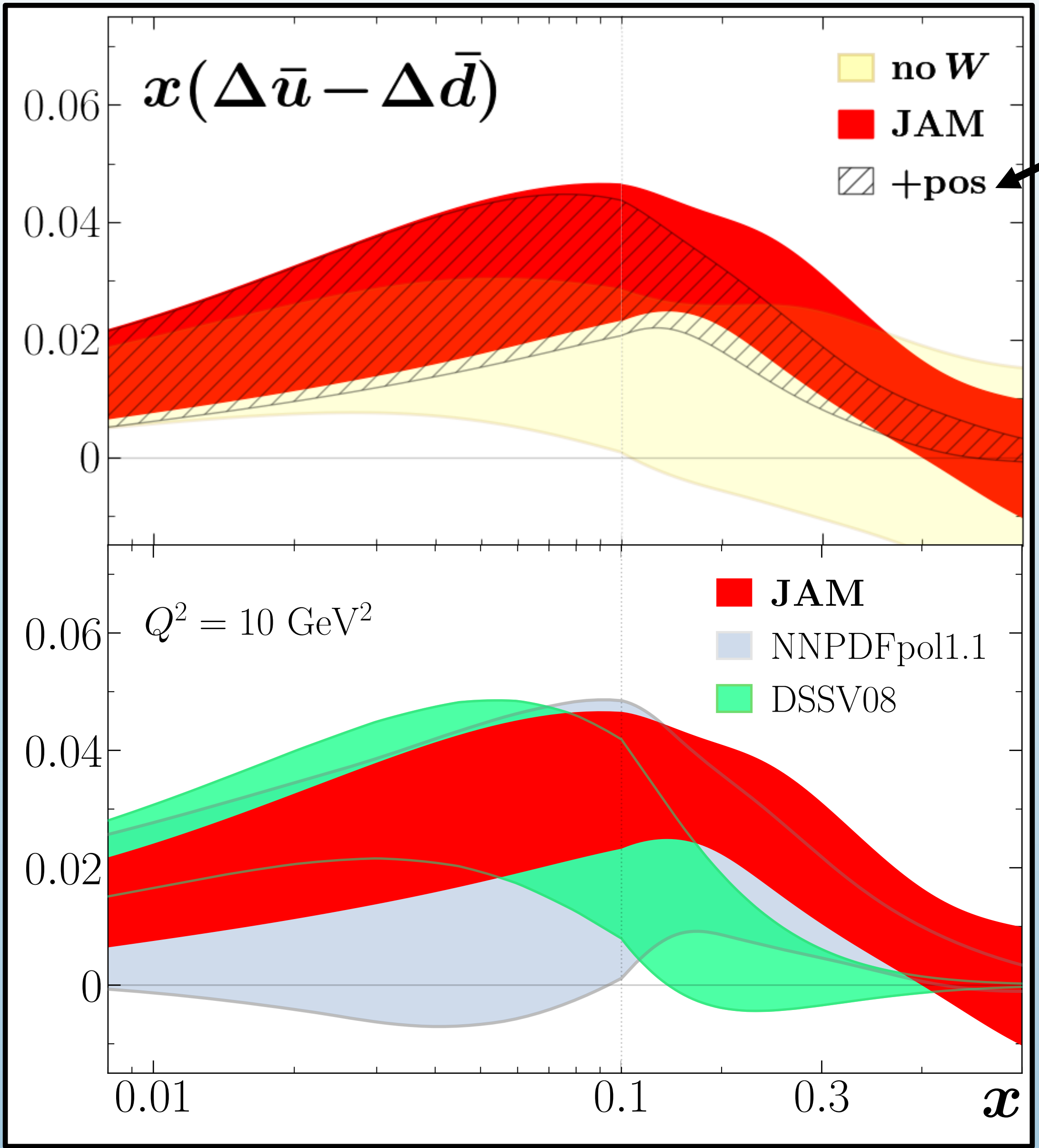

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Can $\overline{\text{MS}}$ parton distributions be negative?
Alessandro Candido, Stefano Forte and Felix Hekhorn

Positivity and renormalization of parton densities
John Collins, Ted C. Rogers, Nobuo Sato

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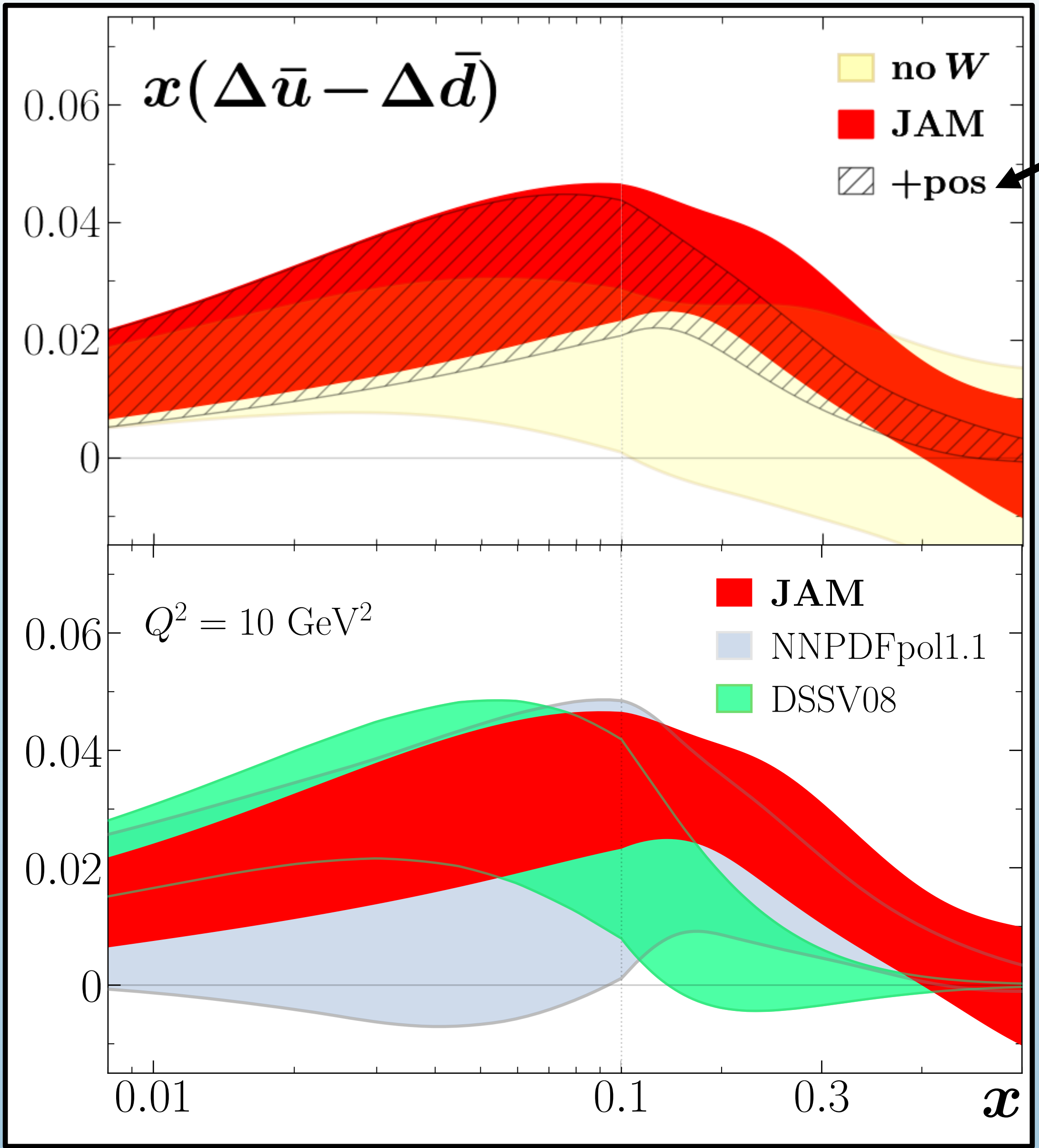

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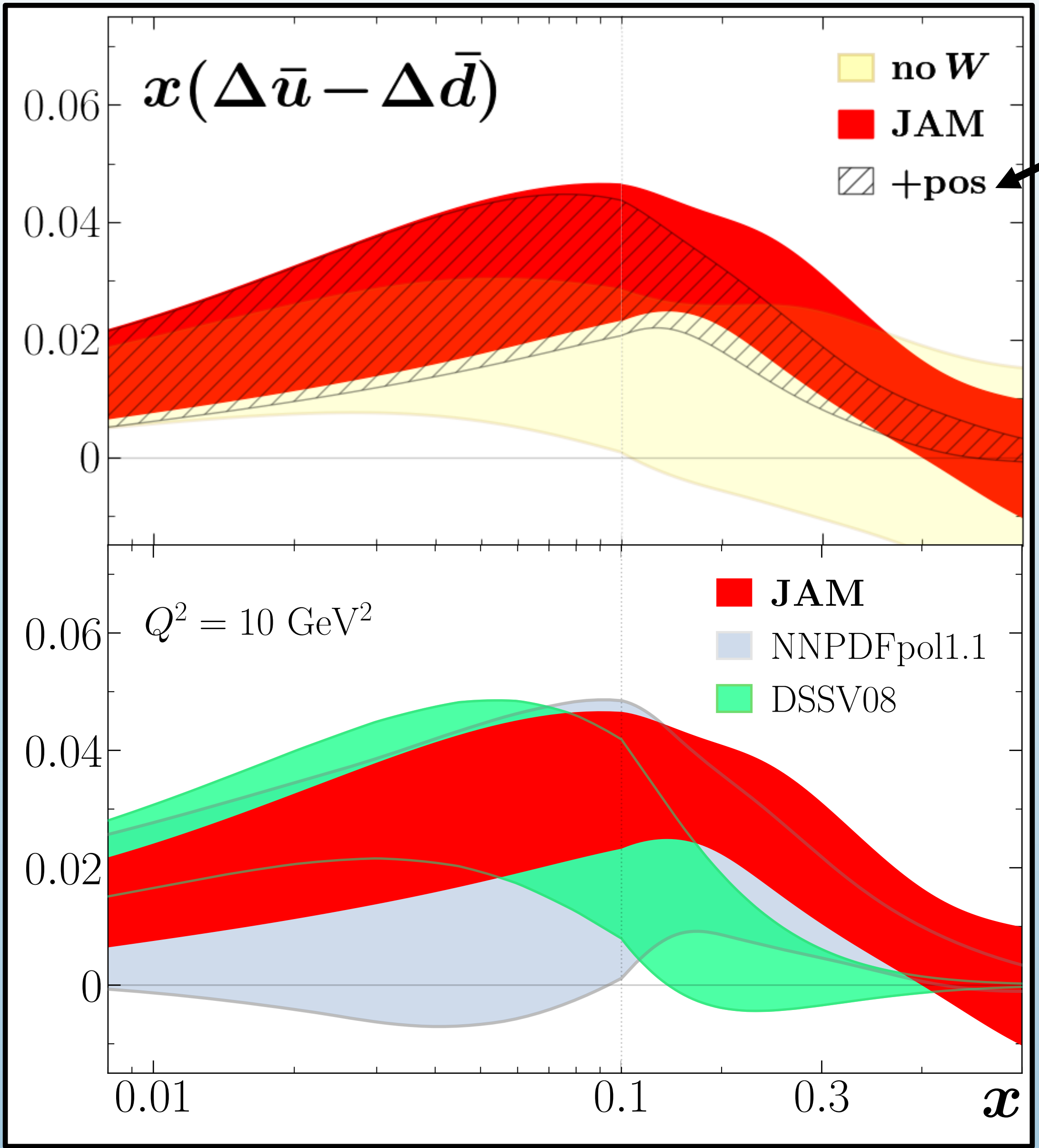

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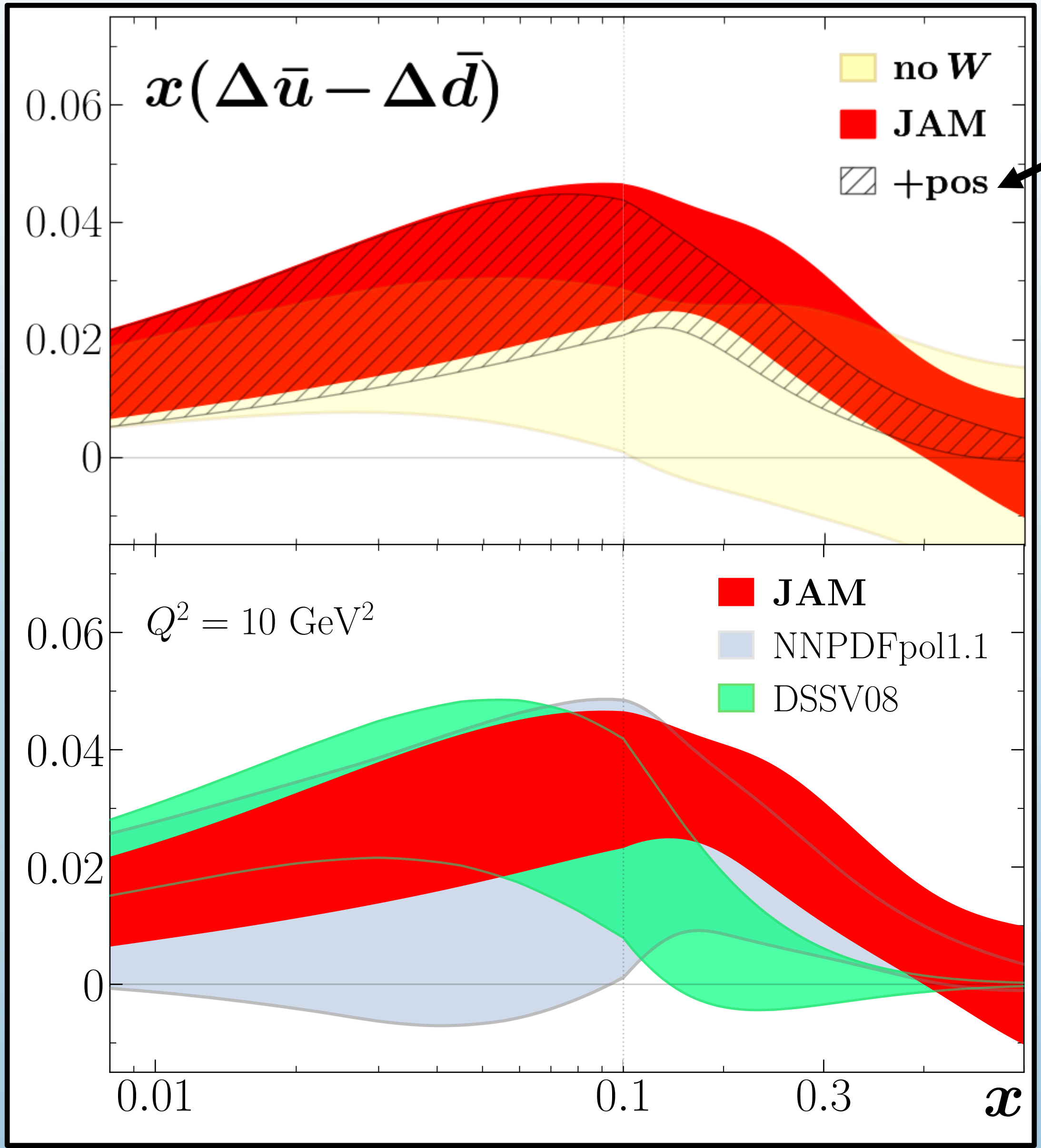

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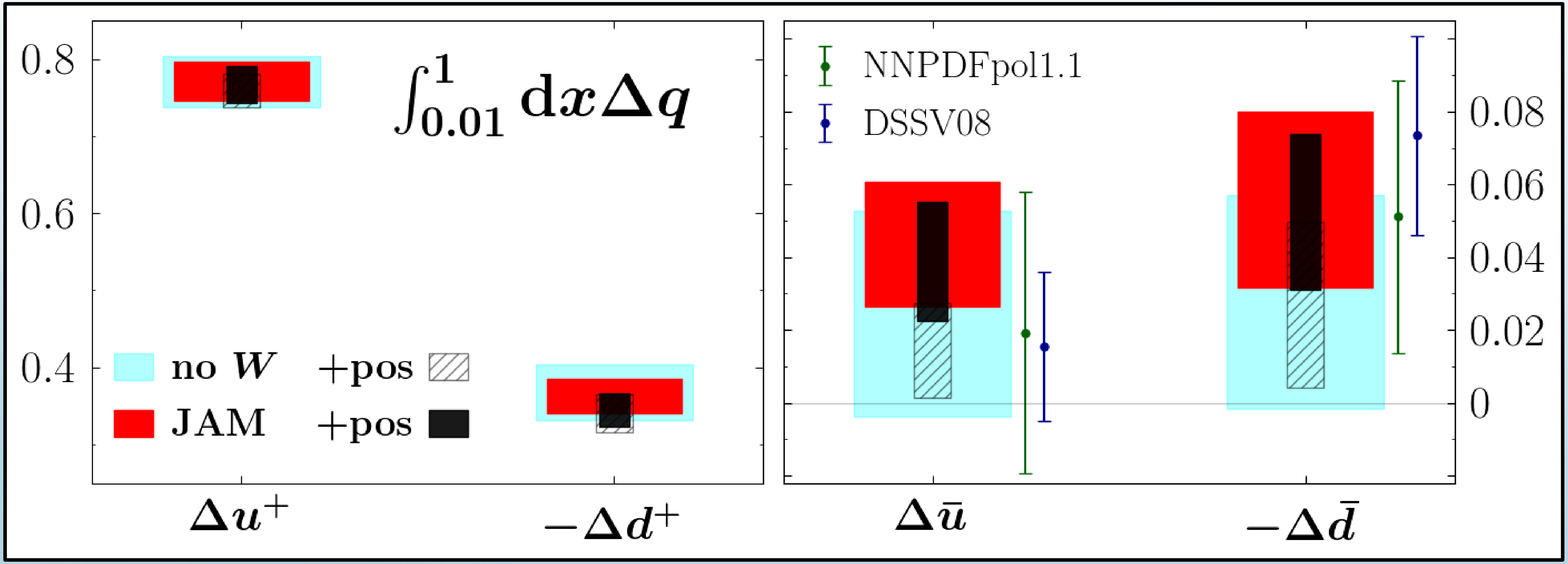
Our result is strongly positive in both regions of x

Proton Spin Contributions



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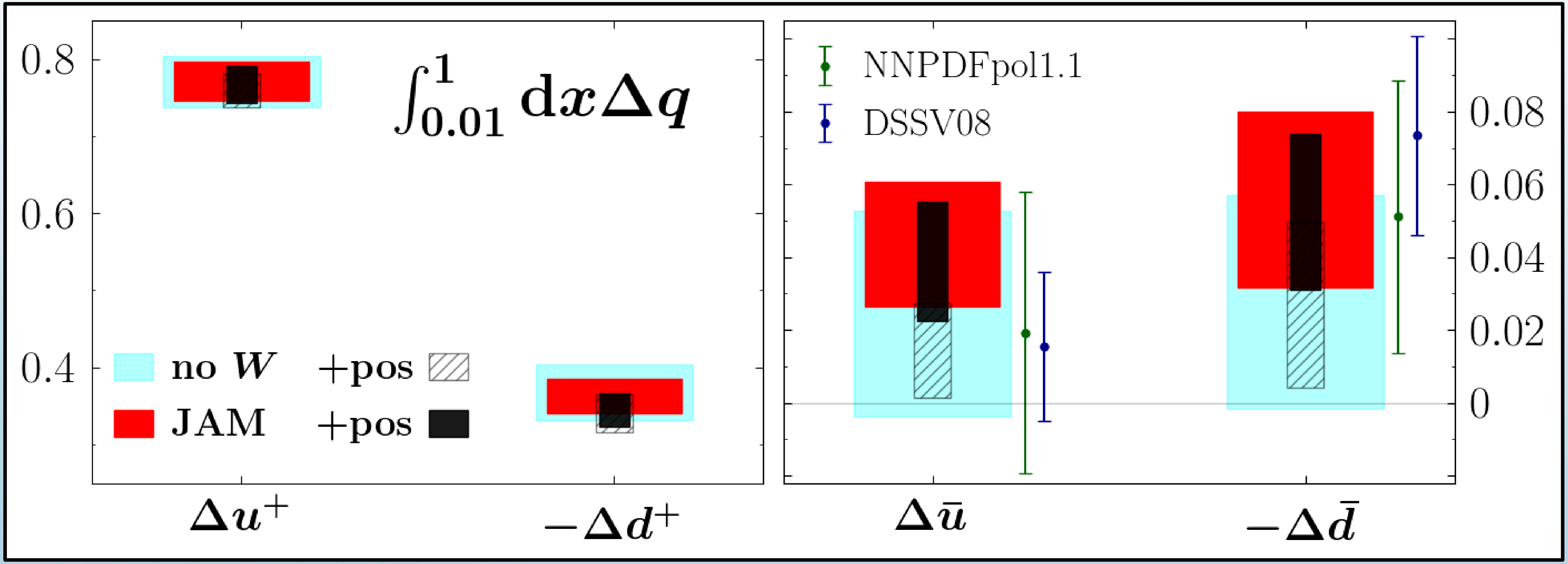

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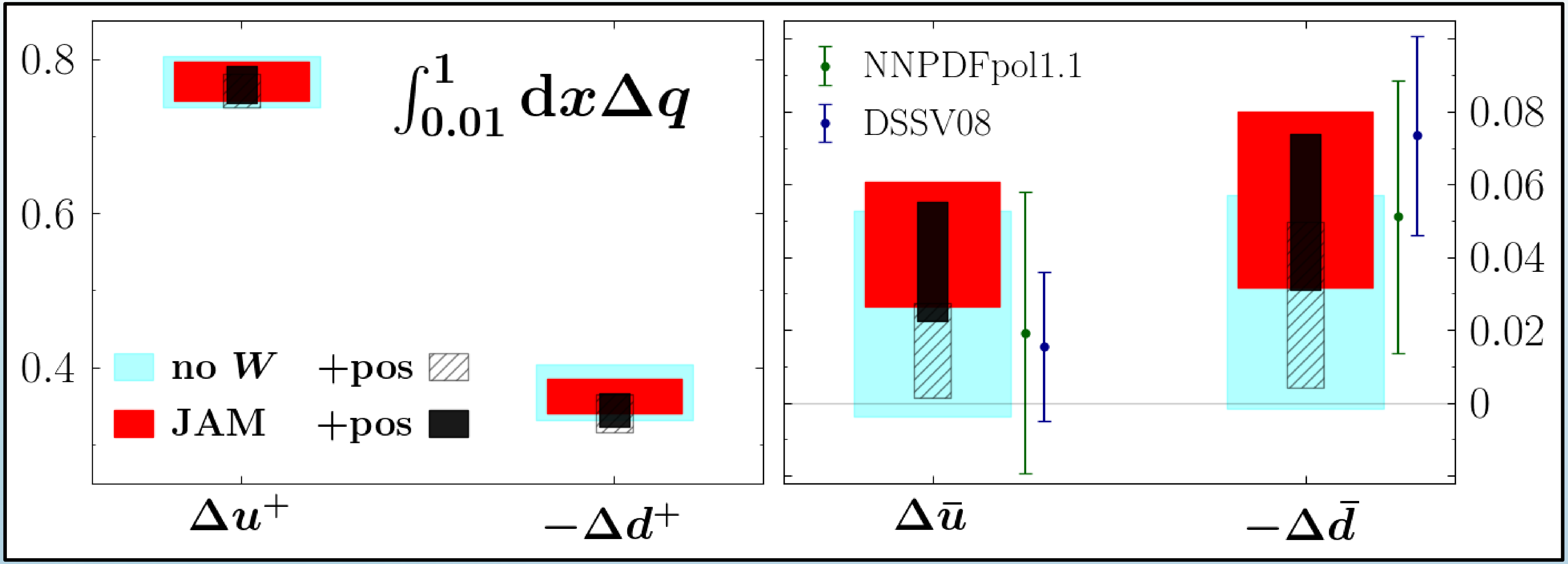

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Flavor	JAM moment (truncated)	Lattice Moment (full)	Difference
Δu^+	0.779(34)	0.864(16)	10%
Δd^+	-0.370(40)	-0.426(16)	13%

C. Alexandrou *et al.*, Phys. Rev. D **101**, 094513 (2020)

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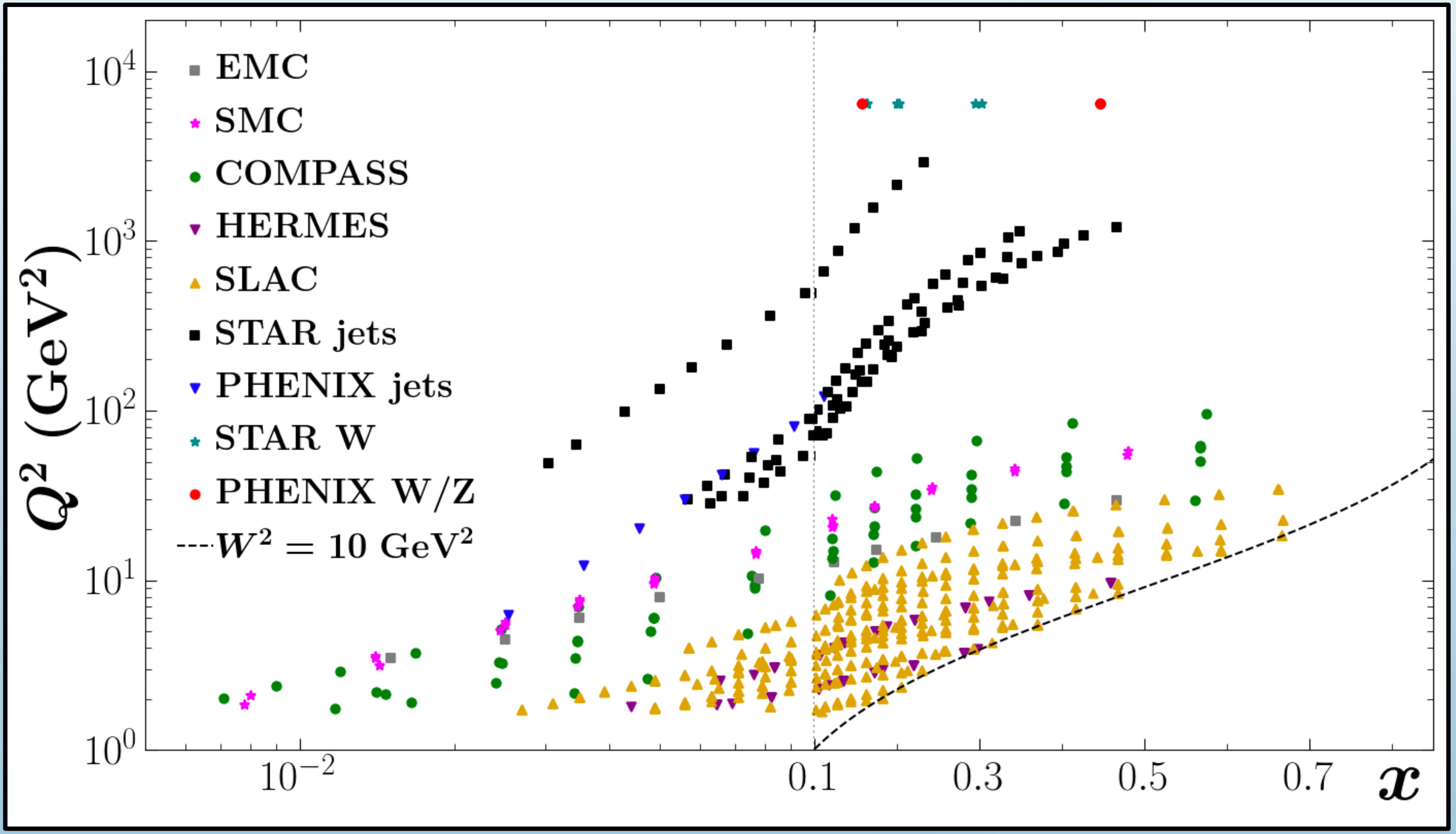

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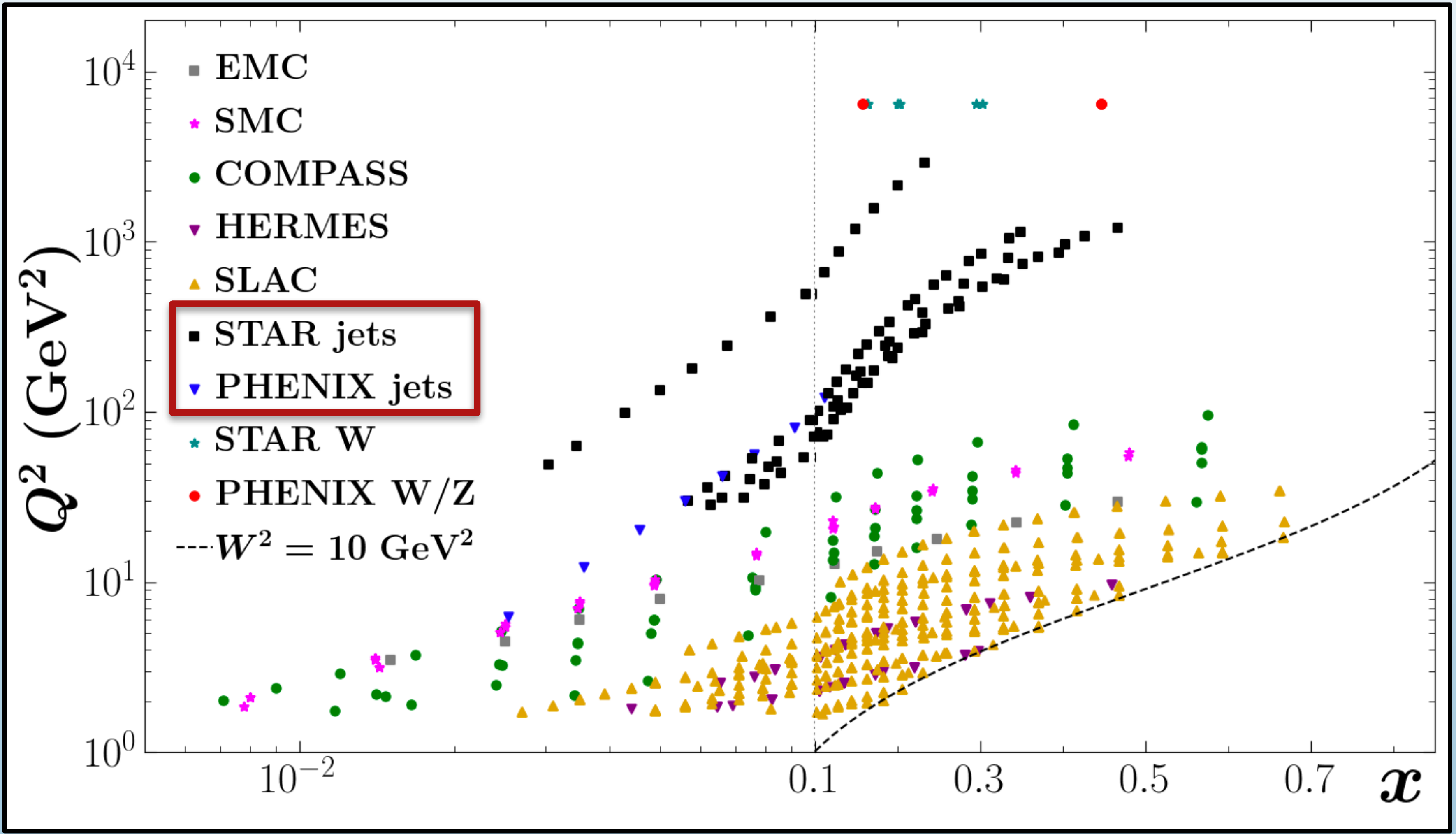
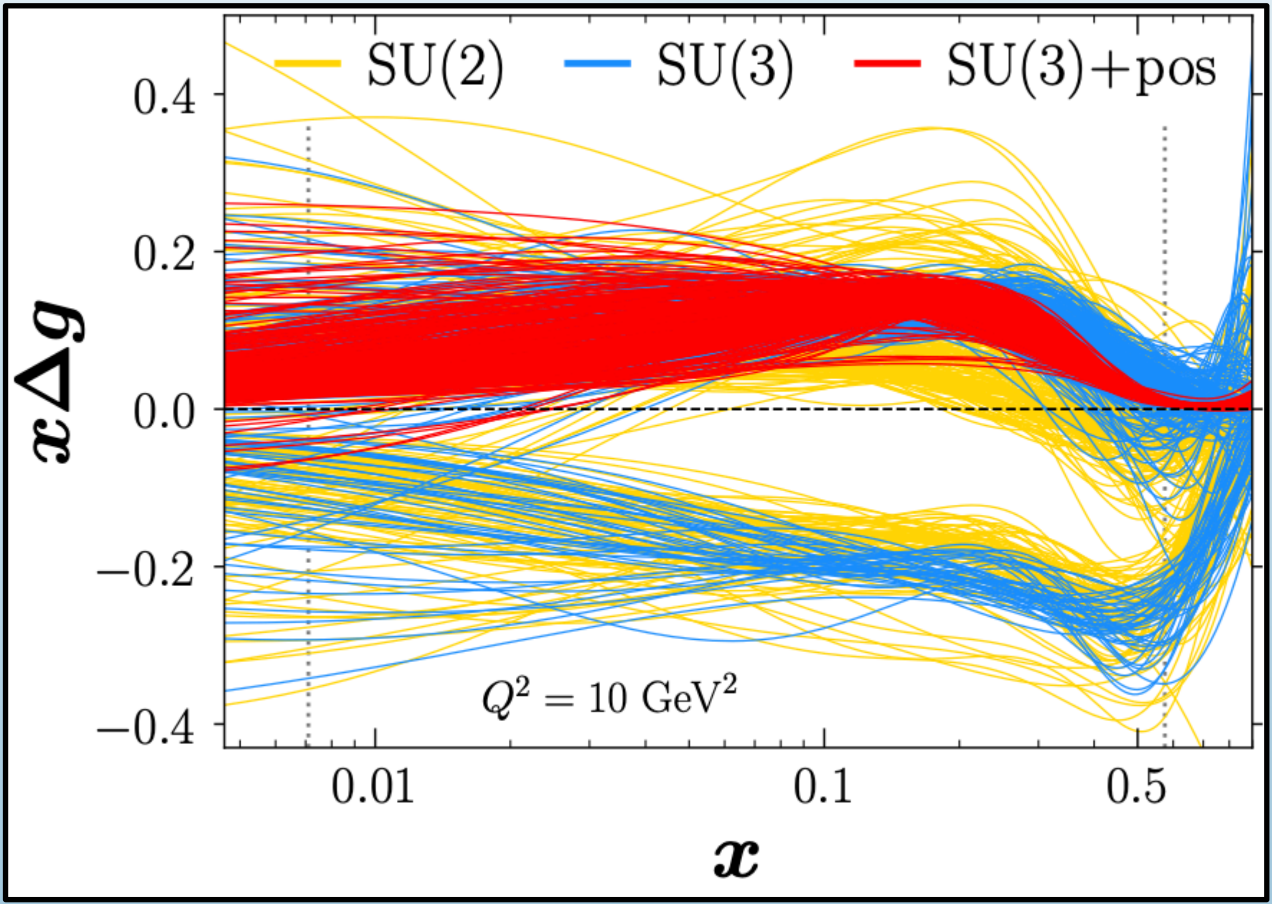
Jets provide most direct constraints on gluon distribution



How well do we know the gluon polarization in the proton? #1

Jefferson Lab Angular Momentum (JAM) Collaboration • Y. Zhou (South China Normal U. and UCLA and William-Mary Coll. and Jefferson Lab) et al. (Jan 6, 2022)

Published in: *Phys.Rev.D* 105 (2022) 7, 074022 • e-Print: 2201.02075 [hep-ph]



Gluon Helicity

How well do we know the gluon polarization in the proton?

#1

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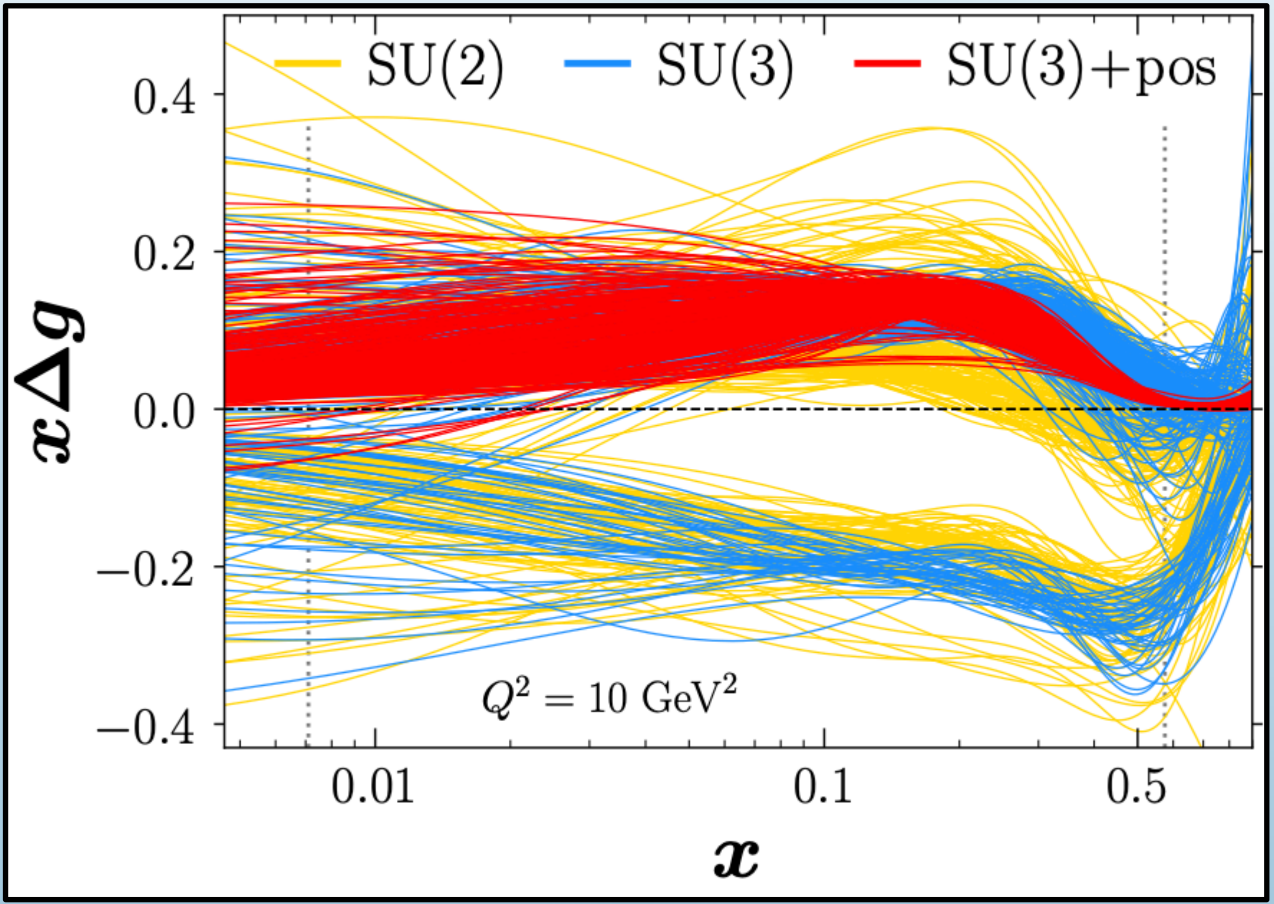

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Can $\overline{\mathrm{MS}}$ parton distributions be negative?

Alessandro Candido, Stefano Forte and Felix Hekhorn

Positivity and renormalization of parton densities

John Collins, Ted C. Rogers, Nobuo Sato

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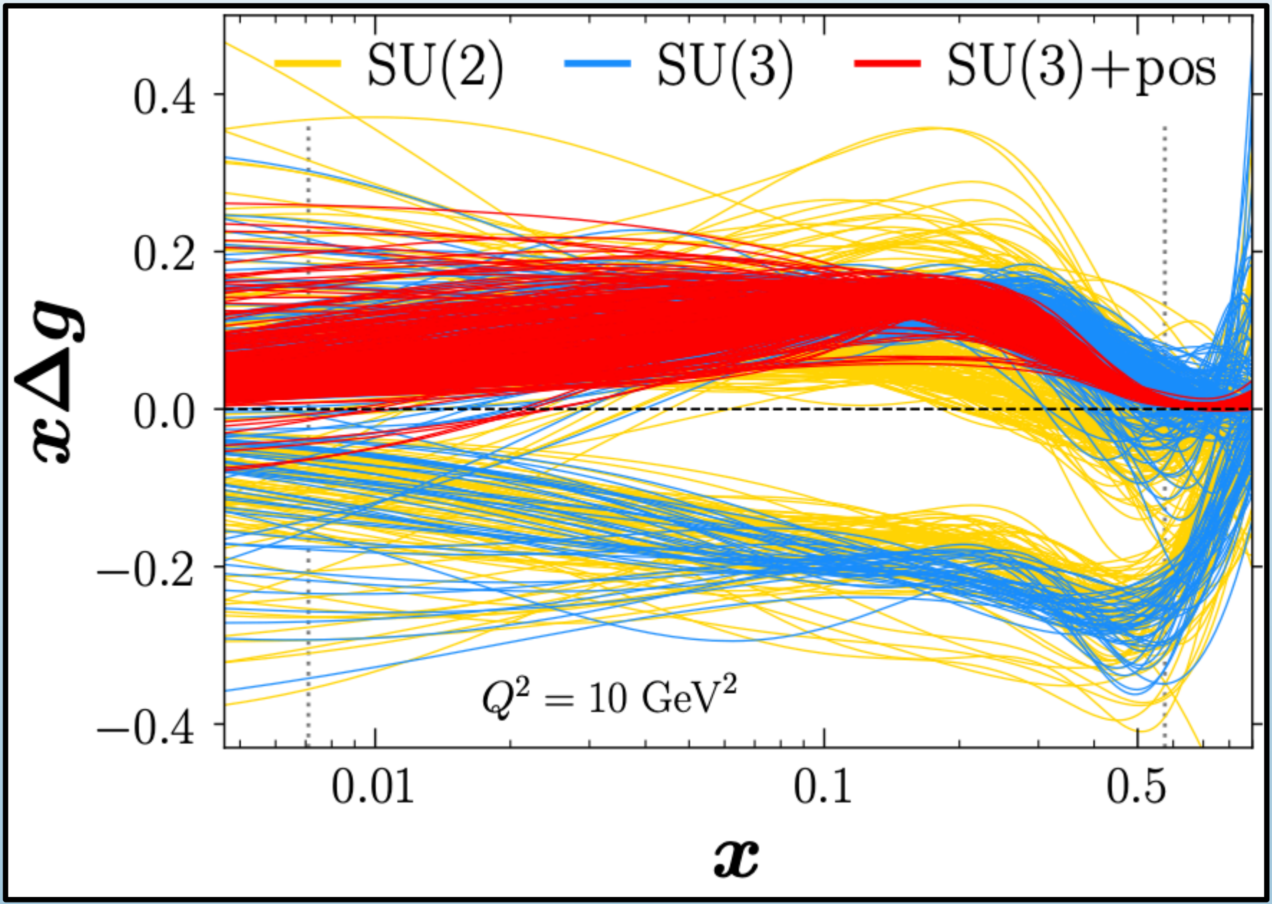

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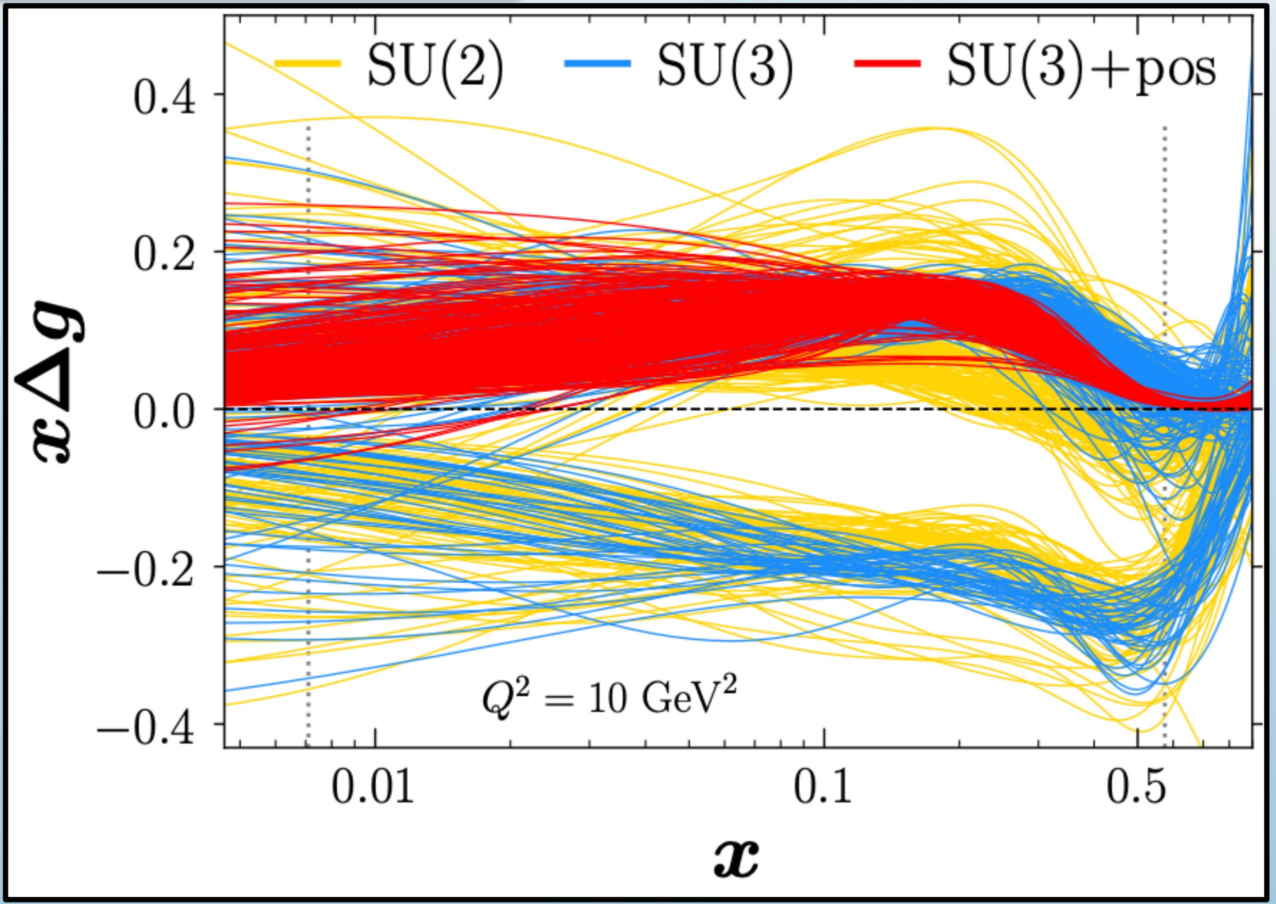

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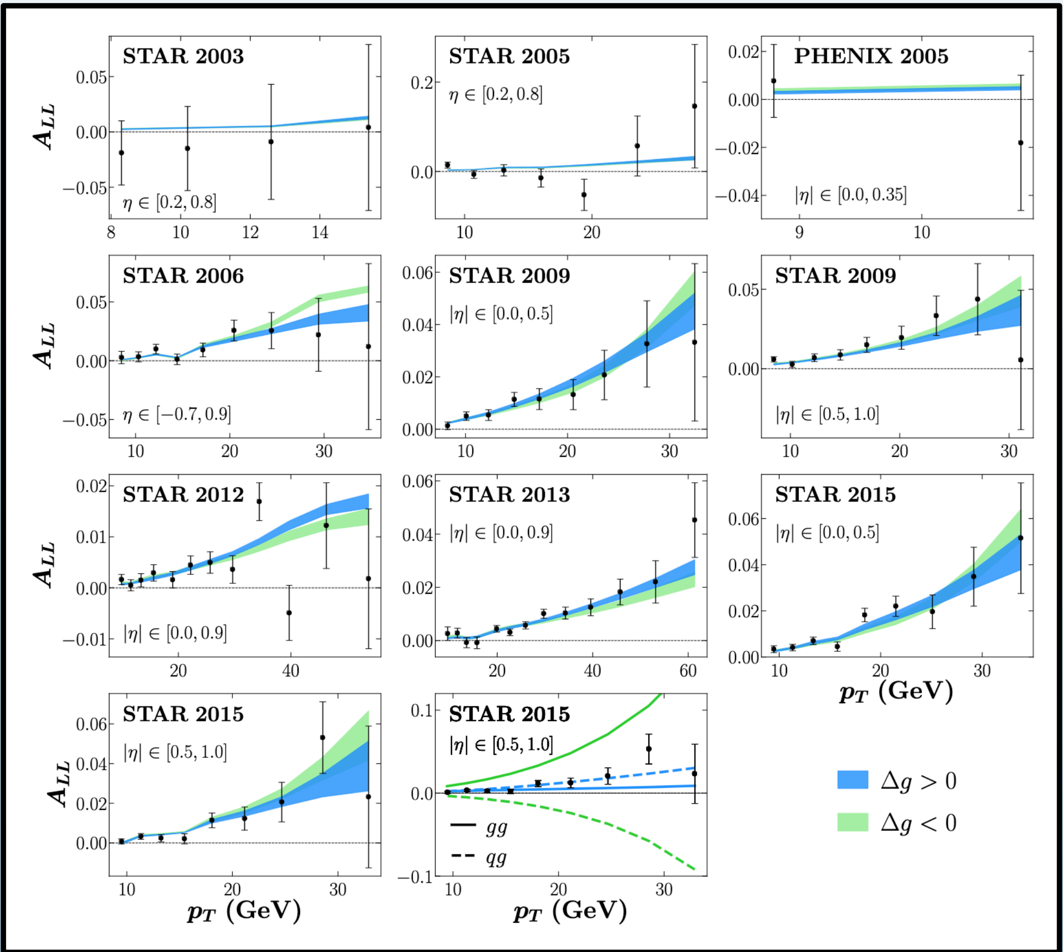

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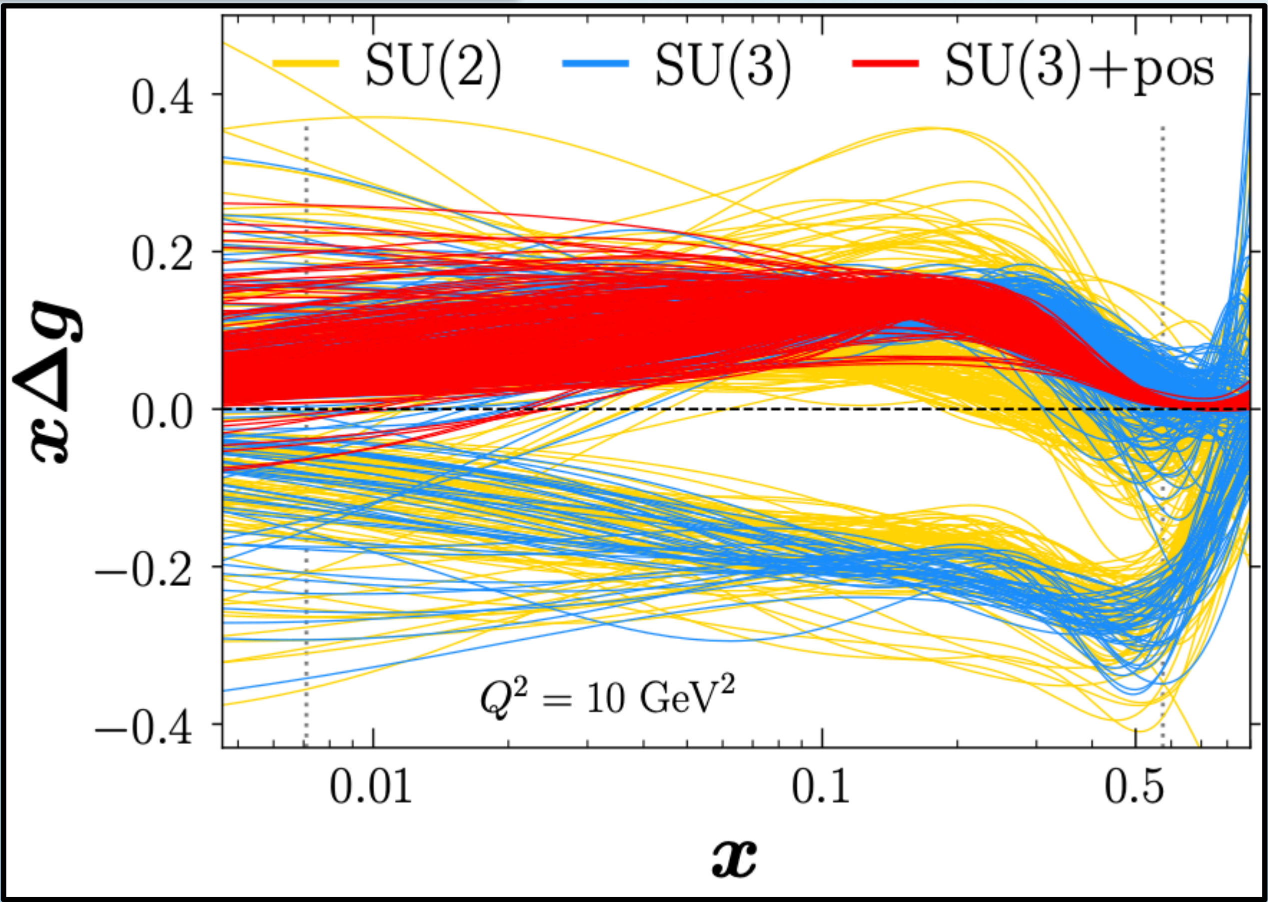


$$A_{LL}^{\text{jet}} \sim (\Delta g)^2 + \Delta q \Delta g + \ldots$$

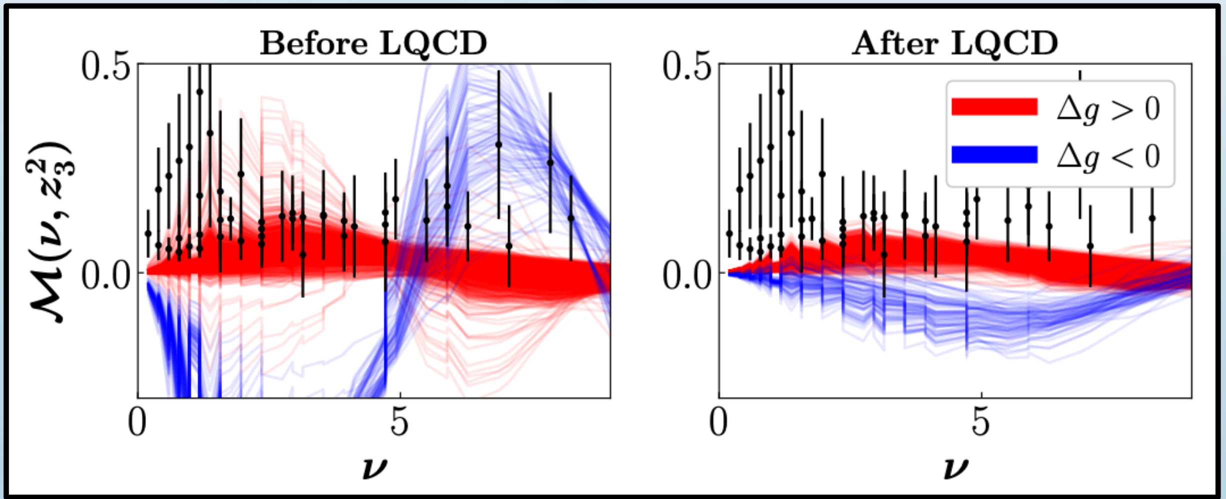
Gluon Helicity

Gluon helicity from global analysis of experimental data and lattice QCD Ioffe time distributions

J. Karpie (Jefferson Lab), R.M. Whitehill (Old Dominion U.), W. Melnitchouk (Jefferson Lab), C. Monahan (Jefferson Lab and William-Mary Coll.), K. Orginos (Jefferson Lab and William-Mary Coll.) Show All(9)

Oct 27, 2023

$$\mathcal{M}(\nu, z_3^2) = \int_0^1 dx \ x \ sin(x\nu)\delta g(x) \quad \text{(LO)}$$



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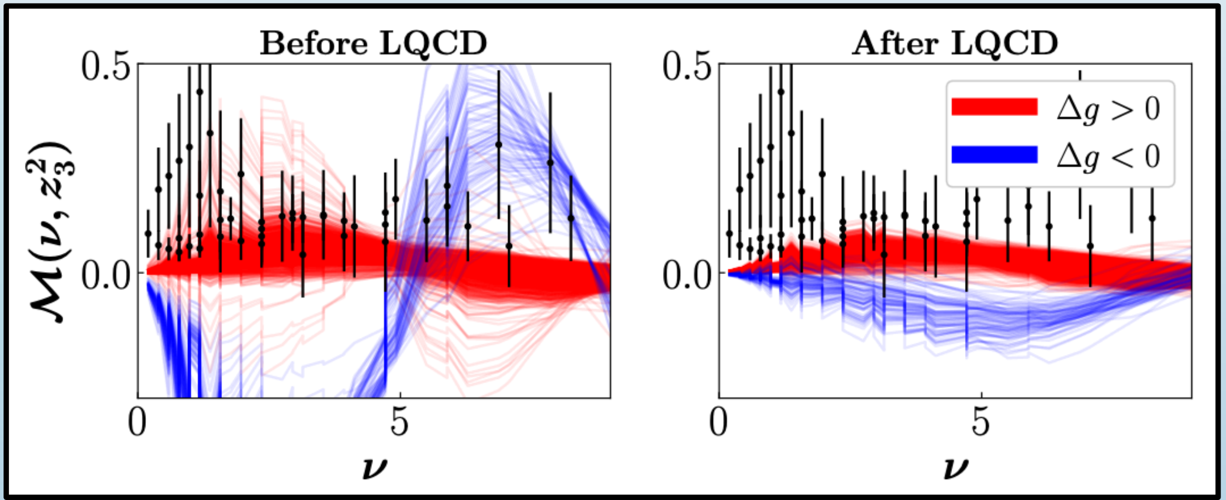

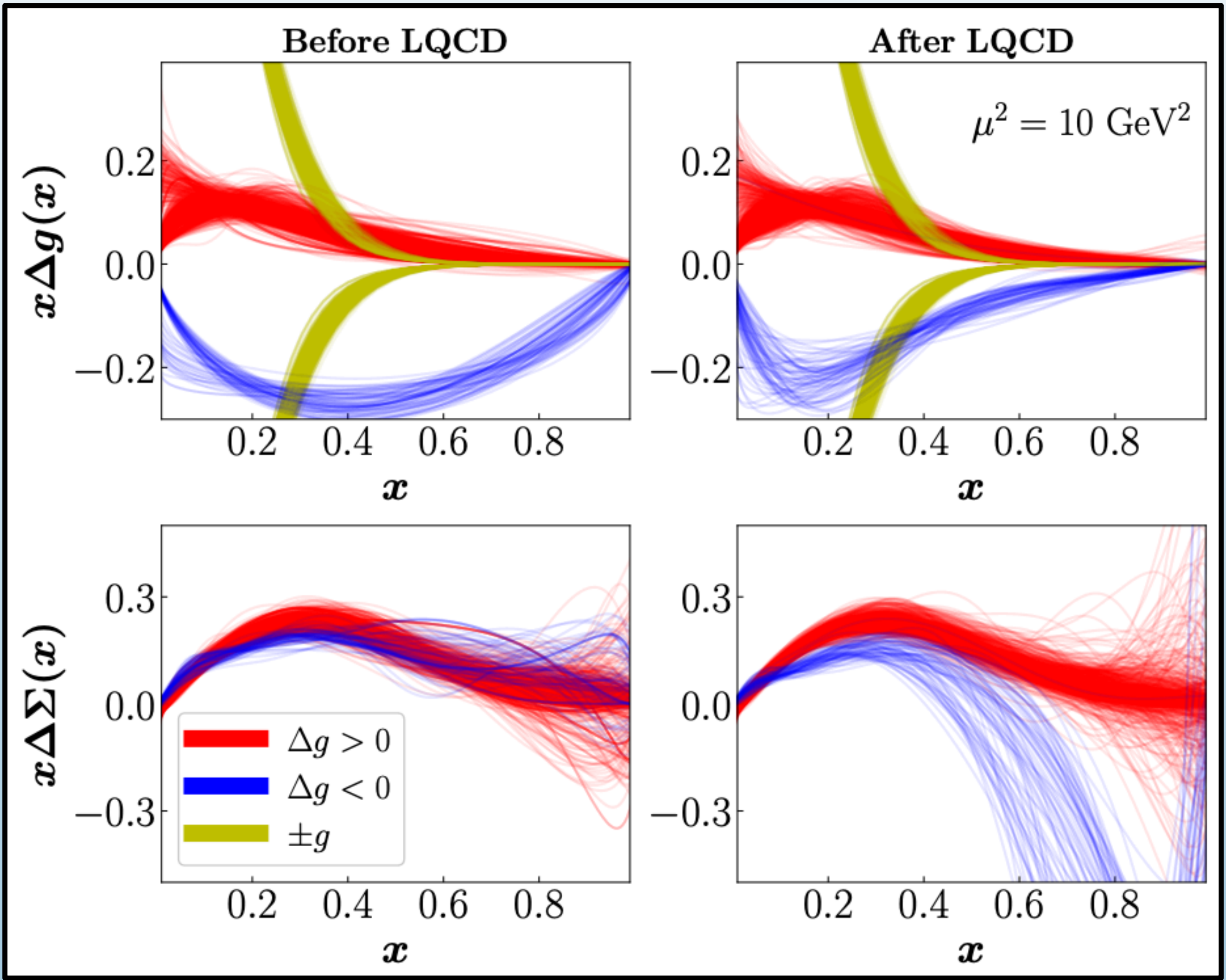

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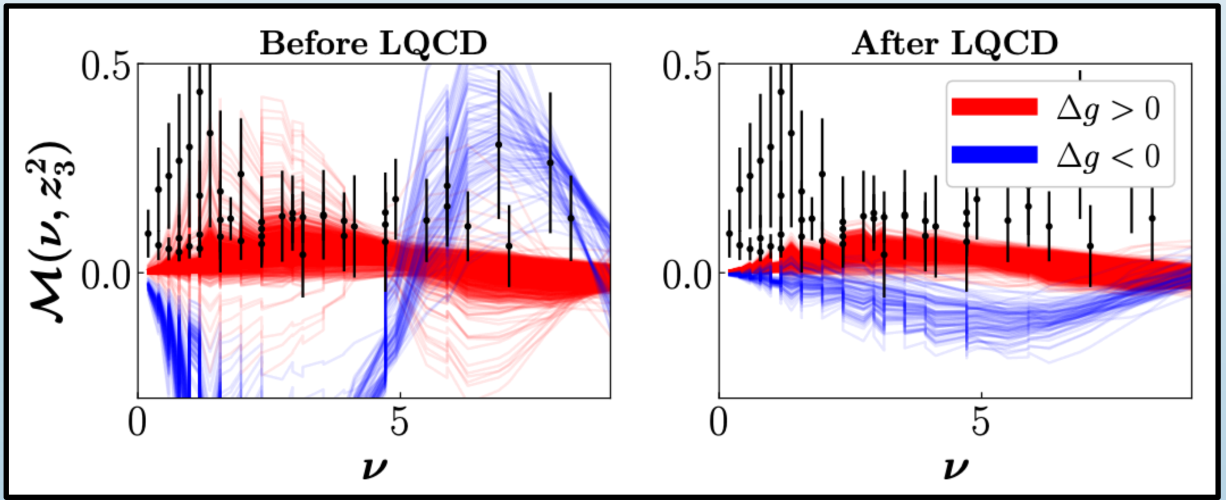
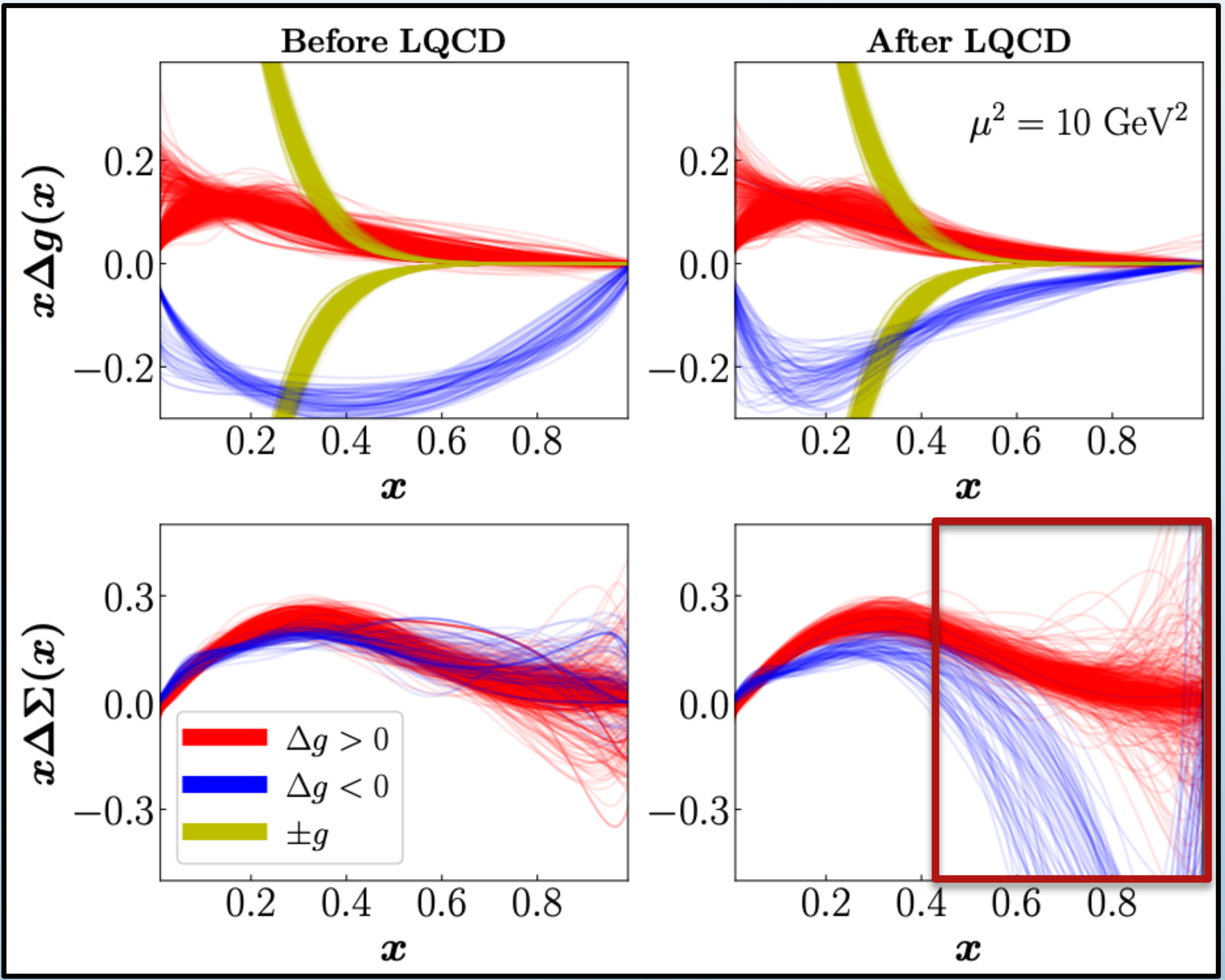




LQCD data does not rule out negative gluon, but leads to wild behavior for $\Delta\Sigma$ at large x

Gluon Helicity

Measurement of charged pion double spin asymmetries at midrapidity in longitudinally polarized $p+p$ collisions at $\sqrt{s}$ = 510 GeV

PHENIX Collaboration • U.A. Acharya (Georgia State U.) et al. (Apr 6, 2020)

Published in: *Phys.Rev.D* 102 (2020) 3, 032001 • e-Print: 2004.02681 [hep-ex]

Charged-pion cross sections and double-helicity asymmetries in polarized p+p collisions at $\sqrt{s}$=200 GeV

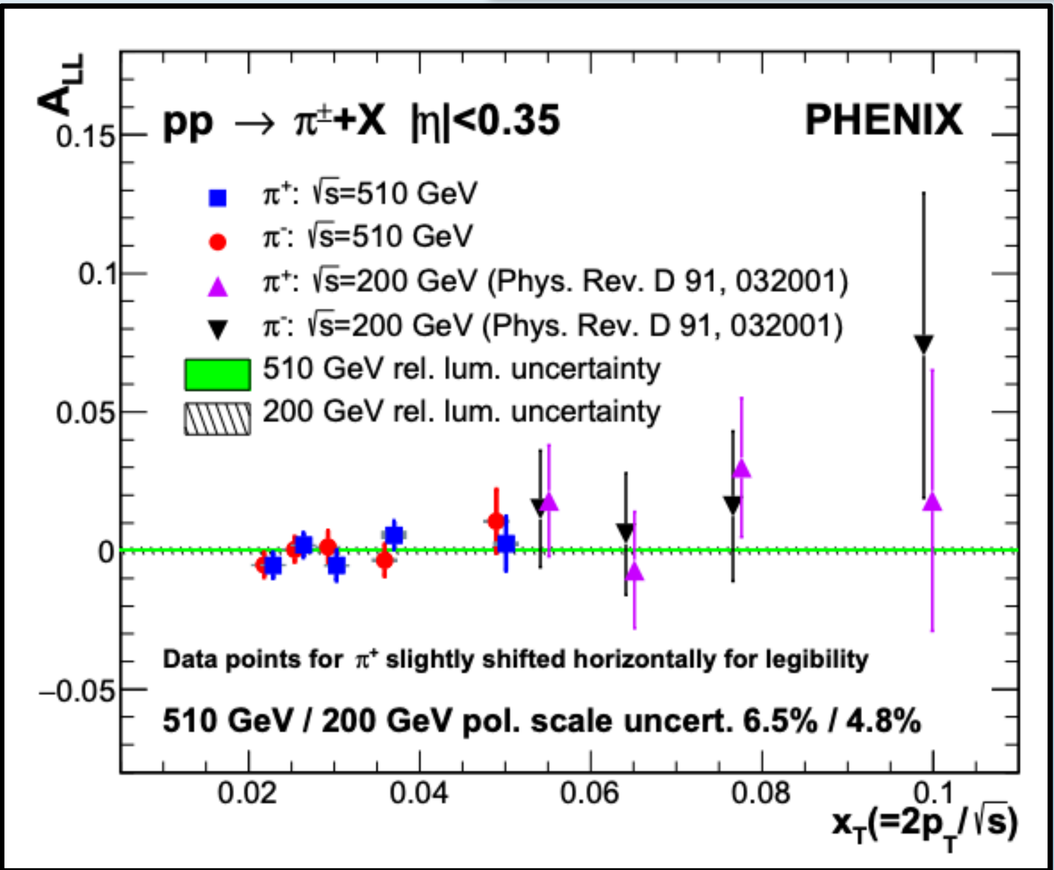
PHENIX Collaboration • A. Adare (Colorado U.) et al. (Sep 5, 2014)

Published in: *Phys.Rev.D* 91 (2015) 3, 032001 • e-Print: 1409.1907 [hep-ex]

$$\vec{p} + \vec{p} \rightarrow \pi^{\pm} + X$$

Charge ordering:

$$\text{If } \Delta g > 0 : A_{LL}^{\pi^+} > A_{LL}^{\pi^0} > A_{LL}^{\pi^-}$$



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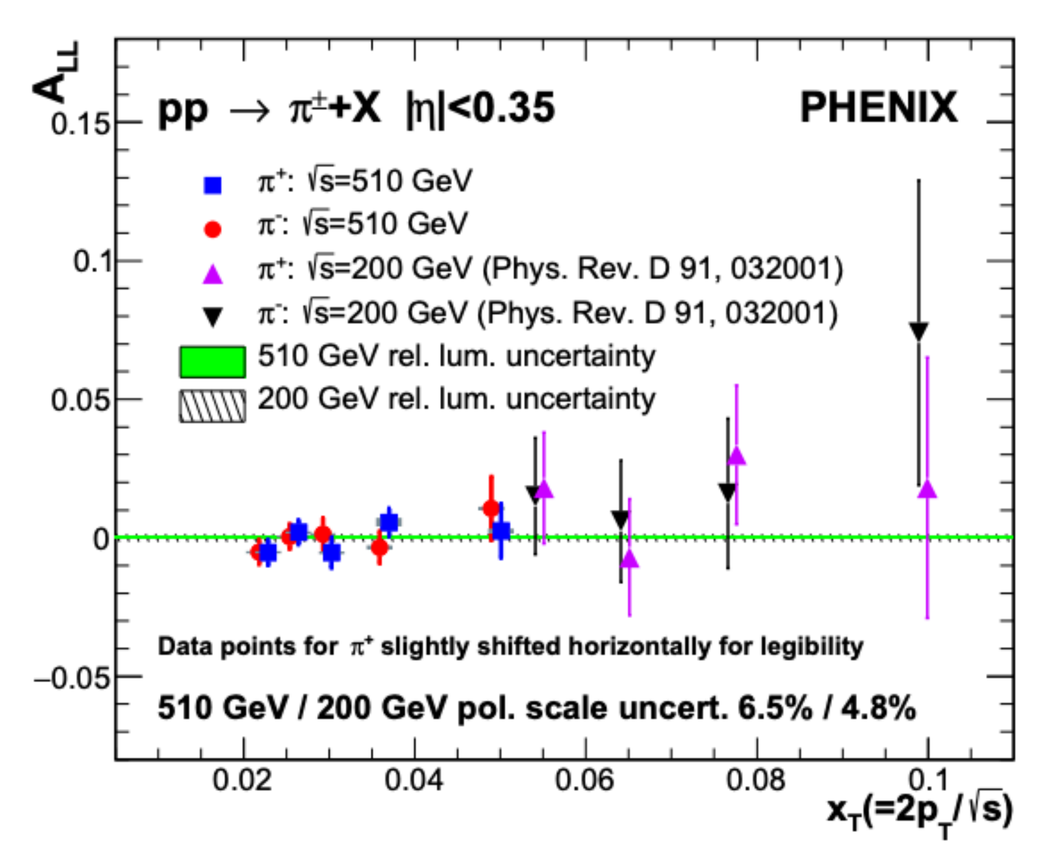
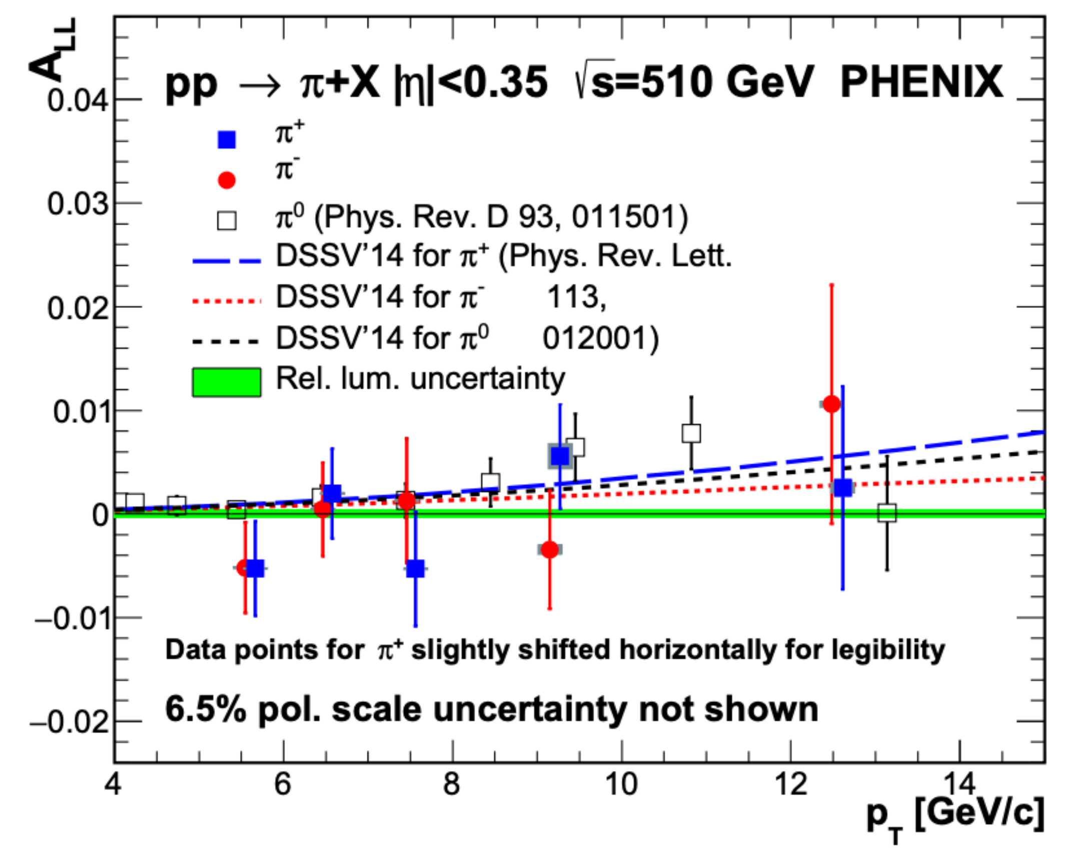
Charge ordering:
If $\Delta g > 0 : A_{LL}^{\pi^+} > A_{LL}^{\pi^0} > A_{LL}^{\pi^-}$

Consistent with DSSV14 analysis (which included 210 GeV data) with $\Delta g > 0$





Phys.Rev.Lett. 113 (2014) 1, 012001

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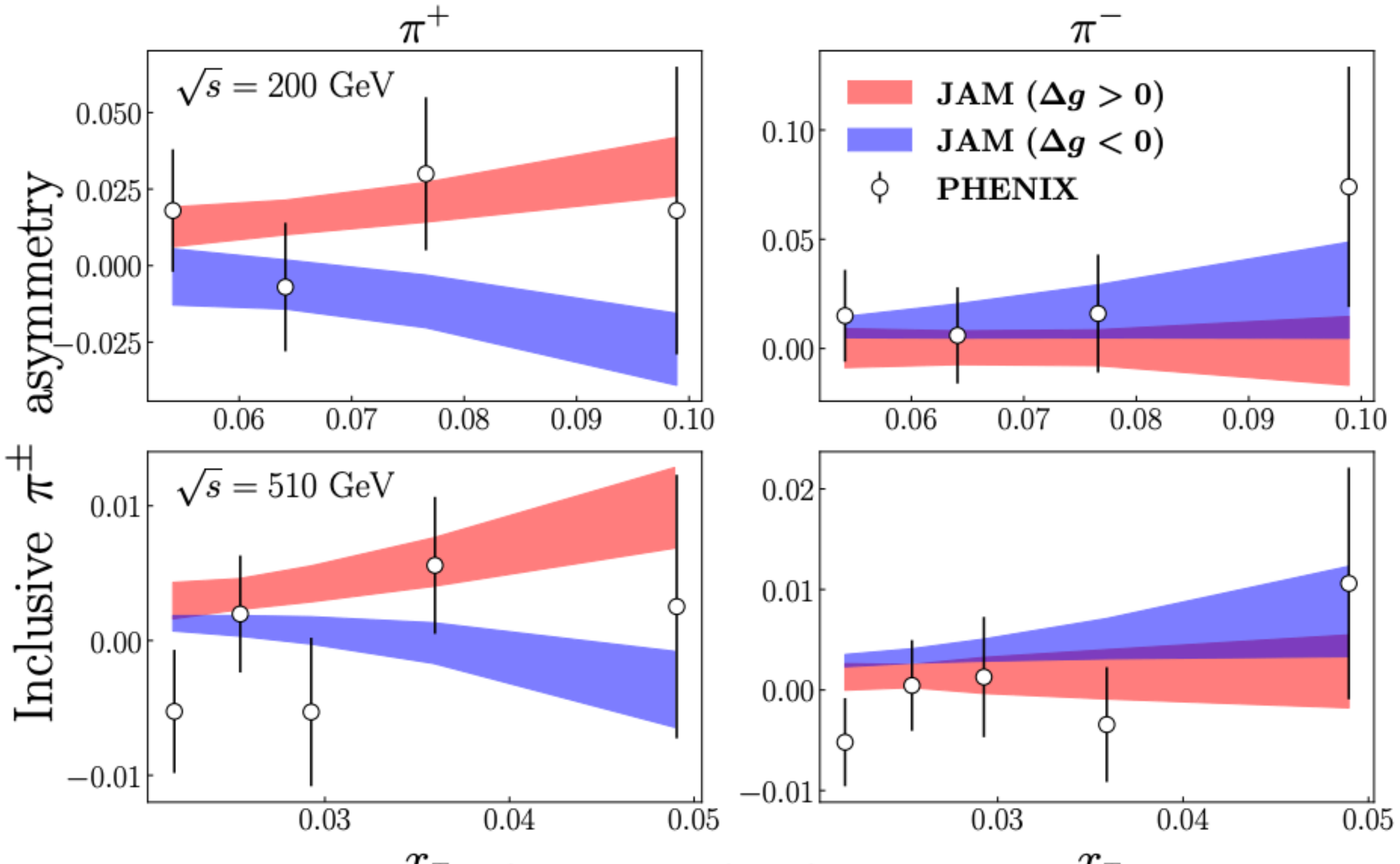

Phys.Rev.D 107 (2023) 3, 034033

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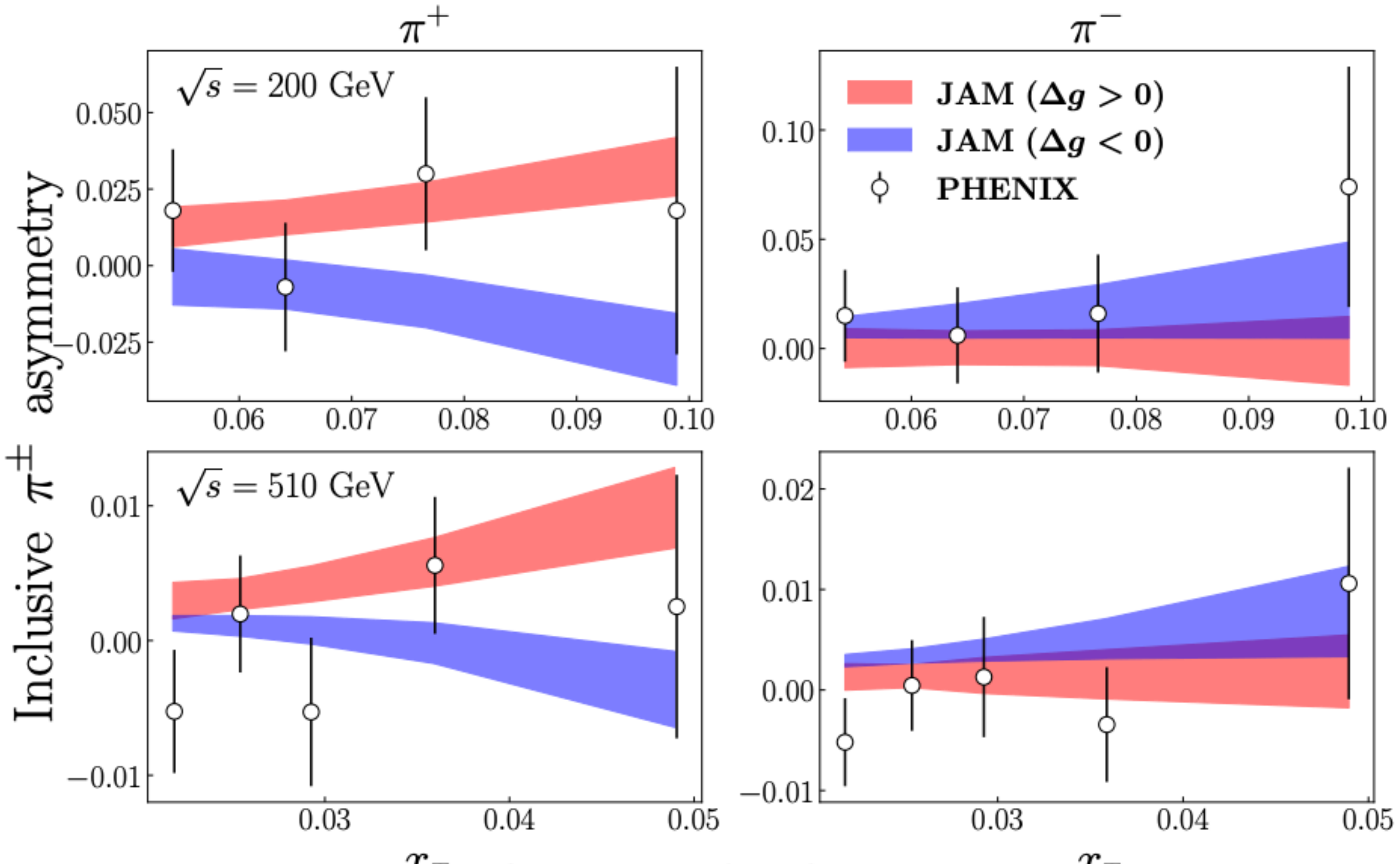

Phys.Rev.D 107 (2023) 3, 034033

It is inconclusive whether data can distinguish between two solutions

Gluon Helicity

Measurement of Direct-Photon Cross Section and Double-Helicity Asymmetry at $\sqrt{s} = 510$ GeV in $\vec{p} + \vec{p}$ Collisions

PHENIX Collaboration • U. Acharya (Georgia State U., Atlanta) et al. (Feb 16, 2022)

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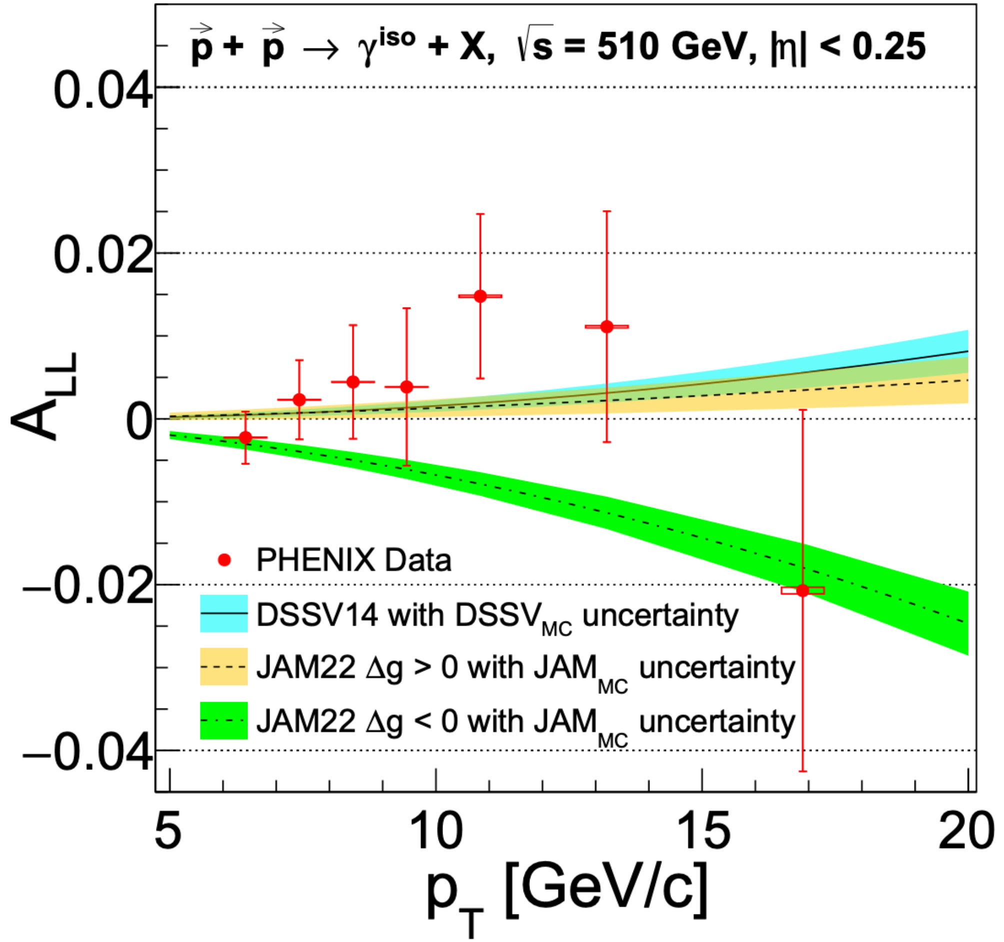

Direct sensitivity to the sign of Δg!

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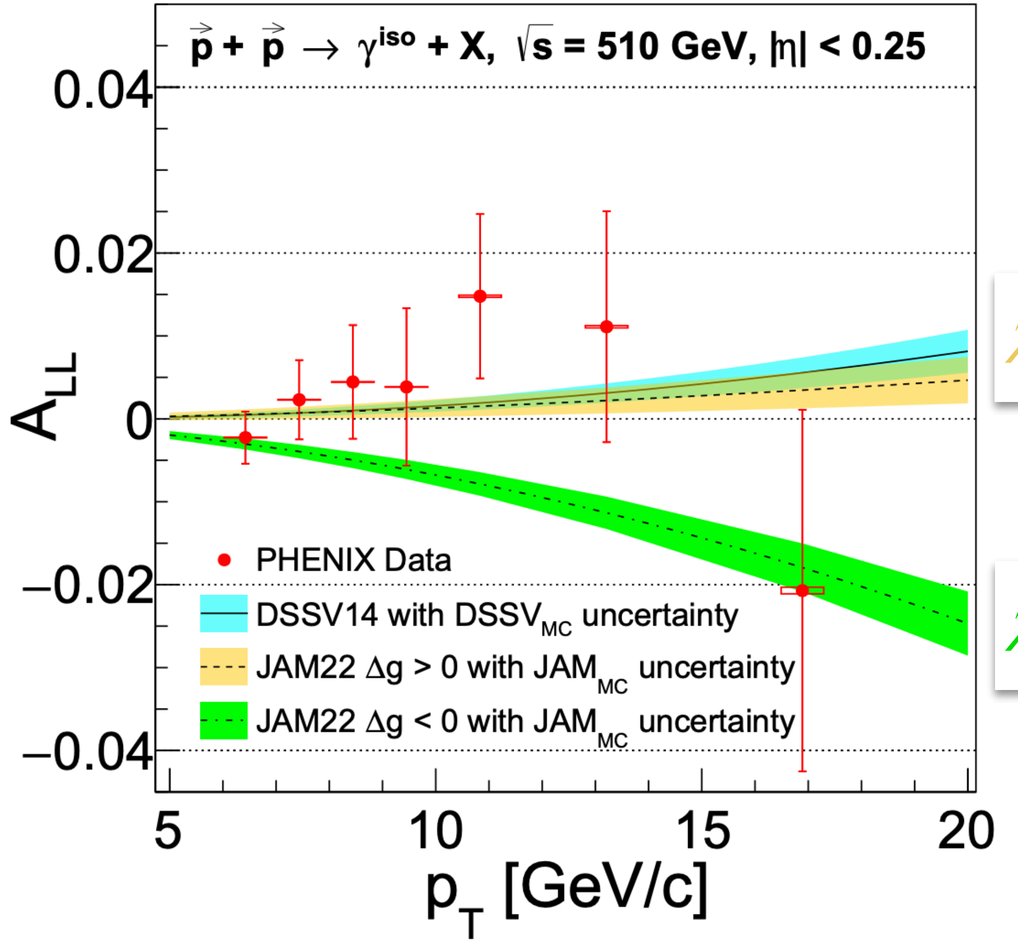

$\chi^2 = 4.7$

$\chi^2 = 12.6$

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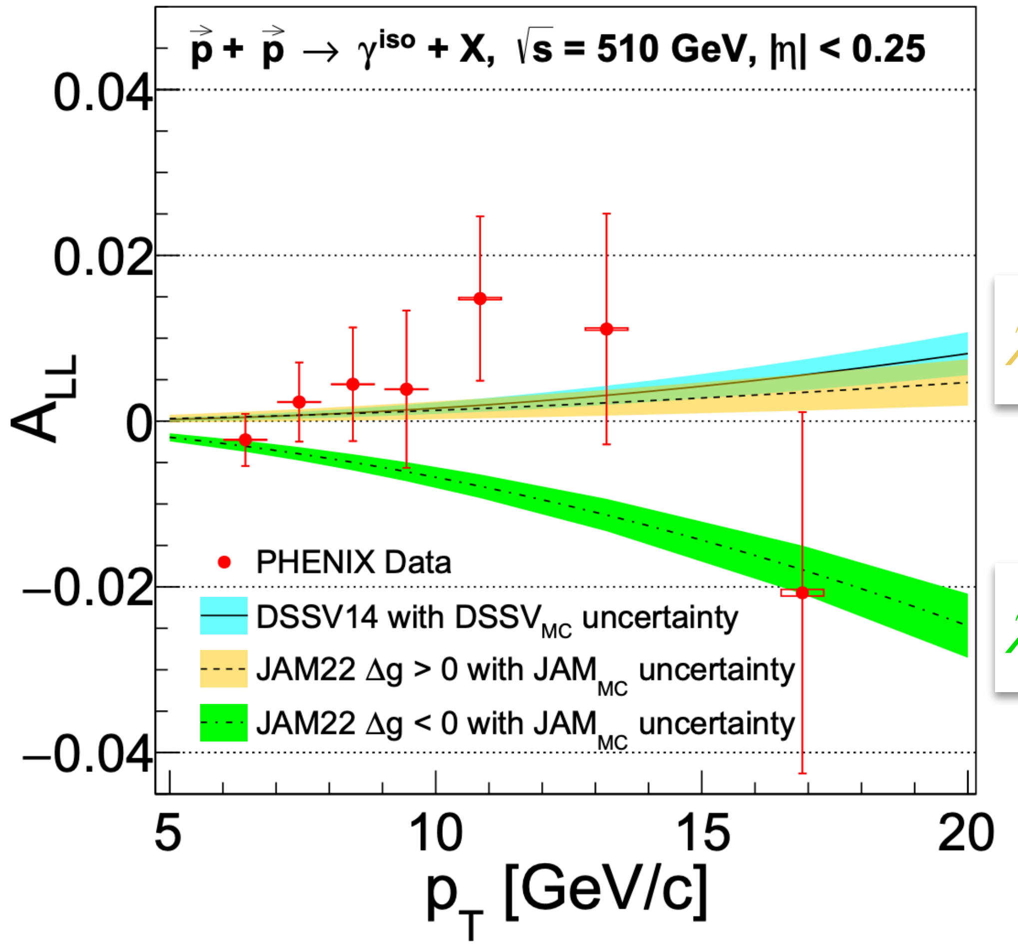

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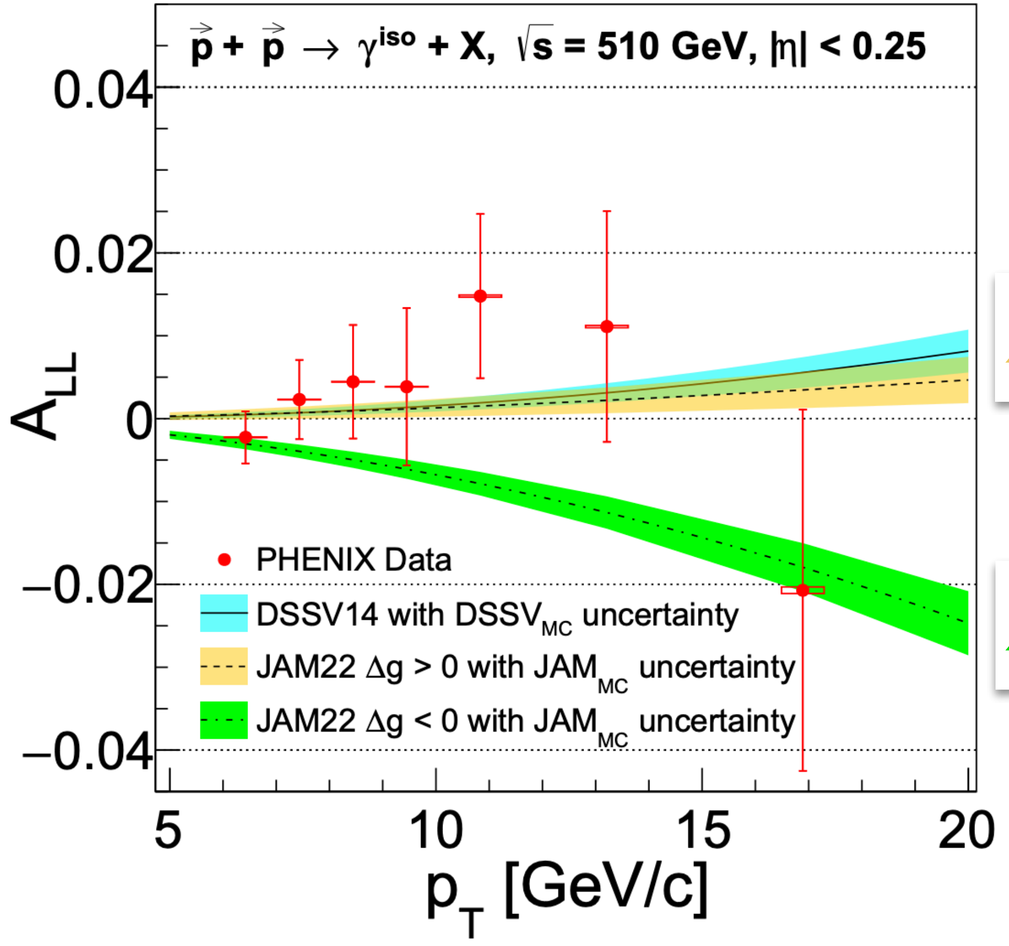

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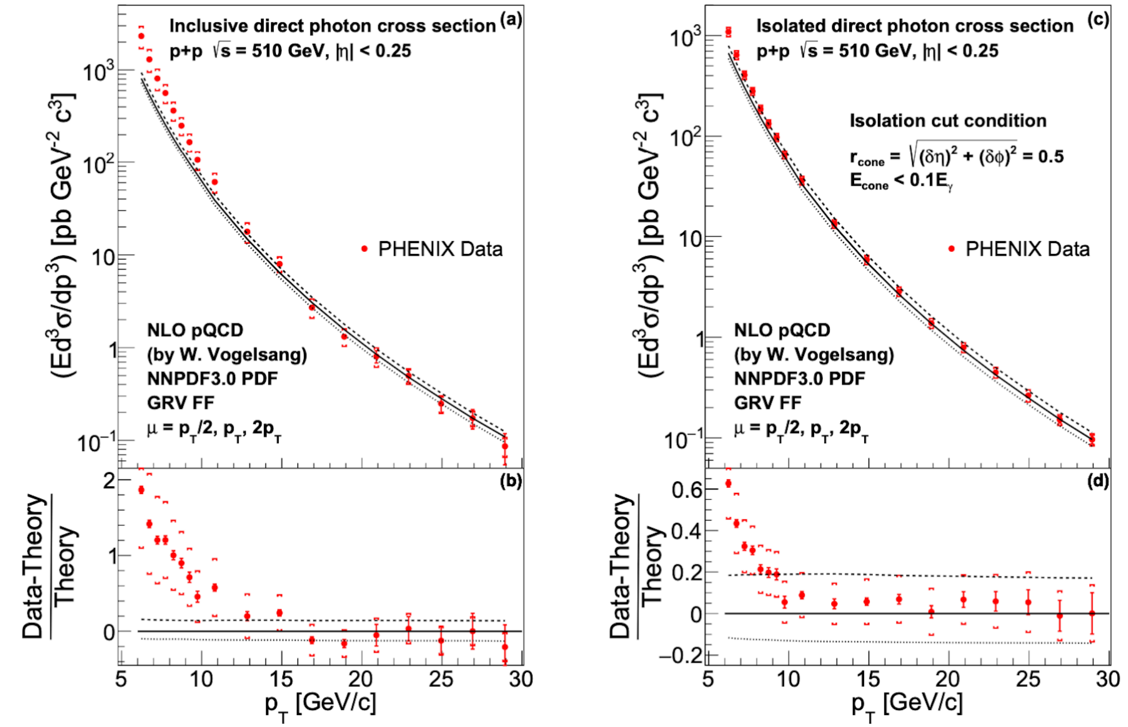

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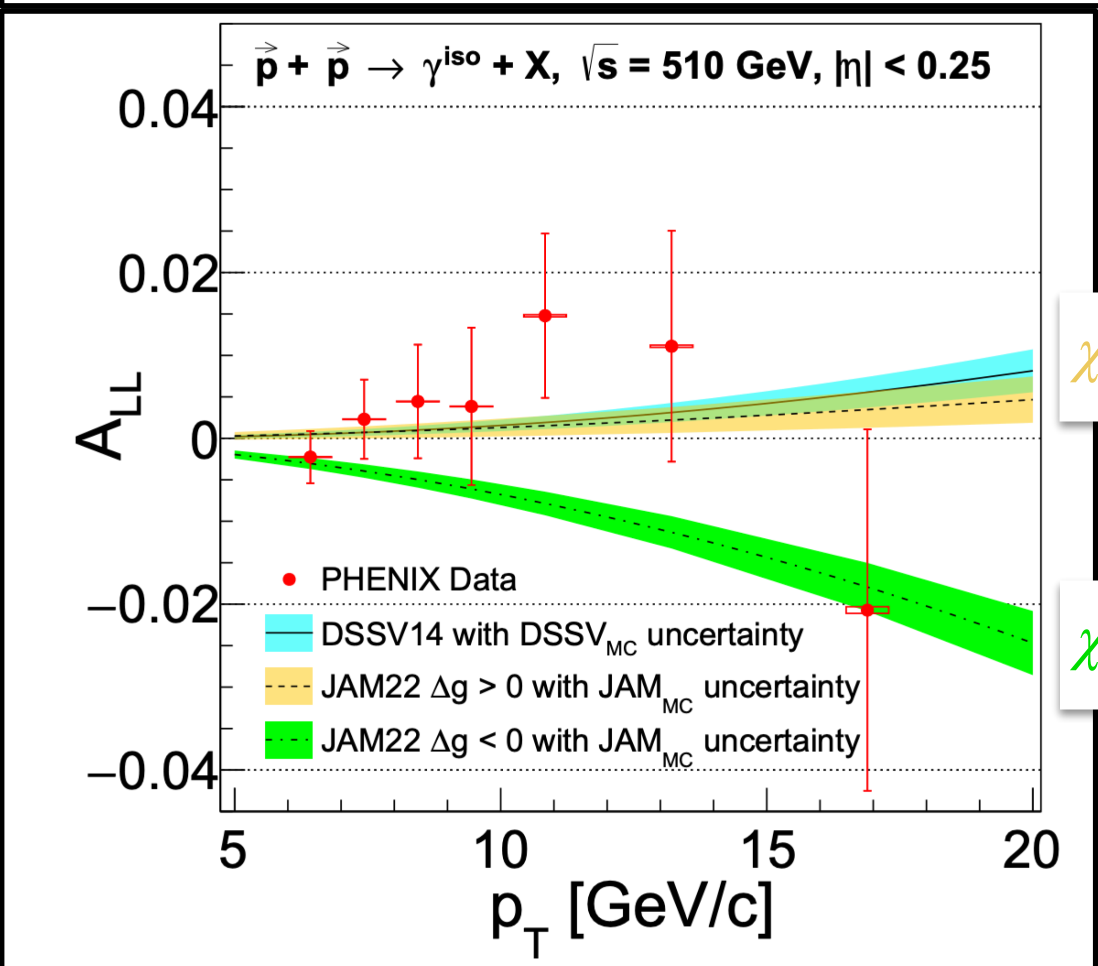

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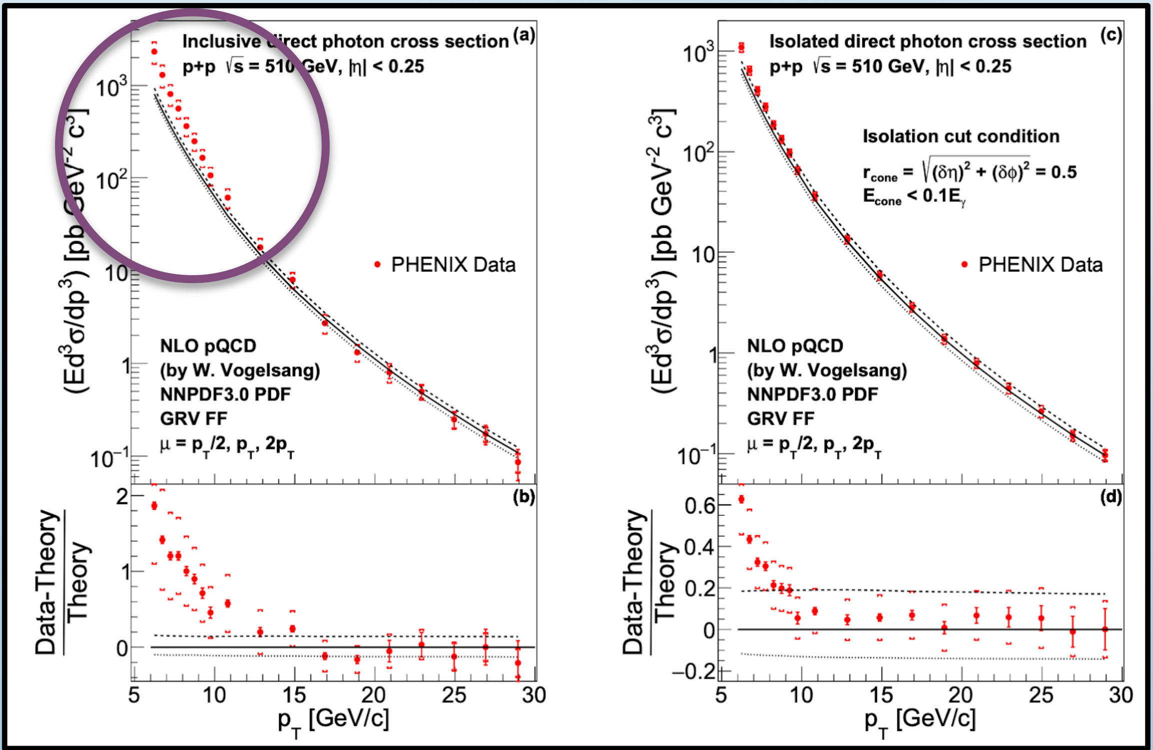
Potential issues at $P_T < 10$



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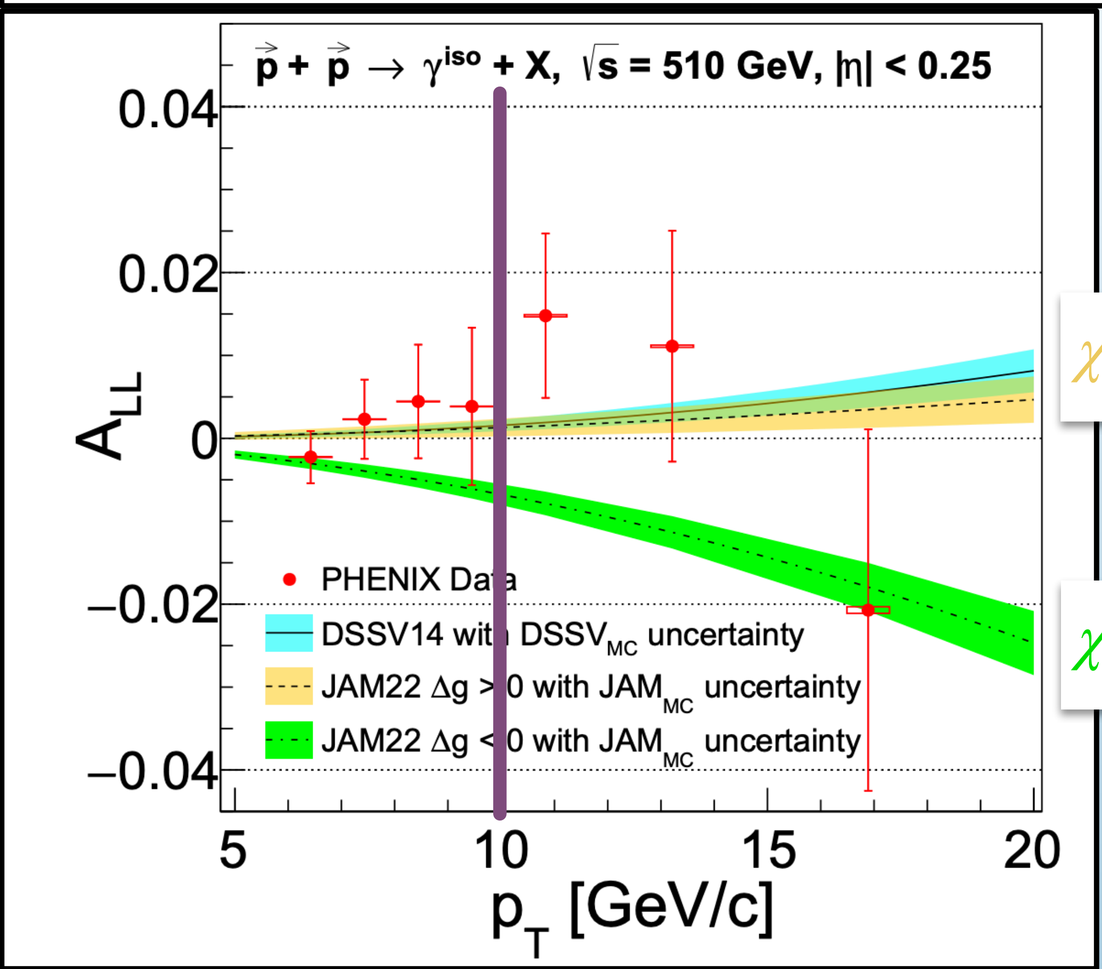

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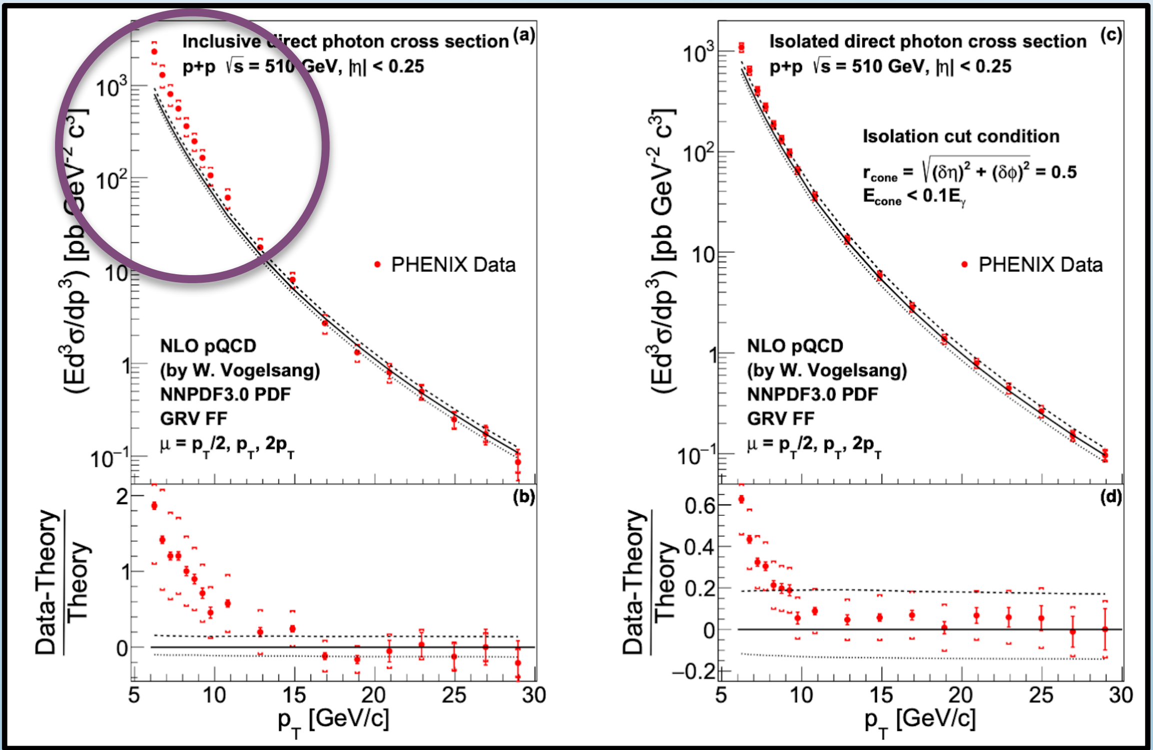

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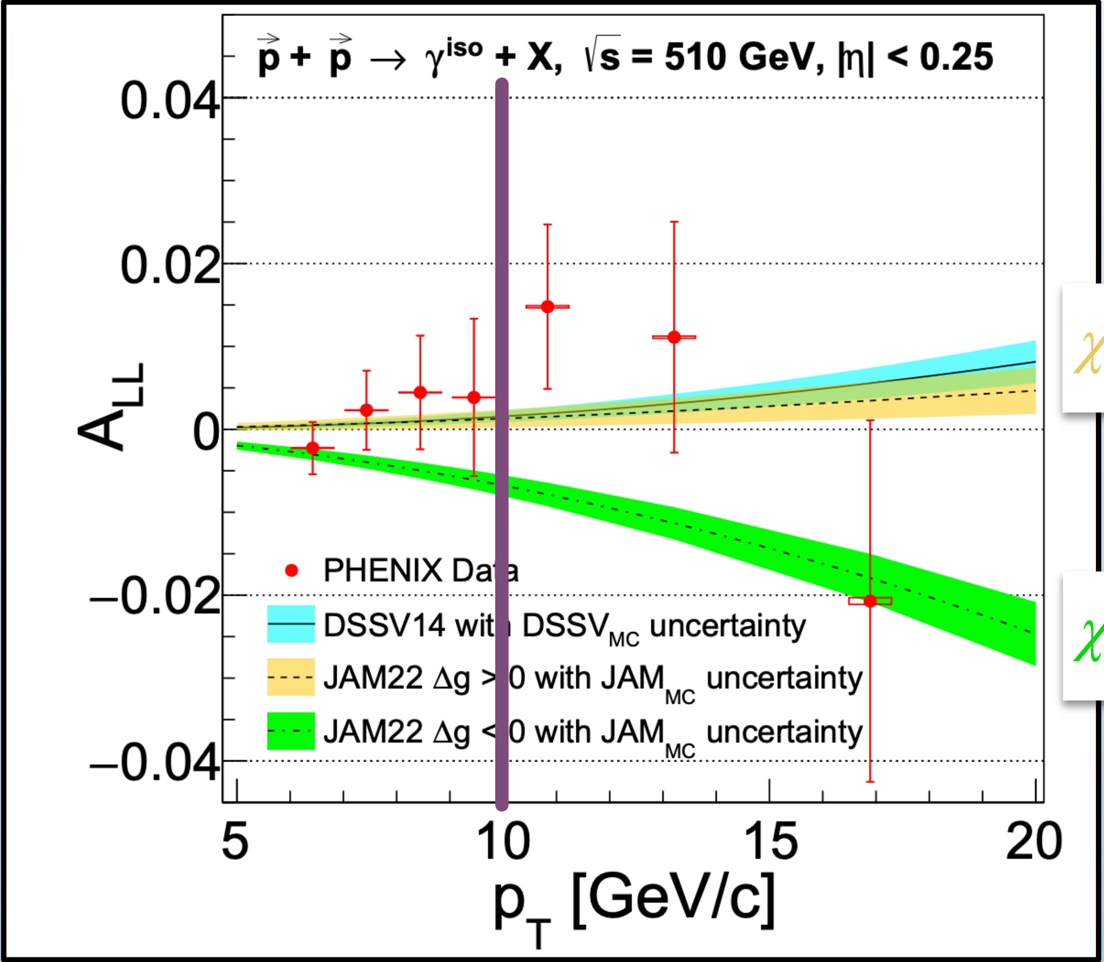

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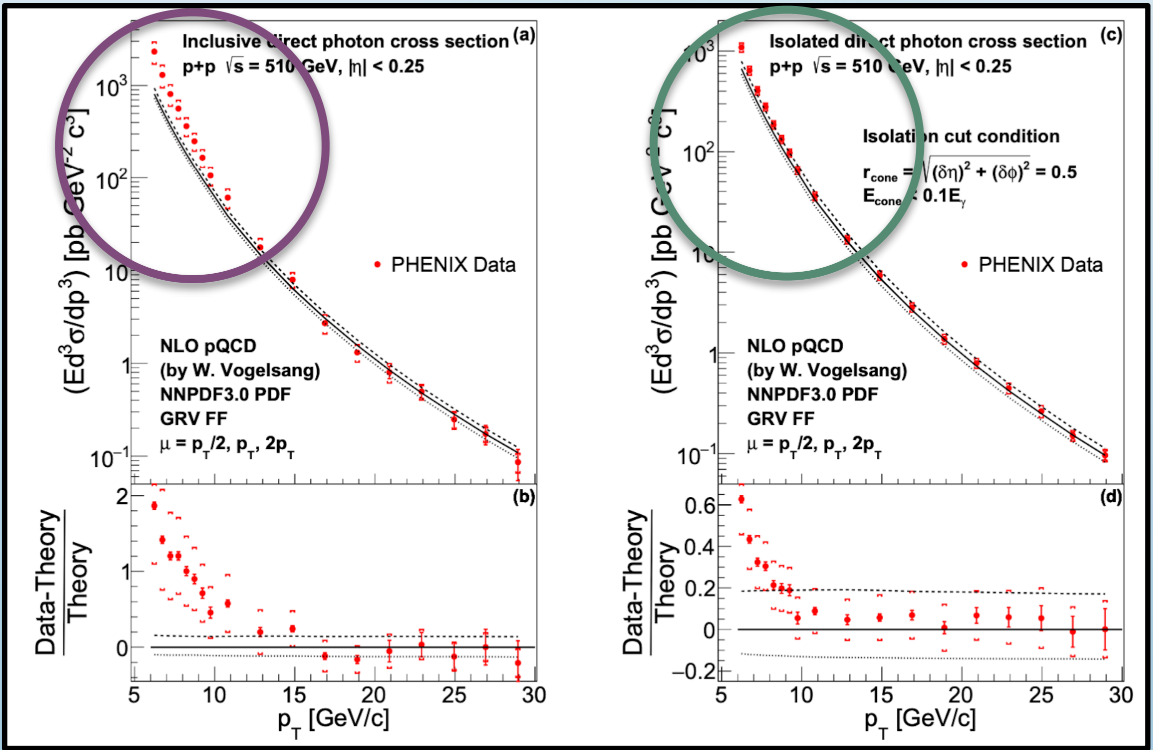
Direct sensitivity to the sign of Δg!

May be aided by isolation cut

Potential issues at $P_T < 10$



The RHIC Cold QCD Program

White Paper

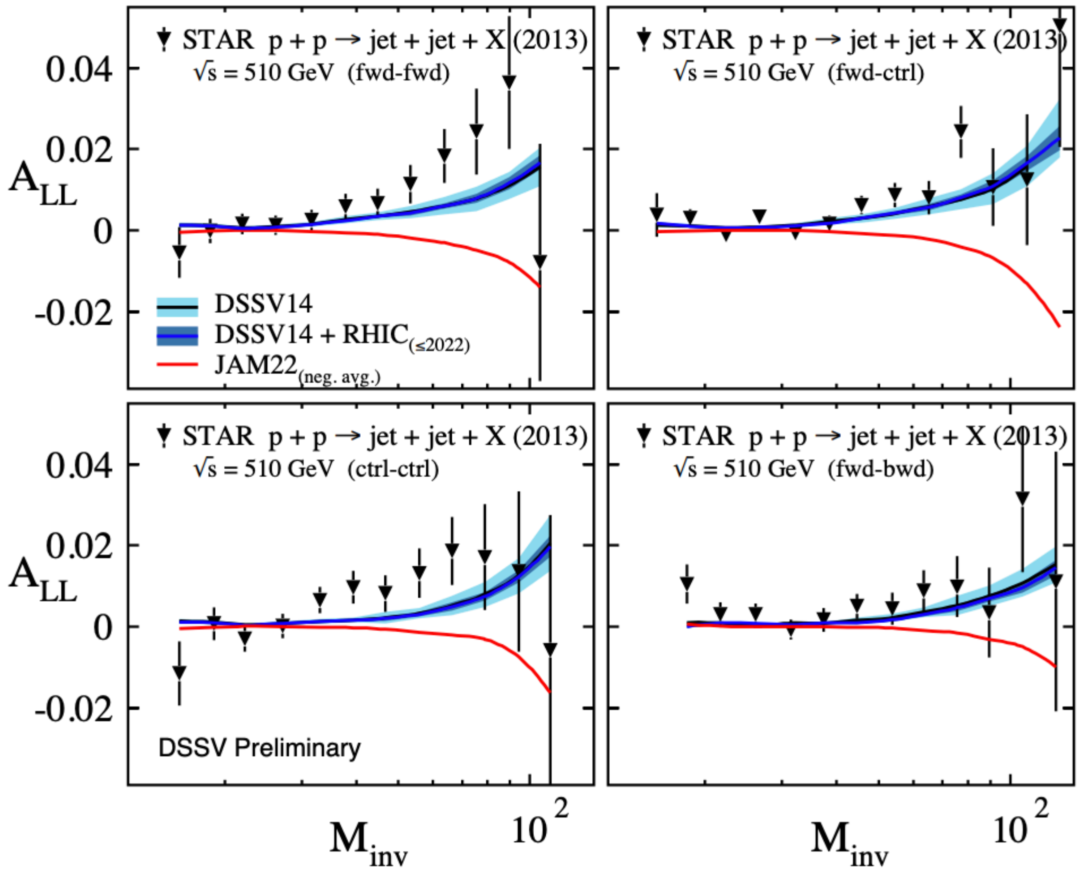


Figure 8: STAR double-helicity asymmetries A_{LL} for dijet production vs dijet invariant mass M_{inv} in polarized *pp* collisions at $\sqrt{s}$=510 GeV at midrapidity from 2013 data set [21]. DSSV14 evaluation [17] is plotted as the black curve with the 1σ uncertainty band marked in light blue. The blue curve with 1σ uncertainty band in dark blue shows the impact of all the data sets included in the new preliminary DSSV fit [2] as in Fig. 6. The red curves show the JAM $\Delta g < 0$ solution [41] calculated by the DSSV group.

Higgs production at RHIC and the positivity of the gluon helicity distribution

Daniel de Florian, Stefano Forte, Werner Vogelsang

Jan 19, 2024

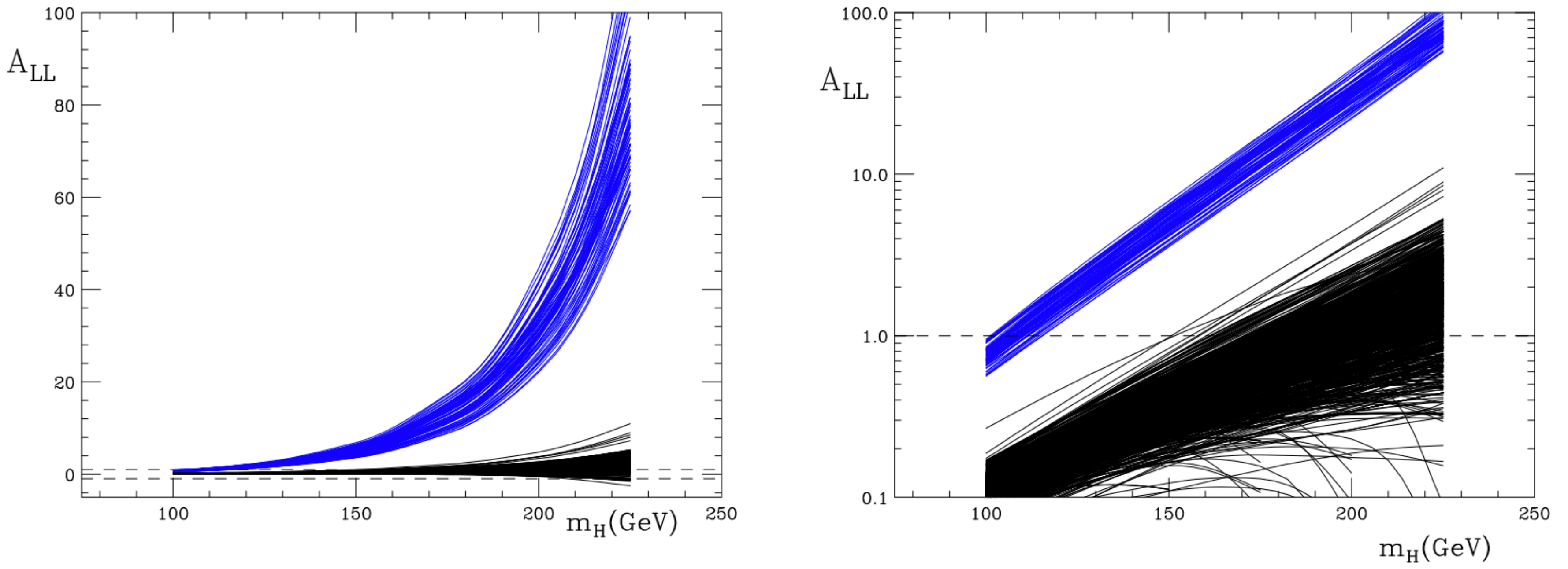


Figure 2: *Double-helicity asymmetry for Higgs production at RHIC as a function of the Higgs mass, with a linear (left) or logarithmic (right) scale on the vertical axis. The upper bands show A_{LL} as obtained for the gluon distribution shown in Fig. 1, while the lower bands provide the corresponding result for the sets of [7] with $\Delta g \geq 0$. In both plots, the dashed lines show the physical limit given by $|A_{LL}| = 1$.*

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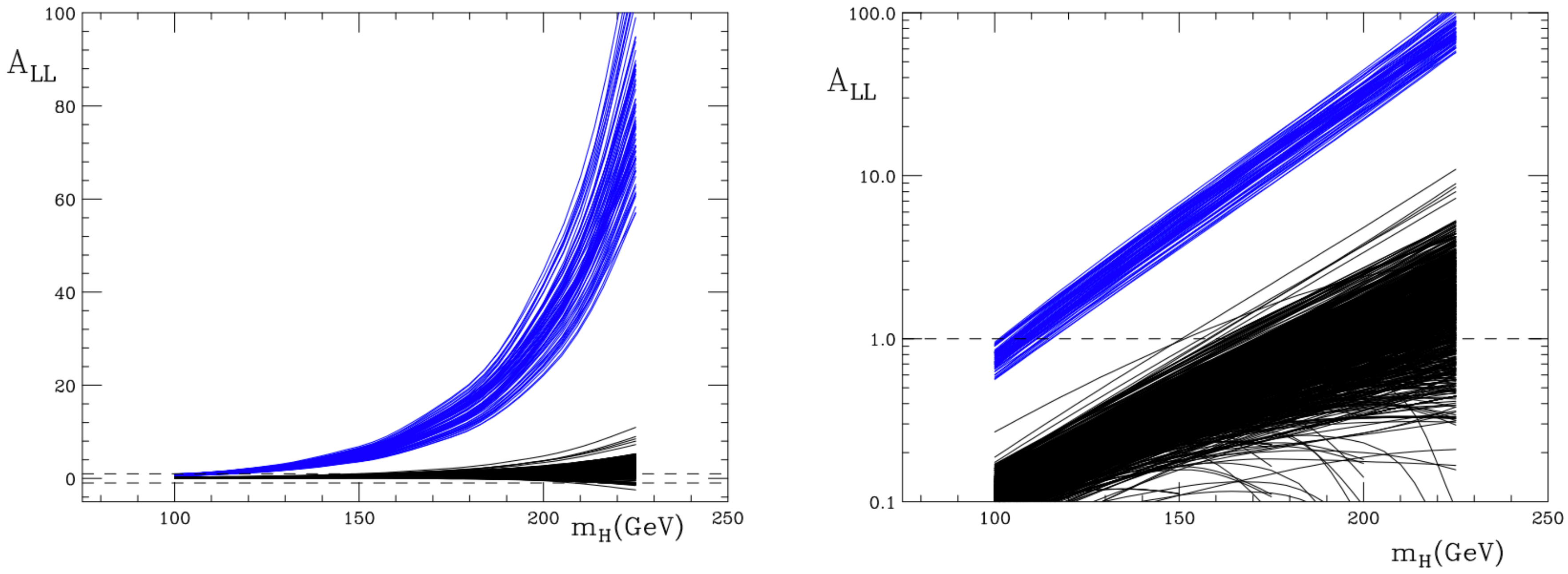

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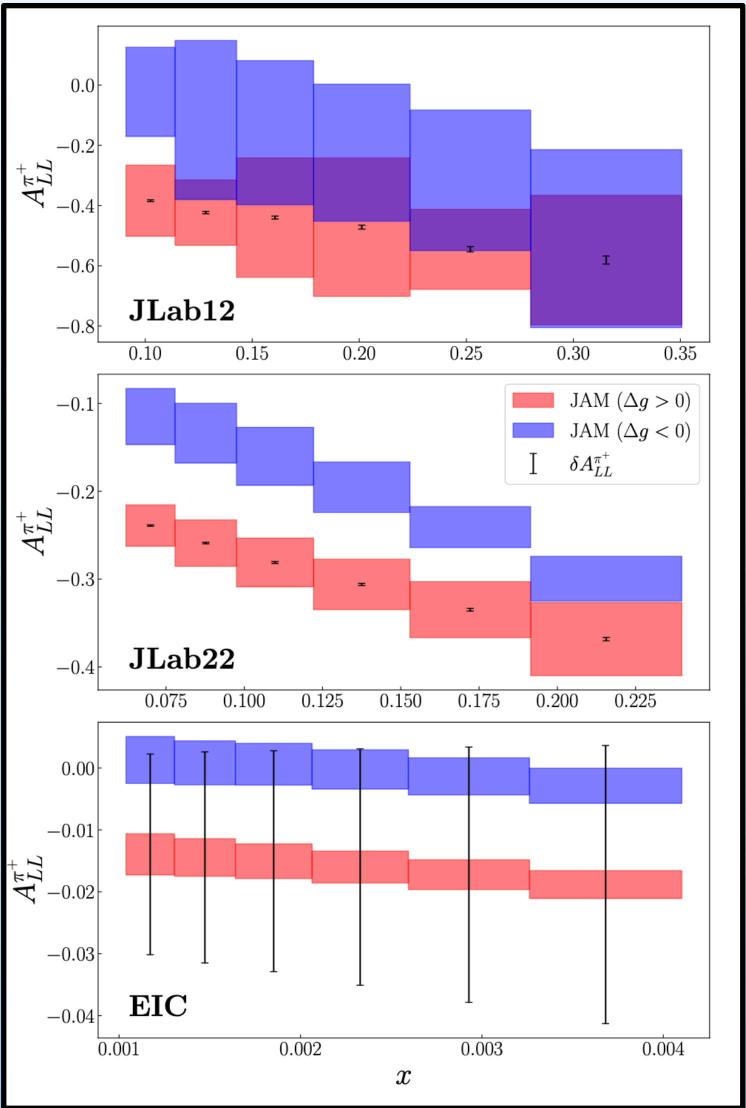
Negative solution (blue) strongly violates physical limit. Positive solution (black) far less so.

Accessing gluon polarization with high-P_T hadrons in SIDIS

Jefferson Lab Angular Momentum (JAM) Collaboration • R.M. Whitehill (Wichita State U.) et al. (Oct 21, 2022)

Published in: *Phys.Rev.D* 107 (2023) 3, 034033 • e-Print: 2210.12295 [hep-ph]

$$\vec{l} + \vec{N} \rightarrow l' + h + X$$



$\mathscr{L} = 86\ \mathrm{fb}^{-1}$ for JLab

$\mathscr{L} = 10\ \mathrm{fb}^{-1}$ for EIC

Future Experiments

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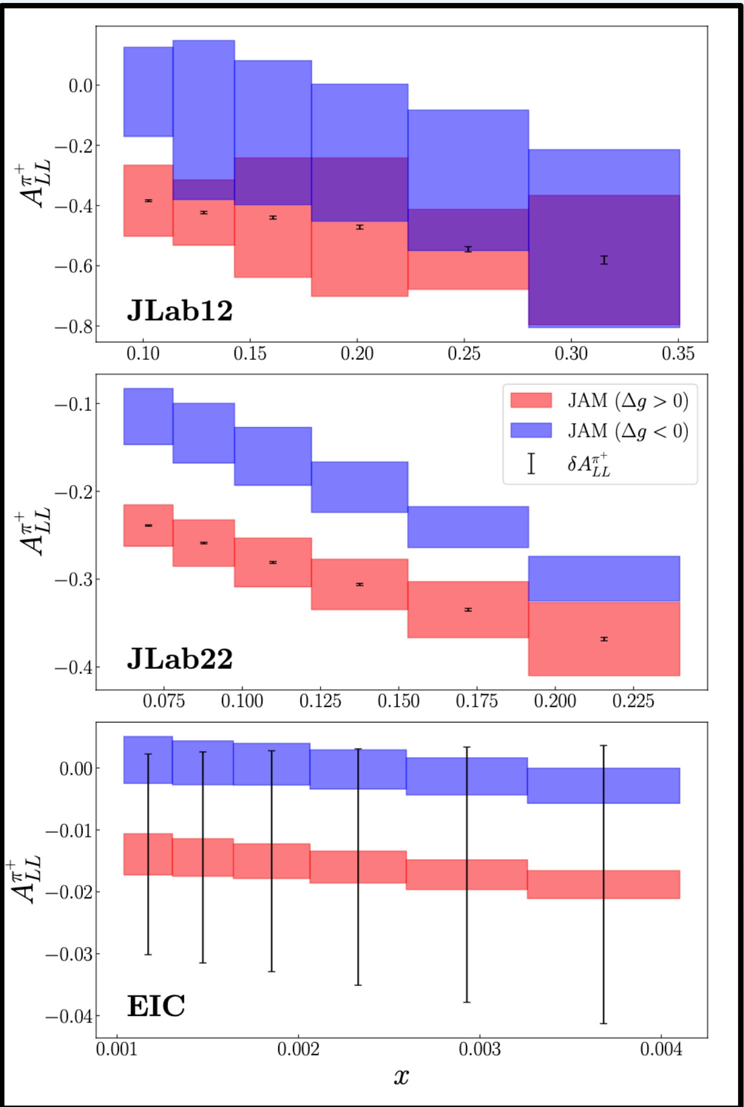

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JLab22 has stronger distinguishing power due to more evolution and access to smaller x

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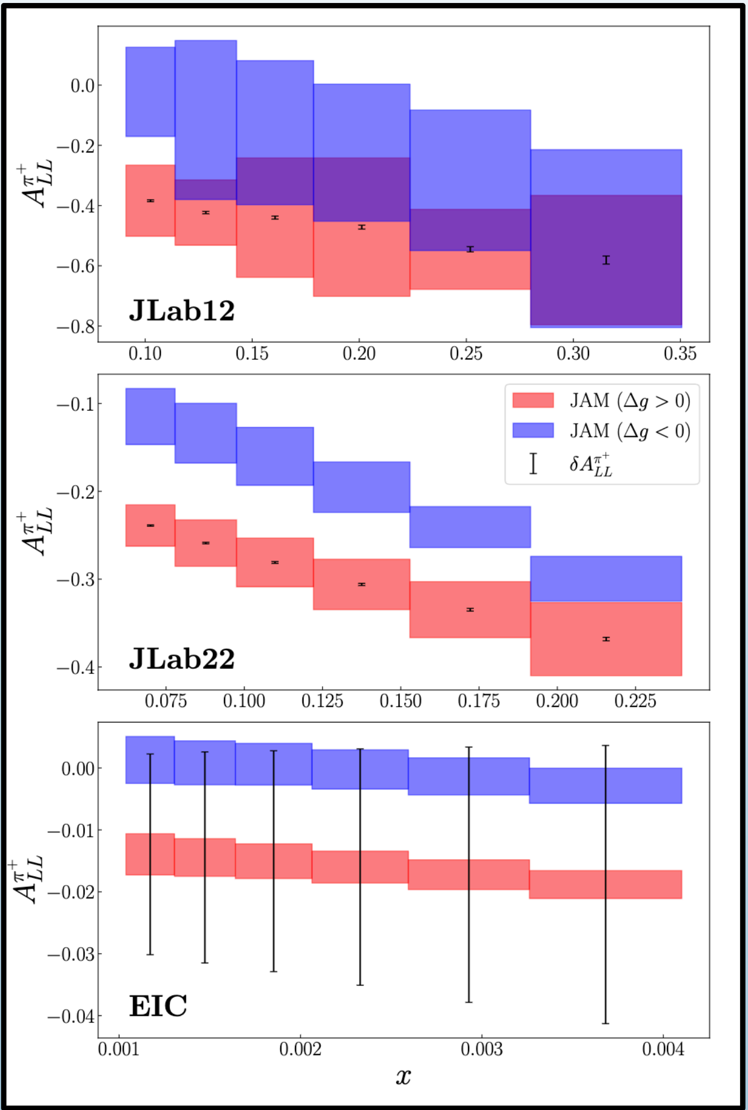
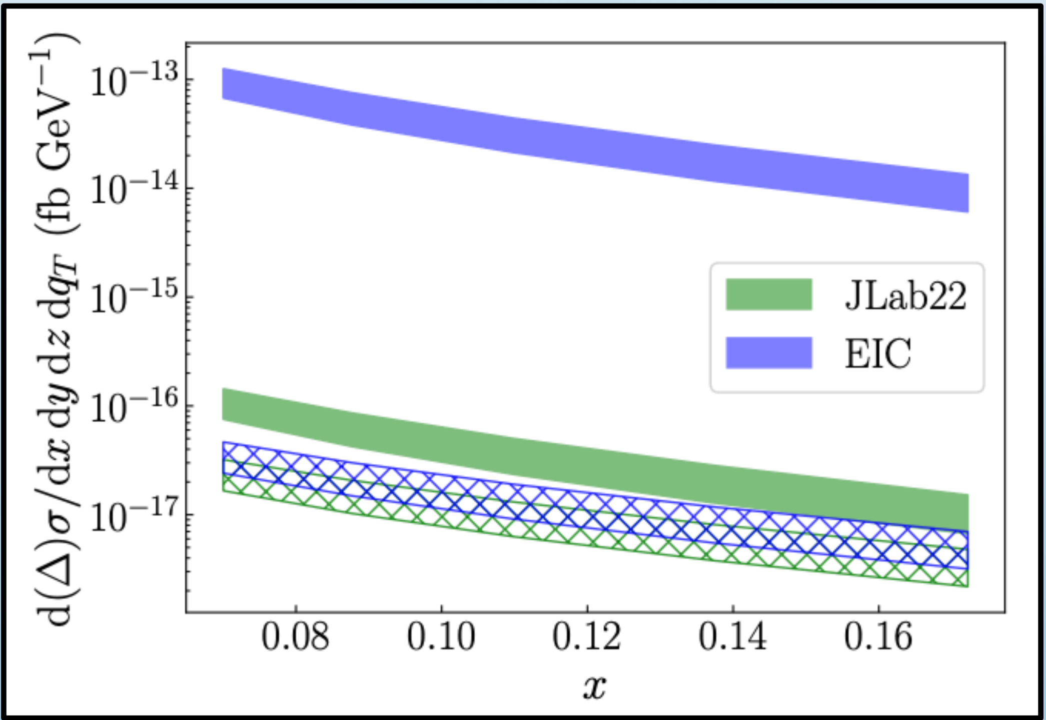


$\mathscr{L} = 86\ \text{fb}^{-1}$ for JLab
$\mathscr{L} = 10\ \text{fb}^{-1}$ for EIC

EIC asymmetry is small due to scaling behavior of unpolarized cross section

JLab22 has stronger distinguishing power due to more evolution and access to smaller x



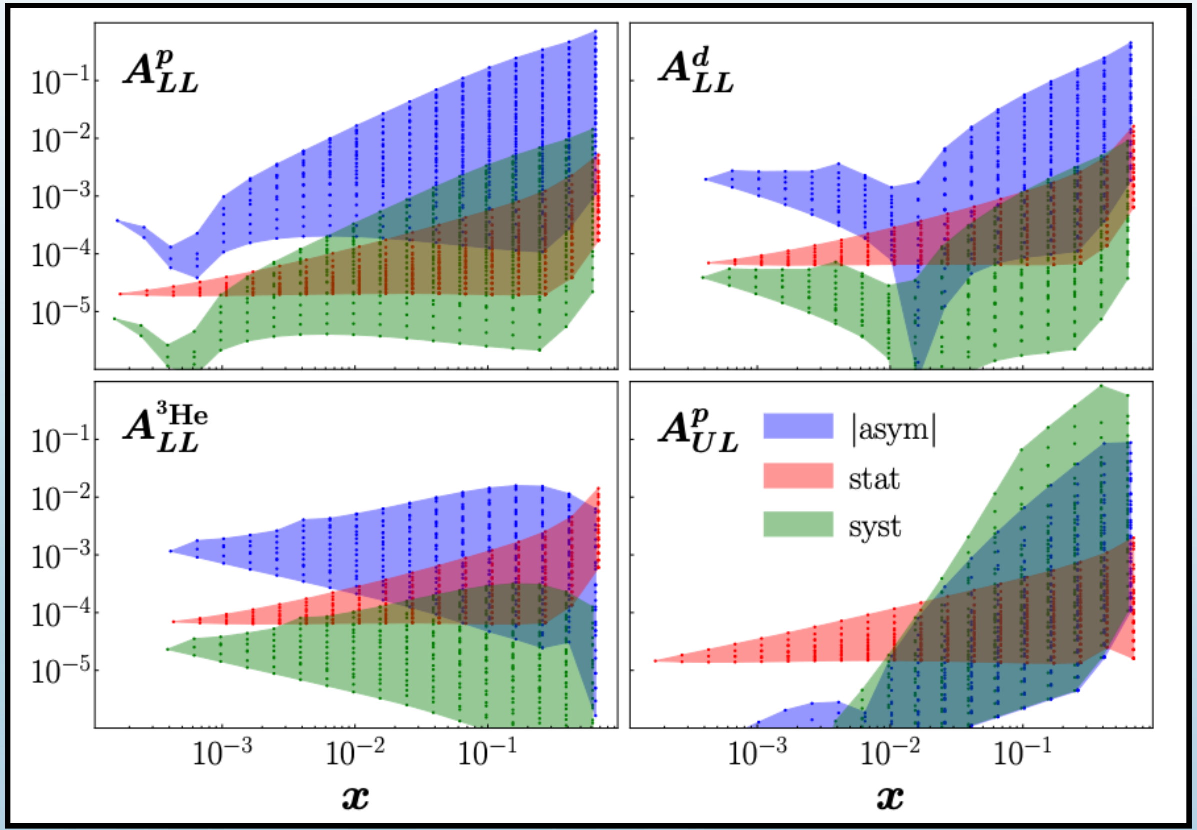
Future Experiments

Revisiting quark and gluon polarization in the proton at the EIC

Jefferson Lab Angular Momentum (JAM) Collaboration • Y. Zhou (William-Mary Coll.) et al. (May 10, 2021)

Published in: *Phys.Rev.D* 104 (2021) 3, 034028 • e-Print: 2105.04434 [hep-ph]

$$\vec{l} + \vec{N} \rightarrow l' + X$$



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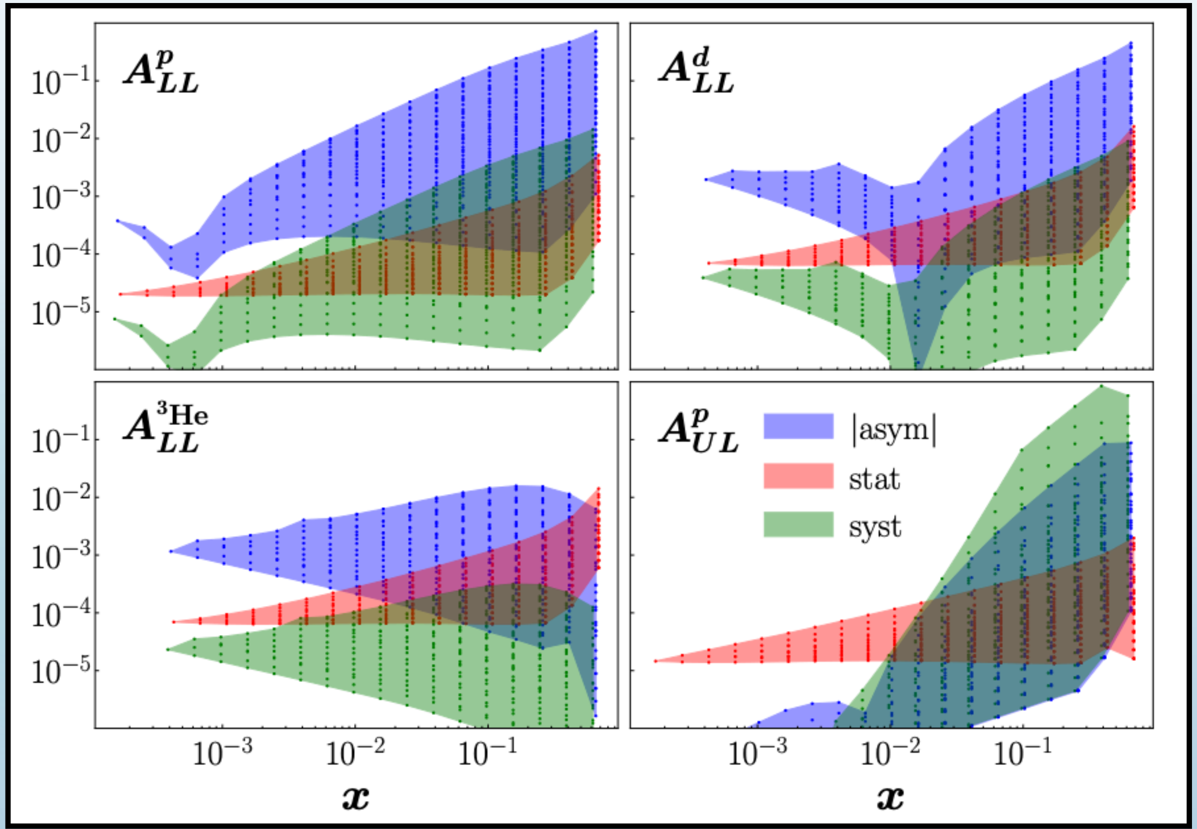
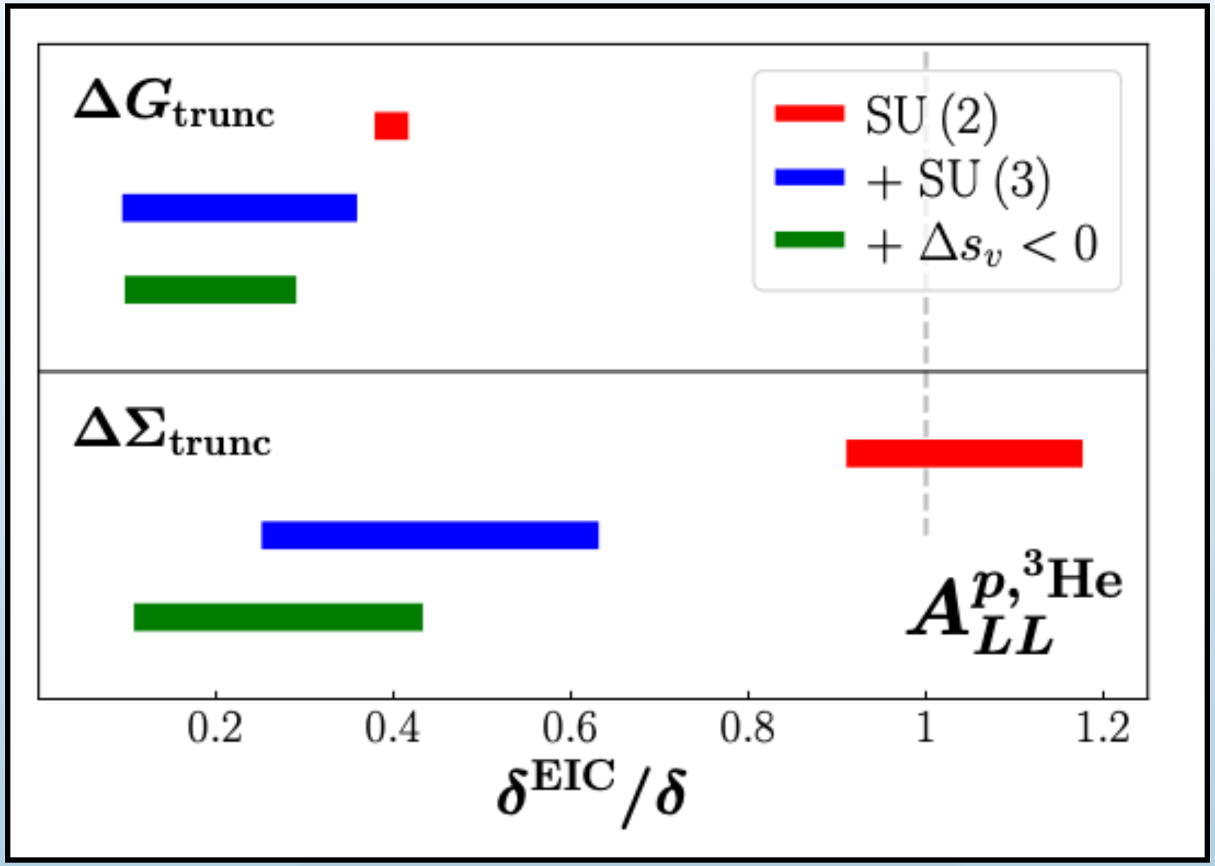
Published in: *Phys.Rev.D* 104 (2021) 3, 034028 • e-Print: 2105.04434 [hep-ph]

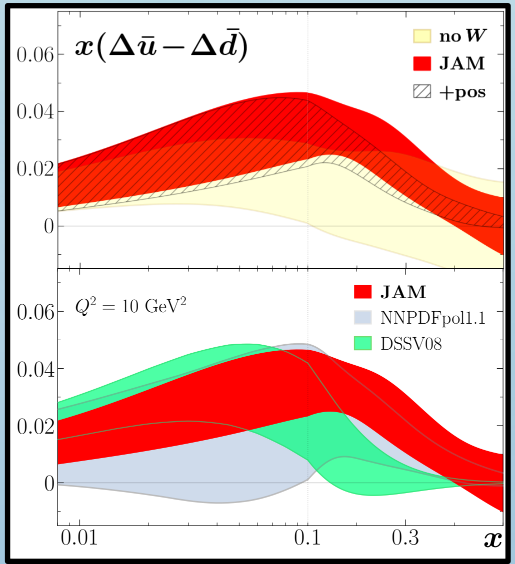
$$\vec{l} + \vec{N} \rightarrow l' + X$$





$$\Delta G_{\text{trunc}} = \int_{10^{-4}}^{1} \mathrm{d}x \Delta g(x)$$

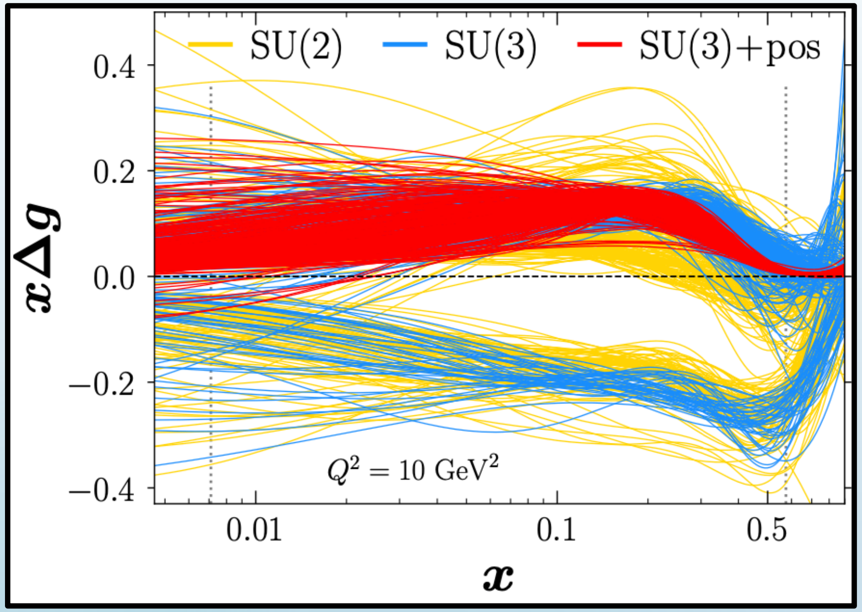
Conclusions and Outlook

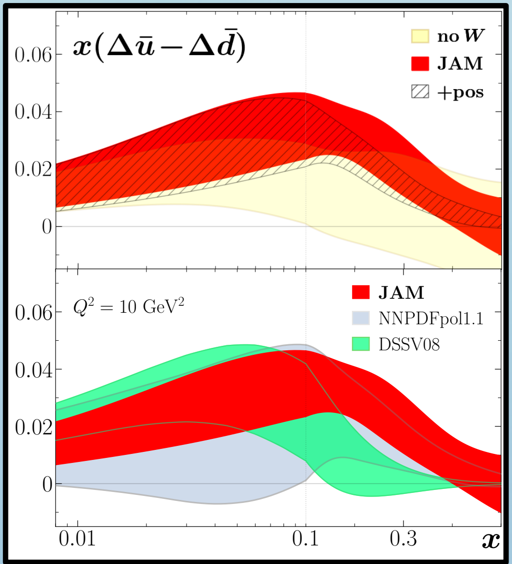


Nonzero sea asymmetry

Conclusions and Outlook

Current JAM analyses have two gluon solutions





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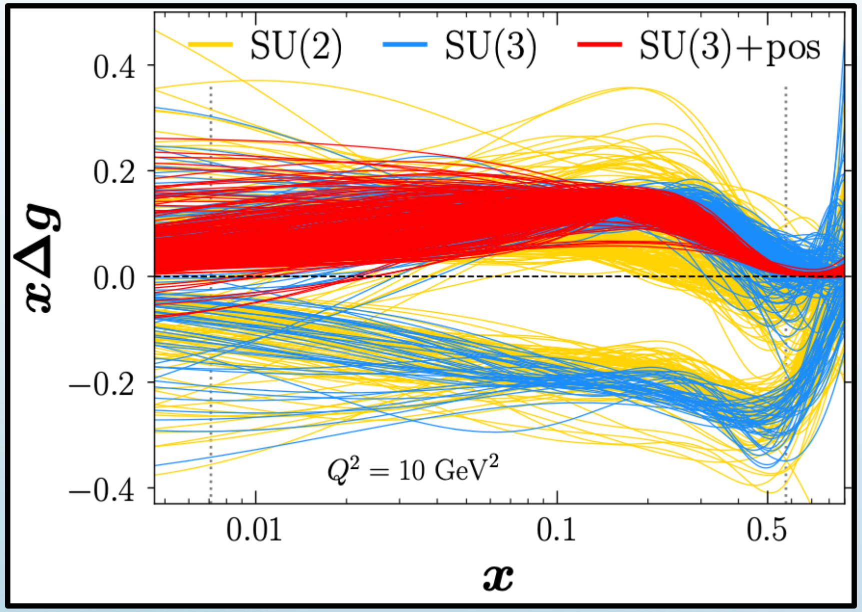
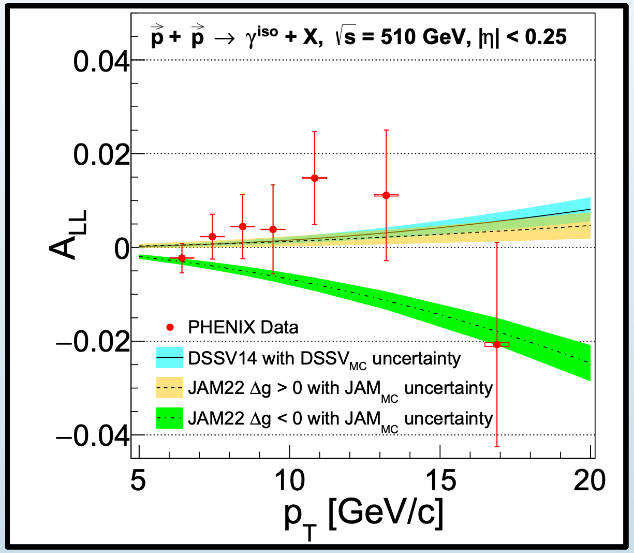
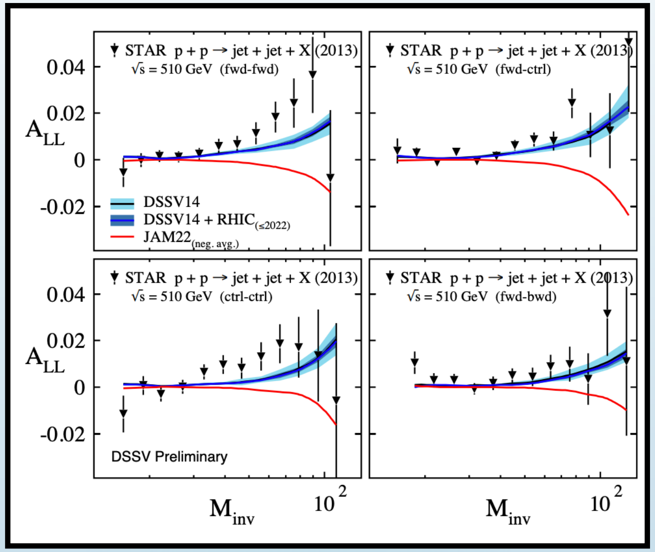
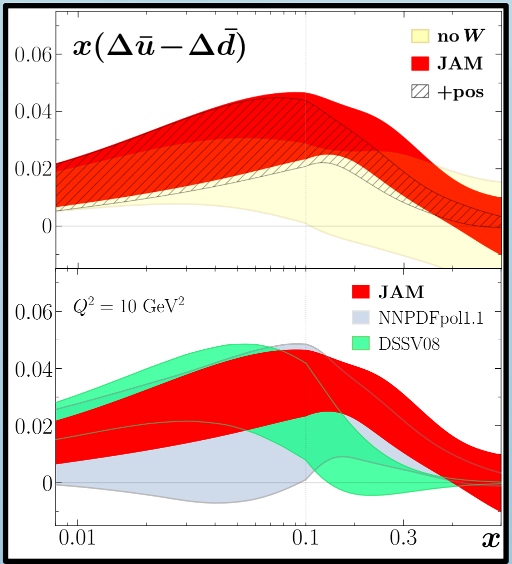


New data from RHIC may help distinguish them







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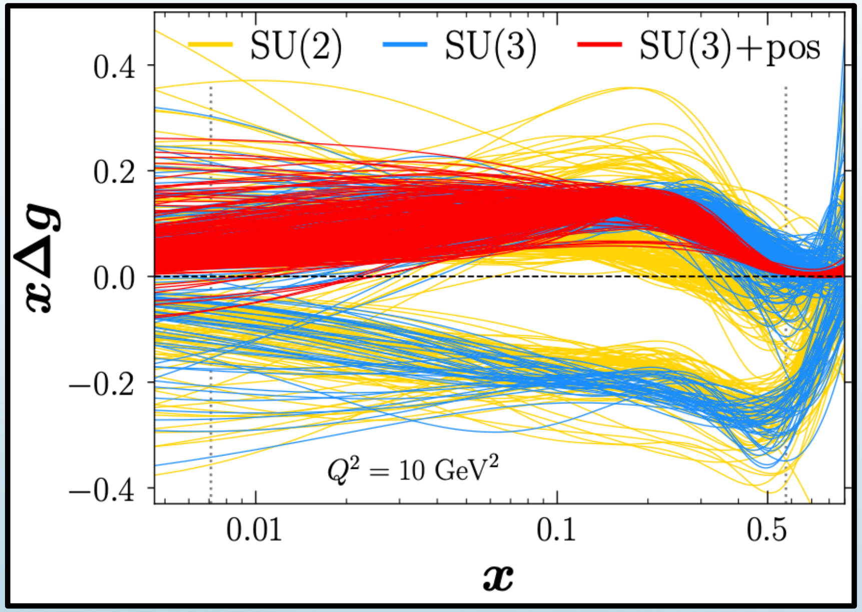

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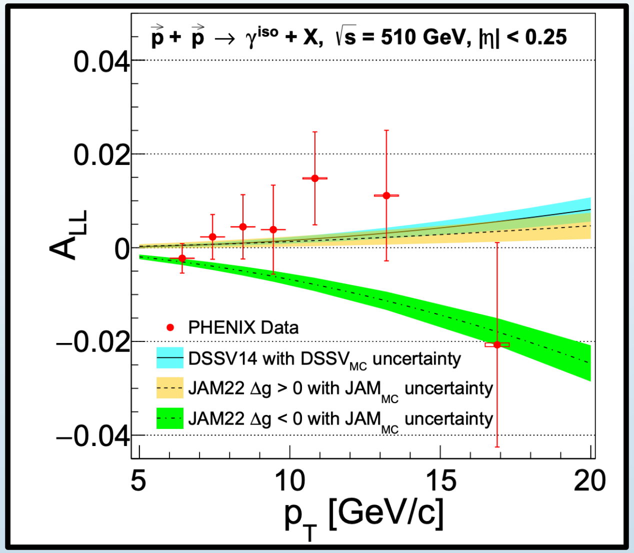

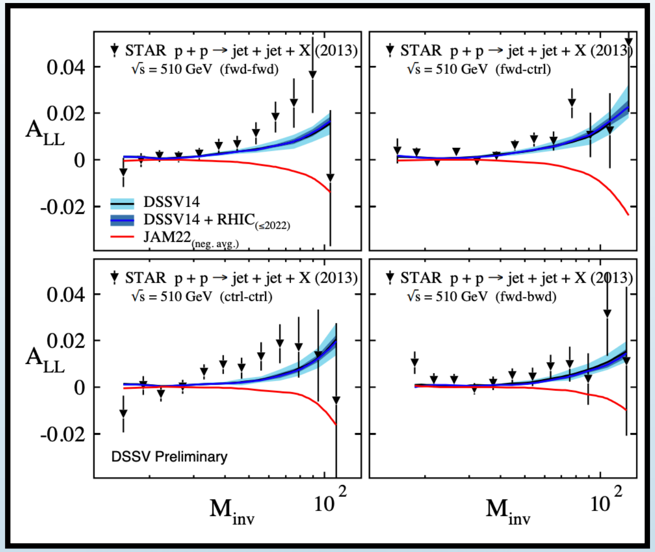

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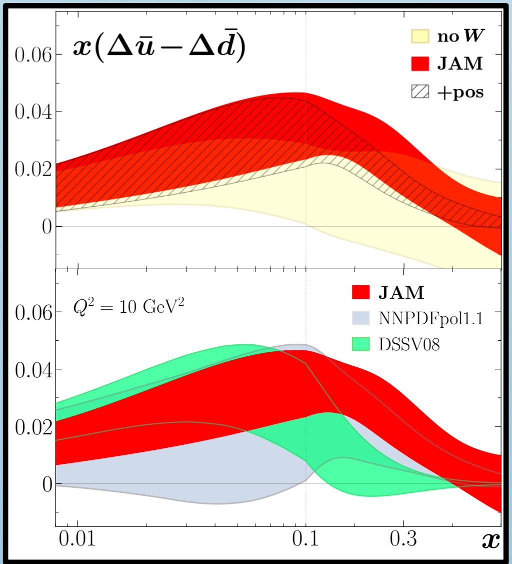
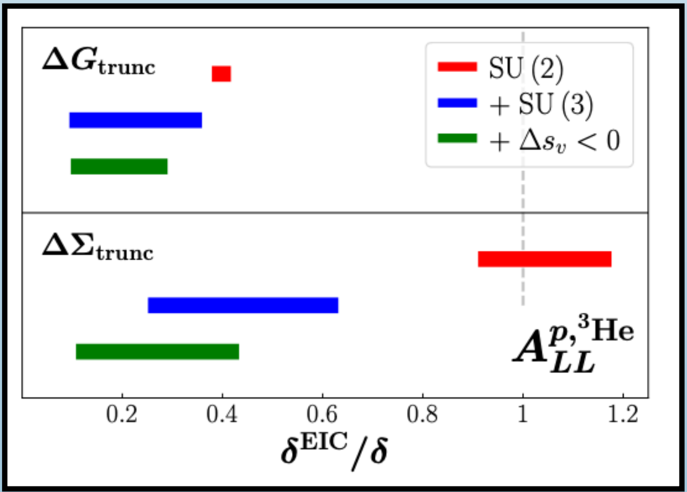
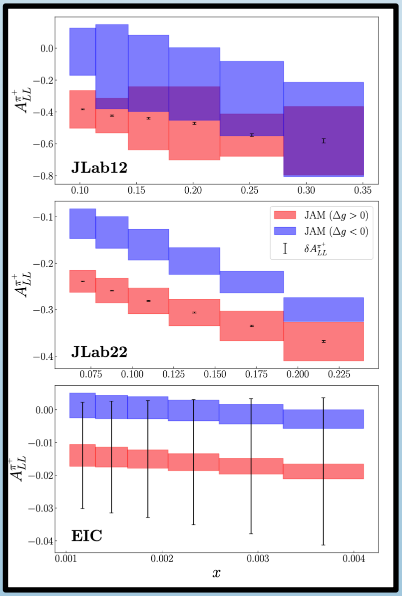


Future data from the EIC and JLab should provide new information





Extra Slides

Parameterize PDFs at input scale $Q_0^2 = m_c^2$

$$f_i(x) = Nx^{\alpha}(1-x)^{\beta}(1+\gamma\sqrt{x}+\eta x)$$

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Evolve PDFs using DGLAP

$$\frac{\mathrm{d}}{\mathrm{d}\,\ln(\mu^2)} f_i(x,\mu) = \sum_j \int_x^1 \frac{dz}{z} P_{ij}(z,\mu) f_j(\frac{x}{z},\mu)$$

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Calculate Observables

$$d\sigma^{pp} = \sum_{ij} H_{ij}^{pp} \otimes f_i \otimes f_j$$

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Mellin Space Techniques

$$\mathrm{d}\sigma^{pp} = \sum_{ijkl} \frac{1}{(2\pi i)^2} \int \mathrm{d}N \int \mathrm{d}M\, \tilde{f}_j(N,\mu_0) \tilde{f}_l(M,\mu_0) \otimes \left[x_1^{-N} x_2^{-M} \tilde{\mathcal{H}}_{ik}^{pp}(N,M,\mu) U_{ij}^S(N,\mu,\mu_0) U_{kl}^S(M,\mu,\mu_0) \right]$$

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$$\sigma = \sum_{ij} H_{ij} \otimes f_i \otimes f_j + \mathcal{O}(1/Q)$$

Experimentally measured cross-section

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"Hard part" (process dependent)
Cross-section at parton level
Calculated in perturbative QCD

Experimentally measured cross-section

"Soft part" (process independent)
Describes internal structure

$$\sigma = \sum_{ij} H_{ij} \otimes f_i \otimes f_j + \mathcal{O}(1/Q)$$

"Hard part" (process dependent)
Cross-section at parton level
Calculated in perturbative QCD

Now that the observables have been calculated…

$$\chi^2(\boldsymbol{a}) = \sum_{i,e} \left(\frac{d_{i,e} - \sum_k r_e^k \beta_{i,e}^k - T_{i,e}(\boldsymbol{a})/N_e}{\alpha_{i,e}} \right)^2 + \sum_k \left(r_e^k \right)^2 + \left(\frac{1 - N_e}{\delta N_e} \right)^2$$

Now that the observables have been calculated…

Data

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Uncorrelated Uncertainties

Correlated Uncertainties

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Data

Theory

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Uncorrelated Uncertainties

Correlated Uncertainties

Normalization Uncertainty

Now that we have calculated $\chi^2(\boldsymbol{a}, \text{data})$…

Likelihood Function

$$\mathcal{L}(\boldsymbol{a}, \text{data}) = \exp\left(-\frac{1}{2}\chi^2(\boldsymbol{a}, \text{data})\right)$$

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Likelihood Function

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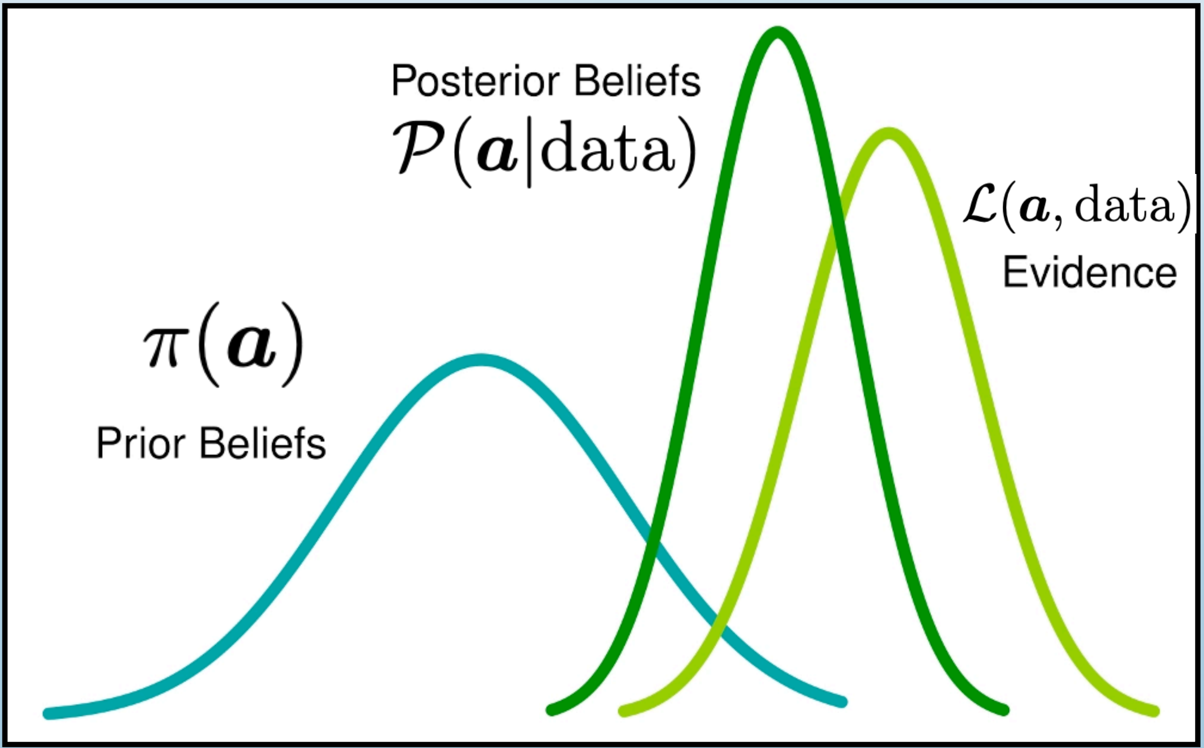
Bayes' Theorem

$$\mathcal{P}(\boldsymbol{a}|\text{data}) \sim \mathcal{L}(\boldsymbol{a}, \text{data})\,\pi(\boldsymbol{a})$$



$$\tilde{\sigma} = \sigma + N(0,1)\ \alpha$$

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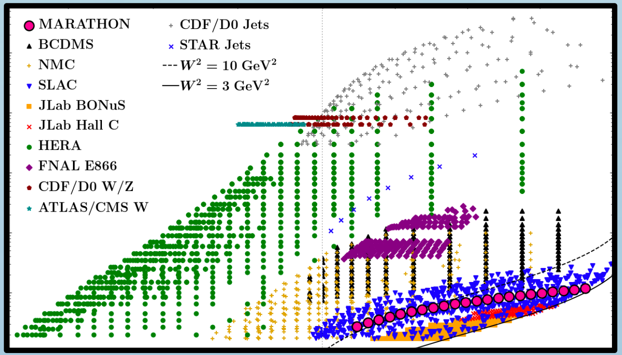
Data

Original Data



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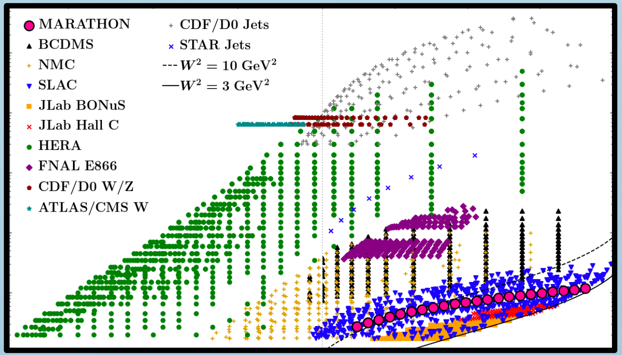
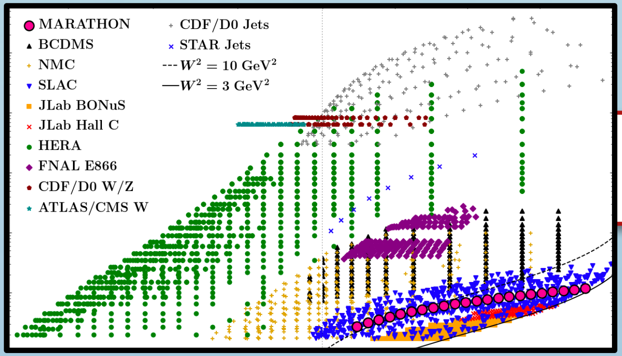


Pseudo-Data → $$\tilde{\sigma} = \sigma + N(0,1)\ \alpha$$ ← Uncorrelated Uncertainties

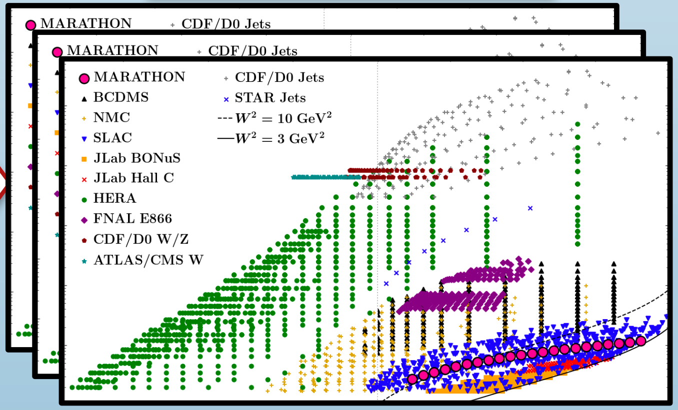
↑ Data

Original Data



DR →

Replica Data



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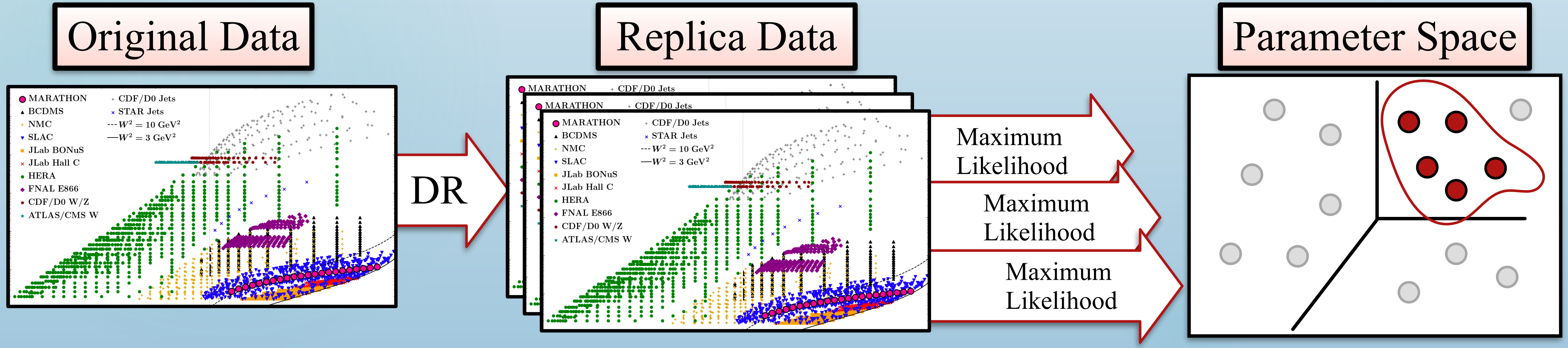

For a quantity $O(\boldsymbol{a})$: (for example, a PDF at a given value of (x, Q^2))

$$E[O] = \int d^n a \; \rho(\boldsymbol{a} \,|\, data) \; O(\boldsymbol{a})$$

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Exact, but
$n = \mathcal{O}(100)$!

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Average over k sets of the parameters (replicas)

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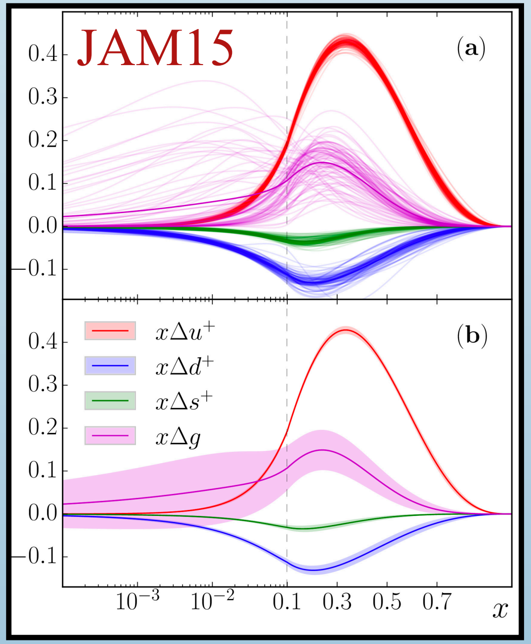
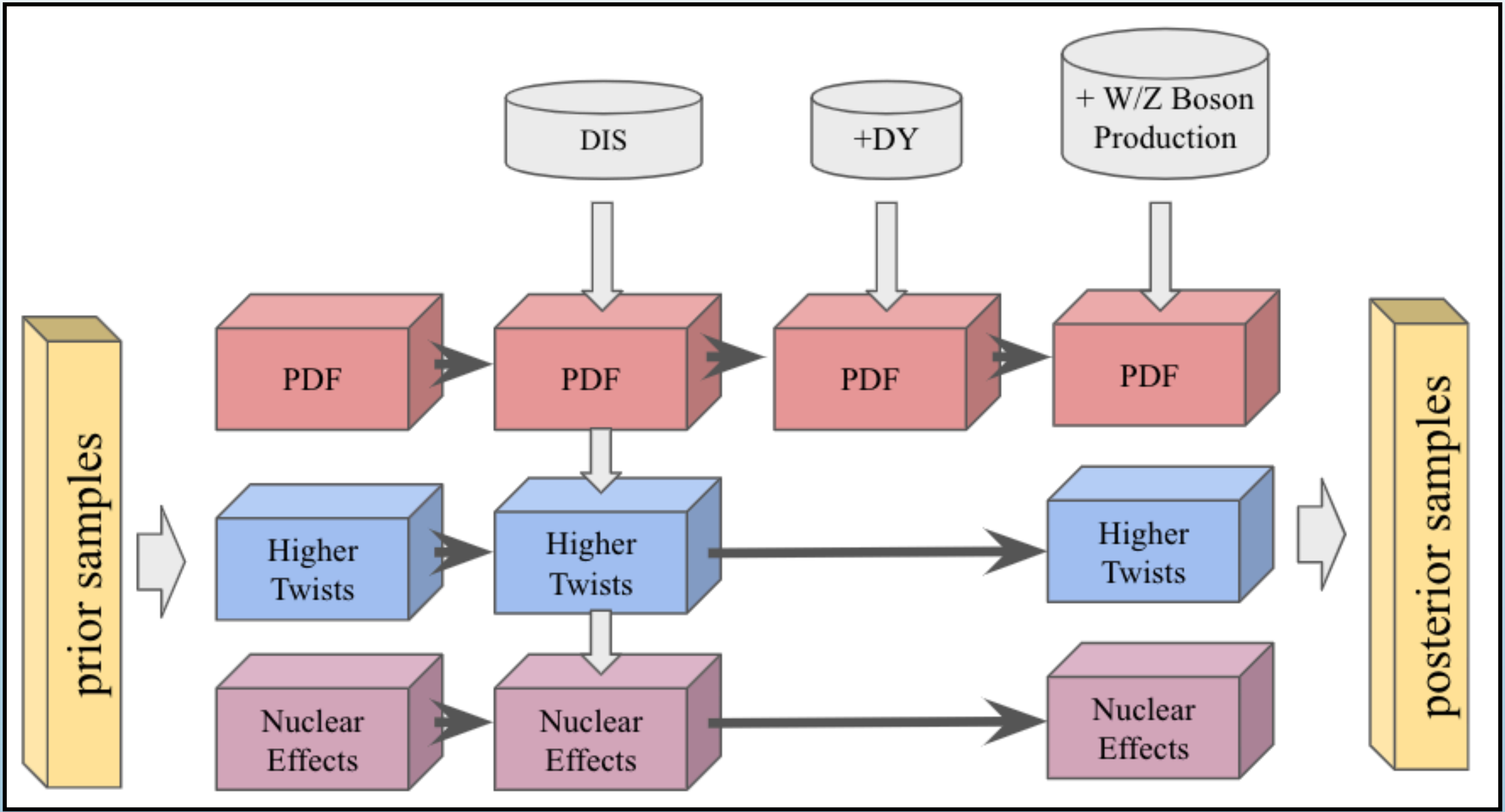
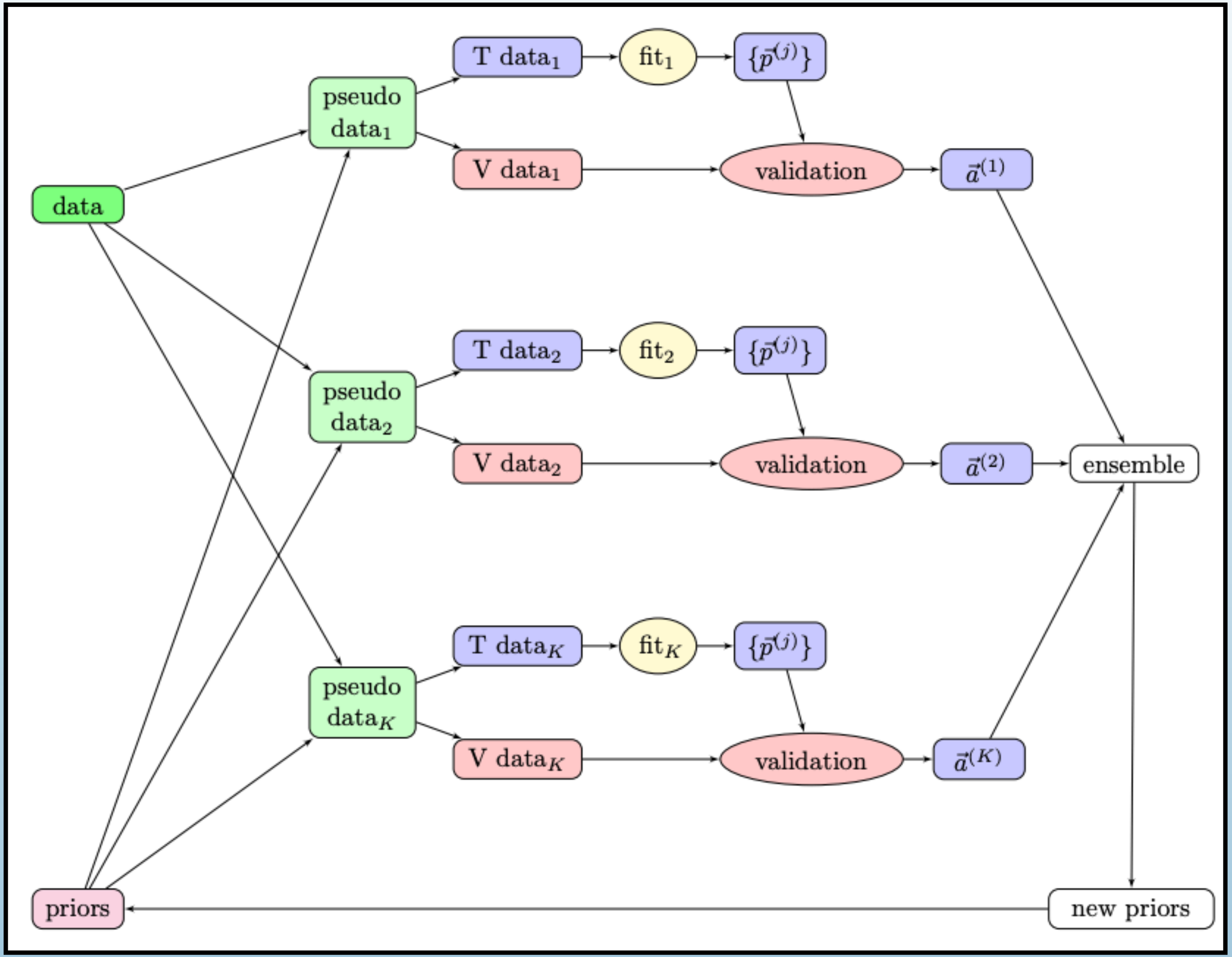
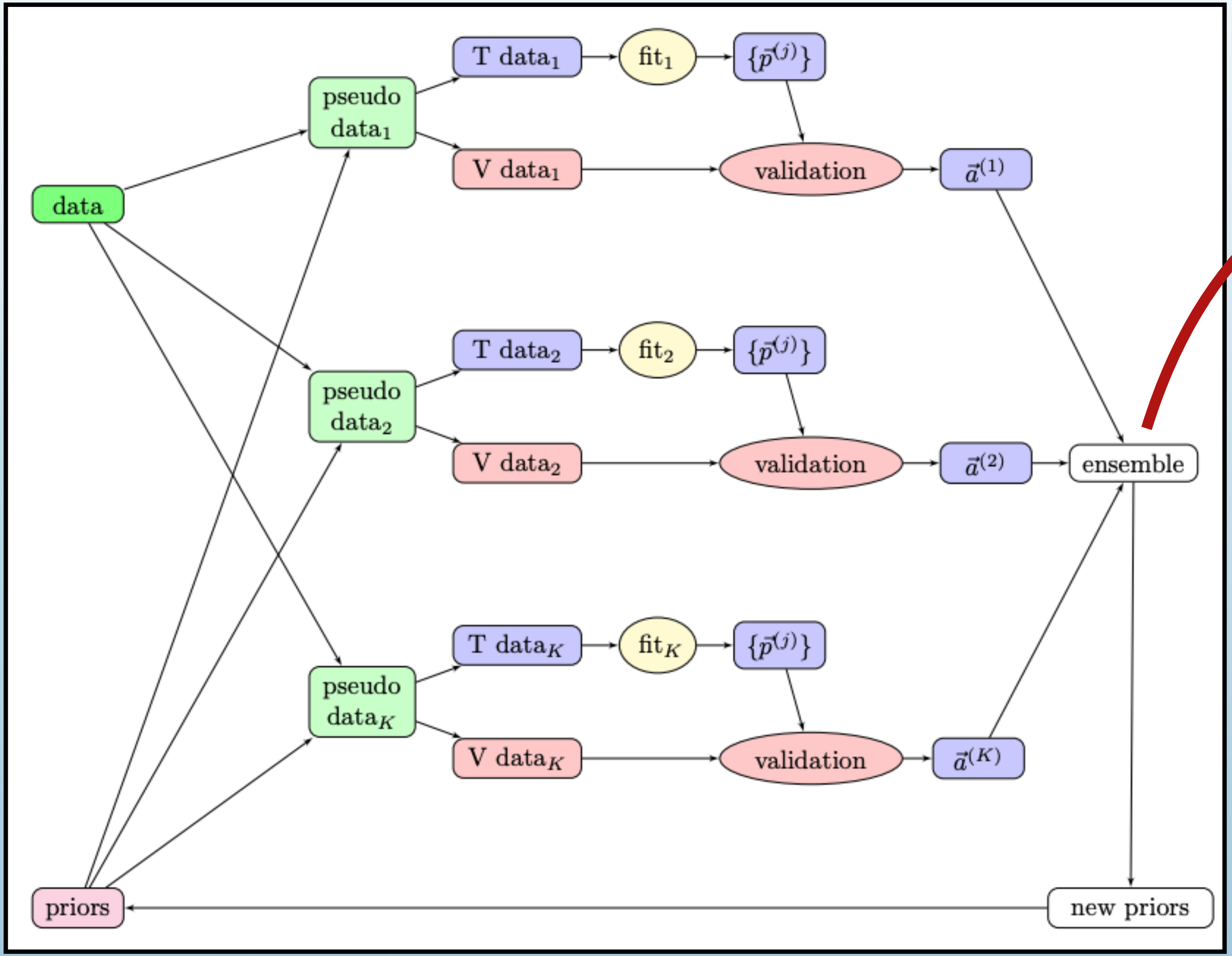



DIS
+DY
+ W/Z Boson Production
prior samples
PDF
PDF
PDF
PDF
Higher Twists
Higher Twists
Higher Twists
Nuclear Effects
Nuclear Effects
Nuclear Effects
posterior samples



data
pseudo data$_1$
pseudo data$_2$
pseudo data$_K$
T data$_1$
V data$_1$
T data$_2$
V data$_2$
T data$_K$
V data$_K$
fit$_1$
fit$_2$
fit$_K$
$\{\vec{p}^{(j)}\}$
$\{\vec{p}^{(j)}\}$
$\{\vec{p}^{(j)}\}$
validation
validation
validation
$\vec{a}^{(1)}$
$\vec{a}^{(2)}$
$\vec{a}^{(K)}$
ensemble
new priors
priors


data
pseudo data$_1$
pseudo data$_2$
pseudo data$_K$
T data$_1$
V data$_1$
T data$_2$
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T data$_K$
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fit$_1$
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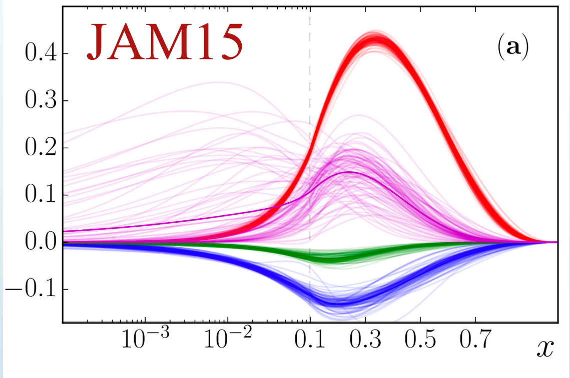
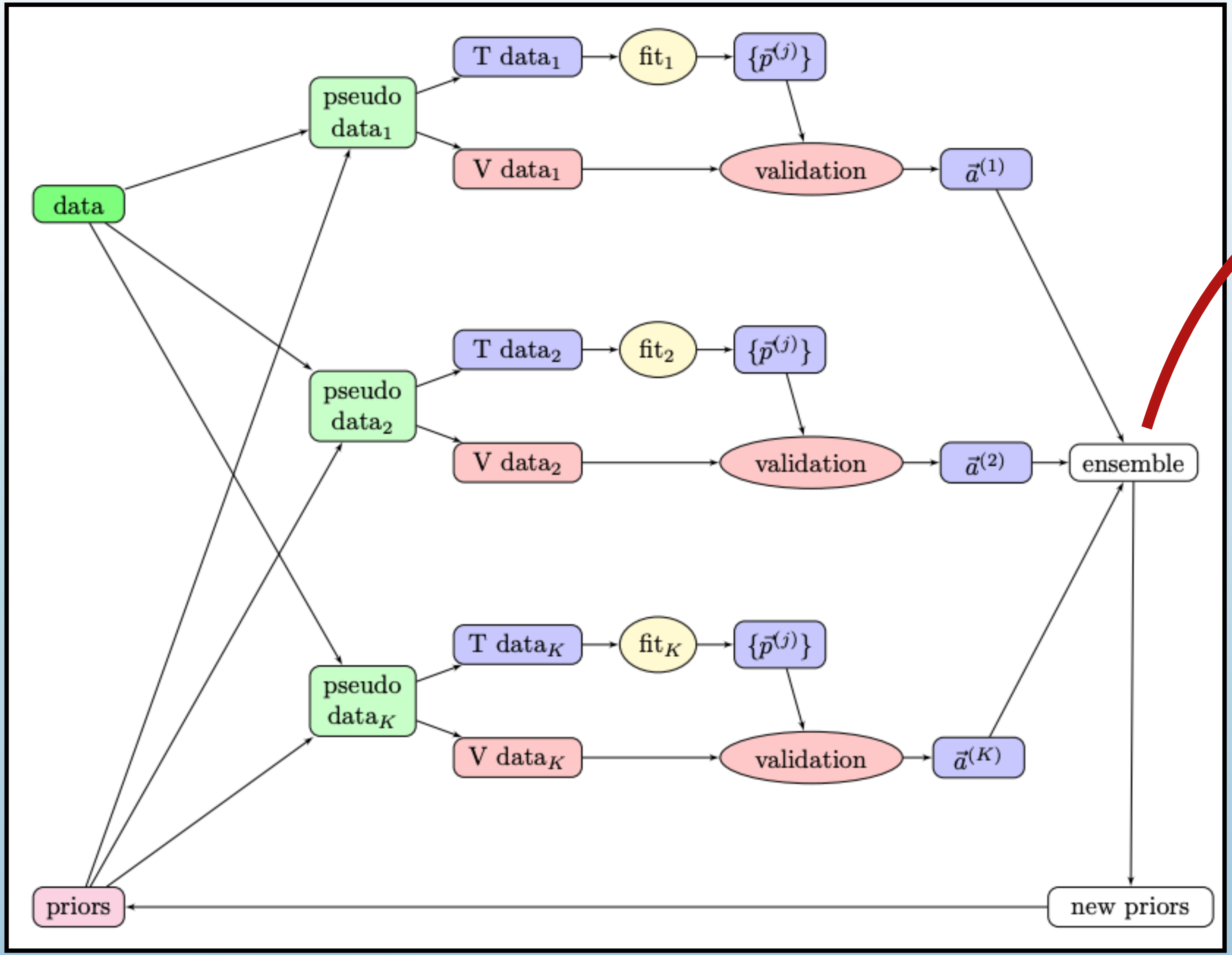
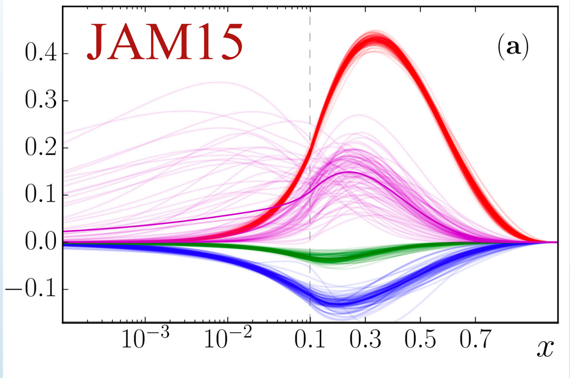

JAM15
(a)
0.4
0.3
0.2
0.1
0.0
−0.1
10^{-3}
10^{-2}
0.1
0.3
0.5
0.7
x





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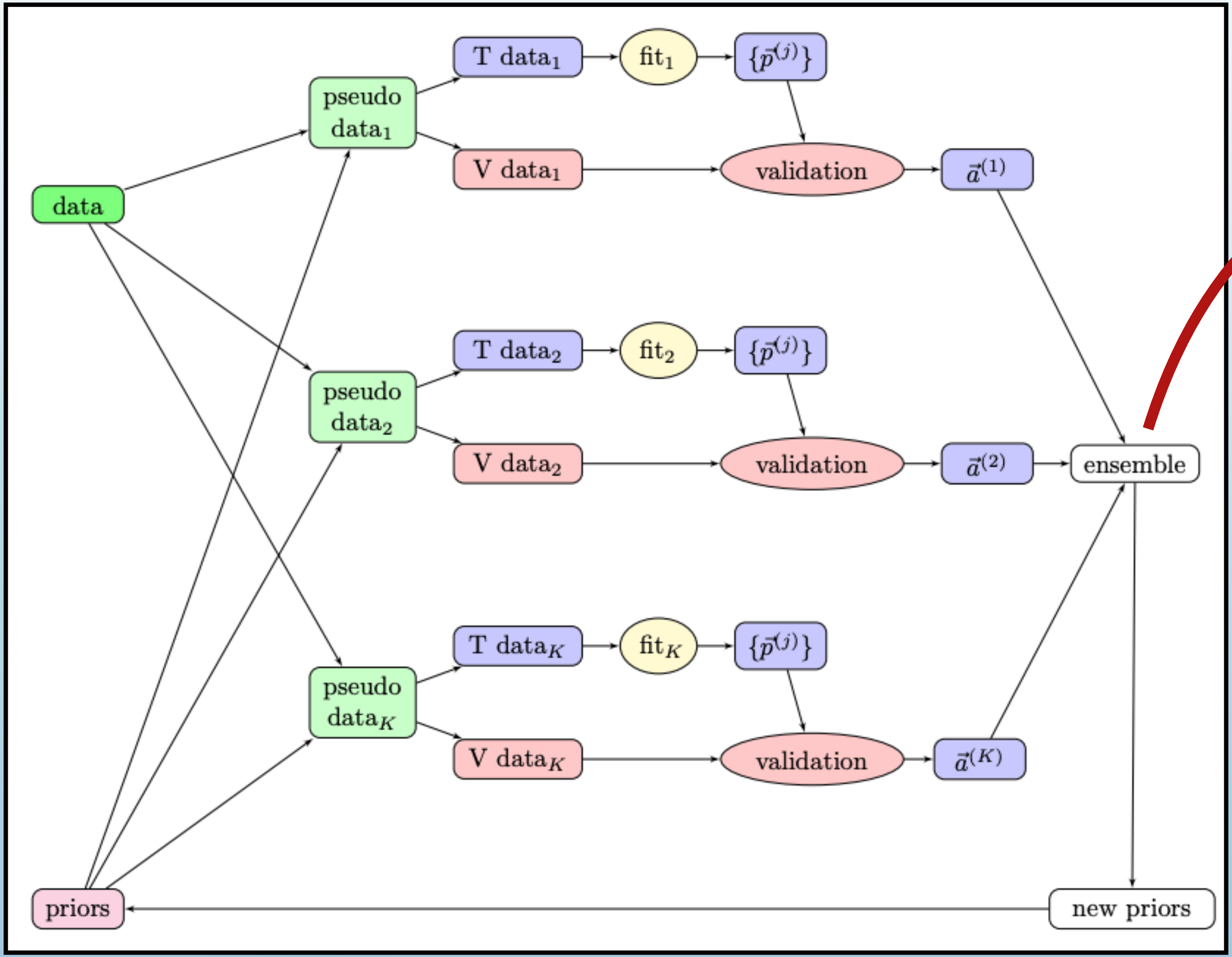

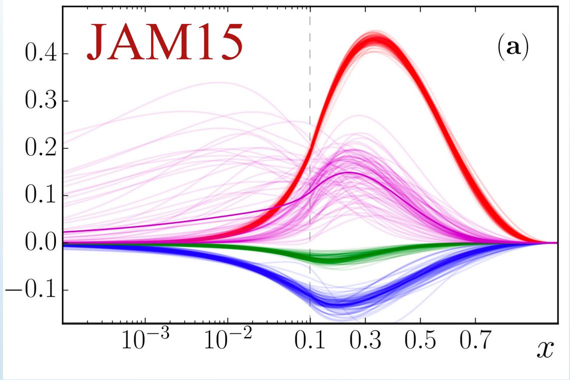

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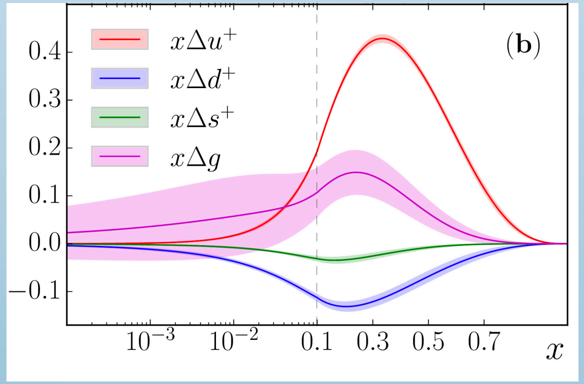

Revisiting helicity parton distributions at a future electron-ion collider #2

Ignacio Borsa (U. Buenos Aires), Gonzalo Lucero (U. Buenos Aires), Rodolfo Sassot (U. Buenos Aires), Elke C. Aschenauer (Brookhaven Natl. Lab.), Ana S. Nunes (Brookhaven Natl. Lab.) (Jul 16, 2020)

Published in: *Phys.Rev.D* 102 (2020) 9, 094018 • e-Print: 2007.08300 [hep-ph]

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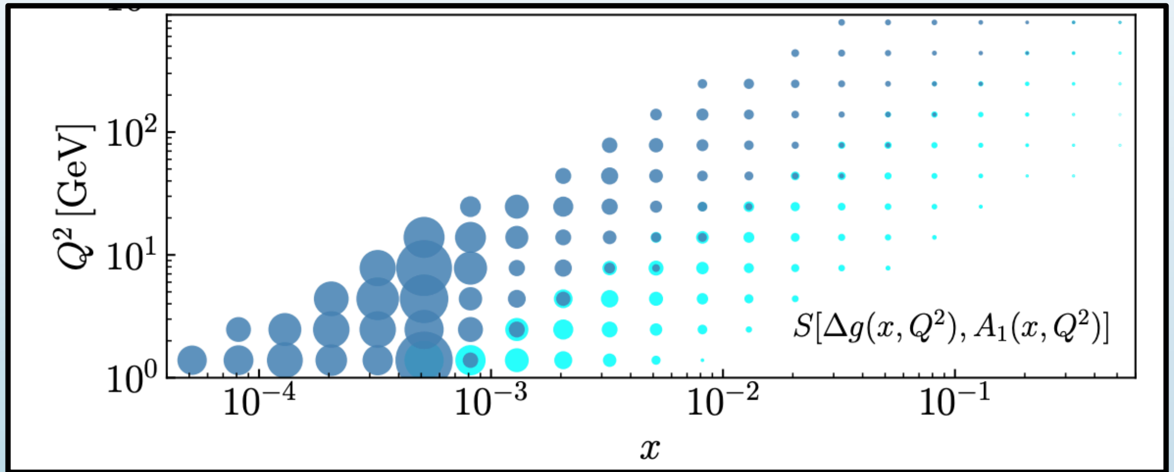

Sensitivity of A_1 to Δg

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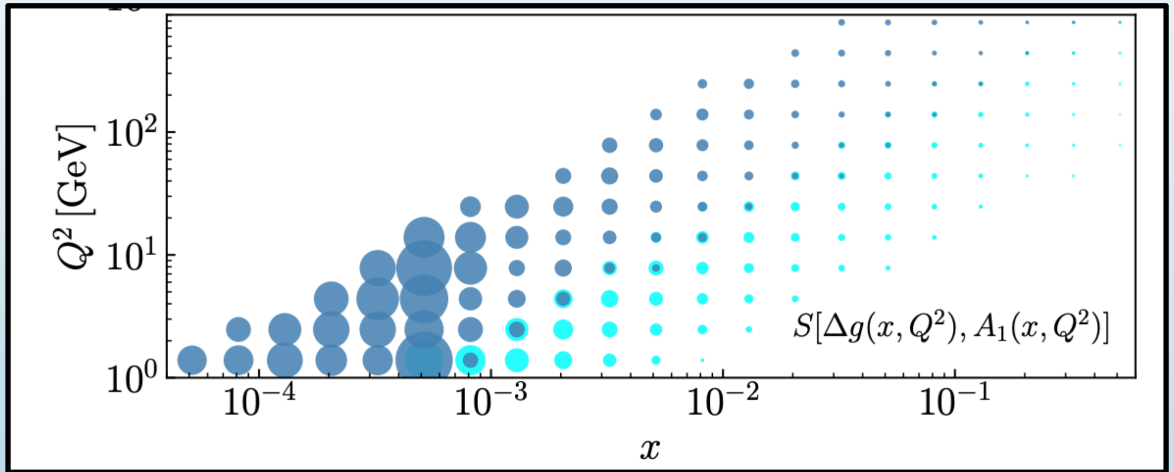

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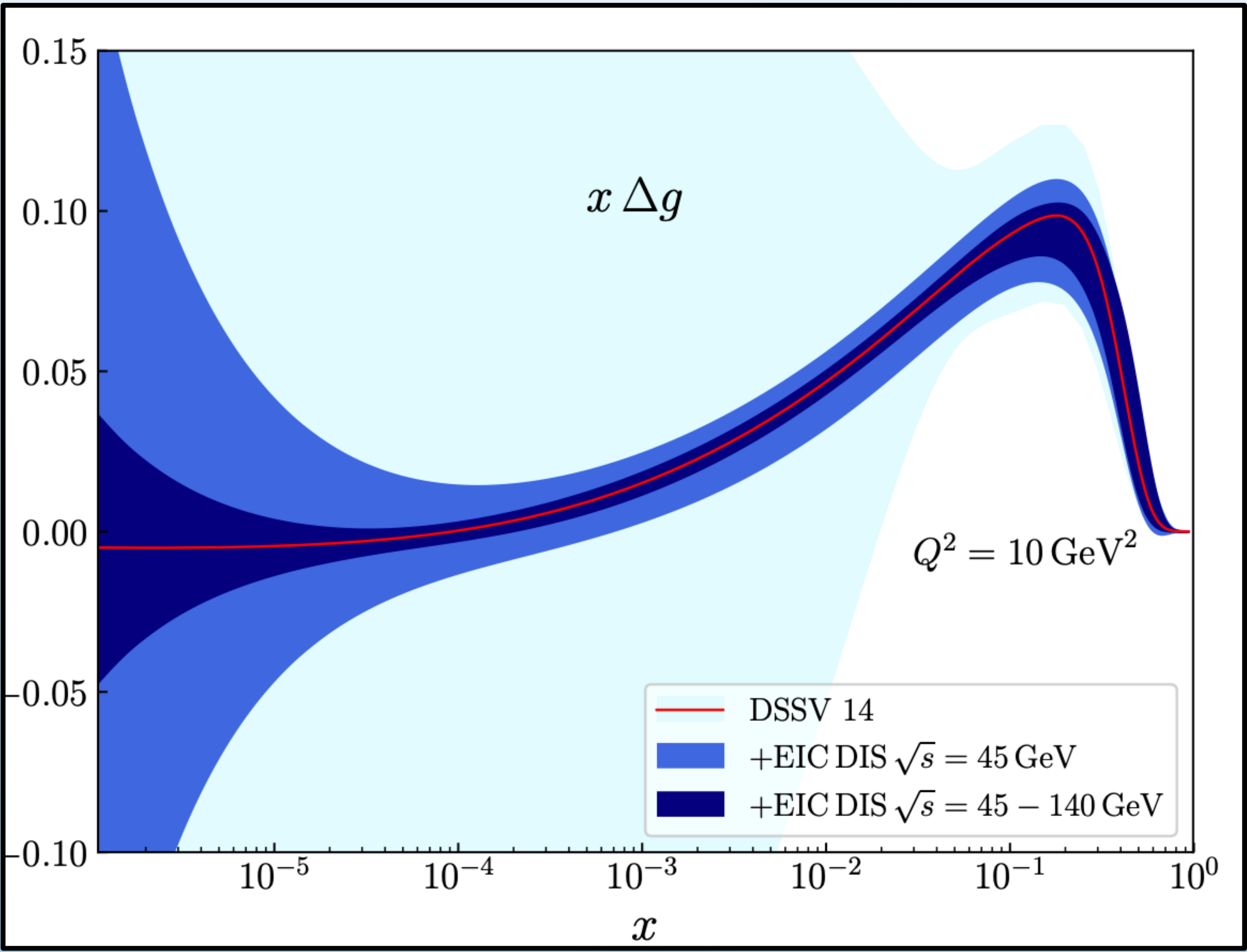

Large impact on Δg predicted, especially below $x \approx 0.01$

Positivity

Positivity and renormalization of parton densities #1

John Collins (Penn State U.), Ted C. Rogers (Old Dominion U. and Jefferson Lab), Nobuo Sato (Jefferson Lab) (Nov 1, 2021)

Published in: *Phys.Rev.D* 105 (2022) 7, 076010 • e-Print: 2111.01170 [hep-ph]

As regards the positivity issue itself, there are several points to make. First, we emphasize that we have not argued that $\overline{\text{MS}}$ pdfs *must* be negative for any particular choice of scales or $\mu_{\overline{\text{MS}}}$. Rather we proved that nothing in the definition of pdfs or in the factorization theorems themselves excludes negativity as a possibility, especially at low or moderate input scales. But we did show arguments that indicate that certain generic situations do tend to lead to negative pdfs of partons with small pdfs, notably for non-valence quarks. Giving a full theoretical answer to the question of whether a particular pdf turns negative depends on its large distance/low energy non-perturbative properties, as the sensitivity to mass scales in the example of Sec. VIII illustrates. Also, the failure of