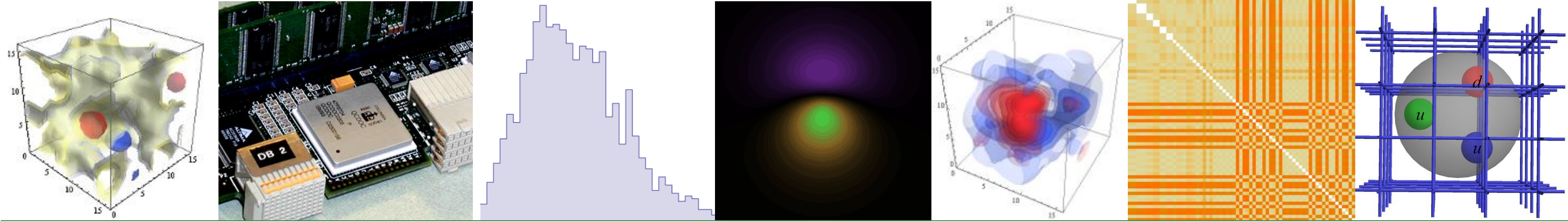


Mapping Meson Parton Distributions with Lattice QCD

HUEY-WEN LIN

CTEQ

This work of HL is supported by the NSF under grant PHY 2209424 & 1653405, DOE under DE-SC0024053 and the Research Corporation for Science Advancement through the Cottrell Scholar Award

NSF

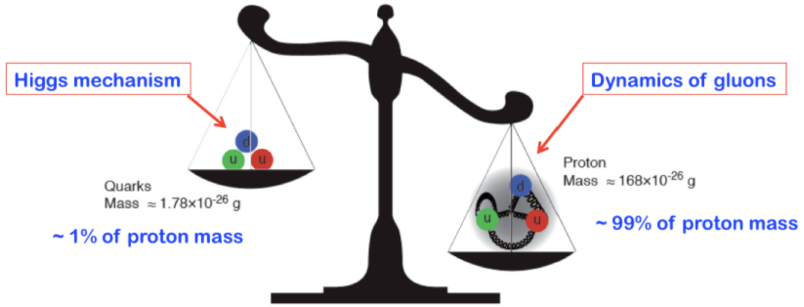
@LinQCD

U.S. DEPARTMENT OF ENERGY

RESEARCH CORPORATION for SCIENCE ADVANCEMENT

Motivation

- Meson structure is crucial to understand the mechanism of emergent hadron mass (EHM)
  - Help decode QCD origin of mass



- Experimentally, meson structure is harder to study
  - LQCD can provide predictions and better precision inputs
  - Quark and gluon parton distribution functions(PDFs), for example

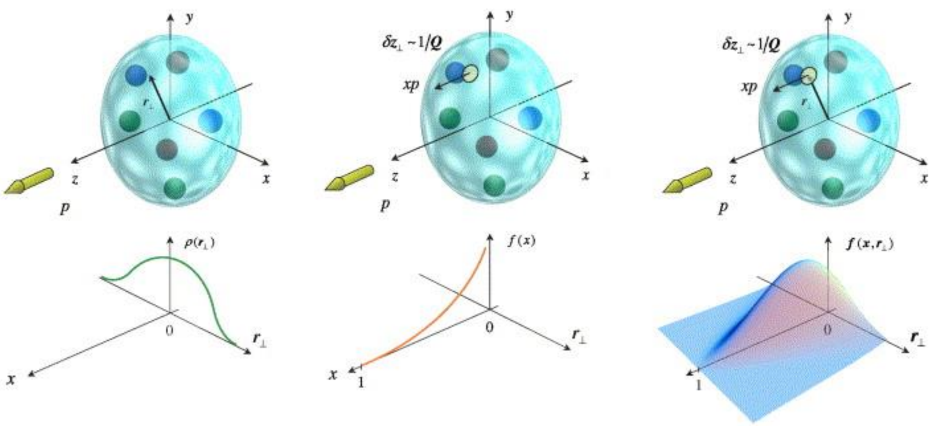


Image from A. Belitskya and A Radyushkin, Physics Report, 416 (2015)

- Generalized parton distributions (GPDs) encode information about the spatial structure & the partonic distribution of spin and orbital angular momenta
  - US EIC, EIcC, LHeC, ...

Outline

§ Lattice QCD in a Nutshell

§ *x*-dependent Meson Structure

- Valence quark distributions of pion and kaon
- Pion and Kaon gluon distribution
- Pion valence-quark GPDs

Biased selected results toward MSULat students and postdocs

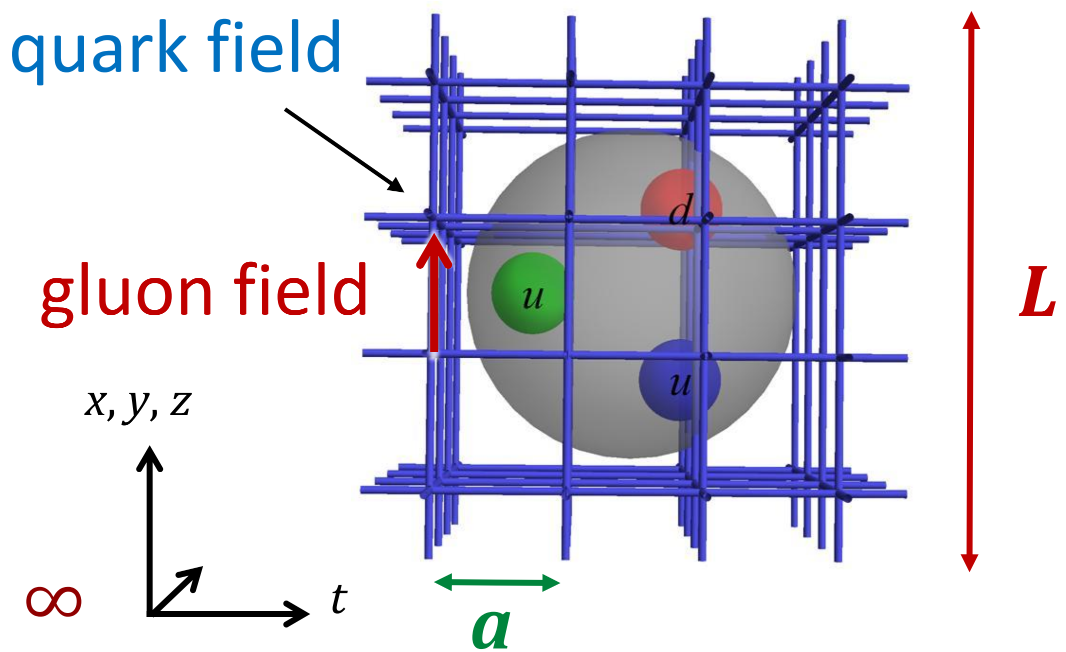
What is Lattice QCD?

- § Lattice QCD is an ideal theoretical tool for investigating the strong-coupling regime of quantum field theories
- § Physical observables are calculated from the path integral

$$\langle 0|O(\bar{\psi},\psi,A)|0\rangle = \frac{1}{Z}\int \mathcal{D}A\,\mathcal{D}\bar{\psi}\,\mathcal{D}\psi\, e^{iS(\bar{\psi},\psi,A)}O\,(\bar{\psi},\psi,A)$$

in **Euclidean** space

- Quark mass parameter (described by m_π)
- Impose a UV cutoff
  discretize spacetime
- Impose an infrared cutoff
  finite volume

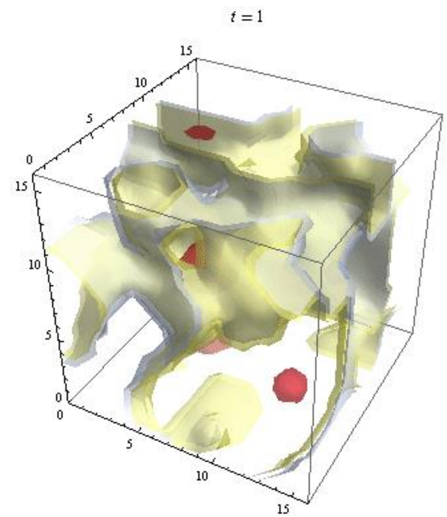


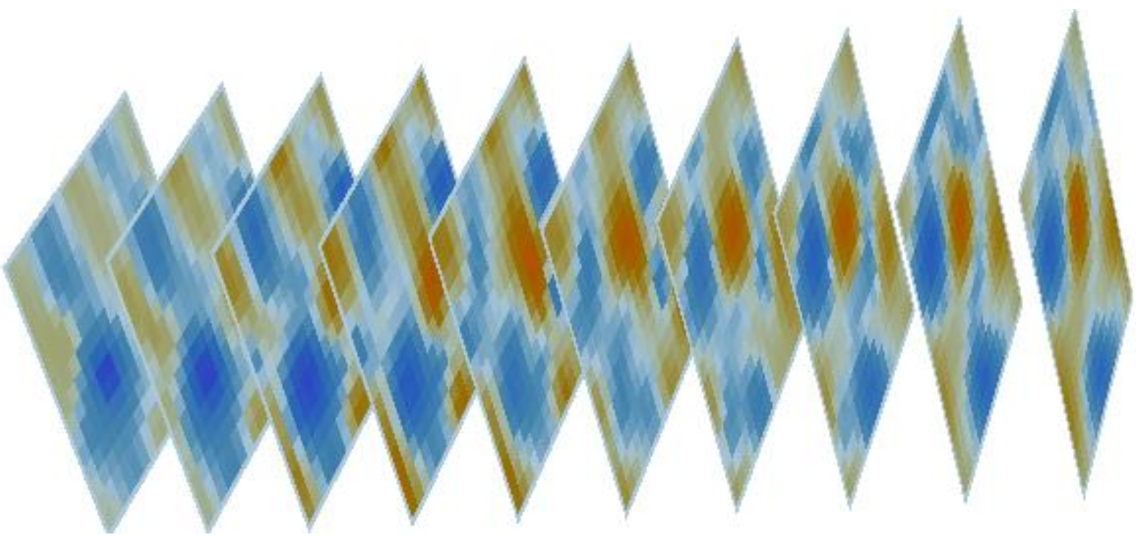
- § Recover physical limit

$m_\pi \to m_\pi^{\text{phys}}$, $a \to 0$, $L \to \infty$

Anatomy of a Lattice Calculation

1. Start with QCD Vacuum (gauge configurations)





Anatomy of a Lattice Calculation

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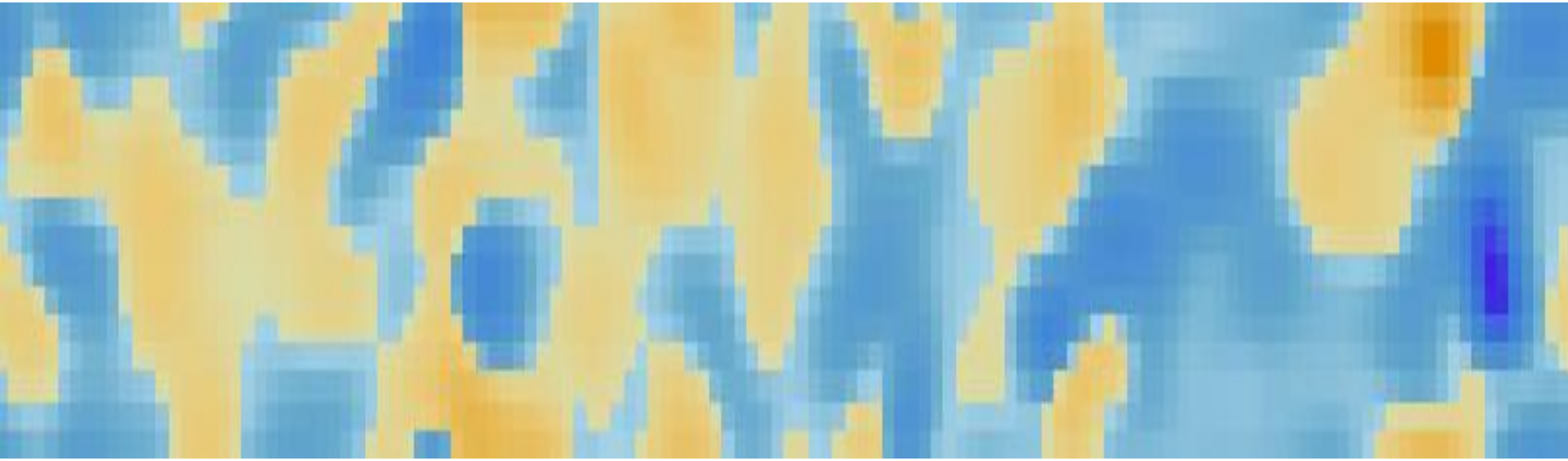
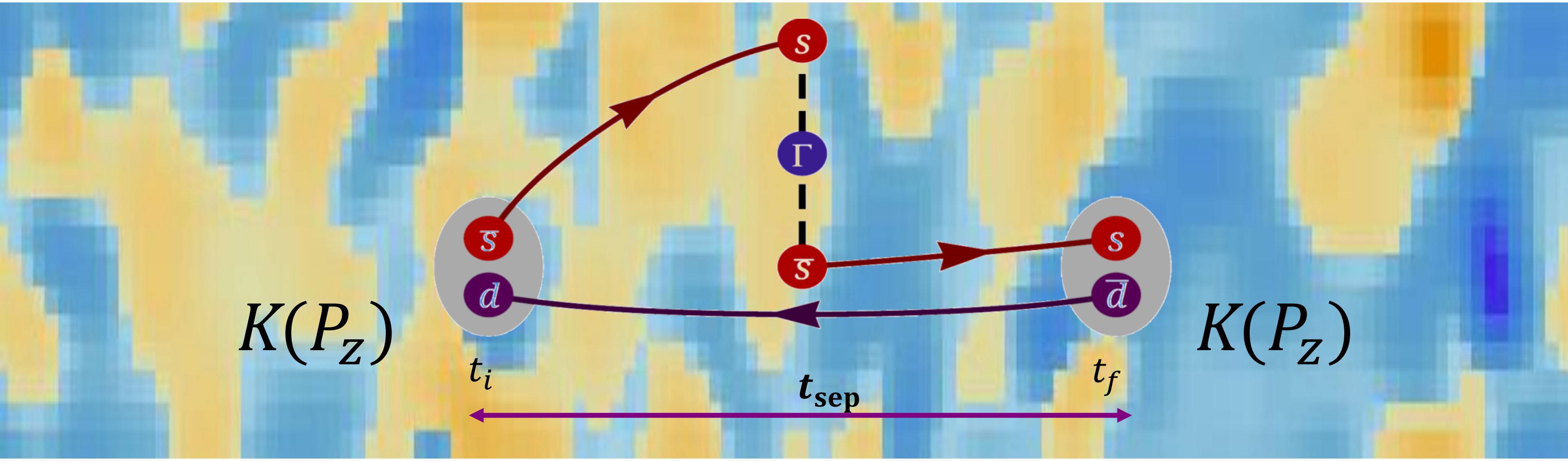


Thanks to MILC collaboration for sharing their 2+1+1 HISQ lattices

Anatomy of a Lattice Calculation

2. Correlators (hadronic observables)

- Invert Dirac operator matrix (rank 10^{12})
- Combine using color, spin and momentum into hadrons

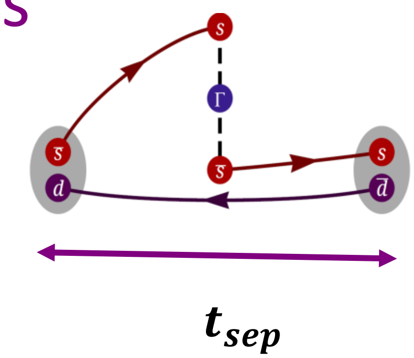


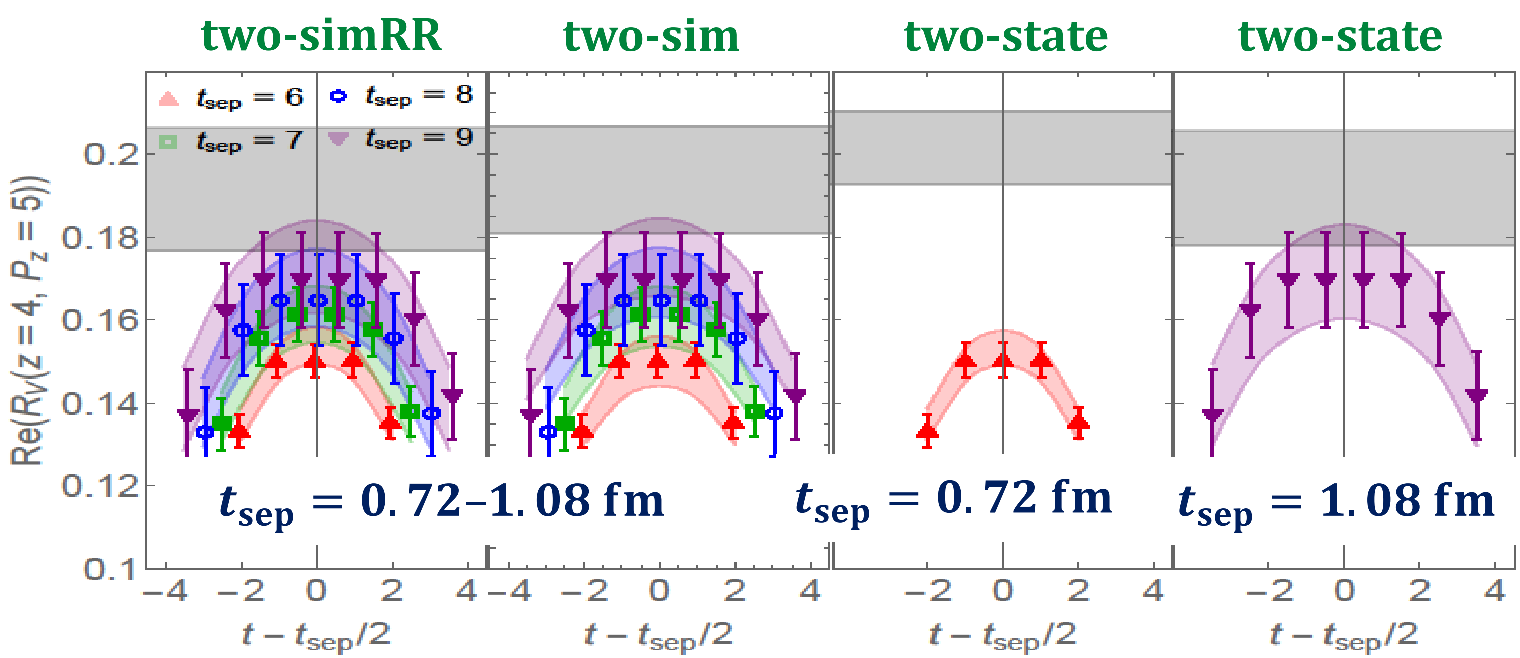
Thanks to MILC collaboration for sharing their 2+1+1 HISQ lattices

3. Extract reliable ground-state matrix elements

- Excited-state removal
  - For example, kaon matrix element at $M_\pi \approx 220$ MeV, $a \approx 0.12$ fm

H. Lin et al. (MSULat), 2003.14128

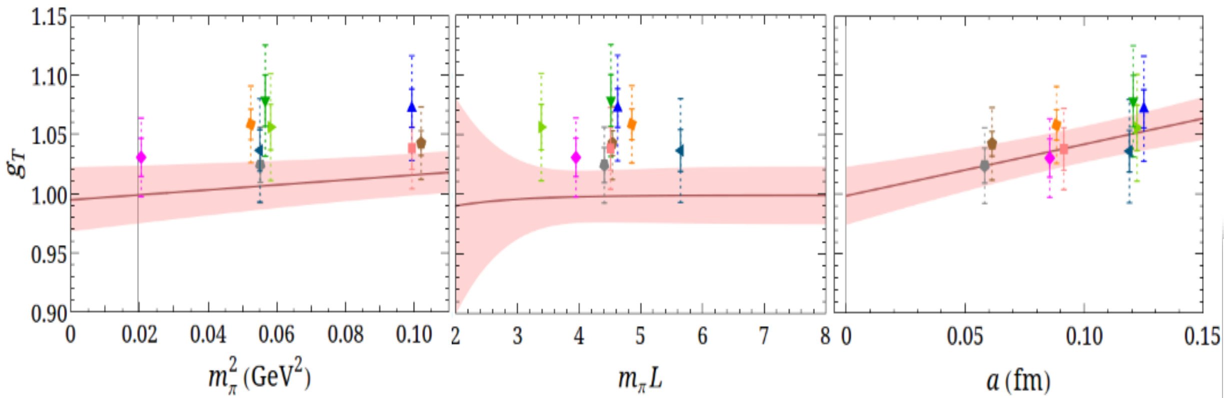


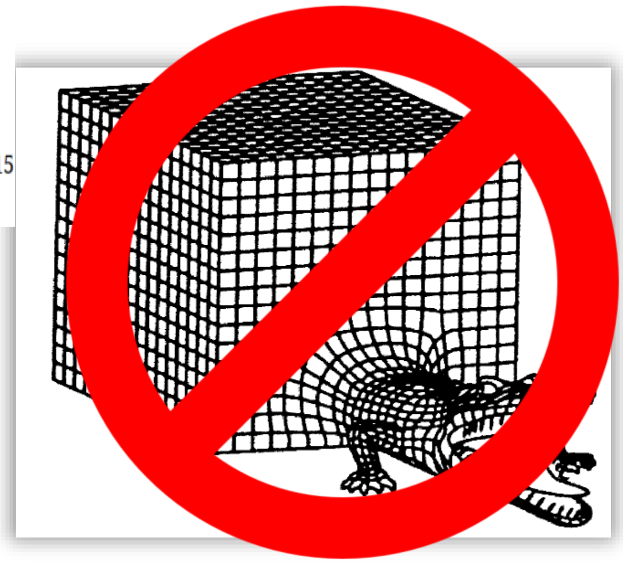


stability in extracting matrix elements

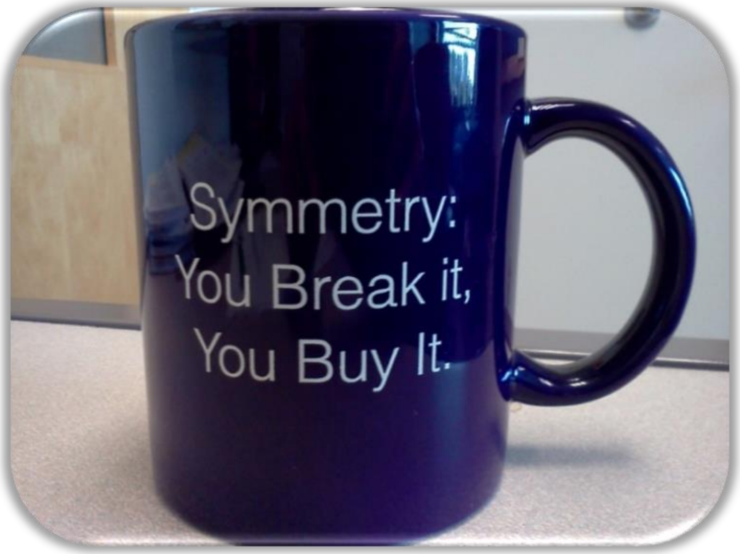
4. Systematic uncertainty (nonzero *a*, finite *L*, etc.)

- Nonperturbative renormalization, etc
- Extrapolation to the continuum limit
  $(m_\pi \to m_\pi^{\text{phys}}, L \to \infty, a \to 0)$





Lattice Structure Limitation



- Lattice calculations rely on operator product expansion, only provide moments

$$\langle x^{n-1}\rangle_q = \int_{-1}^{1} dx\, x^{n-1} q(x)$$

- Longstanding obstacle!
  - Holy grail of structure calculations
- Applies to many structure quantities:
  - Parton Distribution Functions (PDFs)
  - Generalized parton distributions (GPD)
  - Transverse-momentum distributions (TMD)
  - ……..

A NEW HOPE

It is a period of war and economic uncertainty.

Turmoil has engulfed the galactic republics.

Basic truths at foundation of the human civilization are disputed by the dark forces of the evil empire.

A small group of QCD Knights from United Federation of Physicists has gathered in a remote location on the third planet of a star called Sol on the inner edge of the Orion–Cygnus arm of the galaxy.

The QCD Knights are the only ones who can tame the power of the Strong Force, responsible for holding atomic nuclei together, for giving mass and shape to matter in the Universe.

They carry secret plans to build the most powerful

Direct x-Dependent Structure

§ Longstanding obstacle to lattice calculations!

Quantities that can be calculated on the lattice today = $\sum$ Wanted PDFs, GPDs, etc. × pQCD-calculated kernel

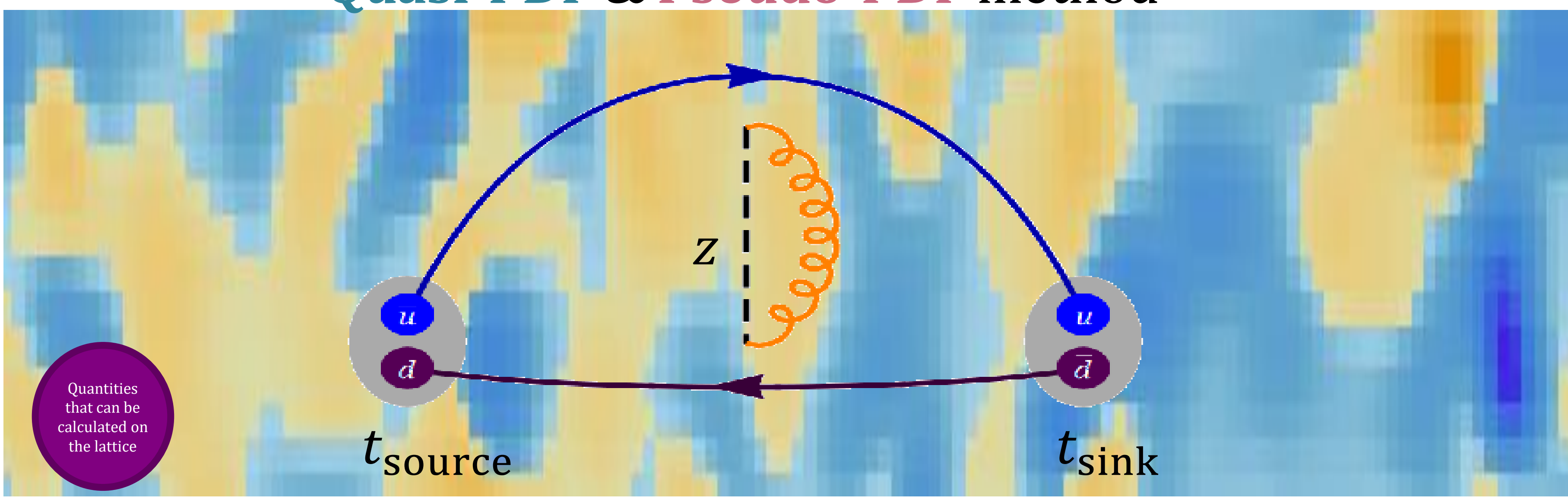
- **Quasi-PDF**/large-momentum effective theory (LaMET) (X. Ji, 2013; See 2004.03543 for review)
- **Pseudo-PDF** method: differs in FT (A. Radyushkin, 2017)
- Lattice cross-section method (**LCS**) (Y Ma and J. Qiu, 2014, 2017)
- Hadronic tensor currents (Liu et al., hep-ph/9806491, … 1603.07352)
- Euclidean correlation functions (RQCD, 1709.04325)
- …

Direct x-Dependent Structure

- Longstanding obstacle to lattice calculations!

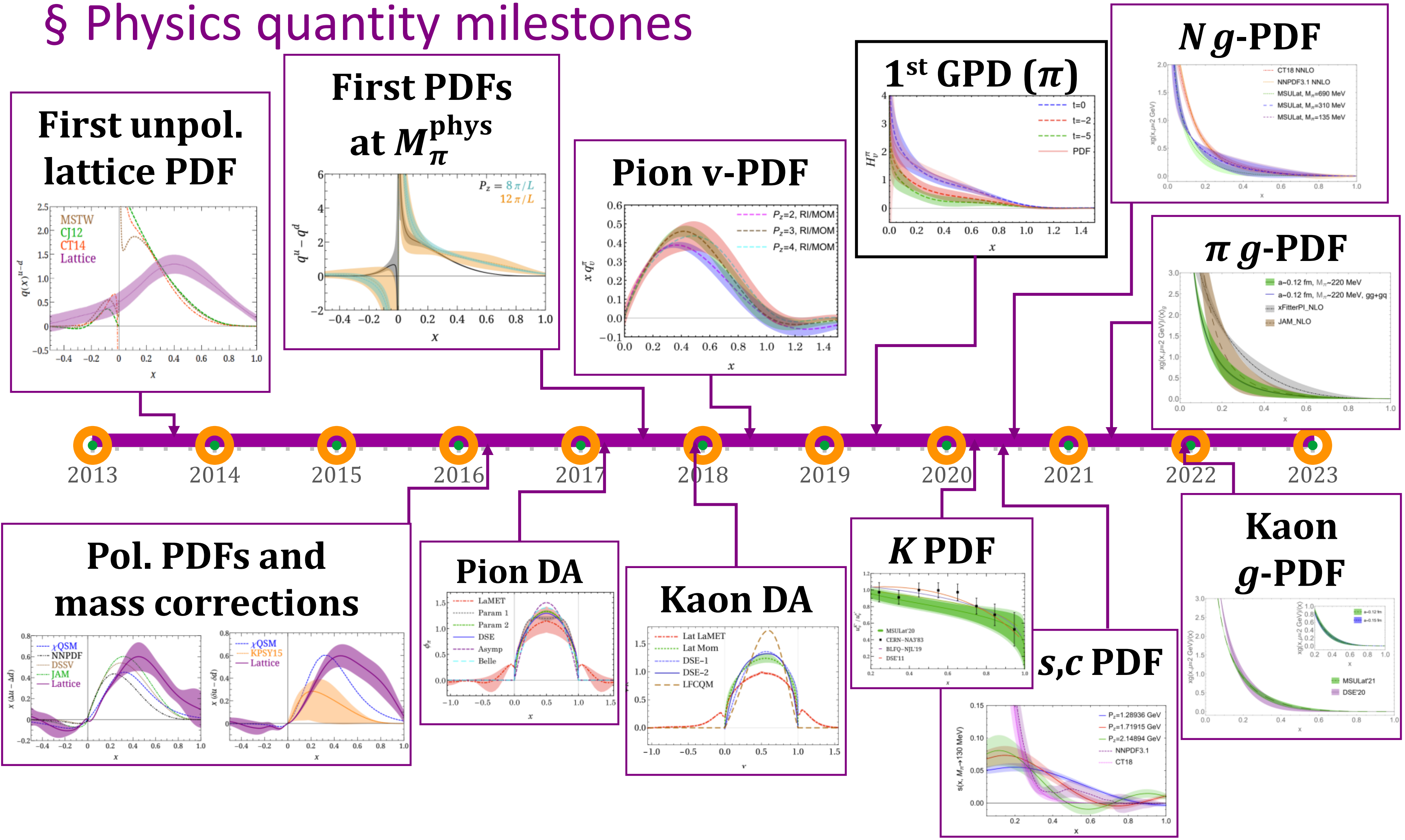
Quantities that can be calculated on the lattice today $= \sum$ Wanted PDFs, GPDs, etc. $\times$ pQCD-calculated kernel

Quasi-PDF & **Pseudo-PDF** method



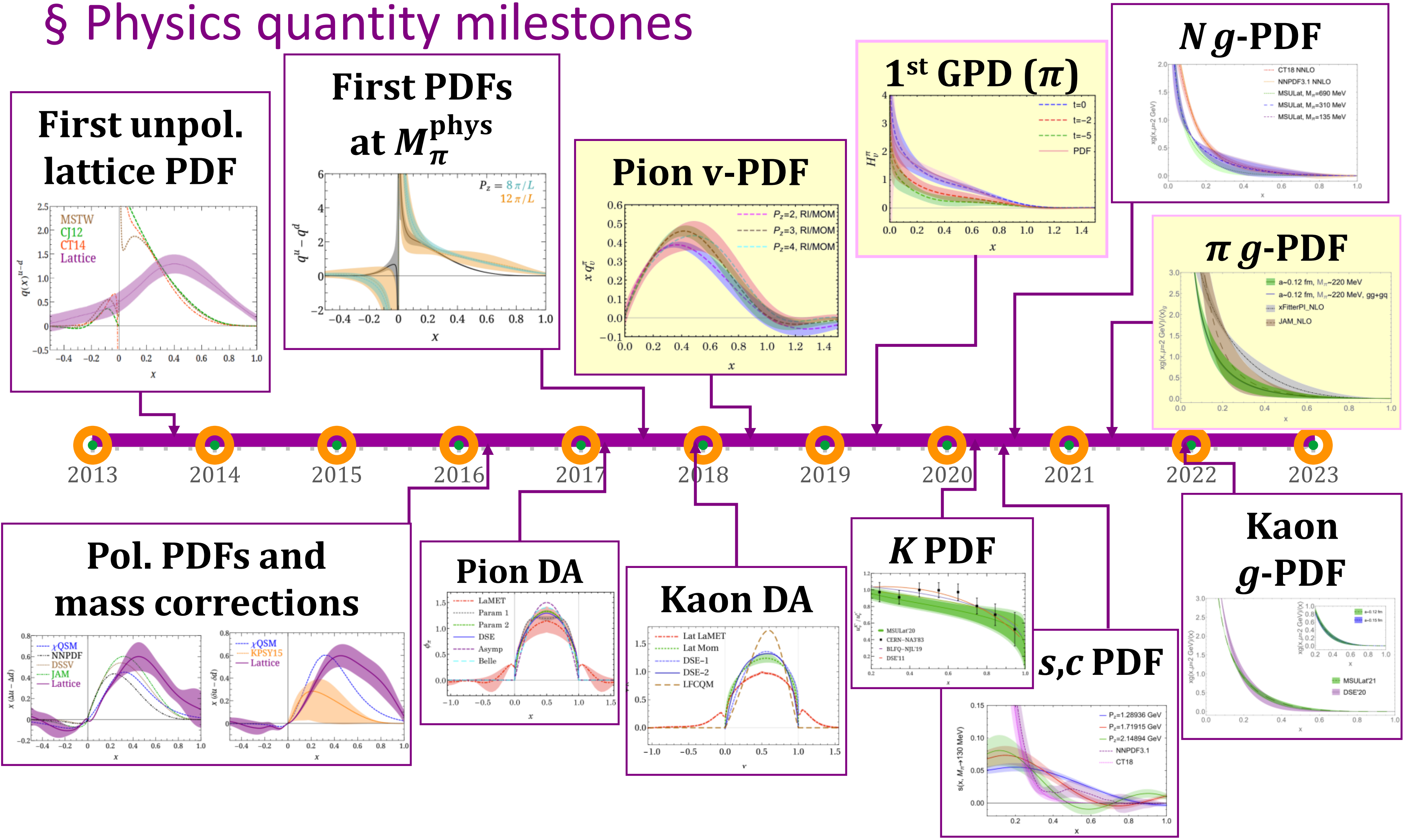
Lattice Parton Calculations

§ Physics quantity milestones



Lattice Parton Calculations

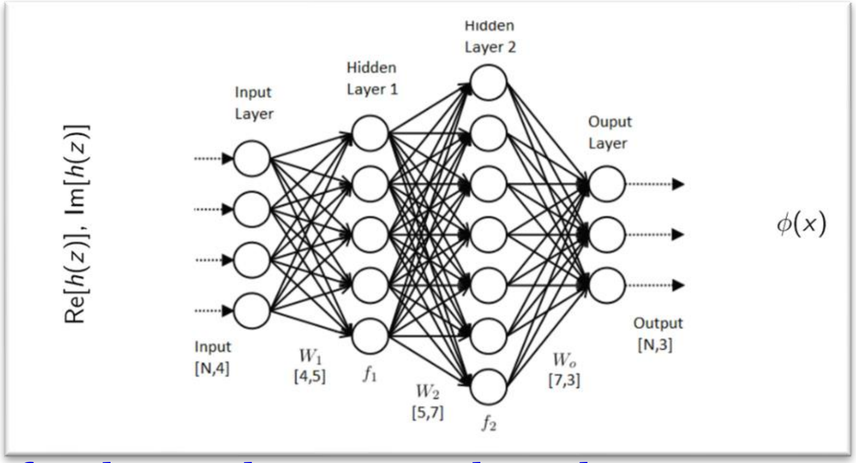
§ Physics quantity milestones



MSULat Pion/Kaon Structure

§ Meson distribution amplitude

- Pion Distribution Amplitude from Lattice,
  Phys. Rev. D 95 (2017) 9, 094514
- Kaon Distribution Amplitude from Lattice QCD and the Flavor SU(3) Symmetry, Nucl. Phys. B 939 (2019) 429-446
- Pion and kaon distribution amplitudes in the continuum limit,
  Phys. Rev. D 102 (2020) 9, 094519



- Precision control in lattice calculation of x-dependent pion distribution amplitude, Nucl. Phys. B 993 (2023) 116282

§ Miscellaneous

- Machine-learning prediction for quasiparton distribution function matrix elements, Phys. Rev. D 101 (2020) 3, 034516

MSULat Pion/Kaon Structure

§ Pion/kaon PDFs

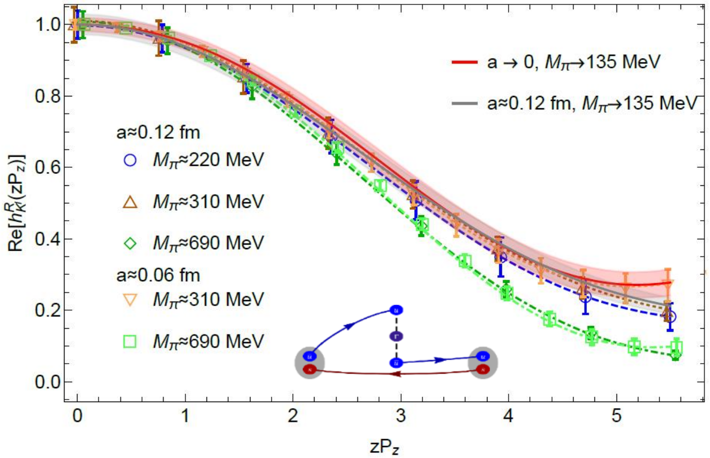
- First direct lattice-QCD calculation of the *x*-dependence of the pion parton distribution function, Phys. Rev. D 100 (2019) 3, 034505
- Valence-Quark Distribution of the Kaon and Pion from Lattice QCD, Phys. Rev. D 103 (2021) 1, 014516
- Gluon parton distribution of the pion from lattice QCD, Phys. Lett. B 823 (2021) 136778
- First glimpse into the kaon gluon parton distribution using lattice QCD, Phys. Rev. D 106 (2022) 9, 094510
- The Gluon Moment and Parton Distribution Function of the Pion from N_f=2+1+1 Lattice QCD, 2310.12034 [hep-lat]
- Pion valence quark distribution at physical pion mass of N_f=2+1+1 LQCD

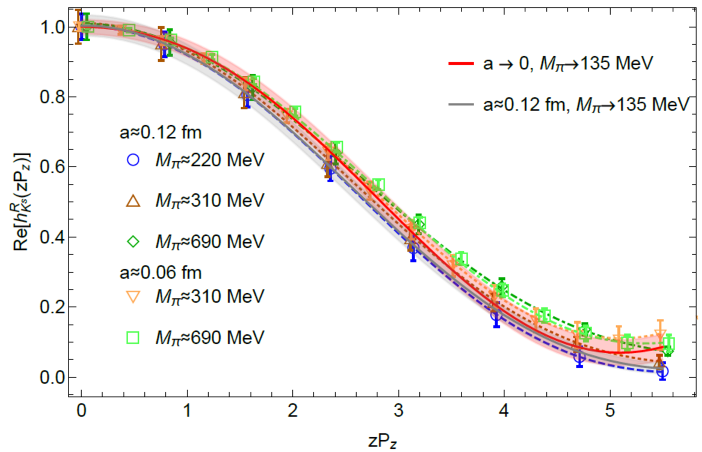
§ Pion GPD

- Pion generalized parton distribution from lattice QCD, Nucl. Phys. B 952 (2020) 114940
- Pion valence-quark generalized parton distribution at physical pion mass , Phys. Lett. B 846 (2023) 138181

Meson Valence-quark PDFs

§ Pion/kaon PDFs using quasi-PDF in the continuum limit

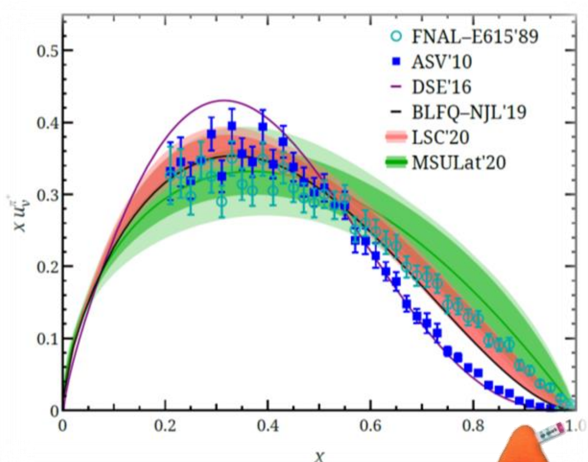


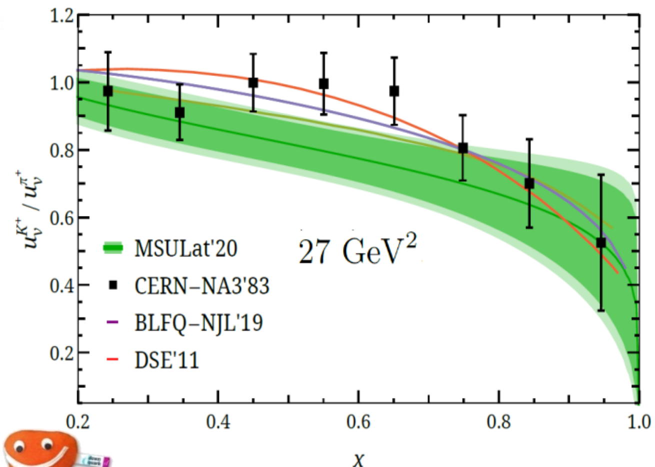


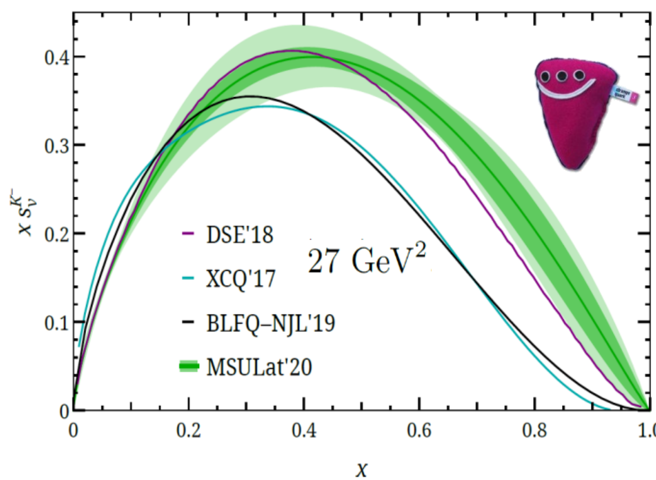
Quantities that can be calculated on the lattice

Wanted PDFs, GPDs, etc…

MSULat, 2003.14128



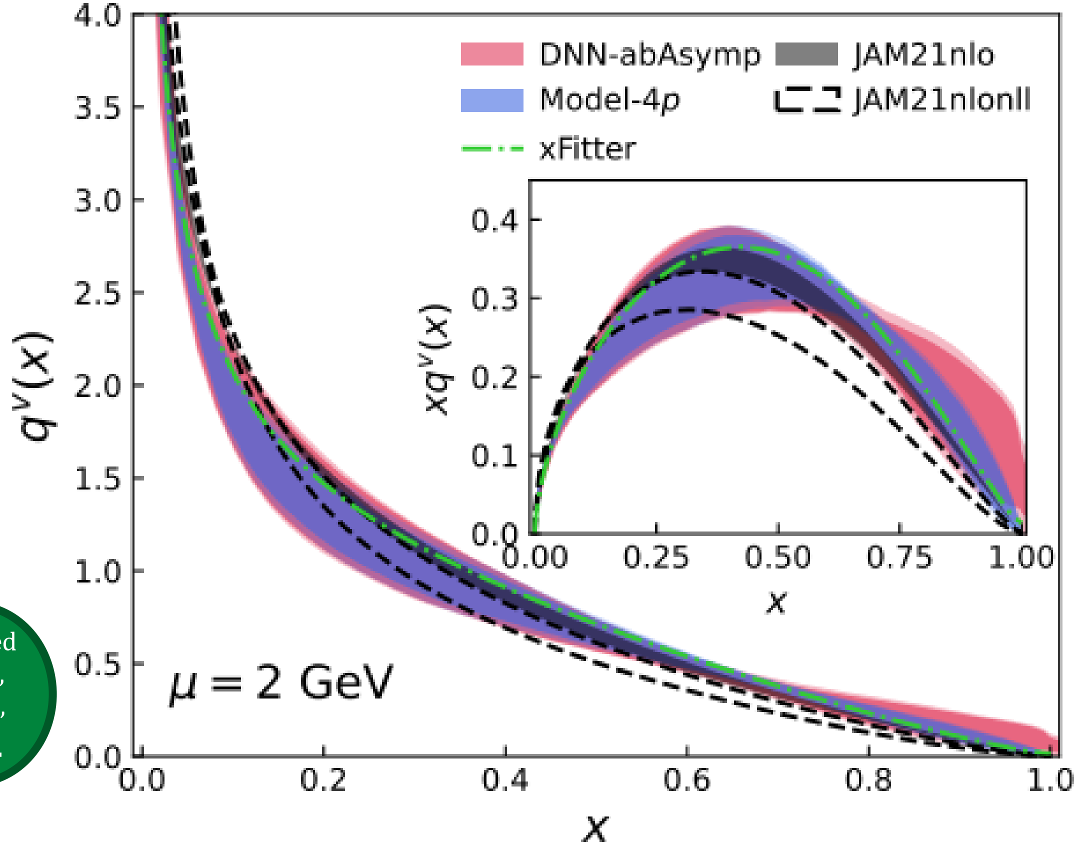




Valence-quark PDFs Update

§ Pion PDFs calculated directly at physical pion mass

- with NNLO matching



- N_f=2+1 clover/HISQ
  a~0.076 fm

Wanted PDFs, GPDs, etc…

ANL/BNL, Phys. Rev. D 106, 114510 (2022)

Valence-quark PDFs Update

§ Pion PDFs calculated directly at physical pion mass

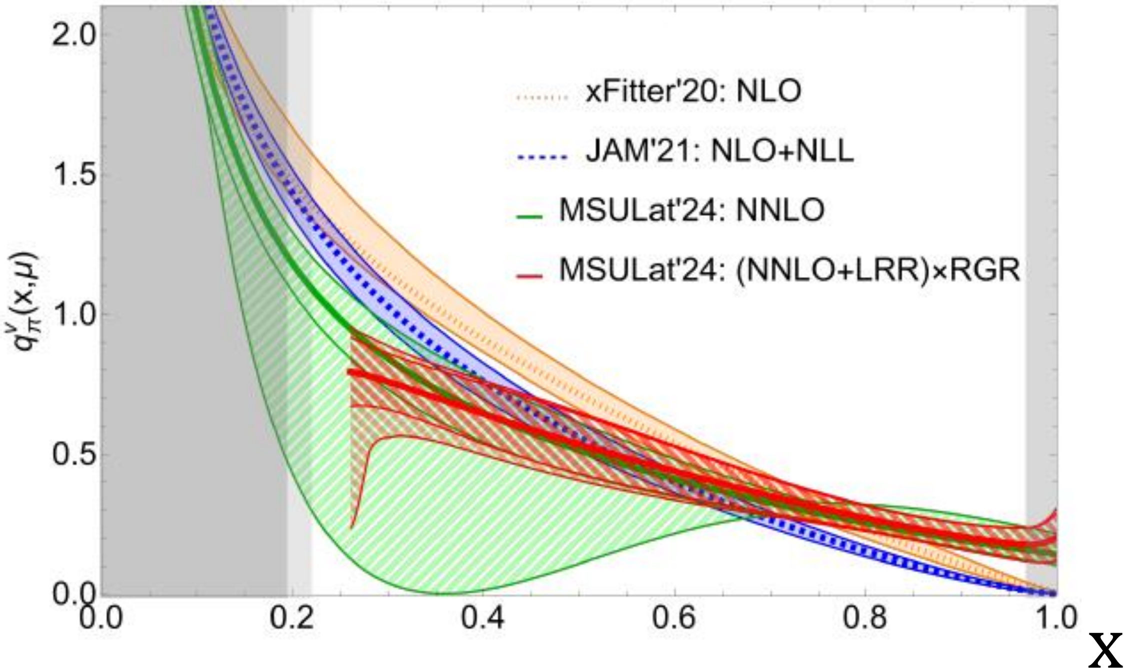
- NNLO matching & treat leading-renormalon effects
  - Leading-renormalon resummation (LRR)
  - Renormalization-group resummation (RGR)

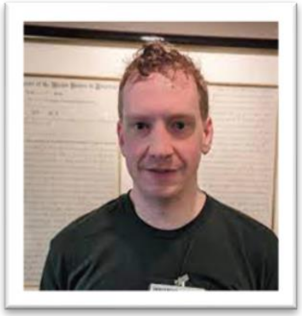
R. Zhang, et. al.
PLB 844, 138081 (2023)

- N_f=2+1+1 clover/HISQ, a~0.09 fm

J. Holligan, HL (MSULat), 10.1088/1361-6471/ad3162

Wanted PDFs, GPDs, etc…

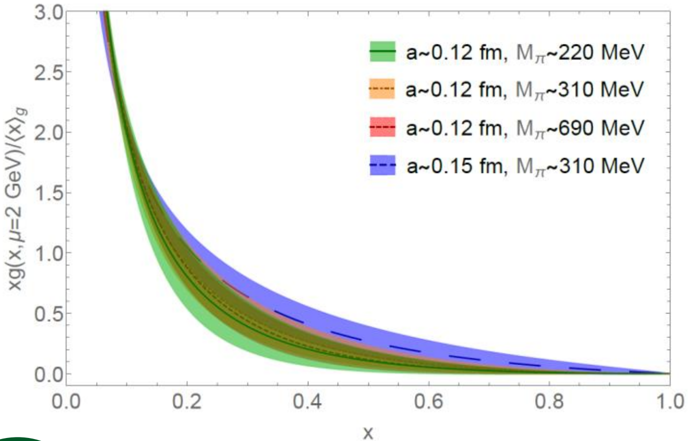


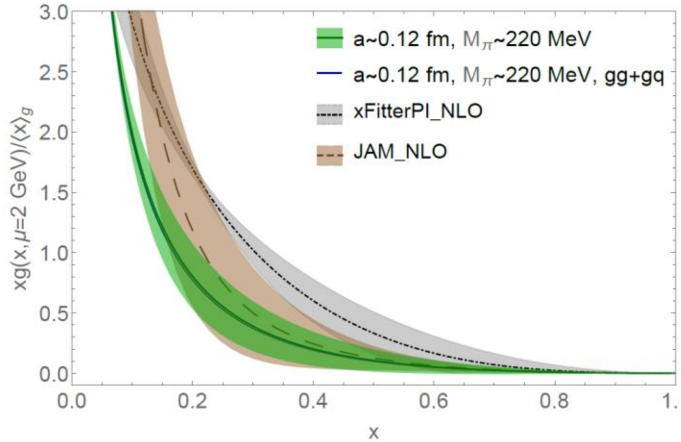


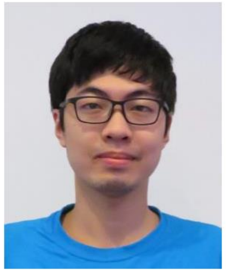
P: Jack Holligan

Meson Gluon PDFs

§ First pion and kaon gluon PDFs $g(x)/\langle x\rangle$ using pseudo-PDF



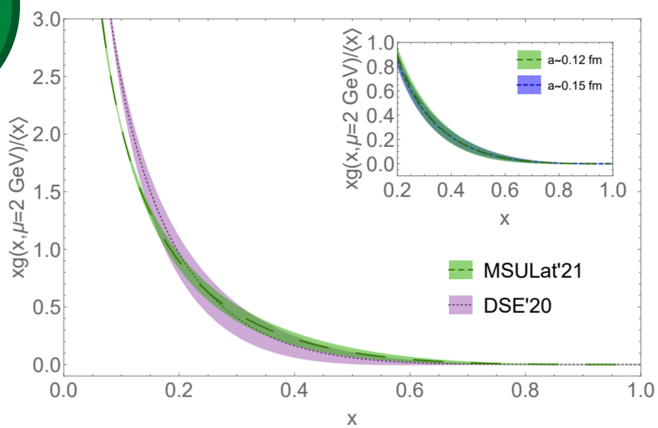




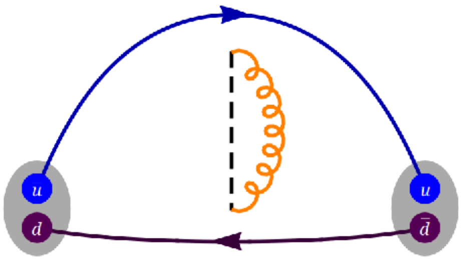
G: Zhouyou Fan

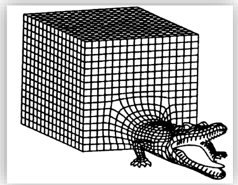
Wanted PDFs, GPDs, etc…

2104.06372, Fan et al. (MSULat); 2112.03124, Salas-Chavira et al. (MSULat)



G: Alejandro Salas-Chavira





finite-volume, discretization, heavy quark mass, …

§ What does lattice QCD say about $g(x)$?

Pion Gluon PDF Update

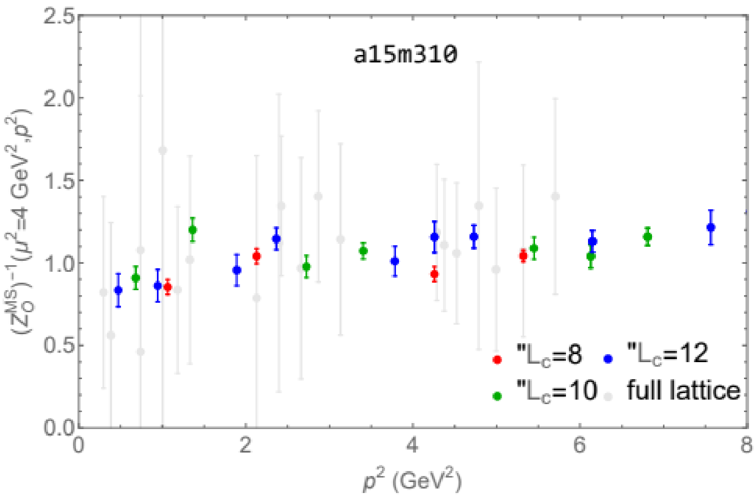
§ Nonperturbatively renormalized $\langle x \rangle_{\{\pi,g\}}$ at the finer lattice spacing at lighter pion mass is nontrivial

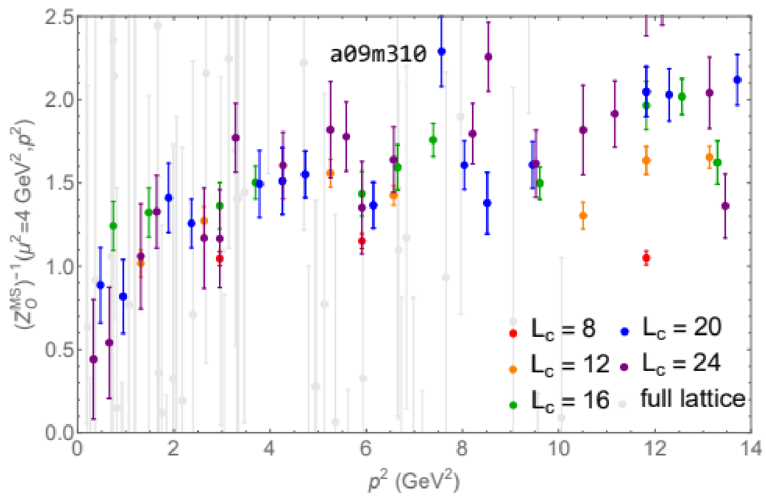
- Using cluster-decomposition error reduction (CDER) to enhance the signal-to-noise ratio

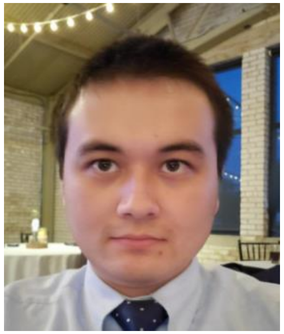
  1805.00531, Y. Yang et al. (χQCD)

- Lattice details: clover/HISQ, a ~{0,15, 0.12, 0.09} fm

  2208.00980, Fan et al. (MSULat)

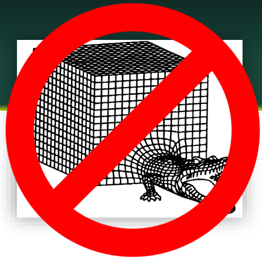




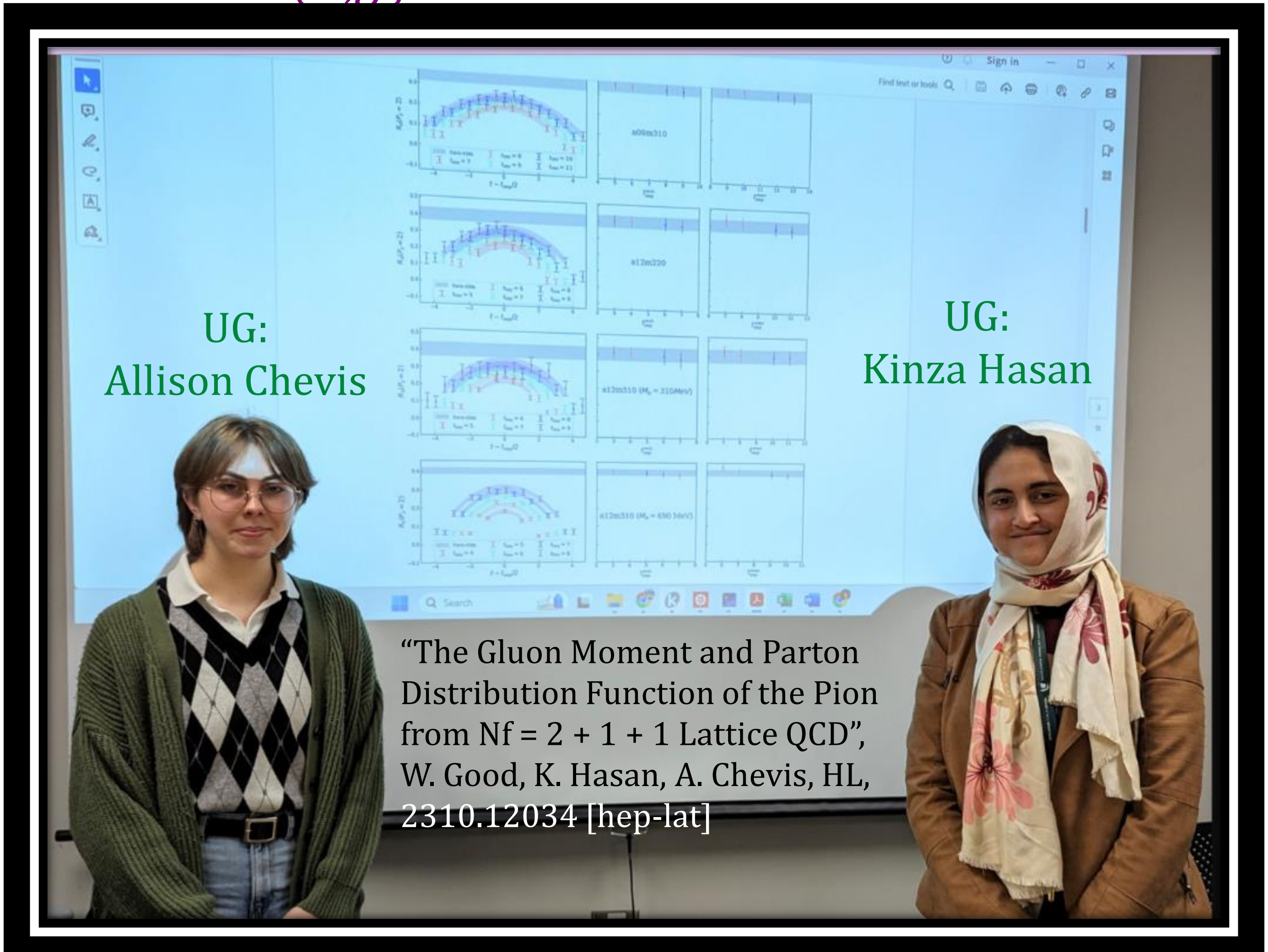


G: Matthew Zeilbeck

Pion Gluon PDF Update

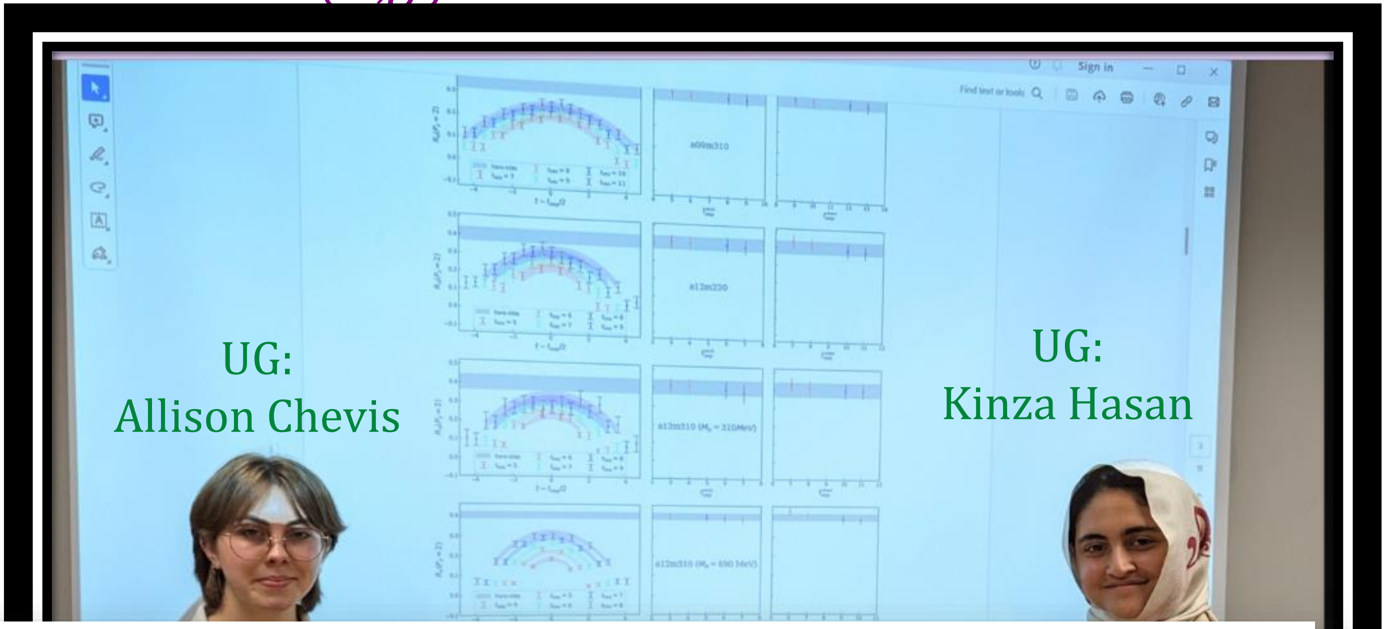


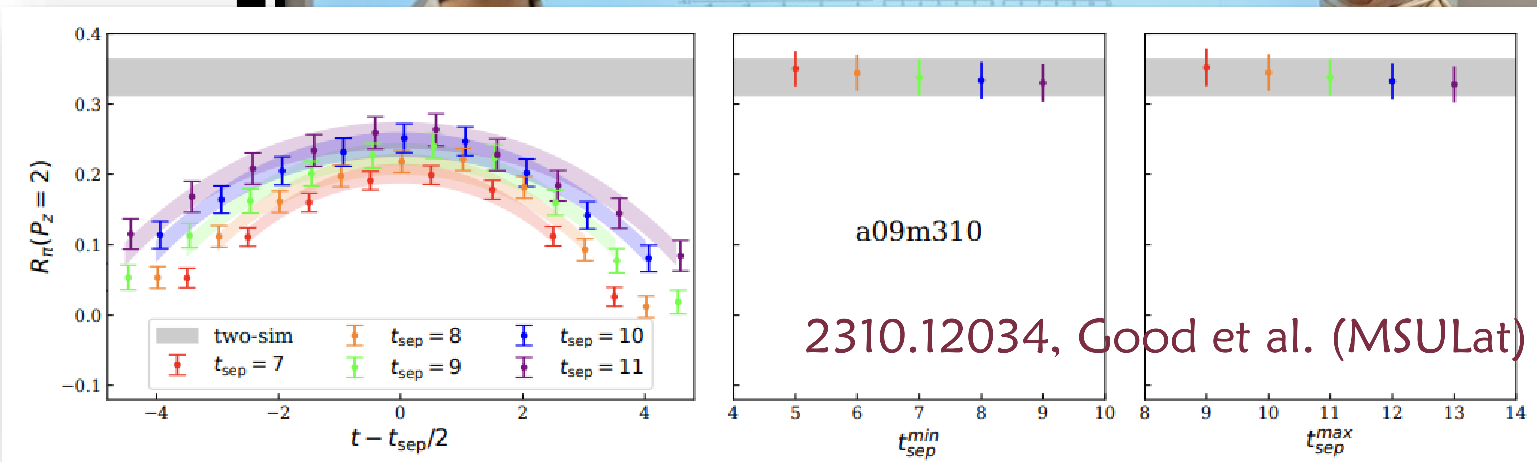
§ Study Bare $\langle x \rangle_{\{\pi,g\}}$



Pion Gluon PDF Update

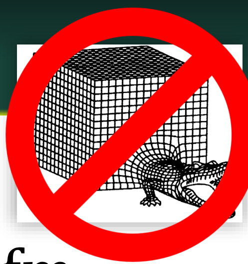
§ Study Bare $\langle x \rangle_{\{\pi,g\}}$





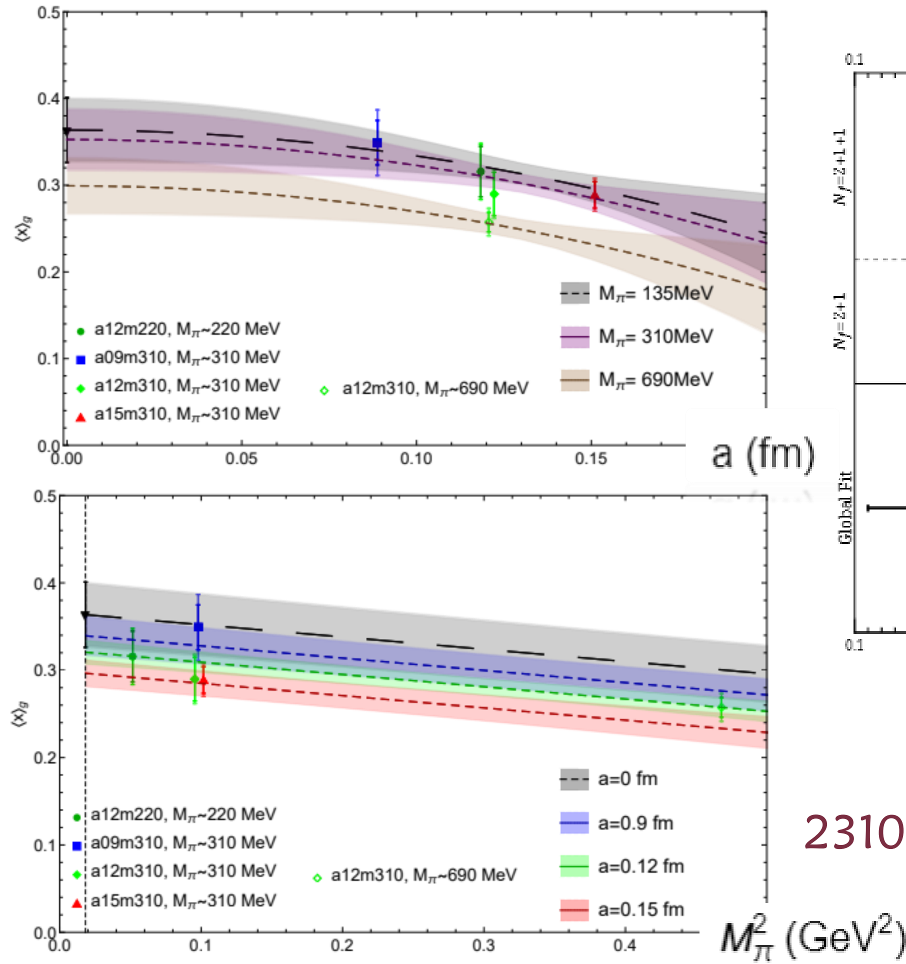
2310.12034, Good et al. (MSULat)

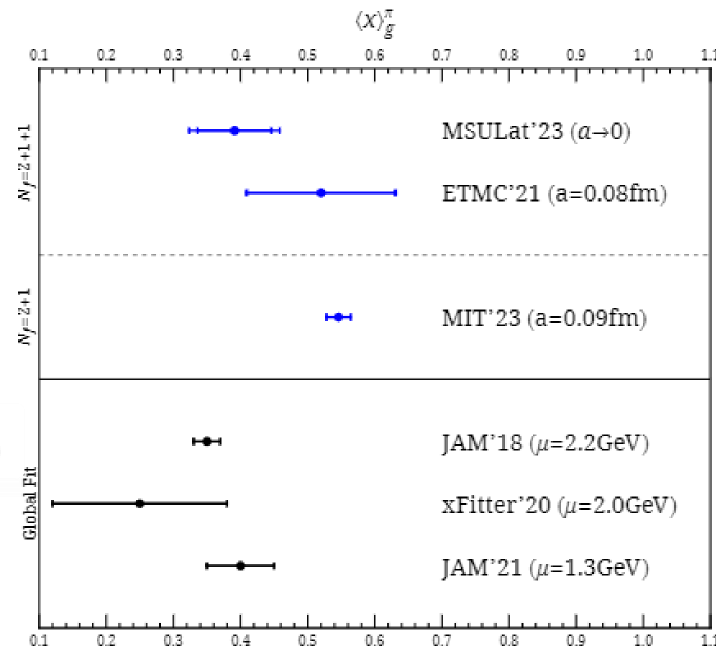
Pion Gluon PDF Update



§ Study discretization systematic in $\langle x \rangle_{\{\pi,g\}}$

- Lattice details: clover/HISQ, HISQ, a ~{0.15, 0.12, 0.09} fm





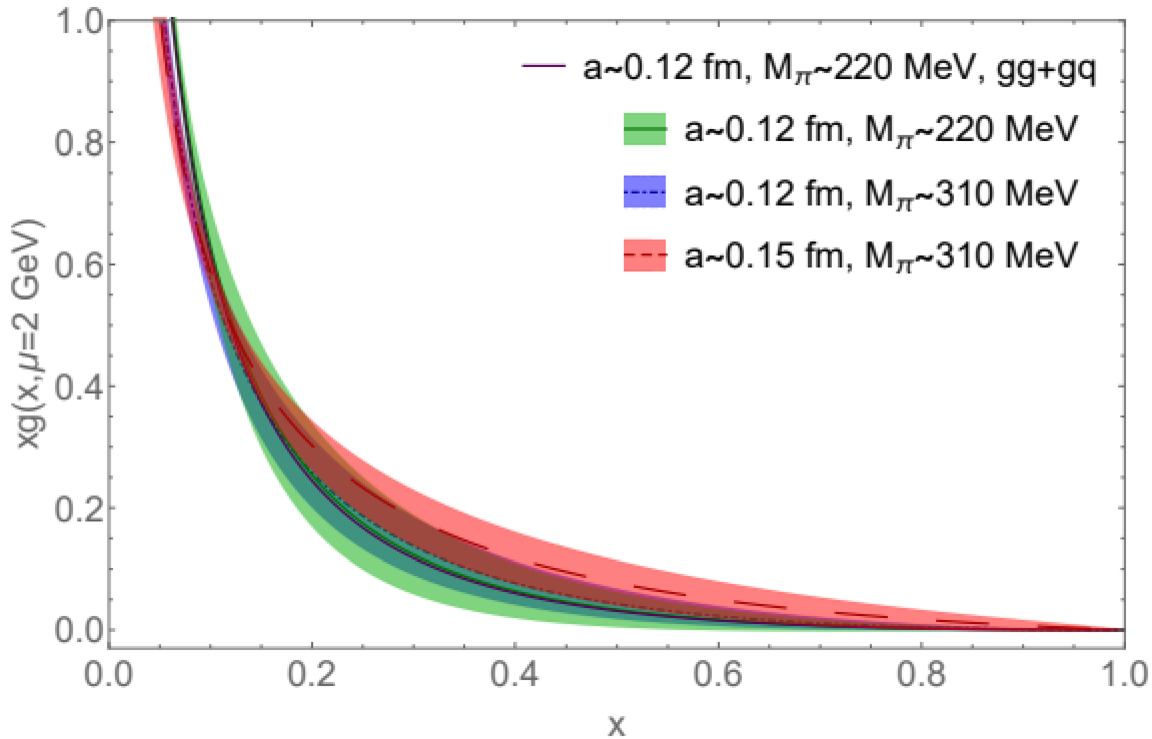
2310.12034, Good et al. (MSULat)

G: Bill Good

Pion Gluon PDF Update

§ Back to Pion gluon PDF $g(x)$

- Update previous calculated $g(x)/\langle x\rangle$ in 2021



2310.12034, Good et al. (MSULat)

G: Bill Good

Pion Gluon PDF Update

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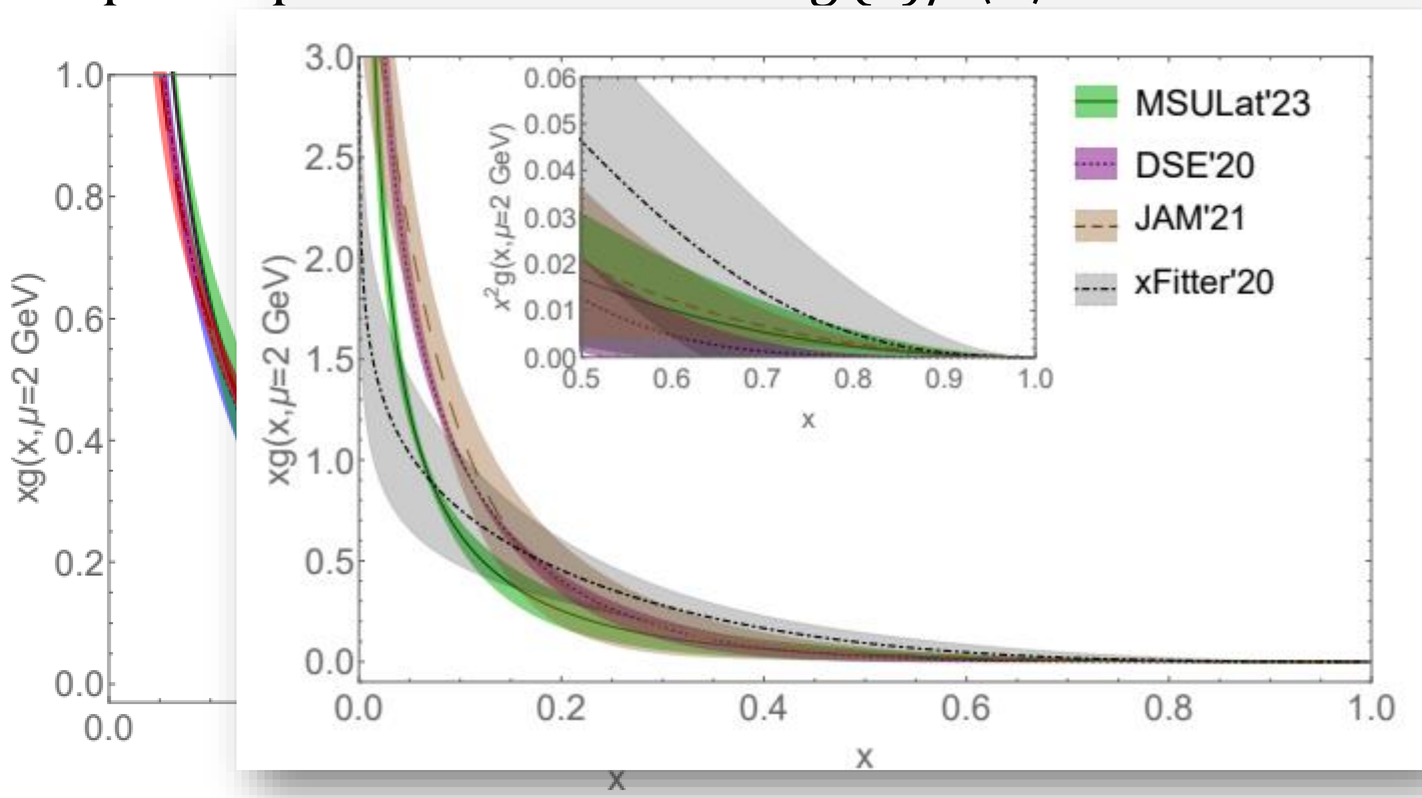


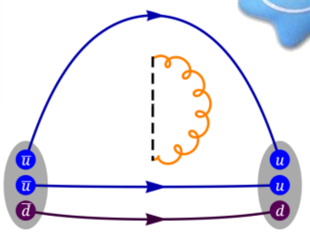
2310.12034, Good et al. (MSULat)

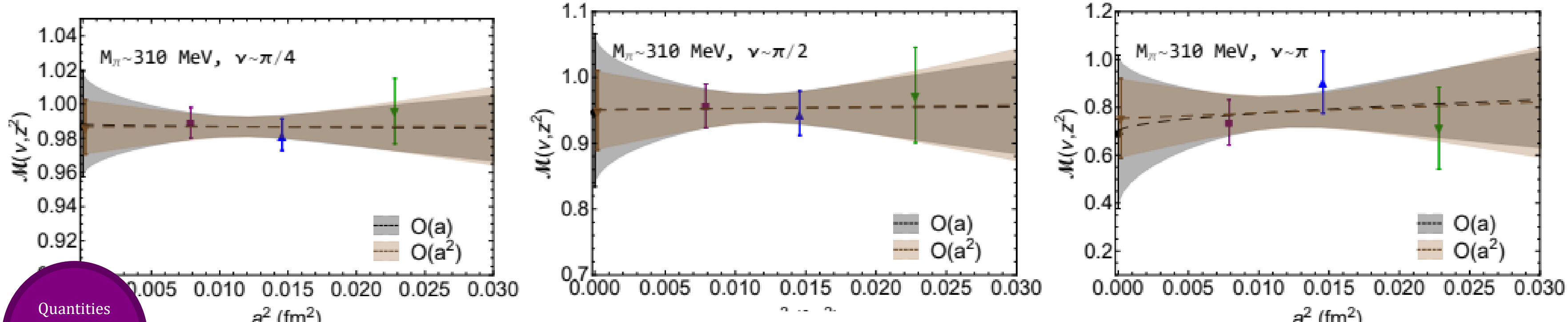
G: Bill Good

Gluon PDF in Nucleon

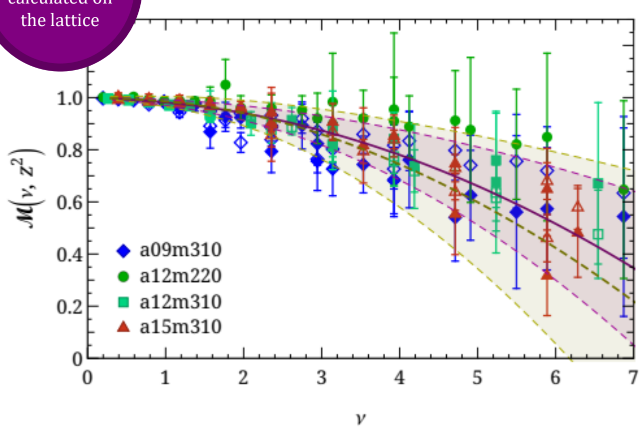
§ Continuum Gluon PDF w/ pseudo-PDF

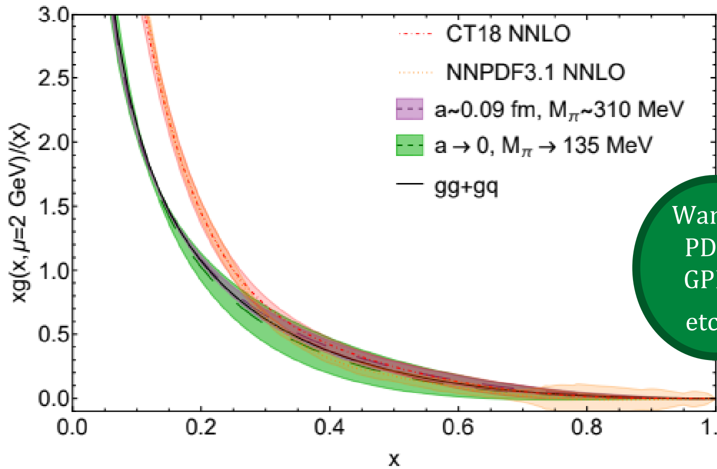
- 2+1+1 HISQ {0.09, 0.12, 0.15} fm, [220,310,700]-MeV pion, 10^5-10^6 statistics
  [arXiv:2210.09985](https://arxiv.org/abs/2210.09985)





Quantities that can be calculated on the lattice



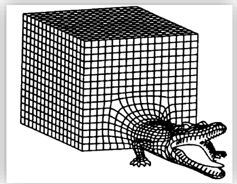


Wanted PDFs, GPDs, etc…

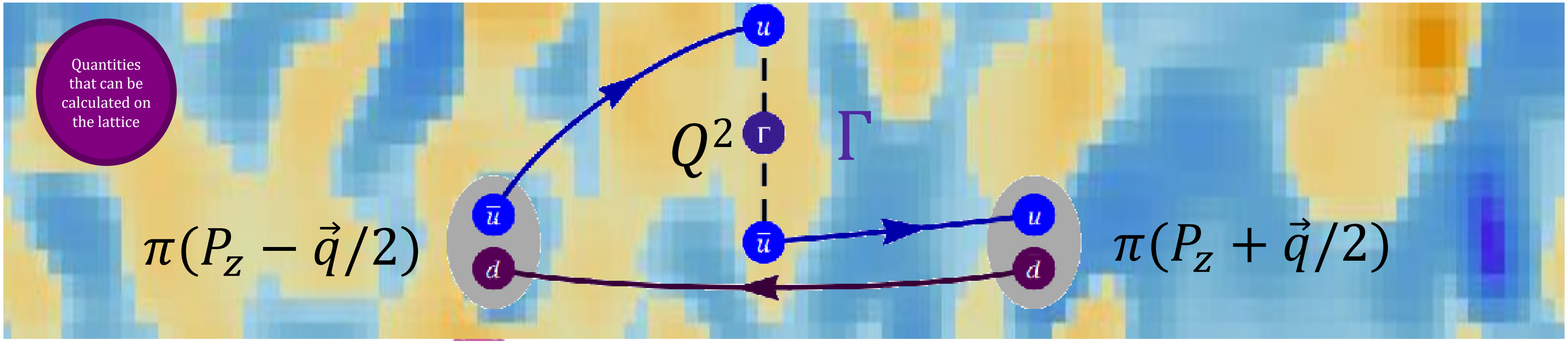
G: Bill Good

Generalized Parton Distributions

Single-ensemble result



finite-volume,
discretization,
heavy quark mass,
...



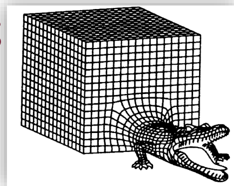
First Lattice GPDs

§ First glimpse into pion GPD using **Quasi-PDF**/LaMET

- Lattice details: clover/HISQ, **0.12fm**, **310-MeV** pion mass

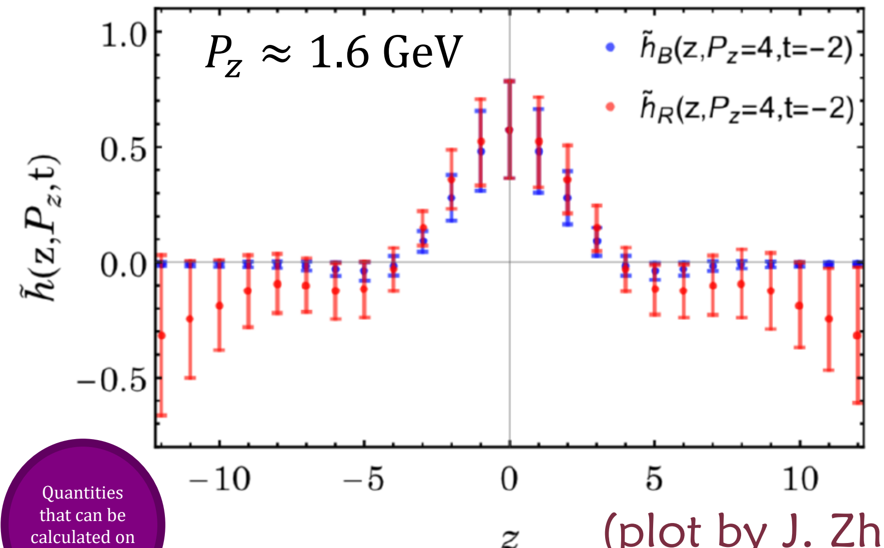
$P_z \approx 1.3, 1.6$ GeV

MILC, Phys. Rev. D, 82 (2010), 074501;
Phys. Rev. D, 87 (2013), 0545056



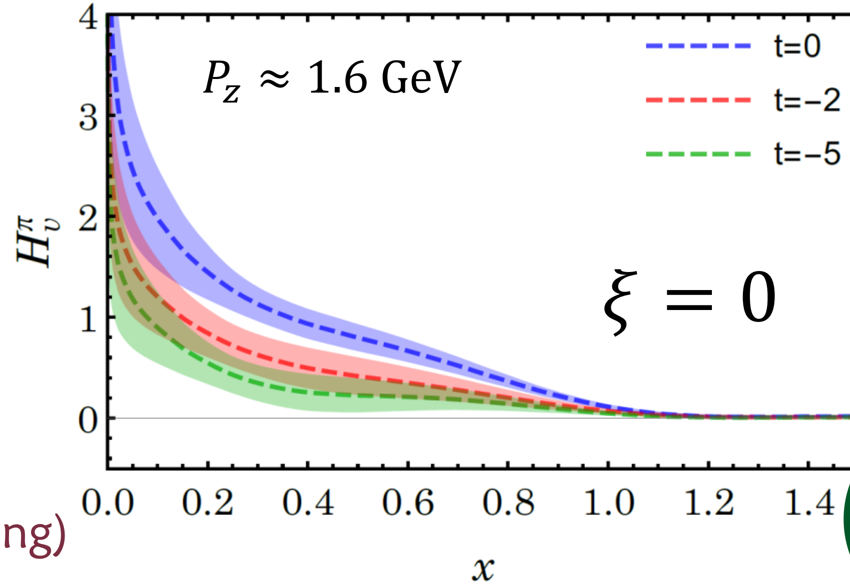
J. Chen, HL, J. Zhang, 1904.1237;

$$H_q^\pi(x,\xi,t,\mu) = \int \frac{d\eta^-}{4\pi} e^{-ix\eta^- P^+} \left\langle \pi(P+\Delta/2) \left| \bar{q}\left(\frac{\eta^-}{2}\right) \gamma^+ \Gamma\left(\frac{\eta^-}{2}, -\frac{\eta^-}{2}\right) q\left(-\frac{\eta^-}{2}\right) \right| \pi(P-\Delta/2) \right\rangle$$



Quantities that can be calculated on the lattice

(plot by J. Zhang)

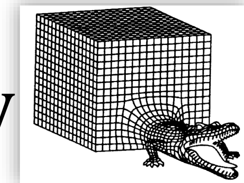


Wanted PDFs, GPDs, etc…

Valence-Quark Pion GPD

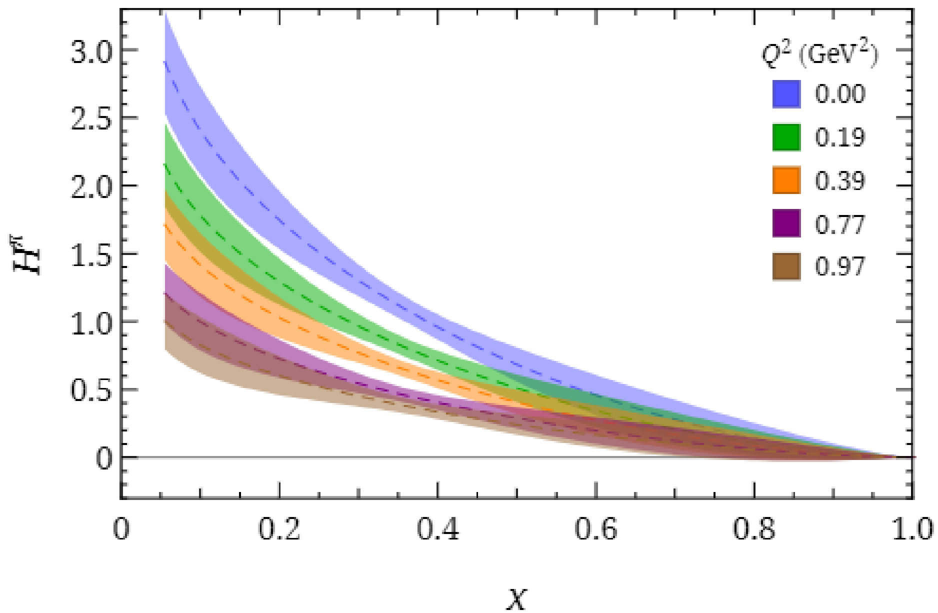
§ Pion GPD (H^π) using quasi-PDFs at physical pion mass

- Lattice details: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 1.7$ GeV
- $\xi = 0$ valence-quark Pion GPD results

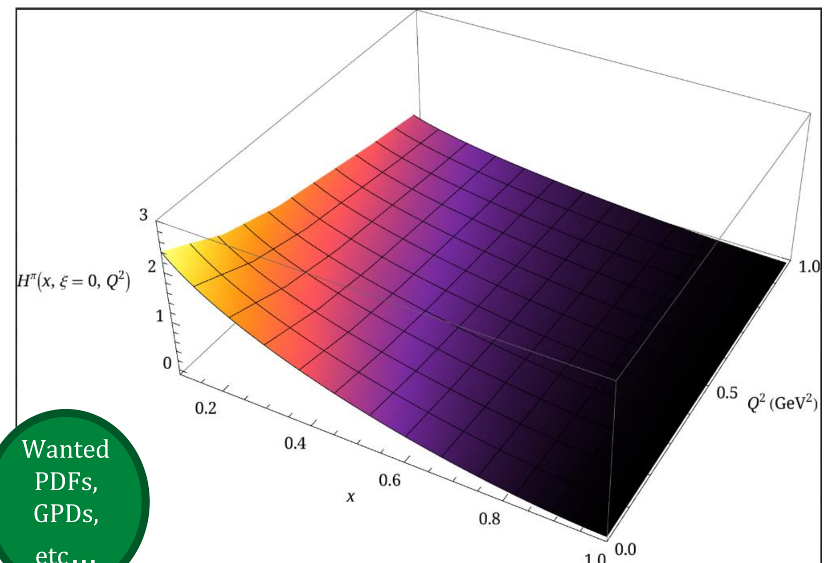


finite-volume,
discretization,

HL (MSULat), Phys. Lett. B 846 (2023) 138181



Wanted PDFs, GPDs, etc…



Valence-Quark Pion GPD

§ Pion GPD (H^π) using quasi-PDFs at physical pion mass

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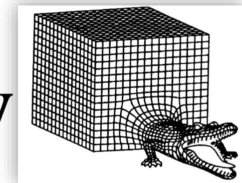
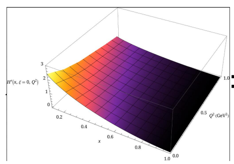
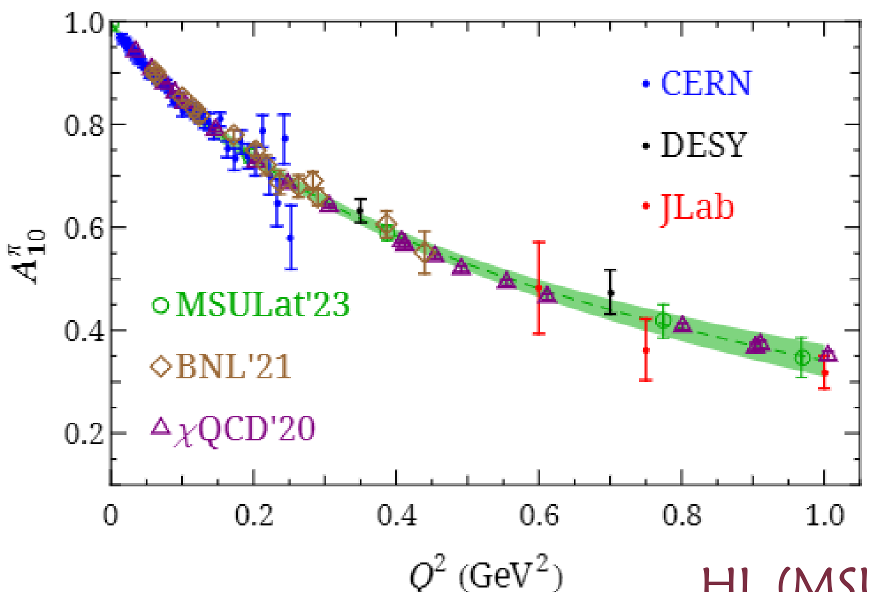
  0.09 fm, 135-MeV pion mass, $P_z \approx 1.7$ GeV



finite-volume,
discretization,

- $\xi = 0$ valence-quark Pion GPD results

$\int_{-1}^{+1} dx\, x^{n-1}$  $= A^{\pi}_{ni}(t)$

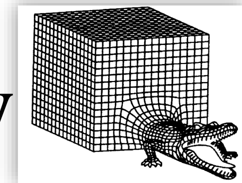


HL (MSULat), Phys. Lett. B 846 (2023) 138181

Pion Tomography

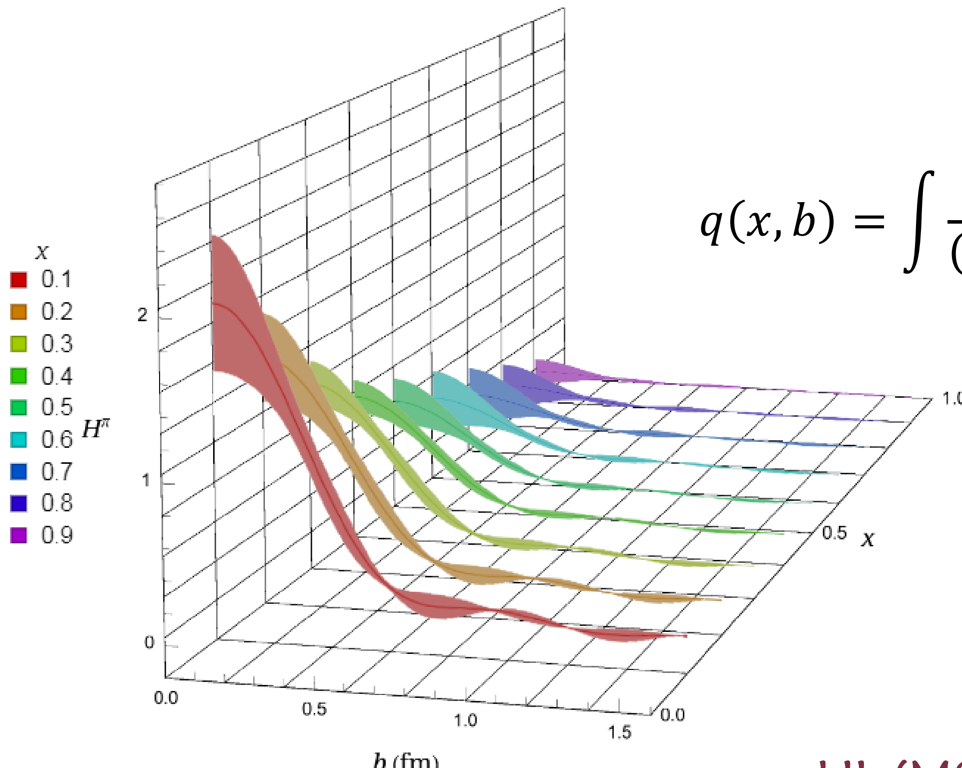
§ Nucleon GPD using quasi-PDFs at physical pion mass

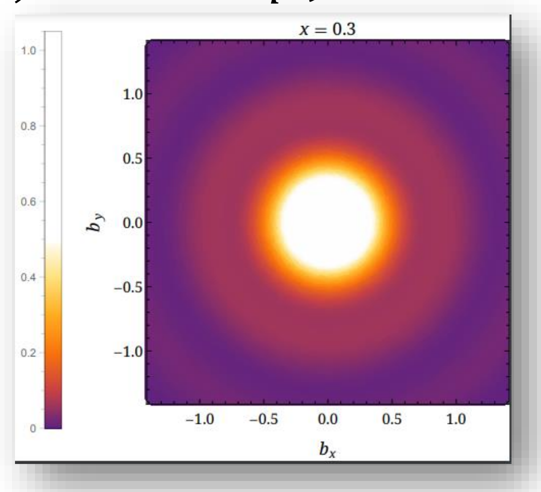
- Lattice details: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 1.7$ GeV
- $\xi = 0$ valence-quark Pion GPD results



finite-volume,
discretization,

$$q(x,b) = \int \frac{d\vec{q}}{(2\pi)^2} H(x, \xi = 0, t = -\vec{q}^2) e^{i\vec{q}\cdot\vec{b}}$$

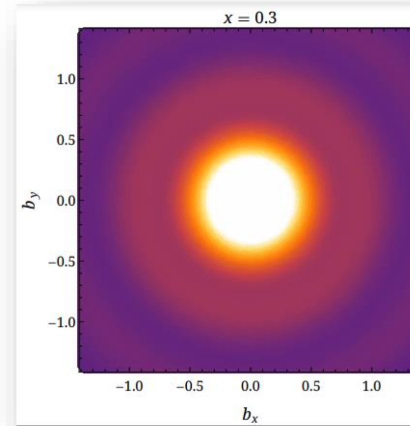




HL (MSULat), Phys. Lett. B 846 (2023) 138181

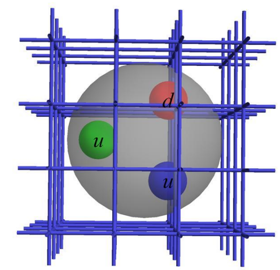
Summary and Outlook

- § Exciting era using LQCD to study *x*-dependent pion/kaon structure
- § Overcoming longstanding limitations
  - Nonperturbative renormalization for gluon operators (not limited to small volume nor too-heavy pion mass)
  - Bjorken-*x* dependence of parton distributions now widely studied
  - More study of systematics planned for the near future



- § Precision and progress are limited by resources
- § In the future

Theory Input → **Global Analysis of PDFs/GPDs** ← **Exp't Input** +



EXCLAIM

Thanks to MILC collaboration for sharing their 2+1+1 HISQ lattices & USQCD/NSF/DOE for computational resources
This work is partially sponsored by grants NSF PHY 1653405 & 1653405, DOE DE-SC0024053 & RCSA Cottrell Scholar Award

Students Wanted

LGT4HEP website: https://lgt4hep.github.io/

Backup Slides

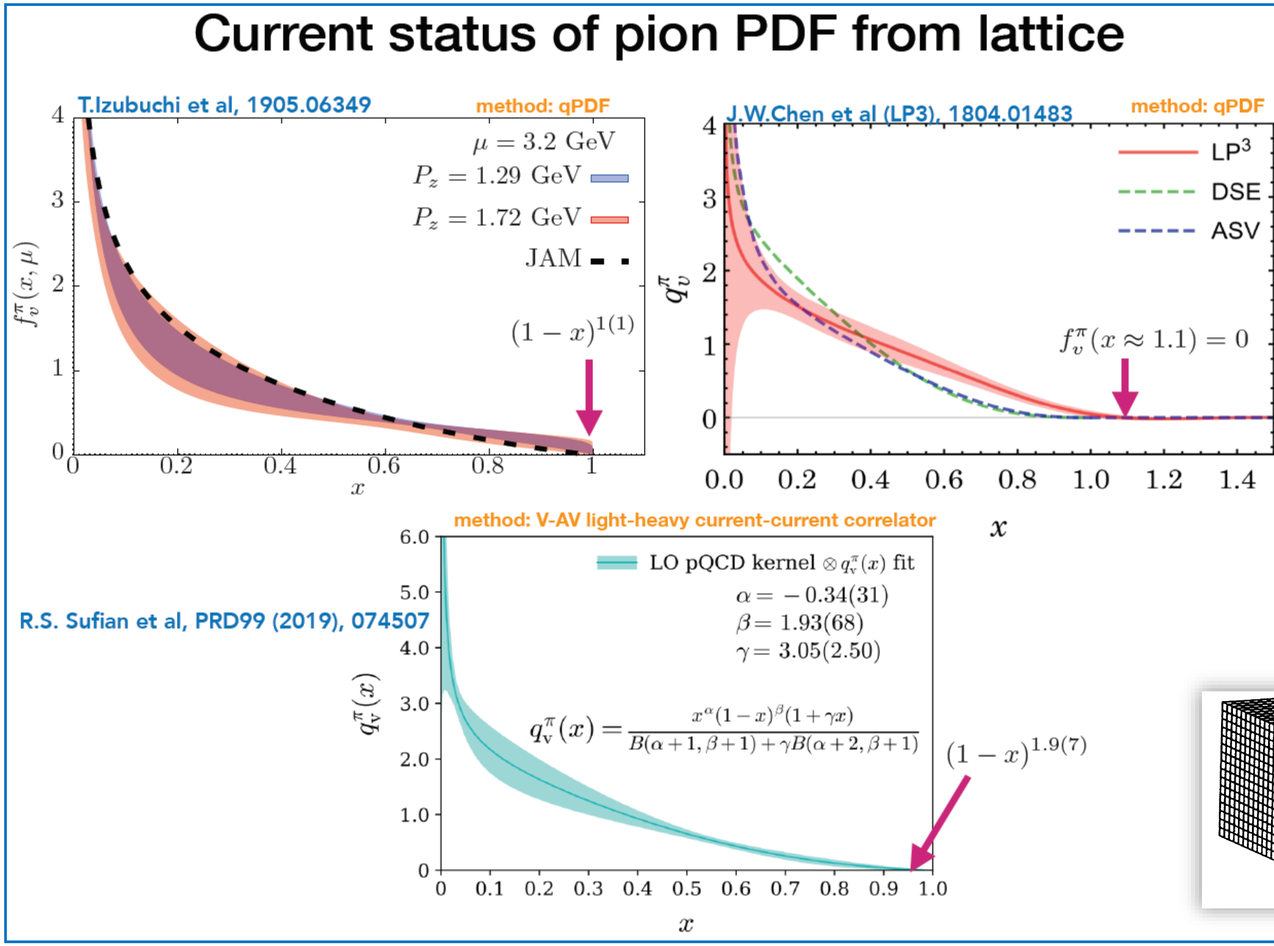
Pion Valence-Quark PDF

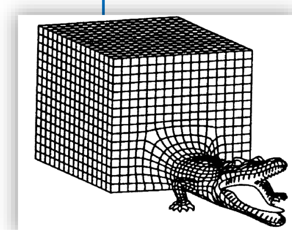
§ Status as of Summer 2019

Slide by Nikhil Karthik @ Lattice 2019

$M_\pi \approx 310$ MeV

$M_\pi \approx 426$ MeV





§ Single-ensemble calculation

- Non-physical pion mass, single lattice spacing, single volume

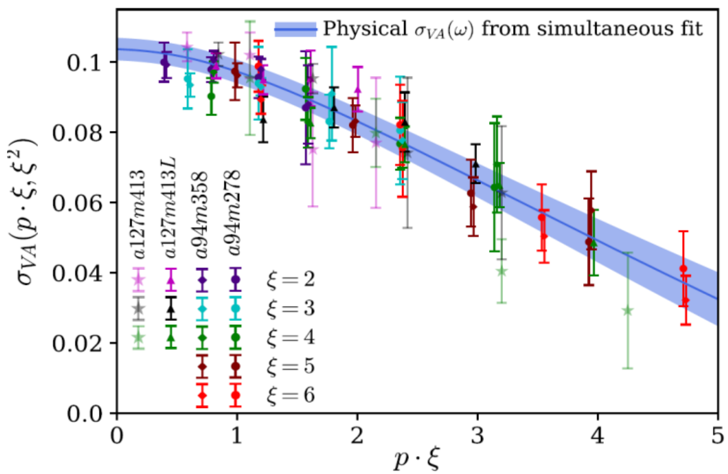
Pion Valence-Quark PDF

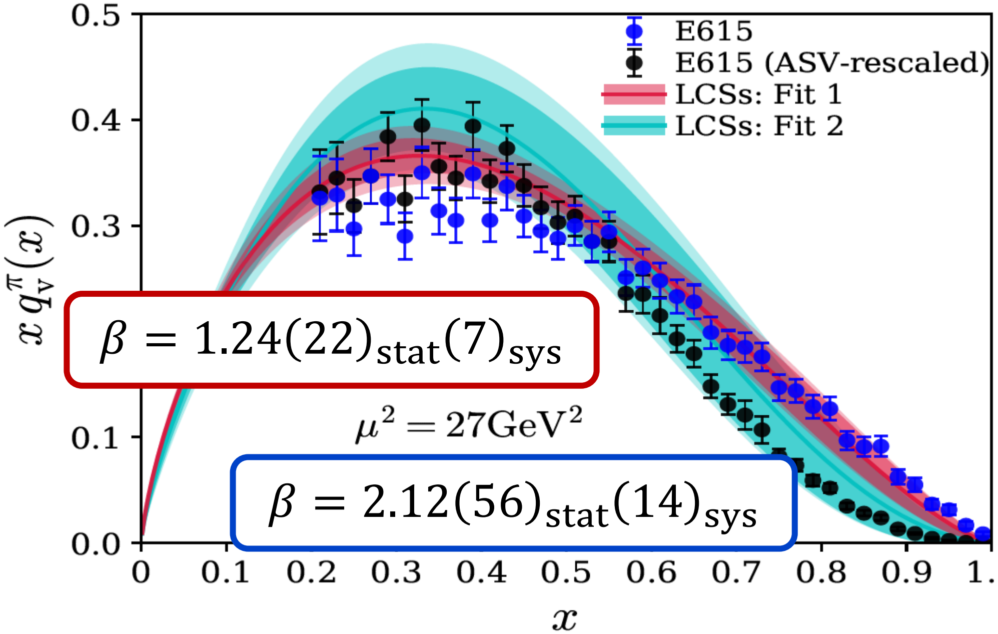
§ Results from JLab-W&M/ **LCS** method

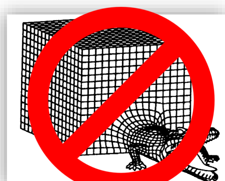
- $M_\pi = \mathbf{278}, 358, 413$ MeV with $a = 0.094, 0.127$ fm
- Extrapolated to physical limit (shown as blue band)
- Renormalized $Z_{V,A}$ in RI/MOM, matched to $\overline{\text{MS}}$, run to 27 GeV^2

R. S. Sufian, et al, 2001.04960

$$q_v^\pi(x) = \frac{x^\alpha(1-x)^\beta(1+\gamma x)}{B(\alpha+1,\beta+1)+\gamma B(\alpha+2,\beta+1)}$$





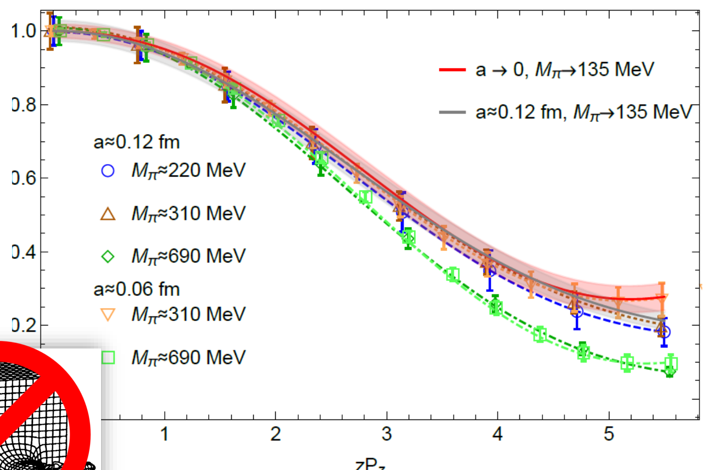


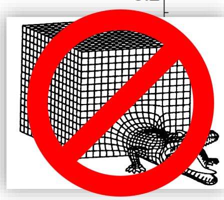
Pion Valence-Quark PDF

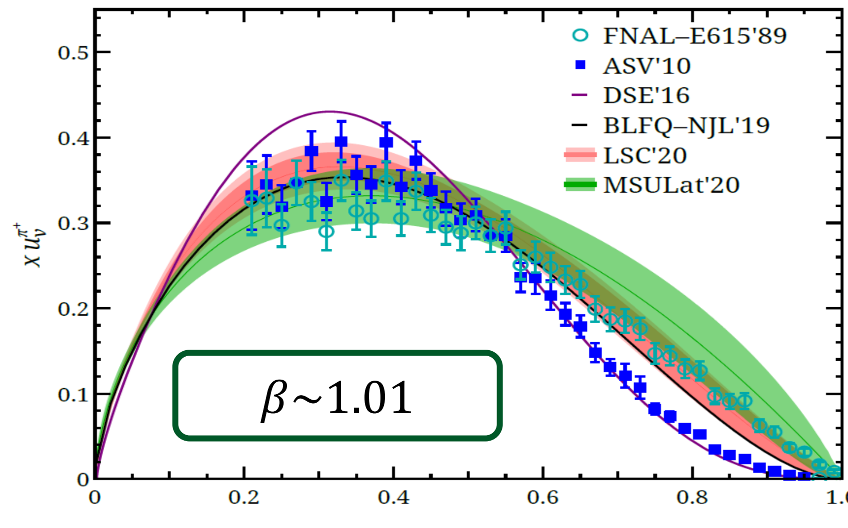
§ Results from MSULat/quasi-PDF method

- $M_\pi = \mathbf{220}, 310, 790$ MeV with $a = 0.06, 0.12$ fm
- Extrapolated to physical limit (shown as pink/green band)
- Renormalized in RI/MOM, matched to $\overline{\text{MS}}$, run to 27 GeV^2

H. Lin et al. (MSULat), 2003.14128







J. S. Conway et al., PRD39, 92 (1989).
M. Aicher et al, PRL105, 252003 (2010), 1009.2481.
C. Chen et al, PRD93, 074021 (2016), 1602.01502.

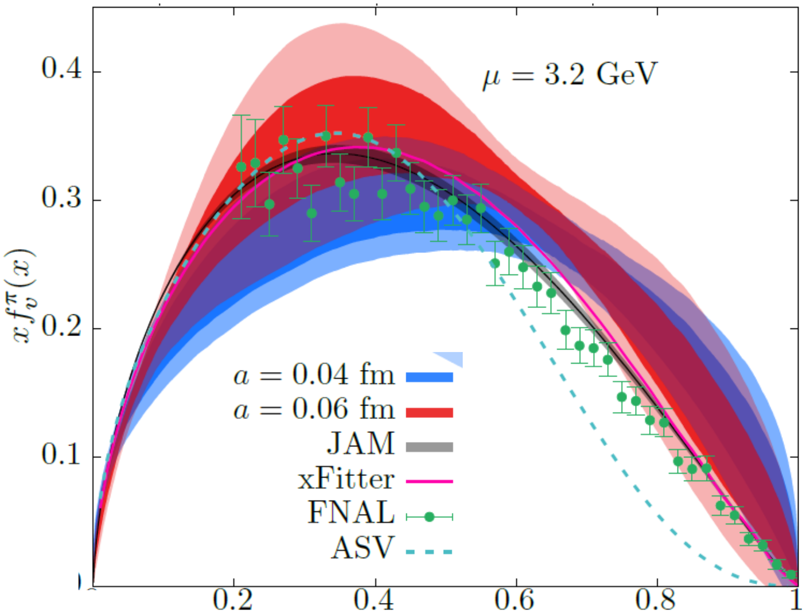
J. Lan, et al, PRL122, 172001 (2019), 1901.11430;
PRD101, 034024 (2020), 1907.01509.
R. S. Sufian, et al, 2001.04960

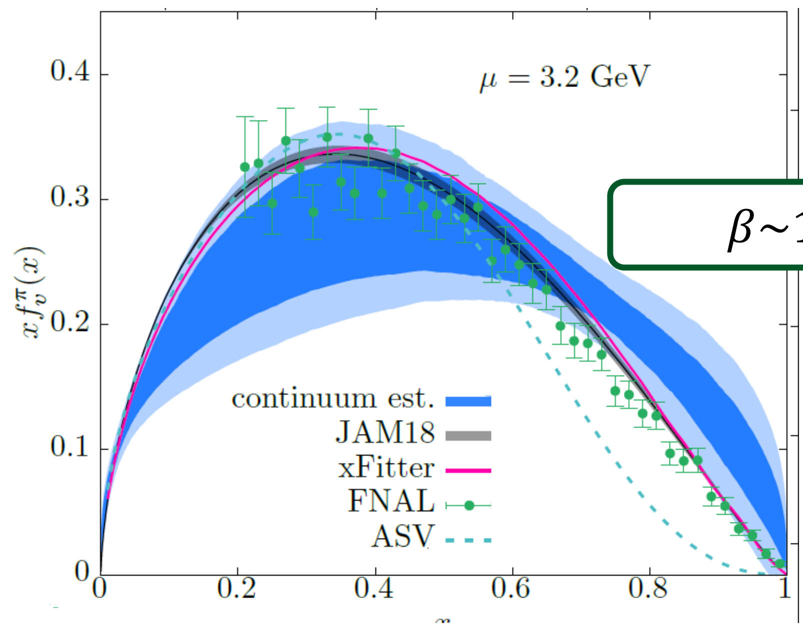
Pion Valence-Quark PDF

§ Results from BNL/quasi-PDF method

- $M_\pi = \mathbf{300}$ MeV with $a = 0.04, 0.06$ fm
- Extrapolated to continuum limit
- Renormalized in RI/MOM, matched to $\overline{\text{MS}}$ at 10 GeV^2

X. Gao et al. 2007.06590



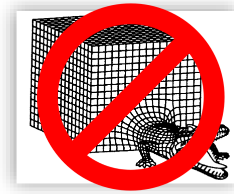


$\beta \sim 1$

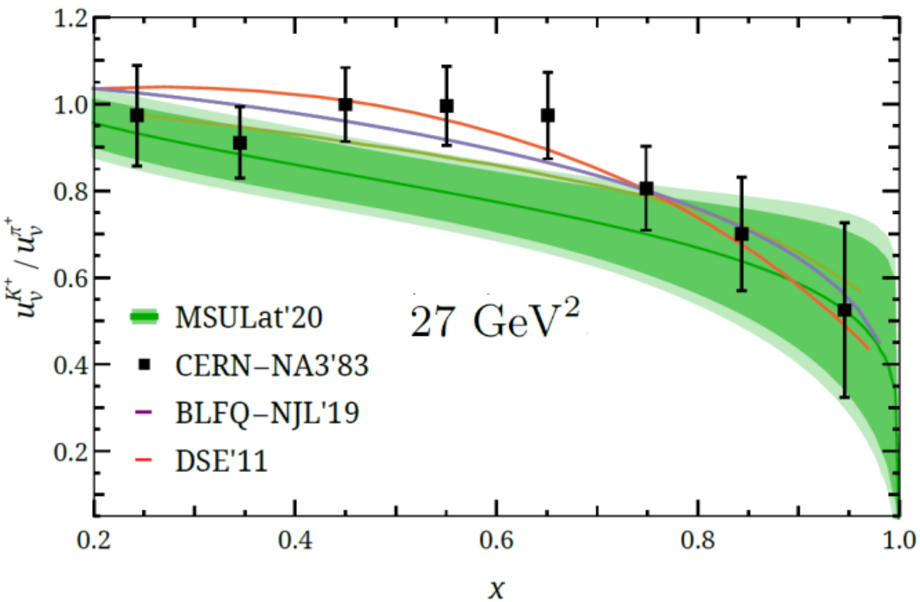
Kaon Valence-Quark PDFs

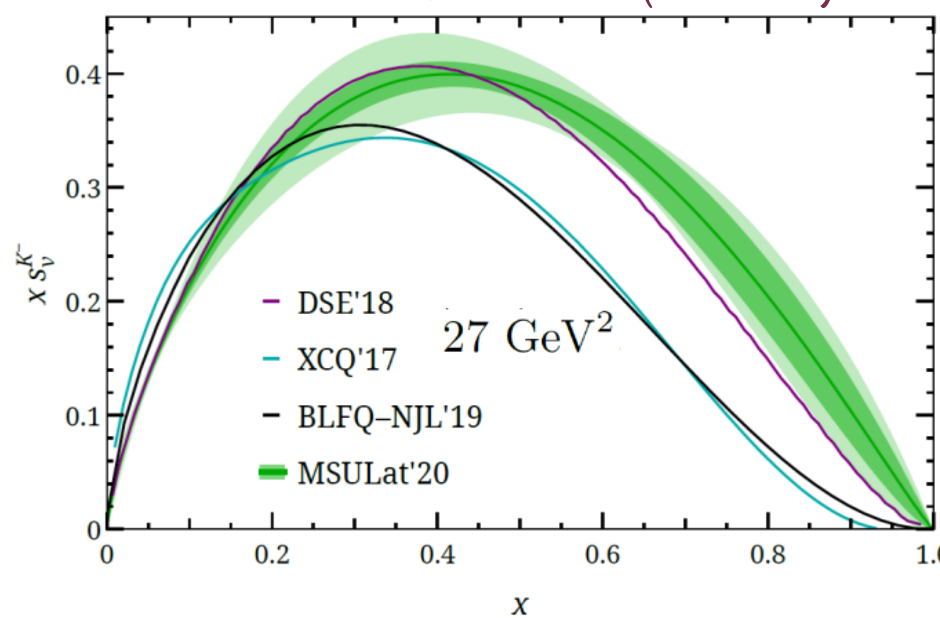
§ Pion/**kaon** PDFs using quasi-PDF in the continuum limit

- Lattice details: clover/2+1+1 HISQ (MSULat)
  $a \approx \{0.06, 0.12\}$ fm,
  $M_\pi \in \{220, 310, 690\}$-MeV pion
  $P_z \approx \{1.3, 1.7\}$ GeV



2003.14128 HL et al (MSULat)

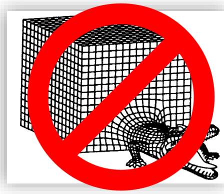




Kaon Valence-Quark PDFs

§ Pion/**kaon** PDFs using quasi-PDF in the continuum limit

- Lattice details: clover/2+1+1 HISQ (MSULat)
  $a \approx \{0.06, 0.12\}$ fm,
  $M_\pi \in \{220, 310, 690\}$-MeV pion
  $P_z \approx \{1.3, 1.7\}$ GeV



§ First LQCD calculation $\langle x^n \rangle$ of $u_v^{K^+}$ and $s_v^{K^-}$

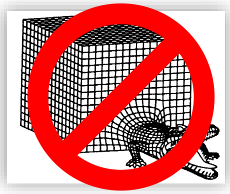
2003.14128
HL et al (MSULat)

n	$\langle x^n \rangle (u_v^{K^+})$	$\langle x^n \rangle (s_v^{K^-})$
1	$0.192(8)_{\text{stat}}(6)_{\text{syst}}$	$0.261(8)_{\text{stat}}(8)_{\text{syst}}$
2	$0.080(7)_{\text{stat}}(6)_{\text{syst}}$	$0.120(7)_{\text{stat}}(9)_{\text{syst}}$
3	$0.041(6)_{\text{stat}}(4)_{\text{syst}}$	$0.069(6)_{\text{stat}}(8)_{\text{syst}}$

Kaon Valence-Quark PDFs

§ Pion/**kaon** PDFs using quasi-PDF in the continuum limit

- Lattice details: clover/2+1+1 HISQ (MSULat)
  $a \approx \{0.06, 0.12\}$ fm,
  $M_\pi \in \{220, 310, 690\}$-MeV pion
  $P_z \approx \{1.3, 1.7\}$ GeV



§ First LQCD calculation $\langle x^n \rangle$ of $u_v^{K^+}$ and $s_v^{K^-}$

§ Later ETMC **260**-MeV results on $\langle x^n \rangle$ of $u_v^{K^+}$ and $s_v^{K^-}$

2003.14128 HL et al (MSULat)

n	$\langle x^n \rangle (u_v^{K^+})$	$\langle x^n \rangle (s_v^{K^-})$
1	$0.192(8)_{\text{stat}}(6)_{\text{syst}}$	$0.261(8)_{\text{stat}}(8)_{\text{syst}}$
2	$0.080(7)_{\text{stat}}(6)_{\text{syst}}$	$0.120(7)_{\text{stat}}(9)_{\text{syst}}$
3	$0.041(6)_{\text{stat}}(4)_{\text{syst}}$	$0.069(6)_{\text{stat}}(8)_{\text{syst}}$

2010.0349, 2104.02247

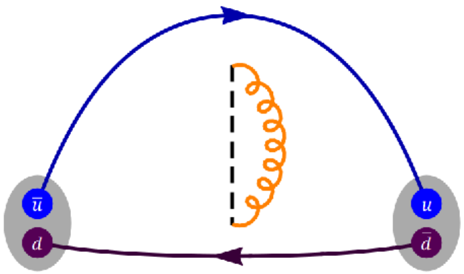
n	$\langle x^n \rangle (u_v^{K^+})$	$\langle x^n \rangle (s_v^{K^-})$
1	$0.246(2)_{\text{stat}}(2)_{\text{syst}}$	$0.317(2)_{\text{stat}}(1)_{\text{syst}}$
2	$0.093(5)_{\text{stat}}(3)_{\text{syst}}$	$0.134(5)_{\text{stat}}(2)_{\text{syst}}$
3	$0.035(6)_{\text{stat}}(3)_{\text{syst}}$	$0.075(5)_{\text{stat}}(1)_{\text{syst}}$

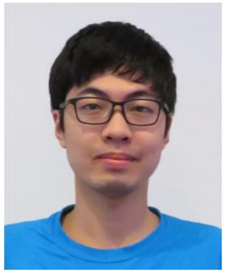
First Pion Gluon PDF

§ Pion GLUON PDFs using pseudo-PDF

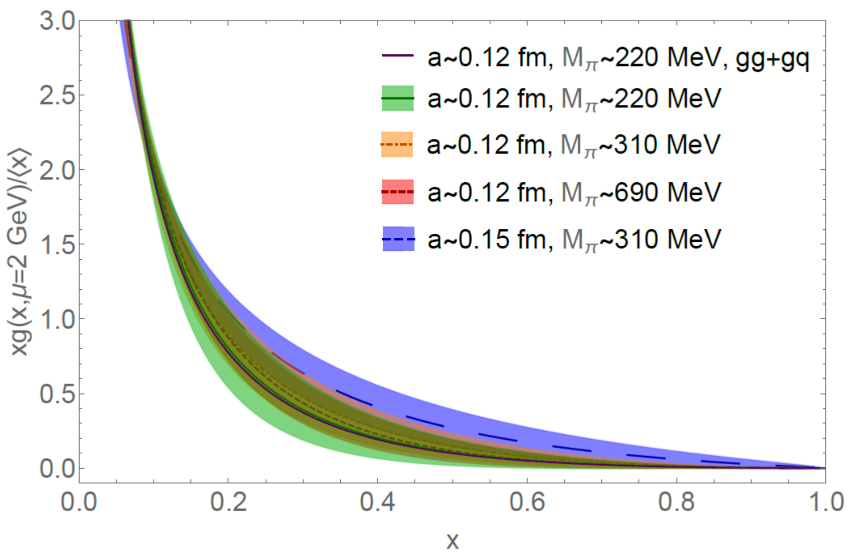
- Lattice details: clover/2+1+1 HISQ (MSULat)
  $a \approx \{0.12, 0.15\}$ fm,
  $M_\pi \in \{220, 310, 690\}$-MeV pion
  $P_{z,\max} \approx 2.3$ GeV

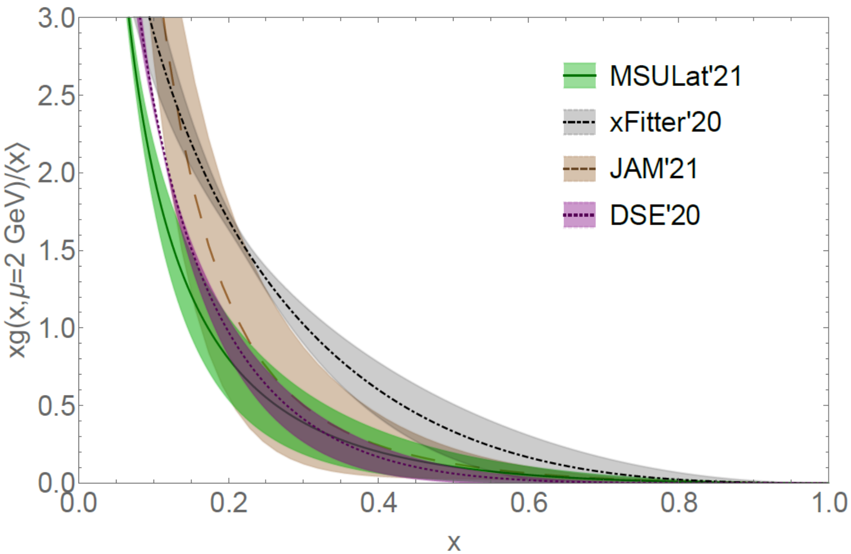
  2104.06372, Fan, HL(MSULat)





Zhouyou Fan (MSU)



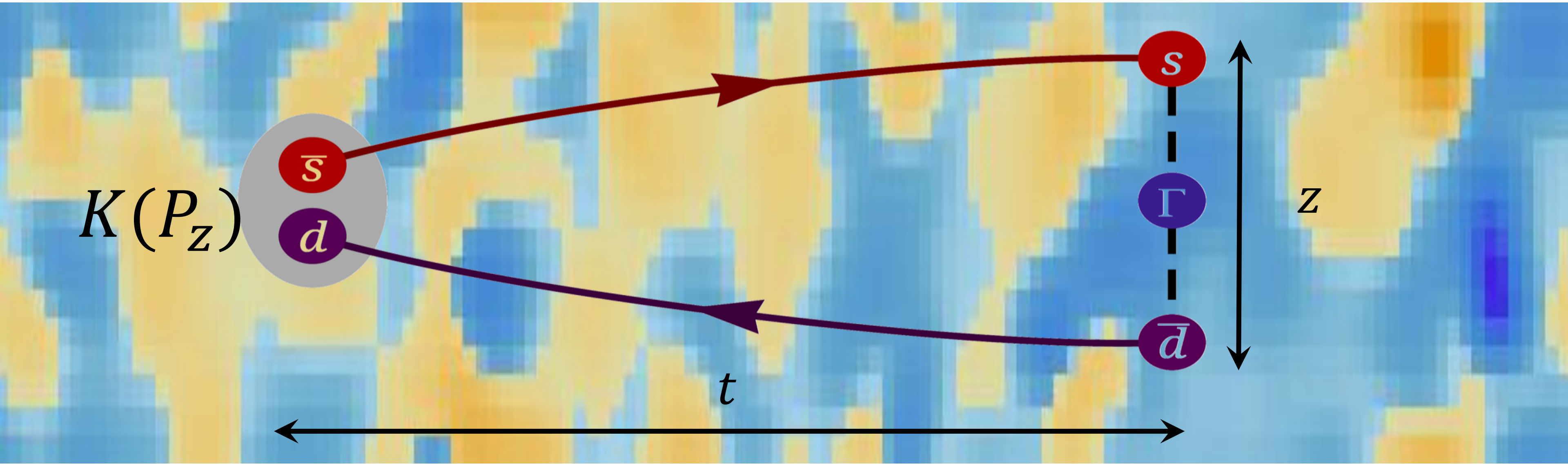


Pion and Kaon DA

§ The first continuum-limit study of *x*-dependent meson DA on the lattice

- $M_\pi \in \{\mathbf{310}, 690\ (\eta_s)\}$ MeV
- $a \in \{0.06, 0.09, 0.12\}$ fm
- $M_\pi^{\min} L = 4.5$

$$C_M^{\mathrm{DA}}(z,P,t) = \left\langle 0 \left| \int d^3y\, e^{i\vec{P}\cdot\vec{y}} \bar{\psi}_1(\vec{y},t)\gamma_z\gamma_5 U(\vec{y},\vec{y}+z\,\hat{z})\psi_2(\vec{y}+z\,\hat{z},t)\bar{\psi}_2(0,0)\gamma_5\psi_1(0,0) \right| 0 \right\rangle$$



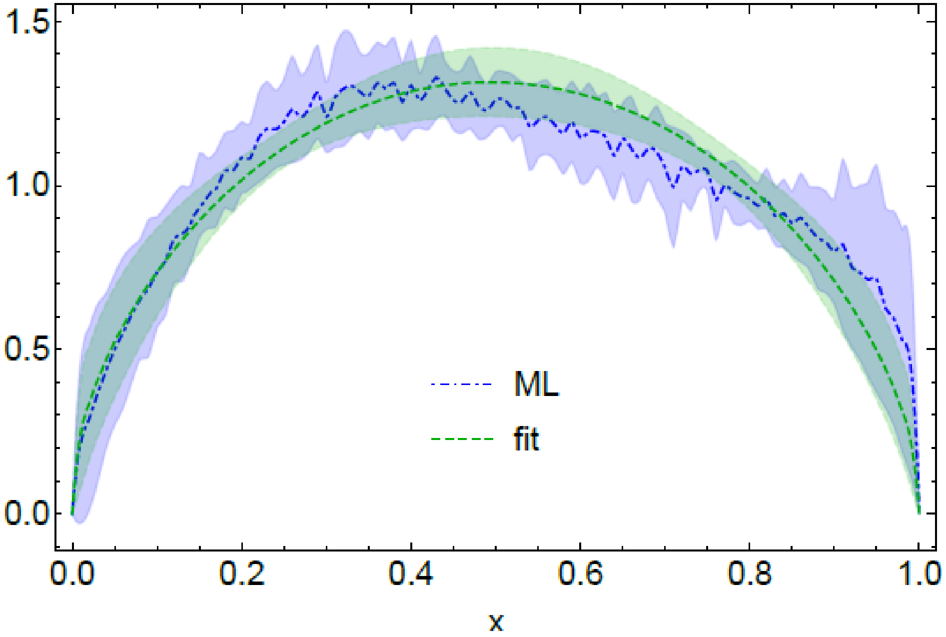
Pion and Kaon DA

§ Extract the DA distribution from the physical-continuum matrix elements

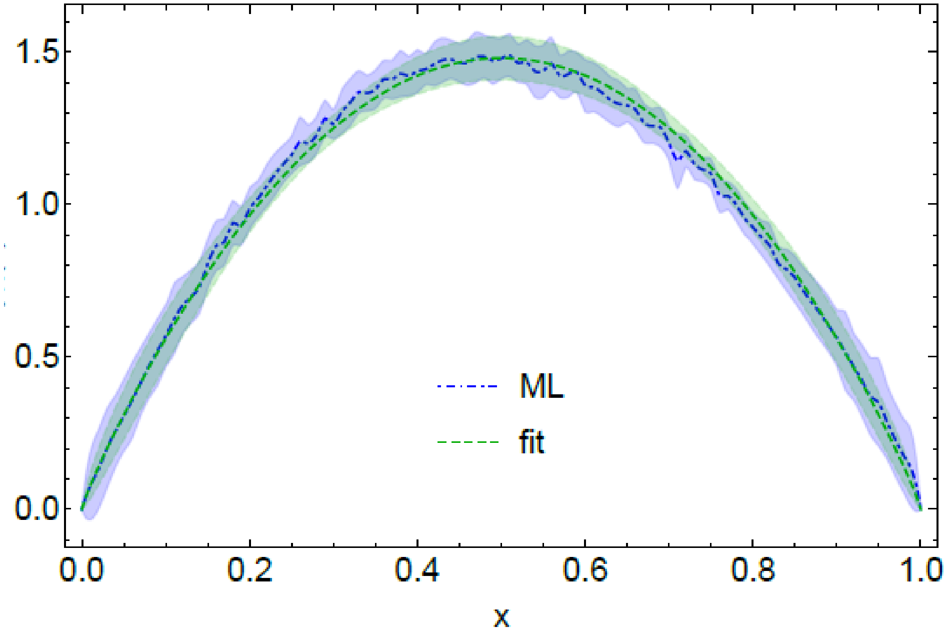
R. Zhang et al. (MSULat), 2005.13955

$$h(z,\mu^R,p_z^R,P_z) = \int_{-\infty}^{\infty} dx \int_0^1 dy\, C\left(x,y,\left(\frac{\mu^R}{p_z^R}\right)^2,\frac{P_z}{\mu^R},\frac{P_z}{p_z^R}\right) f_{m,n}(y) e^{i(1-x)zP_z}$$

Pion



Kaon



Pion and Kaon DA

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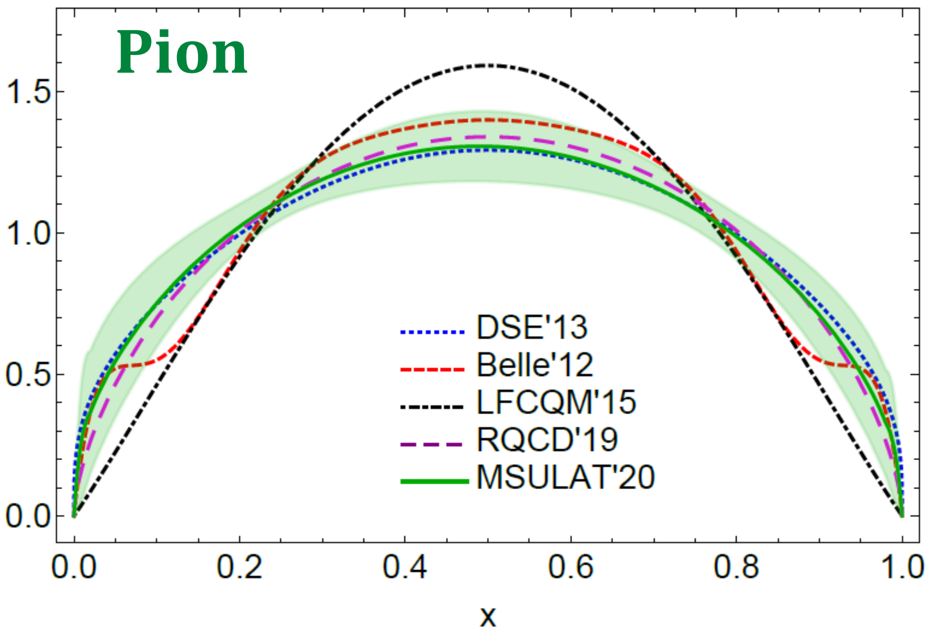
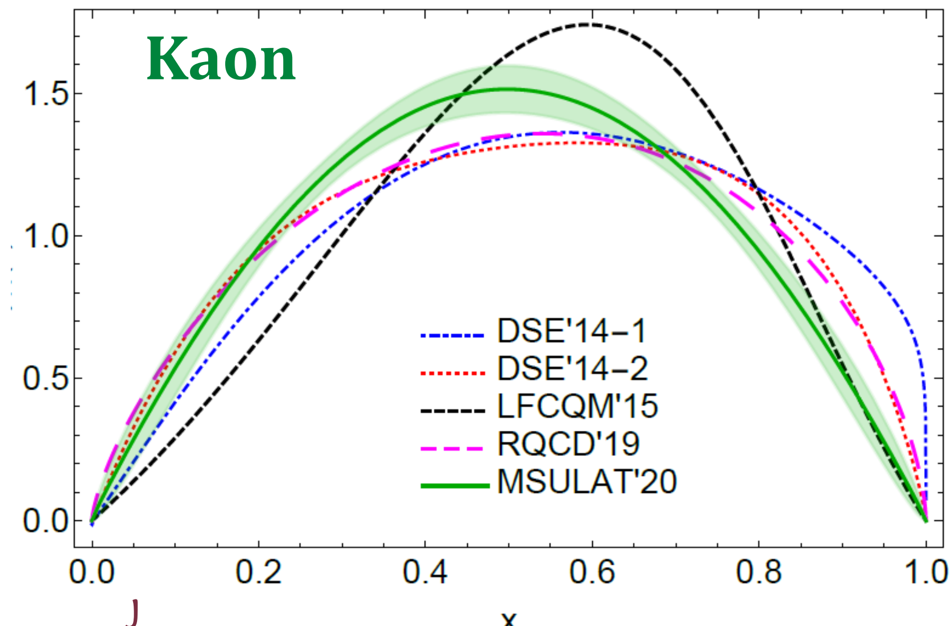
$$h(z,\mu^R,p_z^R,P_z) = \int_{-\infty}^{\infty} dx \int_0^1 dy\, C\left(x,y,\left(\frac{\mu^R}{p_z^R}\right)^2,\frac{P_z}{\mu^R},\frac{P_z}{p_z^R}\right) f_{m,n}(y) e^{i(1-x)zP_z}$$



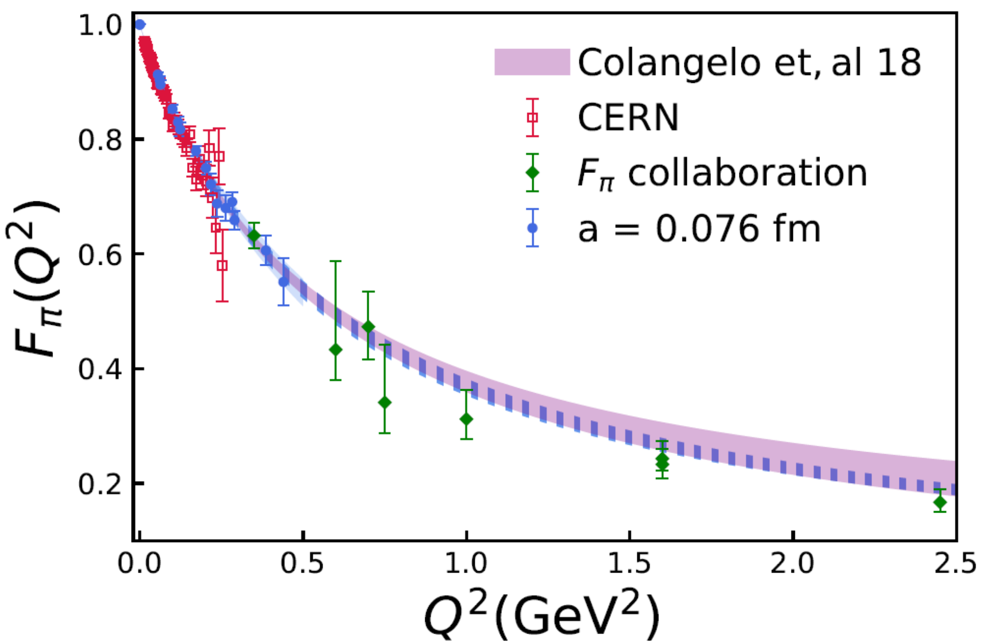


DES'13: L. Chang et al., Phys. Rev. Lett. 110, 132001 (2013); C. Shi et al., Phys. Lett. B738, 512 (2014)
Belle'12: S. Agaev et al., Phys. Rev. D86, 077504 (2012);
LFCQM'15:J. P. B. C. de Melo et al., AIP Conf. Proc. 1735, 080012 (2016);
RQCD'19: G. S. Bali et al.,J HEP 08, 065 (2019); DSE'14:

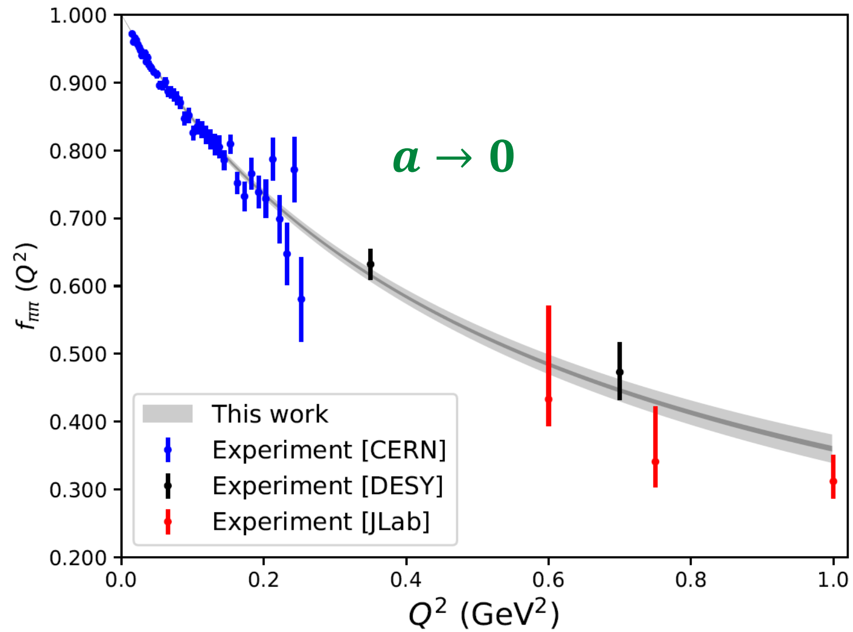
Pion Form Factors

§ Two new lattice pion form factors calcs at physical pion

- χQCD: 2+1f, overlap/DWF,
  $a \approx [0.08, 0.2]$ fm, $M_\pi \in [139, 340]$-MeV
- BNL: 2+1+1,clover/HISQ,
  $a \approx [0.04, 0.08]$ fm, $M_\pi \in \{135, 300\}$-MeV



X. Gao et al (BNL), 2102.06047

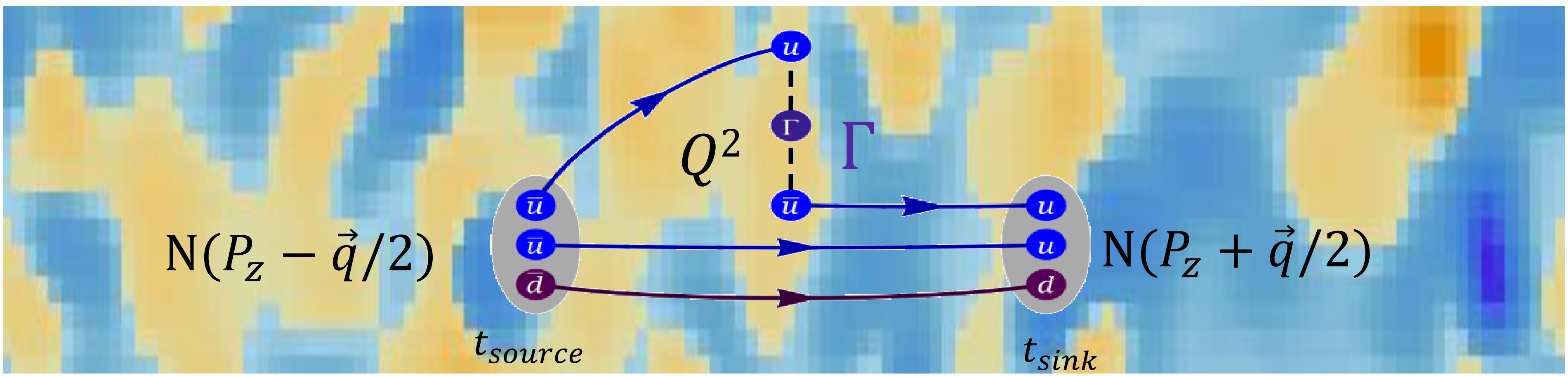


G. Wang et al (XQCD), 2006.05431

2020: Isovector Nucleon GPDs

§ Nucleon GPD using quasi-PDFs at **physical pion mass**

- MSULat: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon GPD results



$$\tilde{F}(x,\tilde{\xi},t,\bar{P}_z) = \frac{\bar{P}_z}{\bar{P}_0}\int \frac{dz}{4\pi} e^{ixz\bar{P}_z}\langle P'|\tilde{O}_{\gamma_0}(z)|P\rangle = \frac{\bar{u}(P')}{2\bar{P}^0}\left(H(x,\tilde{\xi},t,\bar{P}_z)\gamma^0 + E(x,\tilde{\xi},t,\bar{P}_z)\frac{i\sigma^{0\mu}\Delta_\mu}{2M}\right)u(P'')$$

$$p^\mu = \frac{p''^\mu + p'^\mu}{2}, \qquad \Delta^\mu = p''^\mu - p'^\mu, \qquad t = \Delta^2, \qquad \xi = \frac{p''^+ - p'^+}{p''^+ + p'^+}$$

HL, Phys.Rev.Lett. 127 (2021) 18, 182001

2020: Isovector Nucleon GPDs

§ Nucleon GPD using quasi-PDFs at **physical pion mass**

- MSULat: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 2$ GeV
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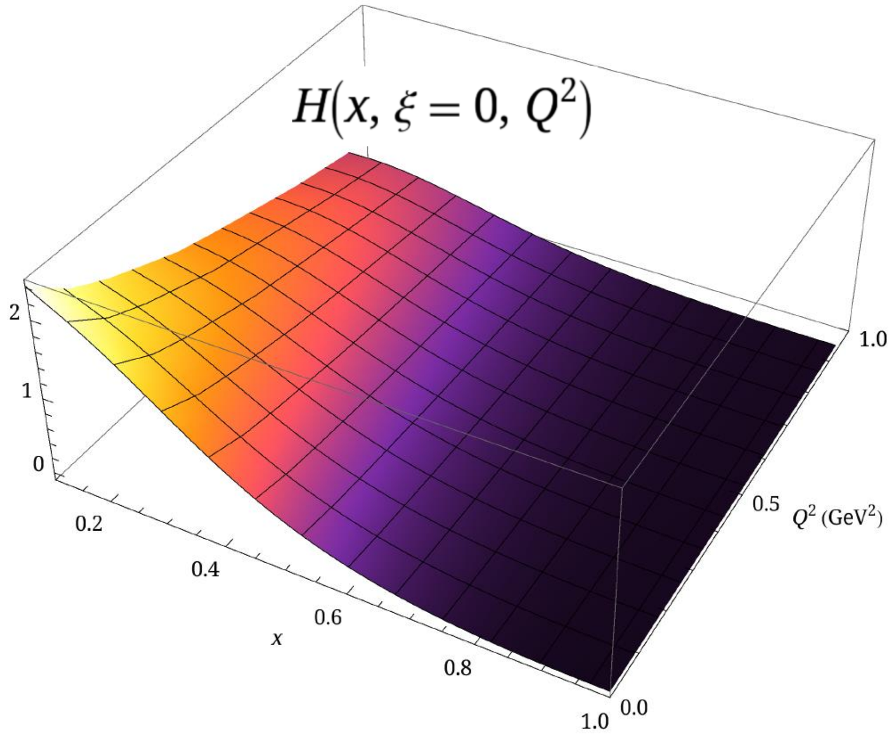
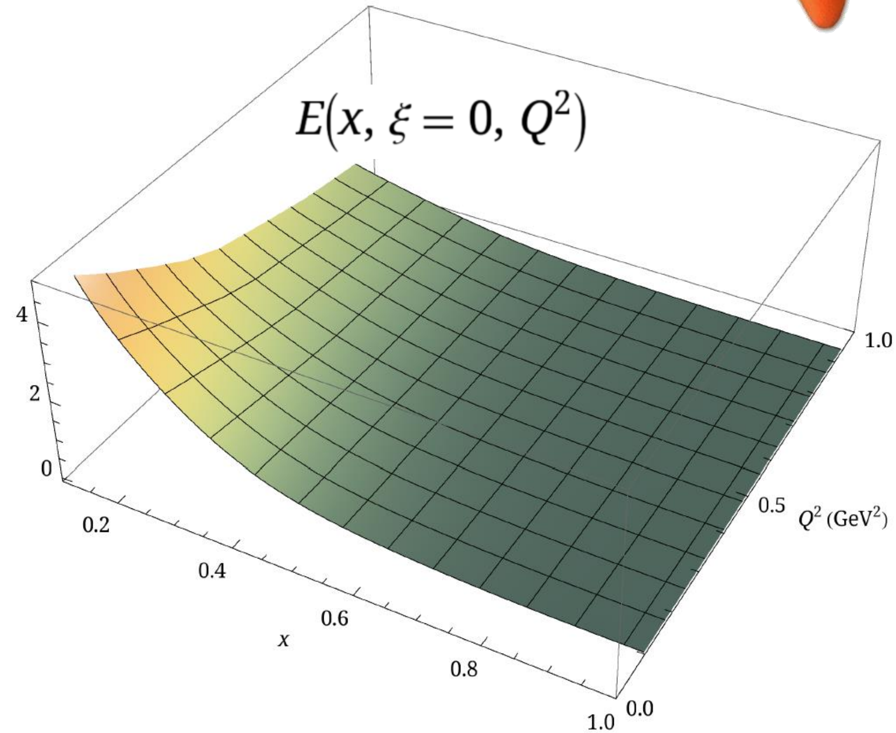




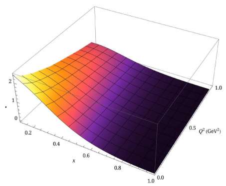
HL, Phys.Rev.Lett. 127 (2021) 18, 182001

2020: Isovector Nucleon GPDs

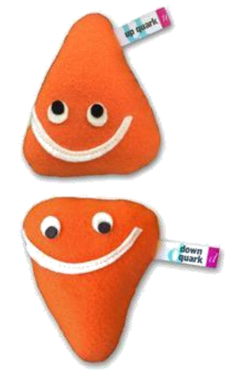
§ Nucleon GPD using quasi-PDFs at physical pion mass

- Lattice details: clover/2+1+1 HISQ (MSULat)
  0.09 fm, **135-MeV** pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon GPD results

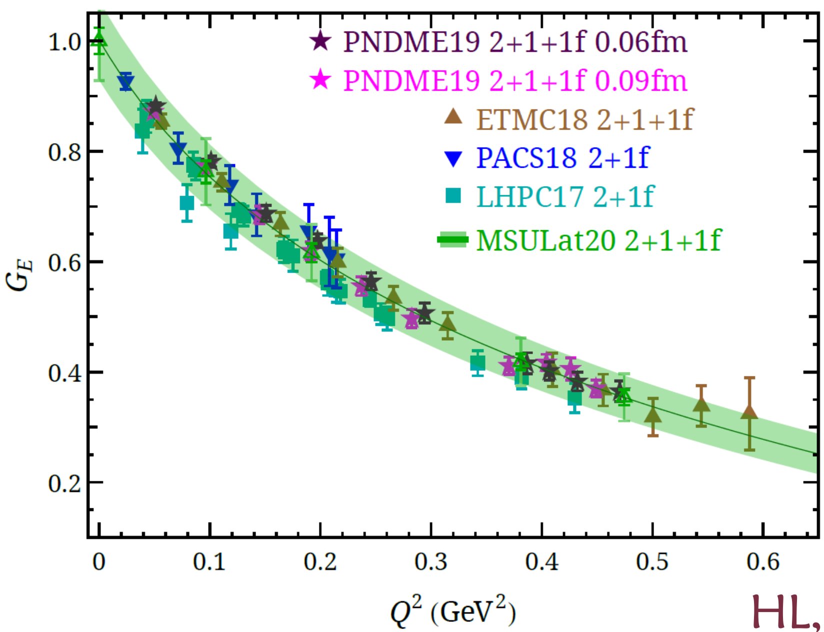
$$\int_{-1}^{+1} dx\, x^{n-1}$$



$$= \sum_{i=0,\text{even}}^{n-1} (-2\xi)^i A^q_{ni}(t) + (-2\xi)^n C^q_{n0}(t)\Big|_{n\ \text{even}}$$



$n = 1$



HL, Phys.Rev.Lett. 127 (2021) 18, 182001

2020: Nucleon GPDs

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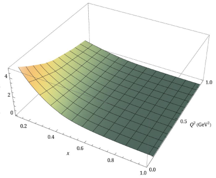
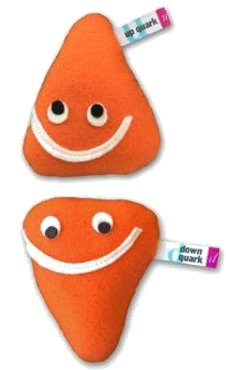
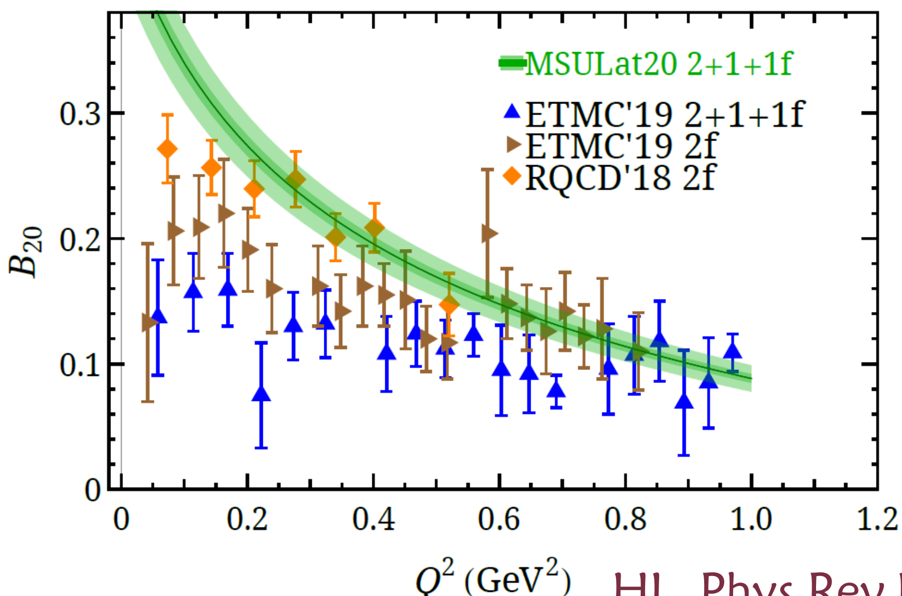
$\int_{-1}^{+1} dx\, x^{n-1}$  $= \sum_{i=0,\text{even}}^{n-1} (-2\xi)^i B_{ni}^q(t) - (-2\xi)^n C_{n0}^q(t) \Big|_{n \text{ even}}$



$n = 2$

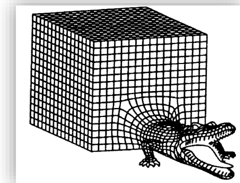


HL, Phys.Rev.Lett. 127 (2021) 18, 182001

2020: Nucleon Tomography

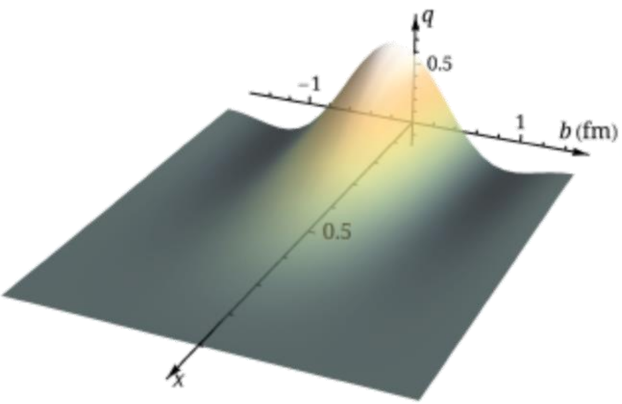
§ Nucleon GPD using quasi-PDFs at physical pion mass

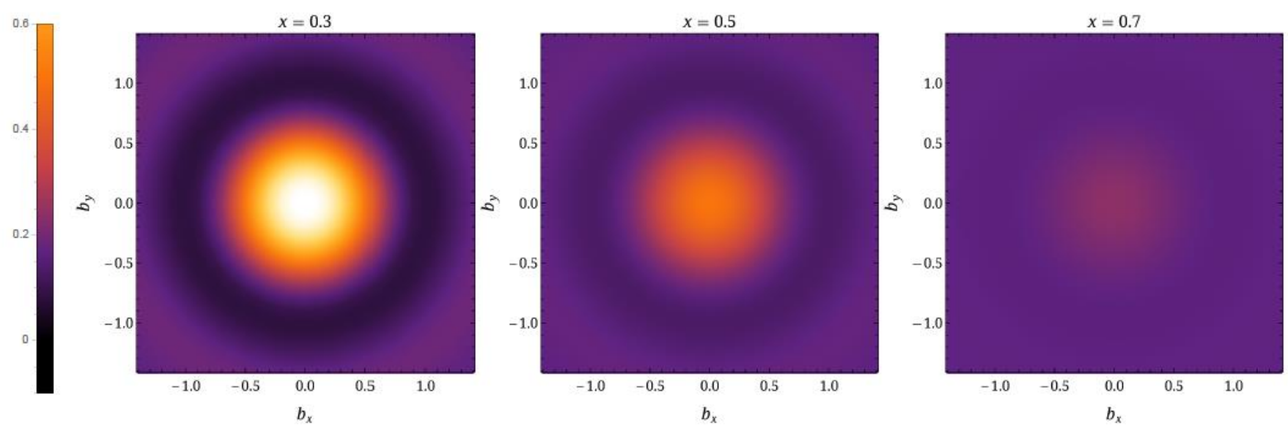
- Lattice details: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon GPD results



finite-volume,
discretization,

$$q(x,b) = \int \frac{d\vec{q}}{(2\pi)^2} H(x, \xi = 0, t = -\vec{q}^2) e^{i\vec{q}\cdot\vec{b}}$$





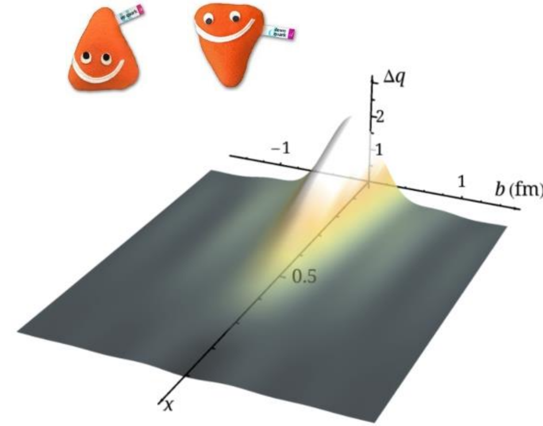
HL, Phys.Rev.Lett. 127 (2021) 18, 182001

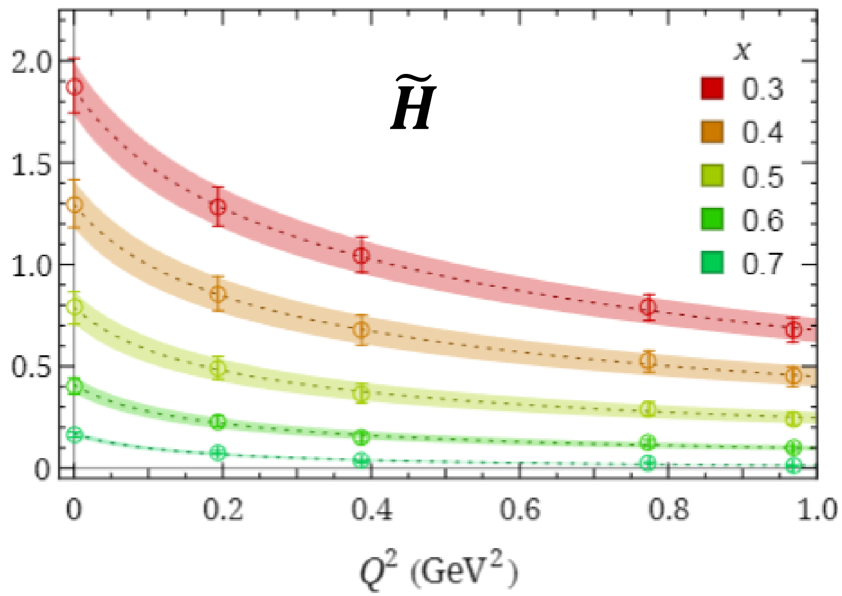
Nucleon Polarized GPDs

§ Helicity GPD ($\widetilde{H}$)using quasi-PDFs at **physical pion mass**

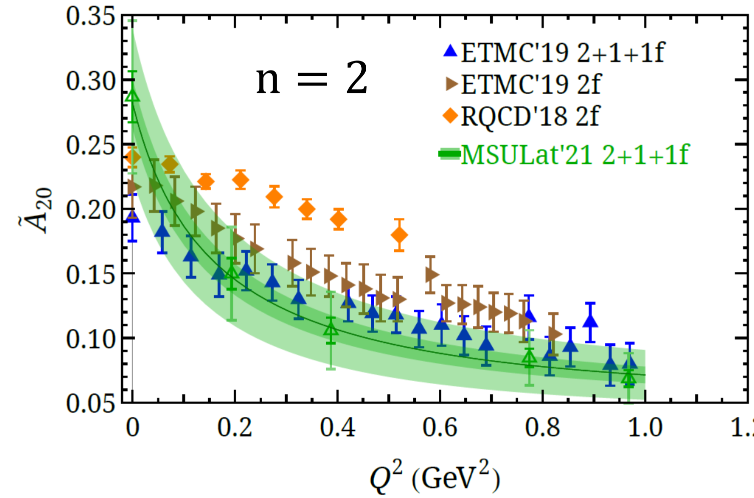
- MSULat: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon (quasi-)GPD results

HL (MSULat), Phys.Lett.B 824 (2022) 136821





- Take the integral to form moments



Caveats

§ Systematics in our earlier quasi-PDF calculation

- Renormalization: non-perturbative RI/MOM renormalization
  - State of the art: hybrid-ratio renormalization

X. Ji et. al. NPB 964, 115311 (2021)

- Next-leading order (NLO) matching only
  - State of the art: NNLO matching kernel available

X. Gao, PRL 128, 142003 (2022)

- Did not teat leading-renormalon effects
  - Leading-renormalon resummation (LRR)
  - Renormalization-group resummation (RGR)

R. Zhang, et. al. PLB 844, 138081 (2023)

- For the rest of this presentation, we will focus on the uncertainties from the above (rather than typical lattice-calculation precision or systematics)

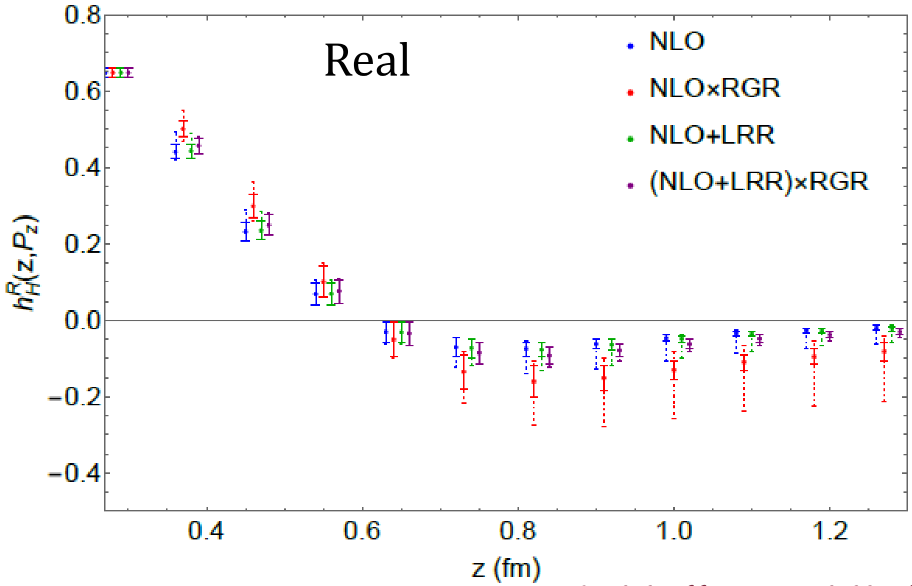
Forward-Limit Case: PDF

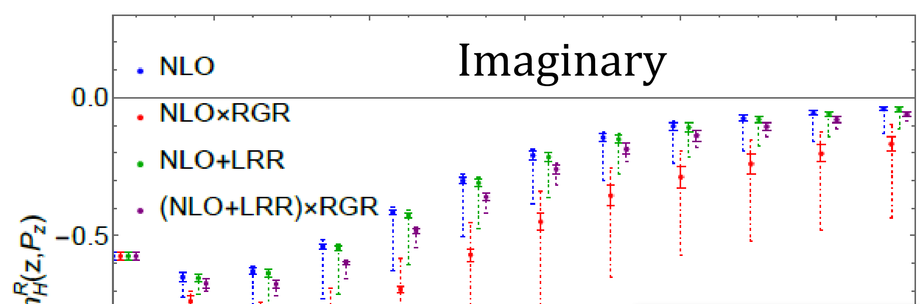
§ NLO hybrid-ratio renormalized matrix elements

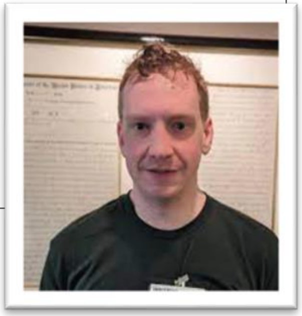
$$h^R(z,P_z) = \begin{cases} N \frac{h^B(z,P_z)}{h^B(z,P_z=0)} & \text{for } z < z_s \\ N e^{(\delta m + m_0)(z-z_s)} \frac{h^B(z,P_z)}{h^B(z,P_z=0)} & \text{for } z \geq z_s \end{cases}$$

Remove the linear divergence & renormalon ambiguity at large distances

- Vary the scale within $[0.75, 1.5]$: $\approx 15\%$ variation $\alpha_s(\mu = 2.0\text{ GeV})$
- Systematic errors shown below:





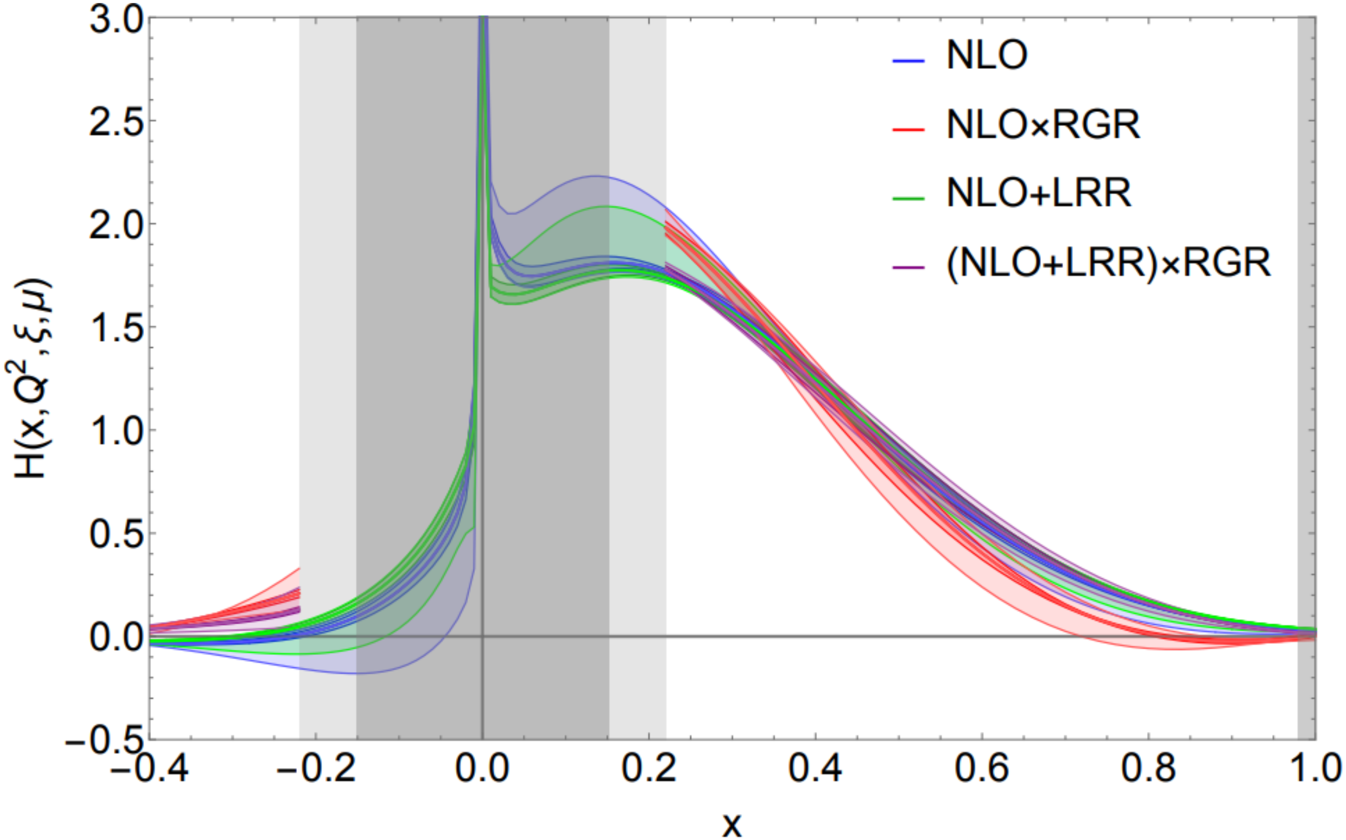


J. Holligan, HL (MSULat), 2312.10829 [hep-lat]

Forward Limit Case: PDF

§ NLO isovector nucleon $H(\xi = 0, Q^2 = 0, x)$

- RGR process: DGLAP equation breaks down for $|x| \lesssim 0.2$ with $\mu = c' \times 2xP_z$



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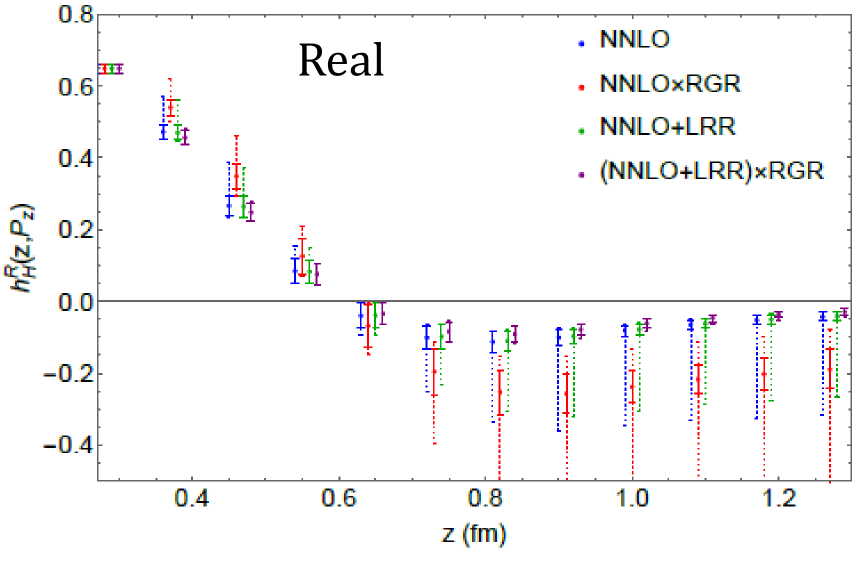

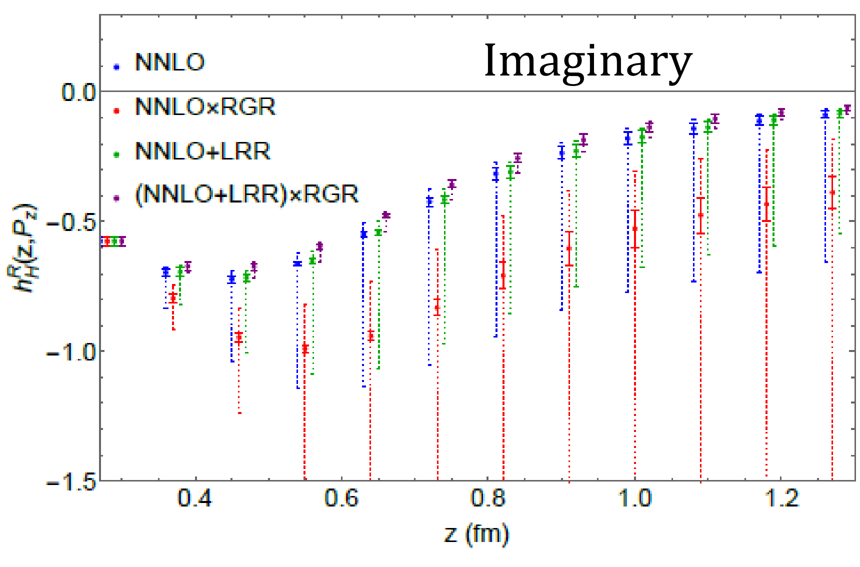

J. Holligan, HL (MSULat), 2312.10829 [hep-lat]

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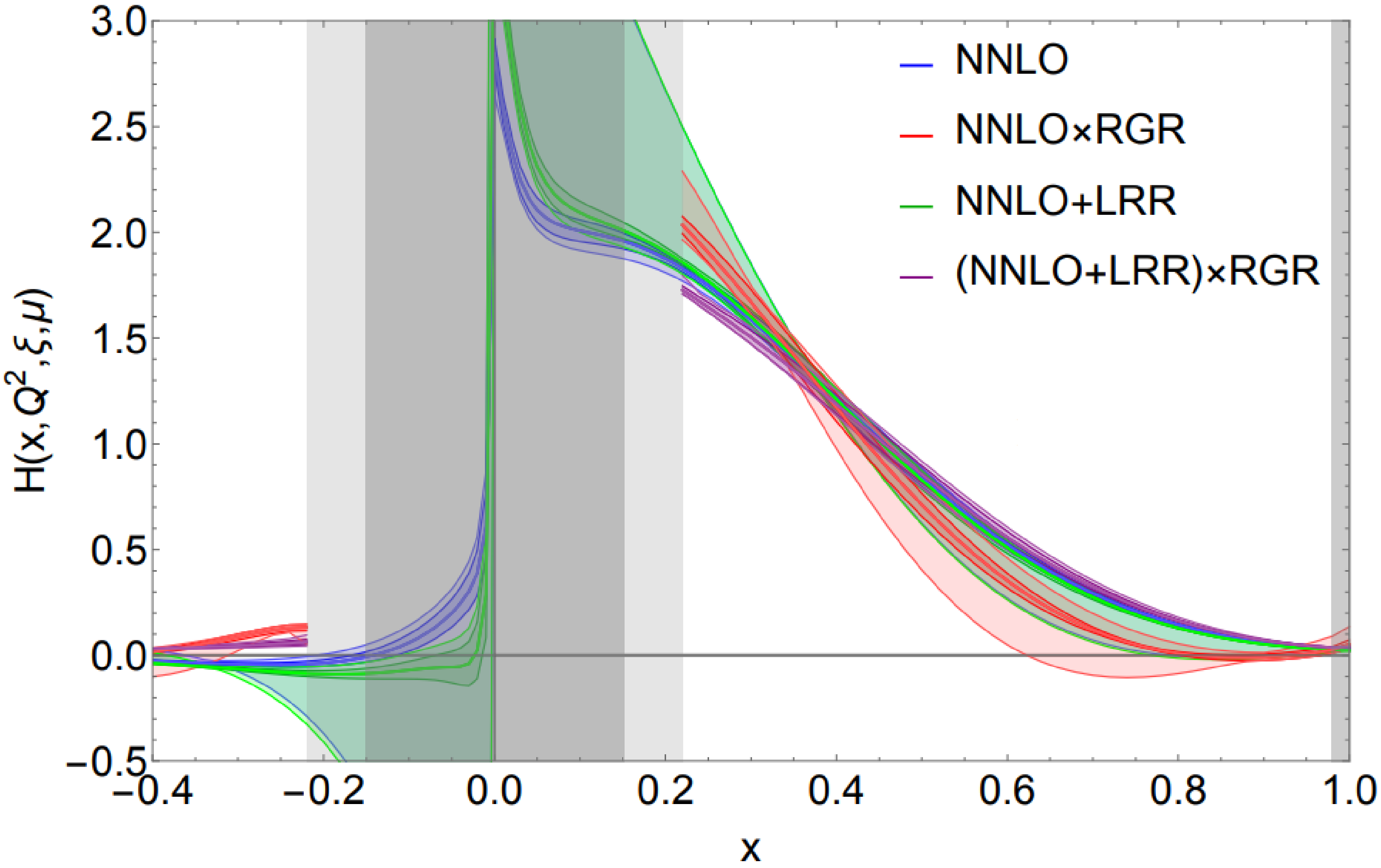
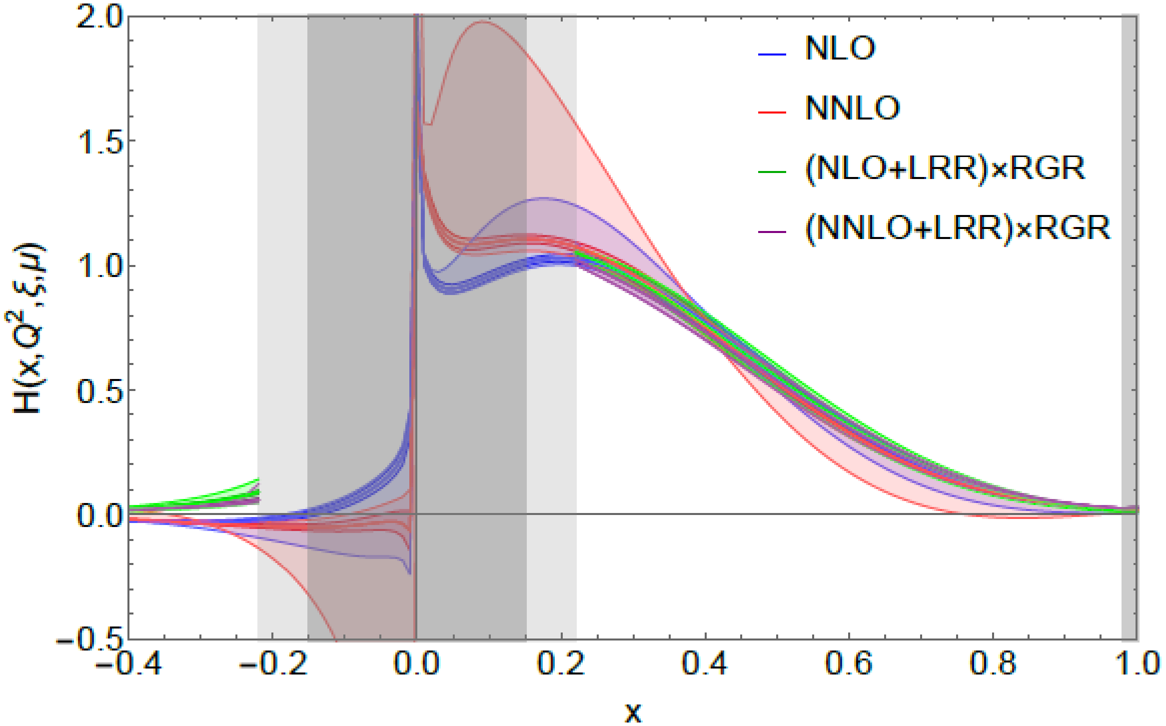


J. Holligan, HL (MSULat), 2312.10829 [hep-lat]

$\xi=0$, $Q^2=0.39$ GeV^2 GPDs

§ Repeat the procedure for nonzero transfer momentum



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ξ=0, Q^2=0.39 GeV² GPDs

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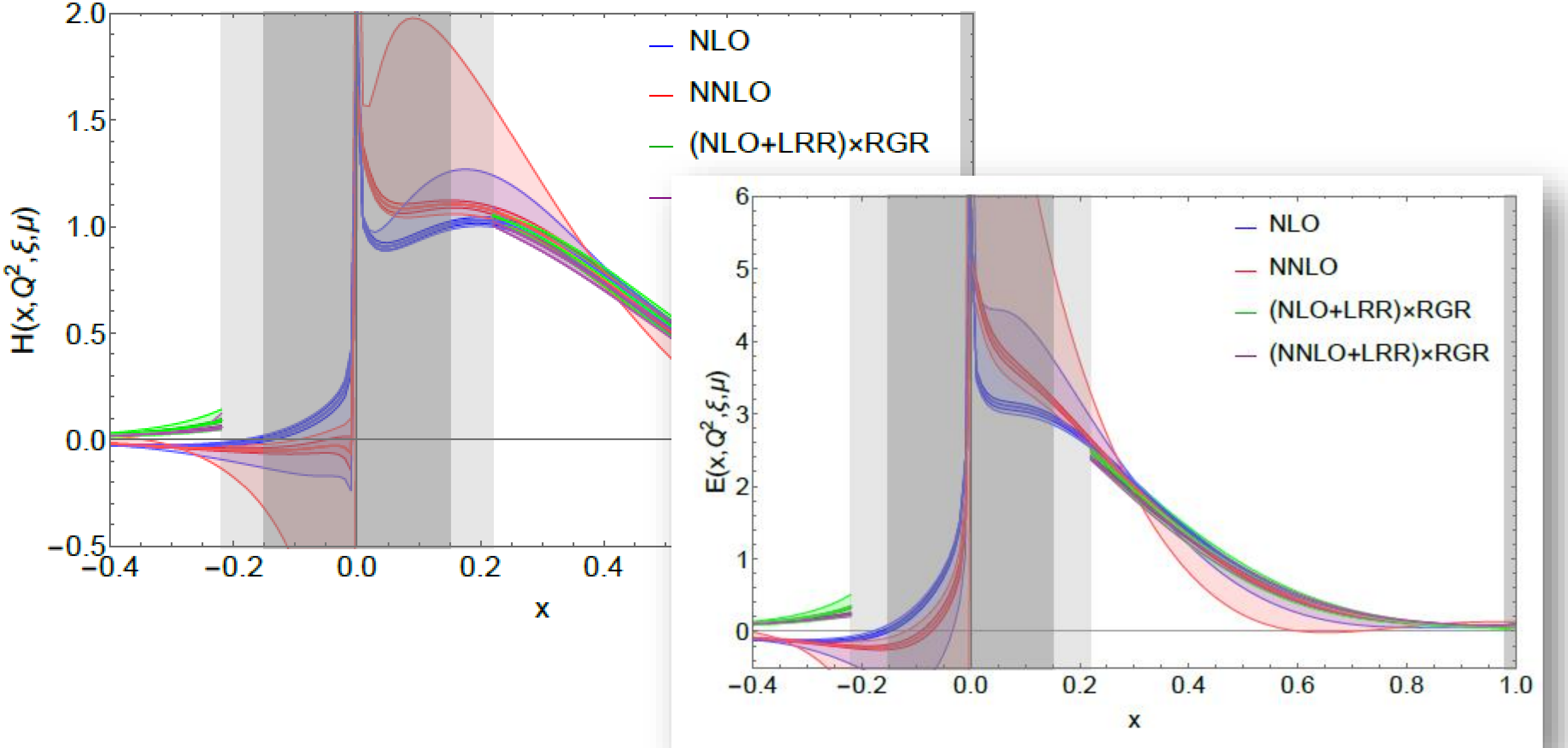

J. Holligan, HL (MSULat), 2312.10829 [hep-lat]

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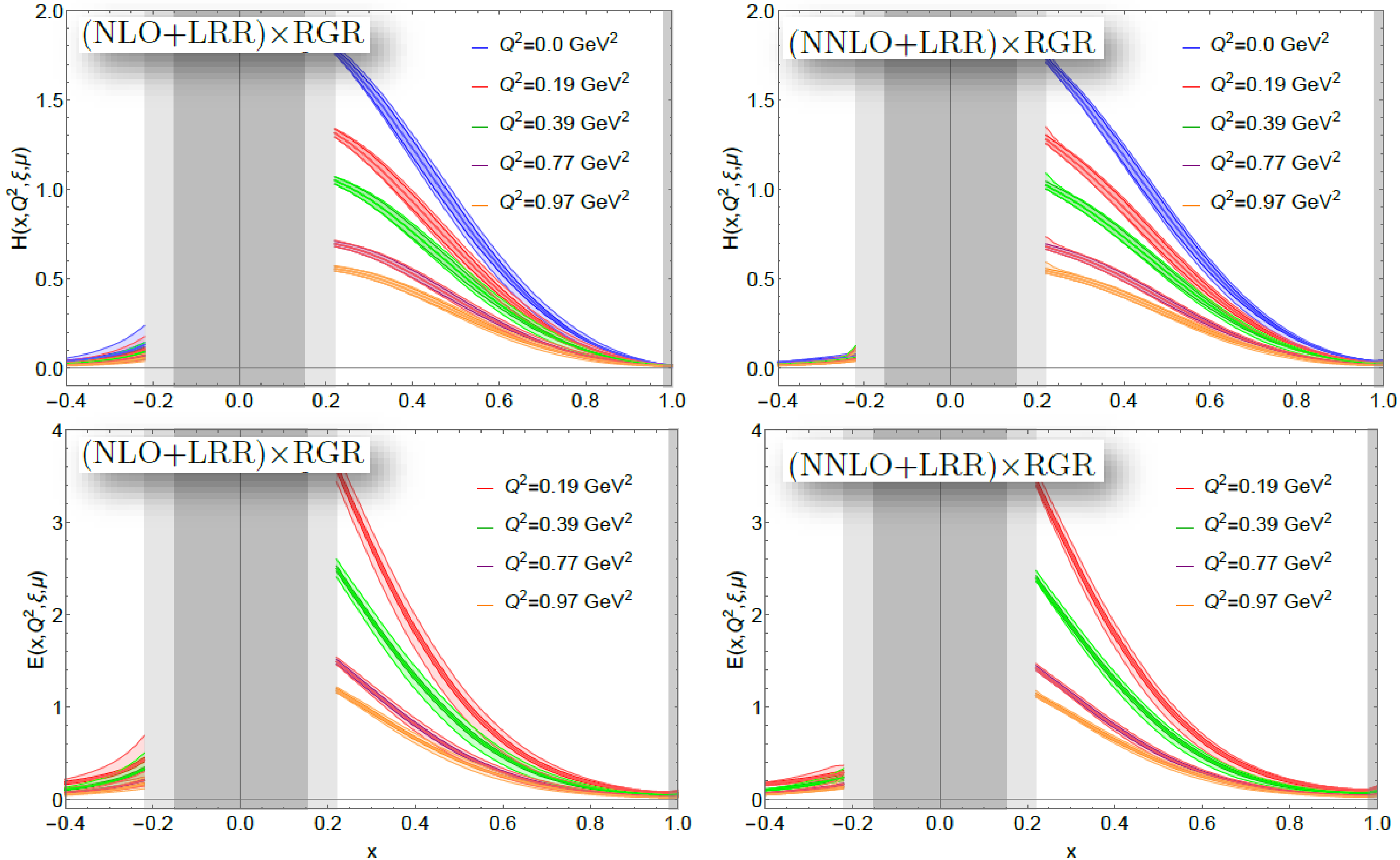


ξ≠0 GPDs

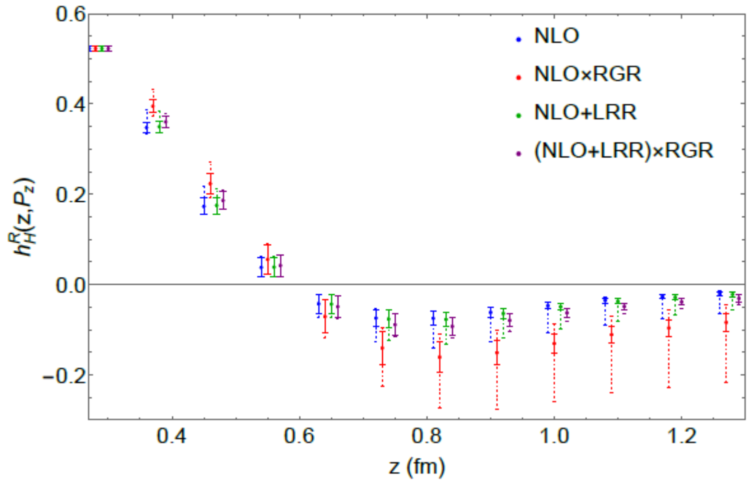
§ Only the NLO matching kernel is available

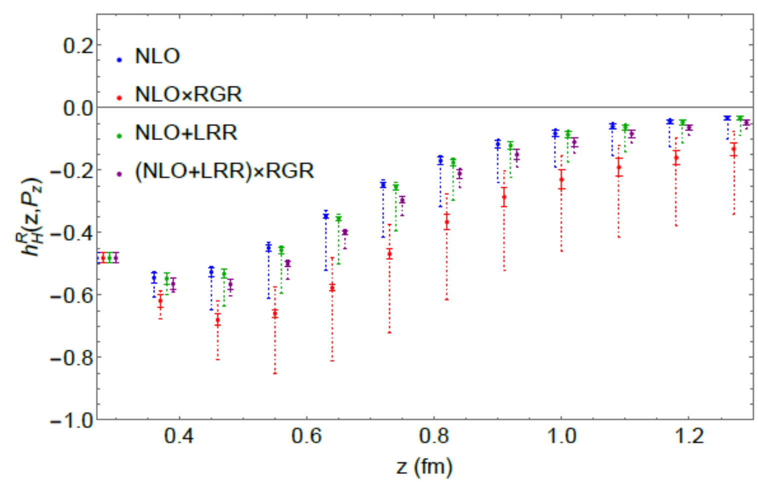
$$
\begin{aligned}
&\mathcal{K}(x, y, \mu, \xi, P_z) \\
&= \delta(x-y) \\
&+ \frac{\alpha_s C_F}{4\pi}\left[\left(\frac{|\xi+x|}{2\xi(\xi+y)} + \frac{|\xi+x|}{(\xi+y)(y-x)}\right)\left(\ln\left(\frac{4y^2(\xi+x)^2 P_z^2}{\mu^2}\right) - 1\right)\right. \\
&+ \left(\frac{|\xi-x|}{2\xi(\xi-y)} + \frac{|\xi-x|}{(\xi-y)(x-y)}\right)\left(\ln\left(\frac{4y^2(\xi-x)^2 P_z^2}{\mu^2}\right) - 1\right) \\
&+ \left.\left(\left(\frac{\xi+x}{\xi+y} + \frac{\xi-x}{\xi-y}\right)\frac{1}{|x-y|} - \frac{|x-y|}{\xi^2-y^2}\right)\left(\ln\left(\frac{4y^2(x-y)^2 P_z^2}{\mu^2}\right) - 1\right)\right]
\end{aligned}
$$

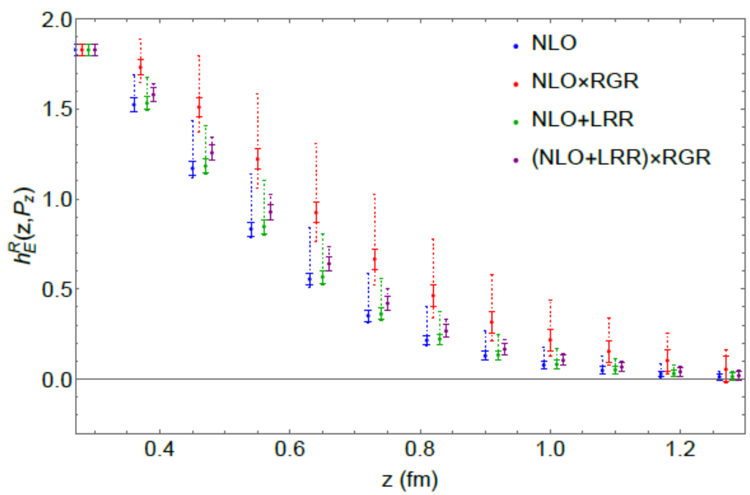
F. Yoa et al, JHEP 11(2023) 021

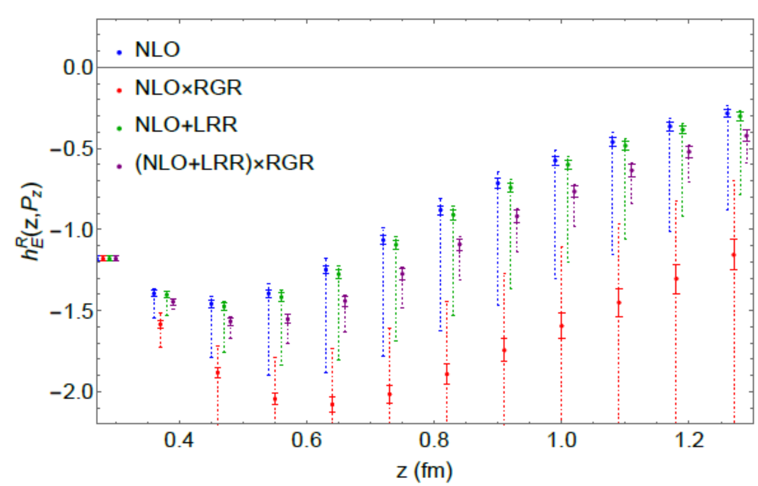
$\xi \neq 0$ GPDs

§ NLO $\xi = 0.1$, $Q^2 = 0.23$ GeV^2









J. Holligan, HL (MSULat), in preparation

ξ≠0 GPDs

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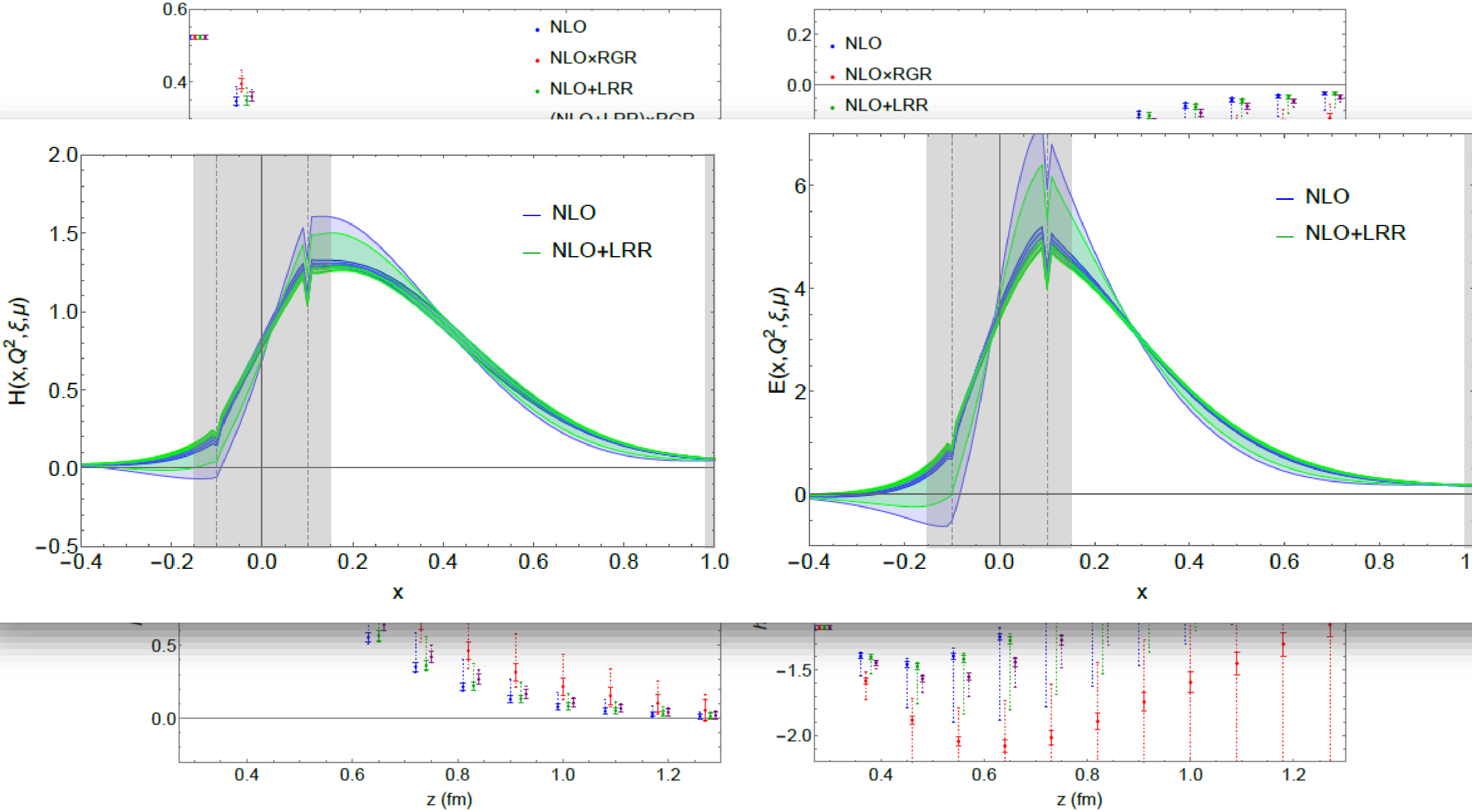

J. Holligan, HL (MSULat),2312.10829 [hep-lat]

Challenges

- § Large momentum is essential
  - With sufficient statistics nucleons may reach 5 GeV
- § Renormalization of linear divergence
  - Wilson-line ops have linear divergences that must be subtracted
- § Methods for signal-to-noise improvement
  - Gluonic observables, new ideas for large momentum
- § Inverse problems PDF extraction in SDF
  - Remove the model/preconditioner-choice dependence
- § Reaching long-range correlations in LaMET
  - For small-x physics, new methods for calculating longer-range correlations must be developed

Whitepaper: Lattice QCD Calculations of Parton Physics, 2202.07193