Parametrizing Pion Structure Functions within a Light-Front Framework

PAW'24 - Physics at AMBER international Workshop 2024

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MAP Collaboration

March 19th

Istituto Nazionale di Fisica Nucleare

UNIVERSITÀ DI PAVIA

Outline

- Model Construction
- Fit of pion collinear PDFs
- Fit of e.m. Form Factors
- Work on pion TMD PDFs

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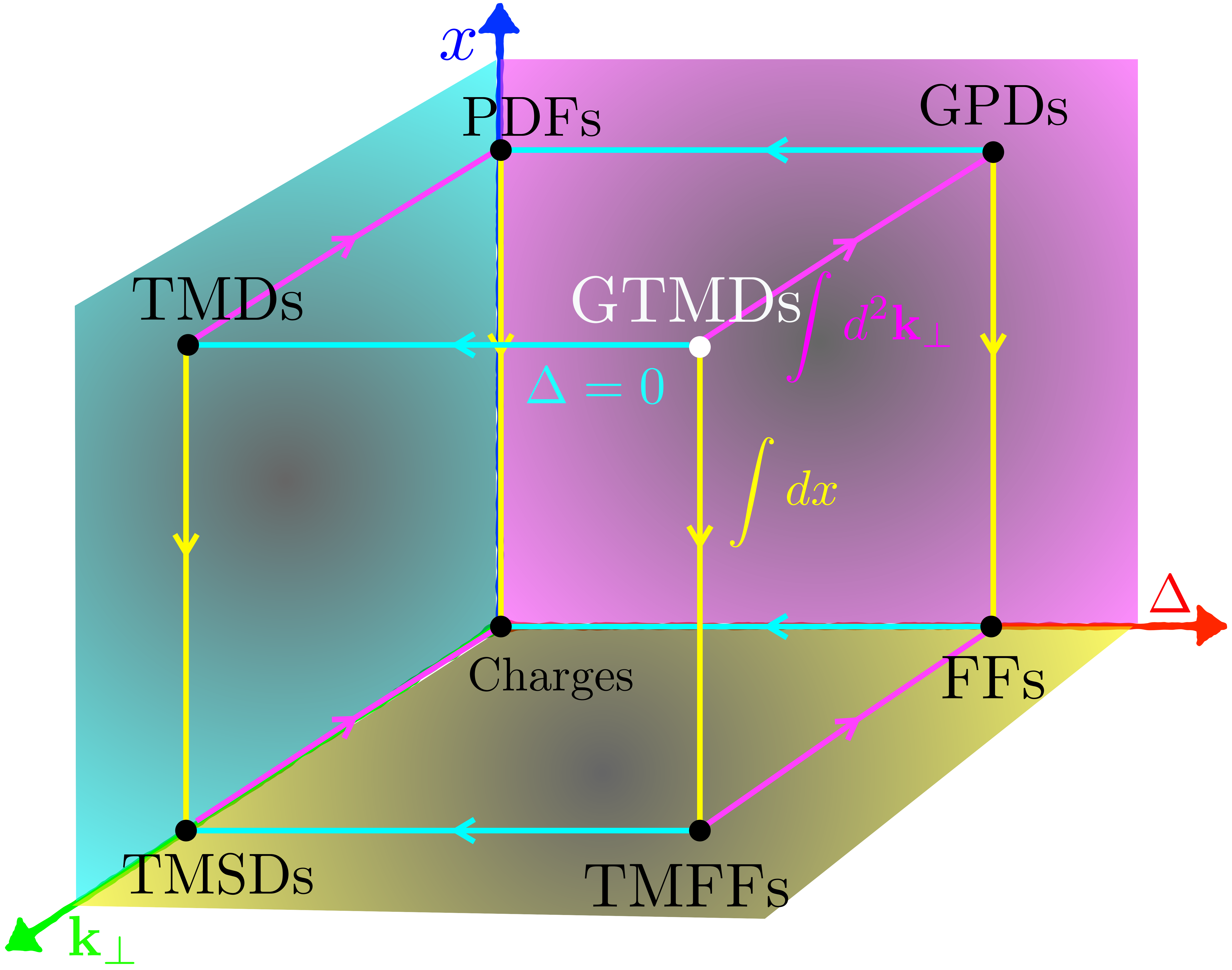
Model Construction



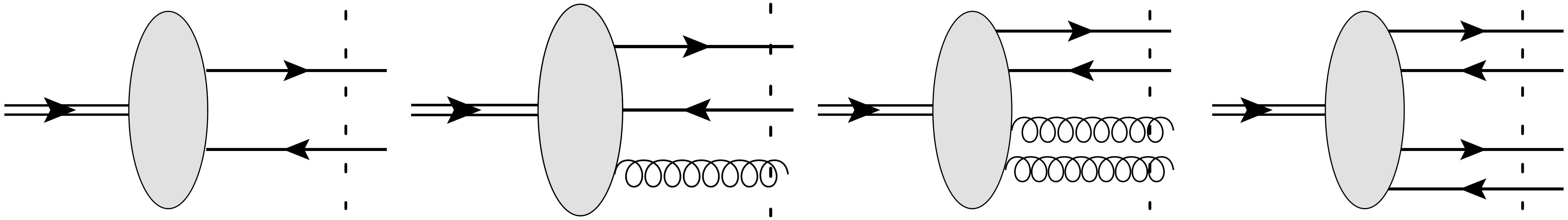
Model Construction

Fock state decomposition

Model Construction

Fock state decomposition

$$|\pi(P)\rangle = \psi_{q\bar{q}}|\pi(P)_{q\bar{q}}\rangle + \psi_{q\bar{q}g}|\pi(P)_{q\bar{q}g}\rangle + \psi_{q\bar{q}gg}|\pi(P)_{q\bar{q}gg}\rangle + \sum_{\{s\bar{s}\}} \psi_{q\bar{q}s\bar{s}}|\pi(P)_{q\bar{q}\{s\bar{s}\}}\rangle$$



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Model Construction

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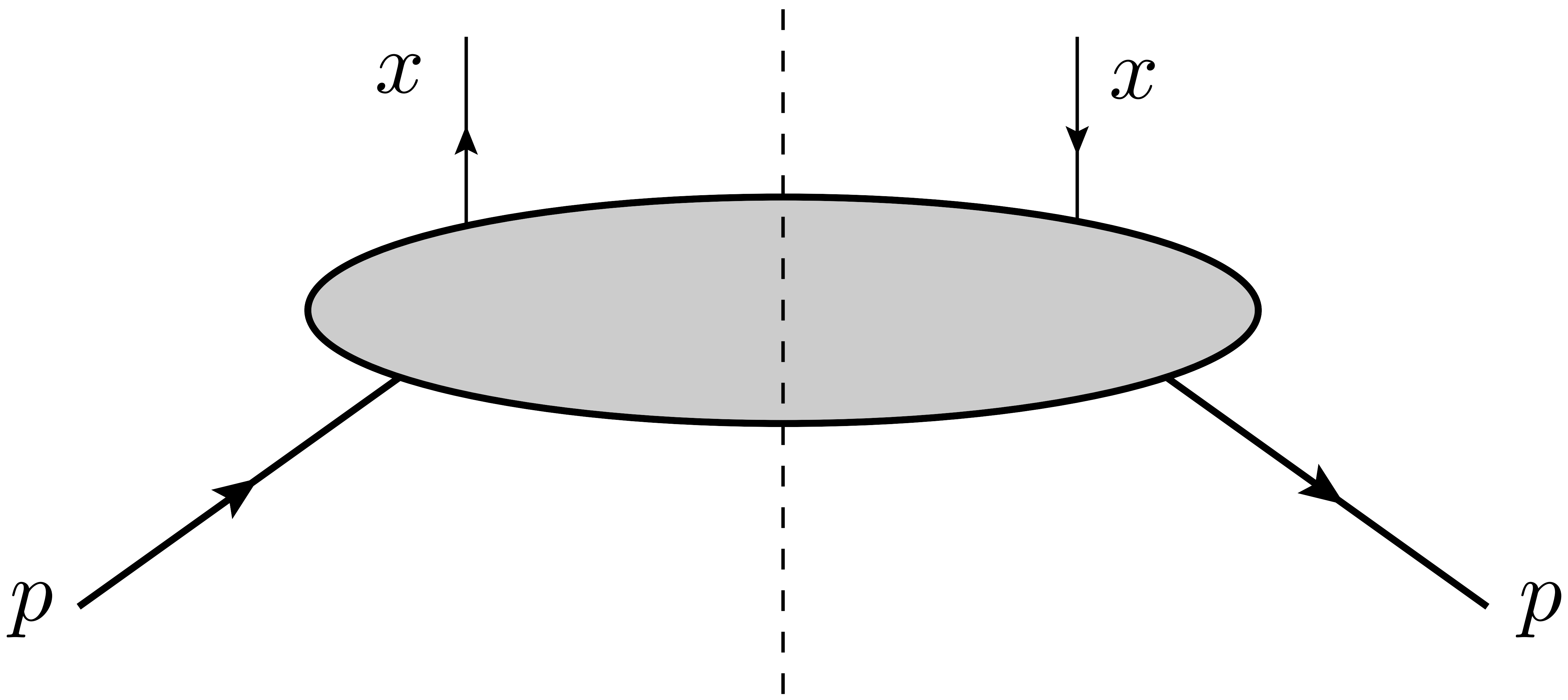
Parton distribution functions

Model Construction

Parton distribution functions

$$f_1^q(x) = \int \frac{d\zeta^-}{2(2\pi)} e^{ik^+\zeta^-} \langle \pi(p)|\bar{\Psi}^q(0)\gamma^+\Psi^q(\zeta)|\pi(p)\rangle \Big|_{\substack{\zeta^+=0 \\ \boldsymbol{\zeta}_\perp=\mathbf{0}}}$$



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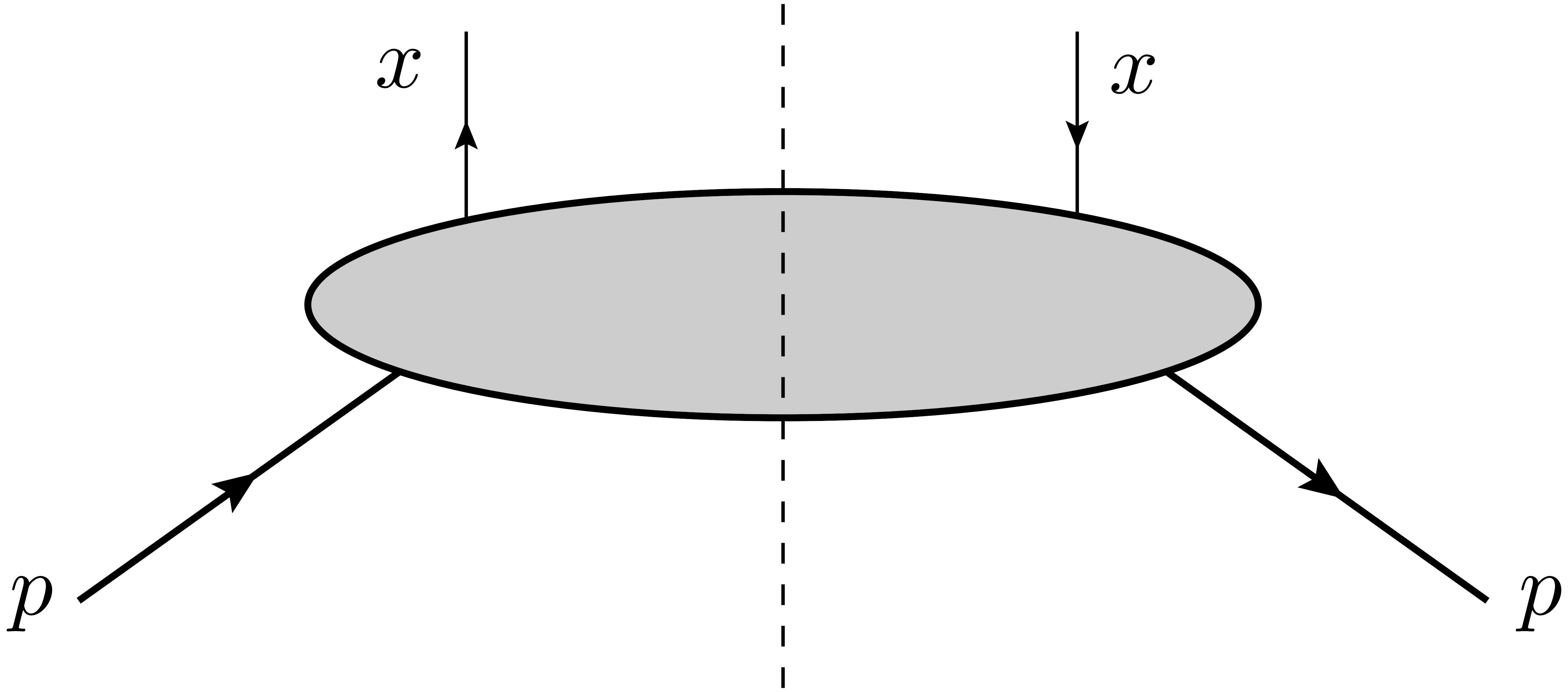

Model Construction

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Model for LFWFs

Model Construction

Model for LFWFs

$$\psi_{q\bar{q}}(1,2) = \phi_{q\bar{q}}(x_1,x_2)\,\Omega_{q\bar{q}}(x_1,x_2,\boldsymbol{k}_{\perp 1},\boldsymbol{k}_{\perp 2})$$

$$\psi_{q\bar{q}g}(1,2,3) = \phi_{q\bar{q}g}(x_1,x_2,x_3)\,\Omega_{q\bar{q}g}(x_1,x_2,x_3,\boldsymbol{k}_{\perp 1},\boldsymbol{k}_{\perp 2},\boldsymbol{k}_{\perp 3})$$

$$\psi_{q\bar{q}gg}(1,2,3,4) = \phi_{q\bar{q}gg}(x_1,x_2,x_3,x_4)\,\Omega_{q\bar{q}gg}(x_1,x_2,x_3,x_4,\boldsymbol{k}_{\perp 1},\boldsymbol{k}_{\perp 2},\boldsymbol{k}_{\perp 3},\boldsymbol{k}_{\perp 4})$$

$$\psi_{q\bar{q}s\bar{s}}(1,2,3,4) = \phi_{q\bar{q}s\bar{s}}(x_1,x_2,x_3,x_4)\,\Omega_{q\bar{q}s\bar{s}}(x_1,x_2,x_3,x_4,\boldsymbol{k}_{\perp 1},\boldsymbol{k}_{\perp 2},\boldsymbol{k}_{\perp 3},\boldsymbol{k}_{\perp 4})$$

Model Construction

Model for LFWFs

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$\phi_{q\bar{q}}, \phi_{q\bar{q}g}, \phi_{q\bar{q}gg}, \phi_{q\bar{q}s\bar{s}}$ ⟷ **Pion Distribution Amplitudes**

$$\langle 0|\bar{u}(z)\,\Gamma\,\mathcal{U}_{(z,-z)}\, d(-z)|\pi^-(p)\rangle$$

Model Construction

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Model Construction

Model for LFWFs $\prod_{i=1}^{N} x_i^{2j_i-1}, \quad j_i = \begin{cases} 1, & i = q, \bar{q} \\ 3/2, & i = g \end{cases}$ lowest conformal spin representation

V. M. Braun and I. E. Filyanov, Z. Phys. C 48, 239 (1990)

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$$\phi_{q\bar{q}}(x_1, x_2) = \mathcal{N}_{q\bar{q}}(x_1 \; x_2)^{\gamma_q} \sum_{n=1}^{2} C_{q_n} \left(\mathcal{G}_n^{\gamma_q+\frac{1}{2}} (2x_1 - 1) + \mathcal{G}_n^{\gamma_q+\frac{1}{2}} (2x_2 - 1) \right)$$

$$\phi_{q\bar{q}g}(x_1, x_2, x_3) = \mathcal{N}_{q\bar{q}g} \; x_1 x_2 \; x_3^2 \sum_{N=1} \sum_{n=1} C_n^N \Bigg((1 - x_3)^n \mathcal{J}_{N-n}^{(2,6)}(1 - 2x_3) \mathcal{J}_n^{(1,1)}\left(\frac{x_2 - x_1}{1 - x_3}\right) + (1 - x_3)^n \mathcal{J}_{N-n}^{(2,6)}(1 - 2x_3) \mathcal{J}_n^{(1,1)}\left(\frac{x_1 - x_2}{1 - x_3}\right) \Bigg)$$

$$\phi_{q\bar{q}gg}^{(1)}(x_1, x_2, x_3, x_4) = N_{qqgg}^{(1)} x_1 x_2 (x_3 x_4)^2 \qquad \phi_{q\bar{q}gg}^{(2)}(x_1, x_2, x_3, x_4) = N_{q\bar{q}gg}^{(2)}(x_1 - x_2)\sqrt{x_1 x_2}\, x_3 x_4 (x_3 - x_4)(x_3 + x_4)$$

$$\phi_{q\bar{q}s\bar{s}}^{(1)}(x_1, x_2, x_3, x_4) = N_{q\bar{q}s\bar{s}}^{(1)} x_1 x_2 \sqrt{x_3 x_4}(x_3 - x_4) \quad \phi_{q\bar{q}s\bar{s}}^{(2)}(x_1, x_2, x_3, x_4) = N_{q\bar{q}s\bar{s}}^{(2)} x_3 x_4 \sqrt{x_1 x_2}(x_1 - x_2) \quad \phi_{q\bar{q}s\bar{s}}^{(3)}(x_1, x_2, x_3, x_4) = N_{q\bar{q}s\bar{s}}^{(3)} x_1 x_2 x_3 x_4$$

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2 (+1) parameters

$$\phi_{q\bar{q}g}(x_1, x_2, x_3) = \mathcal{N}_{q\bar{q}g}\ x_1 x_2\ x_3^2 \sum_{N=1} \sum_{n=1} C_n^N \Big((1 - x_3)^n \mathcal{J}_{N-n}^{(2,6)}(1 - 2x_3) \mathcal{J}_n^{(1,1)}\left(\frac{x_2 - x_1}{1 - x_3}\right)$$

$$+ (1 - x_3)^n \mathcal{J}_{N-n}^{(2,6)}(1 - 2x_3) \mathcal{J}_n^{(1,1)}\left(\frac{x_1 - x_2}{1 - x_3}\right) \Big)$$

1 (+1) parameters

$$\phi^{(1)}_{q\bar{q}gg}(x_1, x_2, x_3, x_4) = N^{(1)}_{qqgg} x_1 x_2 (x_3 x_4)^2 \qquad \phi^{(2)}_{q\bar{q}gg}(x_1, x_2, x_3, x_4) = N^{(2)}_{q\bar{q}gg}(x_1 - x_2)\sqrt{x_1 x_2}\, x_3 x_4 (x_3 - x_4)(x_3 + x_4)$$

$$\phi^{(1)}_{q\bar{q}s\bar{s}}(x_1, x_2, x_3, x_4) = N^{(1)}_{q\bar{q}s\bar{s}} x_1 x_2 \sqrt{x_3 x_4}(x_3 - x_4) \quad \phi^{(2)}_{q\bar{q}s\bar{s}}(x_1, x_2, x_3, x_4) = N^{(2)}_{q\bar{q}s\bar{s}} x_3 x_4 \sqrt{x_1 x_2}(x_1 - x_2) \quad \phi^{(3)}_{q\bar{q}s\bar{s}}(x_1, x_2, x_3, x_4) = N^{(3)}_{q\bar{q}s\bar{s}} x_1 x_2 x_3 x_4$$

1 parameters

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Model Construction

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Model for LFWFs

$$\Omega_{N,\beta}\left(x_1, \boldsymbol{k}_{\perp 1}, x_2, \boldsymbol{k}_{\perp 2}, \ldots, x_N, \boldsymbol{k}_{\perp N}\right) = \frac{(16\pi^2 a_\beta^2)^{N-1}}{\prod_{i=1}^{N} x_i} \exp\left(-a_\beta^2 \sum_{i=1}^{N} \frac{\boldsymbol{k}_{\perp i}^2}{x_i}\right)$$

S. J. Brodsky, T. Huang, P. Lepage (1993)

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$$\int \left[d^2 \boldsymbol{k}_\perp\right]_N \Omega_{N,\beta} = 1$$

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collinear PDFs

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Model Construction

"State of the art" of the model

$$\mathcal{A}^L = \{\gamma_q, d_{q1}, d_{g1}, \alpha_1, \alpha_2, \alpha_3\}$$

$$\mathcal{A}^T = \{a^{(1)}_{q\bar{q}}, a^{(1)}_{q\bar{q}g}, a^{(1)}_{q\bar{q}gg}, a^{(2)}_{q\bar{q}gg}, a^{(1)}_{q\bar{q}s\bar{s}}, a^{(2)}_{q\bar{q}s\bar{s}}, a^{(3)}_{q\bar{q}s\bar{s}}\}$$

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Fit of collinear PDFs ($\mathcal{A}^L$)

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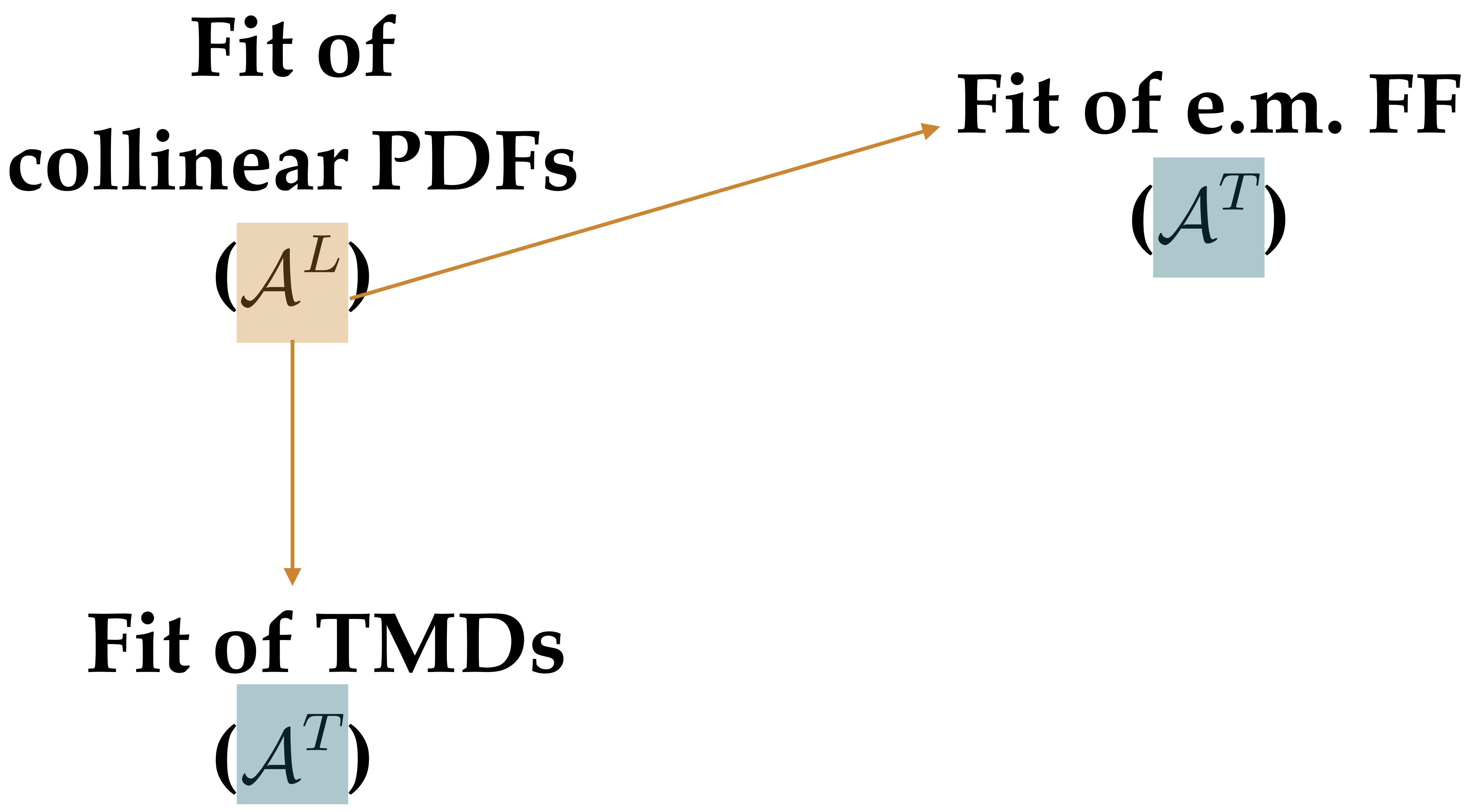

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Fit of collinear PDFs ($\mathcal{A}^L$)

Fit of e.m. FF ($\mathcal{A}^T$)

Fit of TMDs ($\mathcal{A}^T$)

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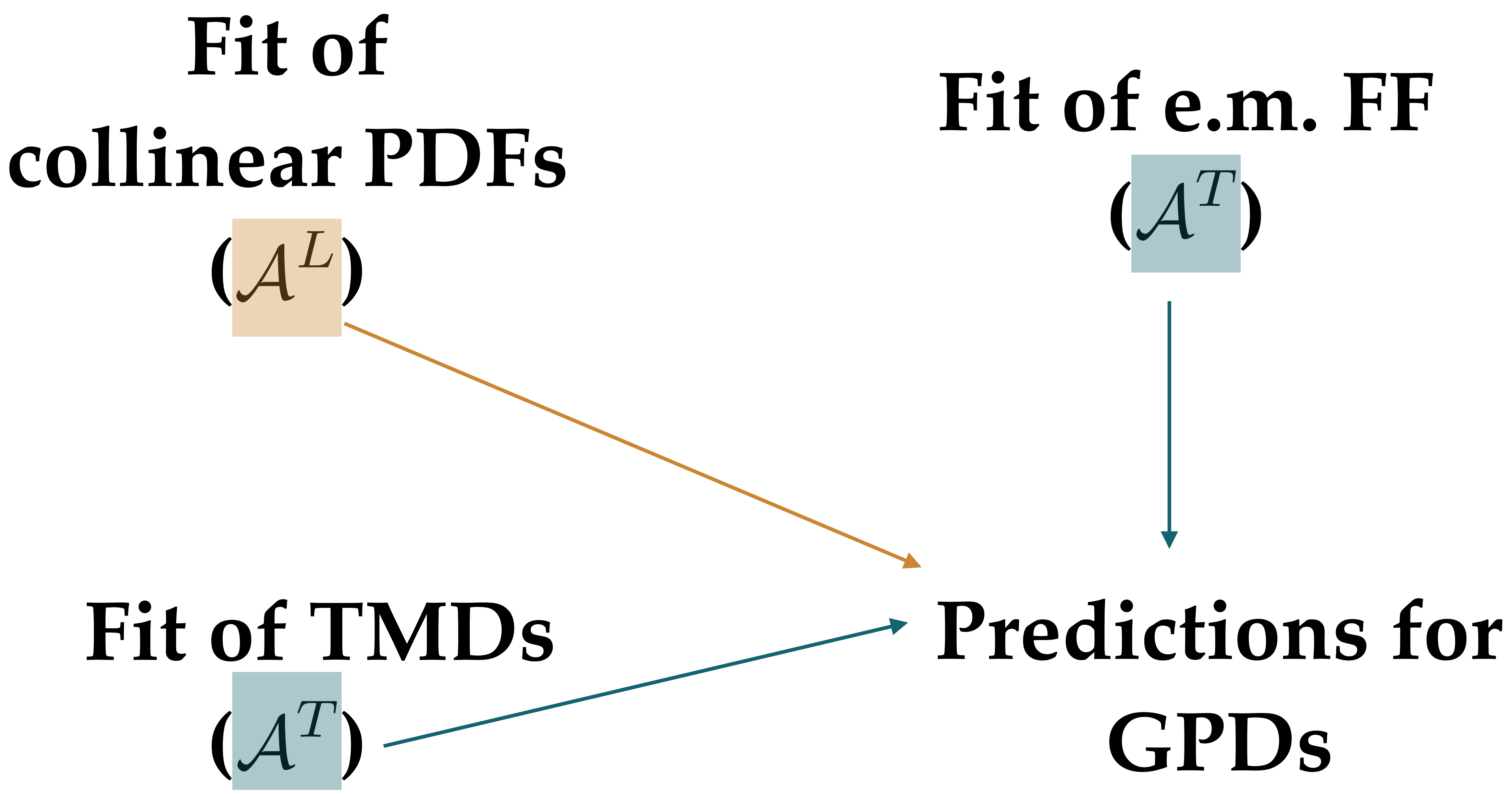

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Fit of pion collinear PDFs

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Fit of pion collinear PDFs

Experimental data:

N_{points}	N_{par}	$\hat{\chi}^2/N_{\text{d.o.f.}}$
260	**6**	**0.884**

↕

NA10	E615	WA70
70	91	99

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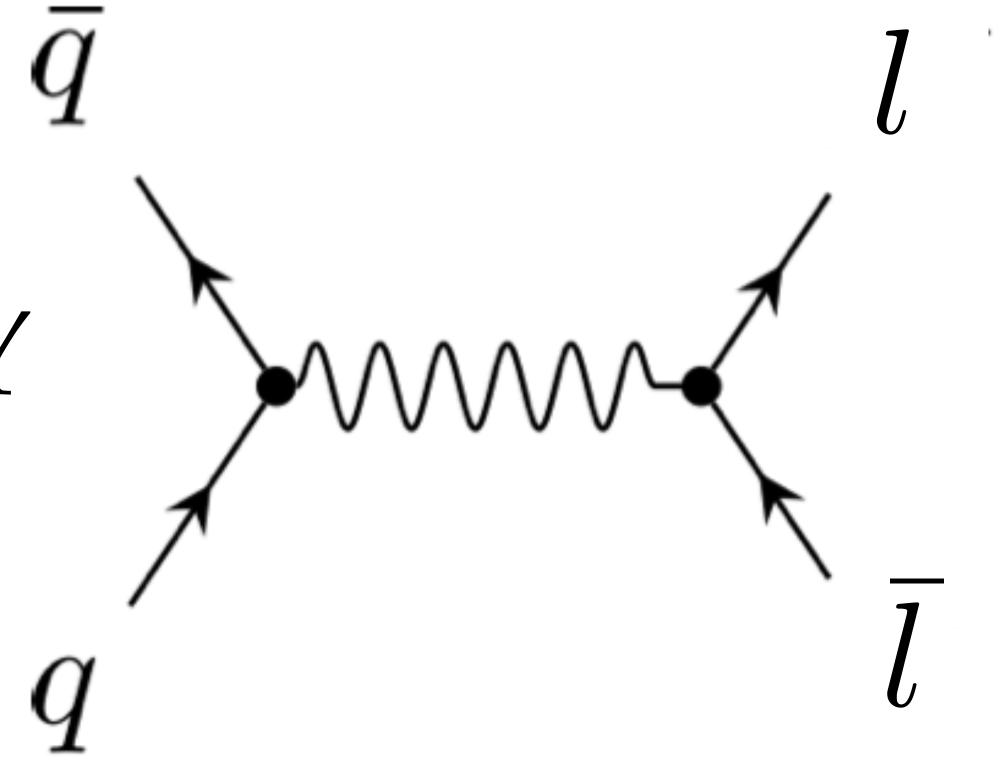

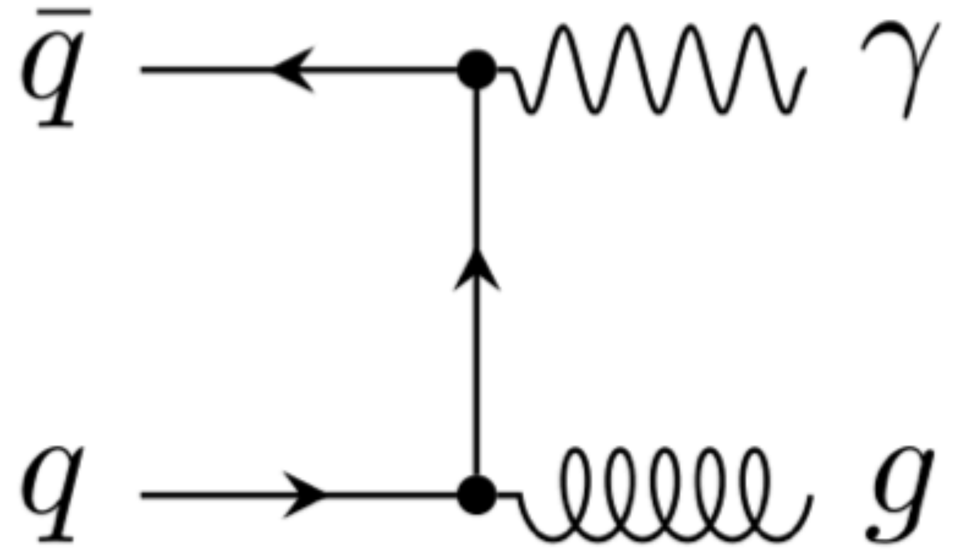

Prompt photon production

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Fit of pion collinear PDFs

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xFitter

Phys. Rev. D 102 (2022) 1, 014040

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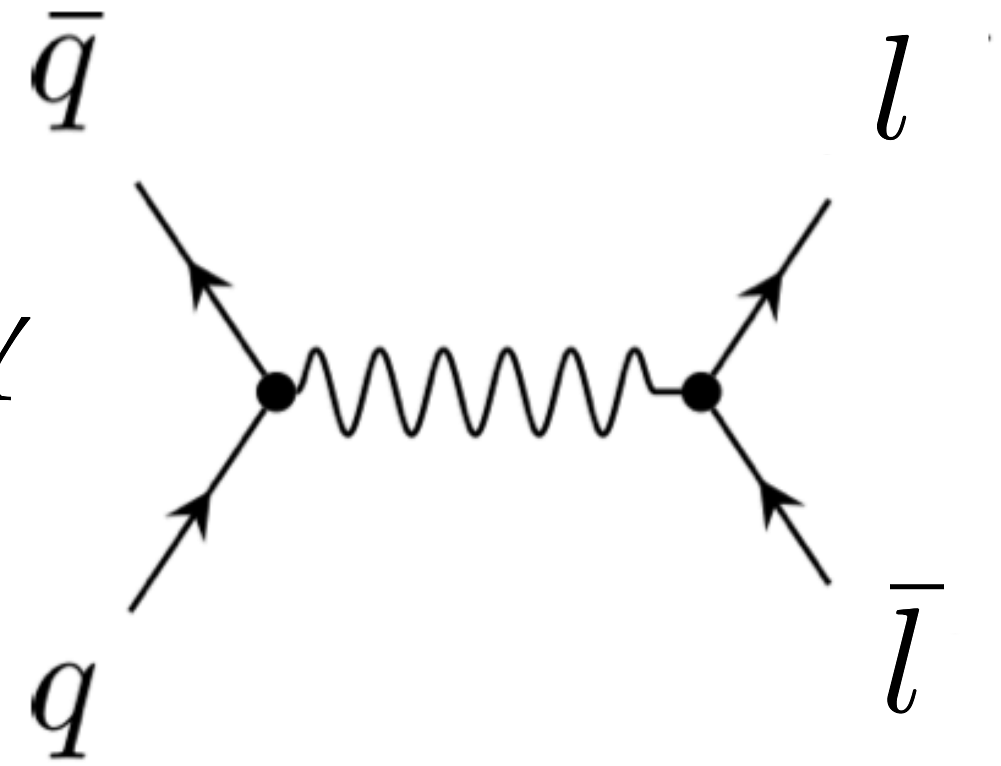

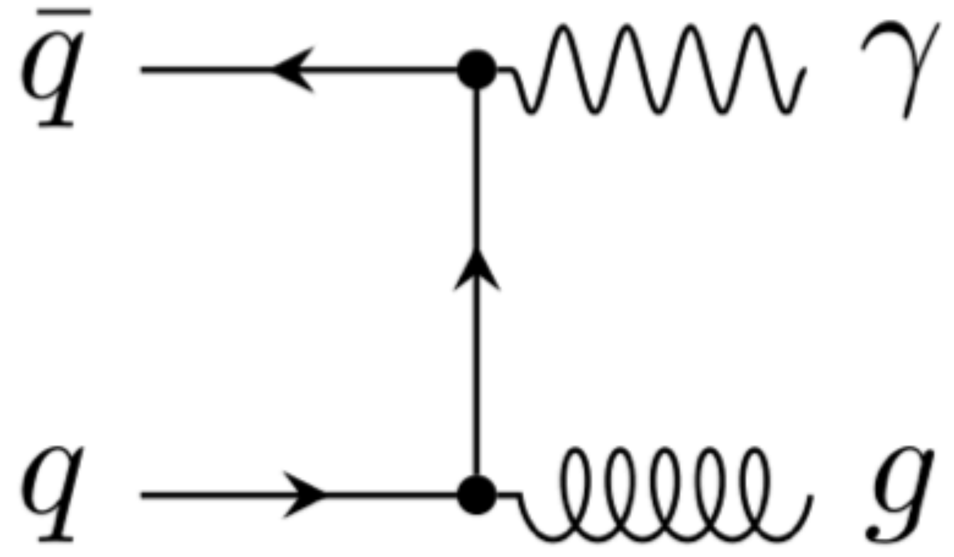

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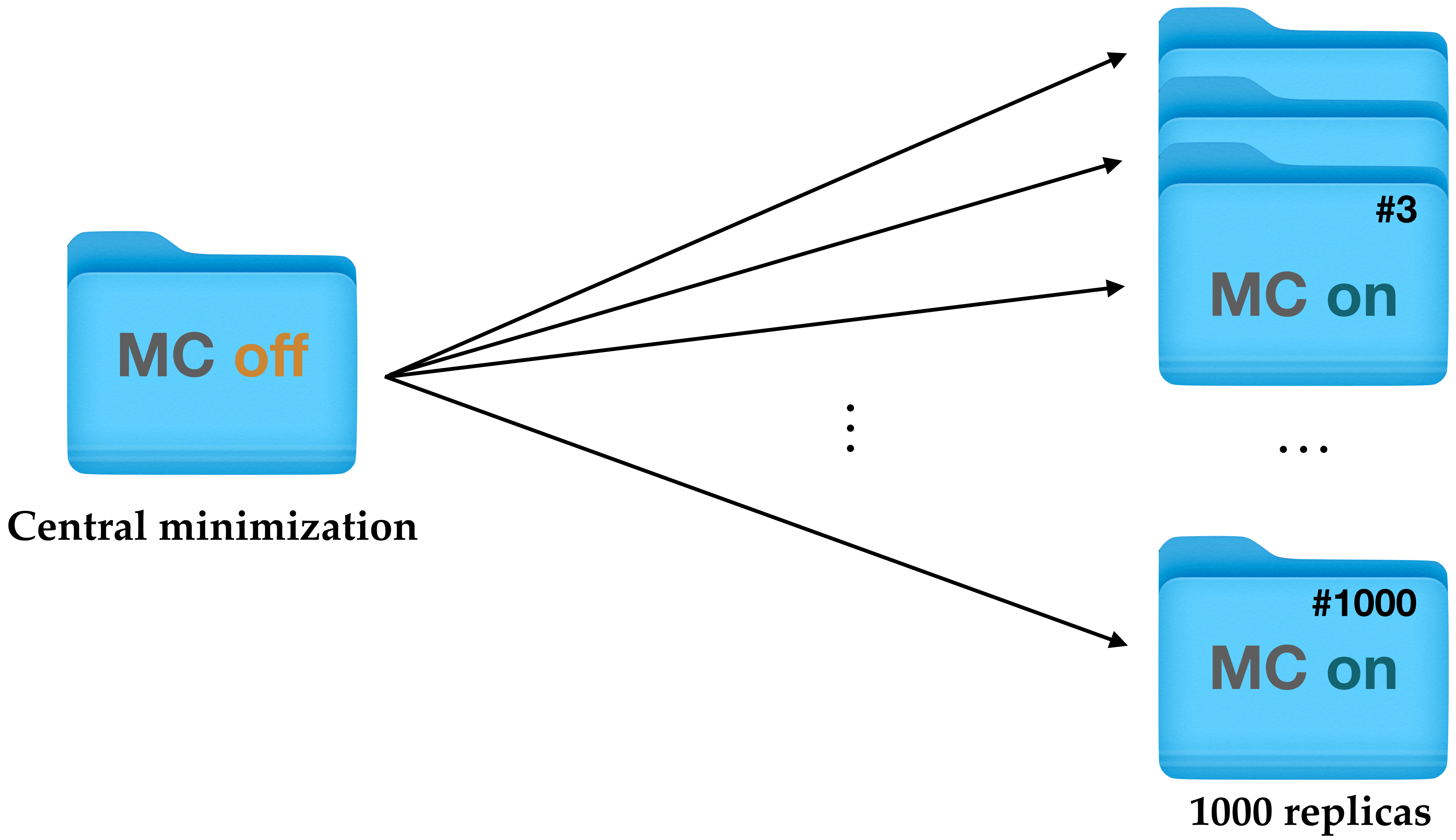

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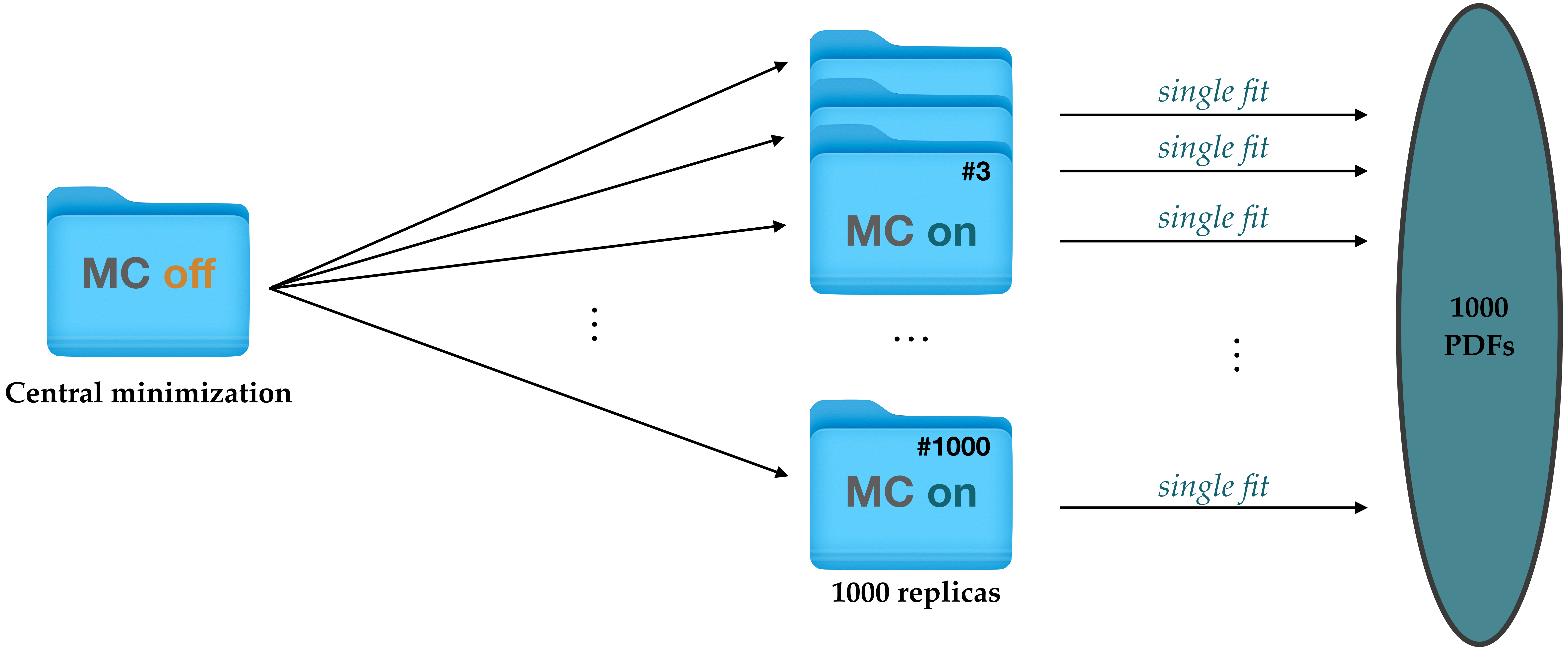

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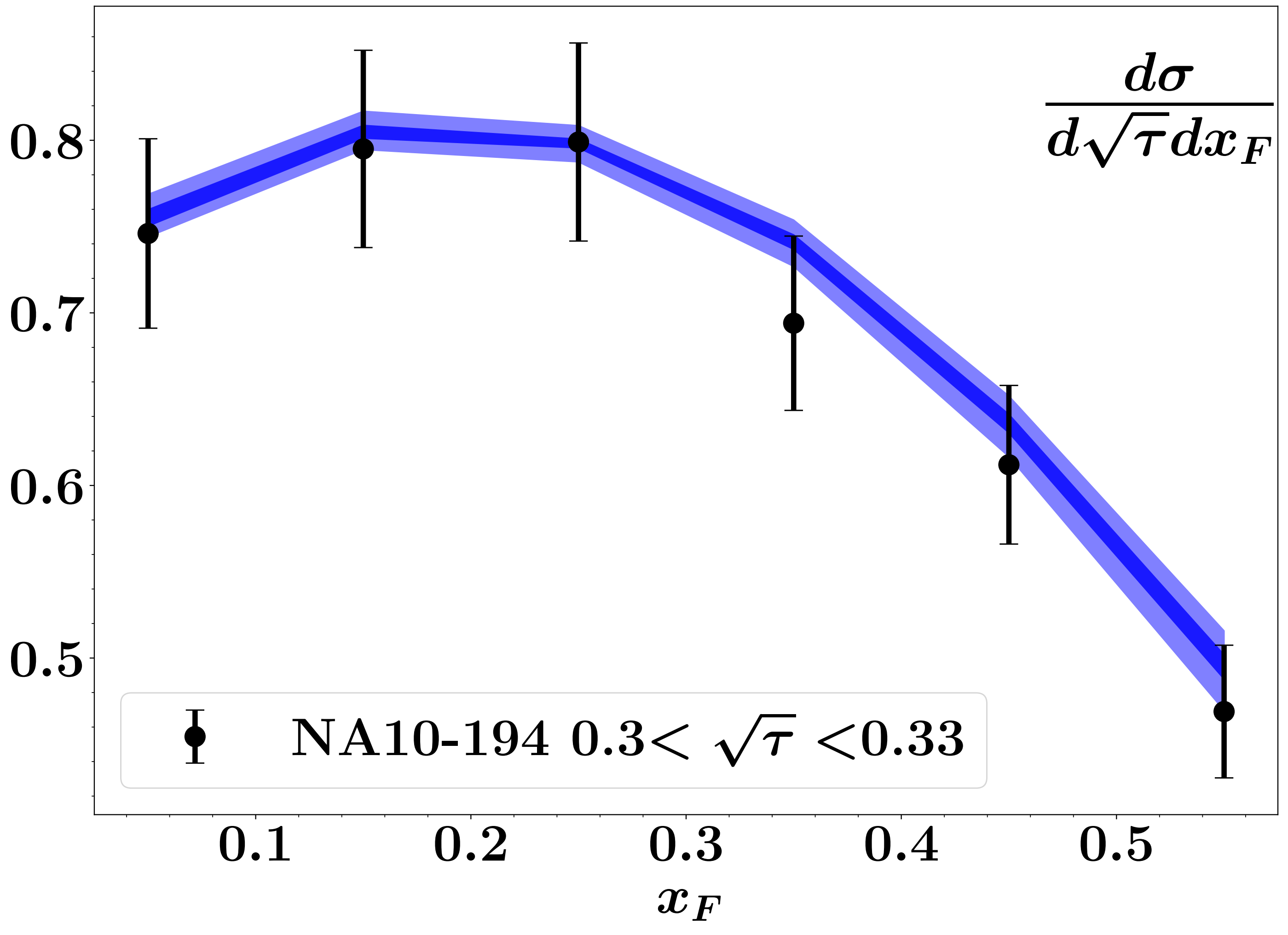
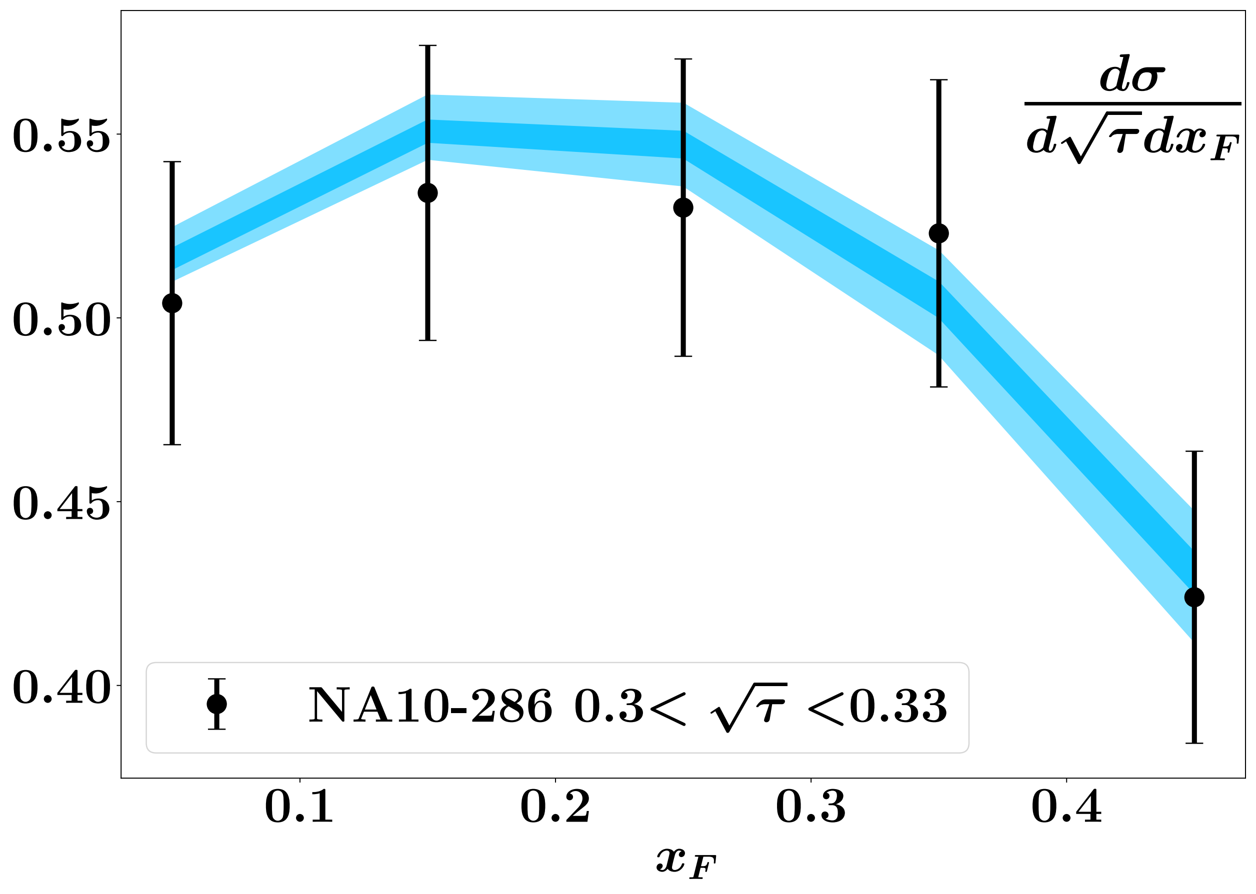
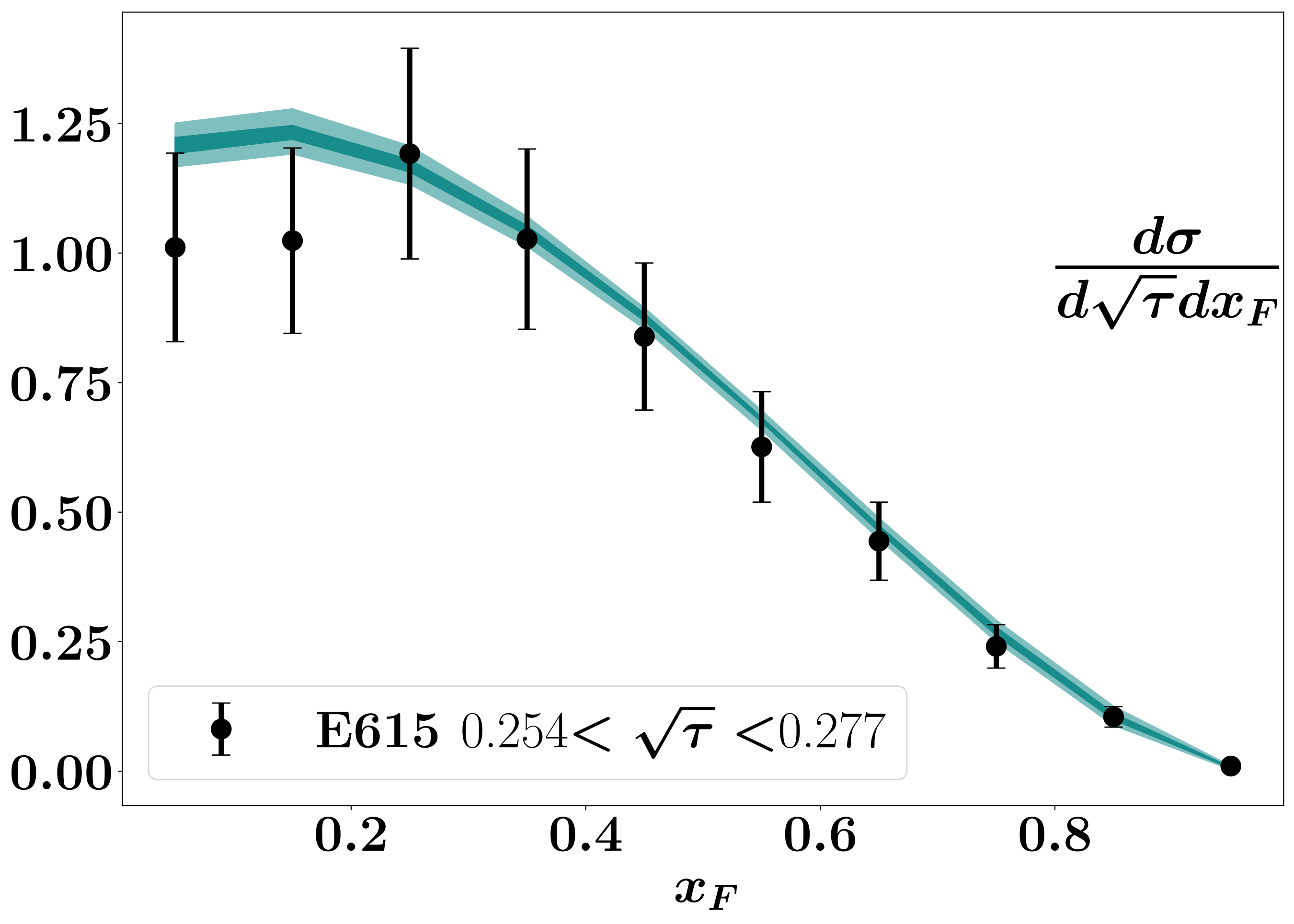
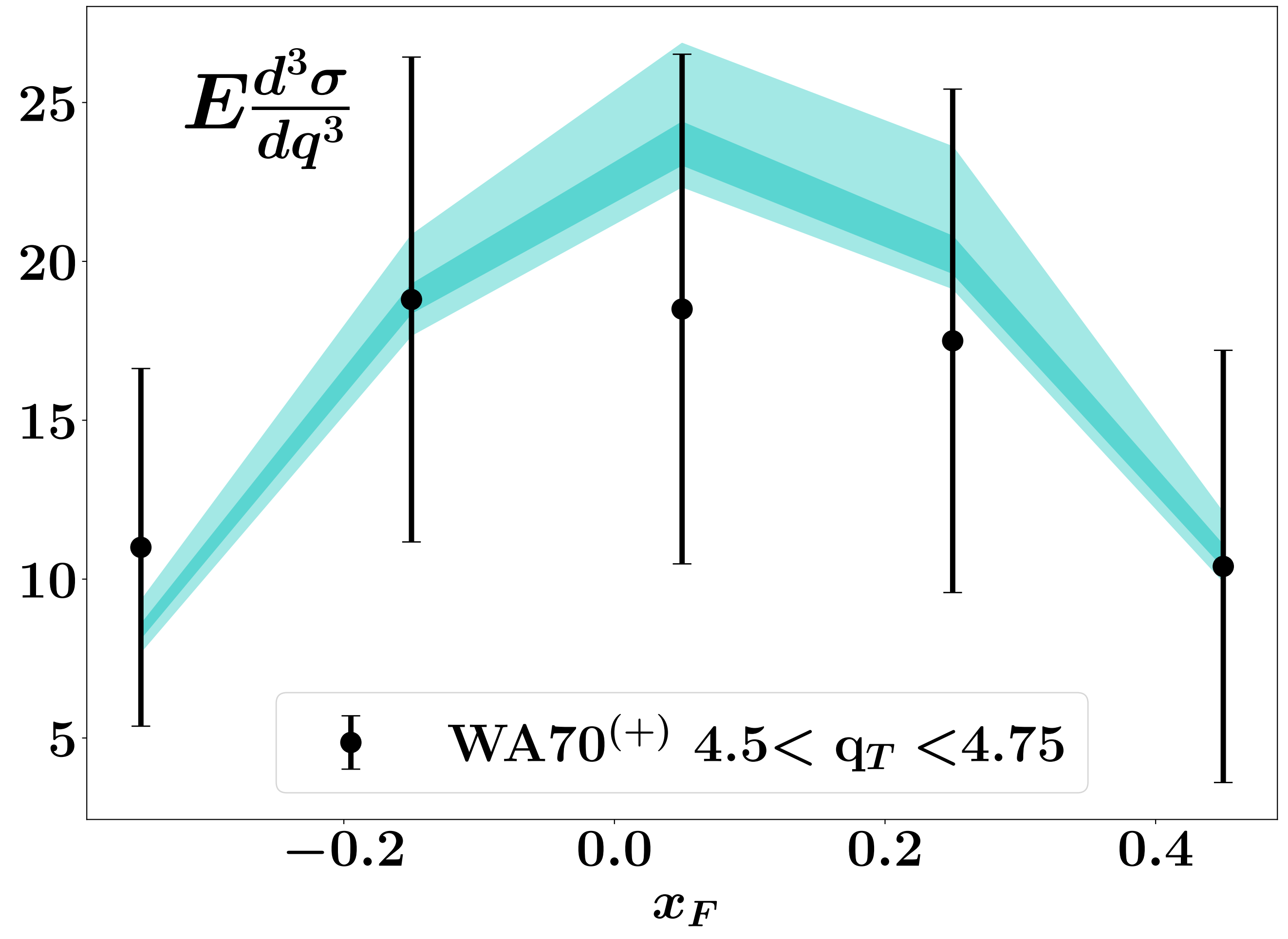


Z. Phys. C 28 (1985) 9.

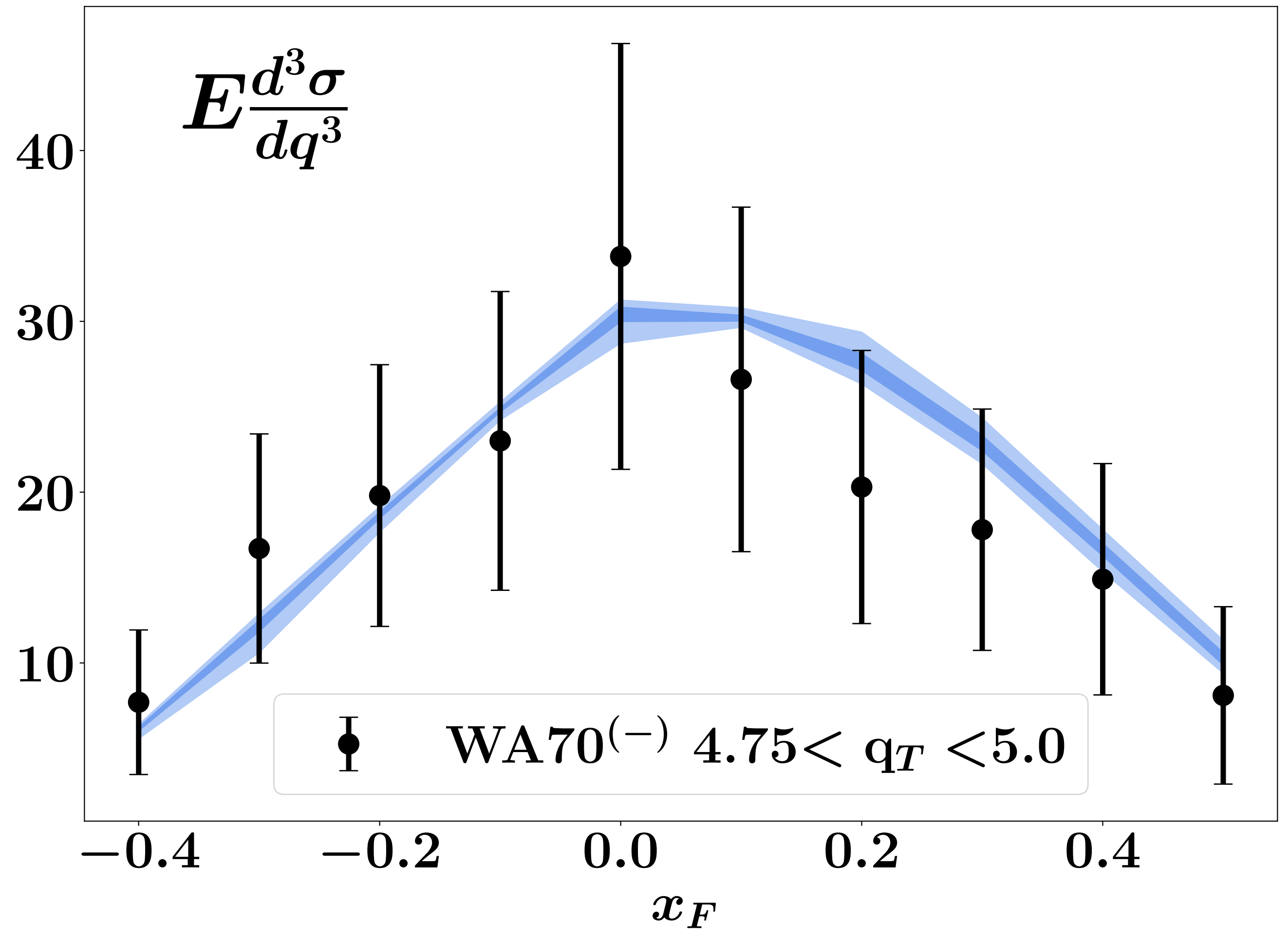




Phys. Rev. D 39 (1989) 92.



Z. Phys. C 37 (1988) 535.



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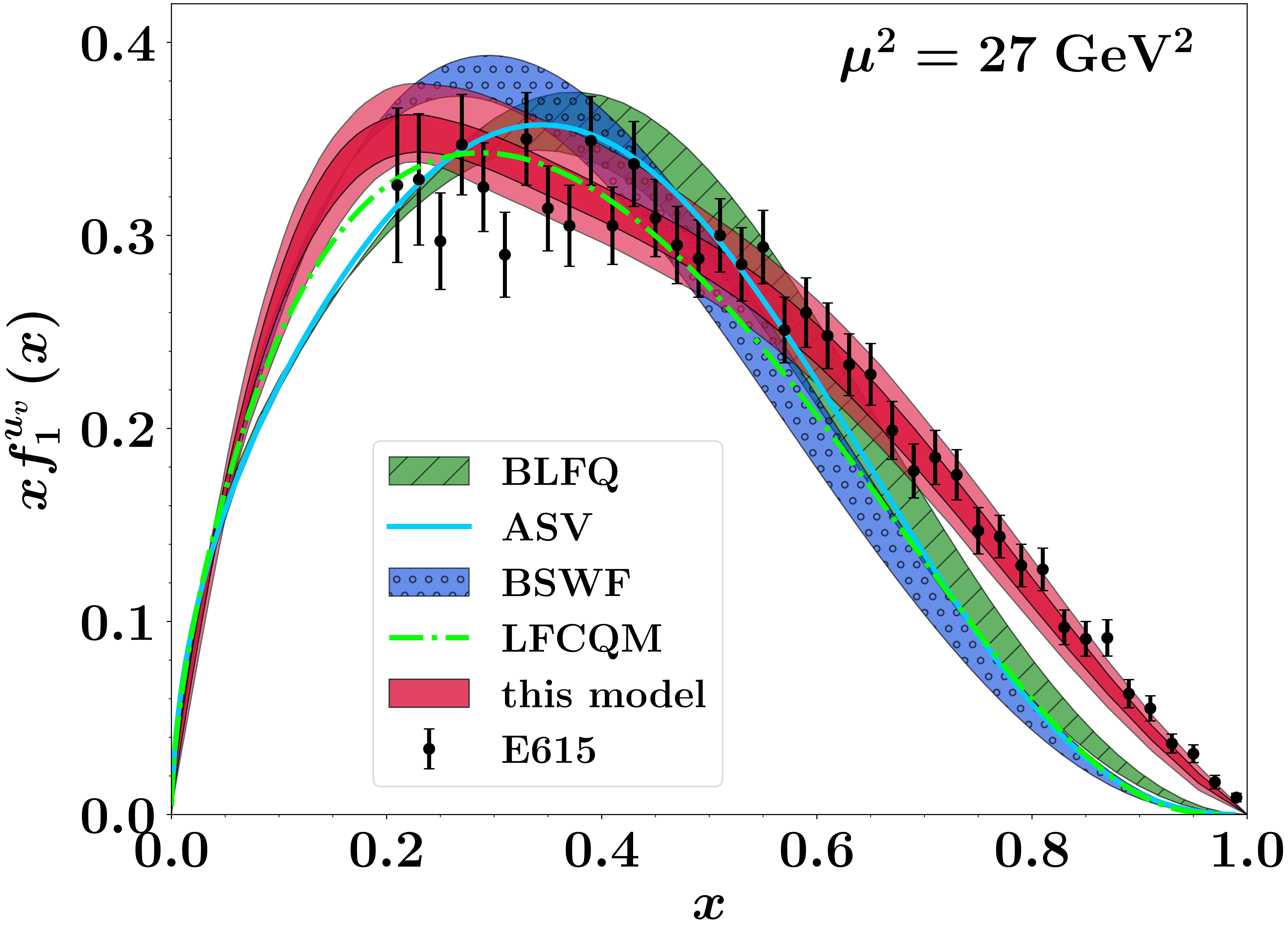
Fit of pion collinear PDFs



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M. Aicher, A. Schafer, W. Vogelsang, *Phys.Rev.Lett. 105, 252003* (2010)

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B. Pasquini, P. Schweitzer, *Phys.Rev.D 90, 014050* (2014)

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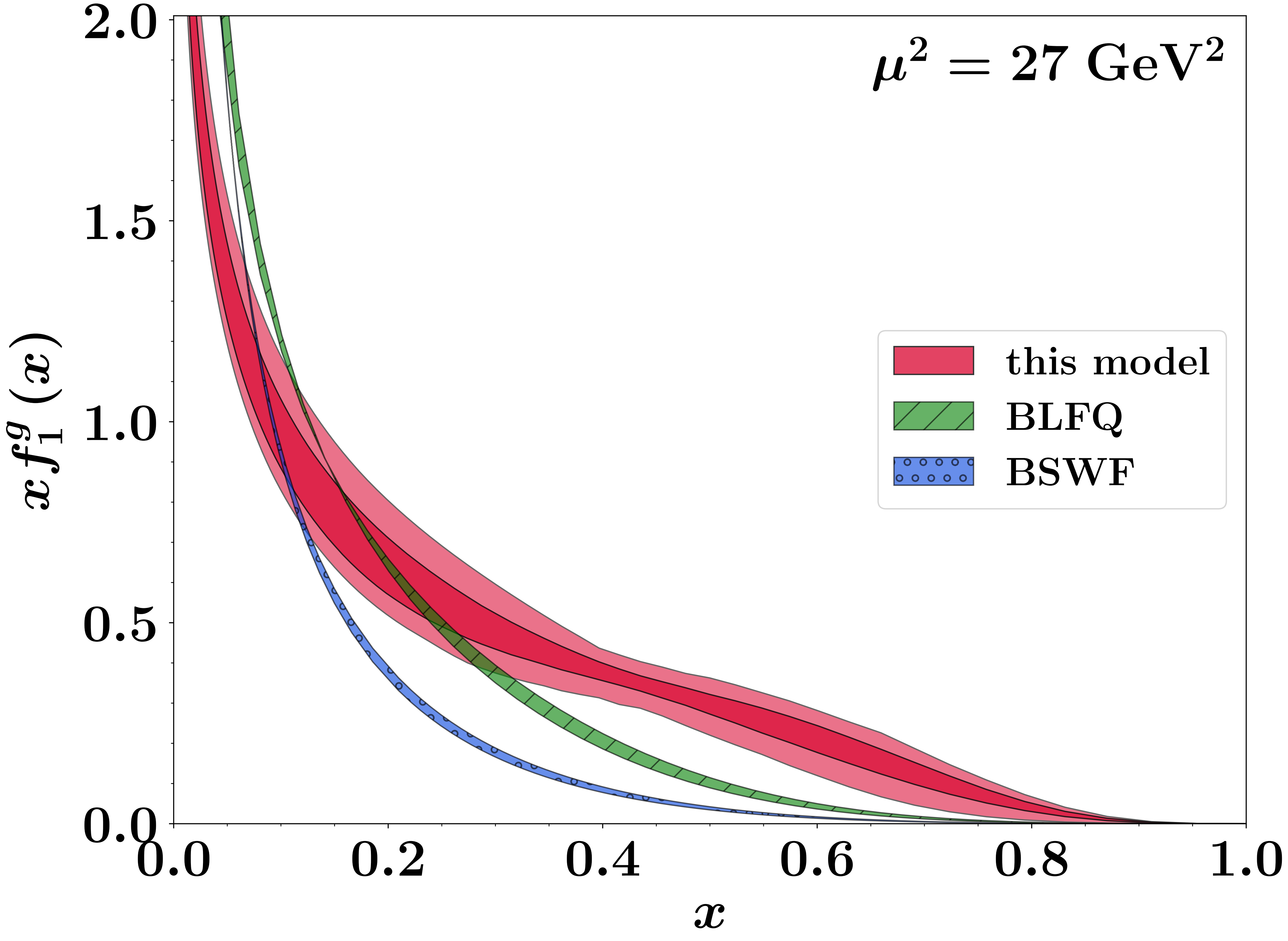
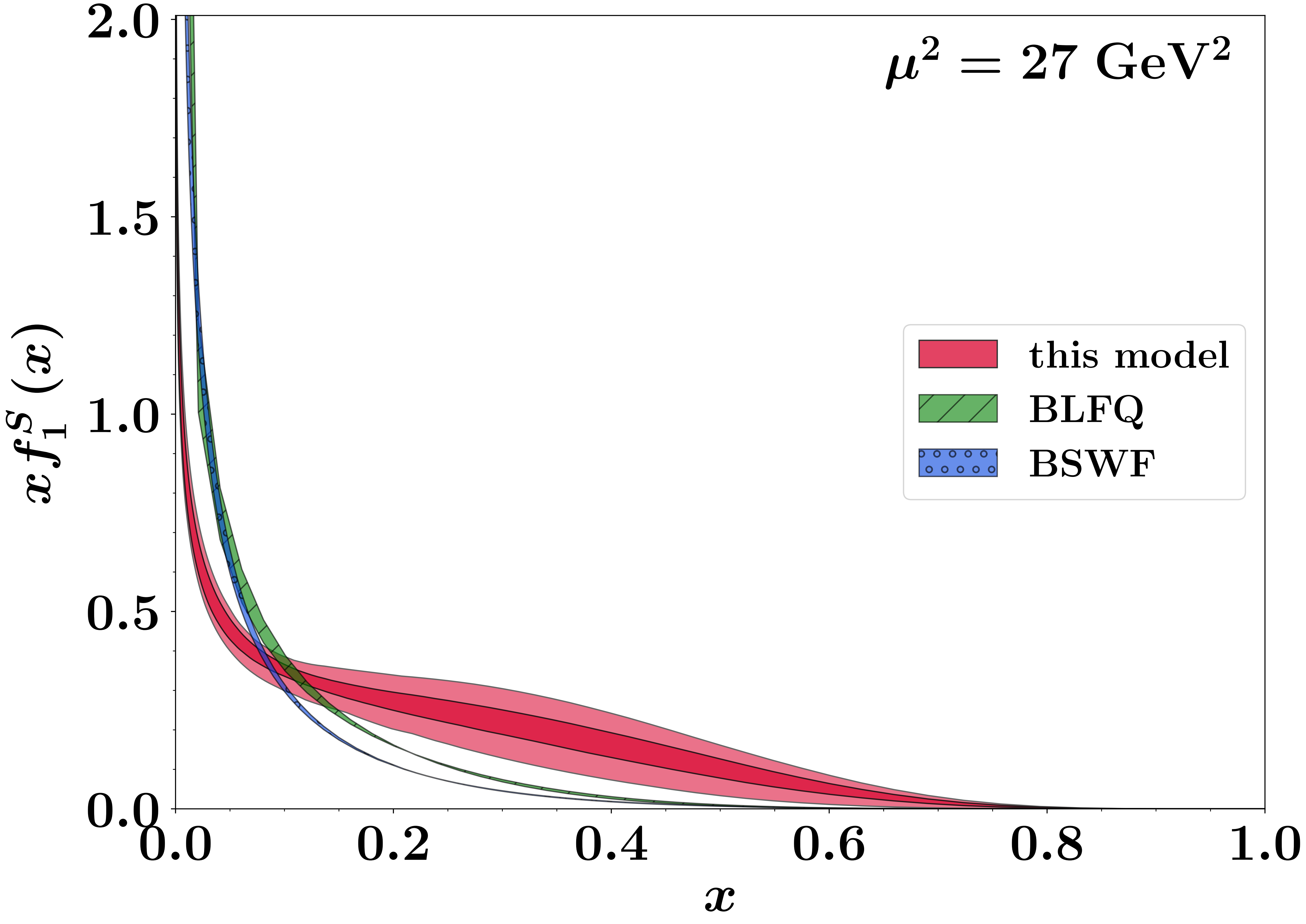
Fit of pion collinear PDFs





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Outline

- Model Construction
- Fit of pion collinear PDFs
- Fit of e.m. Form Factors

  MAP Collaboration, PRD 107 (2023) 11, 114023
- Work on pion TMD PDFs

Fit of e.m. Form Factors

MAP Collaboration, PRD 107 (2023) 11, 114023

Fit of e.m. Form Factors

$$F_1(\Delta) = \frac{1}{2P^+} \langle \pi(p') | \, \bar{\Psi}^q(0) \gamma^+ \Psi^q(0) \, | \pi(p) \rangle$$

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Fit of e.m. Form Factors

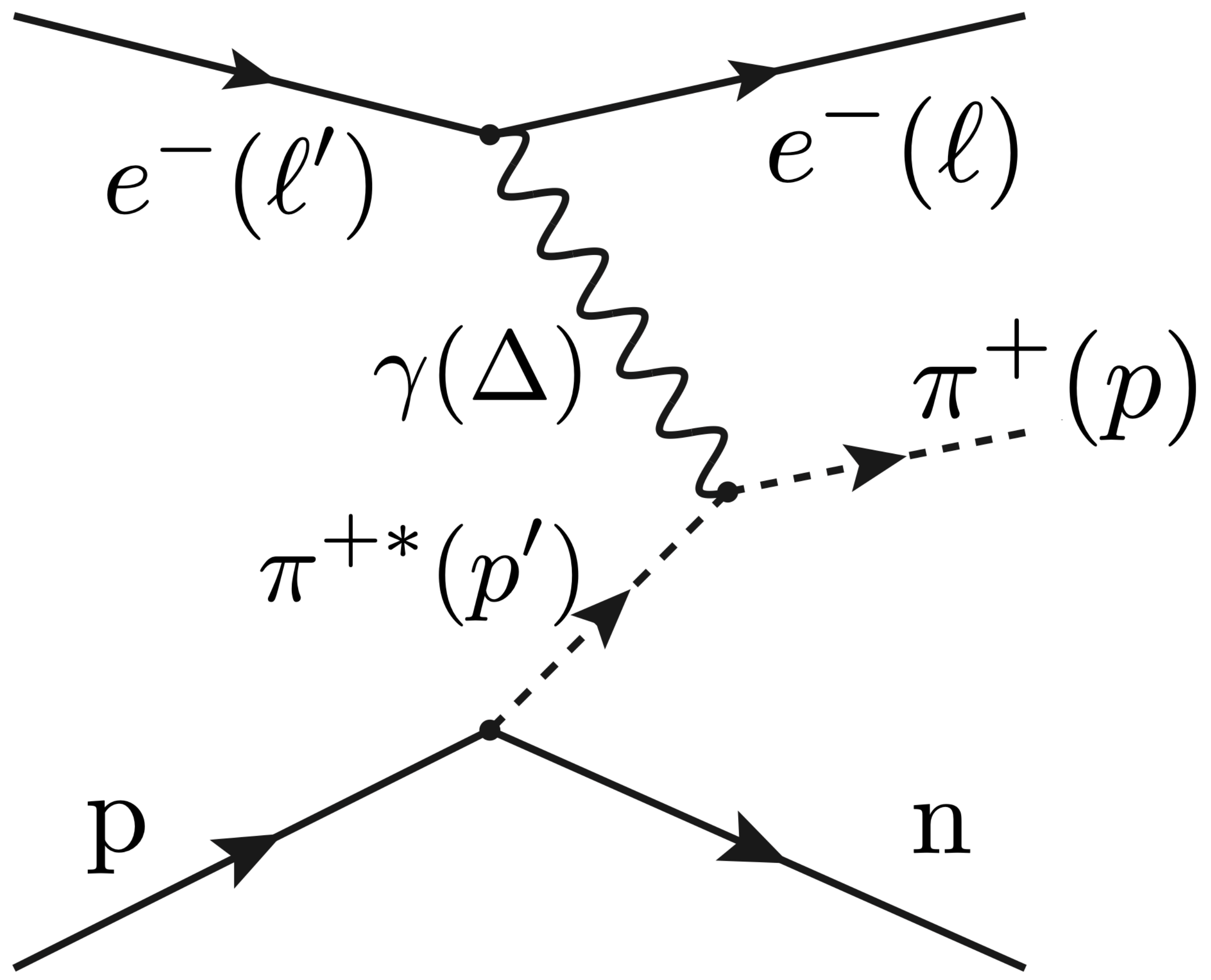
$$F_1(\Delta) = \frac{1}{2P^+} \langle \pi(p')| \bar{\Psi}^q(0)\gamma^+\Psi^q(0) |\pi(p)\rangle$$

$$p = P - \frac{\Delta}{2},$$
$$p' = P + \frac{\Delta}{2}$$

$$Q^2 = -\Delta^2 = \boldsymbol{\Delta}_\perp^2$$
$$\Delta \equiv (0, 0, \boldsymbol{\Delta}_\perp)$$

Sullivan process

J. D. Sullivan, Phys. Rev. D 5 (1972) 1732.



Fit of e.m. Form Factors

$N_{\rm points}$	$N_{\rm par}$	$\hat{\chi}^2/N_{\rm d.o.f.}$
100	**3 (4)**	**1.194**

Experimental data

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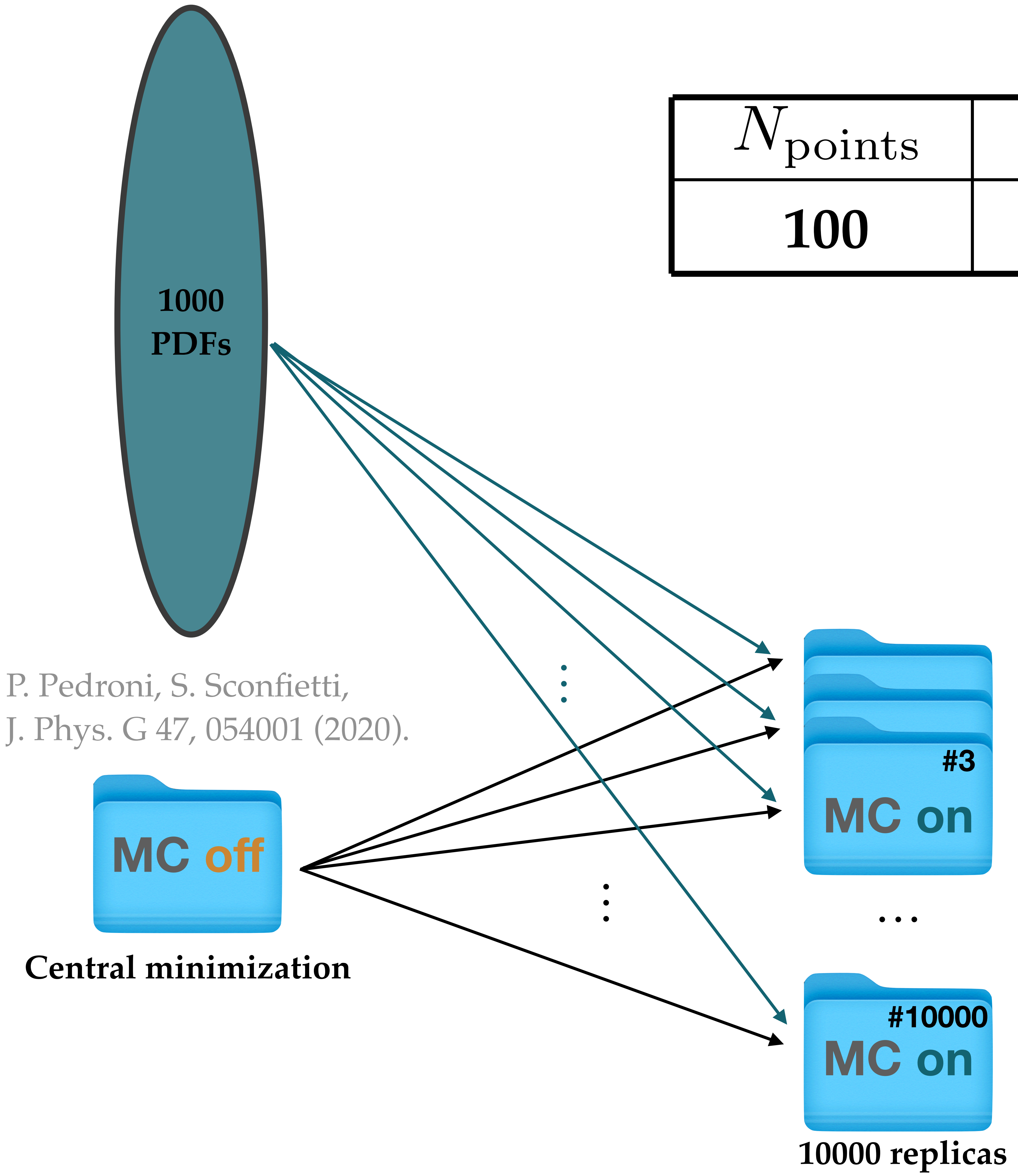
Fit of e.m. Form Factors



N_{points}	N_{par}	$\hat{\chi}^2/N_{\text{d.o.f.}}$
100	**3 (4)**	**1.194**

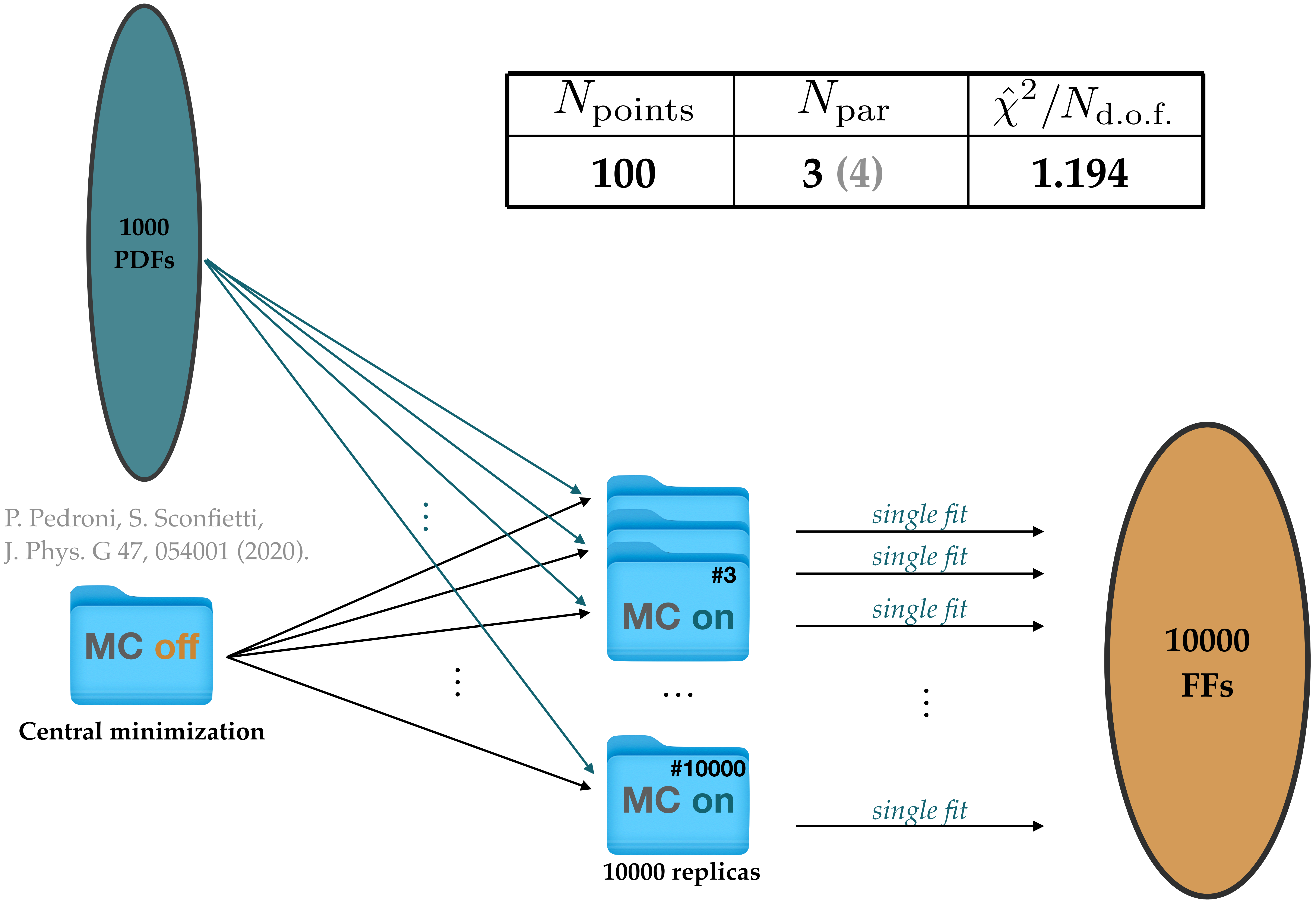
Experimental
data

MAP Collaboration, PRD 107 (2023) 11, 114023

Fit of e.m. Form Factors

N_{points}	N_{par}	$\hat{\chi}^2/N_{\text{d.o.f.}}$
100	**3 (4)**	**1.194**

Experimental data



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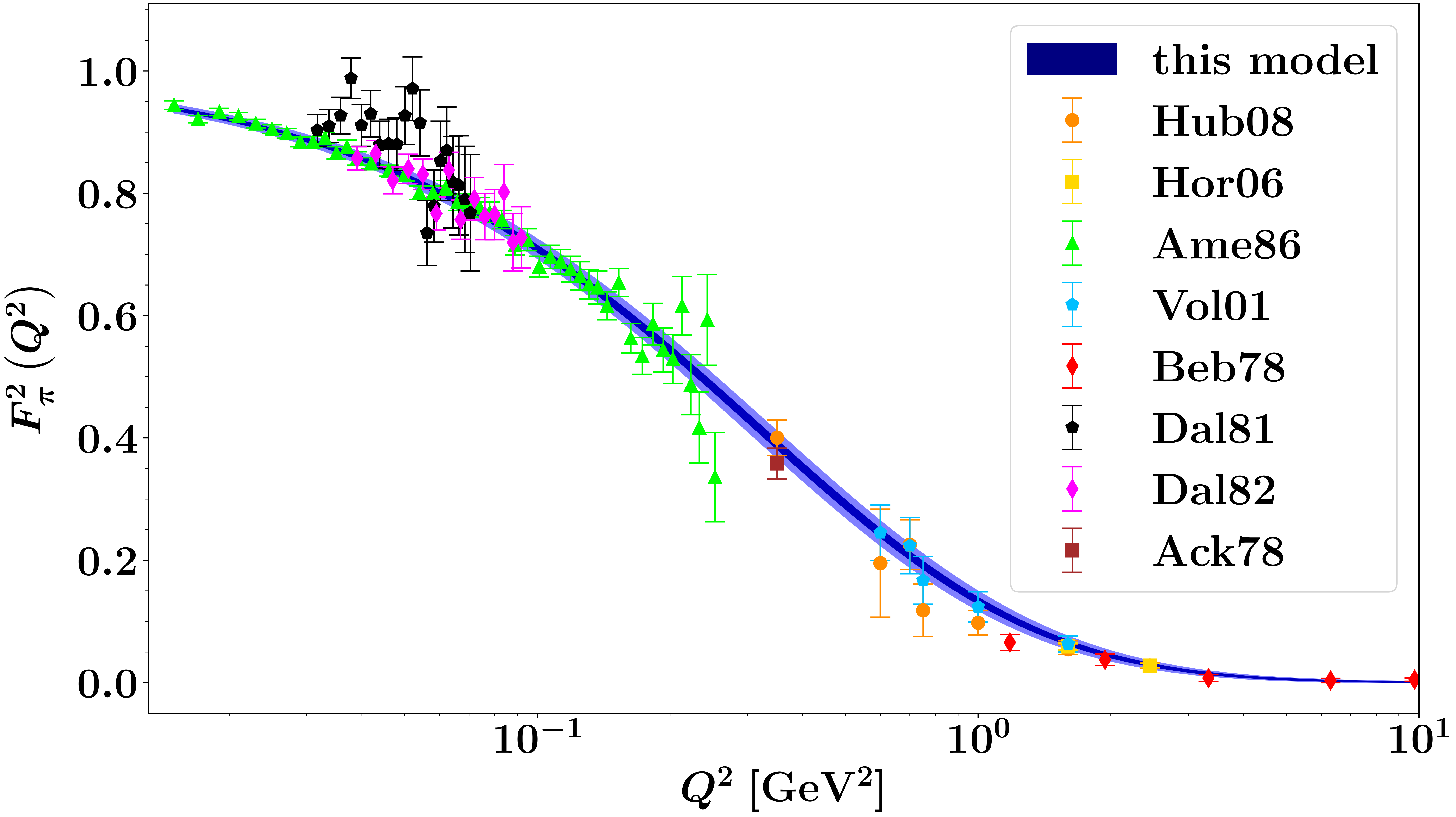
Fit of e.m. Form Factors



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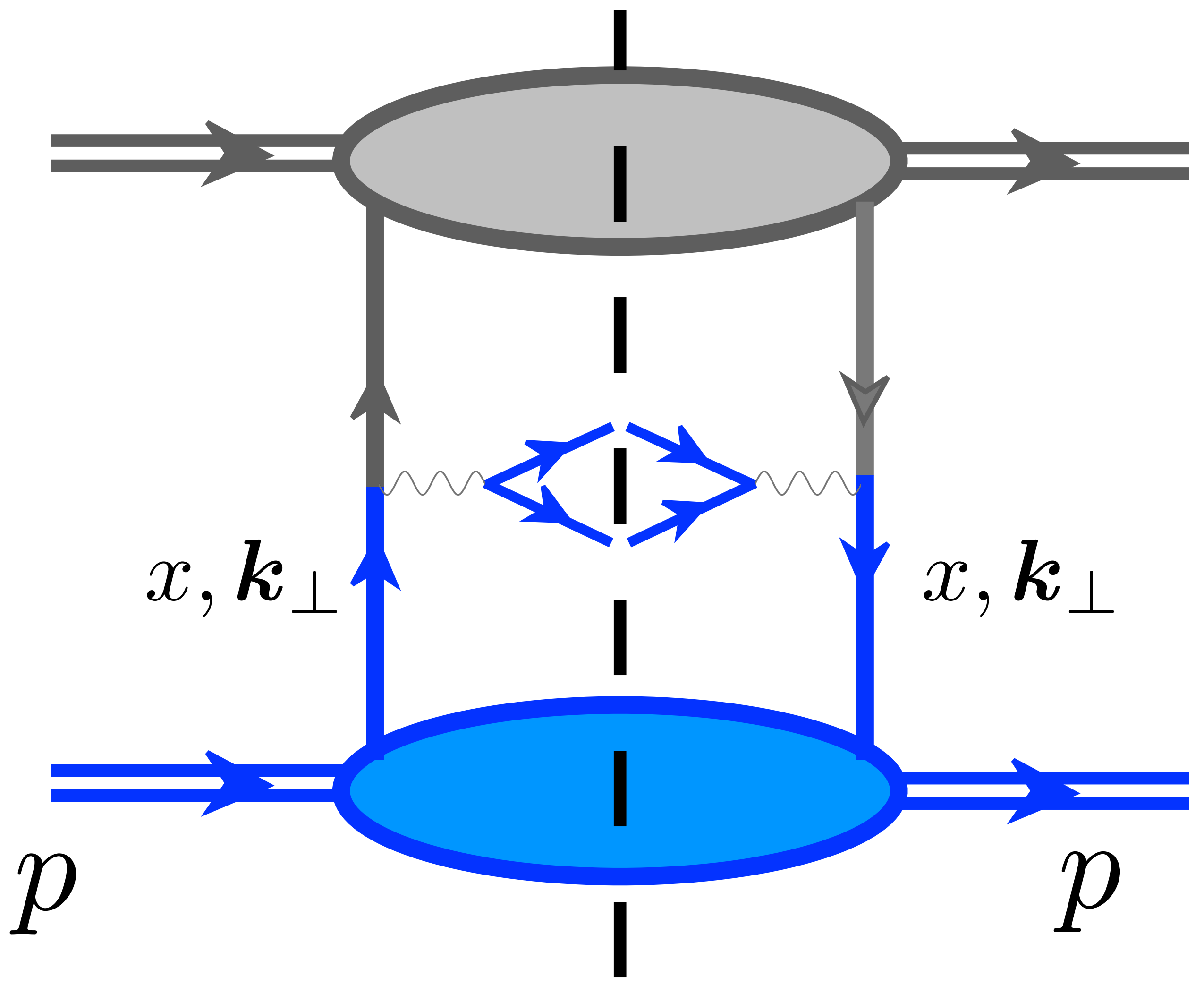
Outline

- Model Construction
- Fit of pion collinear PDFs
- Fit of e.m. Form Factors
- Work on pion TMD PDFs

Work on pion TMD PDFs

Work on pion TMD PDFs

$$h_A(p_A) + h_B(p_B) \to \gamma^*(q) + X \to \ell(l') + \bar{\ell}(l) + X$$

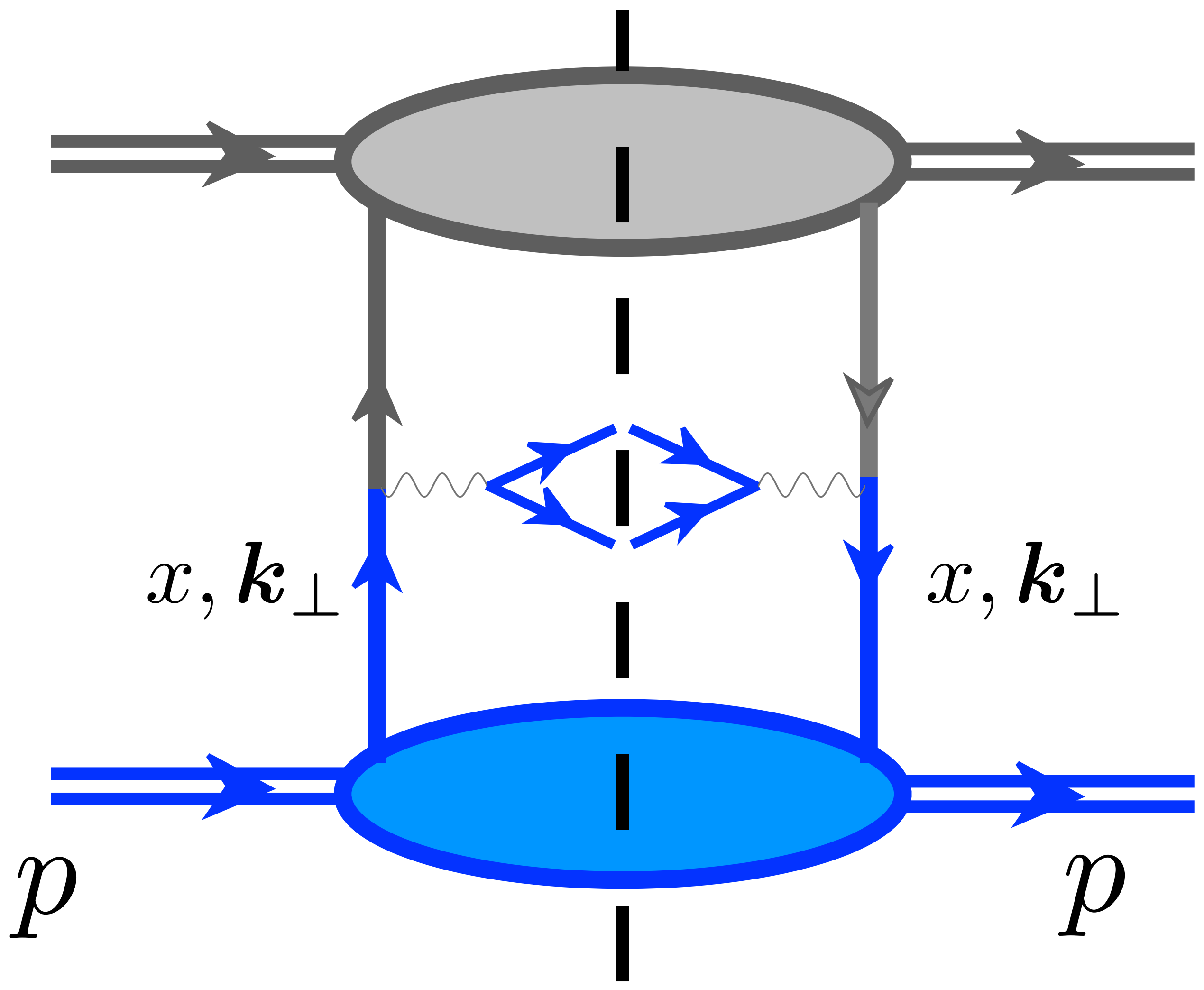


$|\boldsymbol{q}_T| << Q$

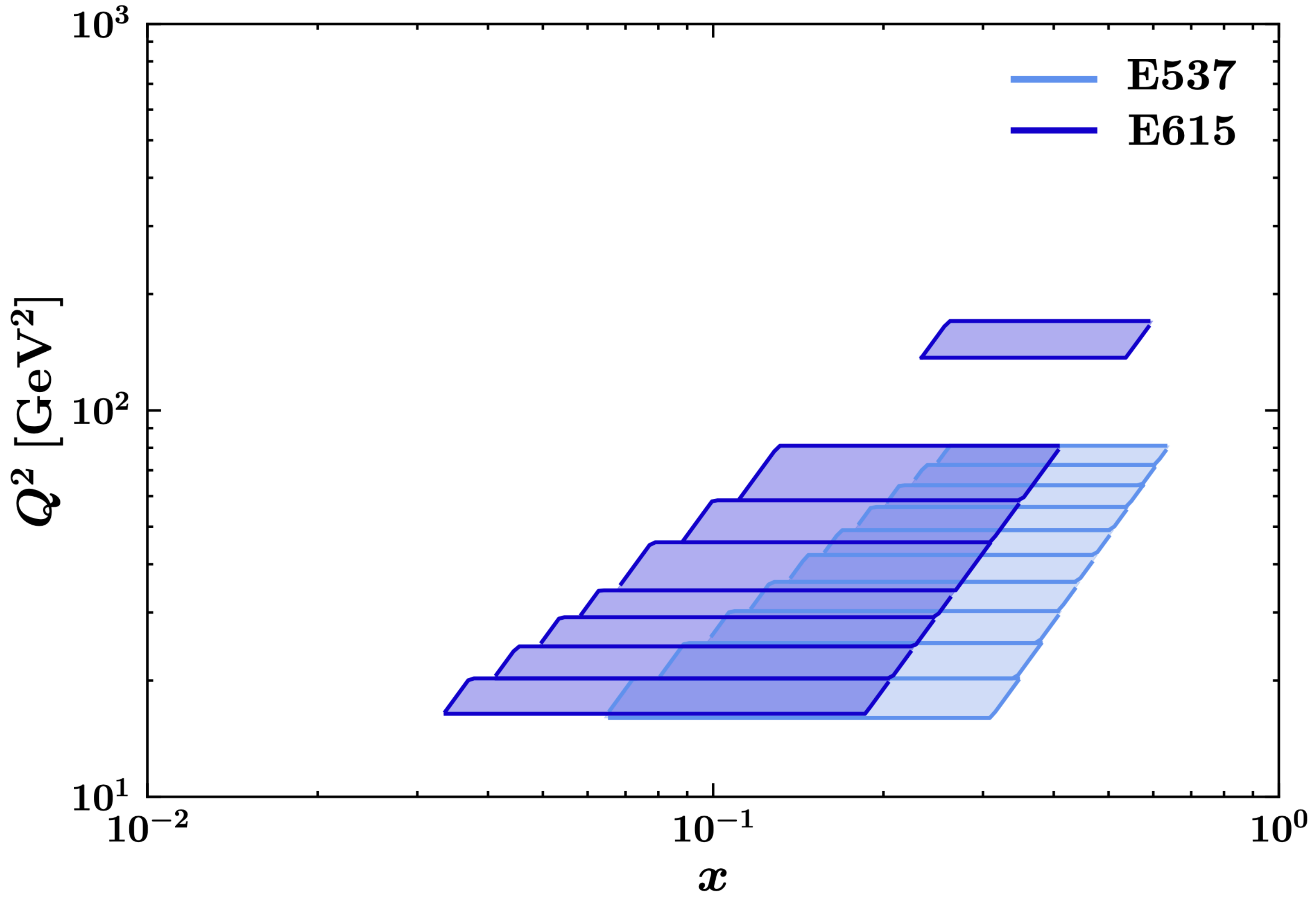
Work on pion TMD PDFs

$$h_A(p_A) + h_B(p_B) \to \gamma^*(q) + X \to \ell(l') + \bar{\ell}(l) + X$$

Experiment	N_{dat}	N_{surv}	Observable	$\sqrt{s}$ [GeV]	x_F range
E615	155	74	$\frac{d^2\sigma}{dQd\lvert\mathbf{q}_T\rvert}$	21.8	0.0 - 1.0
E537	150	58	$\frac{d^2\sigma}{dQd\lvert\mathbf{q}_T\rvert^2}$	15.3	0.0 - 1.0



$|\boldsymbol{q}_T| << Q$



Work on pion TMD PDFs

Work on pion TMD PDFs

Nanga Parbat:

a TMD fitting framework

https://github.com/MapCollaboration/NangaParbat

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Phenomenological approach

Work on pion TMD PDFs

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Phenomenological approach

↓

$$f^{\pi}_{1NP}(x, \boldsymbol{b}_T^2; \zeta)$$

Work on pion TMD PDFs

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Phenomenological approach

↓

$$f^{\pi}_{1NP}(x, \boldsymbol{b}_T^2; \zeta)$$

First fit including DY points
from both E615 and E537

N_{surv}	$\chi^2_{0\,\text{unc}}/N_{\text{surv}}$	$\chi^2_{0\,\text{pen}}/N_{\text{surv}}$	χ^2_0/N_{surv}
132	0.63	0.92	1.55

MAP Collaboration, PRD 107 (2023) 1, 014014

Work on pion TMD PDFs

Nanga Parbat:
a TMD fitting framework

https://github.com/MapCollaboration/NangaParbat

Phenomenological approach

$$f_{1NP}^{\pi}(x, \boldsymbol{b}_T^2; \zeta)$$

First fit including DY points from both E615 and E537

$N_{\rm surv}$	$\chi^2_{0\,\rm unc}/N_{\rm surv}$	$\chi^2_{0\,\rm pen}/N_{\rm surv}$	$\chi^2_0/N_{\rm surv}$
132	0.63	0.92	1.55

MAP Collaboration, PRD 107 (2023) 1, 014014

LFWF approach

Work on pion TMD PDFs

Nanga Parbat:
a TMD fitting framework

https://github.com/MapCollaboration/NangaParbat

Phenomenological approach

↓

$$f^{\pi}_{1NP}(x, \boldsymbol{b}_T^2; \zeta)$$

First fit including DY points from both E615 and E537

$N_{\rm surv}$	$\chi^2_{0\,\rm unc}/N_{\rm surv}$	$\chi^2_{0\,\rm pen}/N_{\rm surv}$	$\chi^2_0/N_{\rm surv}$
132	0.63	0.92	1.55

MAP Collaboration, PRD 107 (2023) 1, 014014

LFWF approach

↓

Adapt the LFWA model to the framework of NangaParbat

Work on pion TMD PDFs

Nanga Parbat:
a TMD fitting framework

https://github.com/MapCollaboration/NangaParbat

Phenomenological approach

↓

$f_{1NP}^{\pi}(x, \boldsymbol{b}_T^2; \zeta)$

First fit including DY points from both E615 and E537

$N_{\rm surv}$	$\chi^2_{0\,\rm unc}/N_{\rm surv}$	$\chi^2_{0\,\rm pen}/N_{\rm surv}$	$\chi^2_0/N_{\rm surv}$
132	0.63	0.92	1.55

→ $\chi^2_0/N_{dat} \simeq 1.26$

MAP Collaboration, PRD 107 (2023) 1, 014014

LFWF approach

↓

Adapt the LFWA model to the framework of NangaParbat

Work on pion TMD PDFs

Nanga Parbat:
a TMD fitting framework

https://github.com/MapCollaboration/NangaParbat

Phenomenological approach

↓

$f^{\pi}_{1NP}(x, \boldsymbol{b}_T^2; \zeta)$

First fit including DY points
from both E615 and E537

$N_{\rm surv}$	$\chi^2_{0\,\rm unc}/N_{\rm surv}$	$\chi^2_{0\,\rm pen}/N_{\rm surv}$	$\chi^2_0/N_{\rm surv}$
132	0.63	0.92	1.55

→ $\chi^2_0/N_{dat} \simeq 1.26$

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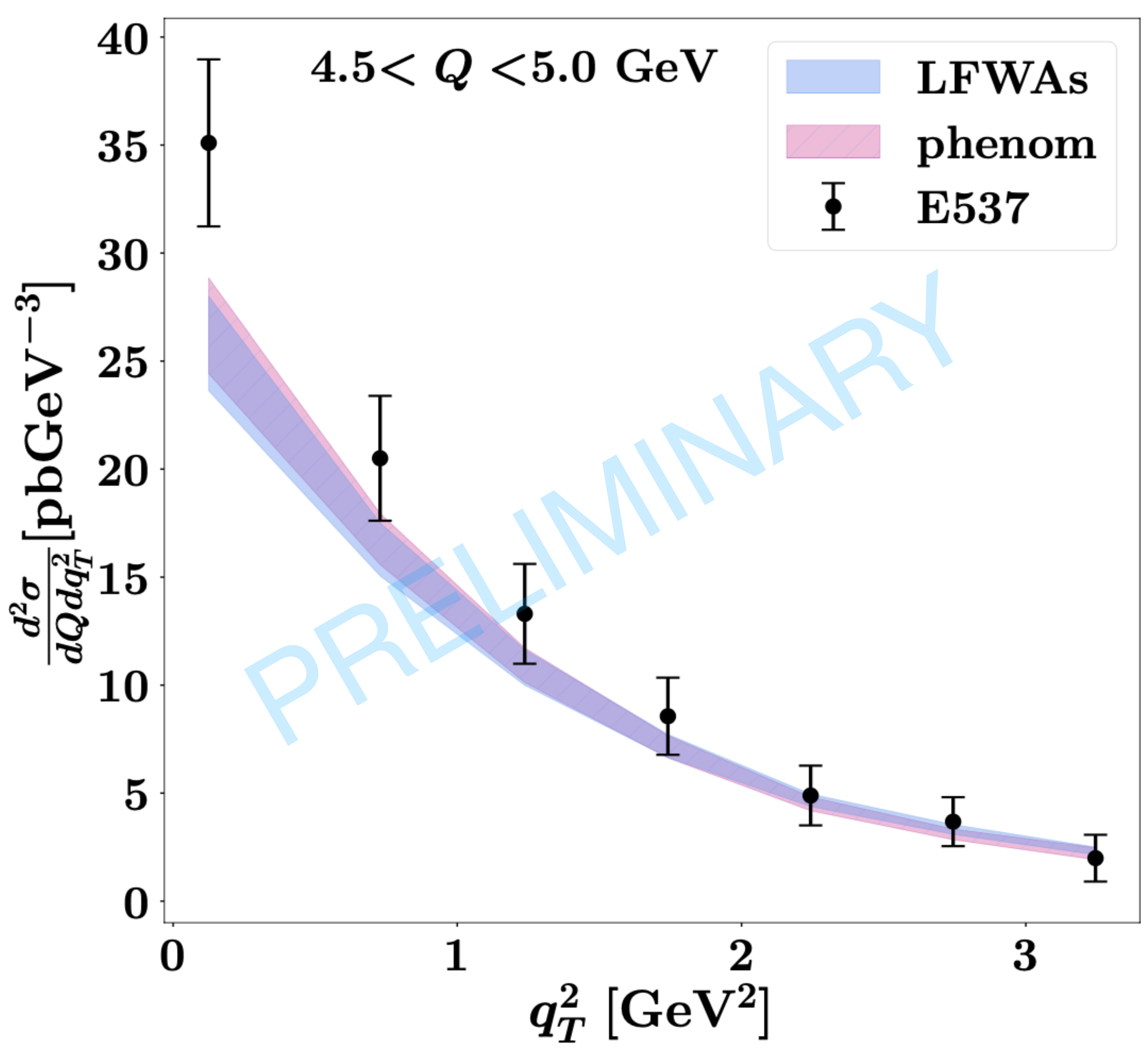
LFWF approach

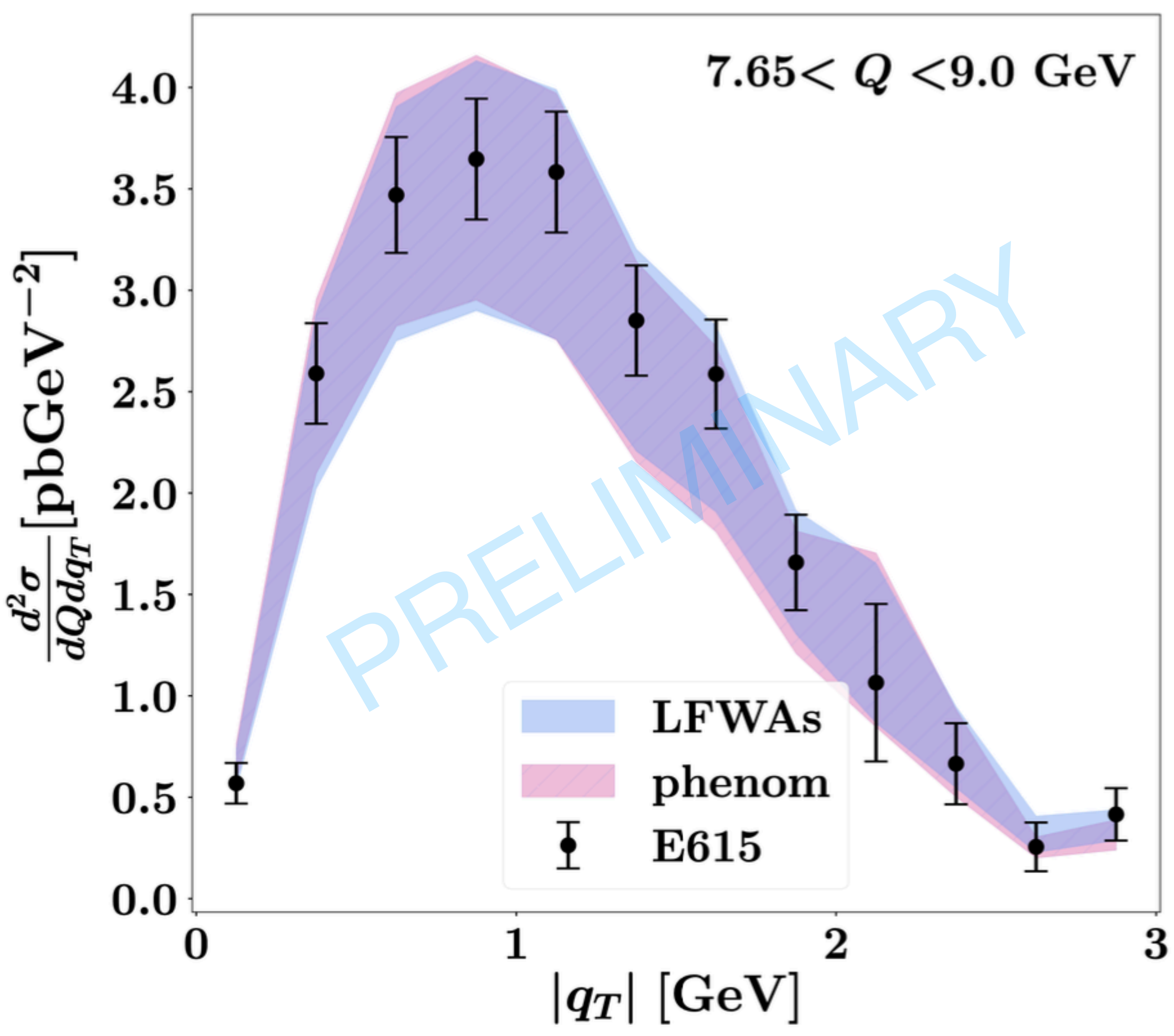
↓

Adapt the LFWA model to the
framework of NangaParbat

similar χ^2_0

Work on pion TMD PDFs





Conclusions and outlook

Conclusions and outlook

- Adaptability of the model

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- Adaptability of the model
  - Fit of pion PDFs

Conclusions and outlook

- Adaptability of the model
  - Fit of pion PDFs
  - Fit of pion e.m. FFs

Conclusions and outlook

- Adaptability of the model
  - Fit of pion PDFs
  - Fit of pion e.m. FFs
- Fit of pion TMDs

Conclusions and outlook

- Adaptability of the model
  - Fit of pion PDFs
  - Fit of pion e.m. FFs
- Fit of pion TMDs
- Prediction of pion GPDs

Conclusions and outlook

- Adaptability of the model
  - Fit of pion PDFs
  - Fit of pion e.m. FFs
- Fit of pion TMDs
- Prediction of pion GPDs
- Need for new experimental data

Backup

Pion PDF: LFWF overlap representation

$$f^{v}_{1,u\bar{d}}(x) = 4\int d[1]d[2]\sqrt{x_1x_2}\,\delta(x-x_1)|\psi^{(1)}_{u\bar{d}}(1,2)|^2$$

$$f^{v}_{1,u\bar{d}g}(x) = 4\int d[1]d[2]d[3]\sqrt{x_1x_2x_3}\,\delta(x-x_1)|\psi^{(1)}_{u\bar{d}g}(1,2,3)|^2$$

$$f^{v}_{1,u\bar{d}gg}(x) = 16\int d[1]d[2]d[3]d[4]\sqrt{x_1x_2x_3x_4}\,\delta(x-x_1)\Big[|\psi^{(1)}_{u\bar{d}gg}(1,2,3,4)|^2 + |\psi^{(2)}_{u\bar{d}gg}(1,2,3,4)|^2\Big]$$

$$f^{v}_{1,u\bar{d}\{s\bar{s}\}}(x) = 8\int d[1]d[2]d[3]d[4]\sqrt{x_1x_2x_3x_4}\,\delta(x-x_1)\Big[|\psi^{(1)}_{u\bar{d}\,s\bar{s}}(1,2,3,4)|^2 + |\psi^{(2)}_{u\bar{d}\,s\bar{s}}(1,2,3,4)|^2 + \frac{1}{2}|\psi^{(3)}_{u\bar{d}\,s\bar{s}}(1,2,3,4)|^2\Big]$$

$$f^{S}_{1,u\bar{d}\{s\bar{s}\}}(x) = 4\int d[1]d[2]d[3]d[4]\sqrt{x_1x_2x_3x_4}\,\delta(x-x_3)\Big[|\psi^{(1)}_{u\bar{d}\,s\bar{s}}(1,2,3,4)|^2 + |\psi^{(2)}_{u\bar{d}\,s\bar{s}}(1,2,3,4)|^2 + \frac{1}{2}|\psi^{(3)}_{u\bar{d}\,s\bar{s}}(1,2,3,4)|^2\Big]$$

$$f^{g}_{1,u\bar{d}g}(x) = 2\int d[1]d[2]d[3]\sqrt{x_1x_2x_3}\,\delta(x-x_3)|\psi^{(1)}_{u\bar{d}g}(1,2,3)|^2$$

$$f^{g}_{1,u\bar{d}gg}(x) = 16\int d[1]d[2]d[3]d[4]\sqrt{x_1x_2x_3x_4}\,\delta(x-x_3)\Big[|\psi^{(1)}_{u\bar{d}gg}(1,2,3,4)|^2 + |\psi^{(2)}_{u\bar{d}gg}(1,2,3,4)|^2\Big]$$

Backup

E.m. FF: LFWF overlap representation

$$F_{1,u\bar{d}}(Q^2) = 2\int \mathrm{d}[1]\mathrm{d}[2]\sqrt{x_1 x_2}\, \psi^{*(1)}_{q\bar{q}}(x_1, x_2, \boldsymbol{k}'_{\perp 1}, \boldsymbol{k}'_{\perp 2})\, \psi^{(1)}_{q\bar{q}}(x_1, x_2, \boldsymbol{k}_{\perp 1}, \boldsymbol{k}_{\perp 2})$$

$$F_{1,u\bar{d}g}(Q^2) = 2\int \mathrm{d}[1]\mathrm{d}[2]\mathrm{d}[3]\sqrt{x_1 x_2 x_3}\, \psi^{*(1)}_{q\bar{q}g}(x_1, x_2, x_3, \boldsymbol{k}'_{\perp 1}, \boldsymbol{k}'_{\perp 2}, \boldsymbol{k}'_{\perp 3})\, \psi^{(1)}_{q\bar{q}g}(x_1, x_2, x_3, \boldsymbol{k}_{\perp 1}, \boldsymbol{k}_{\perp 2}, \boldsymbol{k}_{\perp 3})$$

$$\begin{aligned} F_{1,u\bar{d}gg}(Q^2) = 4\int & \mathrm{d}[1]\mathrm{d}[2]\mathrm{d}[3]\mathrm{d}[4]\sqrt{x_1 x_2 x_3 x_4} \\ & \times \Big[\psi^{*(1)}_{q\bar{q}gg}(x_1, x_2, x_3, x_4, \boldsymbol{k}'_{\perp 1}, \boldsymbol{k}'_{\perp 2}, \boldsymbol{k}'_{\perp 3}, \boldsymbol{k}'_{\perp 4})\ \psi^{(1)}_{q\bar{q}gg}(x_1, x_2, x_3, x_4, \boldsymbol{k}_{\perp 1}, \boldsymbol{k}_{\perp 2}, \boldsymbol{k}_{\perp 3}, \boldsymbol{k}_{\perp 4}) \\ & \quad + \psi^{*(1)}_{q\bar{q}gg}(x_1, x_2, x_4, x_3, \boldsymbol{k}'_{\perp 1}, \boldsymbol{k}'_{\perp 2}, \boldsymbol{k}'_{\perp 4}, \boldsymbol{k}'_{\perp 3})\ \psi^{(1)}_{q\bar{q}gg}(x_1, x_2, x_3, x_4, \boldsymbol{k}_{\perp 1}, \boldsymbol{k}_{\perp 2}, \boldsymbol{k}_{\perp 3}, \boldsymbol{k}_{\perp 4}) \\ & \quad + \psi^{*(2)}_{q\bar{q}gg}(x_1, x_2, x_3, x_4, \boldsymbol{k}'_{\perp 1}, \boldsymbol{k}'_{\perp 2}, \boldsymbol{k}'_{\perp 2}, \boldsymbol{k}'_{\perp 4})\ \psi^{(2)}_{q\bar{q}gg}(x_1, x_2, x_3, x_4, \boldsymbol{k}_{\perp 1}, \boldsymbol{k}_{\perp 2}, \boldsymbol{k}_{\perp 3}, \boldsymbol{k}_{\perp 4}) \\ & \quad - \psi^{*(2)}_{q\bar{q}gg}(x_1, x_2, x_4, x_3, \boldsymbol{k}'_{\perp 1}, \boldsymbol{k}'_{\perp 2}, \boldsymbol{k}'_{\perp 4}, \boldsymbol{k}'_{\perp 3})\ \psi^{(2)}_{q\bar{q}gg}(x_1, x_2, x_3, x_4, \boldsymbol{k}_{\perp 1}, \boldsymbol{k}_{\perp 2}, \boldsymbol{k}_{\perp 3}, \boldsymbol{k}_{\perp 4})\Big] \end{aligned}$$

$$\begin{aligned} F_{1,u\bar{d}\{s\bar{s}\}}(Q^2) = 4\int & \mathrm{d}[1]\mathrm{d}[2]\mathrm{d}[3]\mathrm{d}[4]\sqrt{x_1 x_2 x_3 x_4} \\ & \times \Big[\psi^{*(1)}_{q\bar{q}\,s\bar{s}}(x_1, x_2, x_3, x_4, \boldsymbol{k}'_{\perp 1}, \boldsymbol{k}'_{\perp 2}, \boldsymbol{k}'_{\perp 3}, \boldsymbol{k}'_{\perp 4})\, \psi^{(1)}_{q\bar{q}\,s\bar{s}}(x_1, x_2, x_3, x_4, \boldsymbol{k}_{\perp 1}, \boldsymbol{k}_{\perp 2}, \boldsymbol{k}_{\perp 3}, \boldsymbol{k}_{\perp 4}) \\ & \quad + \psi^{*(2)}_{q\bar{q}\,s\bar{s}}(x_1, x_2, x_3, x_4, \boldsymbol{k}'_{\perp 1}, \boldsymbol{k}'_{\perp 2}, \boldsymbol{k}'_{\perp 3}, \boldsymbol{k}'_{\perp 4})\, \psi^{(2)}_{q\bar{q}\,s\bar{s}}(x_1, x_2, x_3, x_4, \boldsymbol{k}_{\perp 1}, \boldsymbol{k}_{\perp 2}, \boldsymbol{k}_{\perp 3}, \boldsymbol{k}_{\perp 4}) \\ & \quad + \psi^{*(3)}_{q\bar{q}\,s\bar{s}}(x_1, x_2, x_3, x_4, \boldsymbol{k}'_{\perp 1}, \boldsymbol{k}'_{\perp 2}, \boldsymbol{k}'_{\perp 3}, \boldsymbol{k}'_{\perp 4})\, \psi^{(3)}_{q\bar{q}\,s\bar{s}}(x_1, x_2, x_3, x_4, \boldsymbol{k}_{\perp 1}, \boldsymbol{k}_{\perp 2}, \boldsymbol{k}_{\perp 3}, \boldsymbol{k}_{\perp 4})\Big] \end{aligned}$$

$$\begin{cases} \boldsymbol{k}'_{\perp 1} = \boldsymbol{k}_{\perp 1} + (1 - x_1)\boldsymbol{\Delta}_\perp \\ \boldsymbol{k}'_{\perp i} = \boldsymbol{k}_{\perp i} - x_i\boldsymbol{\Delta}_\perp \qquad \text{for } i \neq 1 \end{cases}$$

Backup

Pion TMDs: LFWF overlap representation

$$f_1^u(x,\boldsymbol{k}_\perp)=f_1^{\bar d}(x,\boldsymbol{k}_\perp)=f_1^s(x,\boldsymbol{k}_\perp)=f_1^{\bar s}(x,\boldsymbol{k}_\perp) \quad =\frac{2}{3}\int \mathrm{d}[1]\mathrm{d}[2]\mathrm{d}[3]\mathrm{d}[4]\sqrt{x_1x_2x_3x_4}\,\delta(x-x_3)\delta^{(2)}\left(\boldsymbol{k}_\perp-\boldsymbol{k}_{\perp 3}\right) \times\left[|\psi^{(1)}_{d\bar u\, s\bar s}(1,2,3,4)|^2+|\psi^{(2)}_{d\bar u\, s\bar s}(1,2,3,4)|^2+\frac{1}{2}|\psi^{(3)}_{d\bar u\, s\bar s}(1,2,3,4)|^2\right]$$

$$\begin{aligned}
f_1^d(x,\boldsymbol{k}_\perp)&=f_1^{\bar u}(x,\boldsymbol{k}_\perp)\\
&=2\int \mathrm{d}[1]\mathrm{d}[2]\sqrt{x_1x_2}\,\delta(x-x_1)\delta^{(2)}(\boldsymbol{k}_\perp-\boldsymbol{k}_{\perp 1})\,|\psi^{(1)}_{d\bar u}(1,2)|^2\\
&\quad+2\int \mathrm{d}[1]\mathrm{d}[2]\mathrm{d}[3]\sqrt{x_1x_2x_3}\,\delta(x-x_1)\,\delta^{(2)}(\boldsymbol{k}_\perp-\boldsymbol{k}_{\perp 1})\,|\psi^{(1)}_{d\bar u g}(1,2,3)|^2\\
&\quad+8\int \mathrm{d}[1]\mathrm{d}[2]\mathrm{d}[3]\mathrm{d}[4]\sqrt{x_1x_2x_3x_4}\,\delta(x-x_1)\delta^{(2)}(\boldsymbol{k}_\perp-\boldsymbol{k}_{\perp 1})\left[|\psi^{(1)}_{d\bar u gg}(1,2,3,4)|^2+|\psi^{(2)}_{d\bar u gg}(1,2,3,4)|^2\right]\\
&\quad+4\int \mathrm{d}[1]\mathrm{d}[2]\mathrm{d}[3]\mathrm{d}[4]\sqrt{x_1x_2x_3x_4}\,\delta(x-x_1)\delta^{(2)}\left(\boldsymbol{k}_\perp-\boldsymbol{k}_{\perp 1}\right)\\
&\qquad\times\left[|\psi^{(1)}_{d\bar u\, s\bar s}(1,2,3,4)|^2+|\psi^{(2)}_{d\bar u\, s\bar s}(1,2,3,4)|^2+\frac{1}{2}|\psi^{(3)}_{d\bar u\, s\bar s}(1,2,3,4)|^2\right]\\
&\quad+\frac{2}{3}\int \mathrm{d}[1]\mathrm{d}[2]\mathrm{d}[3]\mathrm{d}[4]\sqrt{x_1x_2x_3x_4}\,\delta(x-x_3)\delta^{(2)}\left(\boldsymbol{k}_\perp-\boldsymbol{k}_{\perp 3}\right)\\
&\qquad\times\left[|\psi^{(1)}_{d\bar u\, s\bar s}(1,2,3,4)|^2+|\psi^{(2)}_{d\bar u\, s\bar s}(1,2,3,4)|^2+\frac{1}{2}|\psi^{(3)}_{d\bar u\, s\bar s}(1,2,3,4)|^2\right]
\end{aligned}$$

Backup

Matching coeff. (perturbative calculable)

Collinear PDFs (previous fit)

Perturbative Sudakov evolution factor

$$\hat{f}_1^q(x_B, \mathbf{b}_T; \mu_F, \zeta_F) = [C \otimes f_1](x_B, b_\star; \mu_{b_\star}, \mu_{b_\star}^2) \exp\left\{ \int_{\mu_{b_\star}}^{\mu_F} \frac{d\mu'}{\mu'} \gamma(\mu', \zeta_F) \right\}$$

$$\times \left(\frac{\zeta}{\mu_{b_\star}^2} \right)^{K(b_\star, \mu_{b_\star})/2} \left[\frac{\zeta}{Q_0} \right]^{-g_K(\mathbf{b}_T)/2} f_1^{NP}(x, \mathbf{b}_T; \zeta, Q_0)$$

Collins-Soper kernel

NP part of Collins-Soper Kernel

Non perturbative part of TMDs

Fit extraction

Collins, *"Foundations of Perturbative QCD"*