Exclusive heavy quarkonium production in charge-exchange reactions

Maxim Nefedov[1], Lech Szymanowski[1,2]

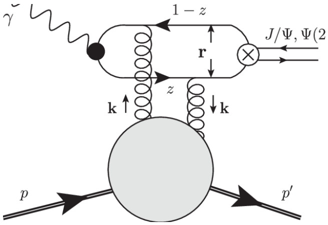
PAW-2024,
CERN

This project is supported in parts by the European Union's Marie Sklodowska Curie action "RadCor4HEF"

(grant agreement No. 101065263)

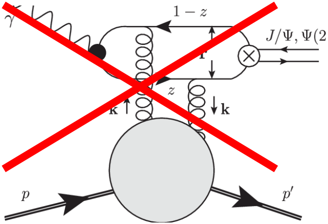
[1]IJClab, Orsay
[2]NCBJ, Warsaw

What this talk is about?



The study of exclusive J/ψ production is often motivated by the fact that it allows one to access poorly known gluon GPDs. However, gluon GPDs are not the only unconstrained objects in exclusive physics.

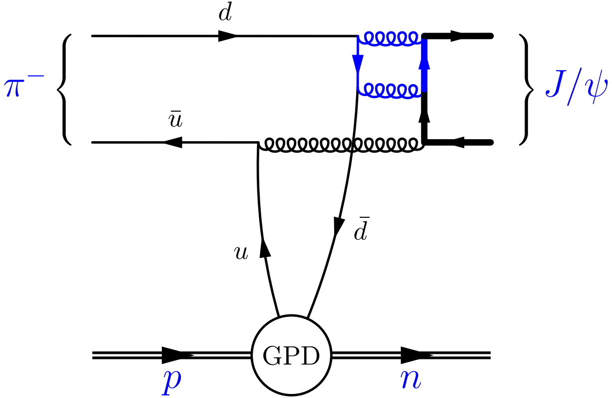
What this talk is about?



So this talk is **not** about this process and **mostly not** about J/ψ because:

- There are several poorly constrained GPDs in quark sector, especially $\tilde{E}_q$
- Quarkonium production physics is **not** simple and often production of quarkonia other than J/ψ brings in *interesting physics puzzles and opportunities*, although maybe experimentally challenging

What this talk is about?



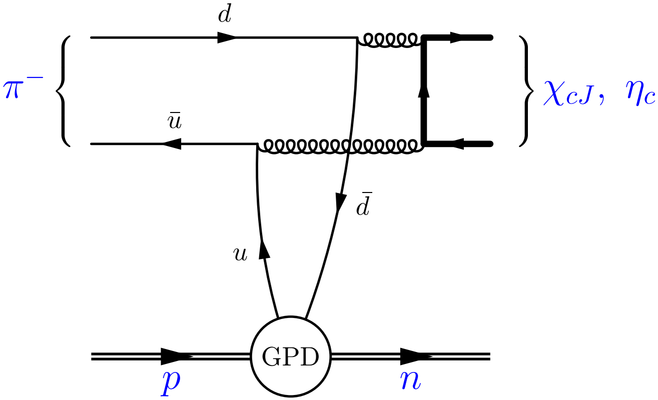
- J/ψ might be more interesting experimentally
- However to produce J/ψ the light and heavy quark subgraphs must be connected by at least 3 gluons (due to $C = -$), which produces a loop.

Can we start with something simpler?

So finally, what this talk is about?

$$\pi^- + p \to \chi_{cJ} + n,$$

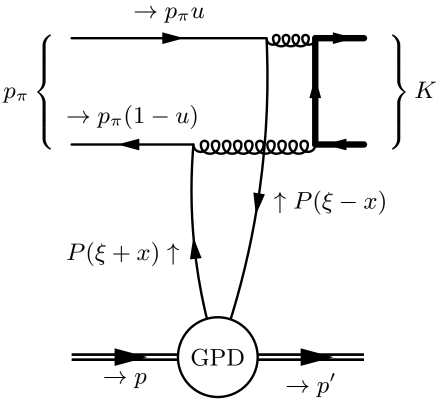
(or $\pi^+ + n \to \chi_{cJ} + p$ with n from a deutron)



Amplitude

$$\mathcal{A}(\pi^- p \to \chi_{cJ} n) = -i f_\pi \int_0^1 du\ \phi_\pi(u) \int_{-1}^{1} dx \left[C_J(x,u) + C_J(-x,1-u)\right]$$

$$\times \frac{1}{P p_\pi} \left\{ \left[\tilde{H}^u(x,\xi,t) - \tilde{H}^d(x,\xi,t)\right] \bar{u}(p') \hat{p}_\pi u(p) + \left[\tilde{E}^u(x,\xi,t) - \tilde{E}^d(x,\xi,t)\right] \bar{u}(p') \frac{i\Delta^\alpha p_\pi^\beta \sigma_{\alpha\beta}}{2M_p} u(p) \right\}, \quad \xi = \frac{p_\pi(p-p')}{p_\pi(p+p')} \simeq \frac{M^2}{2 s_{\pi p}}$$



- In the LO, amplitude depends on **proton** axial GPD combinations: $\tilde{H}^u - \tilde{H}^d$ and $\tilde{E}^u - \tilde{E}^d$
- The amplitude for χ_{c1} is equal to zero in the LO
- Amplitudes for χ_{c0} and *longitudinally polarised* $\chi_{c2}^{\parallel}$ (i.e. $J_z = 0$) are nonzero.
- We take the asymptotic LCDA for pion: $\phi_\pi(u) = 6u(1-u)$. This assumption can be relaxed

Coefficient function and the pinch

For $J = 0$ and $J = 2$, $J_z = 0$ the coefficient function can be represented in a form:

$$
\begin{aligned}
C_0(x,u) &= C^{(\text{reg.})}(x,u) + \Delta C^{(\text{reg.})}(x,u) + C^{(\text{pinch})}(x,u), \\
C_2(x,u) &= \sqrt{2}C^{(\text{reg.})}(x,u) - \frac{1}{\sqrt{2}} C^{(\text{pinch})}(x,u),
\end{aligned}
$$

where $C^{(\text{reg.})}$ and $\Delta C^{(\text{reg.})}$ cause no problems, while:

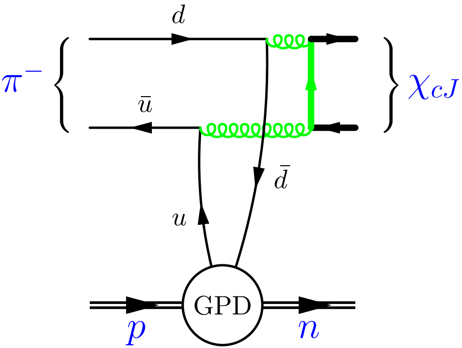
$$
C^{(\text{pinch})}(x,u) \propto \frac{1}{u(1-u)[x(2u-1)+\xi - i\eta]} \frac{1}{(x-\xi-i\eta)(x+\xi+i\eta)},
$$

so e.g. for $u \to 0$:

$$
C^{(\text{pinch})}(x,u) \sim \frac{-1}{u[x-\xi+i\eta]} \frac{1}{(x-\xi-i\eta)(x+\xi+i\eta)},
$$

so the x-integration is pinched at $u \to 0$ and $u \to 1$ and this produces divergent u-integration for any LCDA$\sim u^\alpha (1-u)^\beta$ with $\alpha \leq 1$ and $\beta \leq 1$. **The collinear factorisation is violated.**

Origin of the pinch



The IR problem of P-wave (inclusive case)

It is well known that for the P**-wave** quarkonium **inclusive** production and decays, unexpected divergences appear [Bodwin, '95] .

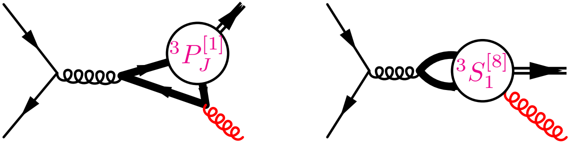
<u>Simple example:</u> The process

$$q + \bar{q} \to Q\bar{Q}[{}^3P_J^{[1]}] + g,$$

naively should not contain IR-divergences, because $g \to Q\bar{Q}[{}^3P_J^{[1]}]$ transition is forbidden. However for $\hat{s} \to M^2$:

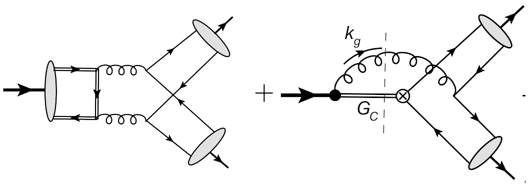
$$|\mathcal{M}(q + \bar{q} \to Q\bar{Q}[{}^3P_J^{[1]}] + g)|^2 \sim \alpha_s (2J+1) \frac{(M^4 - \hat{t}^2)\hat{t}^2}{M^4 (\hat{s} - M^2)^4} \times |\mathcal{M}(q + \bar{q} \to Q\bar{Q}[{}^3S_1^{[8]}])|^2.$$

This new IR divergence can be absorbed through mixing between ${}^3P_J^{[1]}$ and ${}^3S_1^{[8]}$ LDMEs of NRQCD factorisation.



Colour-octet for exclusive processes?

Similar problem arises in exclusive decays of χ_{cJ}, e.g. [N. Kivel, '18]



it is possible to show that the IR divergence cancels between CS and CO contributions in the pNRQCD limit: $m_Q v^2 \ll \Lambda_{\rm QCD} \ll m_Q v$.

For our process, the downside of this solution is that the CO amplitude:

$$\mathcal{A}_J^{[8]} = \langle \chi_{cJ}, n | \mathcal{O}_8(0) | \pi^-, p \rangle,$$

is non-factorisable. The only thing we know about it is the HQSS relation:

$$\mathcal{A}_2^{[8]} = -\frac{1}{\sqrt{2}} \mathcal{A}_0^{[8]} + O(v^2).$$

Everything else is models.

Regularising the pinch

The pinch can be regularised by introducing gluon mass into the gluon propagators ($\mu_g = m_g/M$):

$$C^{(\text{pinch})}(x,u) \propto \frac{1}{u(1-u)[x(2u-1)+\xi-i\eta]}\frac{1}{\left(x-\xi+\frac{2\xi}{u}\mu_g^2-i\eta\right)\left(x+\xi-\frac{2\xi}{1-u}\mu_g^2+i\eta\right)},$$

Which leads to the logarithm of $\mu_g \ll 1$ after integration:

$$\int_{-1}^{1} dx f(x) \int_{0}^{1} du \phi_\pi(u) C^{(\text{pinch})}(x,u) = -48 F_0 i\pi [f(\xi)+f(-\xi)] \ln \mu_g + \text{finite terms},$$

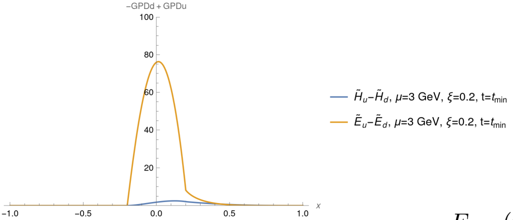
where in our case $f(x)$ is $\tilde{H}^u(x,\xi,t) - \tilde{H}^d(x,\xi,t)$ or $\tilde{E}^u(x,\xi,t) - \tilde{E}^d(x,\xi,t)$.

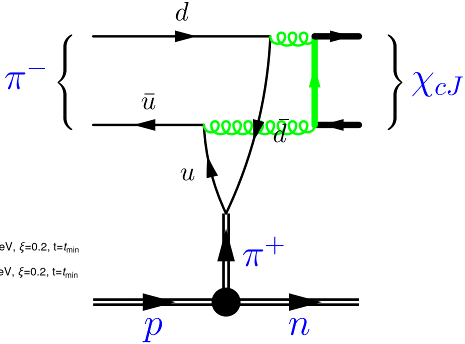
Let's see what happens in the GK model [Goloskokov, Kroll, '11] .

Goloskokov-Kroll model and the pion pole

The GK model includes very large "pion pole" contribution to $\tilde{E}_q$, which was needed to explain the $\gamma^* + p \to \pi^+ + n$ data.

Many thanks to Jakub Wagner (NCBJ) for sharing with us his GK model code!





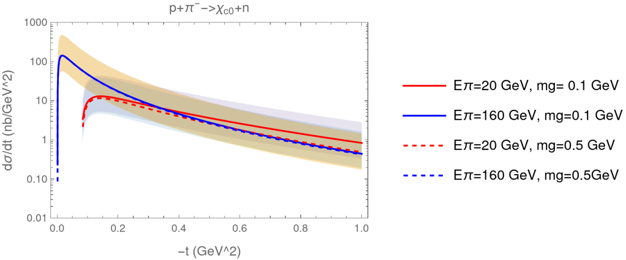
$$\tilde{E}^{(\pi\text{-pole})}_{u,d}(x,\xi,t) = \theta(|x|<\xi)\,\frac{F_{\text{pole}}(t)}{2\xi}\,\phi_\pi\left(\frac{x+\xi}{2\xi}\right),$$

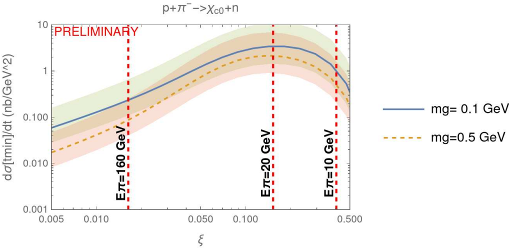
$$F_{\text{pole}}(t) = 2m_p f_\pi \frac{\sqrt{2}g_{\pi pn}F_{p\pi n}(t)}{m_\pi^2 - t}, \quad \phi_\pi(u) = 6u(1-u),$$

$g_{\pi pn} \sim 13$. **Note that** $\tilde{E}^{(\pi\text{-pole})}_{u,d}(\xi,\xi,t) = \tilde{E}^{(\pi\text{-pole})}_{u,d}(-\xi,\xi,t) = 0 \Rightarrow$ **No pinch!**

Differential cross section, χ_{c0}

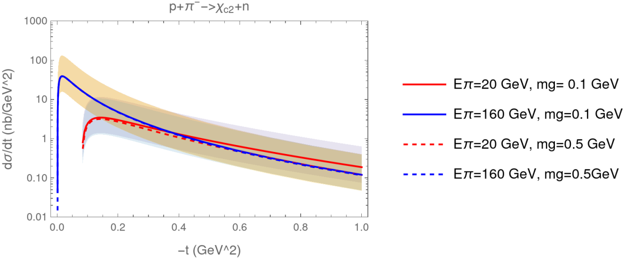
$$\frac{d\sigma}{dt} = \frac{1}{8\pi s^2}\left\{(1-\xi^2)|\mathcal{A}_H|^2 - \left(\xi^2 + \frac{t}{4M_p^2}\right)|\mathcal{A}_E|^2 + 2\xi^2 \mathrm{Re}\left[\mathcal{A}_H \mathcal{A}_E^*\right]\right\}, \quad t_{\min} = -M_p^2 \frac{4\xi^2}{1-\xi^2} \simeq -\frac{M^4 M_p^2}{s_{\pi p}^2}.$$

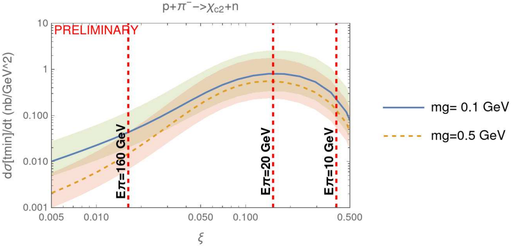




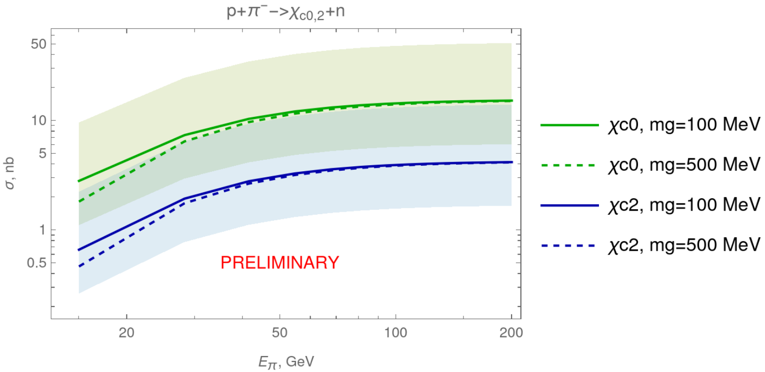
Differential cross section, χ_{c2}

$$\frac{d\sigma}{dt} = \frac{1}{8\pi s^2}\left\{(1-\xi^2)|\mathcal{A}_H|^2 - \left(\xi^2 + \frac{t}{4M_p^2}\right)|\mathcal{A}_E|^2 + 2\xi^2 \mathrm{Re}\left[\mathcal{A}_H \mathcal{A}_E^*\right]\right\}, \quad t_{\min} = -M_p^2 \frac{4\xi^2}{1-\xi^2} \simeq -\frac{M^4 M_p^2}{s_{\pi p}^2}.$$





Total cross section



The flattening of the cross section at high E_π is probably unphysical. It is due to $\tilde{E}_{u,d}^{(\pi\text{-pole})}$ collapsing into $\delta(x)$ at $\xi \ll 1$ ($s_{\pi p} \to \infty$). The cross section should decrease instead.

Conclusions and outlook

- The process $\pi^- + p \to \chi_{cJ} + n$ is sensitive to quark GPDs $\tilde{H}_q$ and $\tilde{E}_q$. Amplitude for $J = 1$ is zero while for $J = 0$ and $J = 2$ with $J_z = 0$ it is nonzero
- But there is a factorisation violation, requiring new (CO) contribution to the amplitude
- However if something similar to the GK pion-pole mechanism for $\tilde{E}_q$ is valid, then the sensitivity to the factorisation violation will be very mild
- The cross section is in nanobarn range
- $d\sigma/dt$ contains nontrivial interference effects. It is sensitive to factorisation violating effects for $t \sim t_{\min}$ and much less sensitive to them at $t \sim 100 - 200$ MeV
- The computation for J/ψ is in our plans, it is not guaranteed that it will be free from factorisation violation (CO) effects
- Can bottomonia be measured at AMBER? η_c? What about photo/electro-production of h_c ?

Thank you for your attention!