Radiative Corrections for Proton Radius Measurements in Scattering Experiments

Andrei Afanasev

The George Washington University, Washington, DC, USA

Physics at AMBER international Workshop

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Plan of talk

Radiative corrections for charged lepton scattering

- Update on modelling and theory uncertainties

Two-photon exchange effects

- Differences between muon scattering and electron scattering on a proton target
- Charge asymmetries

Electron vs muon scattering

- **Summary**

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Proton Radius Measurements/Scattering Kinematics

- Table from review arXiv:2306.14578 (Afanasev, Bernauer, Blunden et al, Radiative Corrections: From Medium to High Energy Experiments EPJA 2024)

Table 1: Characteristic parameters of published and upcoming proton form factor measurements

	Measured particle	E_{beam} [GeV]	θ [degrees]	Q^2 [GeV/c^2]	Effect of radius on form factor	Fractional contribution of G_E to cross section
GMp-12 [63]	e'	2.222 – 10.587	24.25 – 53.5	1.577 – 15.76	>100%	0.2 – 11.9%
MAMI FF [150, 178]	e'	0.180 – 0.855	15.5 – 135	0.0033 – 0.98	2 – 59%	0.23 – 99.4%
MAMI High-Q^2 [179]	e'	0.720 – 1.500	15 – 120	0.35 – 1.95	>100%	1.3 – 92.5%
MAMI ISR [180, 181]	e'	0.195, 0.330	15.21	0.001 – 0.004	0.6 – 2.4%	>98%
MAMI Jet Target [182]	e'	0.315	15 – 40	0.007 – 0.043	6 – 39%	71.2 – 98.5%
PRad [183]	e'	1.1, 2.2	0.7 – 6.5	0.00022 – 0.058	0.13 – 35%	87.6 – 100%
AMBER [176]	$\mu' + p$	60, 100	N/A	0.001 – 0.04	0.6 – 24%	91 – 100%
MESA [184]	e'	0.02 – 0.105	15 – 165	0.000027 – 0.035	0.016-21%	91 – 100%
MUSE [174, 175]	e', μ', π'	0.115 – 0.21	20 – 100	0.0016 – 0.082	0.96 – 4.9%	58 – 99.6%
PRAD-II [185]	e'	0.7, 1.4, 2.1	0.7 – 6.5	0.00007 – 0.056	0.042 – 34%	88.6 – 100%
ULQ2 [186]	e'	0.01 – 0.06	0.7-6.5	0.000027 – 0.012	0.016 – 7.2%	56 – 100%

-H. Gao, M. Vanderhaeghen, The proton charge radius, Rev. Mod. Phys. **94**, 015002 (2022)

-C. Peset, A. Pineda, and O. Tomalak, The proton radius (puzzle?) and its relatives, Prog. Part. Nucl. Phys. **121**, 103901 (2021)

-J.-P. Karr, D. Marchand, E. Voutier, The proton size, Nature Reviews Physics **2**, 601–614 (2020)

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Bremsstrahlung for Relativistic vs Nonrelativistic Lepton Scattering

- Accelerated charge always radiates, but the magnitude of the effect depends on kinematics
- See Bjorken&Drell (Vol.1, Ch.8):
  - For large $Q^2 >> m_l^2$ the rad.correction is enhanced by a large logarithm, $\log(Q^2/m_l^2) \sim 15$ for $1 GeV^2$ momentum transfers by electrons vs 4.6 for muons => rad corrections due to brem for muons are smaller by ~3x
  - For small $Q^2 << m_l^2$, rad.correction suppressed by Q^2/m_e^2
  - For intermediate $Q^2 \sim m_l^2$, neither enhancement nor suppression, rad correction of the order $2\alpha/\pi$

Two-Photon Exchange Overview

Progress in Particle and Nuclear Physics 95 (2017) 245–278

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journal homepage: www.elsevier.com/locate/ppnp

Review

Two-photon exchange in elastic electron–proton scattering

CrossMark

A. Afanasev [a,*], P.G. Blunden [b,c], D. Hasell [d], B.A. Raue [e]

[a] *George Washington University, Washington, DC, USA*
[b] *University of Manitoba, Winnipeg, MB, Canada*
[c] *Jefferson Lab, Newport News, VA, USA*
[d] *Massachusetts Institute of Technology, Cambridge, MA, USA*
[e] *Florida International University, Miami, FL, USA*

McMule extension Engel et al, arXiv:2307.16831
Talukdar et al, Phys. Rev. D 104, 053001 (2021)

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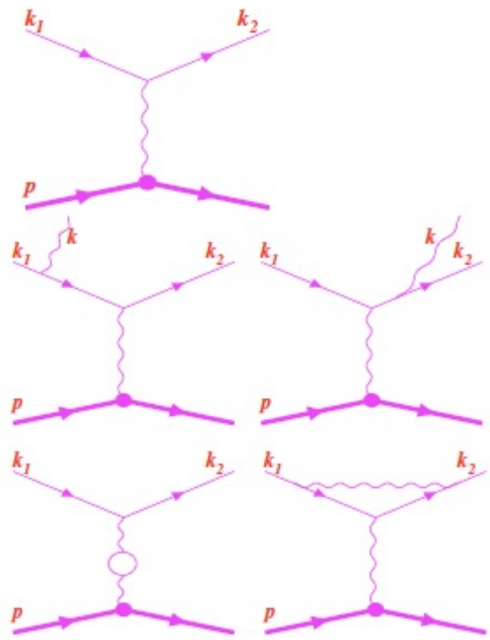
RC to elastic lp scattering
Basic contributions

Born contribution

Real photon emission

Virtual particle contribution



Computational approaches:

Ilyichev (Minsk) and AA: EPJA 58, 156 (2022)
MASSRAD (semi-analytic), updated ELRADGEN Monte Carlo, (Afanasev et al., Czech. J. Phys. 53 (2003) B449; Akushevich et al., Comput. Phys. Commun. 183 (2012) 1448) to include (a) mass effects and (b) two-photon effects (c) hard brem included
+Dedicated MS generators for MAMI (Bernauer), OLYMPUS (Gramolin), PRAD, MUSE,

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Radiative corrected cross section

$l(k_1) + p(p_1) \to l'(k_2) + p'(p_2) + \gamma(k)$

Three contribution $\sigma \equiv d\sigma/d\Omega$

- σ_{Born} Born contribution
- σ_R real photon emission
- σ_V additional virtual particle contributions

σ_R and σ_V are infrared divergent terms.
But $\sigma_R + \sigma_V$ are infrared free!

$\sigma_{tot} = \sigma_{Born} + \sigma_R + \sigma_V = \sigma_{Born} + (\sigma_R - \sigma_{IR}) + (\sigma_V + \sigma_{IR})$

$$\sigma_R = \int dv dt d\phi_k \frac{d\sigma_R}{dv dt d\phi_k}$$

$v = (p_2 + k)^2 - M^2$ — inelasticity, $t = -(p_2 - p_1)^2$,
ϕ_k — angle between $(\vec{k}_1, \vec{k}_2)$ and $(\vec{k}_1, \vec{k})$ planes

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Mo-Tsai and Bardin-Shumeiko methods

$$\frac{d\sigma_R}{d\Omega dE_\gamma} = f_0(E_\gamma) + \frac{f_1(E_\gamma)}{E_\gamma}, \text{where} \frac{d\sigma_R}{d\Omega} = \int_0^{E_\gamma^{max}} dE_\gamma \frac{d\sigma_R}{d\Omega dE_\gamma} = \infty$$

Mo-Tsai: $\frac{d\sigma_R}{d\Omega} \to \frac{d\sigma_R^{soft}(\Delta)}{d\Omega} + \frac{d\sigma_R^{hard}(\Delta)}{d\Omega},$

Direct integration $\frac{d\sigma_R^{hard}(\Delta)}{d\Omega} = \int_\Delta^{E_\gamma^{max}} dE_\gamma \frac{d\sigma_R}{d\Omega dE_\gamma}$

Integration with regularization: $\frac{d\sigma_R^{soft}(\Delta)}{d\Omega} = \int_\lambda^\Delta dE_\gamma \frac{f_1(0)}{E_\gamma}$

Bardin-Shumeiko: $\frac{d\sigma_R}{d\Omega} = \frac{d\sigma_{IR}}{d\Omega} + \frac{d\sigma_R}{d\Omega} - \frac{d\sigma_{IR}}{d\Omega} = \frac{d\sigma_{IR}}{d\Omega} + \frac{d\sigma_F}{d\Omega}$

Infrared part $\frac{d\sigma_{IR}}{d\Omega} = \frac{d\sigma_R^{soft}(\Delta)}{d\Omega} + \frac{d\sigma_{IR}^{hard}(\Delta)}{d\Omega},$

Direct integration $\frac{d\sigma_F}{d\Omega}$ and $\frac{d\sigma_{IR}^{hard}(\Delta)}{d\Omega} = \int_\Delta^{E_\gamma^{max}} dE_\gamma \frac{f_1(0)}{E_\gamma}$

Radiative corrected cross section

$l(k_1) + p(p_1, \eta) \to l'(k_2) + p'(p_2) + \gamma(k)$

$$\sigma_{tot}(v_{cut}) = \int_0^{v_{cut}} dv \left(\frac{d\sigma_R}{dv} - \frac{d\sigma_{IR}}{dv} \right) + (1 + \delta(v_{cut}))\sigma_{Born}$$

For generation of radiative events $v_{cut} = v_{max}$

$$\sigma_{tot}(v_{cut}) = \sigma_r(v_{cut}, v_{min}) + \sigma_n(v_{min})$$

v_{min} missing mass square resolution.
Usually $v_{min} < v_{cut} \ll v_{max}$

$$\sigma_r(v_{min}) = \int_{v_{min}}^{v_{max}} dv \int dt d\phi_k \frac{d\sigma_R}{dv dt d\phi_k}$$

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ELRADGEN Monte Carlo

Input-Output Data

Input:

- 4-momentum of the virtual photon $q = k_1 - k_2$ generated externally;
- missing mass square resolution v_{min};

Output:

- 4-momentum of the real photon k;
- 4-momentum of the virtual photon $q = k_1 - k_2 - k$;
- weight for elastic Born contribution reweighting $w = (\sigma_B + \sigma_{RC})/\sigma_B$.

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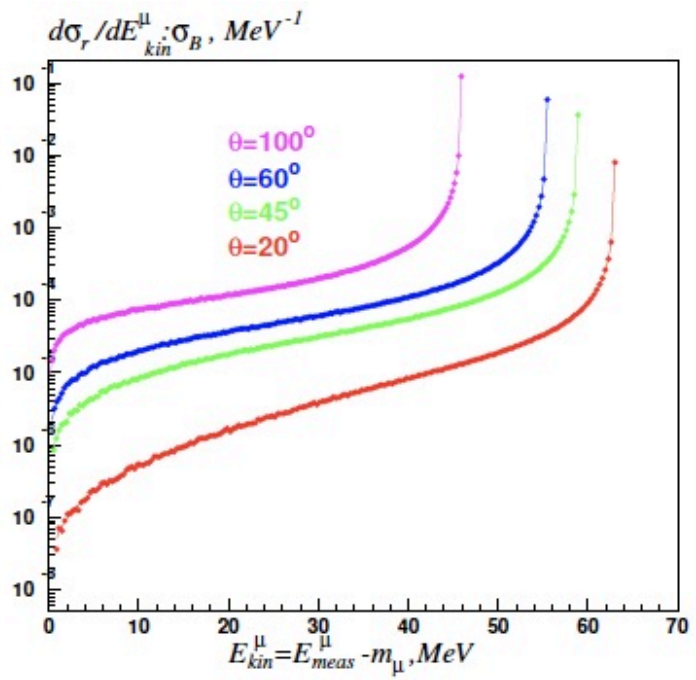
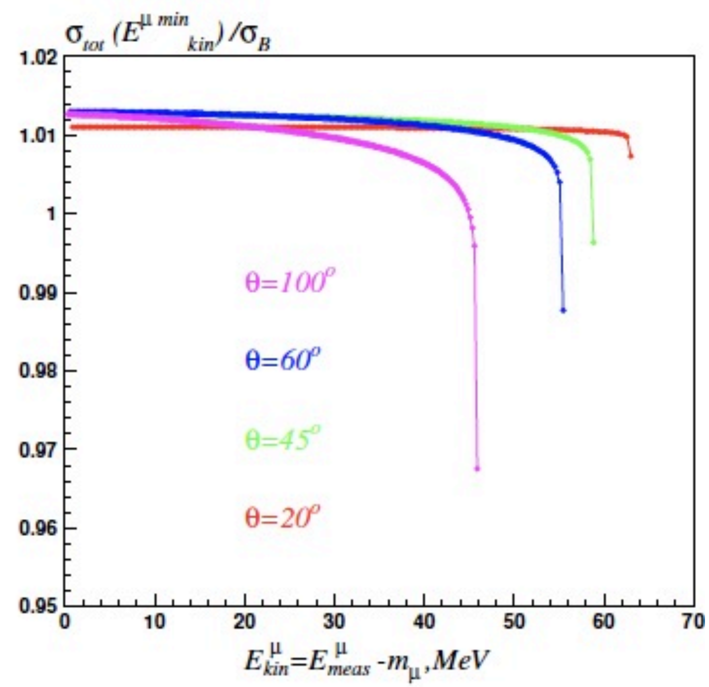
Radiative corrections for muon scattering





$$E^{\mu}_{meas} = E^{\mu}_{beam} - \frac{Q^2(v)+v}{2M_p} \qquad E^{\mu\,min}_{meas} = E^{\mu}_{beam} - \frac{Q^2(v_{cut})+v_{cut}}{2M_p}$$

$$\sigma_{tot}(v_{cut}) = \sigma_r(v_{cut}, v_{min}) + \sigma_n(v_{min})$$

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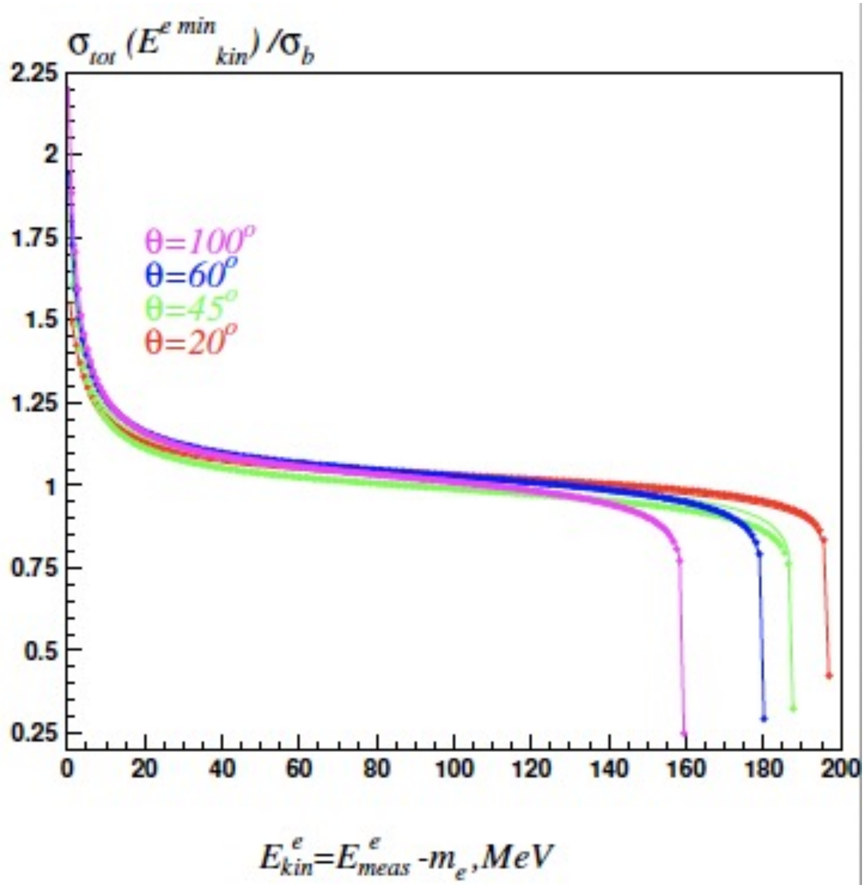
Radiative correction for electrons



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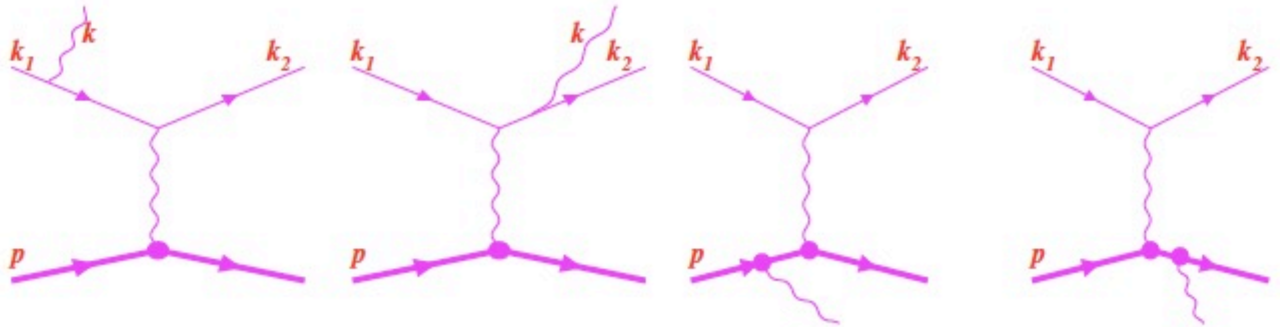
Interference effects between lepton and proton brem



Two assumptions:

- Any exited states in the intermediated proton are not considered that allow us to use the standard fermionic propagator for this particle.
- Extending the on-shell proton vertex on off-shell region.

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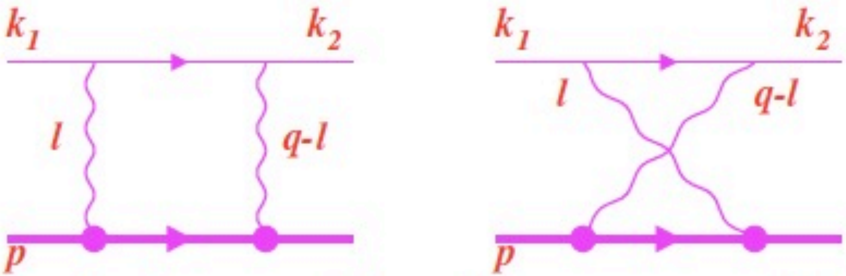
IR divergence cancellation

The infrared divergent term was extracted by Bardin-Shumeiko approach and cancelled with corresponding soft part in two-photon exchange contribution



Soft photon estimation was estimated by A. Afanasev and O. Koshchii in Phys. Rev. D 96, 016005

All calculation has been performed beyond the ultrarelativistic limit keeping lepton mass during the whole process of calculation.

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"Standard" TPE Corrections Calculated with no Ultra-relativistic Approximation

- Koshchii, AA, Phys. Rev. D 96, 016005 (2017)

$$d\sigma_{1\gamma} = \frac{1}{\epsilon_m(1+\tau)}[\tau G_M^2(Q^2) + \epsilon_m G_E^2(Q^2)]d\sigma_M,$$

$$d\sigma_M = \frac{\alpha^2}{Q^4}\frac{(4\varepsilon_1\varepsilon_2 - Q^2)\vec{k}_2^2}{|\vec{k}_1|(|\vec{k}_2| + \frac{\varepsilon_1}{M}|\vec{k}_2| - \frac{\varepsilon_2}{M}|\vec{k}_1|\cos\theta)},$$

$$\epsilon_m^{-1} = \frac{(s-u)^2 + Q^2(4M^2+Q^2) - 4m^2(4M^2+Q^2)}{(s-u)^2 - Q^2(4M^2+Q^2)}$$

One-Photon Exchange modified by the lepton mass

$$\begin{aligned}\delta = -\frac{\alpha}{\pi}\Bigg(&\frac{b_{12}}{\gamma_{12}}\left[\frac{1}{2}\ln(\alpha_{12})\cdot\ln\left(\frac{4\gamma_{12}^2}{m^4\alpha_{12}(1-\alpha_{12})^2}\right) + \mathrm{Li}_2\left(\frac{u}{2\gamma_{12}(1-\alpha_{12})}\right) - \mathrm{Li}_2\left(\frac{u\alpha_{12}}{2\gamma_{12}(1-\alpha_{12})}\right)\right]\\
&-\frac{b_{11}}{\gamma_{11}}\left[\frac{1}{2}\ln(\alpha_{11})\cdot\ln\left(\frac{4\gamma_{11}^2}{m^4\alpha_{11}(1-\alpha_{11})^2}\right) + \mathrm{Li}_2\left(\frac{2m^2+2M^2-s}{2\gamma_{11}(1-\alpha_{11})}\right) - \mathrm{Li}_2\left(\frac{(2m^2+2M^2-s)\alpha_{11}}{2\gamma_{11}(1-\alpha_{11})}\right)\right]\\
&+\frac{2\alpha_{11}b_{11}}{\alpha_{11}^2m^2-M^2}\left[\ln^2\left(\frac{\nu}{M^2}\right) - \ln^2\left(\frac{\nu}{\alpha_{11}mM}\right)\right] - \frac{2\alpha_{12}b_{12}}{\alpha_{12}^2m^2-M^2}\left[\ln^2\left(\frac{\nu}{M^2}\right) - \ln^2\left(\frac{\nu}{\alpha_{12}mM}\right)\right]\\
&+\frac{b_{11}}{2\gamma_{11}}[S_{11}^{(2)} + S_{22}^{(2)}] - \frac{b_{12}}{2\gamma_{12}}[S_{12}^{(2)} + S_{21}^{(2)}]\Bigg).\end{aligned}$$

Lepton charge asymmetry due to (soft) TPE
(Heavy-lepton analogue of Tsai'61 or Maximon-Tjon'00)

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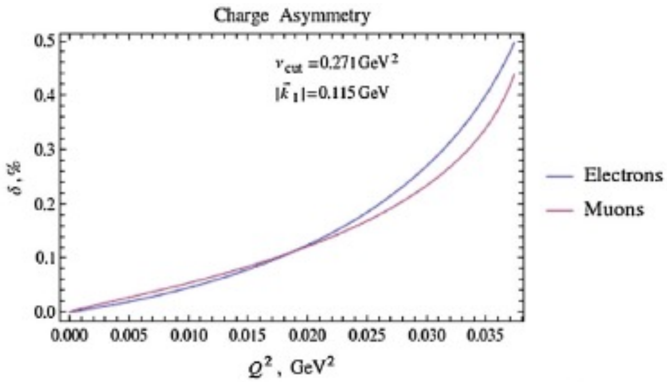
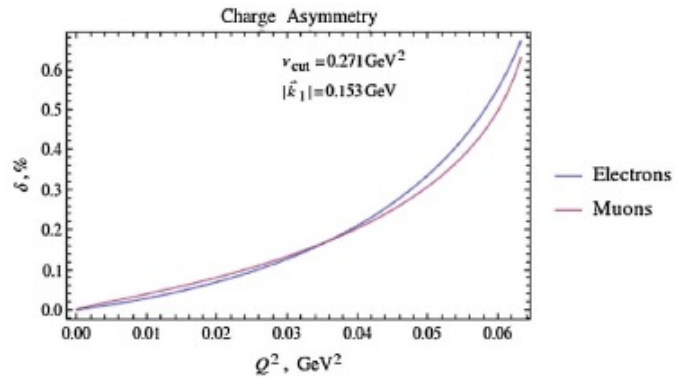
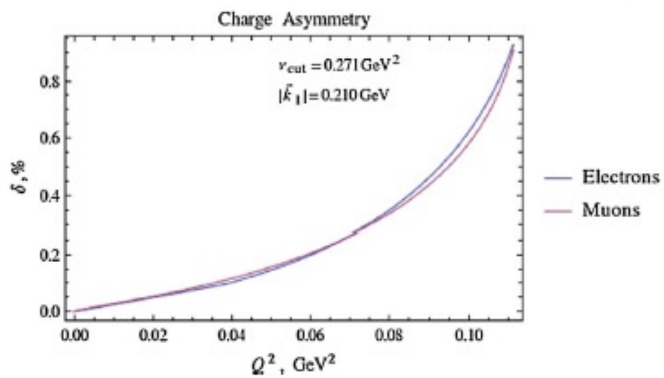
Predicting Charge Asymmetries for MUSE

- MUSE will compare e^+/e^- and μ^+/μ- cross sections
- Koshchii, AA, Phys. Rev. D 96, 016005 (2017)Predicted asymmetries are <1%







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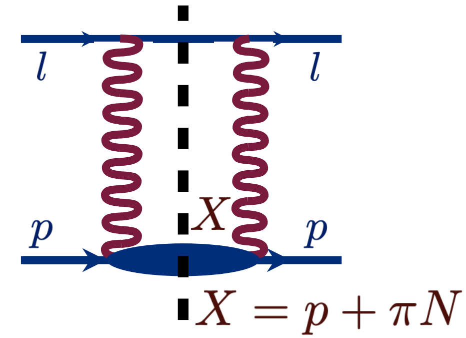
Two-photon exchange correction

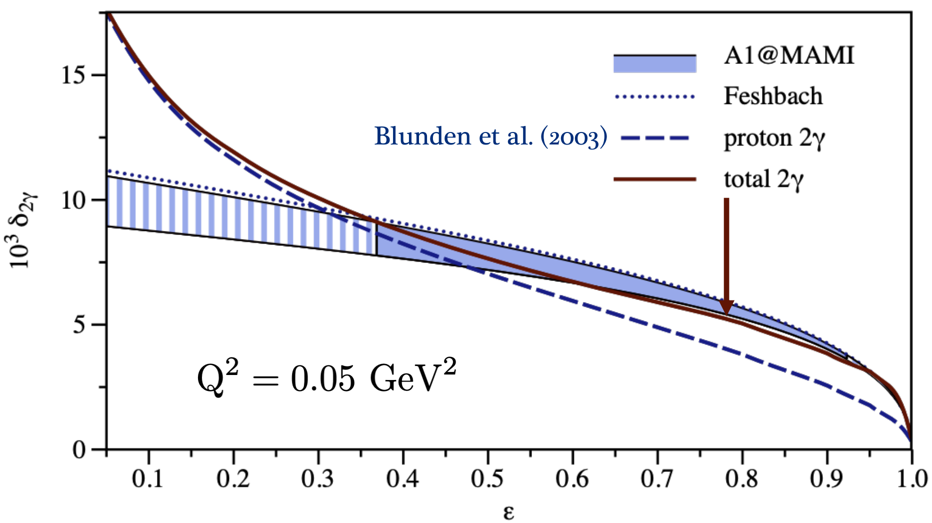
- near-forward scattering at low Q^2:

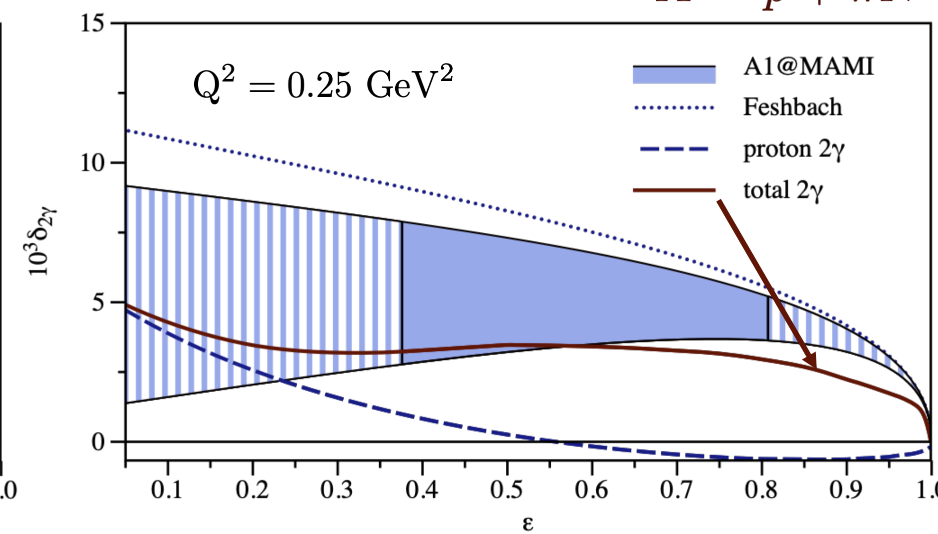
Feshbach inelastic proton

$$\text{ep}: \quad \delta_{2\gamma} \sim a\sqrt{Q^2} + b\, Q^2 \ln Q^2 + c\, Q^2 \ln^2 Q^2$$

R. W. Brown (1970)







O. T. and M. Vanderhaeghen (2016)

ω → Tomalak, Vanderhaeghen ← θ

- all channels at small scattering angles both for e and μ
- 2γ at large ε agrees with empirical fit

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Helicity-Flip in TPE; estimate of inelastic contribution

- New dynamics from scalars (σ, f-mesons). No pseudo-scalar (π^0, η) contribution for unpolarized particles, cf Naik, AA (arXiv:2401.13892)
- Scalar t-channel exchange contributes to TPE (no longer setting m_{lepton} to zero!)

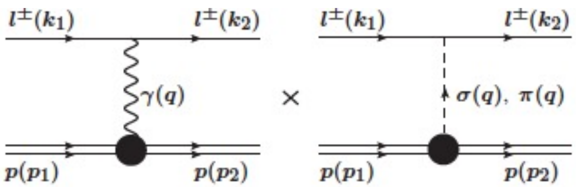
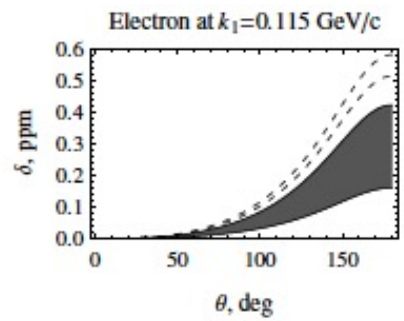
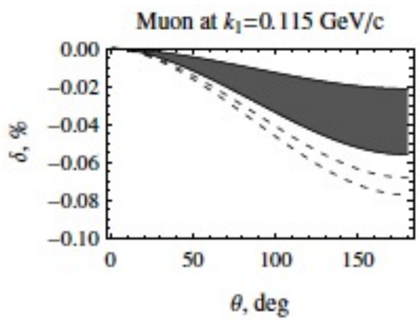


FIG. 1. One-photon and one σ (π) meson exchange diagrams





- No information on $F_{\sigma\mu\mu}$ coupling is available. Need model estimates.
- Theory analysis by AA, Koshchii, Phys.Rev. D 94, 116007 (2016).

<u>Can be studied directly in the ratio of μ+ and μ- cross sections</u>

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Rad Corrections for MUSE vs AMBER

Eur. Phys. J. A (2022) 58:156
https://doi.org/10.1140/epja/s10050-022-00805-8

THE EUROPEAN PHYSICAL JOURNAL A

Regular Article - Theoretical Physics

Charge-asymmetric correlations in elastic lepton- and antilepton-proton scattering from real photon emission

A. Afanasev[1,a], A. Ilyichev[2,3,b]

Important detail:
How is the kinematics fixed for radiative events?

(a) Fix the scattering angle, integrate over energies
(b) Fix Q^2, integrate over angles

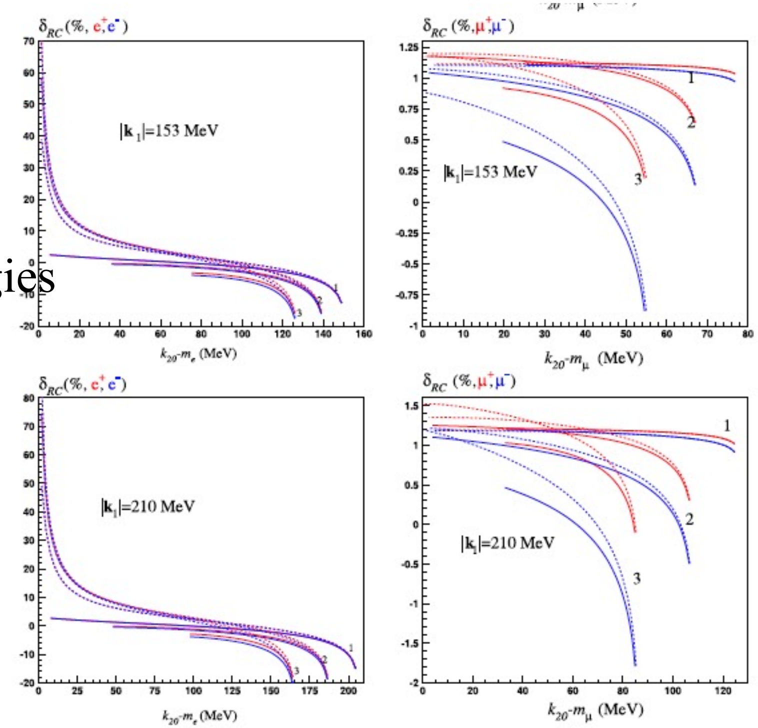

Fig. 4 Relative RC vs the value of the scattering lepton kinetic energy for elastic $e^{\mp}p$ and $\mu^{\mp}p$ scattering, beam momenta is equal to 115 MeV, 153 MeV and 210 MeV for $\theta = 20^o$ (1), 60^o (2), 100^o (3). Blue (red) lines correspond to lepton (antilepton) scattering while solid (dashed) lines correspond to fixed Q^2 ($\cos\theta$)

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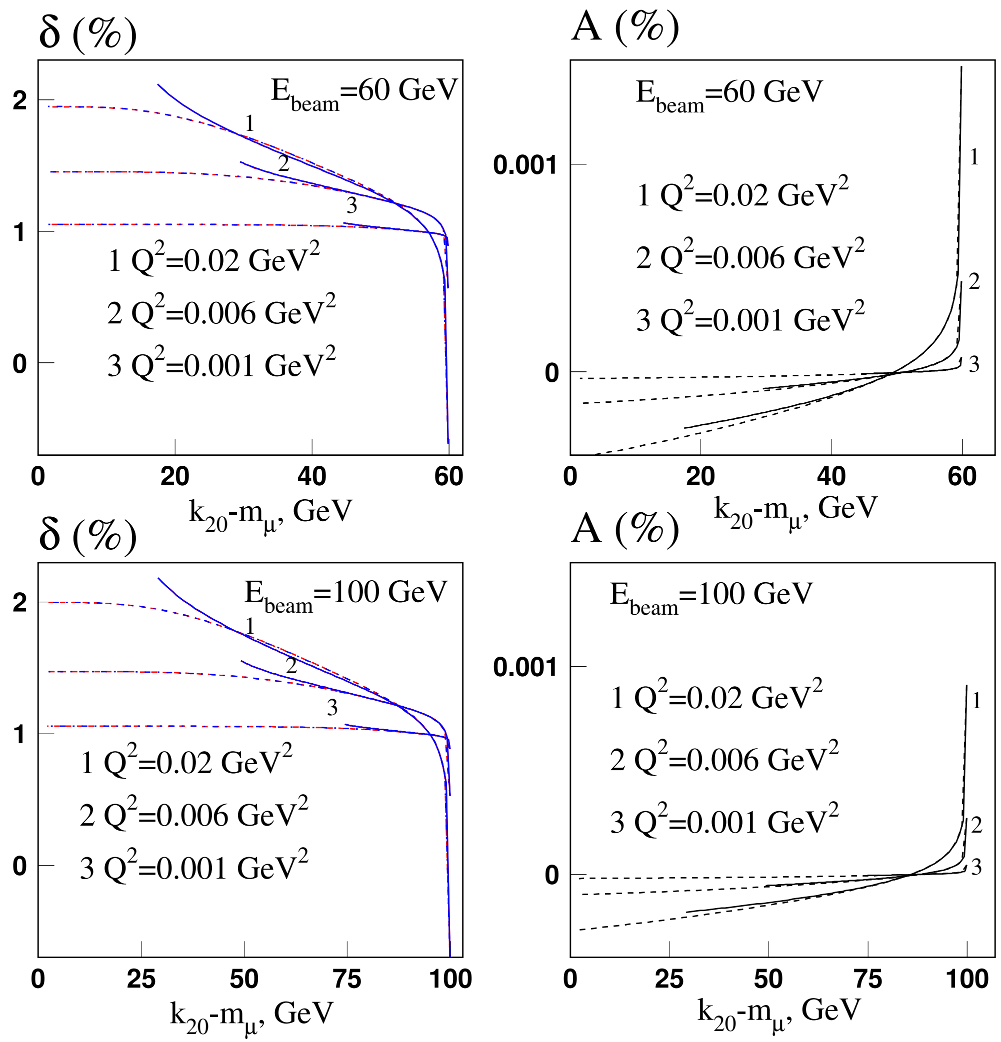
AMBER Kinematics

- RadCor for Cross Section   Charge Asymmetries



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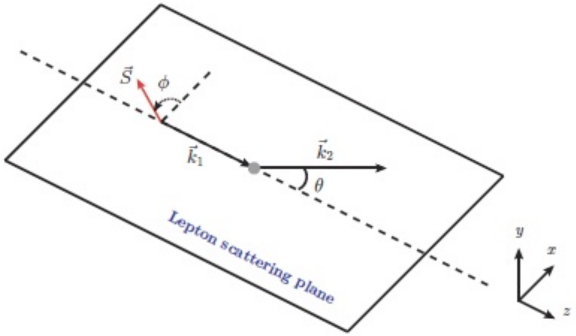
Beam Single Spin Asymmetry: A source of systematics?

- Muons produced in pion decay are spin-polarized due to weak interactions
  - Polarizations are opposite for positive vs negative muons
  - A single-spin scattering asymmetry arises from two-photon exchange and may be a source of systematic effect in MUSE
- This effects was evaluated for MUSE kinematics by Koshchii and AA,

Phys. Rev. **D** 100, 096020 (2019)

$$d\sigma_T(\phi) = d\sigma_U + \frac{\vec{S}\cdot\left(\vec{k}_1\times\vec{k}_2\right)}{\left|\vec{k}_1\times\vec{k}_2\right|}d\sigma_y$$
$$= d\sigma_U + d\sigma_y \sin\phi,$$



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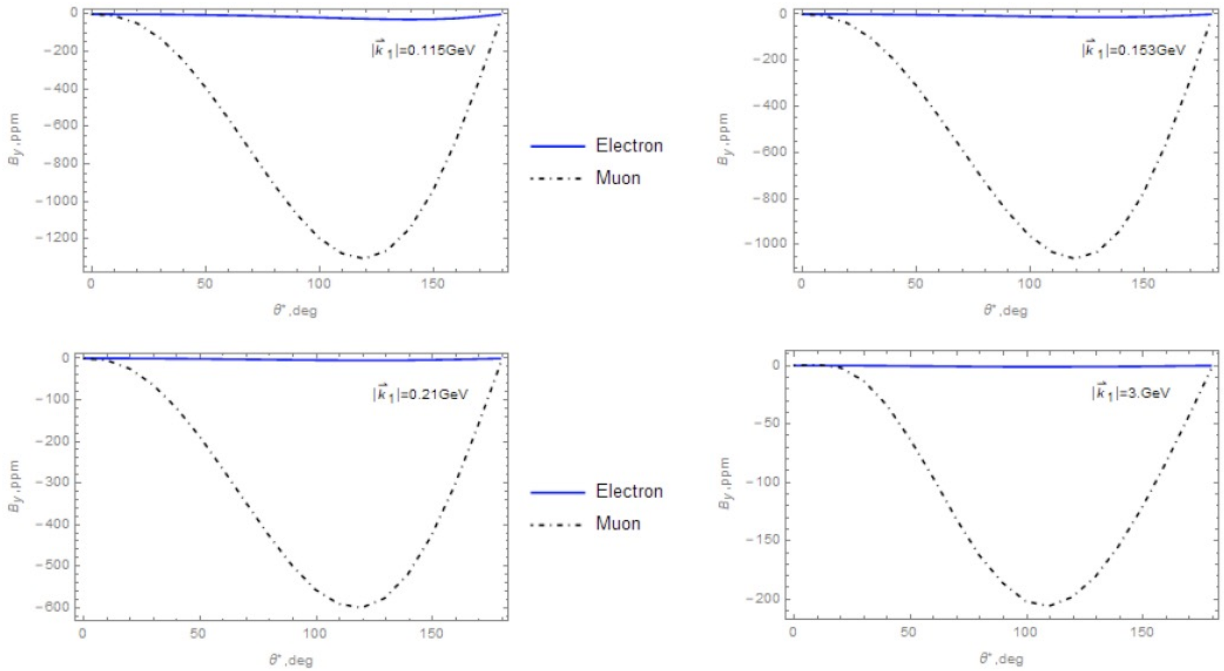
Beam SSA: results

- Koshchii, AA, Phys. Rev. D 100, 096020 (2019) Asymmetry doubles for **μ**+/**μ**- ratio



- Largest asymmetries for MUSE are for the muons, ~0.2%, azimuthal dependence ~sin(**ϕ**), therefore will cancel for azimuthally symmetric detectors. Possible source of systematics to account for.
- Additional suppression at higher energies

No issue for AMBER within ppm

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Rad Corrections for different kinematic settings

- Leptonic variables, p(l,l')γp: detection of the scattered lepton, leaving options for integration over brem photon phase space:
  - Fix the scattering angle for Q^2 of interest in the elastic limit p(l,l')p, then integrate over the final-lepton energy
  - Fix Q^2, then integrate over scattering angles
    - Both options provided by MASSRAD code
- Hadronic variables, p(l,p')γl: detection of recoil proton
  - Rad correction involves integration up to 100% energy loss by the final lepton – a highly singular behavior for electrons, less singular for muons, see AA, Akushevich, Ilyichev, Merenkov, PLB514, 269 (2001)

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Conclusions

- Radiative corrections show significant difference between electron and muon scattering, must be properly accounted for
- Radiative corrections calculated to be about 1-1.5% for muons and varies from -4% to +3% for electrons (MUSE and AMBER)
  - Uncertainties mainly from acceptances, need to include in detector simulations (Strauch et al) Theory uncertainties for MUSE experiment evaluated at <0.1% (muons), <0.5% (electrons)
- Beam SSA is evaluated as a possible source of systematics, <0.2%, events have to be integrated symmetrically over azimuthal angles to cancel SSA
- Two-photon exchange <1% (electrons), <0.5% (muons) <0.1%(inelastic excitations)
- Two-photon effects can be studied directly in the ratio of $\mu+$ and $\mu-$, e^+ and e^- cross sections; TPE cancel in the sum of particle+antiparticle cross sections

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