PAW'24

McMule
Radiative Corrections for Scattering Experiments

Marco Rocco

for the McMule Team

Paul Scherrer Institut

18$^{\text{TH}}$ March 2024

- higher-order predictions and comparison with precision experiments
- focus on $2 \to 2$ low-energy QED+ scattering processes
- **input**: matrix elements by us or others (at NNLO + first visits at N3LO)
- **output**: physical cross section for any physical observable at fixed order
- at present an integrator, generator features under testing

McMule

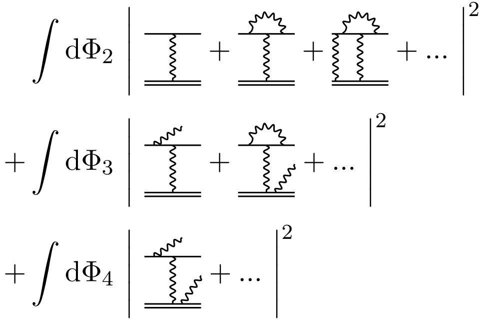
Monte Carlo for MUons and other LEptons

code → `https://mule-tools.gitlab.io/`

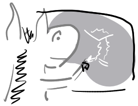
docs → `https://mcmule.readthedocs.io/`

(a taste of) theory

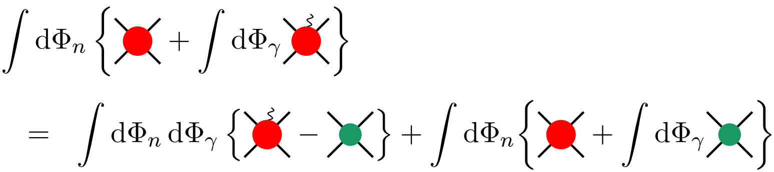
McMule at NNLO :: 3 pillars



1. fully-differential PS integration
   → FKS^ℓ
2. virtual amplitudes with massive particles
   → one-loop: OpenLoops
   → two-loop: massification
3. numerical instabilities due to pseudo-singularities
   → next-to-soft stabilisation



local subtraction of infrared divergences



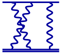
- exploits exponentiation of soft singularities [YFS 61]
- works at all orders in QED [Engel, Signer, Ulrich 19]

- singularities are dealt with locally → stable numerical integration
- subtraction makes negative-weighted events much more frequent
- theory error: 0

full 2-loop amplitude with $M \neq 0$, $m = 0$ $\rightarrow$ [Bonciani et al. 21]

full 2-loop amplitude with $M \neq 0$, $m \neq 0$ $\rightarrow$ [??]

◇ exploit scale hierarchy $m^2 \ll M^2, Q^2$, expand in $m^2/Q^2 \sim 0$

 $$\sim A \log^2 \frac{m^2}{Q^2} + B \log \frac{m^2}{Q^2} + C + \mathcal{O}\Big(\frac{m^2}{Q^2}\Big)$$

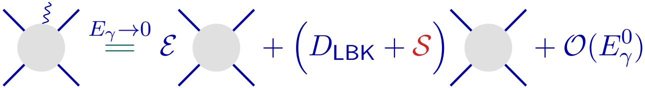
◇ massification: $\mathcal{A}_{mM}(m) = \mathcal{S}' \times Z \times Z \times \mathcal{A}_{mM}(0) + \mathcal{O}(m)$

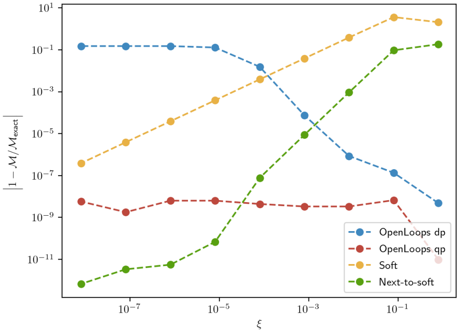
[Penin 06, Becher, Melnikov 07; Engel, Gnendiger, Signer, Ulrich 18]

◇ theory error: $\mathcal{O}(10^{-2})\,@\,\mathrm{NNLO} \sim \mathcal{O}(10^{-5})$

OpenLoops [Buccioni, Pozzorini, Zoller 18, Buccioni et al. 19]

LBK theorem [LBK 58-61, Engel, Signer, Ulrich 21, 2xEngel 23]



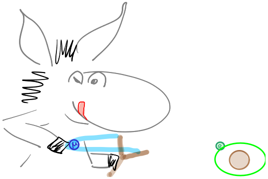


◇ introduce NTS stabilisation [McMule 21, 22]

– if $E_\gamma < E_{\text{NTS}} \sim 10^{-3}\sqrt{s}/2$
  switch to the expansion above

– theory error:
  $\mathcal{O}(10^{-3})\,@\,\text{NNLO} \sim \mathcal{O}(10^{-6})$

– different carbon footprint



phenomenology

disclaimer

what the second part of this talk does *not* contain 

- experimental details
- further technical details on higher-order QED calculations
- studies on two-photon-exchange (TPE) corrections

what the second part of this talk does contain 

- phenomenology tailored to ℓp-scattering experiments
- studies on QED radiative corrections to $\ell^\pm p^* \to \ell^\pm p^*$

lepton-proton scattering

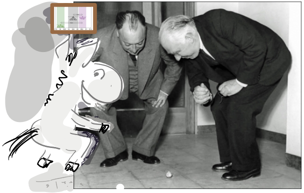
$$\ell\, p \to \ell\, p$$

lepton-proton scattering (*known subset*)

$$\ell\,\ell' \to \ell\,\ell'$$

lepton-proton scattering (*known subset*)

$$\ell\,\ell' \to \ell\,\ell'$$



[2212.06481]

lepton-proton scattering (*one more step*)

$$\ell\, p^{1\gamma} \to \ell\, p^{1\gamma}$$ “single-dipole”

$$\ell\, \boldsymbol{\mu} \to \ell\, \boldsymbol{\mu}$$ “point-like”

lepton-proton scattering @ various experiments

$\ell\, p^{1\gamma} \to \ell\, p^{1\gamma}$ "single-dipole"

$\ell\, \mu \to \ell\, \mu$ "point-like"

lepton-proton scattering @ various experiments

$$\ell\, p^{1\gamma} \to \ell\, p^{1\gamma}$$ “single-dipole”

$$\ell\, \mu \to \ell\, \mu$$ “point-like”

- $\ell = \{e^{\pm},\, \mu^{\pm}\}$

lepton-proton scattering @ various experiments

$$\ell\, p^{1\gamma} \to \ell\, p^{1\gamma}$$ “single-dipole”

$$\ell\, \mu \to \ell\, \mu$$ “point-like”

- $\ell = \{e^{\pm},\, \mu^{\pm}\}$
- $E_{\text{beam}} = 10^1 \div 10^5$ MeV

lepton-proton scattering @ various experiments

$$\ell\, p^{1\gamma} \to \ell\, p^{1\gamma}$$ "single-dipole"

$$\ell\, \boldsymbol{\mu} \to \ell\, \boldsymbol{\mu}$$ "point-like"

- $\ell = \{e^{\pm},\, \mu^{\pm}\}$
- $E_{\text{beam}} = 10^1 \div 10^5$ MeV
- angular/energy window for ℓ

lepton-proton scattering @ various experiments

$$\ell\, p^{1\gamma} \to \ell\, p^{1\gamma}$$ "single-dipole"

$$\ell\, \boldsymbol{\mu} \to \ell\, \boldsymbol{\mu}$$ "point-like"

- $\ell = \{e^{\pm},\, \mu^{\pm}\}$
- $E_{\text{beam}} = 10^1 \div 10^5$ MeV
- angular/energy window for ℓ
- restricting photon emission

MUSE

[2307.16831]

MUSE

[2307.16831]

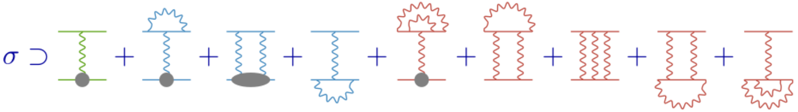
`mule-tools.gitlab.io/user-library/l-p-scattering/muse-tpe`

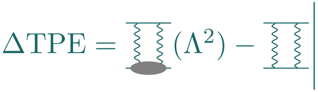
MUSE

[2307.16831]

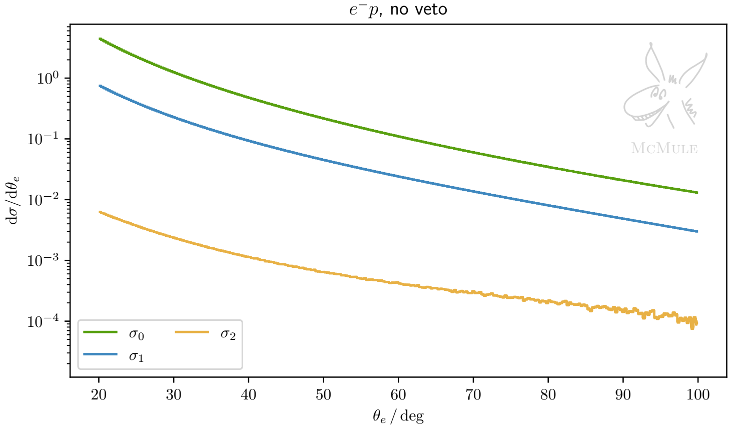
- $E_{\text{beam}} = 210$ MeV
- $|\vec{p}_\ell| > 15$ MeV
- $20 < \theta_\ell < 100$ deg
- (optional) cut events if $E_\gamma > 84$ MeV if $\theta_\gamma < 100$ mrad

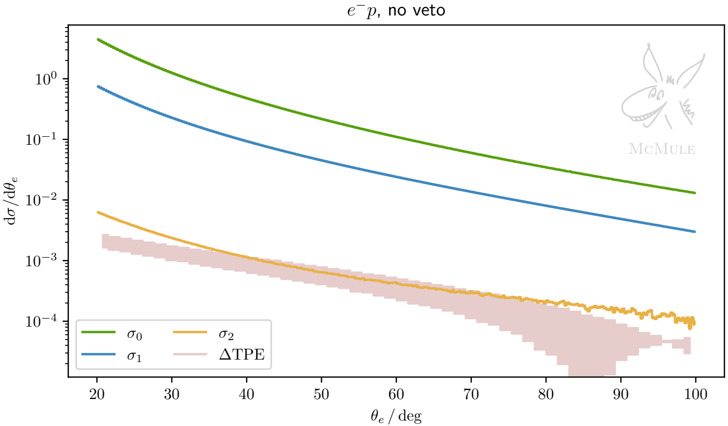
- simple dipole model for proton form factor $G_E = \frac{G_M}{1+\kappa} = \left(1 + \frac{Q^2}{\Lambda^2}\right)^{-2}$



- 
$\Delta\text{TPE} = [\ldots](\Lambda^2) - [\ldots]\Big|_{\Lambda^2=\{0.88,0.71,0.60\}\,\text{GeV}^2}$

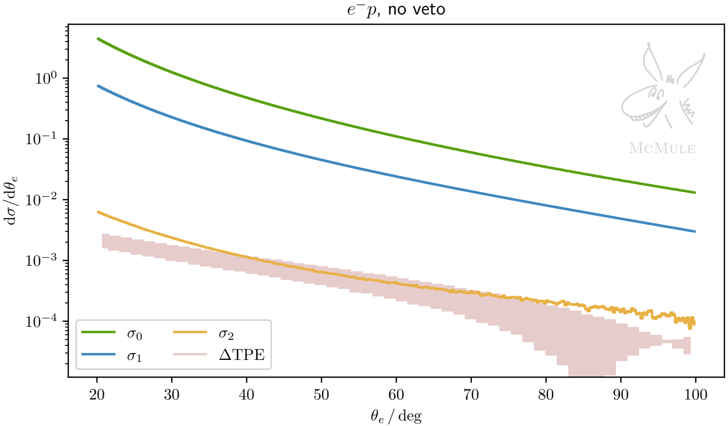
differential results :: electrons @MUSE





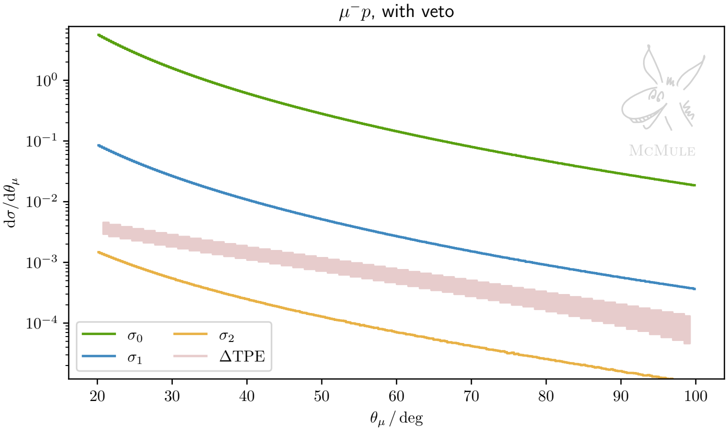
◇ NLO QED $\gtrsim$ LO hadronic

differential results :: electrons @MUSE

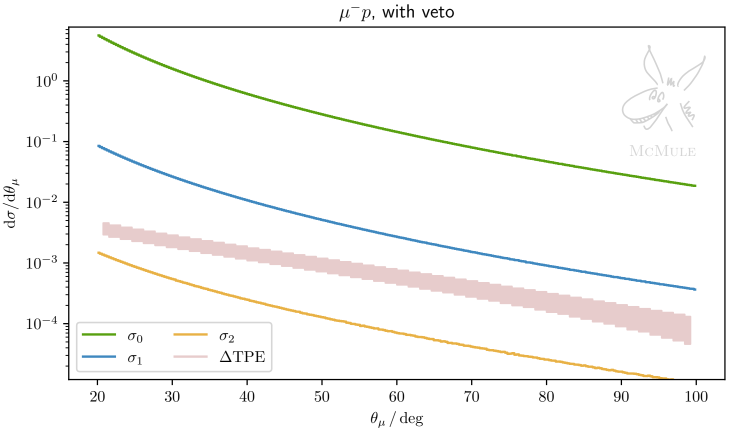


◇ NLO QED $\gtrsim$ LO hadronic

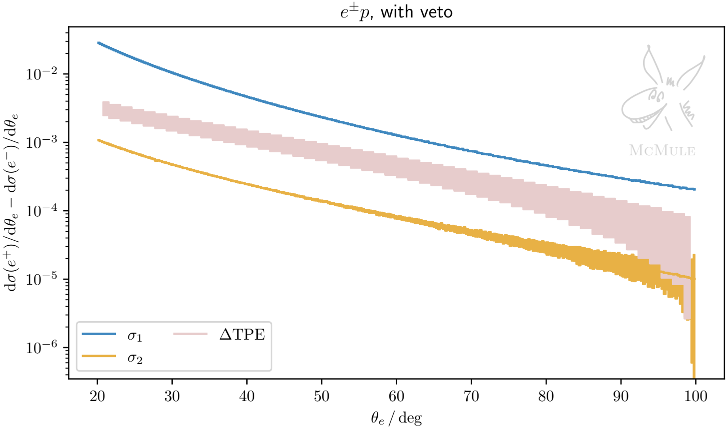
◇ NNLO QED $\sim$ TPE hadronic



◇ remove events inside ecal (~ 100 mrad) w/ $E_\gamma^{\text{tot}} > 84$ MeV



◇ NNLO QED < TPE hadronic



◇ NNLO QED $\lesssim$ TPE hadronic

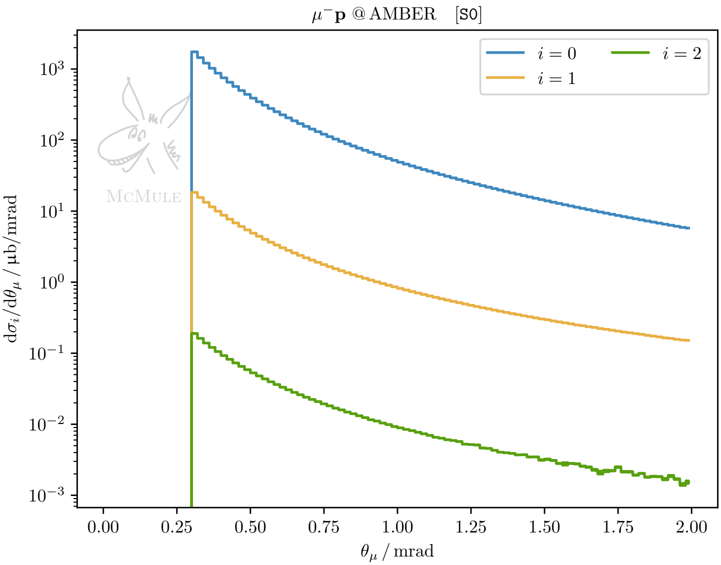
AMBER

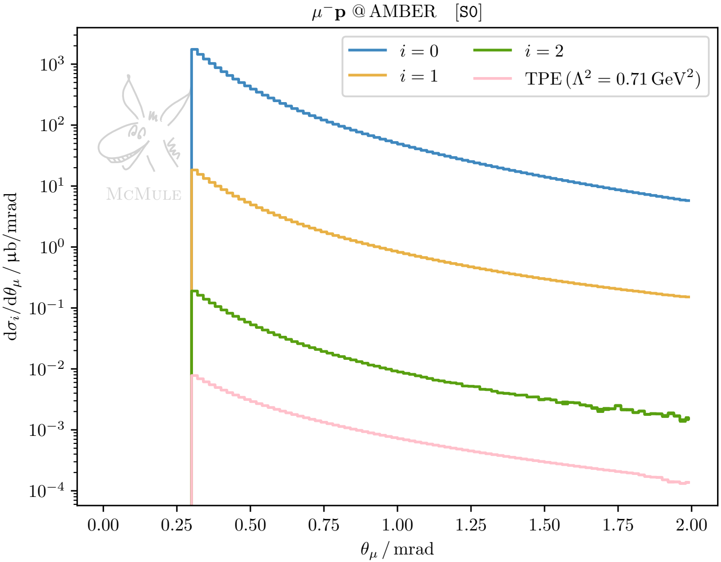
AMBER

mule-tools.gitlab.io/user-library/l-p-scattering/amber (soon here)

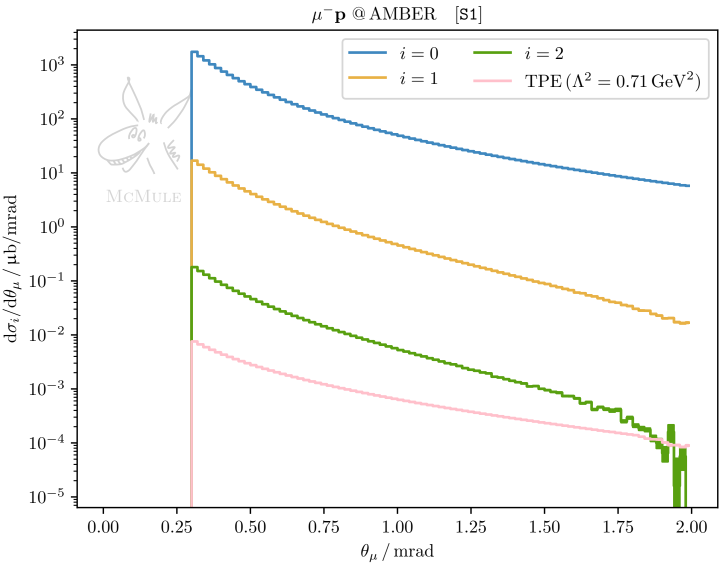
AMBER

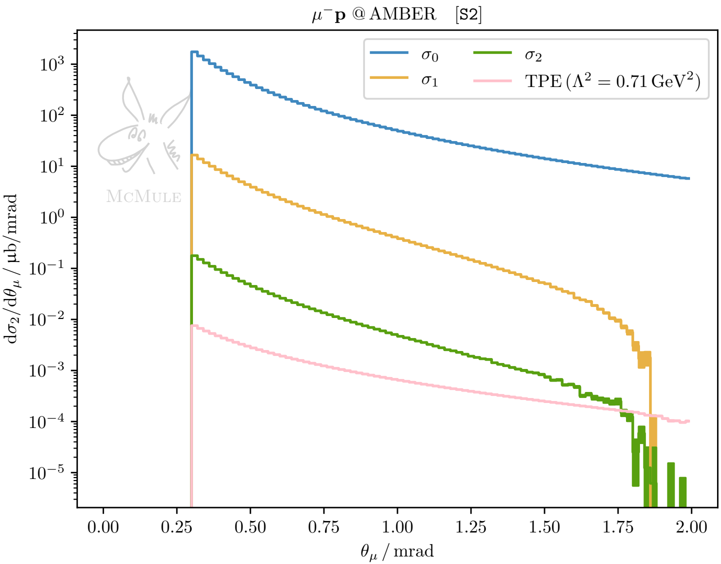
- $E_{\text{beam}} = 100$ GeV
- $0.3 < \theta_\ell < 2$ mrad
- (S1) cut events if $E_\mu < 99$ GeV *or*
- (S2) cut events if $E_\gamma > 0.3$ GeV if $\theta_\gamma < 16$ mrad


$\mu^- \mathbf{p}$ @ AMBER [S0]
$i = 0$
$i = 1$
$i = 2$
McMule
$d\sigma_i / d\theta_\mu$ / μb/mrad
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
θ_μ / mrad


$\mu^- \mathbf{p}$ @ AMBER [S0]
$i = 0$
$i = 1$
$i = 2$
TPE ($\Lambda^2 = 0.71\,\mathrm{GeV}^2$)
McMule
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
$\mathrm{d}\sigma_i / \mathrm{d}\theta_\mu$ / µb/mrad
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
θ_μ / mrad

first studies





$\mu^- \mathbf{p}$ @ AMBER [S2]
σ_0
σ_2
σ_1
TPE ($\Lambda^2 = 0.71\,\mathrm{GeV}^2$)
McMule
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
$\mathrm{d}\sigma_2/\mathrm{d}\theta_\mu$ / μb/mrad
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
θ_μ / mrad

PRad

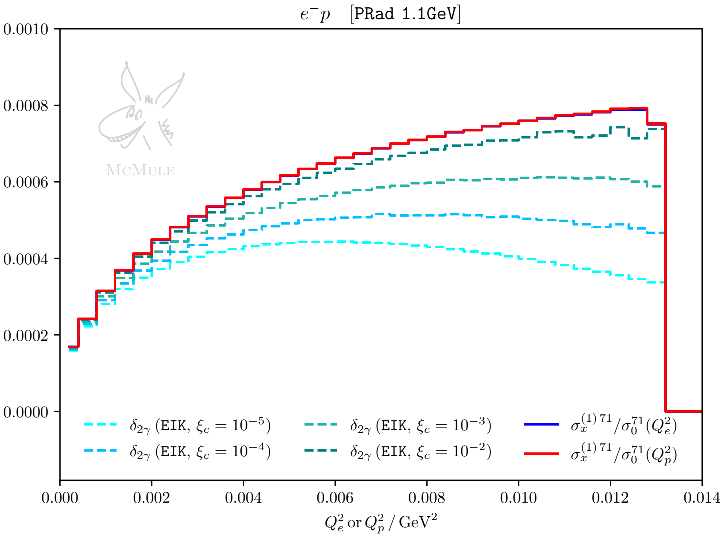
PRad

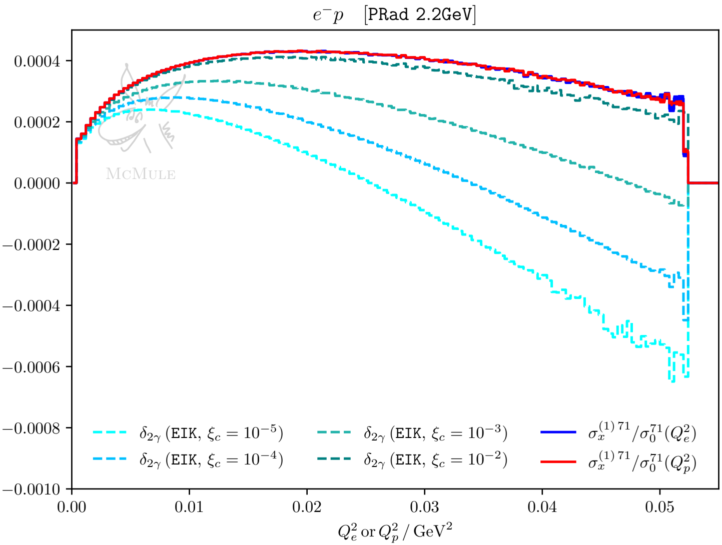
`mule-tools.gitlab.io/user-library/l-p-scattering/prad` (soon here)

PRad

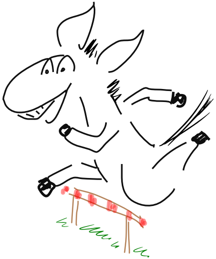
- $E_{\text{beam}} = 1.1$ GeV or 2.2 GeV
- $0.7 < \theta_e < 6$ deg
- cut events if $E_\gamma > 20$ MeV if $\theta_\gamma > 6$ mrad
- Møller scattering background!

focus on subtraction





e^-p [PRad 2.2GeV]
0.0004
0.0002
0.0000
−0.0002
−0.0004
−0.0006
−0.0008
−0.0010
0.00
0.01
0.02
0.03
0.04
0.05
Q_e^2 or Q_p^2 / GeV^2
McMule
$\delta_{2\gamma}$ (EIK, $\xi_c = 10^{-5}$)
$\delta_{2\gamma}$ (EIK, $\xi_c = 10^{-4}$)
$\delta_{2\gamma}$ (EIK, $\xi_c = 10^{-3}$)
$\delta_{2\gamma}$ (EIK, $\xi_c = 10^{-2}$)
$\sigma_x^{(1)\,71}/\sigma_0^{71}(Q_e^2)$
$\sigma_x^{(1)\,71}/\sigma_0^{71}(Q_p^2)$



a mule never stops

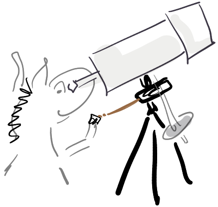
- modern (QCD-inspired) QED can predict NNLO if not more
- assessed importance of NNLO QED wrt TPE (hadronic)

a mule never stops

- modern (QCD-inspired) QED can predict NNLO if not more
- assessed importance of NNLO QED wrt TPE (hadronic)
  $\hookrightarrow$ make sure you are taking care of photons $\rightarrow$ McMule allows you to!

- modern (QCD-inspired) QED can predict NNLO if not more
- assessed importance of NNLO QED wrt TPE (hadronic)

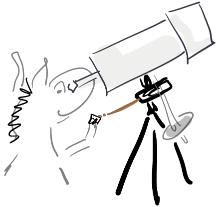
future steps of the mule



- modern (QCD-inspired) QED can predict NNLO if not more
- assessed importance of NNLO QED wrt TPE (hadronic)

future steps of the mule

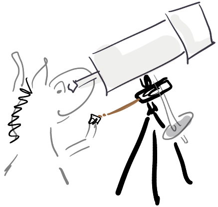
◇ integrator $\longrightarrow$ generator



- modern (QCD-inspired) QED can predict NNLO if not more
- assessed importance of NNLO QED wrt TPE (hadronic)

future steps of the mule

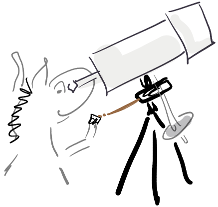
- ◇ integrator $\longrightarrow$ generator
- ◇ N3LO corrections for $\ell\ell \to \gamma^*$



- modern (QCD-inspired) QED can predict NNLO if not more
- assessed importance of NNLO QED wrt TPE (hadronic)

future steps of the mule

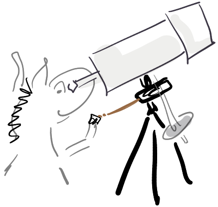
- integrator $\to$ generator
- N3LO corrections for $\ell\ell \to \gamma^*$
- γ^* can be *any* OPE (^1H, ^2H, ^{12}C, $\ldots$)
  $\to$ ULQ^2, MAGIX



- modern (QCD-inspired) QED can predict NNLO if not more
- assessed importance of NNLO QED wrt TPE (hadronic)

future steps of the mule

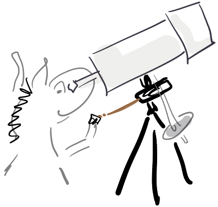
- integrator $\to$ generator
- N3LO corrections for $\ell\ell \to \gamma^*$
- γ^* can be *any* OPE (^1H, ^2H, ^{12}C, $\ldots$)
  $\to$ ULQ^2, MAGIX
- get dirtier with protons (and pions)



- modern (QCD-inspired) QED can predict NNLO if not more
- assessed importance of NNLO QED wrt TPE (hadronic)

future steps of the mule

- integrator $\to$ generator
- N3LO corrections for $\ell\ell \to \gamma^*$
- γ^* can be *any* OPE $({}^1H, {}^2H, {}^{12}C, \ldots)$
  $\to$ ULQ^2, MAGIX
- get dirtier with protons (and pions)
- electroweak corrections

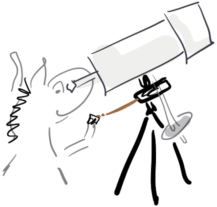


a mule never stops

- modern (QCD-inspired) QED can predict NNLO if not more
- assessed importance of NNLO QED wrt TPE (hadronic)

future steps of the mule

- integrator $\to$ generator
- N3LO corrections for $\ell\ell \to \gamma^*$
- γ^* can be *any* OPE (^1H, ^2H, ^{12}C, $\ldots$)
  $\to$ ULQ^2, MAGIX
- get dirtier with protons (and pions)
- electroweak corrections
- beyond-fixed-order QED (à la YFS)



- modern (QCD-inspired) QED can predict NNLO if not more
- assessed importance of NNLO QED wrt TPE (hadronic)

future steps of the mule

- ◇ integrator $\rightarrow$ generator
- ◇ N3LO corrections for $\ell\ell \rightarrow \gamma^*$
- ◇ γ^* can be *any* OPE (^1H, ^2H, ^{12}C, $\ldots$)
  $\rightarrow$ ULQ^2, MAGIX
- ◇ get dirtier with protons (and pions)
- ◇ electroweak corrections
- ◇ beyond-fixed-order QED (à la YFS)
- ◇ world dominance

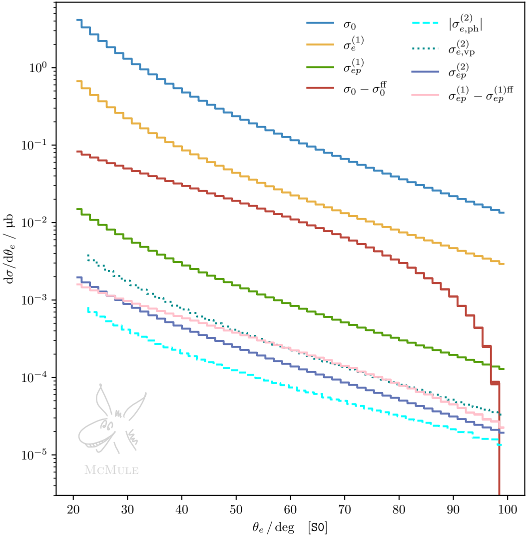
- modern (QCD-inspired) QED can predict NNLO if not more
- assessed importance of NNLO QED wrt TPE (hadronic)

future steps of the mule

- integrator $\to$ generator
- N3LO corrections for $\ell\ell \to \gamma^*$
- γ^* can be *any* OPE (${}^1H$, ${}^2H$, ${}^{12}C$, $\ldots$)
  $\to$ ULQ^2, MAGIX
- get dirtier with protons (and pions)
- electroweak corrections
- beyond-fixed-order QED (à la YFS)

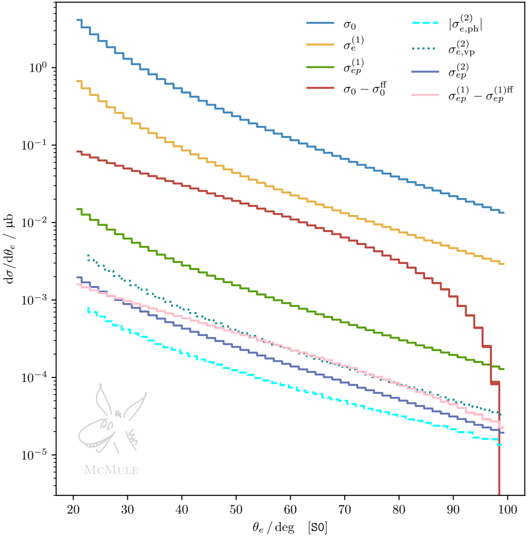
Thank you!

hard cut on hard photons @MUSE

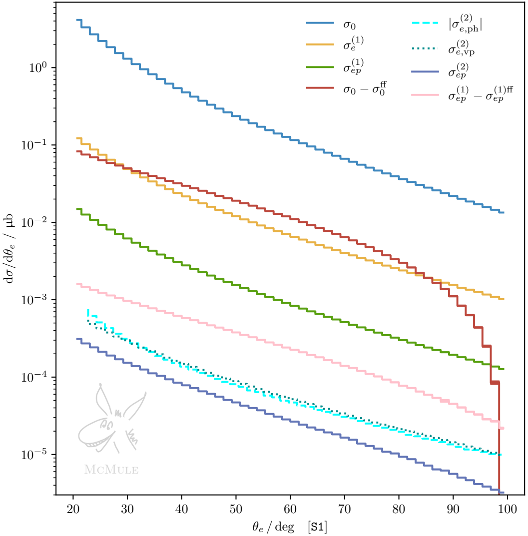


a closer look at MUSE

hard cut on hard photons @MUSE

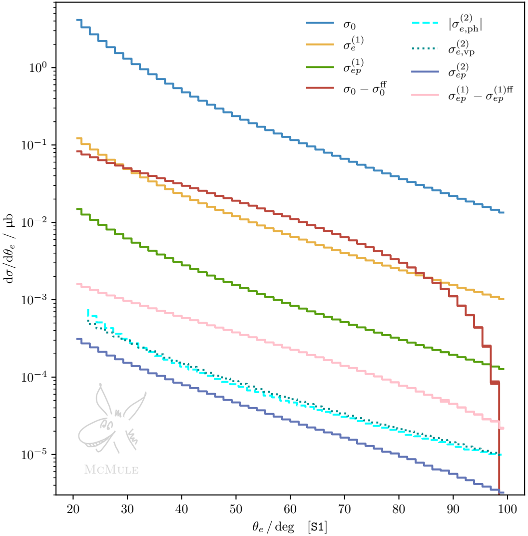


– forward calorimeter ($\sphericalangle \sim 100$ mrad)



- forward calorimeter ($\sphericalangle \sim 100$ mrad)
- remove events inside w/ $E_\gamma^{\rm tot} > 0.4p$ (@MUSE $p = 210$ MeV)

hard cut on hard photons @MUSE



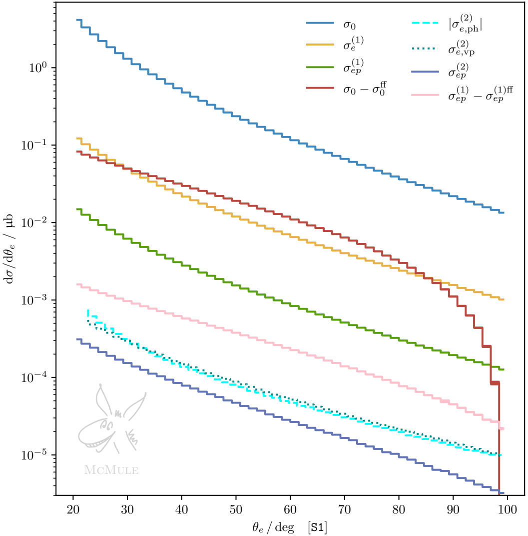
– forward calorimeter ($\sphericalangle \sim 100$ mrad)

– remove events inside w/ $E_\gamma^{\rm tot} > 0.4p$

(@MUSE $p = 210$ MeV)

◇ NLO QED $\sim$ LO hadronic

hard cut on hard photons @MUSE



– forward calorimeter ($\sphericalangle \sim 100$ mrad)

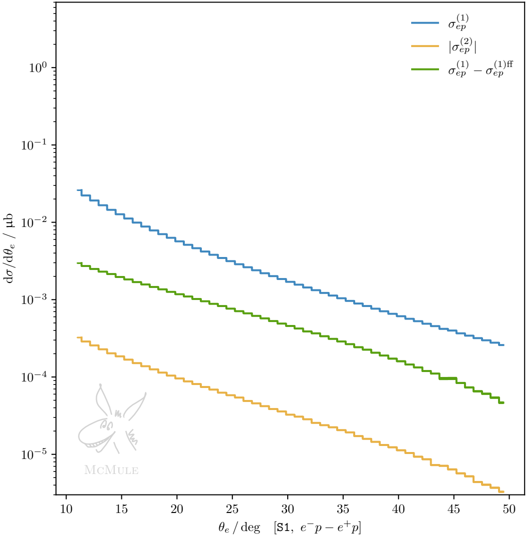
– remove events inside w/ $E_\gamma^{\rm tot} > 0.4p$

(@MUSE $p = 210$ MeV)

◇ NLO QED ∼ LO hadronic

◇ NNLO QED $\lesssim$ TPE hadronic

hard cut on hard photons @MUSE



– forward calorimeter ($\sphericalangle \sim 100$ mrad)

– remove events inside w/ $E_\gamma^{\mathrm{tot}} > 0.4p$

(@MUSE $p = 210$ MeV)

◇ NLO QED $\sim$ LO hadronic

◇ NNLO QED $\lesssim$ TPE hadronic