AMBER
Apparatus for Meson and Baryon Experimental Research

Measuring Hadron Charge Radii with AMBER

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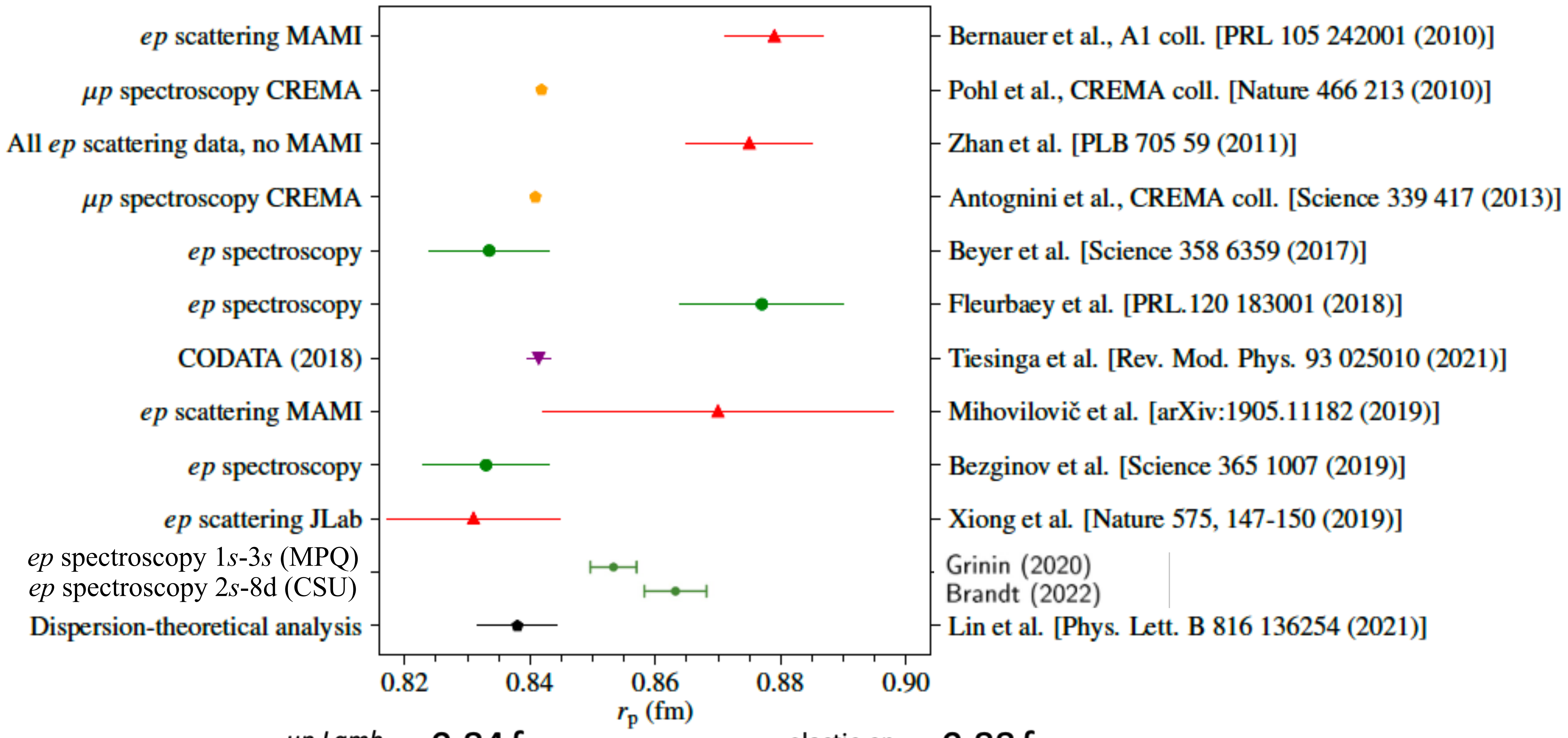
Proton Radius Measurements



$r_p^{\mu p\ Lamb} \approx 0.84\,\mathrm{fm}$ $r_p^{elastic\ ep} \approx 0.88\,\mathrm{fm}$

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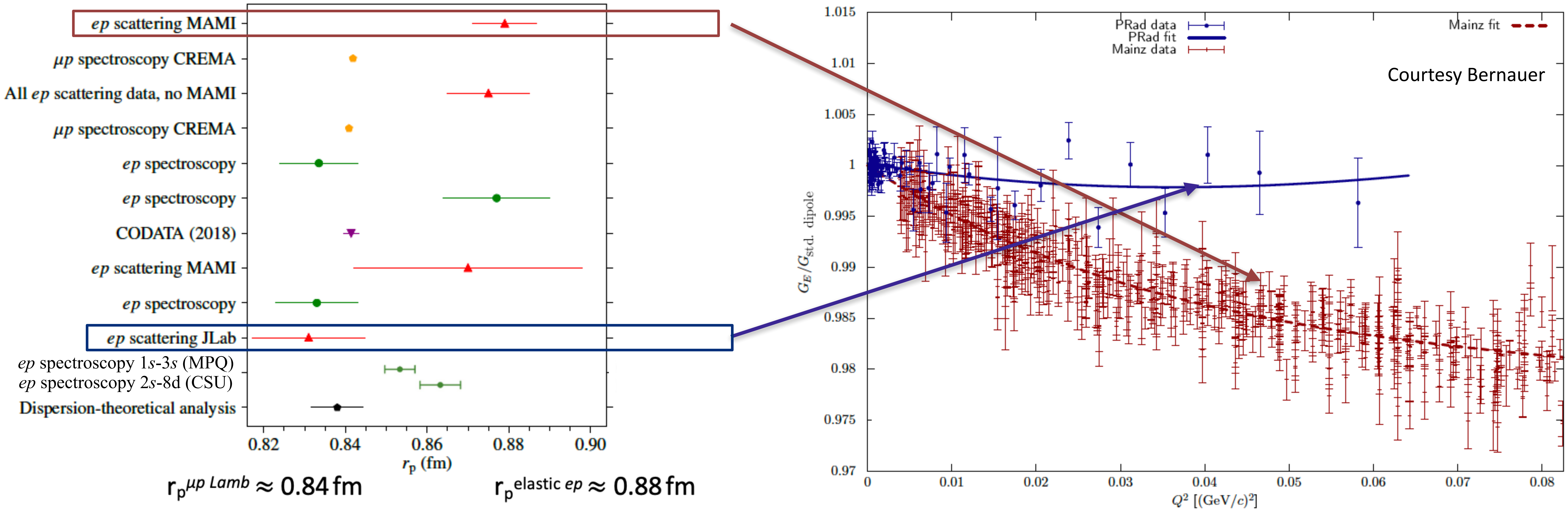

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Alternative techniques

- MUSE: low energy μ and e beams of both polarities
- ULQ2 (Tohoku): very low energy electron scattering (Suda et al.)
- COMPASS: high energy μ beams of both polarities (x 500 beam energy of MUSE!!)
  – beam energy irrelevant.. Q^2 is important variable (see details later)
  – COMPASS has demonstrated excellent Q^2 resolution with Primakoff reactions
  – Coulomb peak from πA scattering $\pi + Z \rightarrow \pi + \gamma + Z_{recoil}$ - $\Delta Q^2 \approx 5 \times 10^{-4} (GeV/c)^2$
  – well performing spectrometer and well understood apparatus

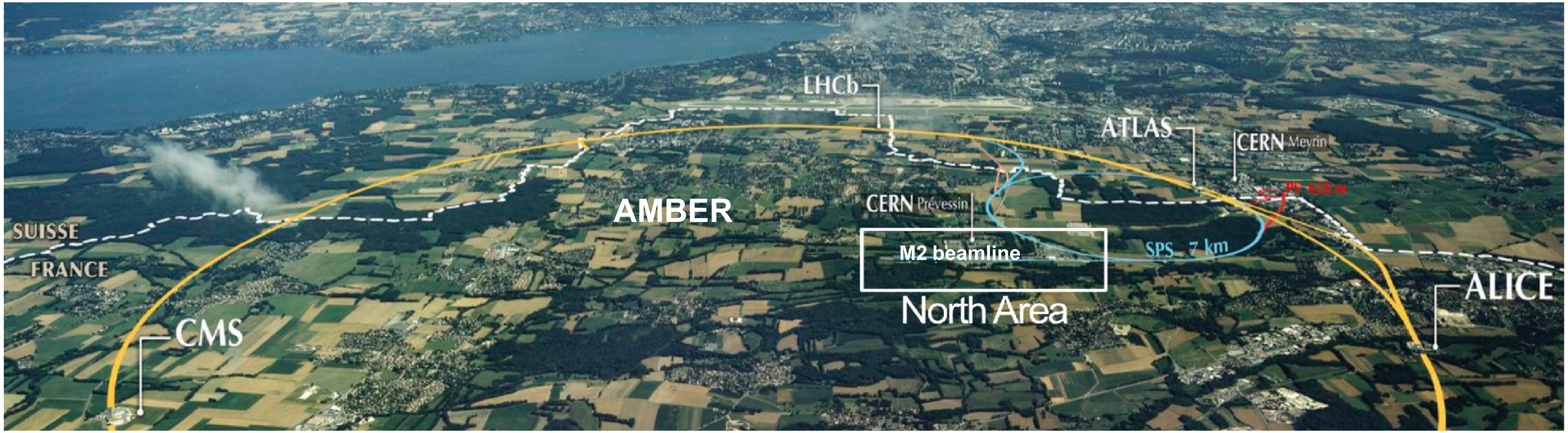
....

Apparatus for Meson and Baryon Experimental Research

Beamline for High-Energy Muon Beams

M2 beamline at *CERN's SPS*

North Area of *CERN* : M2 beamline provides a unique high-intensity muon beam



- Muon momenta up to 200 GeV/*c* - flux up to 10^7 μ/s
- PRM: beam momentum of 100 GeV/*c* and 2 MHz beam rate

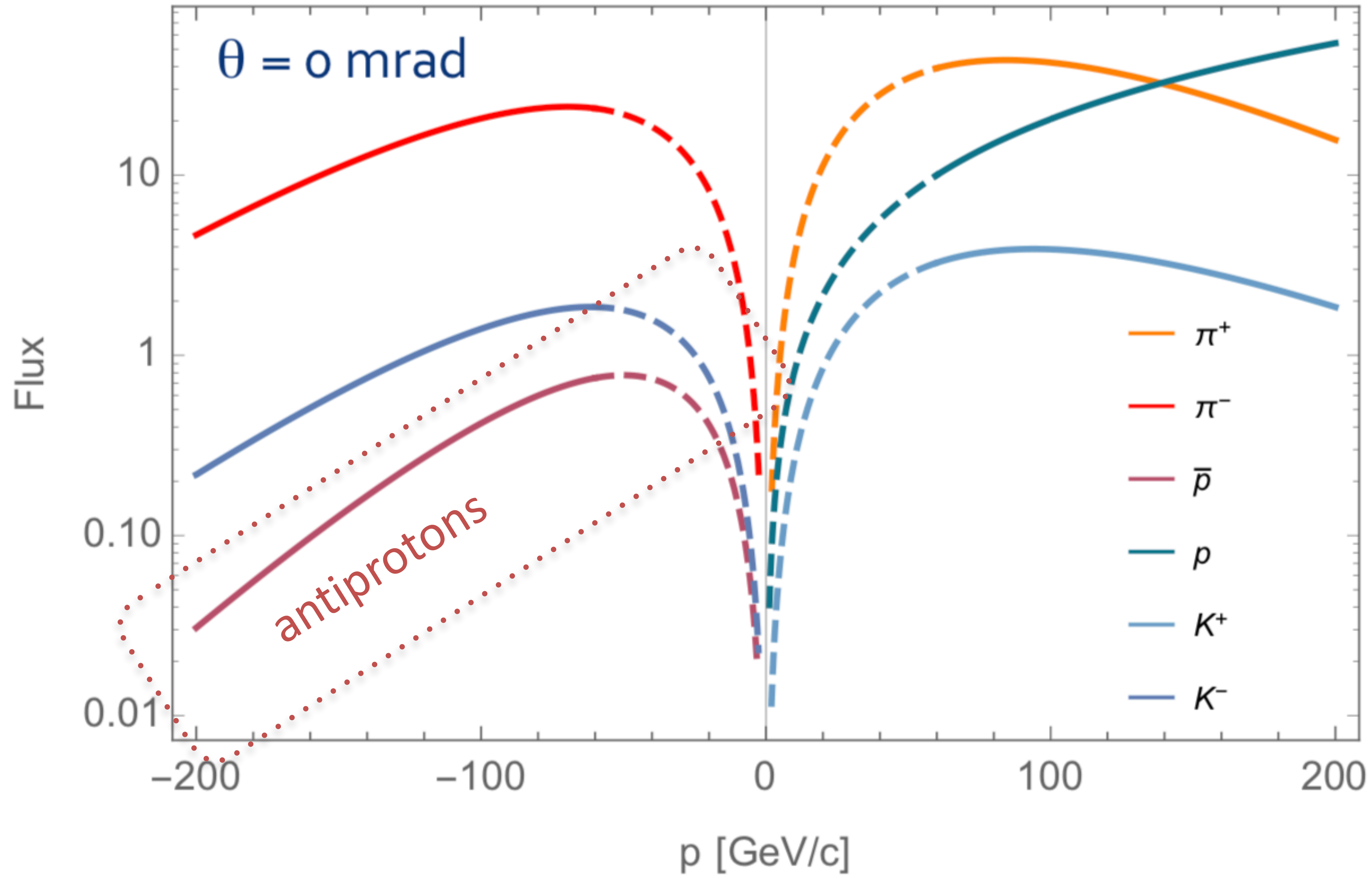
- *AMBER* as successor at *COMPASS* location starting 2023 with the first full PRM pilot run in 10/2023

→ broad physics program: PRM, Drell-Yan, Anti-Proton Cross-Section, use RF separated beams (plan)

See presentation Jan Friedrich

Hadron Radius Measurements with hadron Beams

- M2 beam line is also an excellent hadron beam line



Particle ID

- use 2 Beam CEDARs
- efficiently tag rarest hadron

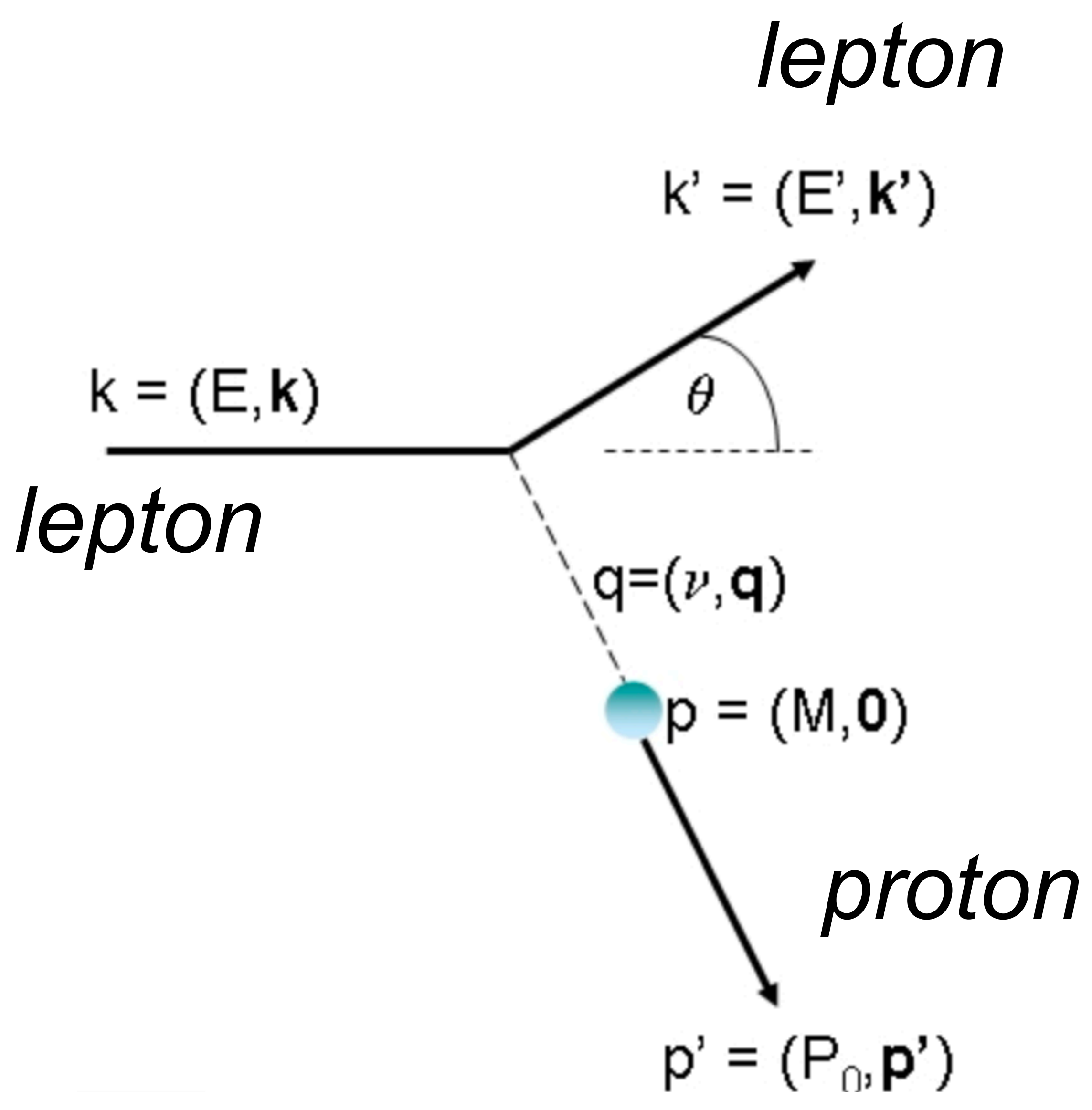
Hadron Charge Radii Through Elastic Hadron-Lepton Scattering at low Q^2

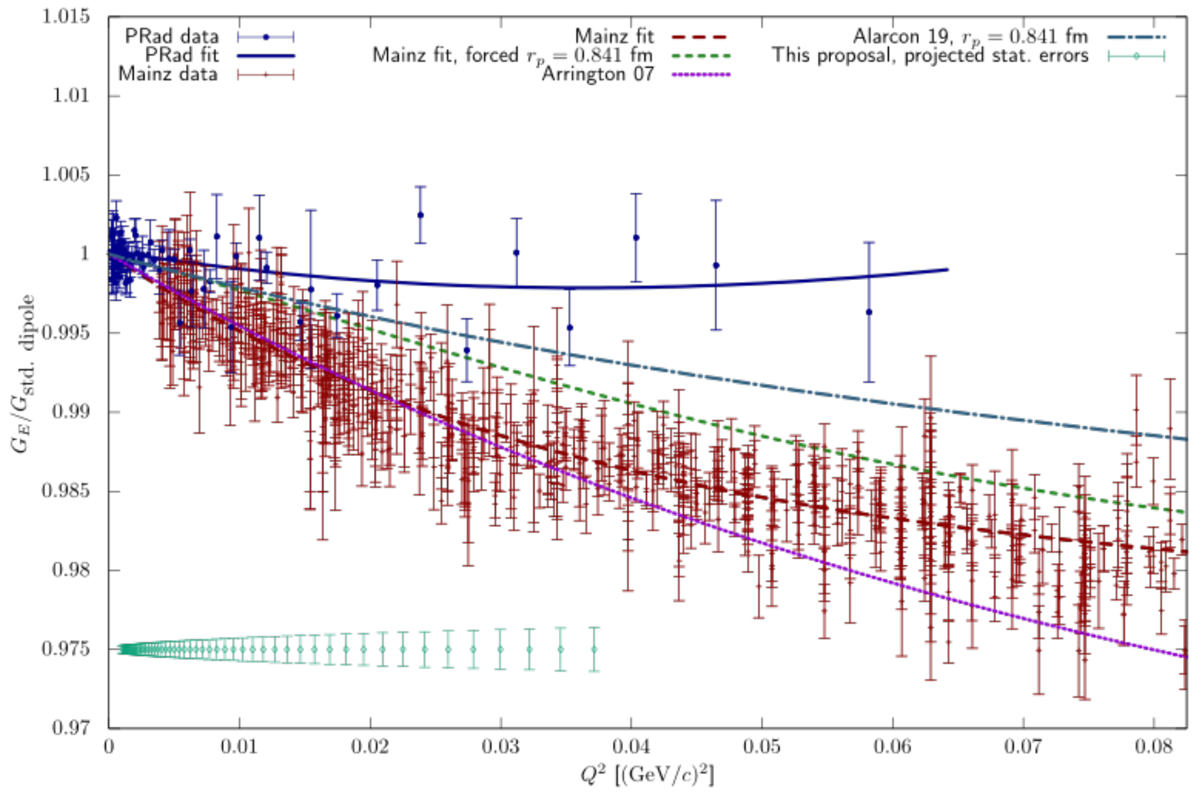
Protons in hydrogen target (or other stable nuclei):
Measurement via elastic electron or muon scattering
Cross section:

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4} R \left(\varepsilon G_E^2 + \tau G_M^2\right)$$

Charge radius from the slope of G_E

$$\langle r_E^2 \rangle = -6\hbar^2 \left. \frac{dG_E(Q^2)}{dQ^2} \right|_{Q^2 \to 0}$$





Hadron Charge Radii Through Elastic Hadron-Lepton Scattering at low Q²

Apparatus for Meson and Baryon Experimental Research

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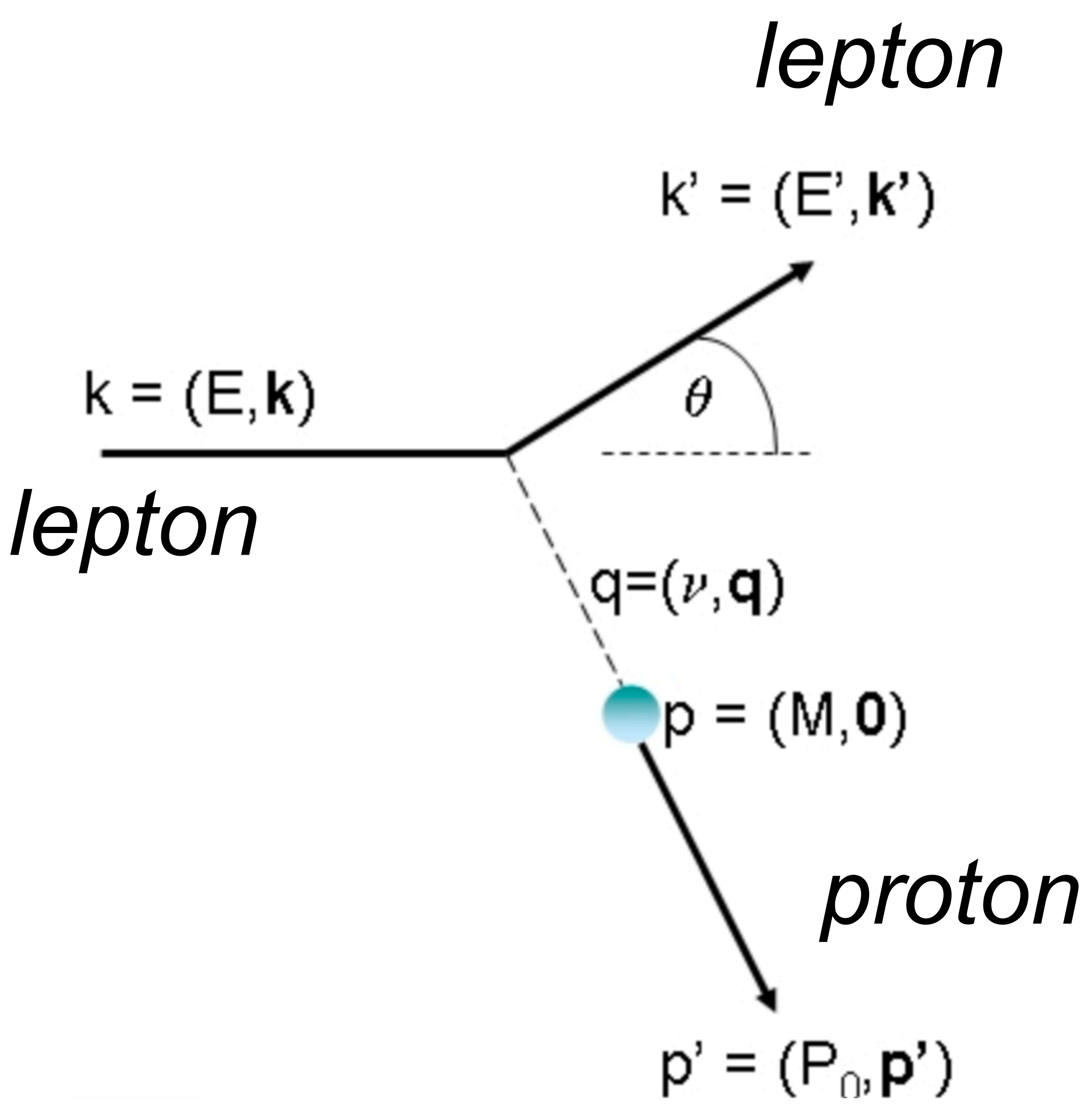
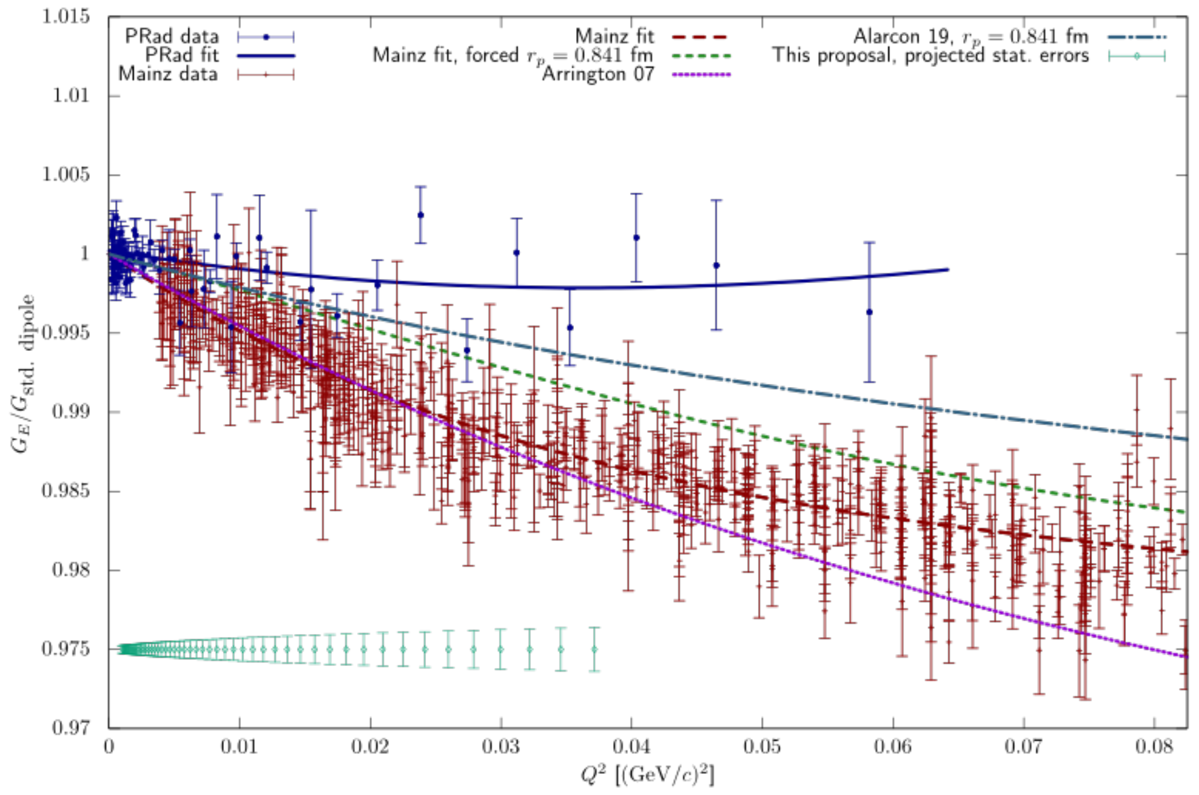
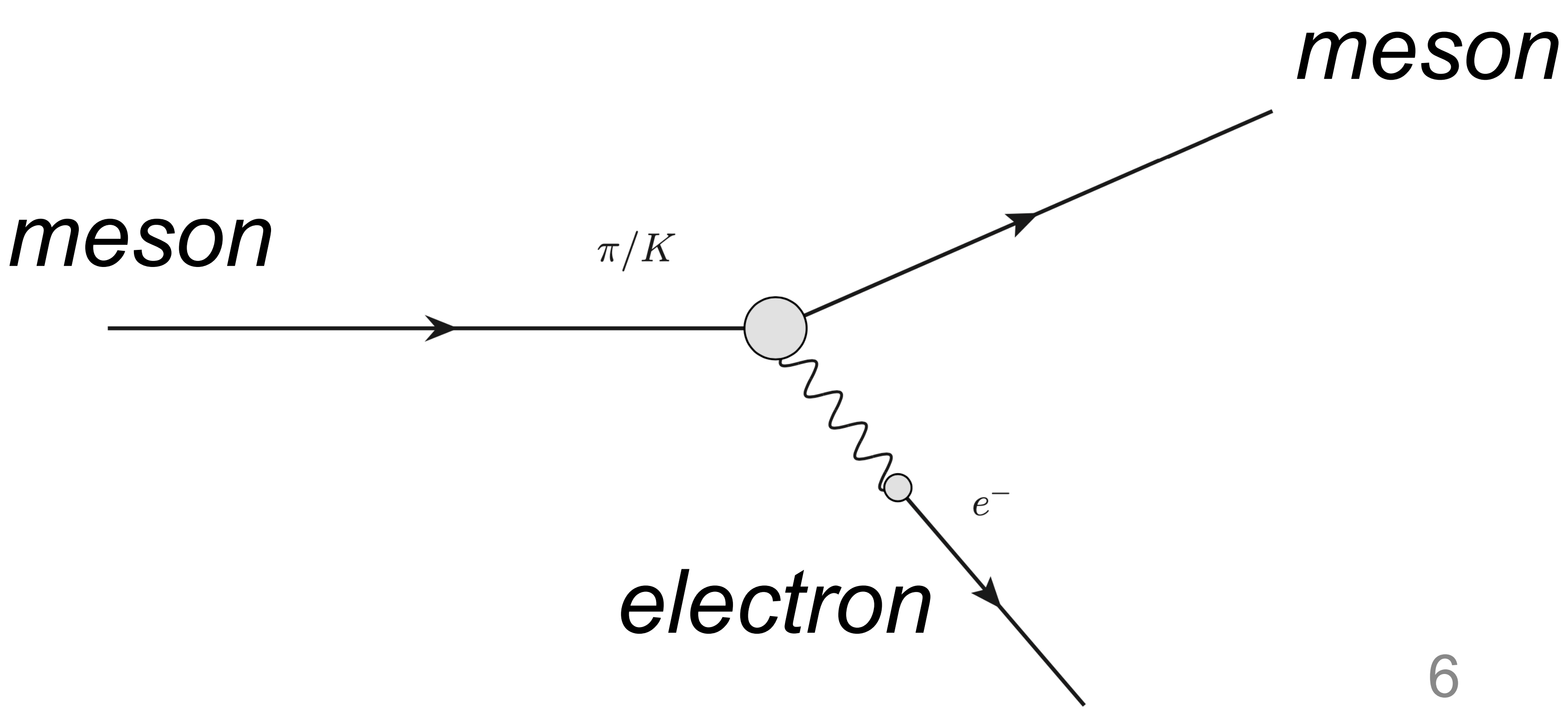




Charge radius from the slope of G_E

$$\langle r_E^2 \rangle = -6\hbar^2 \left. \frac{dG_E(Q^2)}{dQ^2} \right|_{Q^2 \to 0}$$

For unstable particles, electron scattering can only be realised in *inverse kinematics*



Hadron Radius Measurements

From: EPJC 8 (**1999**) 59, The WA89 Collaboration (measurement of Σ^- charge radius) updated 21.6.2022

Measured $\langle r^2_{ch} \rangle$ in fm^2 of various hadrons

	Experiment		experiment year
p	$\approx 0.84 - 0.87$		2023
$\overline{p}$			unmeasured
n	-0.1101 ± 0.0086		2021
Σ^-	$0.61 \pm 0.12 \pm 0.09$		2001
π^-	0.439 ± 0.008 [5]		1986
K^-	0.34 ± 0.02 [6]		1986
K_L^0	$-0.077 \pm 0.007 \pm 0.011$	$K_L^0 \rightarrow \pi^-\pi^+e^+e^-$	1998

comparatively good accuracies (pion radius ~2%) stem from assuming a theoretical shape of the form factor

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Measuring Hadron Charge Radii in Inverse Kinematics

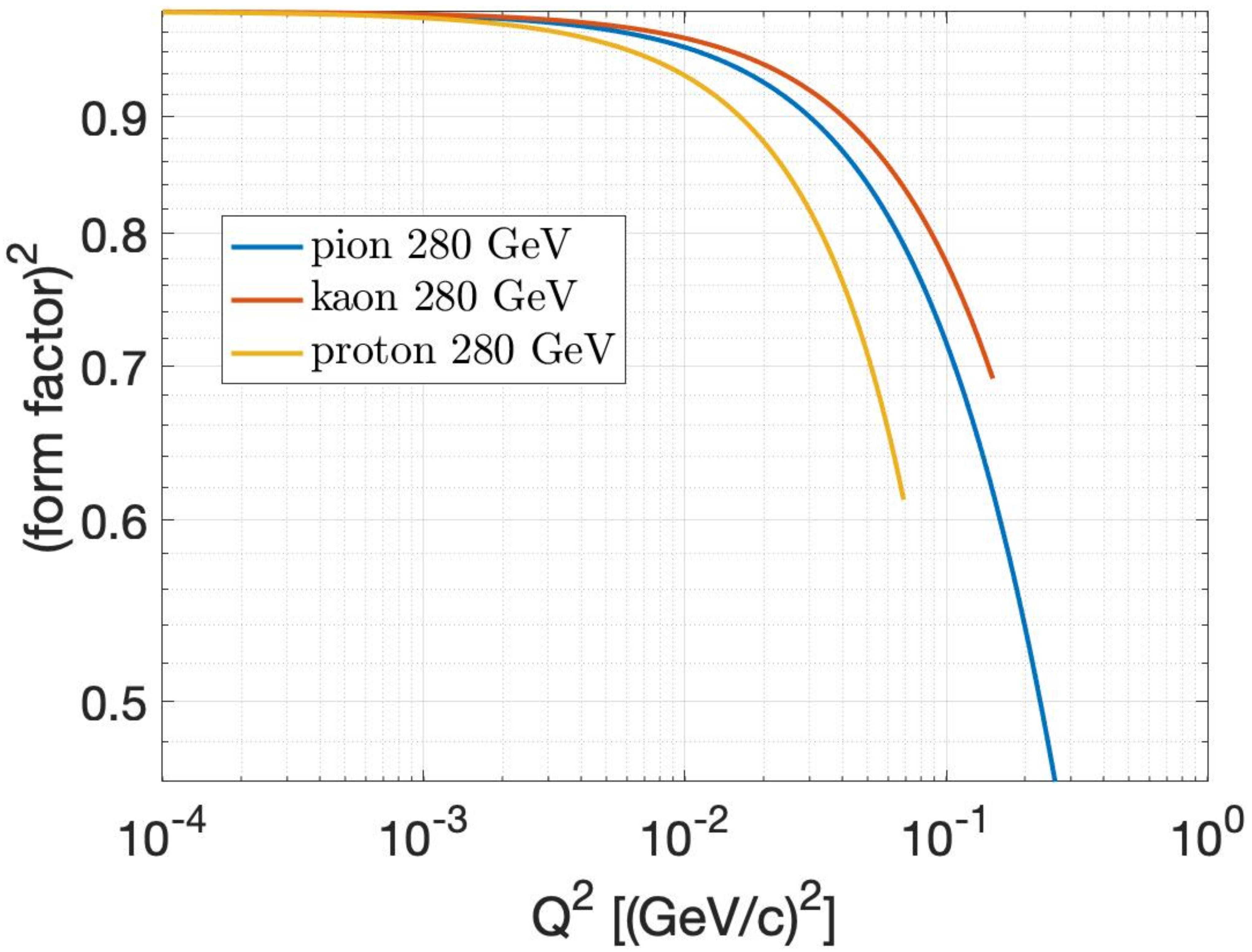
Why using inverse kinematics ?

- with no stable meson target existing - use stable lepton target
  - hadron is beam particle —> reaction in inverse kinematics

- kinematic range experimentally „unreachable“
  - make use of „easily“ measurable quantities to address „difficult regime“ (mostly low Q^2)
- electron initially at rest —> no initial external Bremsstrahlung
- final electron is accelerated —> external Bremsstrahlung for outgoing electron
  - impact on particle momentum
  - Impact on particle trajectory
- internal Bremsstrahlung effects independent of reference system (vertex corrections)

What is the role of Q^2_{max}

- large values of Q²: higher sensitivity to charge distribution —> $< r_E^2 >$
- small values of Q²: smaller extrapolation uncertainties to Q² = 0 and $\frac{dF(Q^2)}{dQ^2}|_{Q^2=0}$

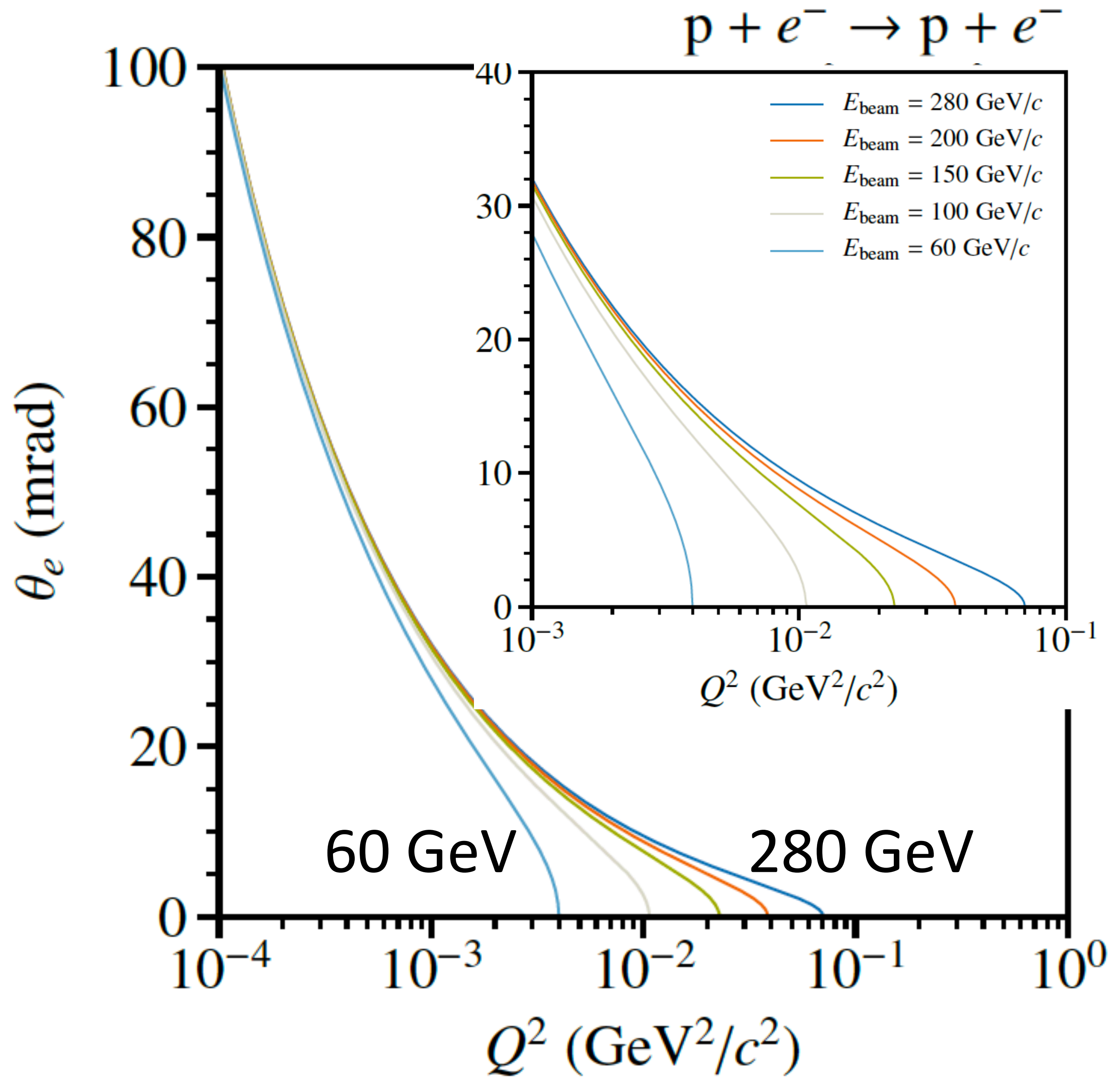


Beam	E_{beam} [GeV]	Q^2_{max} [GeV²]	Relative charge-radius effect on σ(Q²)
π	280	0,268	~54%
K	280	0,15	~30%
K	80	0,021	~5%
K	50	0,009	~2-3%
p	280	0,070	~28%

Apparatus for Meson and Baryon Experimental Research

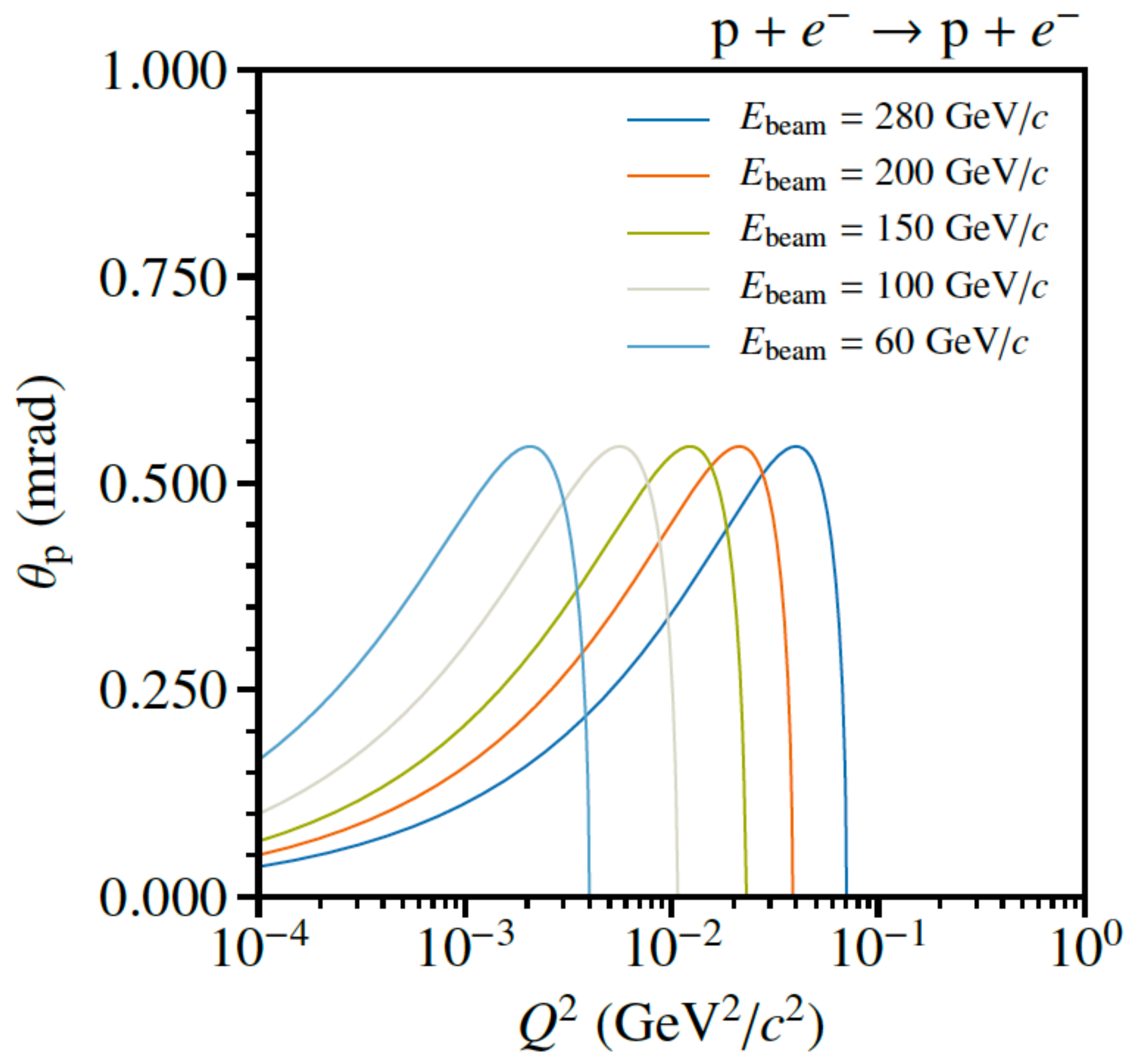
Critical Kinematic Quantities

- largest e^{-}scattering angle θ_e determines Q^2_{min}



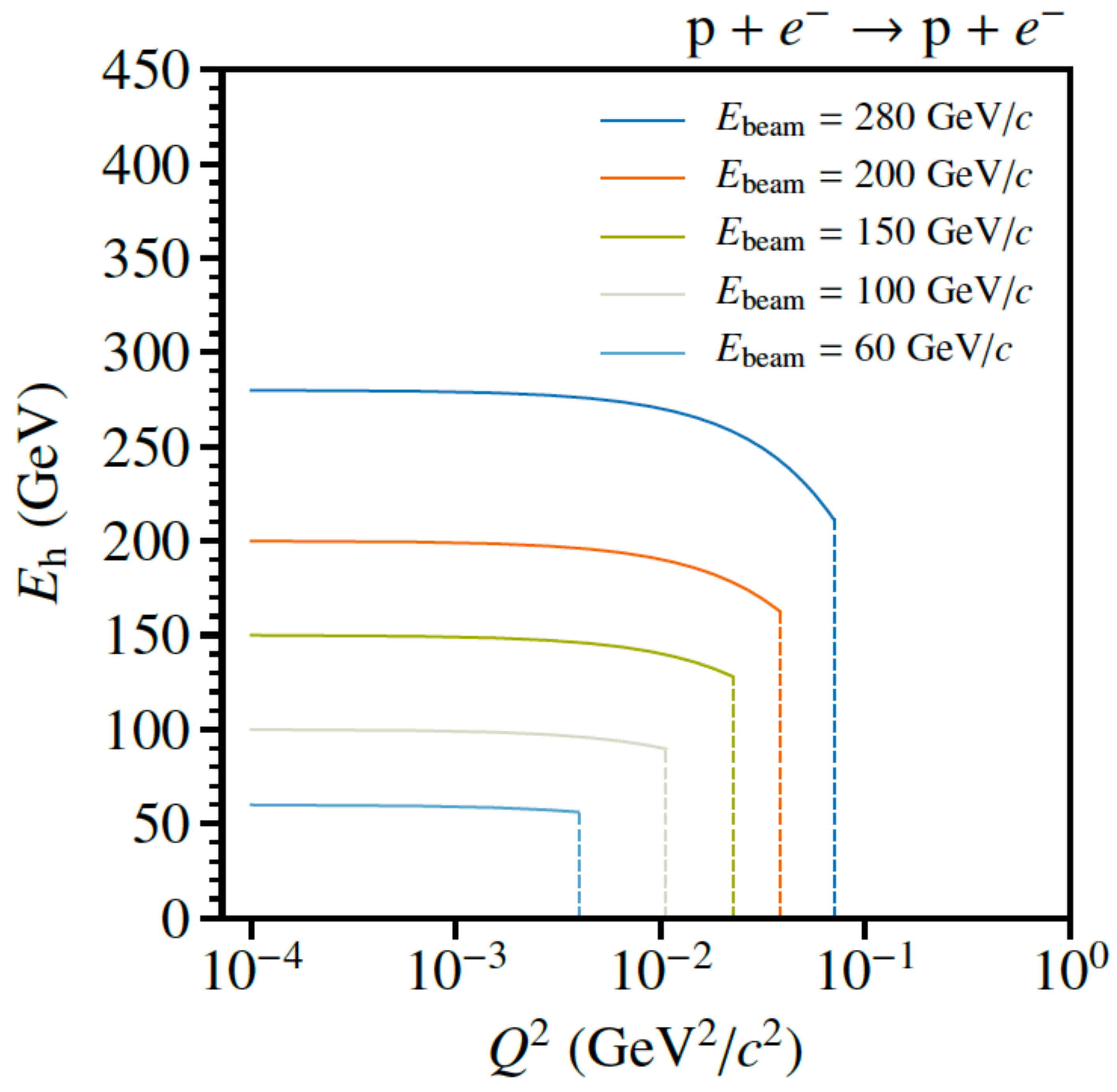
- large acceptance, small multiple scattering

- hadron scattering θ_h angle very small



- high resolution tracking

- scattered hadron used to constrain beam momentum

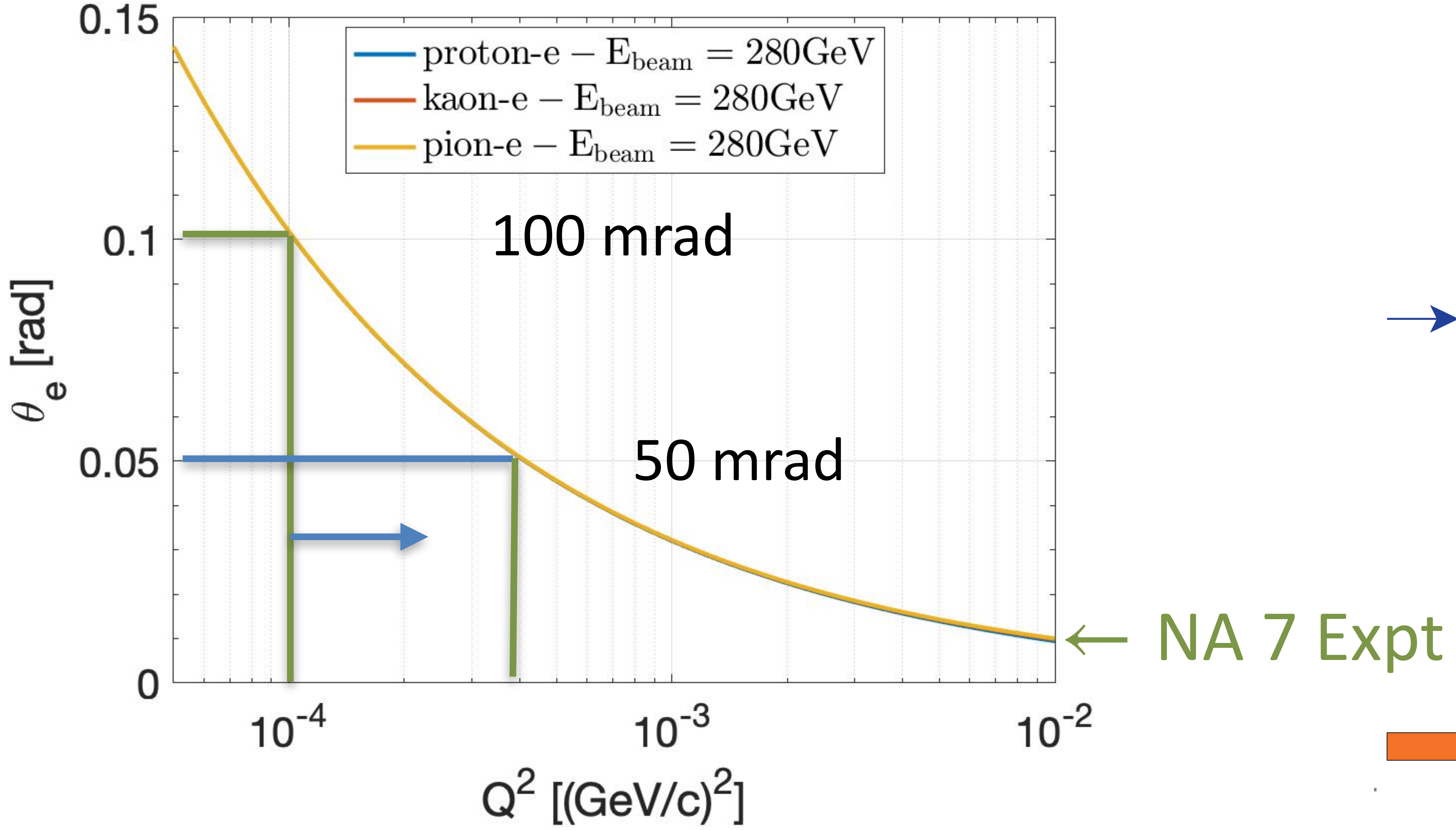


- high momentum resolution $\Delta p/p < 0.3 - 0.4\,\%$

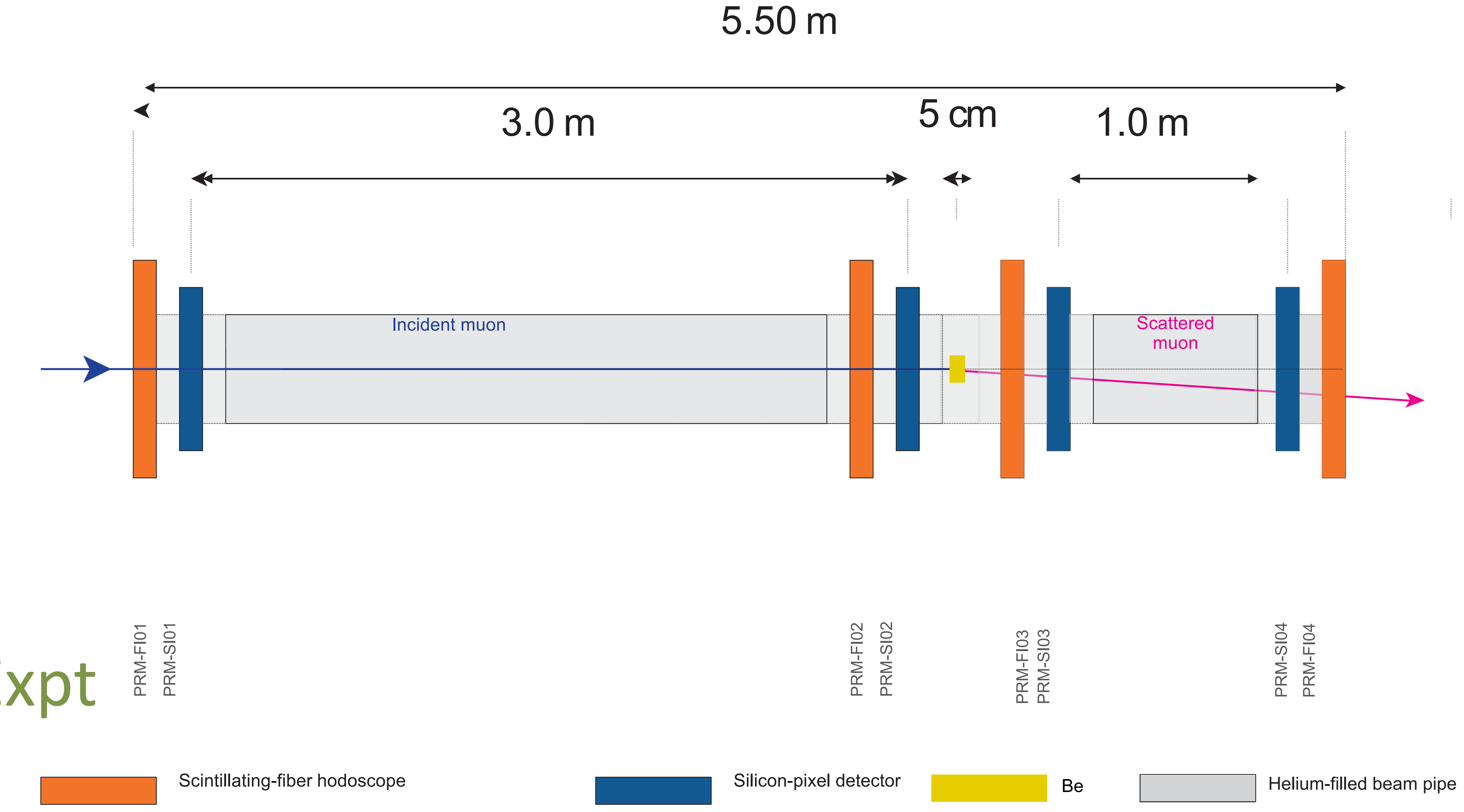
Setup for solid target

- solid target (e.g. 1-25 mm Be) offers large acceptance for outgoing electron
- compress set-up
- Q^2 via three independent measurements - θ_e , θ_p , p'_{hadron}

e^-acceptance effect



little dependence on E_{inc}

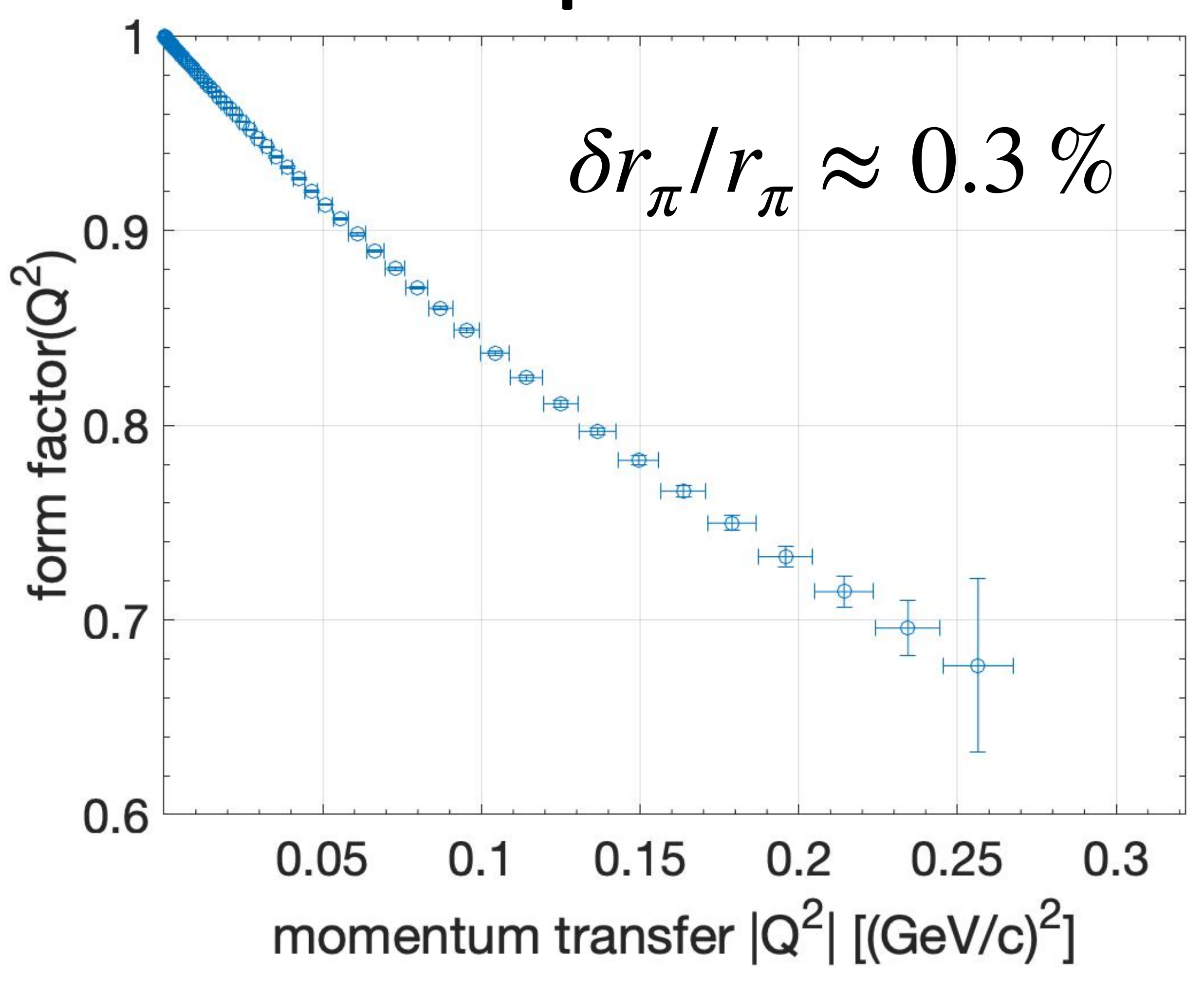


compact set-up

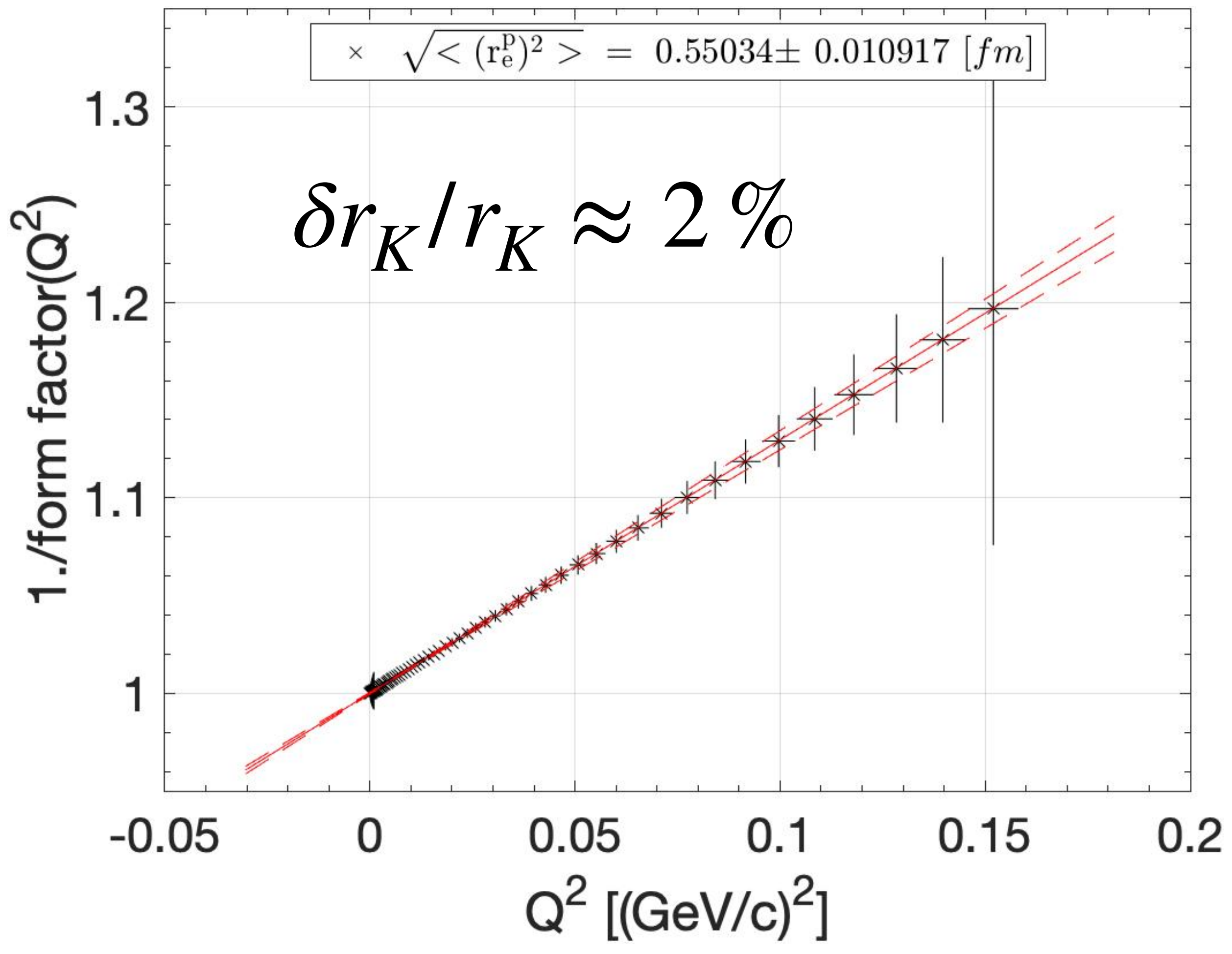
Simulate Results for Kaons and Pions

- Assume 30 days of beam time (100% efficiency) - use pole description for FF

pion



kaon



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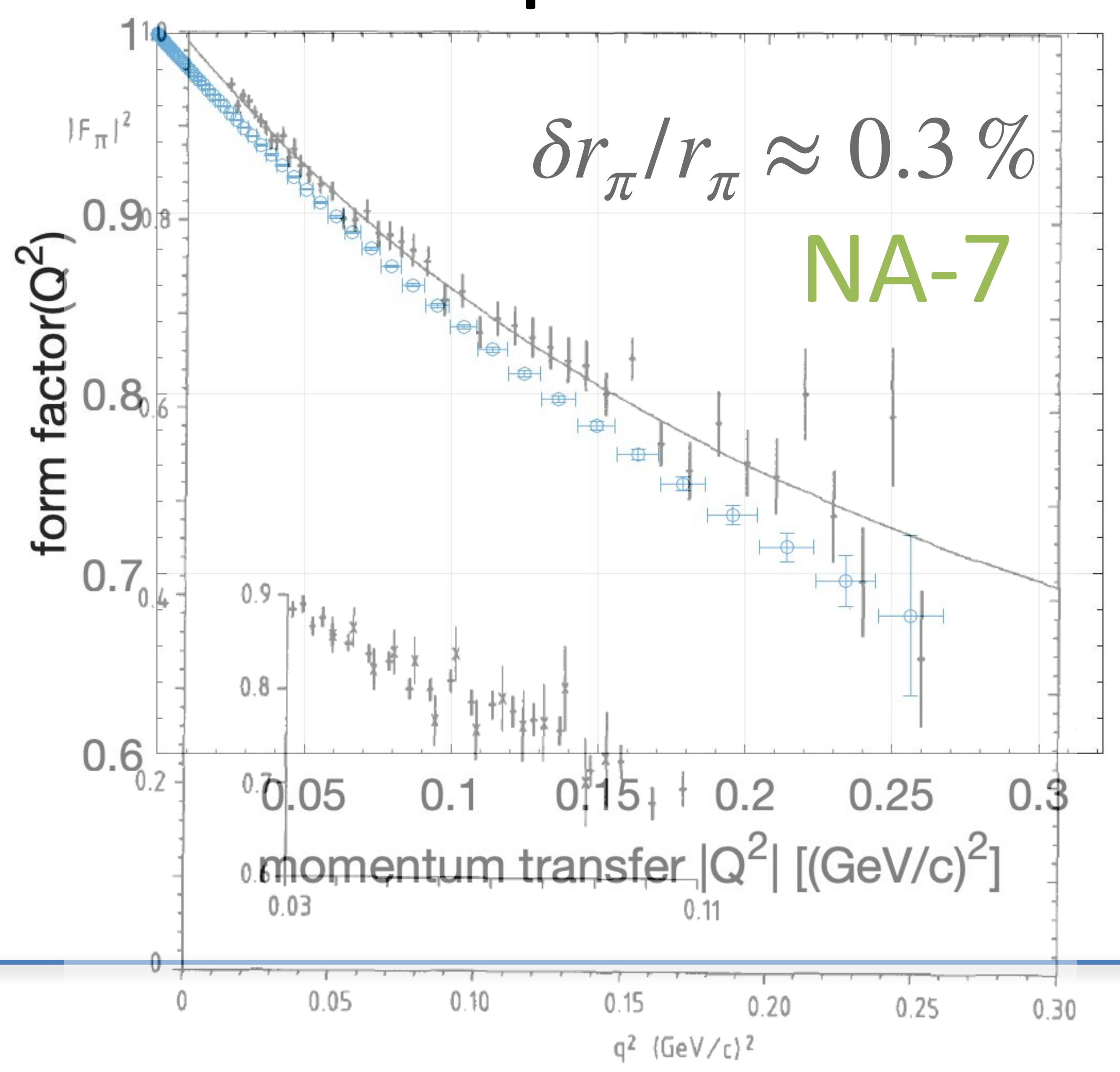
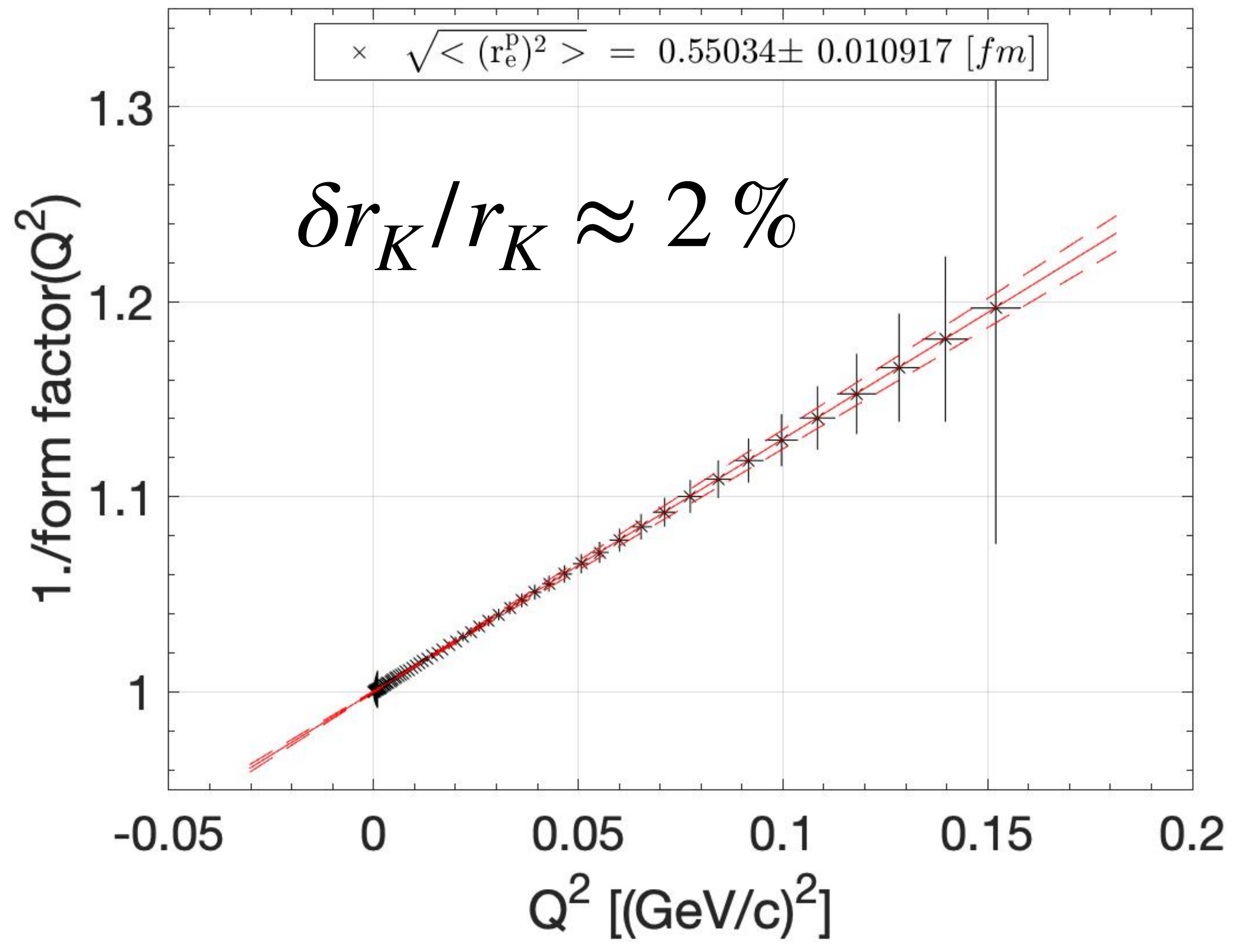
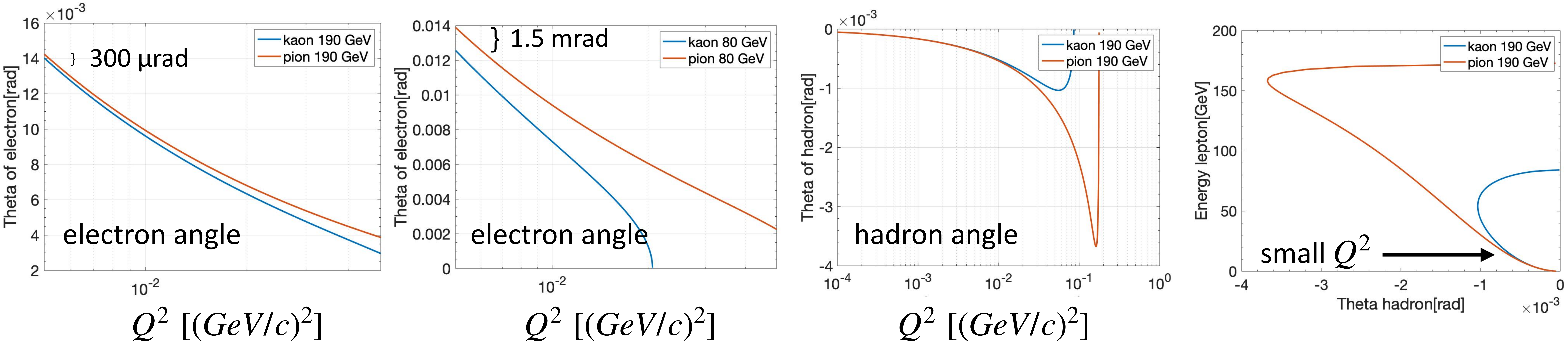


kaon



Separation of Kaon and Pion Induced Reactions

- CEDAR leaves Kaon beam with large pion contamination (about 3%)
- Can we separate kaon and pion induced reactions through kinematics ?
- yes.. but only for $Q^2 > \approx 5 - 10 \cdot 10^{-3}$ (may jeopardize radiative tail detection)

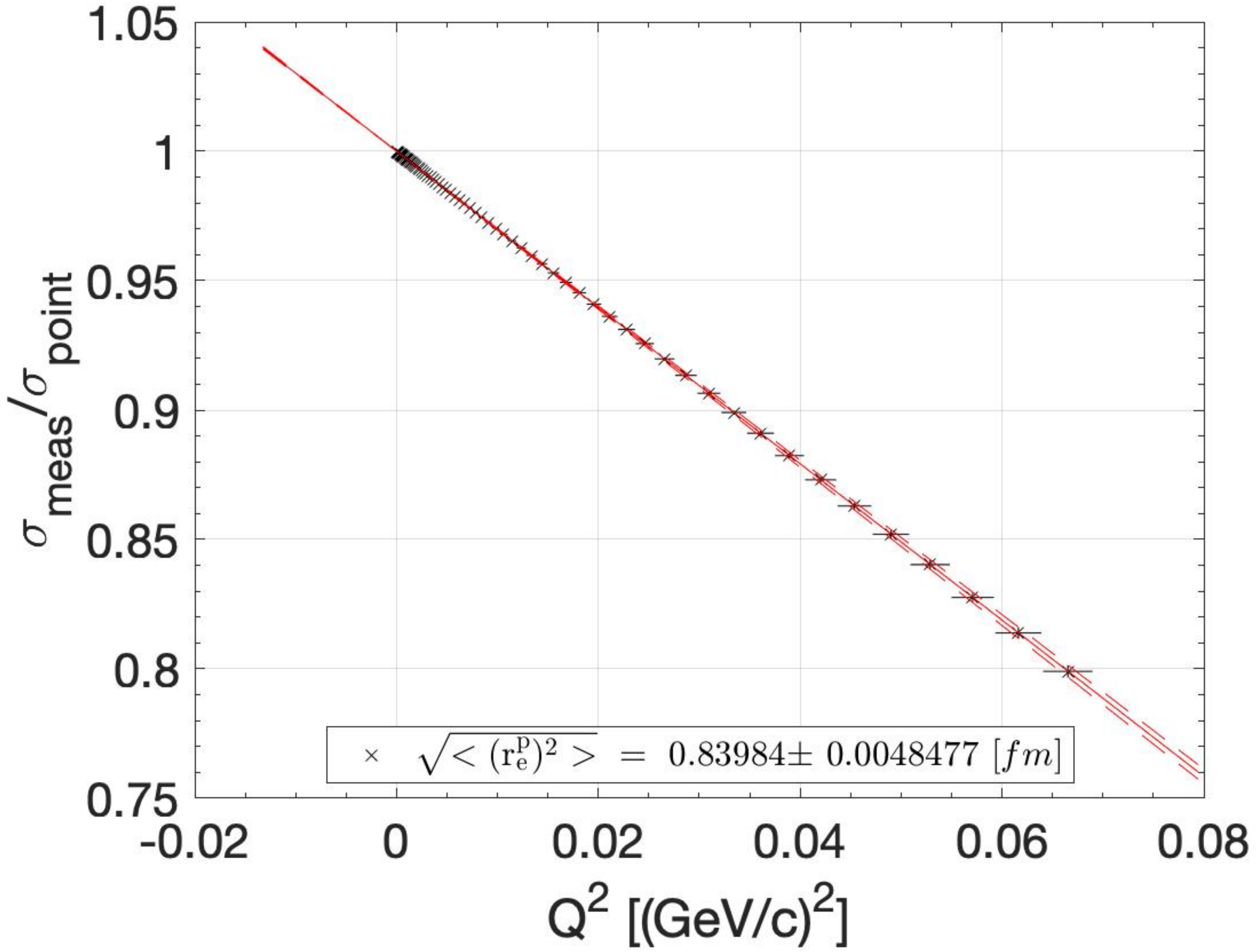


- measure small Q^2 with small beam momenta for kinematic separation

And…The Same With protons

- Two techniques to extract $\sqrt{<(r_e^p)^2>}$:
  - fit for $R_{point} = \sigma(Q^2)_{exp}/\sigma(Q^2)_{point}$
    - small uncertainties (but external input - $G_M(Q^2)$)
    - accuracy limited by resolution δQ^2

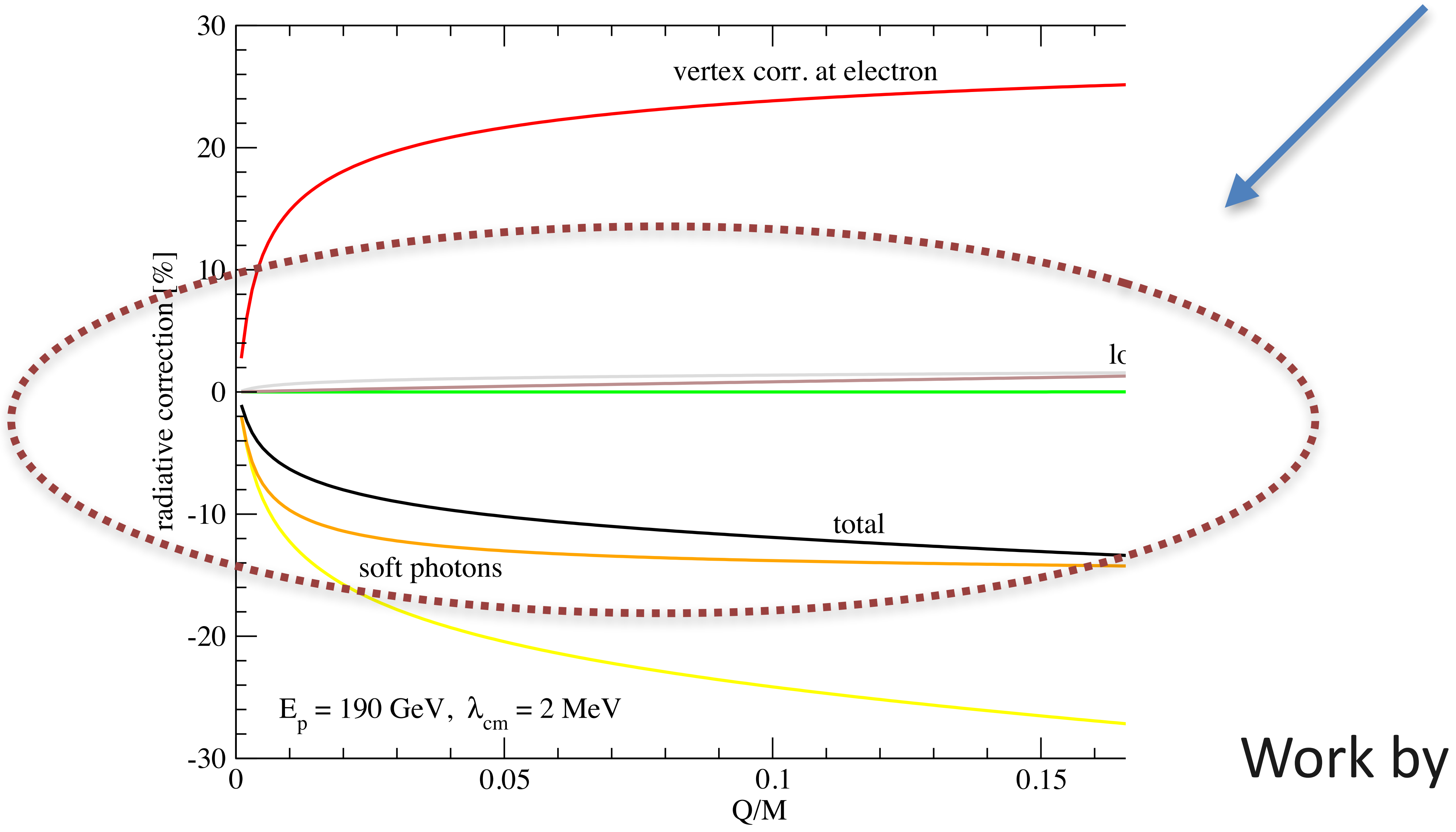
we have to carefully estimate $\delta Q^2(Q^2)$
uncertainties far below 1% seem possible



Radiative Corrections

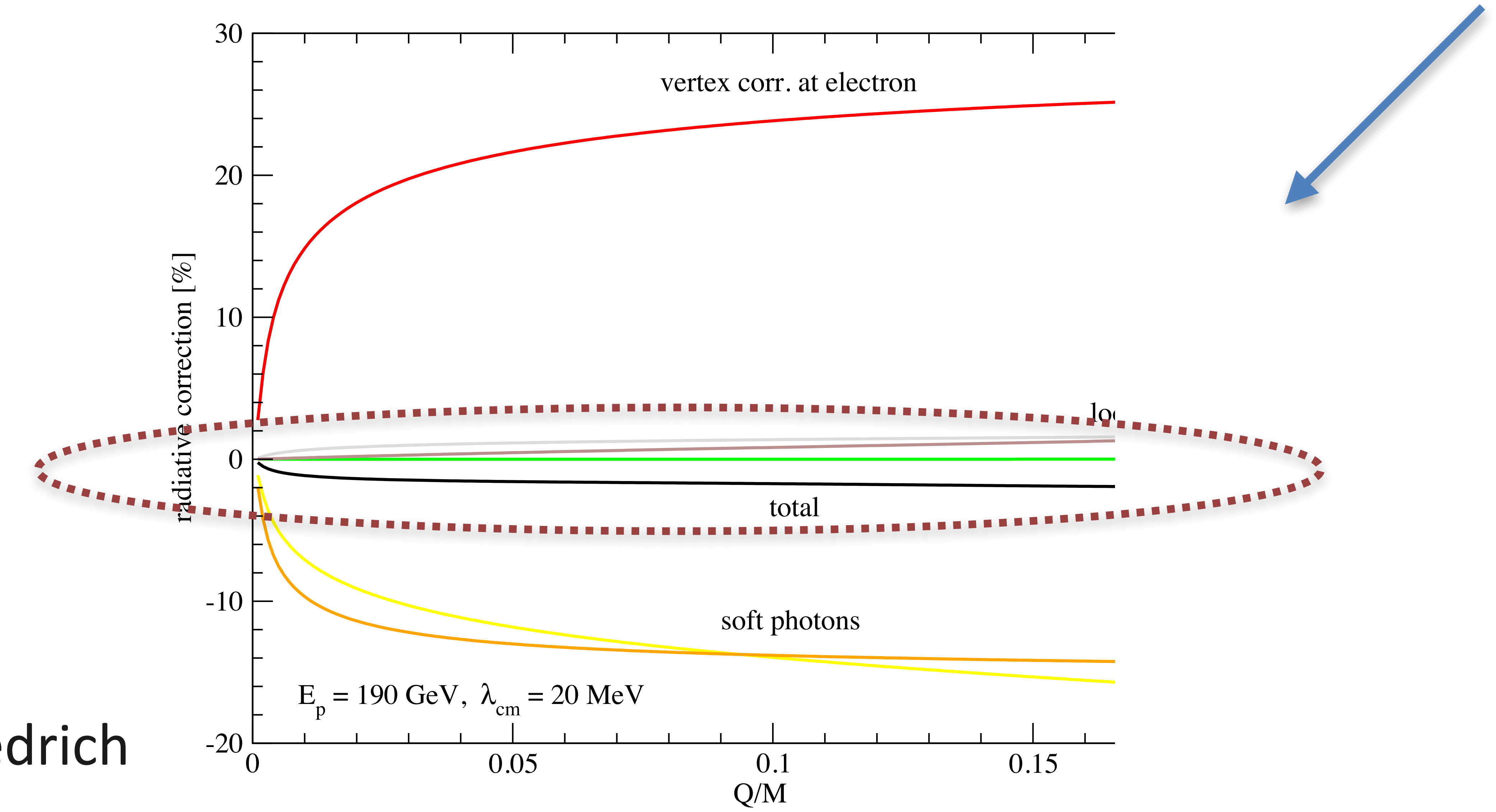
- with 190 GeV protons, we have to consider the case of incoming e^- of 105 MeV beam energy
- Vertex correction and internal Bremsstrahlung enter with opposite sign
- Issue: identification of p-e^- scattering - kinematic correlation of outgoing particles
  - cut in cm on 2% momentum correlation (2 MeV) - cut in cm on 20% momentum correlation (20 MeV)

12%



Work by Kaiser/Friedrich

2%



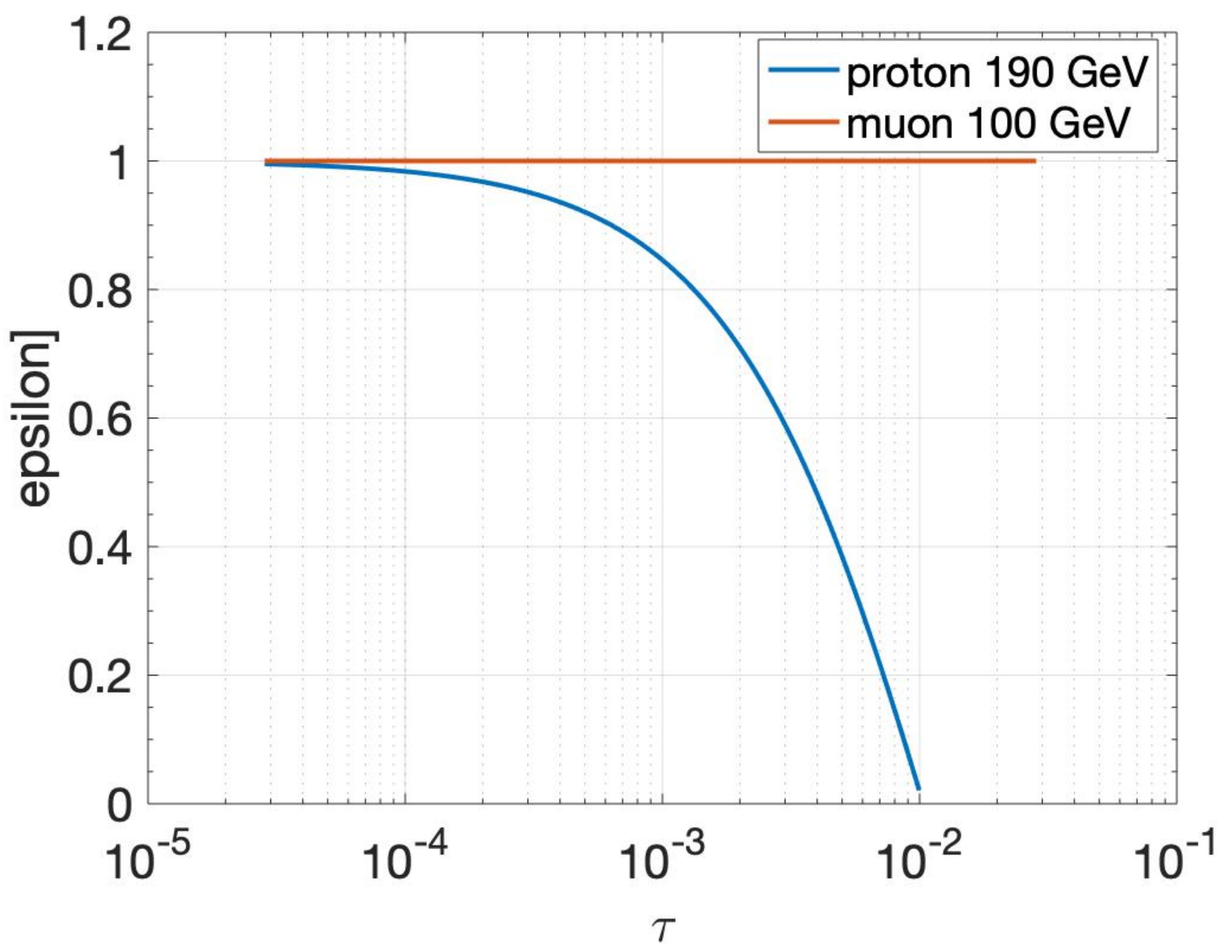
Nucleons in Inverse Kinematics

ϵ : photon polarization
τ : reduced Q^2
R: normalization

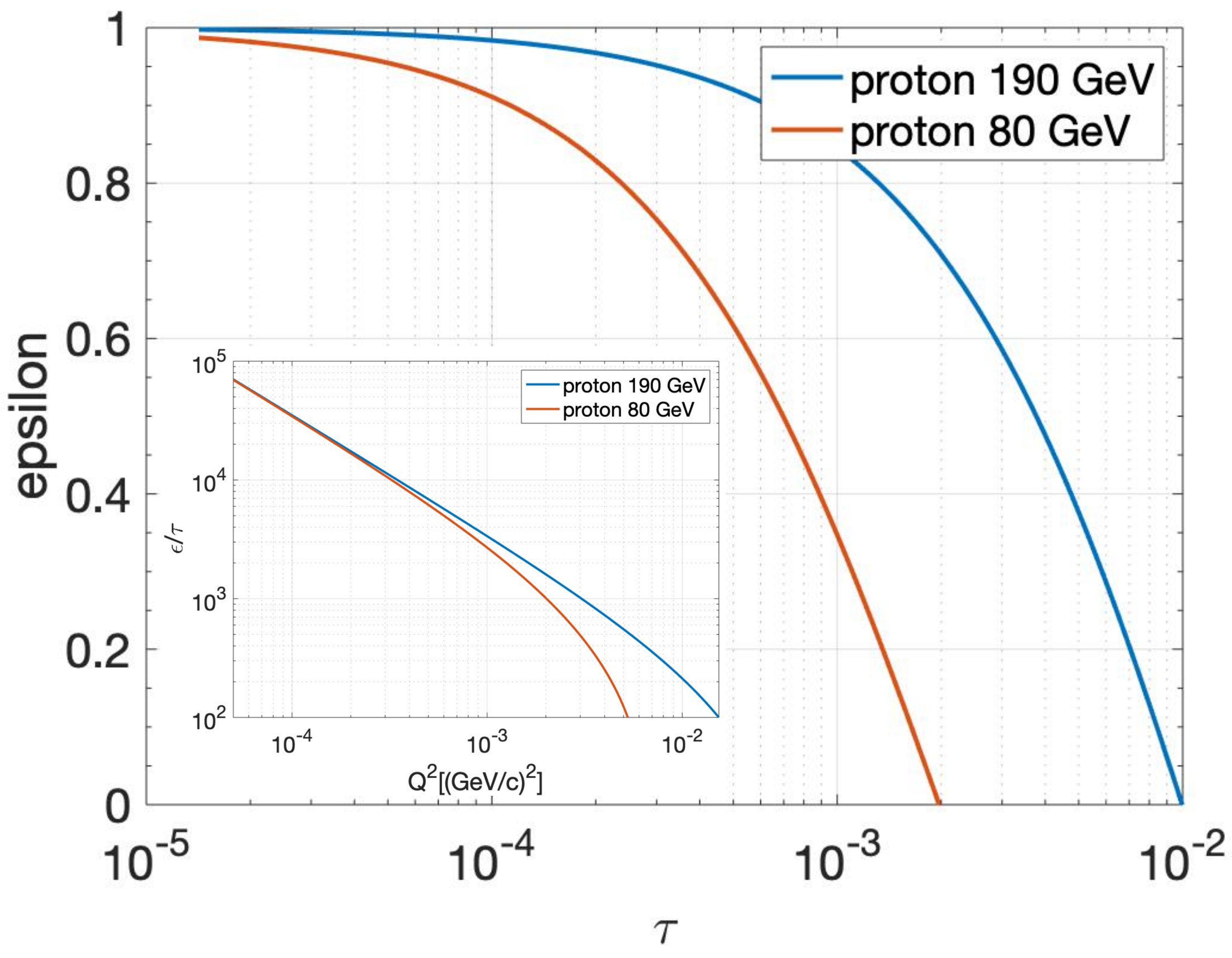
Inverse kinematics allows easy way to access difficult *ep* kinematics

- kinematic variables R, ε, τ $\quad \frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4} R \left(\epsilon \cdot G_E^2 + \tau \cdot G_M^2\right)$
- access Rosenbluth technique through variation of p_{beam}

$$\sigma_R = \left(\frac{d\sigma}{d\Omega}\right)_{exp} / \left(\frac{d\sigma}{d\Omega}\right)_M \frac{\epsilon(1+\tau)}{\tau} = \frac{\epsilon}{\tau} G_E^2 + G_M^2$$



high energy muon scattering: little sensitivity to $G_M^2(Q^2)$



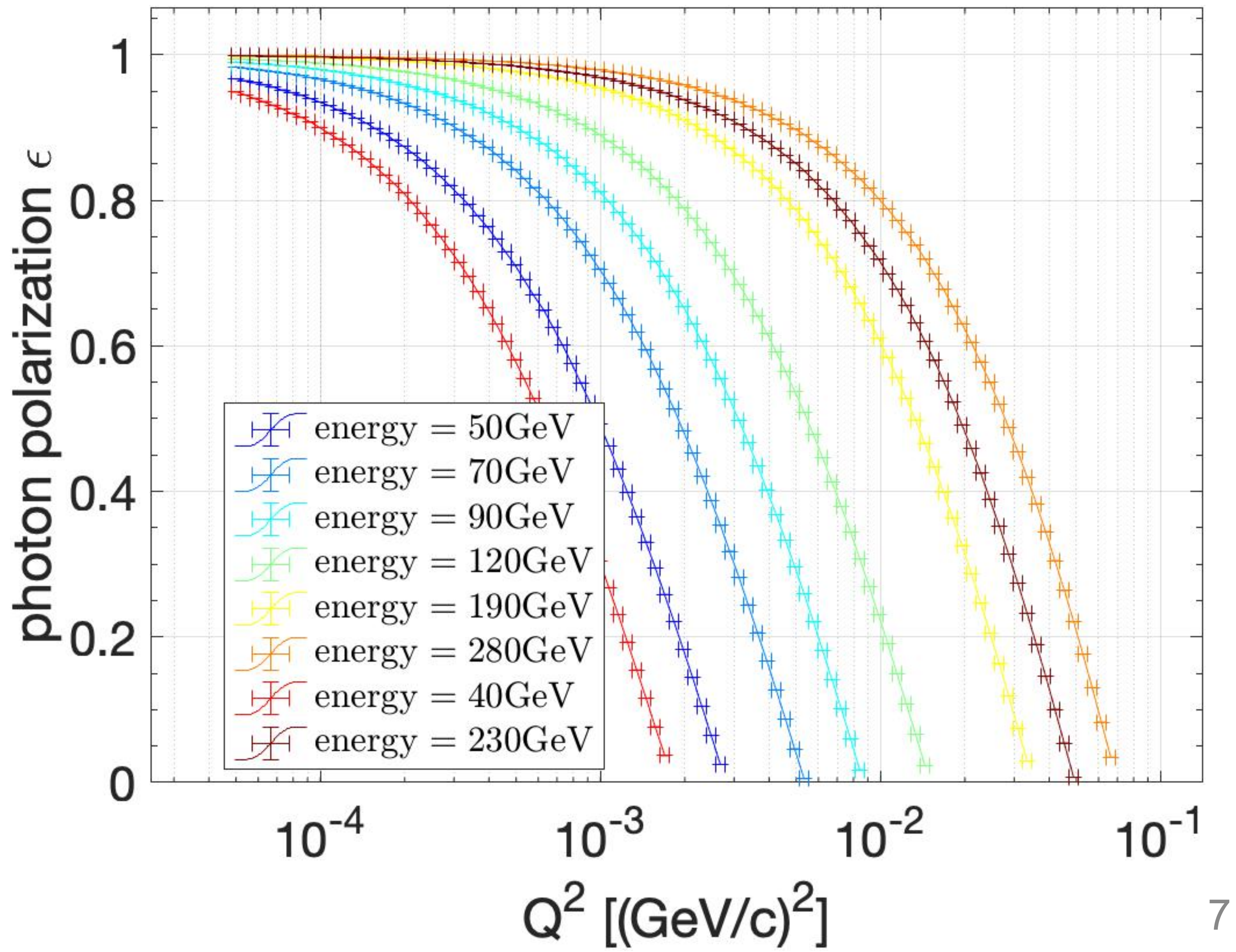
use different nucleon beam momenta to access $G_M^2(Q^2)$

$G_M^p(Q^2)$

- Rosenbluth separation allows for extract $G_M^p(Q^2)$ at low Q^2 !
- presently - knowledge data only for $Q^2 > 0.02(GeV/c)^2$ (Mainz data)
- Inverse kinematics could add information for $0.004 > Q^2 > 0.04(GeV/c)^2$
- first measurement in this kinematic range for this quantity !
- equivalent incoming electron energies: 30-105 MeV

Rosenbluth separation requires variation of ϵ/τ

Requires many beam energies[1]



[1]target thickness $\Delta x(E_{beam})$

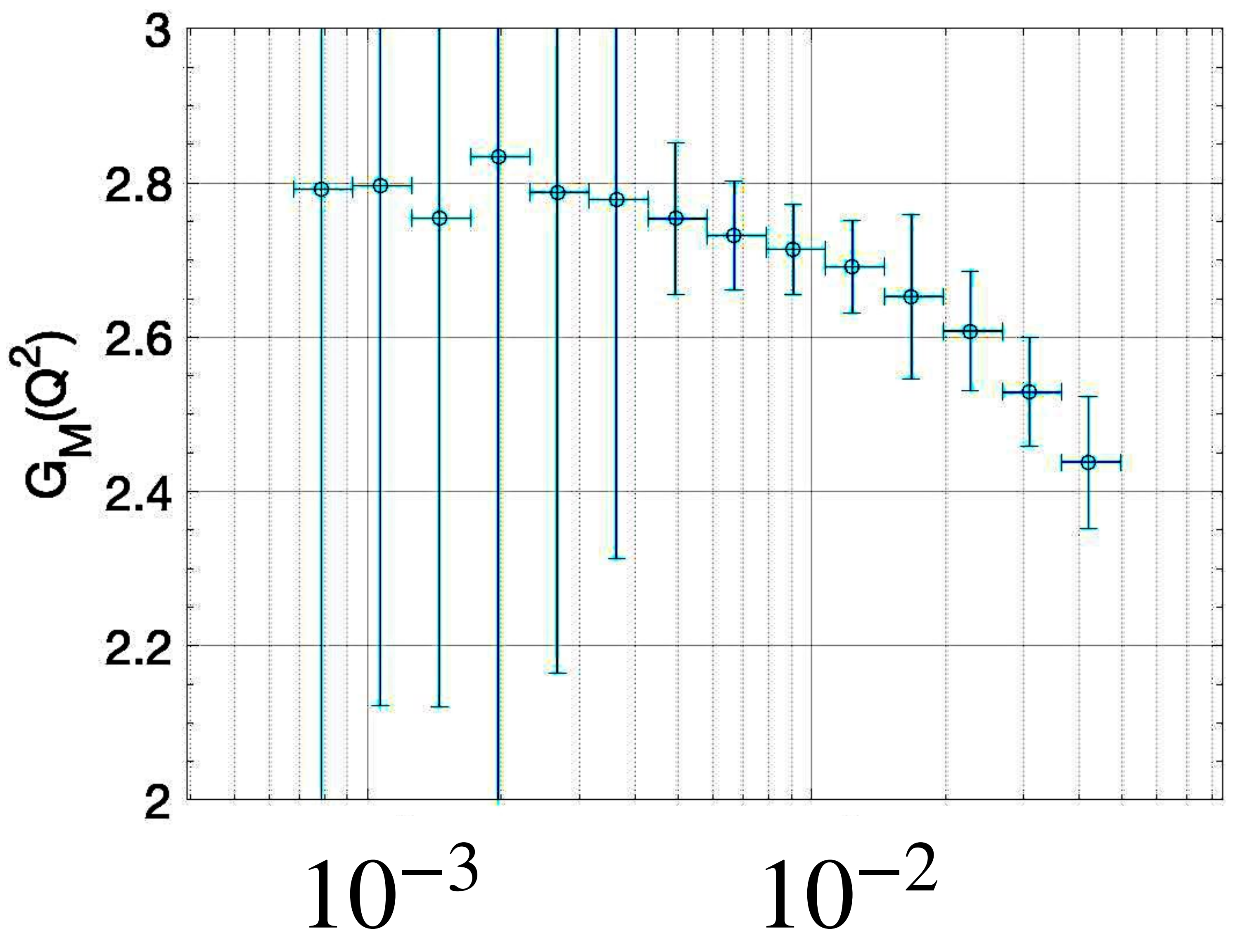
Extraction of $G_M^p(Q^2)$ and $G_E^p(Q^2)$

use 10 different settings (energy/target thickness) - assume 130 days of beam time (100% efficiency)

perform Rosenbluth separation and fit σ_R versus ϵ

$$\sigma_R = \left(\frac{d\sigma}{d\Omega}\right)_{exp} / \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}}$$

- error bars depend on fitting method (very preliminary)



$G_M^p(Q^2)$

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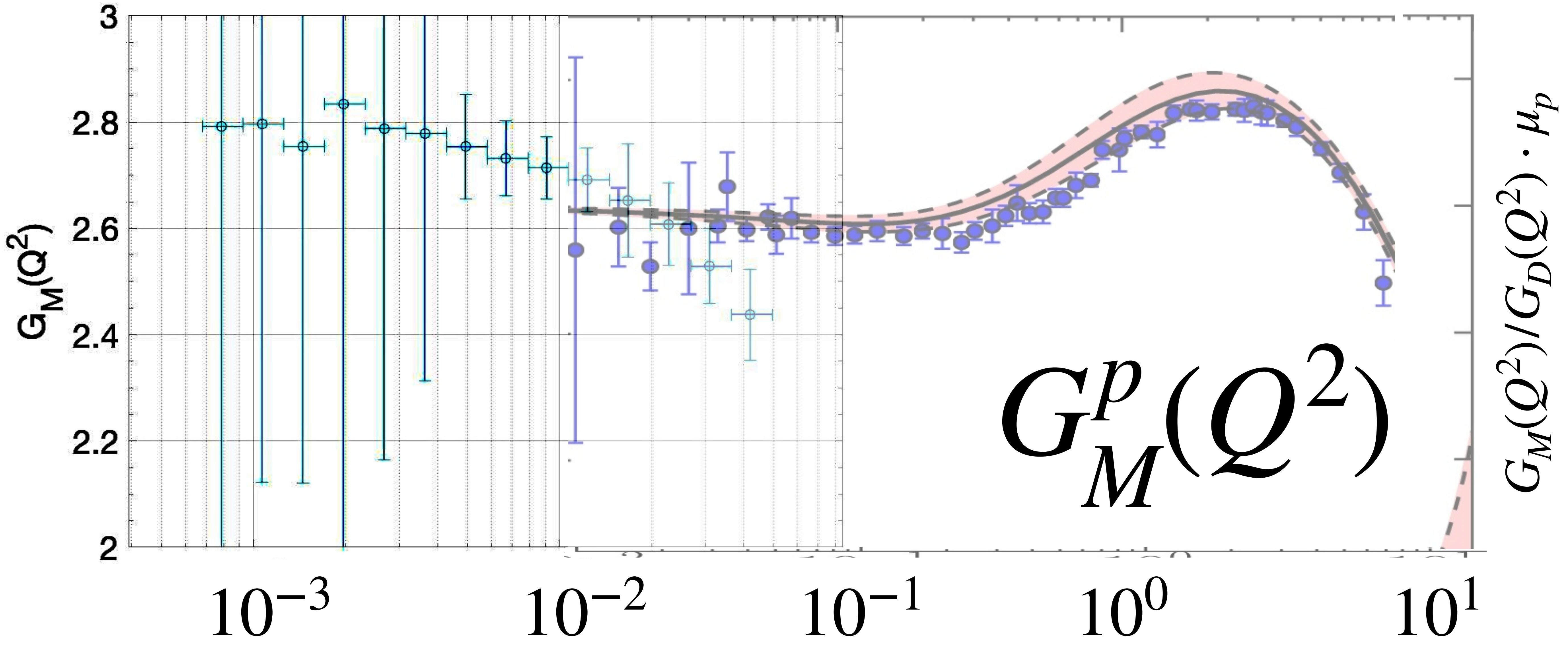

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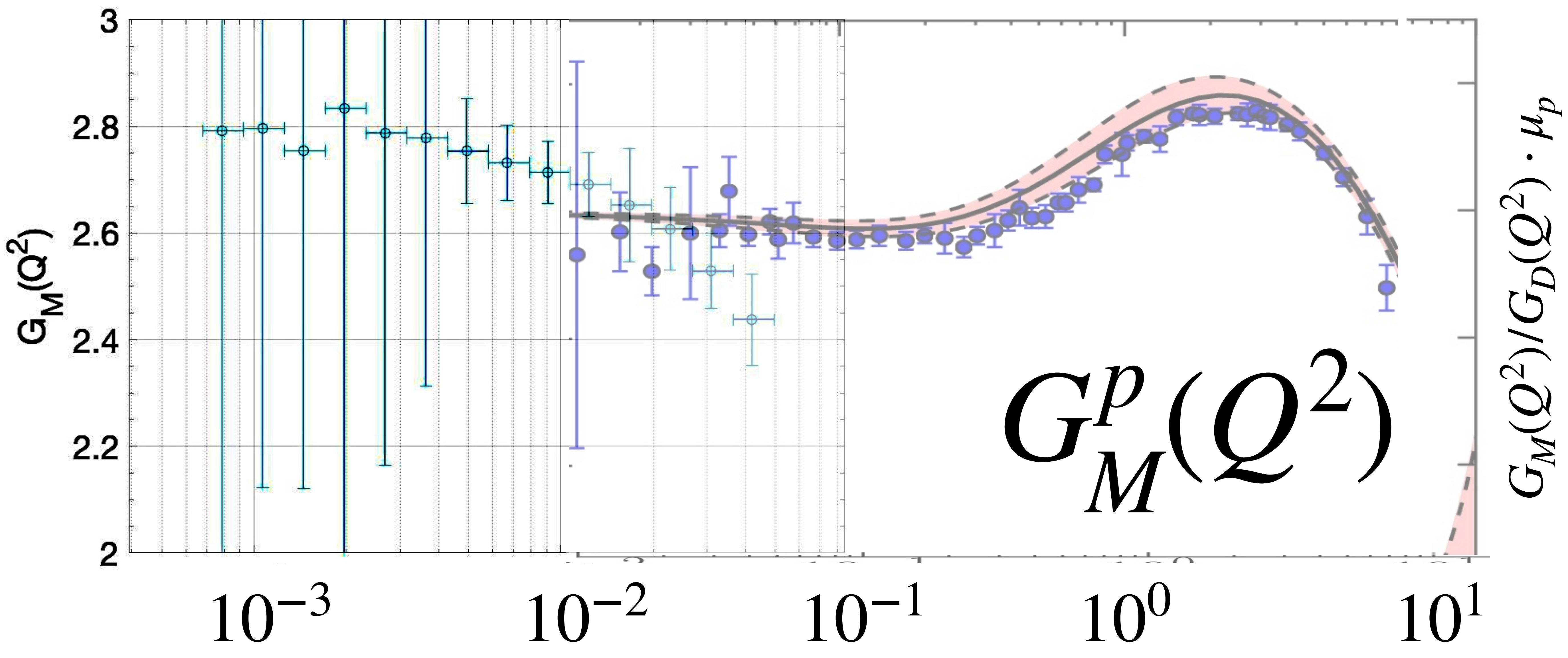
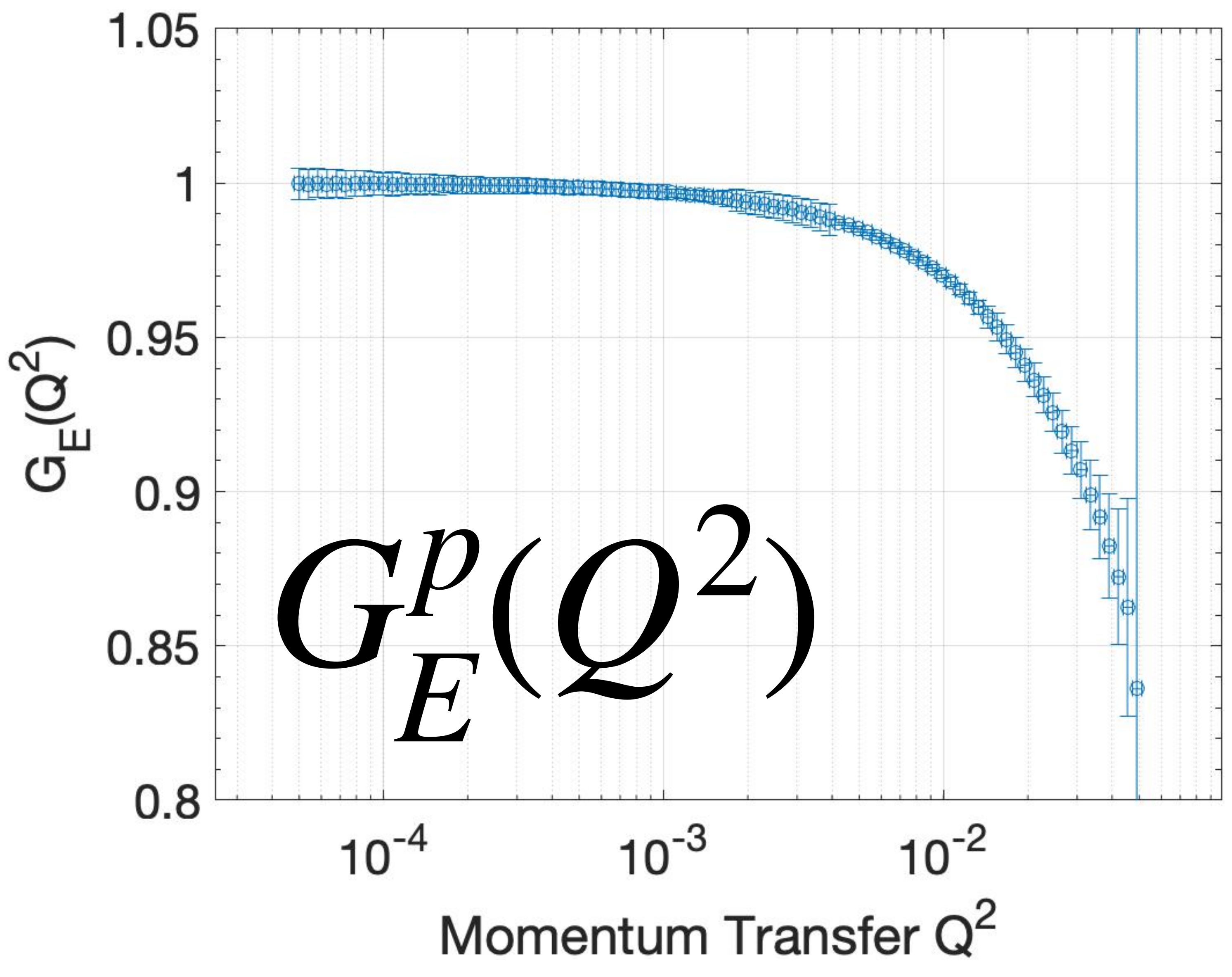
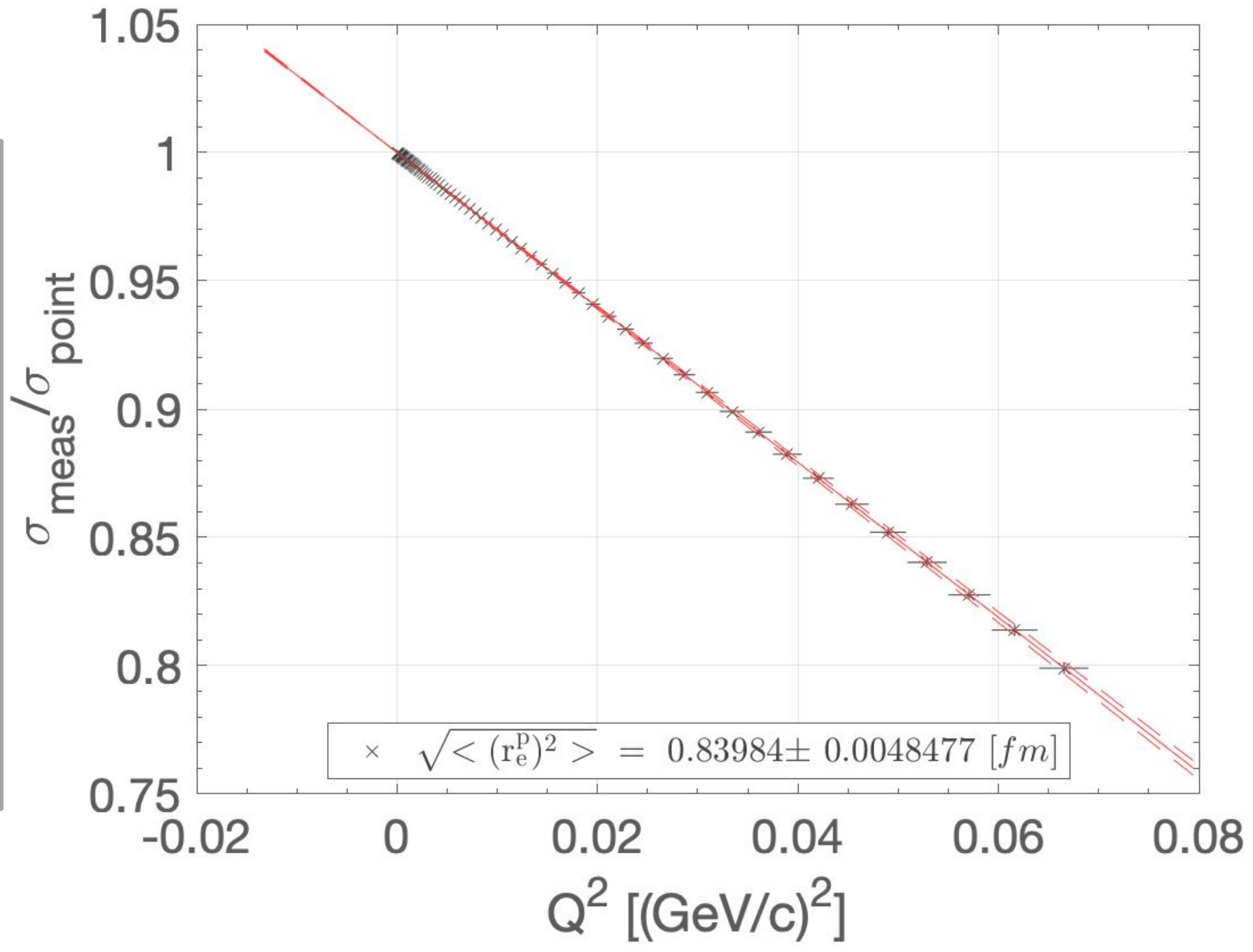




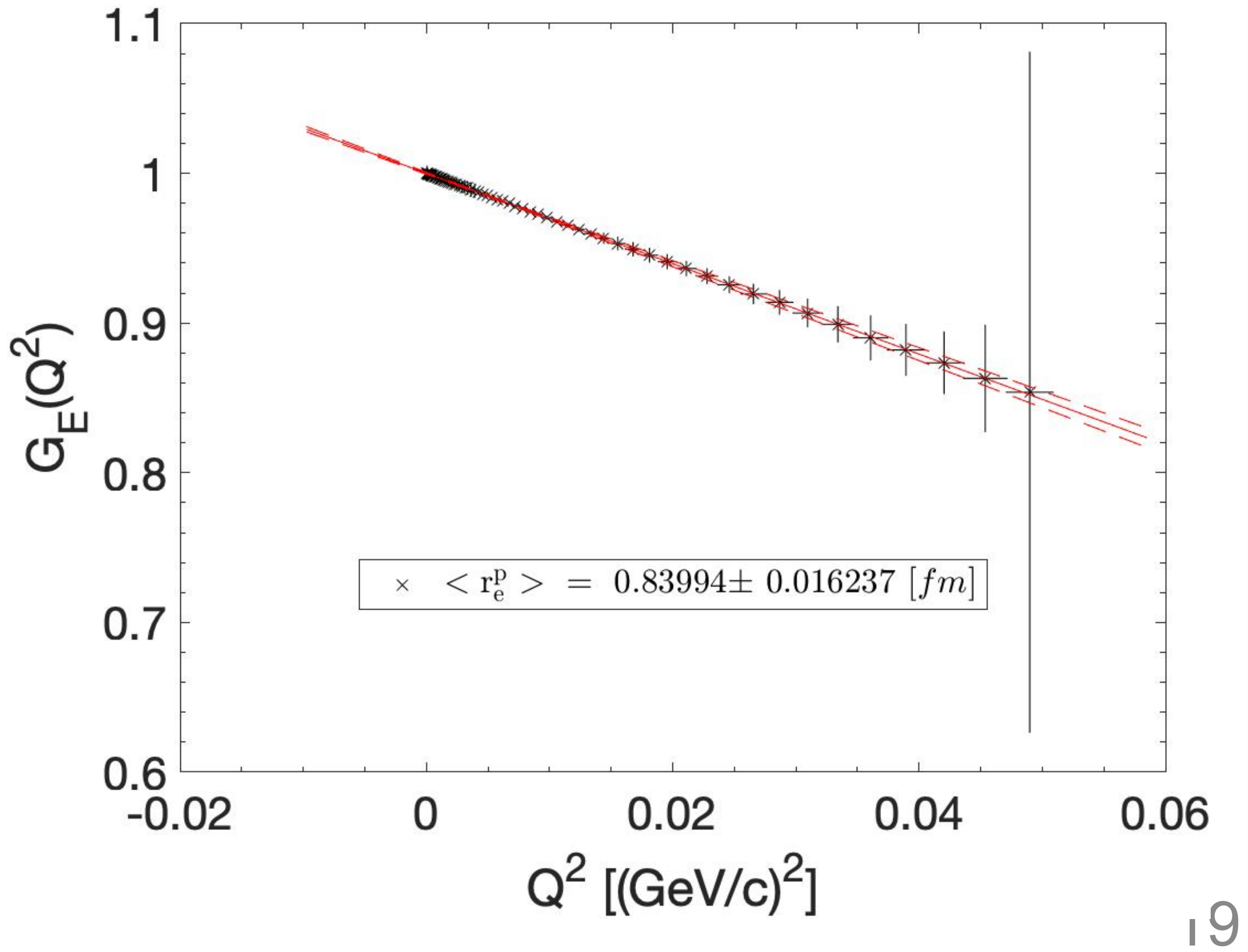
Extract Radius via $G_E(Q^2)$

- Two techniques to extract $\sqrt{<(r_e^p)^2>}$:
  - fit for $R_{point} = \sigma(Q^2)_{exp}/\sigma(Q^2)_{point}$
    - small uncertainties (but external input - $G_M(Q^2)$)
    - accuracy limited by resolution δQ^2
  - fit $G_E(Q^2)$
    - accuracy limited by number of settings (range in ϵ) for Rosenbluth fits and correlation with simultaneous extraction of $G_M(Q^2)$

we have to carefully estimate $\delta Q^2(Q^2)$ uncertainties far below 1% seem possible



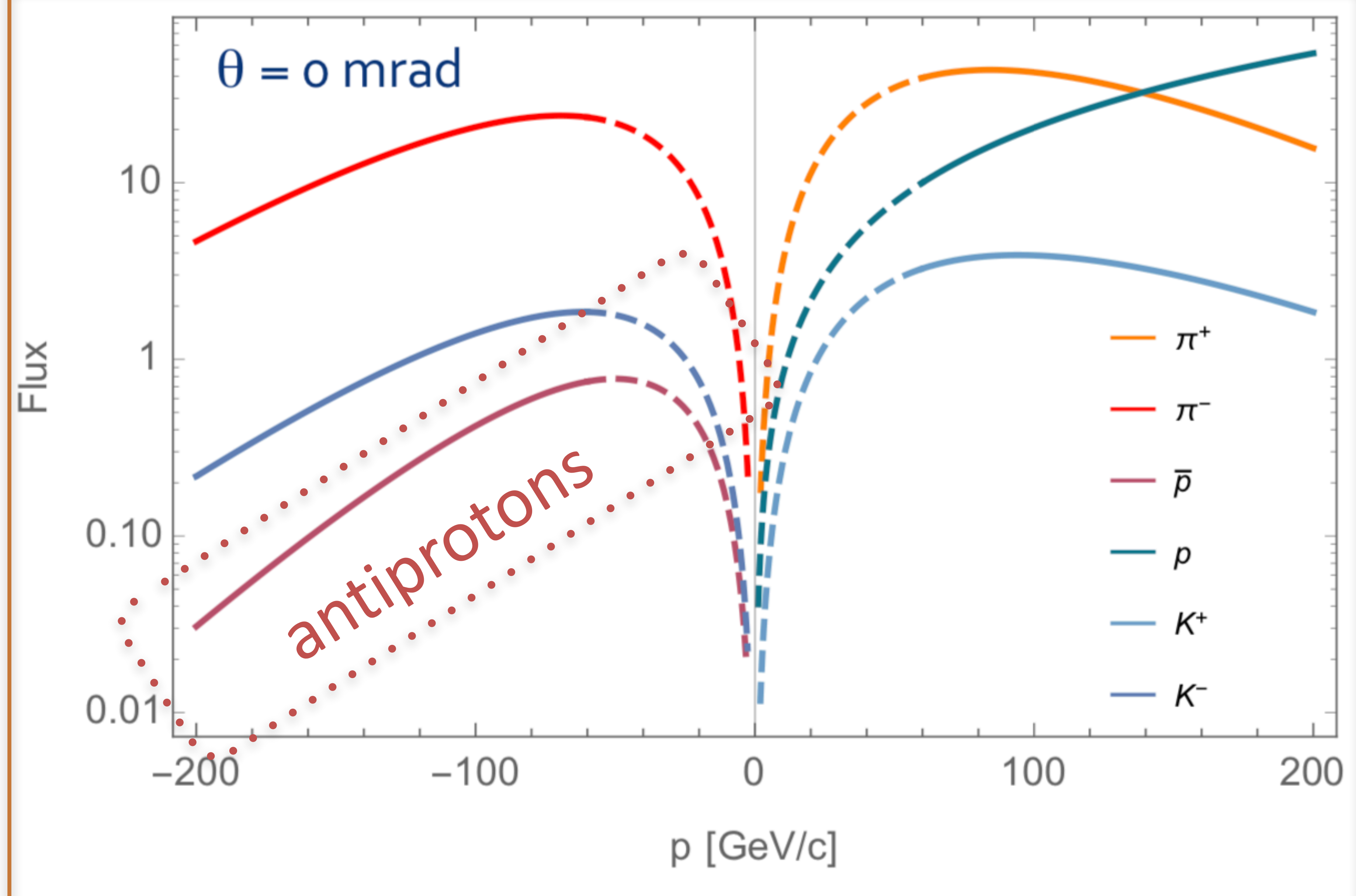
resolution limited owing to limited range in Q^2
varying ϵ leads to smaller Q^2_{max}



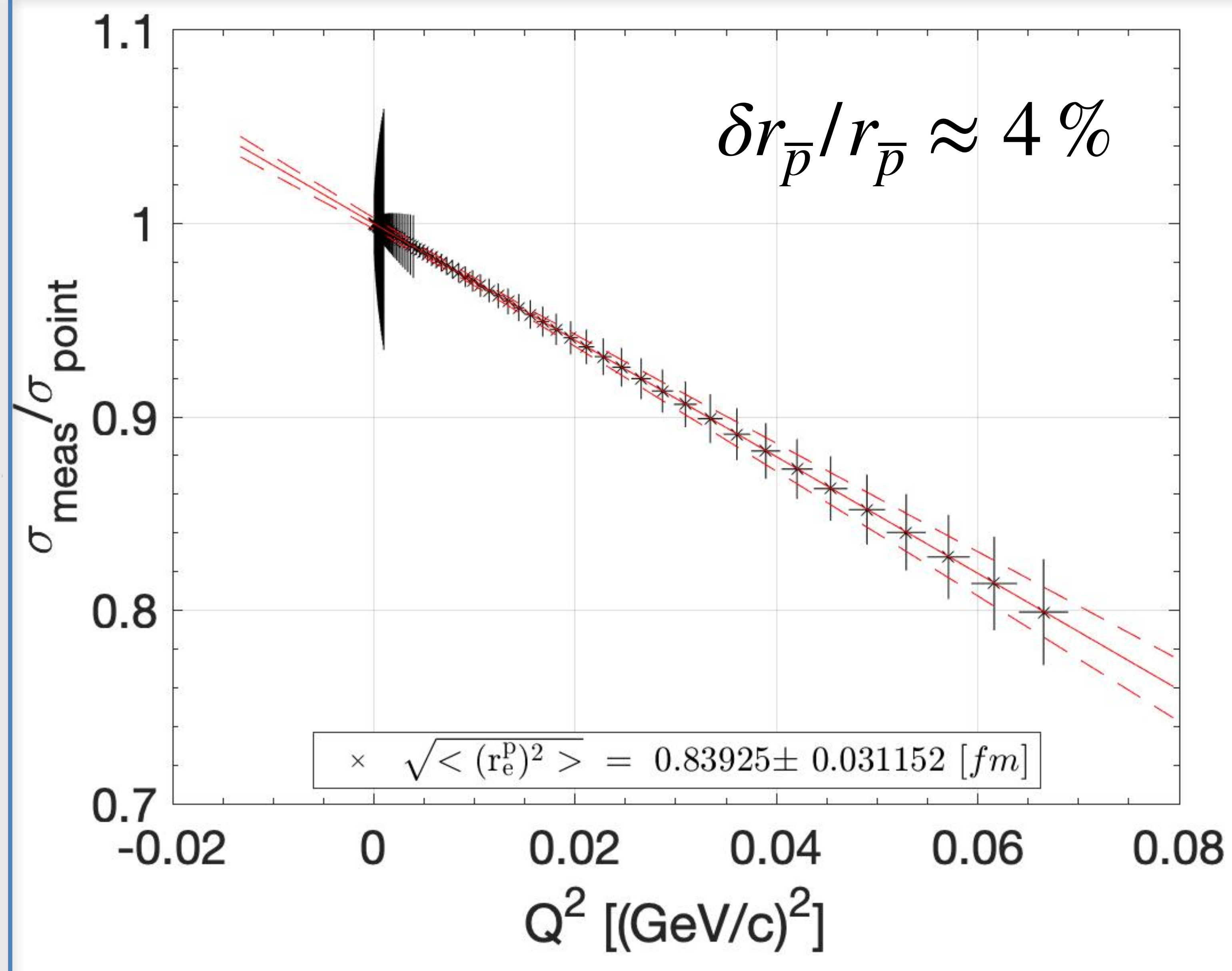
Charge Radius of Antiprotons

A. use data taking mode with pions - assume 30 days (no variation of E_{in}^{beam})

B. use energy dependent fraction of $\overline{p}$ in pion beam



M2 beam @ CERN SPS



parasitic running with pion

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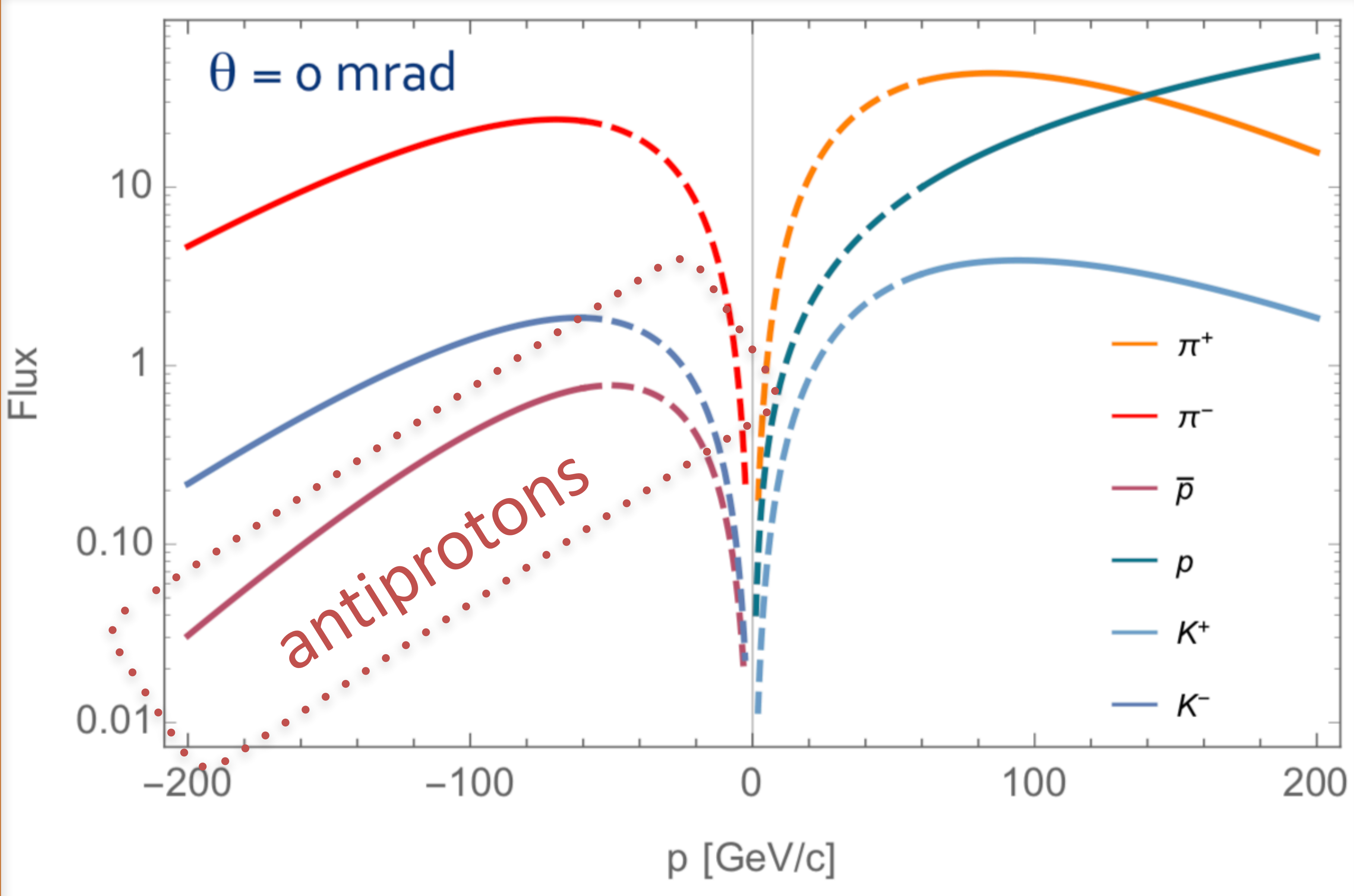

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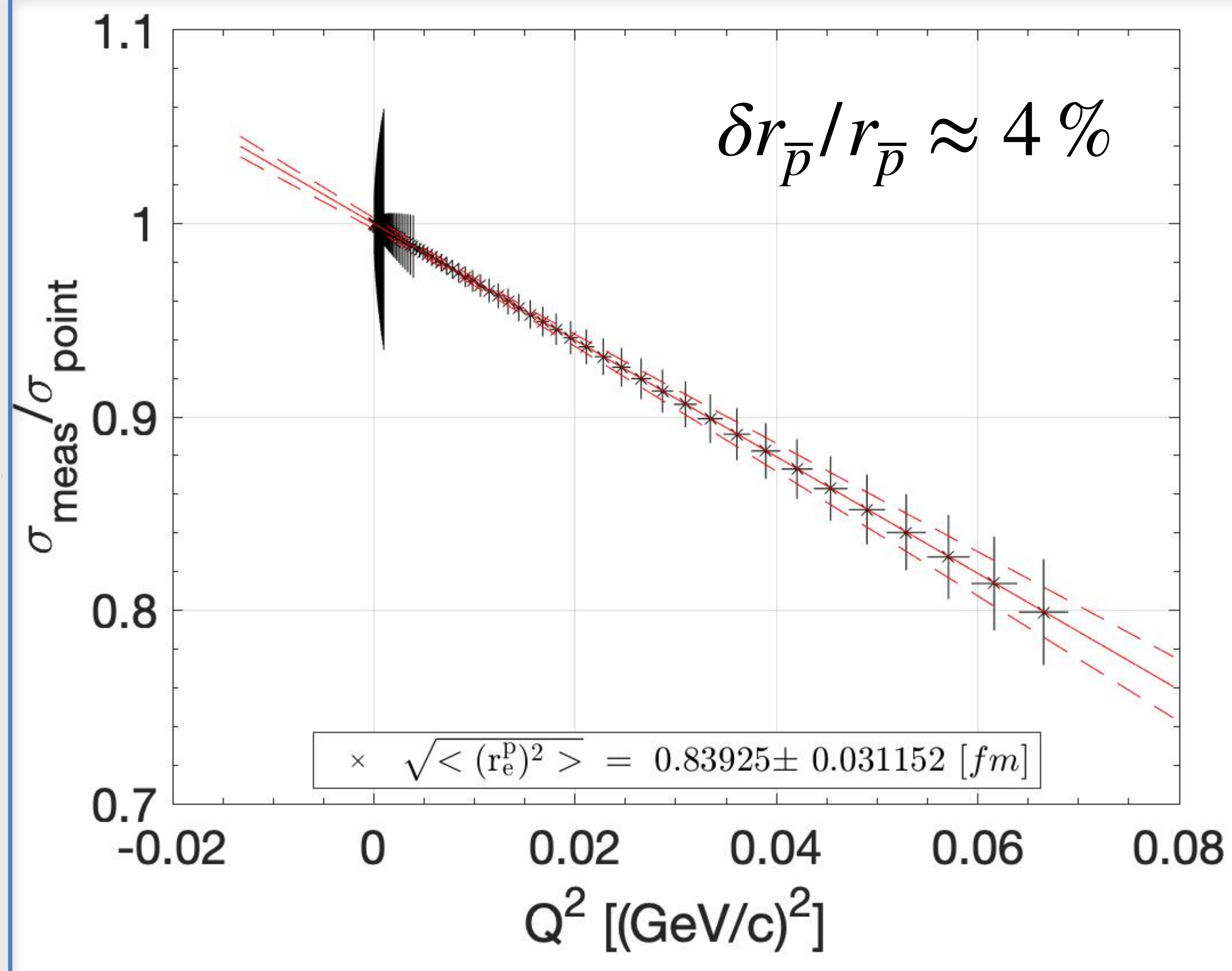

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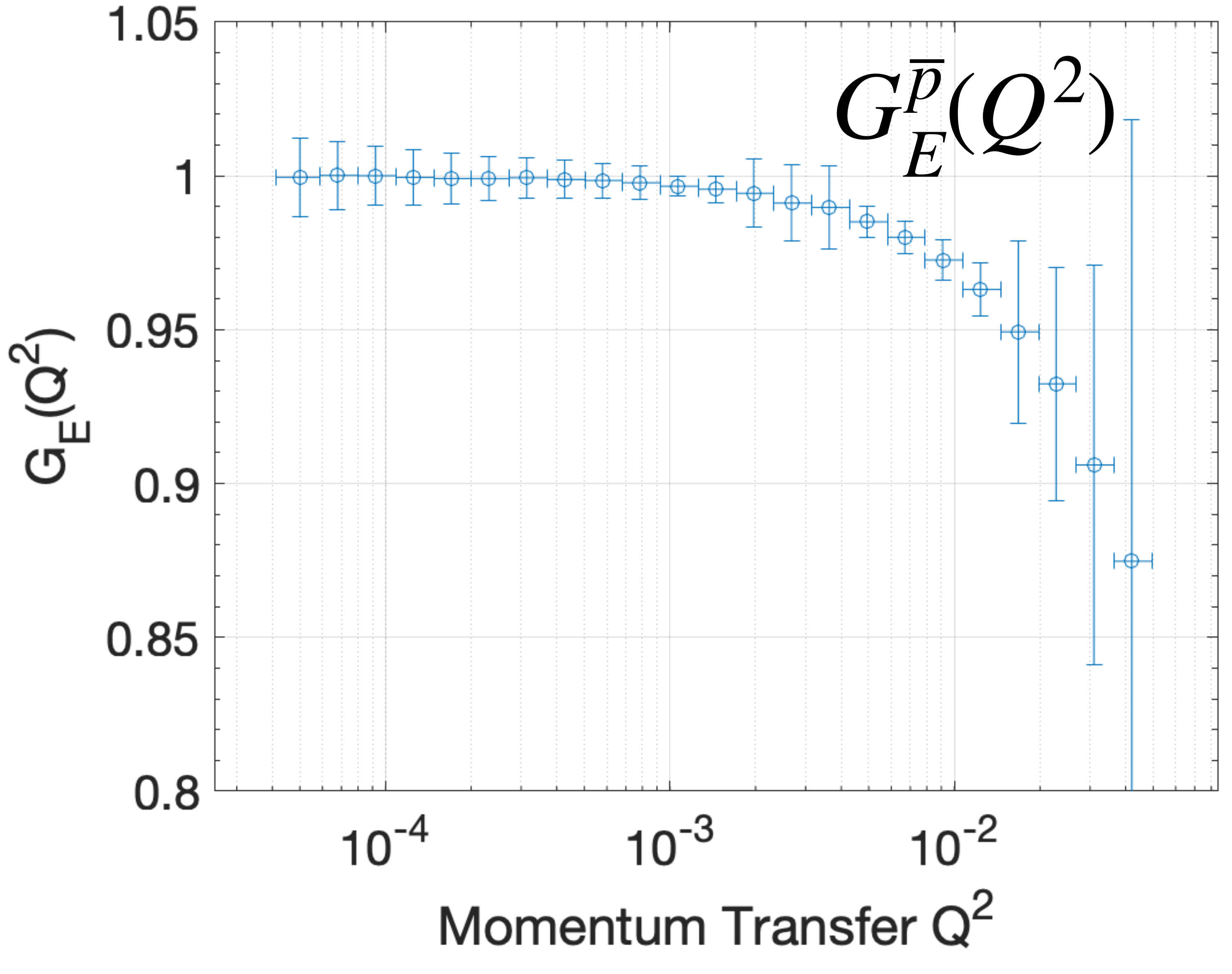


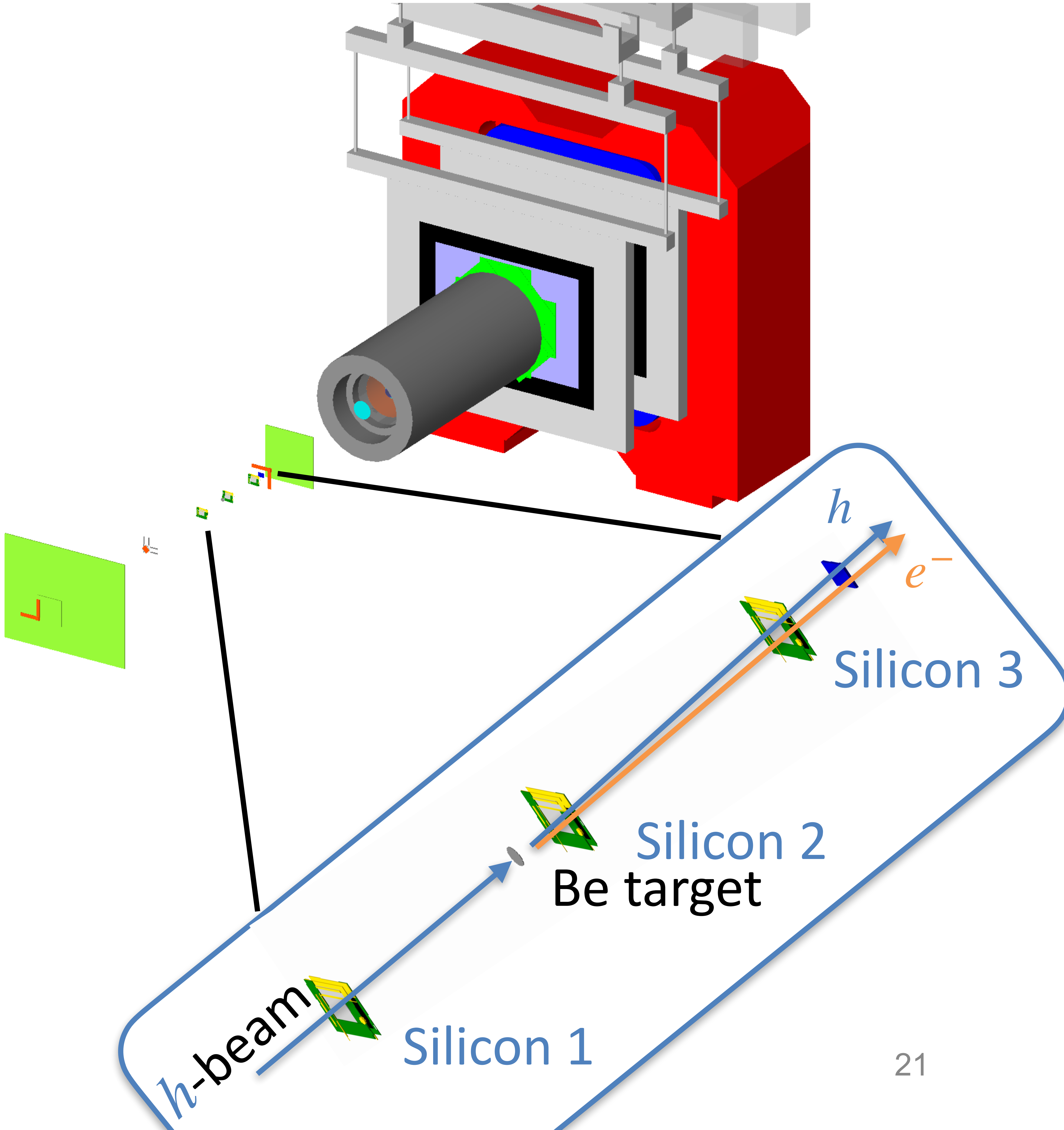
dedicated running like protons

Parasitic Run 2024

First (incomplete) simulation by Ch. Dreisbach

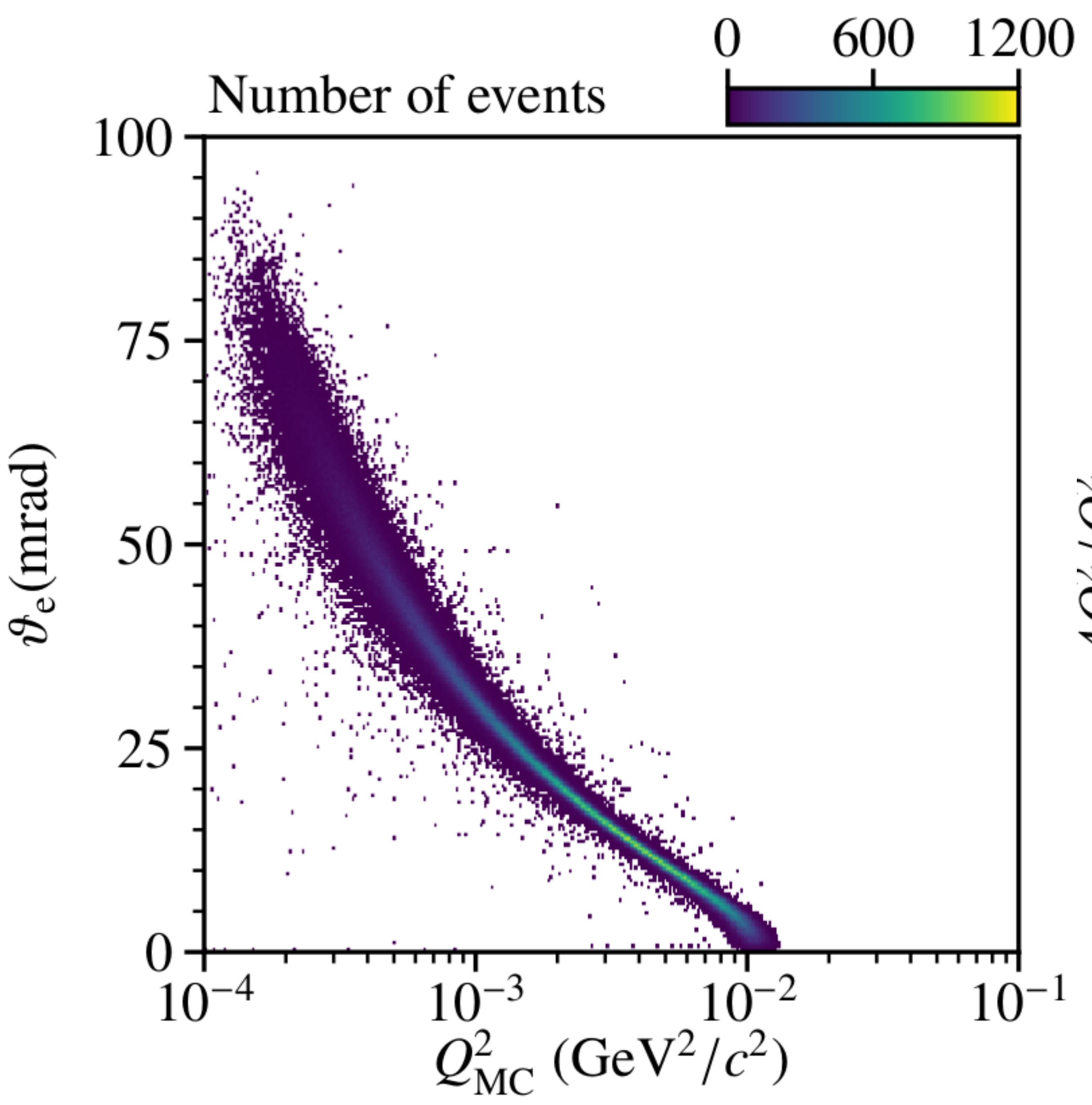
- dynamic range defined by distance Si(2)-Si(3)
- target area upstream of H_2/D_2 target
- trigger
  - use thin trigger scintillator downstream of SI(3)
  - trigger on all incoming kaons/antiprotons (CEDAR)

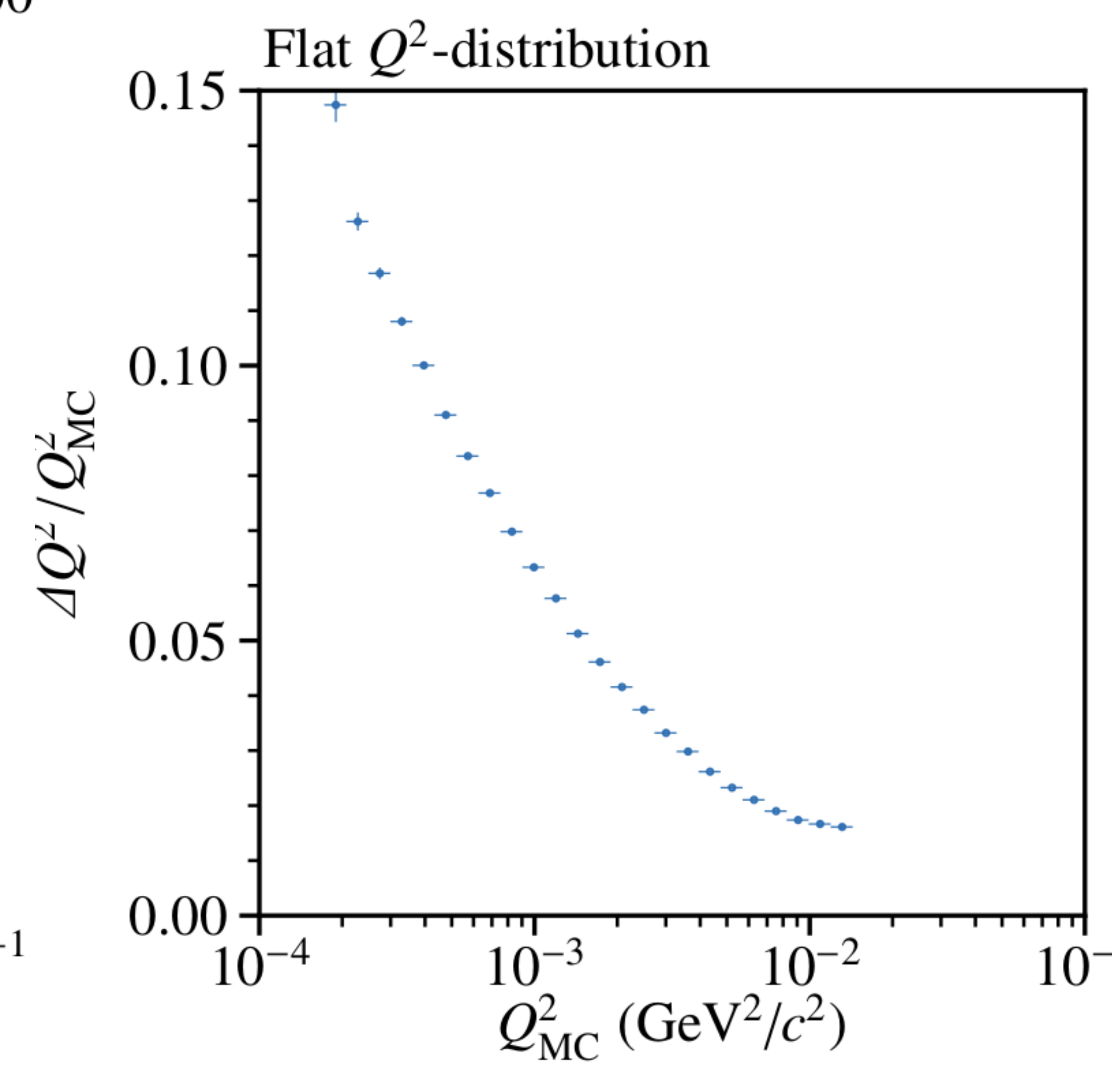
- Test of principle and first measurement

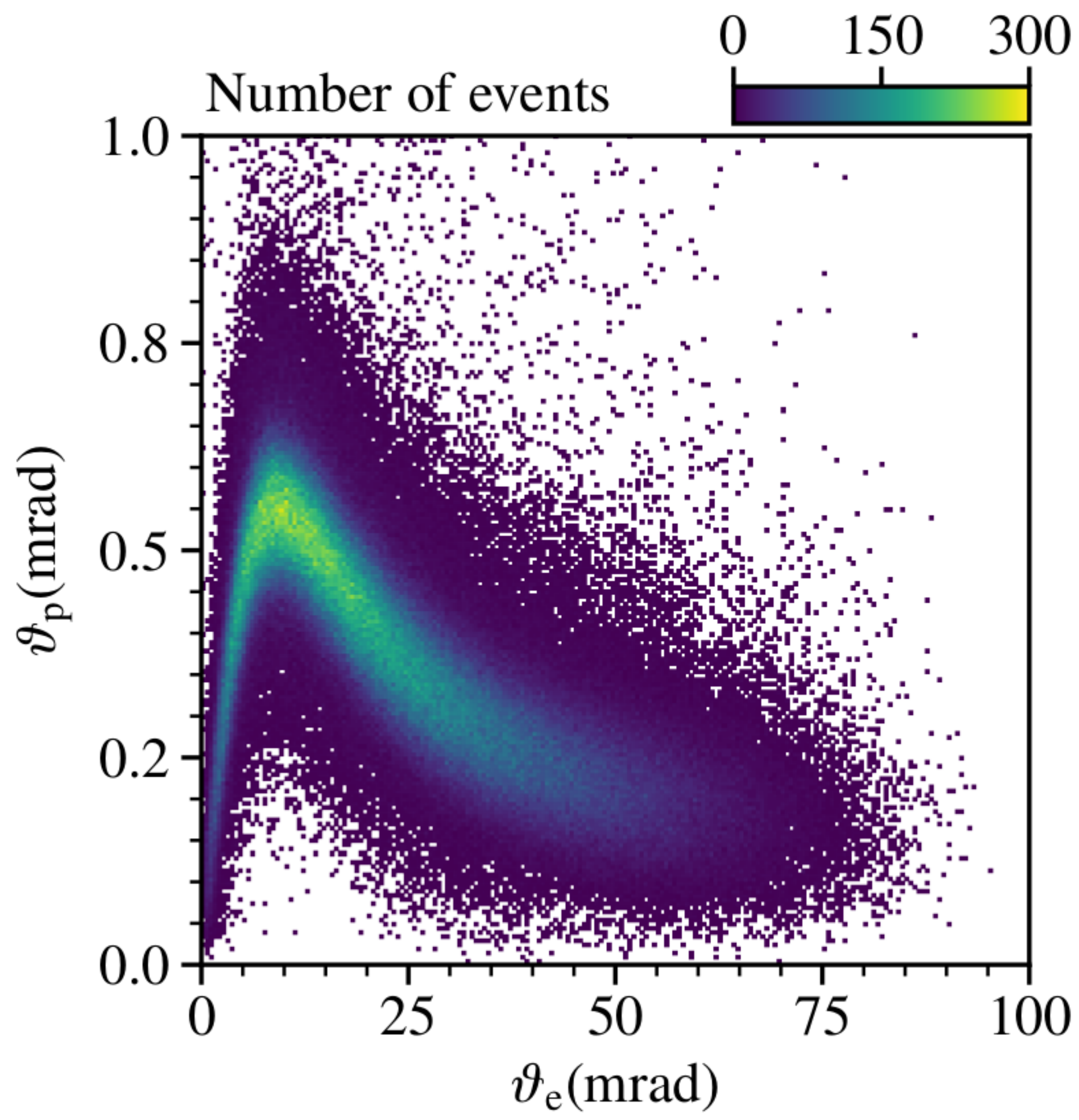


Parasitic Run 2024

- some kinematic distributions for „present“ silicon set-up
  - assumed 100 GeV, distribution $\Delta p/p$ for μ beam, full reconstruction
  - $\Delta Q^2/Q^2 < 7\,\%$ at small Q^2 using full event reconstruction (θ_h, θ_e, $\vec{p_h}$)







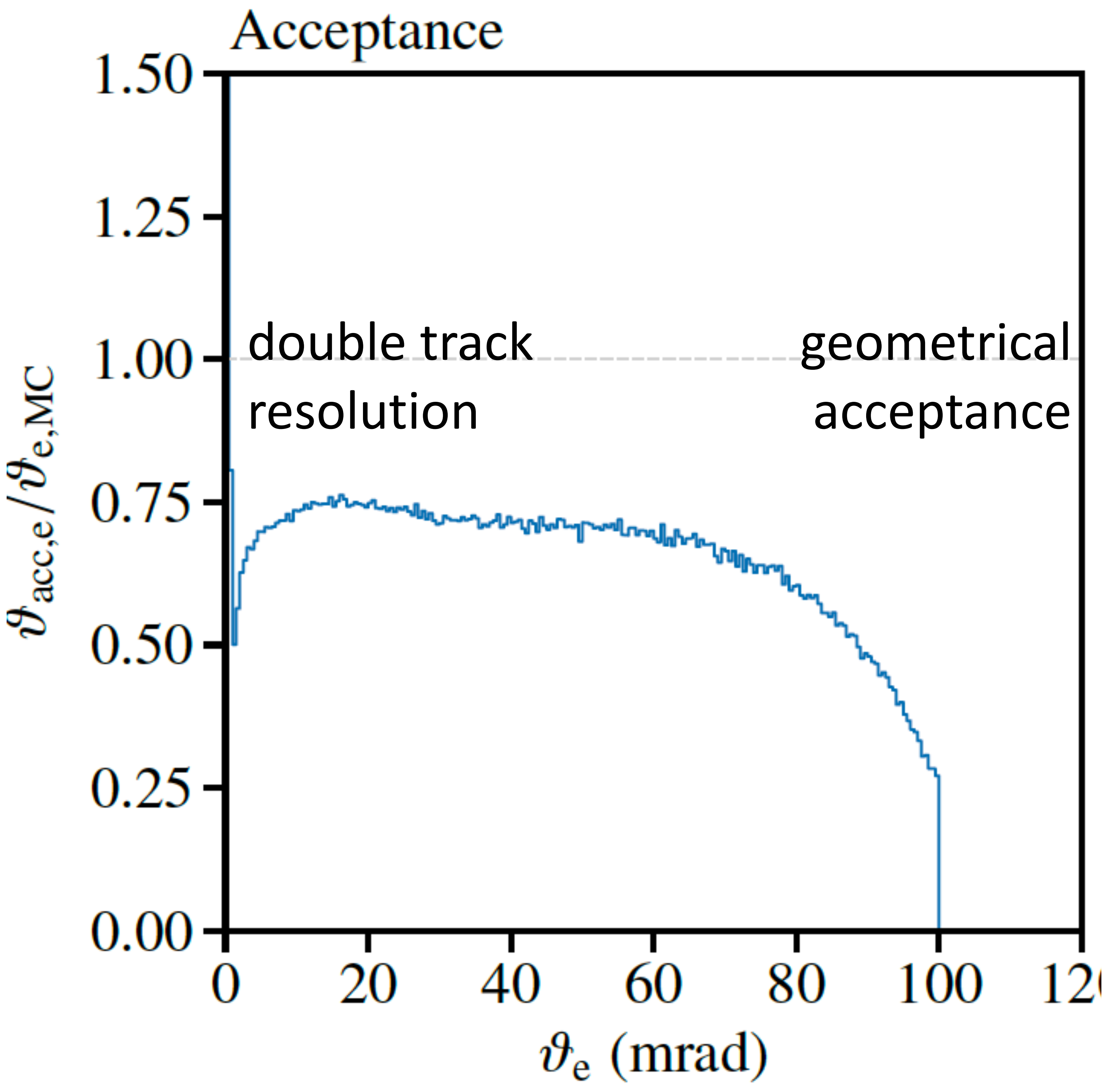
Summary Inverse Kinematics

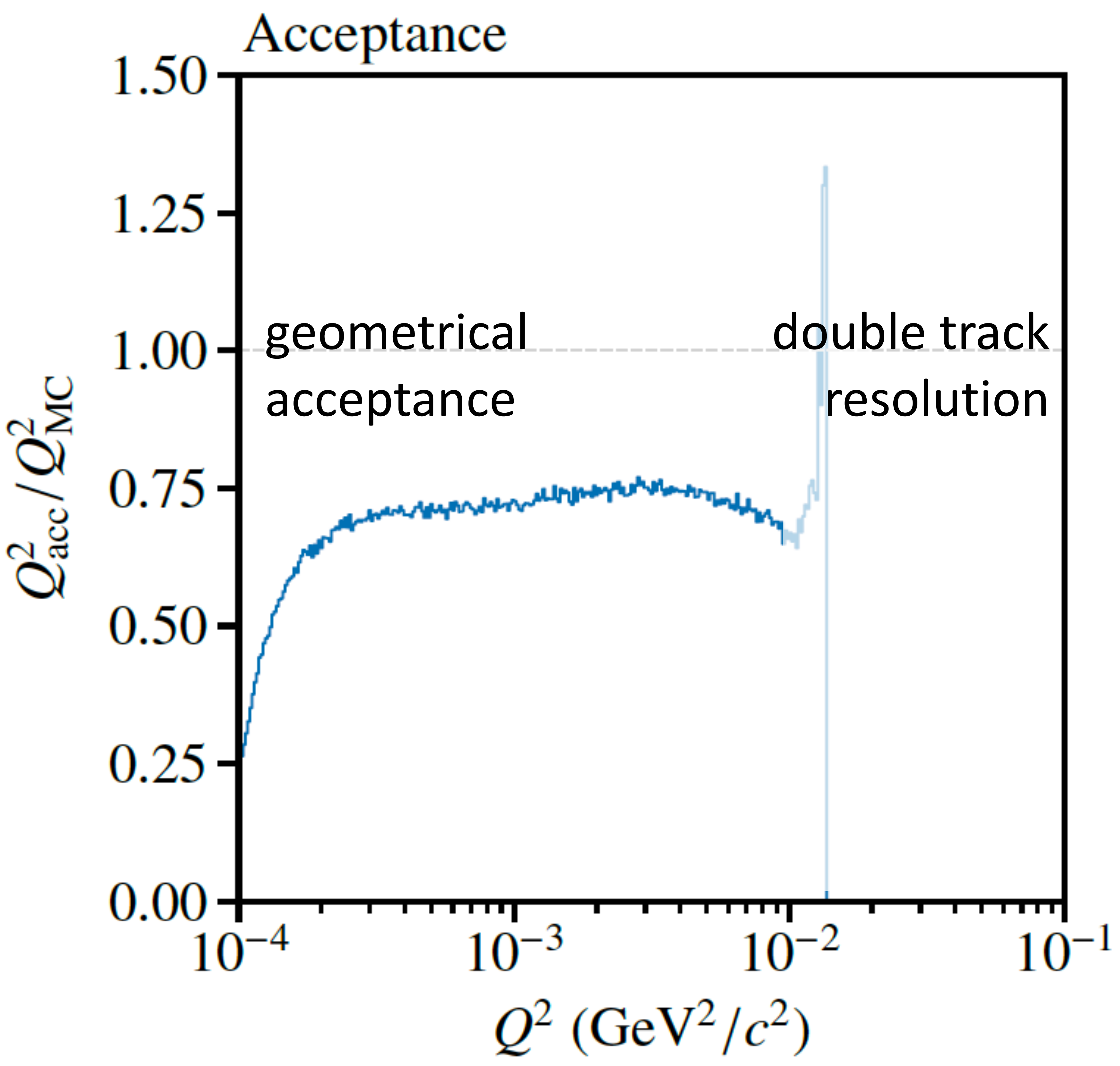
- Meson radii are of key interest in understanding their inner structure
- pions : data of previous experiments can be challenged (statistics !! + systematics)
- kaons : significant increase of the form factor knowledge in the range $10^{-4} < Q^2 < 0.15\ [(GeV/c)^2]$ (factor 10)
- large Q^2 range possible (in particular down to very small Q^2)
  accessible Q^2 range determined by detection requirements for outgoing electron
- Proton inverse kinematics allows low Q^2 kinematics and Rosenbluth separation $G_M^p(Q^2)$
- Antiprotons: First ever measurements of form factors (incl. Rosenbluth separation)
- Comparison of proton - and antiproton - electron scattering sensitive to TPE

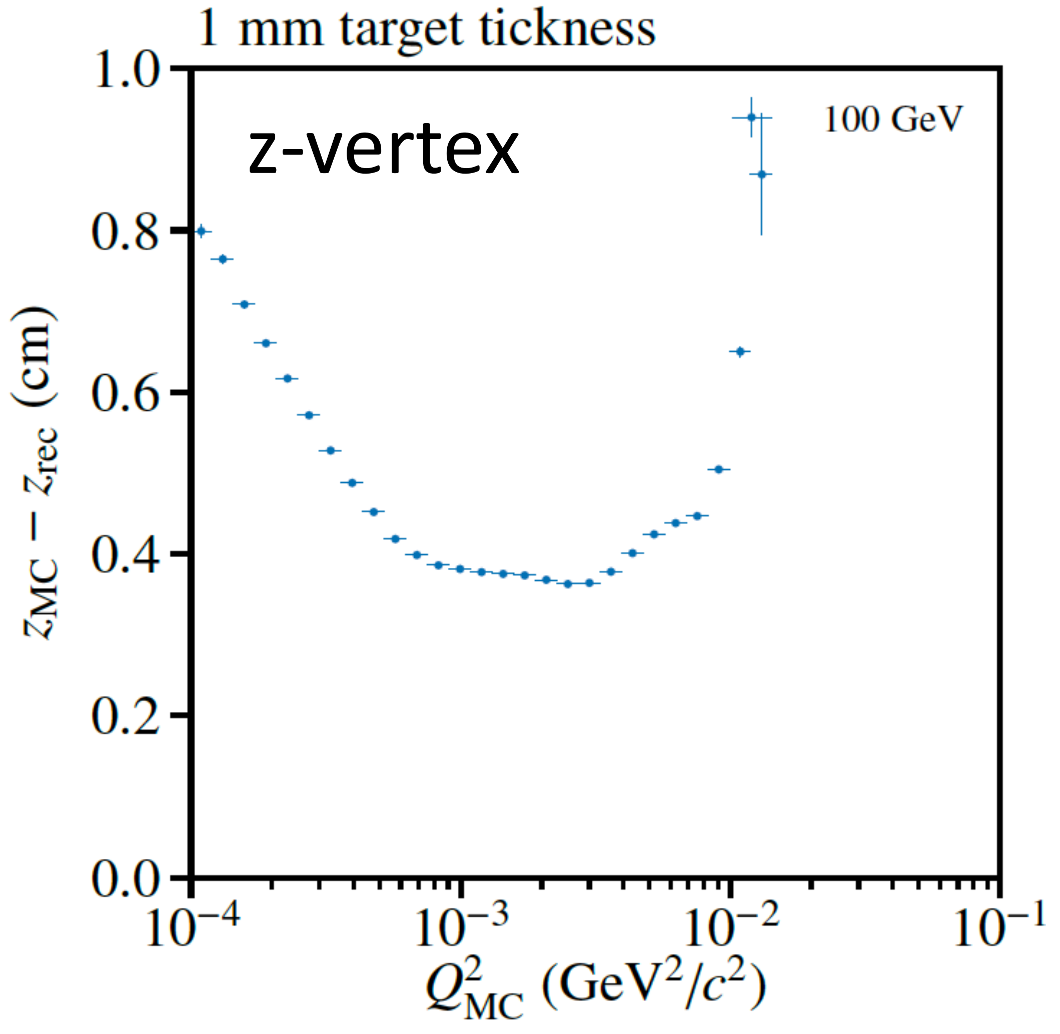
Backup I

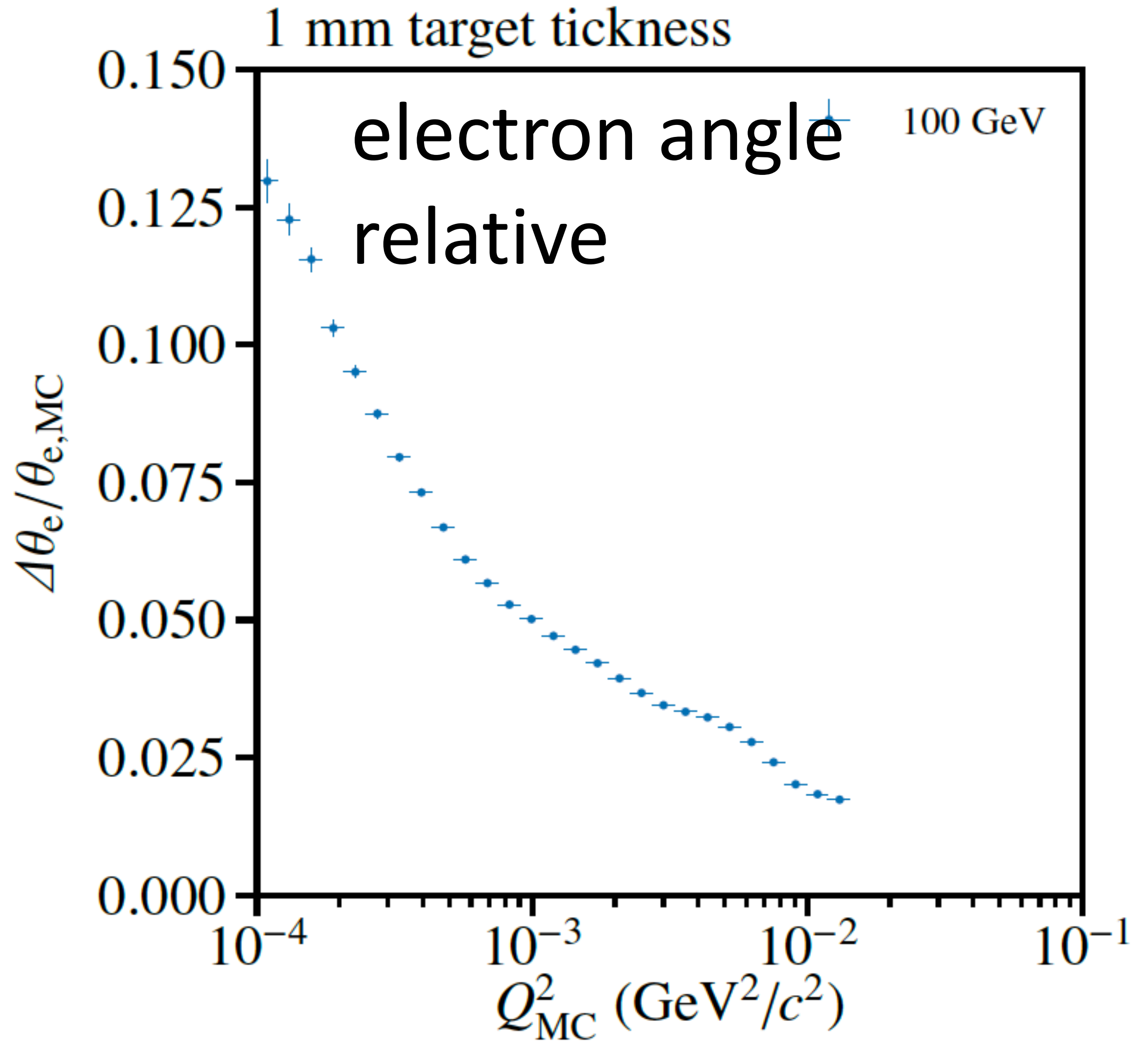
Acceptances - MC studies with Reconstruction Status 2023

- acceptances translating into general efficiencies and Q^2 biases



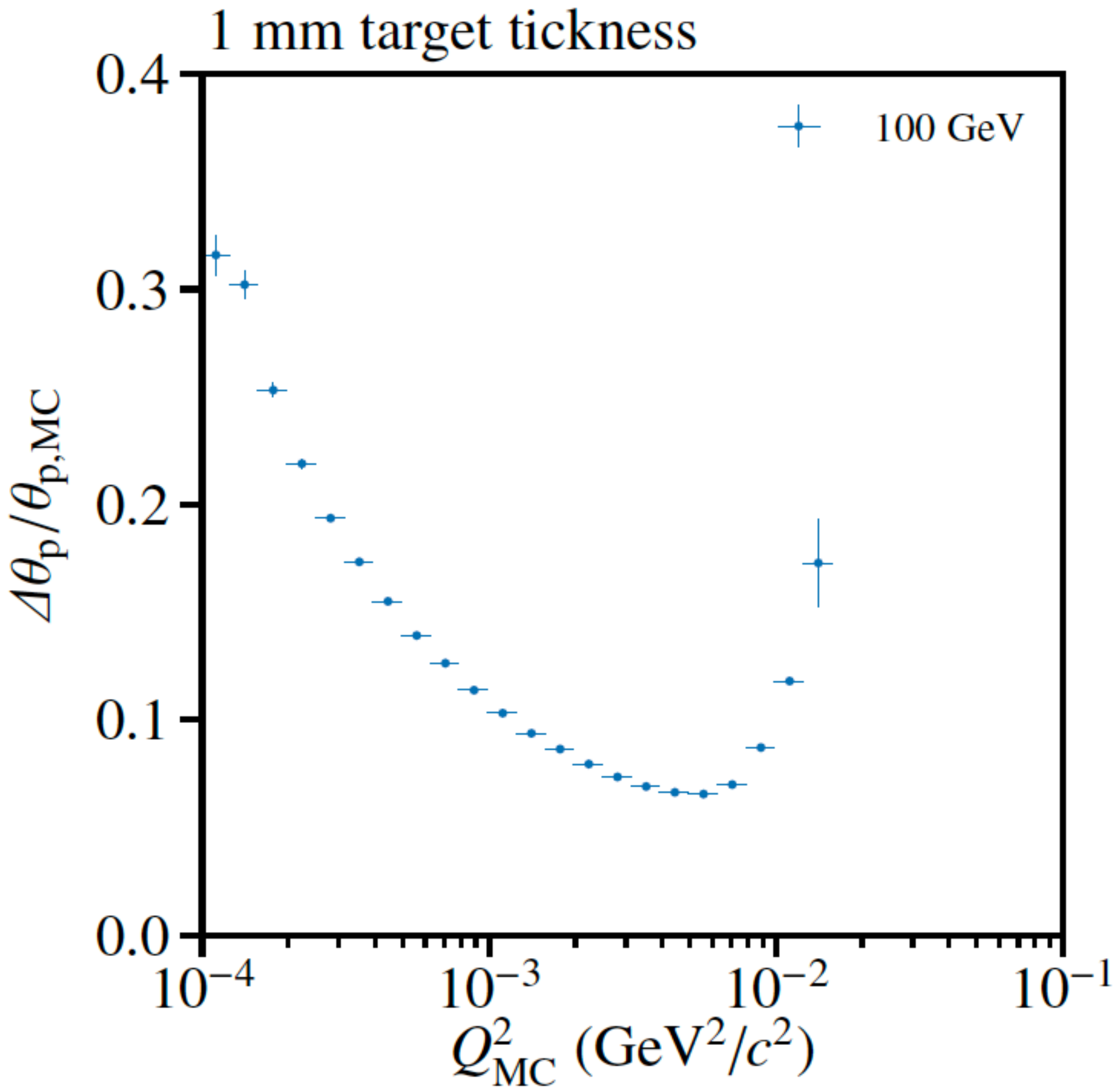






What about Q^2 resolution ?

- Q^2 will be determined mostly by θ_e
- γ radiation by high energy electrons (high Q^2) spoils resolution
- Measurement of θ_h can help
- at very high Q^2 also p_h can be used (but $\Delta p_{beam}/p_{beam} \approx 1\,\%$)



- at present, this effect has not been accounted for
- First estimates could be obtained with McMule (see talk by Marco Rocco)

Apparatus for Meson and Baryon Experimental Research

Backup II

$\overline{p}$ Content of M2-Beam @ SPS

$\overline{p}/\pi^-$ @60 GeV : ≈ 30

$\overline{p}/\pi^-$ @160 GeV : ≈ 90

$\overline{p}/\pi^-$ @190 GeV : ≈ 170

Apparatus for Meson and Baryon Experimental Research

Hadron Radius Measurements

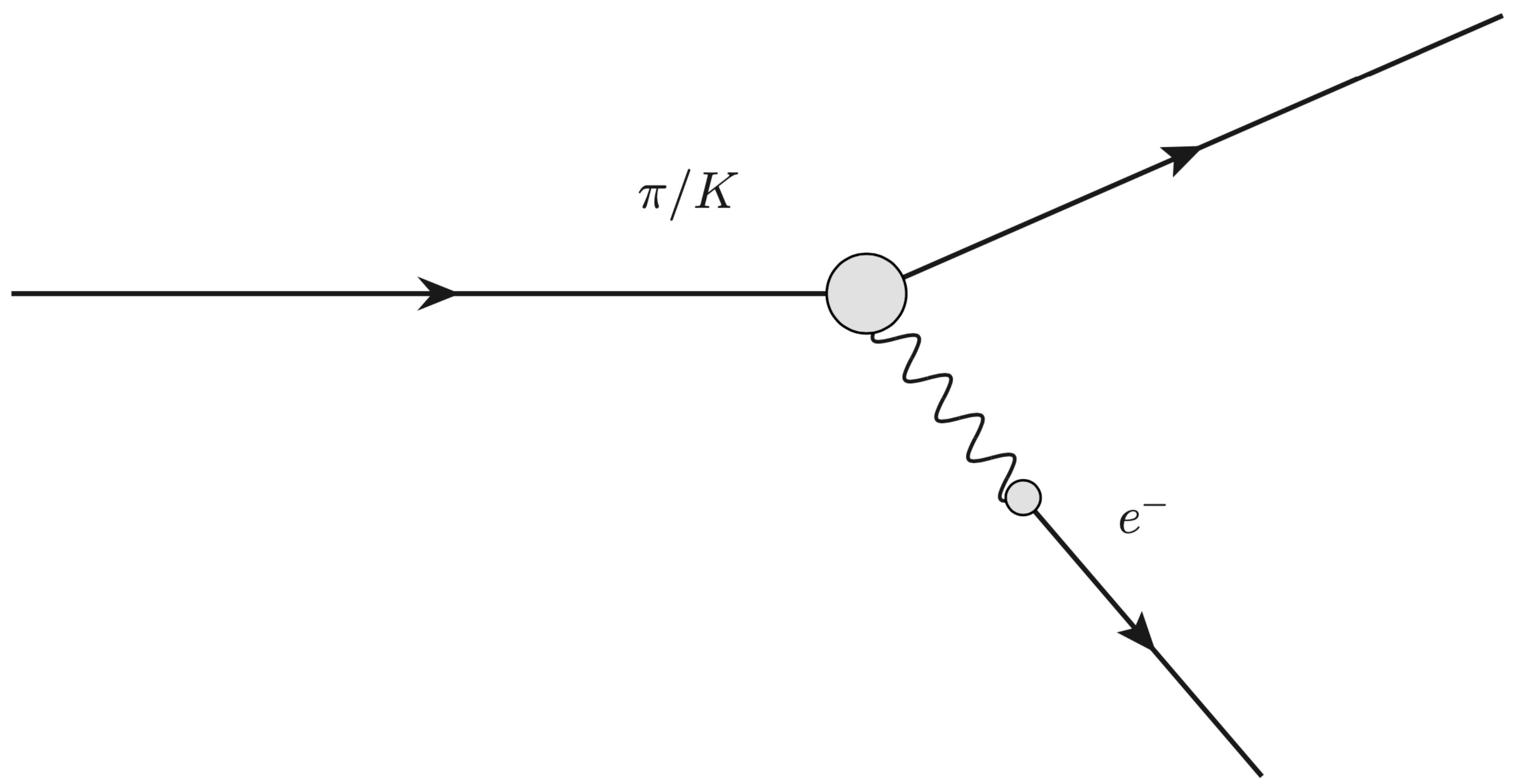
From: EPJC 8 (**1999**) 59, The WA89 Collaboration (measurement of Σ^- charge radius) updated 21.6.2022

Measured $\langle r^2_{ch} \rangle$ in fm^2 of various hadrons

	Experiment	Soliton [7]	Skyrme [8]	non-relat. quark [12]	Skyrme [9]	Cloudy Bag [11]	experiment year
p	$\approx 0.84 - 0.87$	0.78	1.20	0.67	0.775	0.714	2020
n	-0.1101 ± 0.0086	-0.09	-0.15		-0.308	-0.121	2021
Σ^-	$0.61 \pm 0.12 \pm 0.09$	0.75	1.21	0.55	0.751	0.582	2001
π^-	0.439 ± 0.008 [5]	S. R. Amendolia, et al. , Nucl. Phys. B **277**, 168 (1986)					1986
K^-	0.34 ± 0.02 [6]	S. R. Amendolia, et al. , Phys. Lett. B **178**, 435 (1986)					1986
K^0_L	$-0.077 \pm 0.007 \pm 0.011$	$K^0_L \to \pi^-\pi^+e^+e^-$					1998

comparatively good accuracies (pion radius ~1%) stem from assuming a theoretical shape of the form factor

Kinematics



$$K^- \, e^-_{target} \rightarrow K^- \, e^-$$

$$Q^2 \approx 2m_e \cdot E_e$$

$$s = 2E_b m_e + m_b^2 + m_e^2$$

$$Q^2_{max} = \frac{4 \cdot m_e^2 \cdot p_b^2}{s} = 4 \cdot p_{cm}^2$$

Beam	Ebeam [GeV]	Q^2_{max} [GeV²]	$E^{min}_{scatter}$ [GeV]	$E^{electron}_{max}$ Q^2_{max} [GeV]	CM momenta [GeV]
π	190	0,176	17.2	173	0,210
K	190	0,086	105.2	84.7	0,147
K	80	0,021	59.7	20.2	0,072
K	50	0,009	41.3	8.7	0,047
p	190	0,035	155.3	34.3	0,094

Meson Form Factors

- Various analytic functions are discussed for hadronic (mesonic) form factors

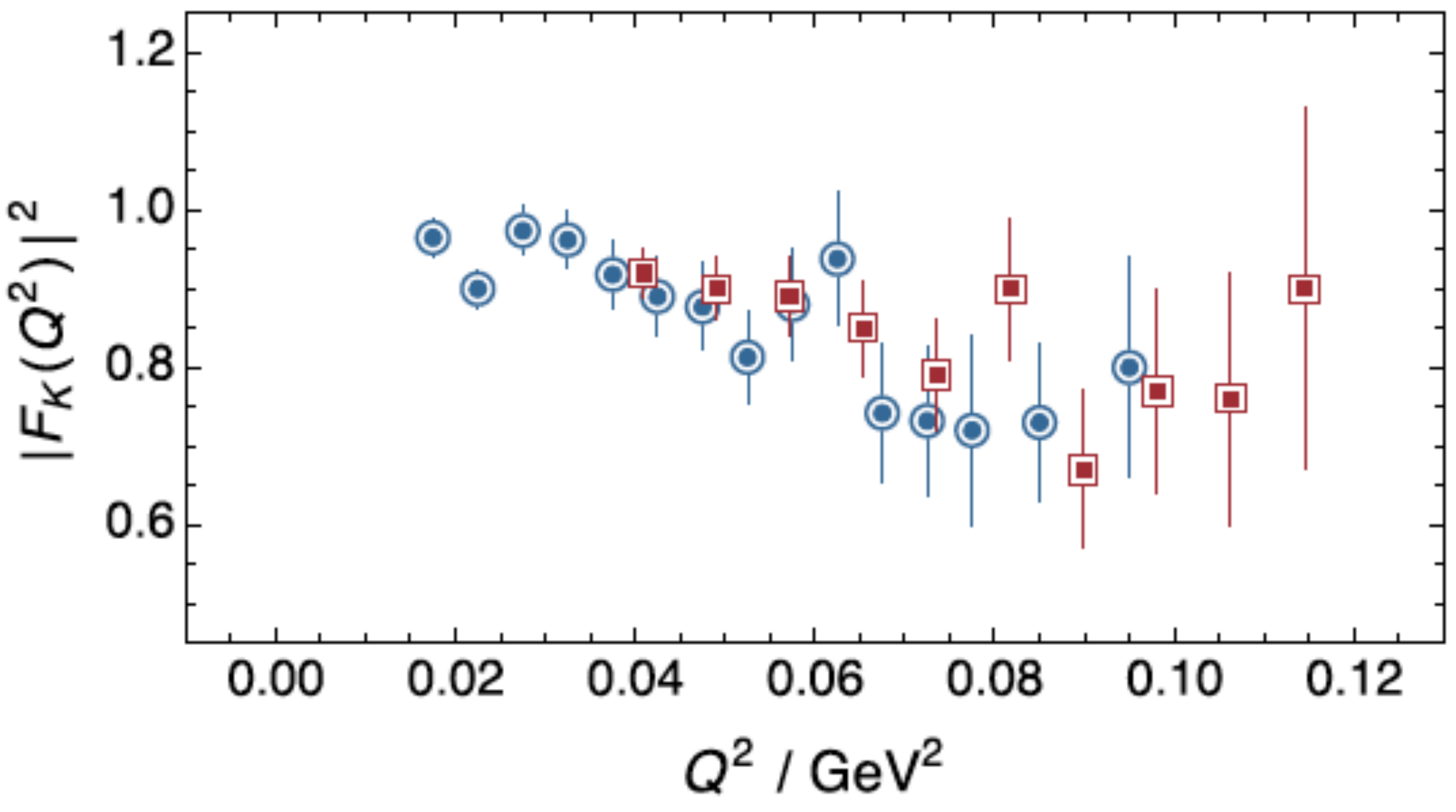
$$F_\pi(Q^2) \propto \begin{cases} \dfrac{1}{1+r_\pi^2 Q^2/6} & \text{monopole} \\ \dfrac{1}{(1+r_\pi^2 Q^2/12)^2} & \text{dipole} \\ e^{-r_\pi^2 Q^2/6} & \text{Gaussian} \end{cases}$$

- usual problem: slope at $Q^2=0$ depends on extrapolation
- various Ansatz:
  – physics motivated Q^2 dependence of FF
  – polynomial or quasi spline fit extrapolation to $Q^2=0$ „without bias“

The Kaon Case

- Only scattering data: NA 7
- 250 GeV beam
- 23 cm LH_2 target
- Beam intensity: 4.5 x 10^4/s

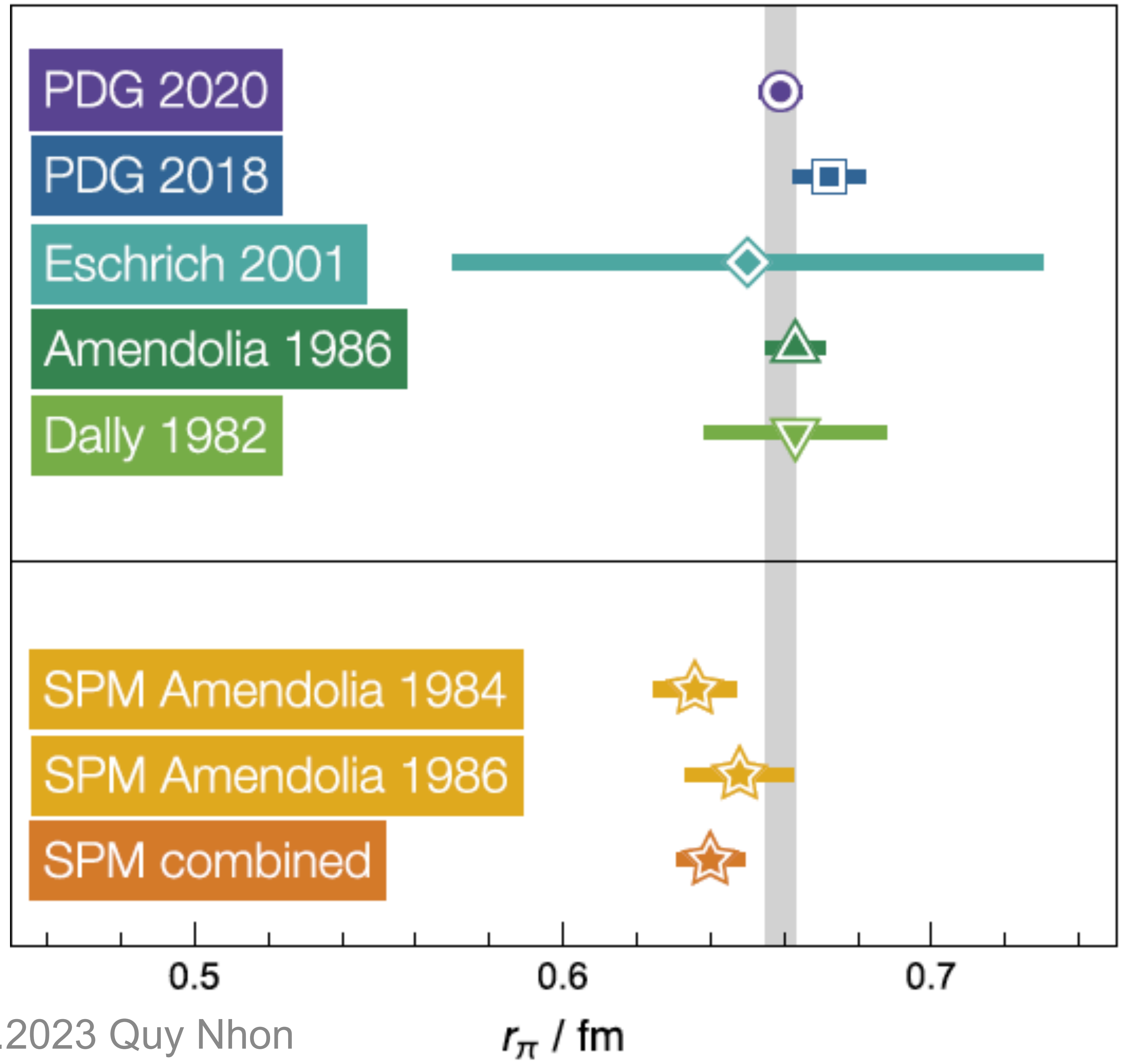
from Physics Letters B 822 (2021) 136631



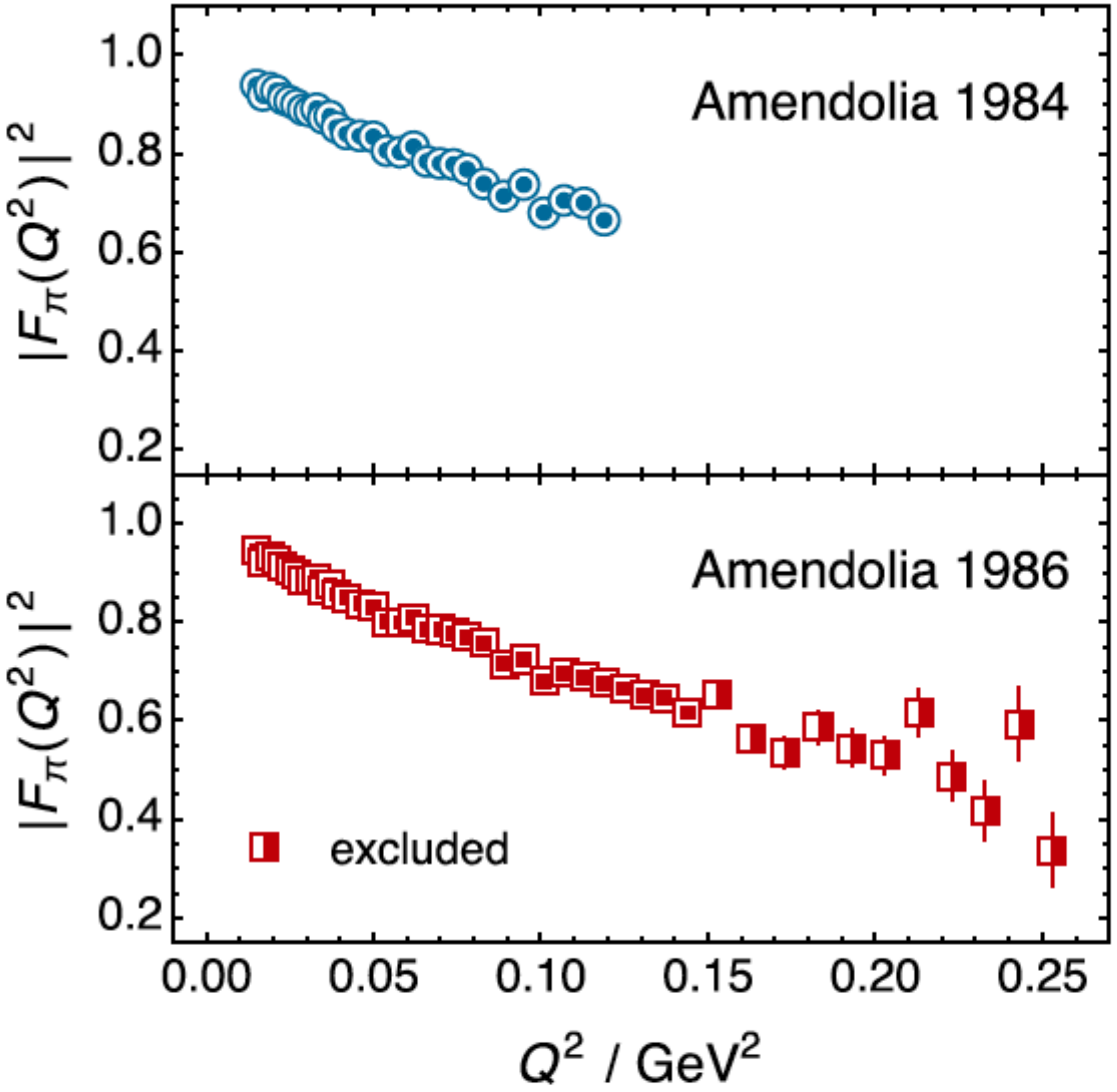
$[(GeV/c)^2]$

Pion-Electron scattering

from Physics Letters B 822 (2021) 136631



from Physics Letters B 822 (2021) 136631



AMBER
Apparatus for Meson and Baryon Experimental Research

Proton-Electron scattering

Why p-e$^-$ scattering ?

- complementary measurement to Mainz, JLAB and PSI
- very different kinematics and twofold reconstruction of Q^2
  - scattered proton (multiple scattering of little issue)
  - outgoing electron (Bremsstrahlung corrections and multiple scattering of low energy electron)
  - high beam quality (small divergence, small beam spot size)

What is the equivalent for electron-proton scattering ?

- assume p^{proton}=190 GeV/c
- equivalent normal kinematics using proton at rest: $p^{electron}$=103.5 MeV/c
- calculate internal Bremsstrahlung for the equivalent kinematics
- variation of beam energy easy