Two-photon physics

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18 - 20 March, 2024, Château de Bossey, Switzerland

Outline

Proton size and electromagnetic structure

- Precision hadronic structure from muonic atom spectroscopy
- Status and prospects

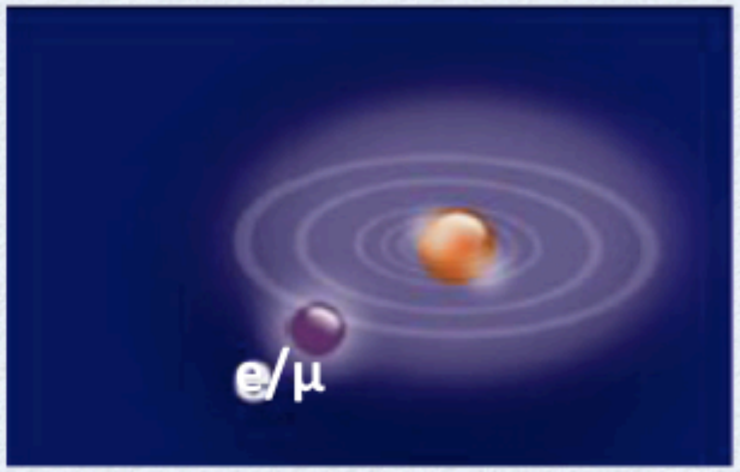
Two-photon exchange in lepton-nucleon scattering

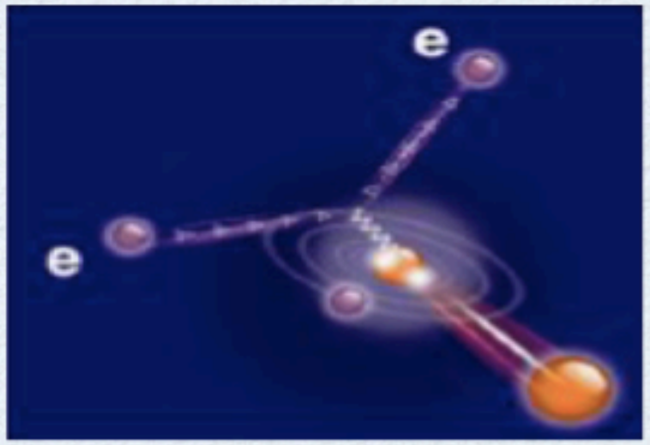
- Formalism for electron and muon scattering
- Comparison with e^- data

Nucleon structure from virtual Compton scattering

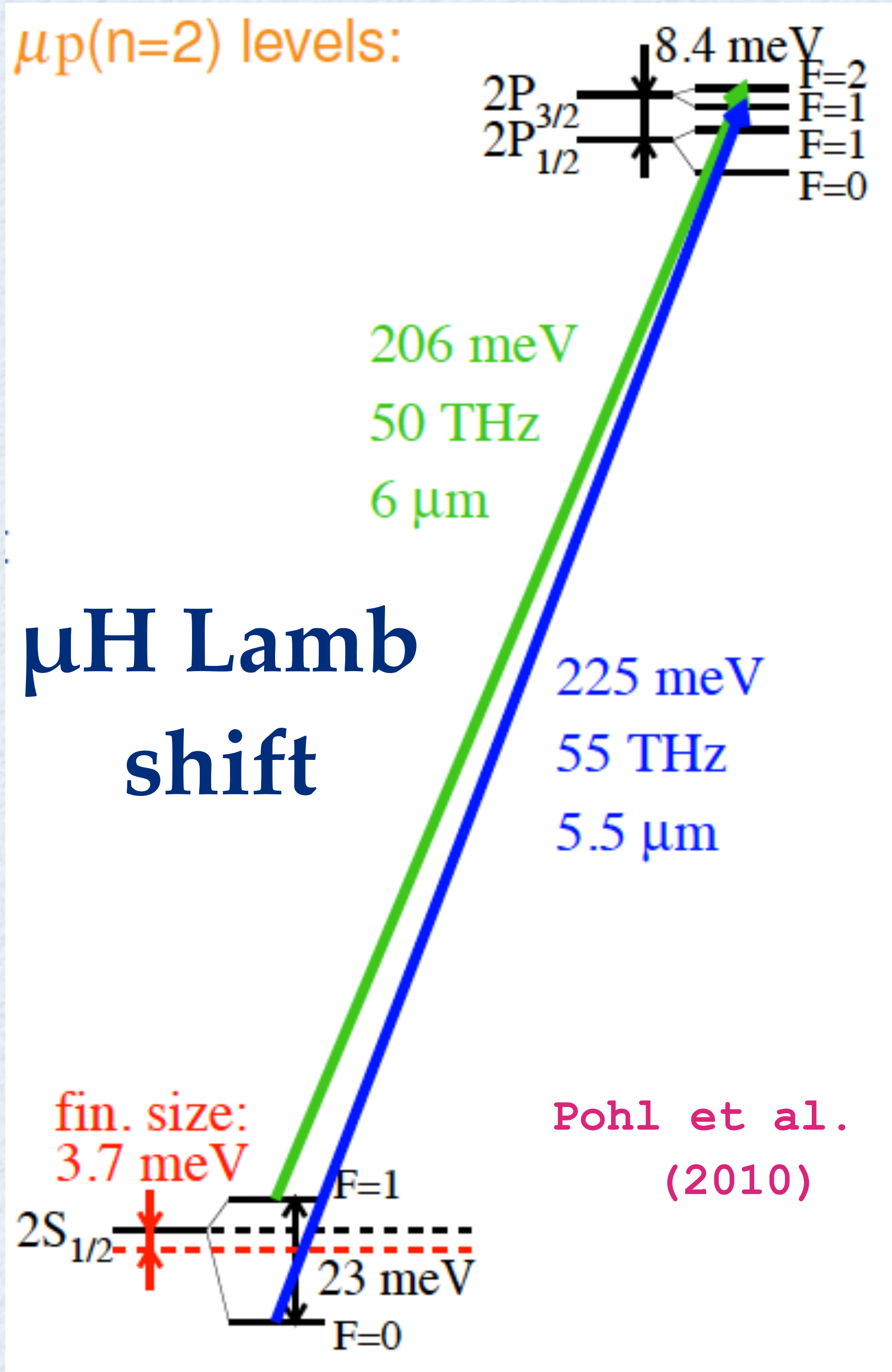
- Possible opportunity for AMBER

Proton size and electromagnetic structure





Proton radius from Hydrogen spectroscopy



$$\Delta E_{LS} = 206.0336\,(15)\ -\ 5.2275\,(10)\ R_E^2\ +\ \Delta E_{TPE} \quad \text{meV}$$

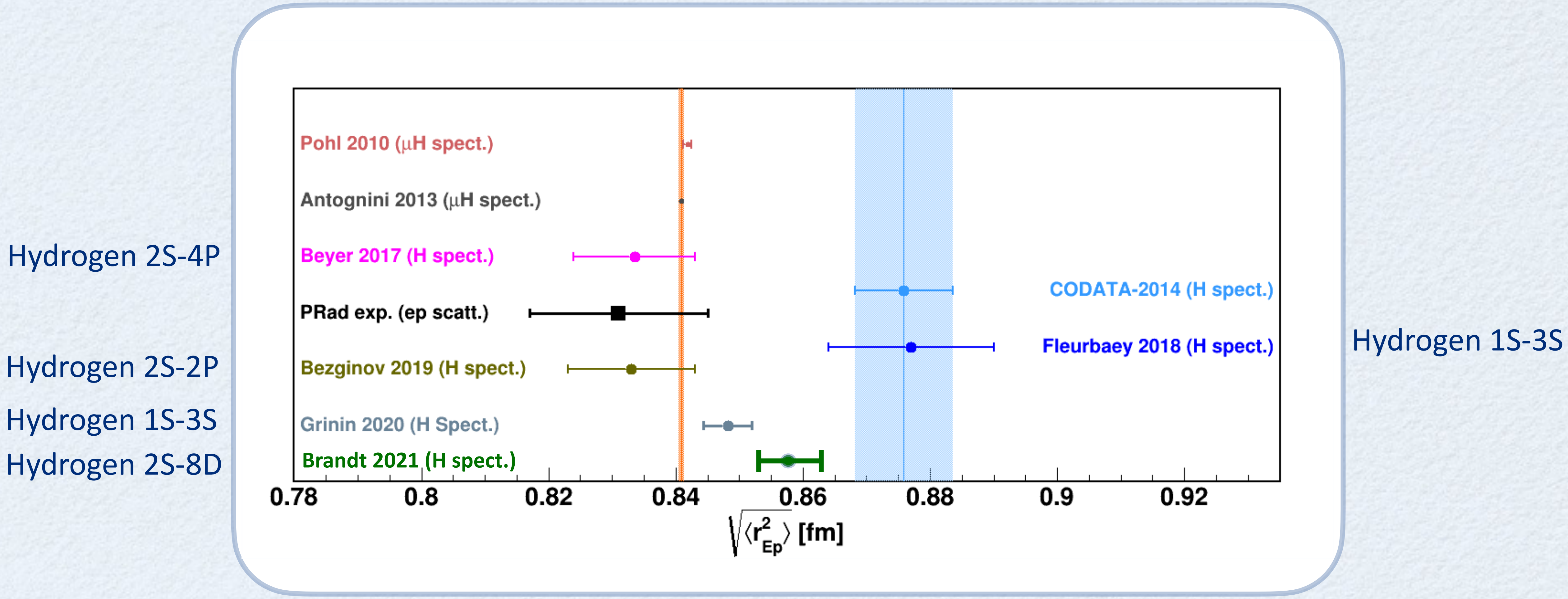
Antognini et al.(2013)

3.70 meV

O(α^5) correction

0.033 (2) meV

Proton charge radius: present experimental status



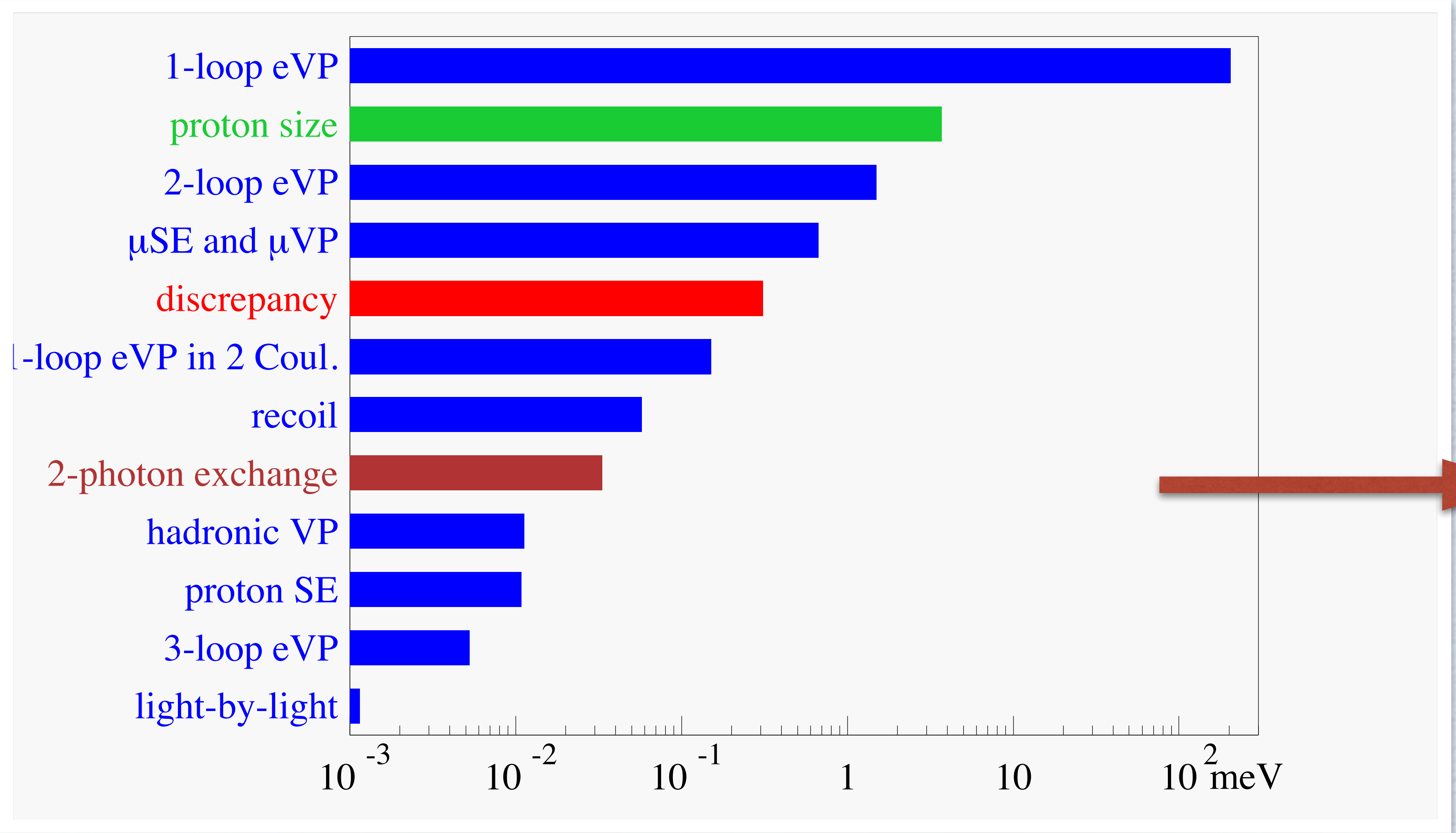
from recent compilation **Rev. Mod. Phys. 94 (2022) 015002**

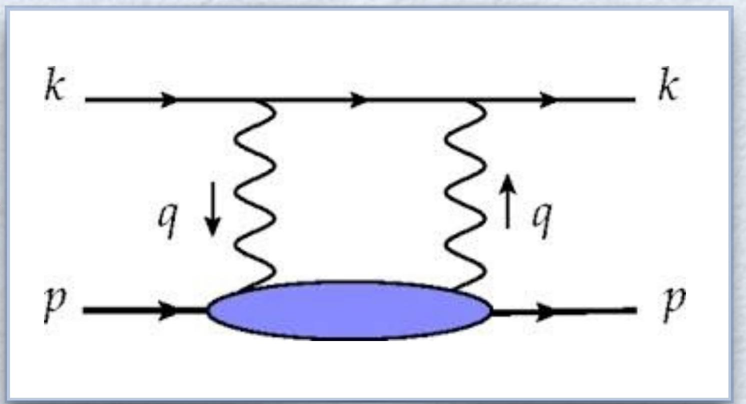
H.Gao, M.Vdh

- 3 out of 6 new results are fully consistent with muonic hydrogen result
- inconsistency between Fleurbaey et al. (Paris) and Grinin et al. (Garching) results for 1S-3S H : Grinin et al.: factor 2 more precise, ~2σ smaller than Fleurbaey et al., ~2σ larger than μH result
- Brandt et al. (Colorado) result is ~3σ larger than CODATA 2018 / muonic atom spect.

Lamb shift: status of theory

μH Lamb shift: summary of corrections





largest theoretical uncertainty

total hadronic correction on Lamb shift

- elastic contribution on 2S level: ΔE_{2S} = -23 μeV
- inelastic contribution: Carlson, Vdh (2011) + Birse, McGovern (2012)

ΔE_{TPE} (2P-2S) = (33 ± 2) μeV

For H: present accuracy comparable with experimental precision δ_{exp} (ΔE_{LS}) = 2.3 μeV

Muonic atom spectroscopy needs nucleon/nuclear input

2S-2P Lamb Shift:

THEORY | **EXPERIMENT**

	$\Delta E_{TPE} \pm \delta_{theo}\,(\Delta E_{TPE})$	Ref.	$\delta_{exp}(\Delta_{LS})$	Ref.
μH	33 μeV $\pm$ 2 μeV	Antognini et al. (2013)	2.3 μeV	Antognini et al. (2013)
μD	1710 μeV $\pm$ 15 μeV	Krauth et al. (2015)	3.4 μeV	Pohl et al. (2016)
μ^3He$^+$	15.30 meV $\pm$ 0.52 meV	Franke et al. (2017)	0.05 meV	
μ^4He$^+$	9.34 meV $\pm$ 0.25 meV -0.15 meV $\pm$ 0.15 meV (3PE)	Diepold et al. (2018) Pachucki et al. (2018)	0.05 meV	Krauth et al. (2020)

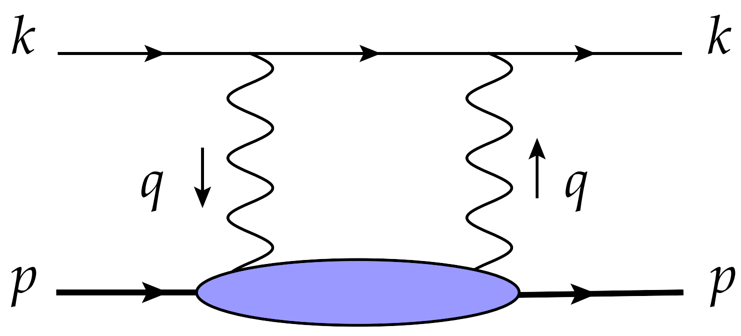
μH:

present accuracy comparable with experimental precision
Future: factor 5 improvement on Lamb shift planned @PSI
CREMA, FAMU, J-PARC: 1S hyperfine splitting in μH to 1ppm

μD, μ^{3}He$^+$, μ^{4}He$^+$:

present accuracy factor 5-10 worse than experimental precision

Two-photon exchange: hadronic corrections



$$T^{\mu\nu}(p,q) = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) T_1(\nu, Q^2) + \frac{1}{M^2}\left(p^\mu - \frac{p\cdot q}{q^2} q^\mu\right)\left(p^\nu - \frac{p\cdot q}{q^2} q^\nu\right) T_2(\nu, Q^2)$$

- Two-photon exchange (TPE): lower blob contains both elastic (nucleon) and inelastic states
- Lamb shift: described by unpolarized amplitudes T_1, T_2 : functions of energy ν and Q^2
- Hyperfine splitting: described by polarized amplitudes S_1, S_2
- Imaginary parts: directly proportional to nucleon structure functions F_1, F_2 resp. g_1, g_2
- Real parts: obtained as dispersion integral over the imaginary parts modulo a subtraction function in case of T_1

$$\Delta E = \Delta E^{el} + \Delta E^{subtr} + \Delta E^{inel}$$

→ Elastic state: involves **nucleon form factors**

→ Subtraction: involves **nucleon polarizabilities**

→ Inelastic state: involves **nucleon structure functions**

Hadron/Nuclear physics input needed !

Two-Photon Exchange (TPE) in Lamb shift

wave function at the origin

$$\Delta E(nS) = 8\pi\alpha m\, \phi_n^2\, \frac{1}{i} \int_{-\infty}^{\infty} \frac{d\nu}{2\pi} \int \frac{d\mathbf{q}}{(2\pi)^3} \frac{\left(Q^2 - 2\nu^2\right) T_1(\nu, Q^2) - (Q^2 + \nu^2)\, T_2(\nu, Q^2)}{Q^4 (Q^4 - 4m^2\nu^2)}$$

dispersion relation
& optical theorem

$$T_1(\nu, Q^2) = T_1(0, Q^2) + \frac{32\pi Z^2 \alpha M \nu^2}{Q^4} \int_0^1 dx \frac{x F_1(x, Q^2)}{1 - x^2(\nu/\nu_{el})^2 - i0^+}$$

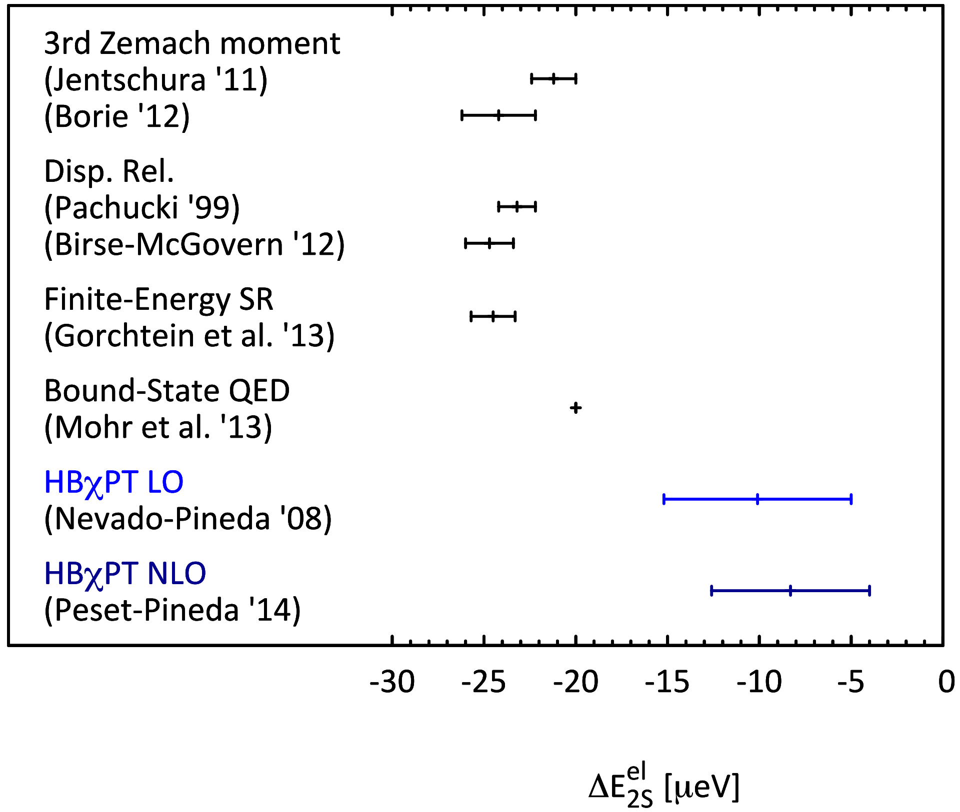
$$T_2(\nu, Q^2) = \frac{16\pi Z^2 \alpha M}{Q^2} \int_0^1 dx \frac{F_2(x, Q^2)}{1 - x^2(\nu/\nu_{el})^2 - i0^+}$$

low-energy expansion:

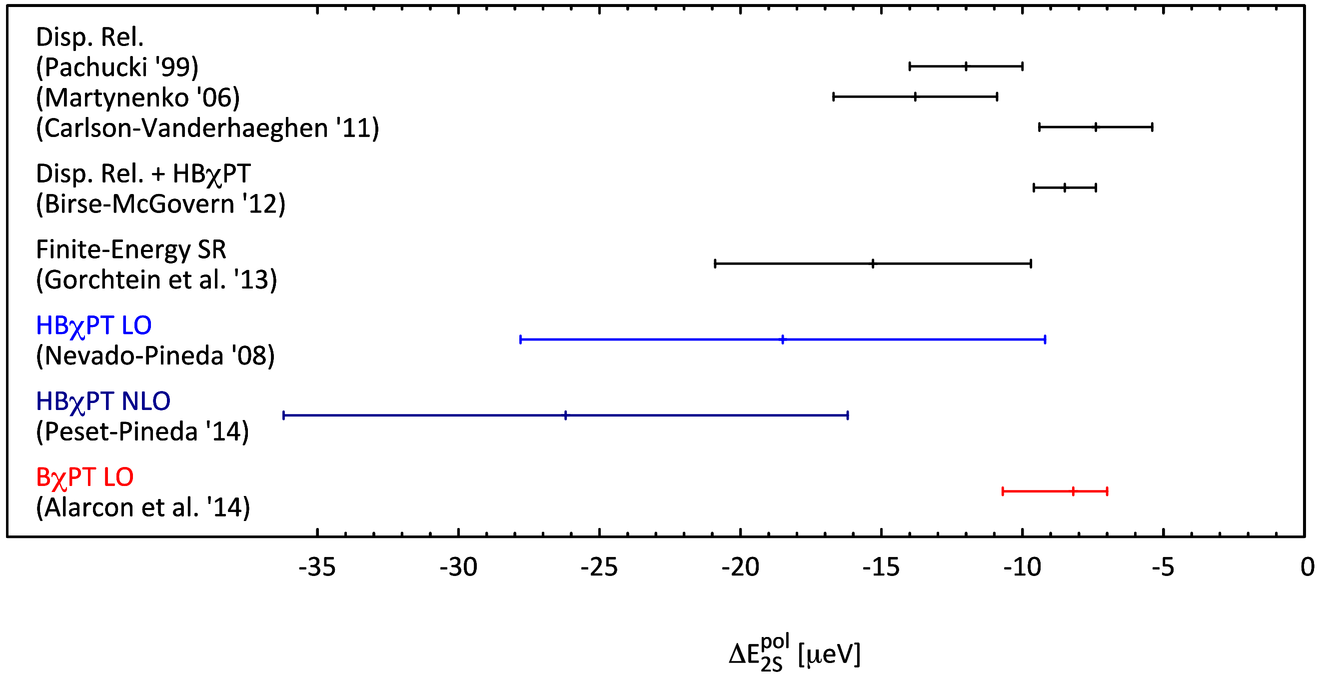
$$\lim_{Q^2 \to 0} \overline{T}_1(0, Q^2)/Q^2 = 4\pi\beta_{M1}$$

Caution:
in the dispersive approach
the $T_1(0,Q^2)$ subtraction function
is model-dependent!

TPE elastic correction:



TPE polarizability correction:



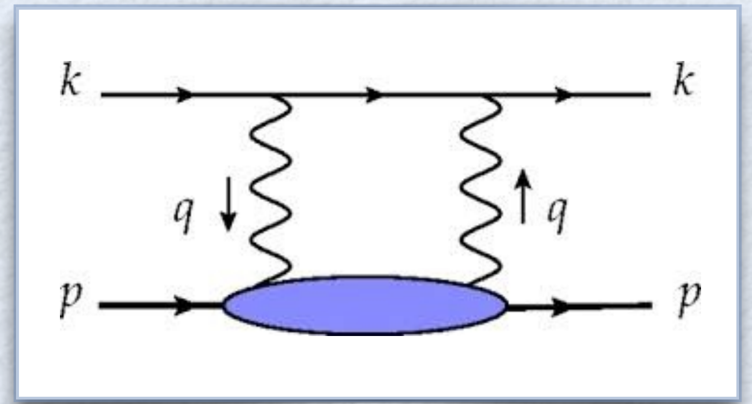
Hagelstein, Miskimen, Pascalutsa (2016)

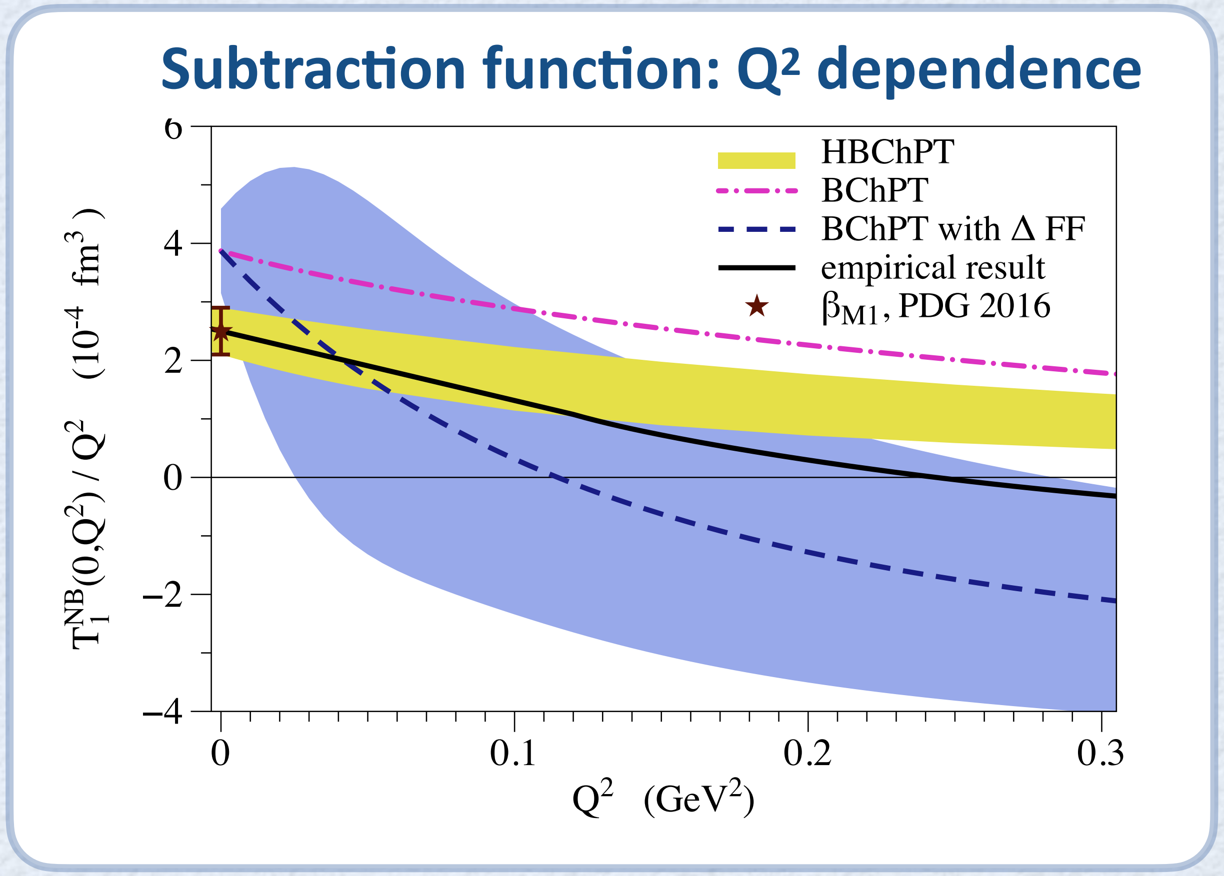
review: Antognini, Hagelstein, Pascalutsa **Ann.Rev.Nucl.Part.Sci. 72 (2022) 389**

Improved determination of subtraction function (Lamb shift)

Future plan @PSI:
factor 5 improvement
on LS for muonic H !

Antognini, Pohl



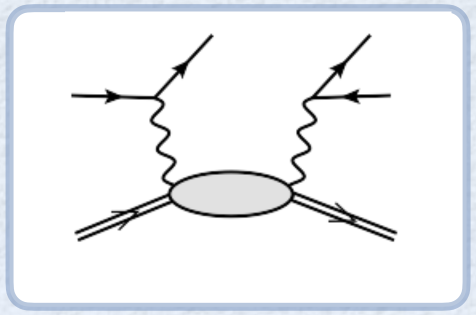


Lensky, Hagelstein, Pascalutsa, Vdh (2018)

To improve on uncertainty due to subtraction function: 3 avenues

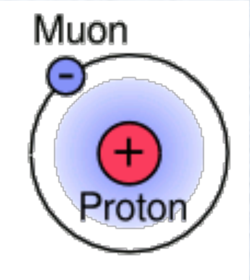
- Full NLO calculation in Baryon ChPT   Pascalutsa et al.
- New prospect for lattice determination of subtraction function   Hagelstein, Pascalutsa (2020)
- Empirical determination of Q^4 term using dilepton production process

Pauk, Carlson, Vdh (2020)

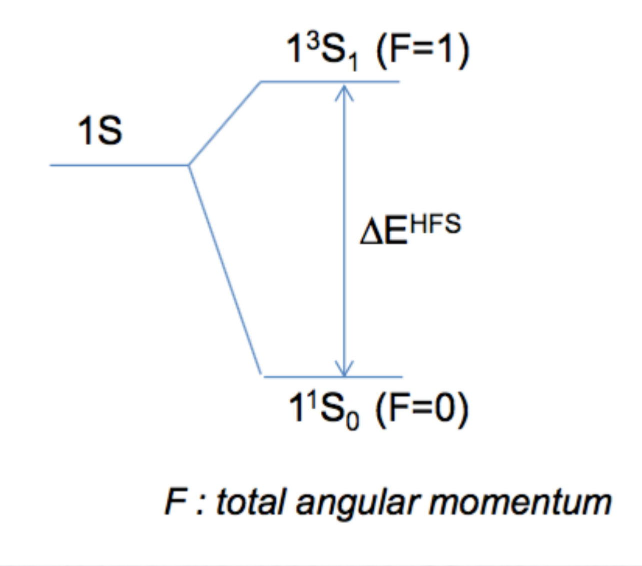


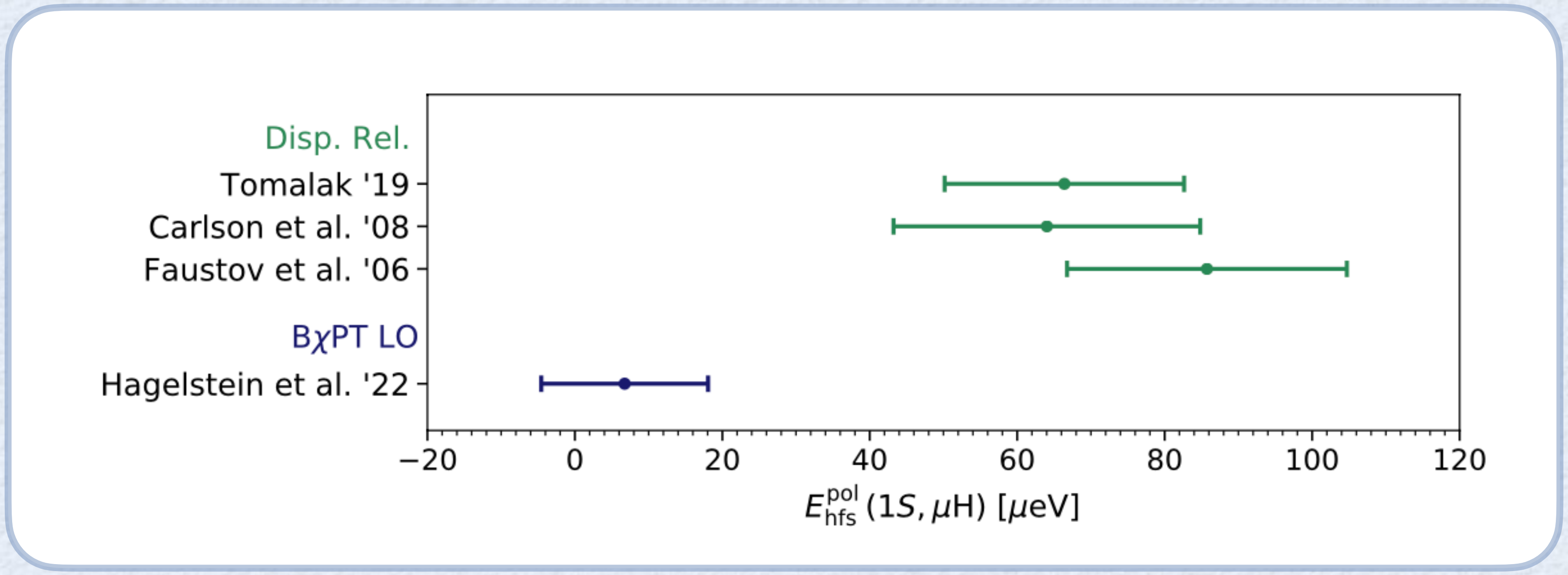
Hyperfine Splitting in muonic Hydrogen

Measurements of the μH ground-state HFS planned by CREMA, FAMU, J-PARC collaborations **precision goal: 1ppm !**



Currently: disagreement between data-driven evaluations and chiral perturbation theory

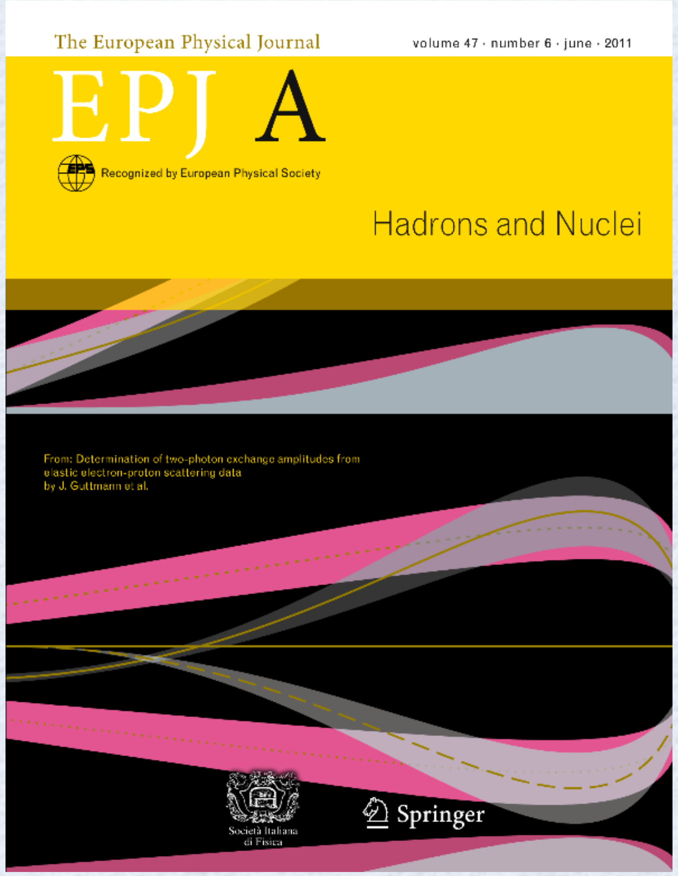




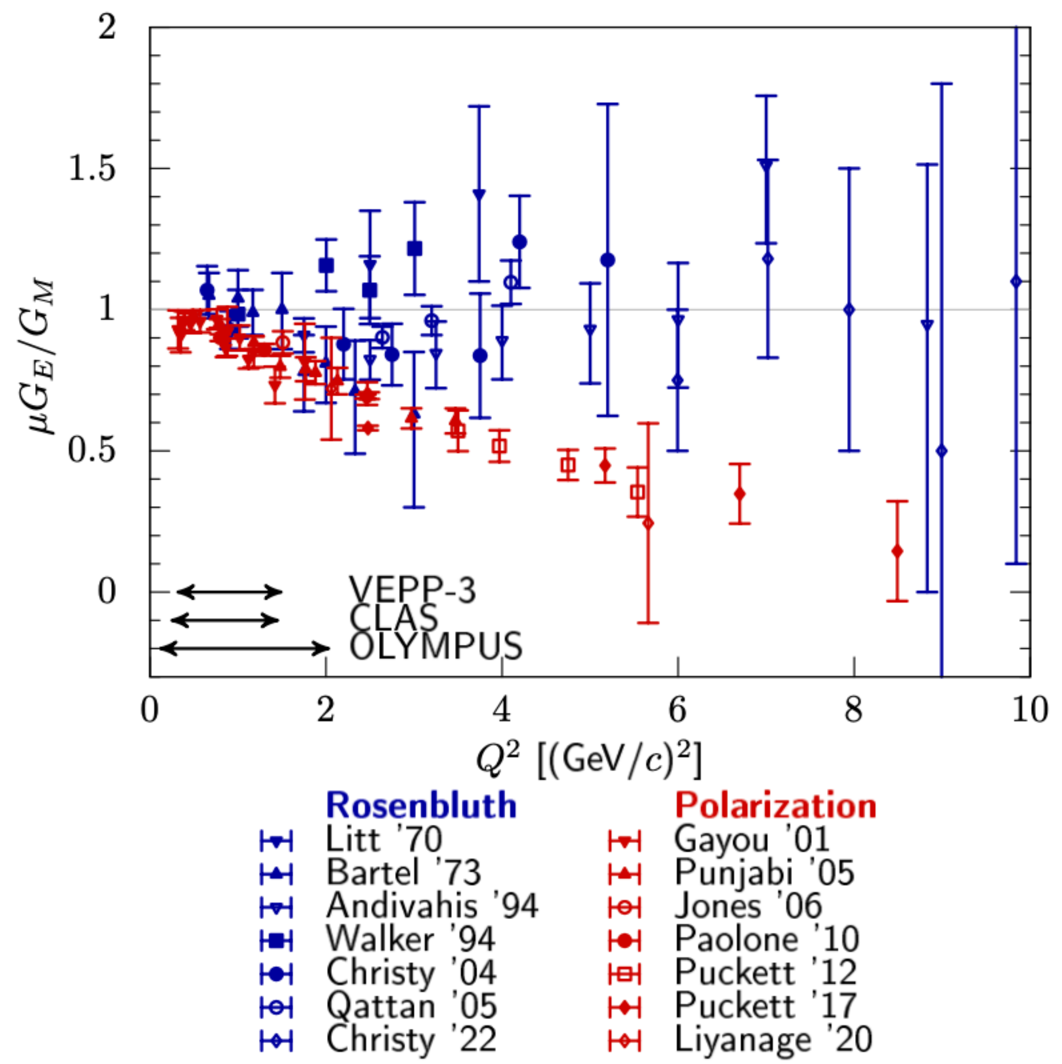
Calls for re-evaluation of empirical parametrizations of nucleon structure functions

Antognini, Hagelstein, Pascalutsa (2022)

Two-photon exchange in lepton-nucleon scattering



Rosenbluth vs polarization transfer measurements of G_E/G_M of proton



Rosenbluth data

SLAC, JLab (Hall A, C)

Polarization data

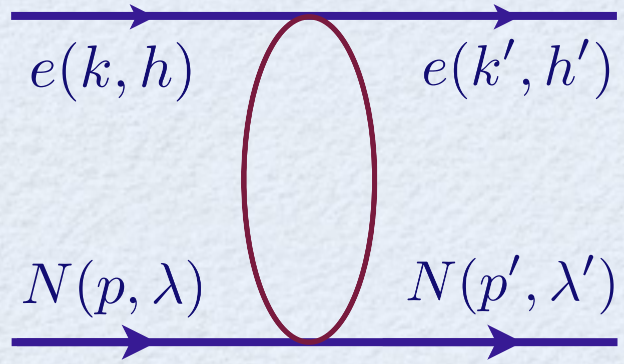
JLab (Hall A, C)

Two methods: two different results

most likely: 2γ-exchange correction

2γ-exchange (TPE) in e$^-$ scattering: general

$$P = \frac{p+p'}{2} \qquad K = \frac{k+k'}{2}$$



$$t = (k-k')^2 \qquad u = (k-p')^2$$

$$s = (p+k)^2 \qquad \nu = \frac{s-u}{4}$$

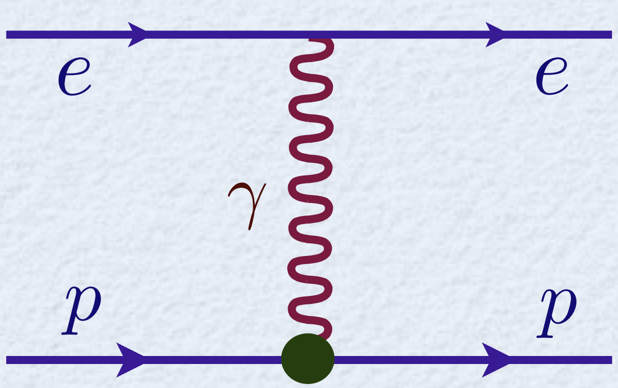
discrete symmetries + $m_e = 0$ ⟶ 3 structure amplitudes

$$T = \frac{e^2}{Q^2}\bar{e}(k',h')\gamma_\mu e(k,h).\bar{N}(p',\lambda')[\mathcal{G}_M(\nu,t)\gamma^\mu - \mathcal{F}_2(\nu,t)\frac{P^\mu}{M} + \mathcal{F}_3(\nu,t)\frac{\hat{K}P^\mu}{M^2}]N(p,\lambda)$$

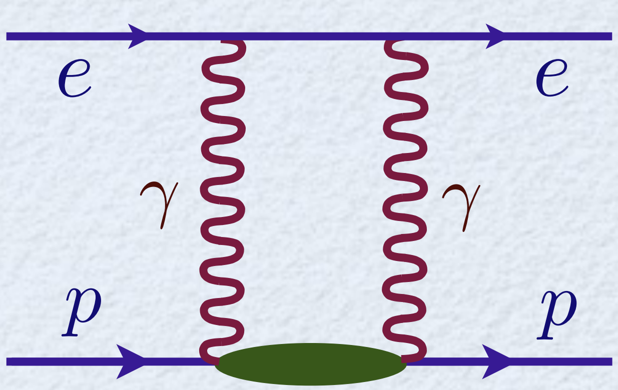
Guichon, Vdh (2003)

Leading contribution to cross section - interference term

1 photon diagram



2 photon exchange diagram



$$\delta_{TPE} \sim \Re\mathcal{G}_M, \Re\mathcal{F}_2, \Re\mathcal{F}_3$$

2γ-exchange at low Q²

2γ blob: near-forward virtual Compton scattering

Feshbach — inelastic — elastic

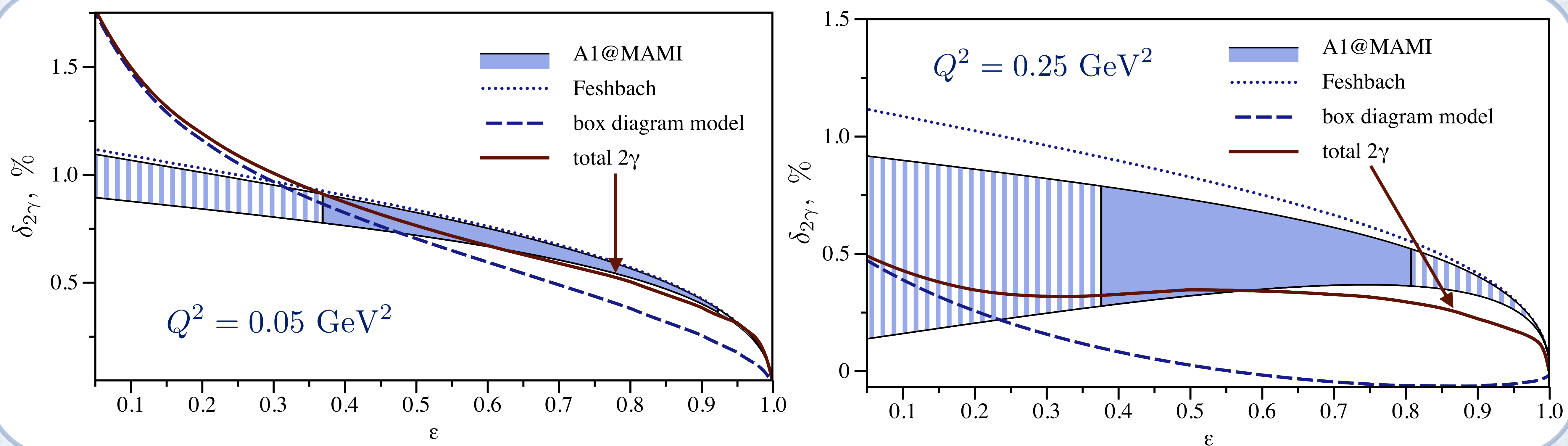
$$\delta_{2\gamma} \sim a\,\sqrt{Q^2} + b\,Q^2 \ln Q^2 + c\,Q^2 \ln^2 Q^2$$

McKinley, Feshbach (1948)
R.W.Brown (1970)
M. Gorchtein (2013)

unpolarized proton structure

Tomalak, Vdh (2016)

$$\delta_{2\gamma} = \int \mathrm{d}\nu_\gamma \mathrm{d}\tilde{Q}^2 (\omega_1(\nu_\gamma, \tilde{Q}^2) \cdot F_1(\nu_\gamma, \tilde{Q}^2) + \omega_2(\nu_\gamma, \tilde{Q}^2) \cdot F_2(\nu_\gamma, \tilde{Q}^2))$$



2γ at large ε agrees with empirical fit

r_E extraction ✓

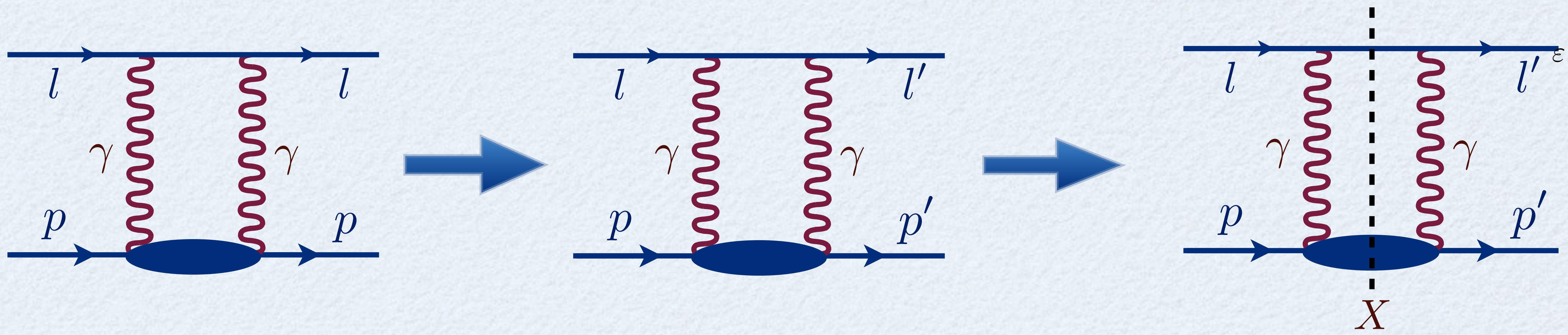
TPE: theoretical approaches

Hadronic approaches: low, intermediate Q^2

forward scattering
(atomic calculations)
structure functions

near-forward scattering
(large ε)
account for all inelastic 2γ

non-forward scattering
(arbitrary ε)
models /
dispersion relations

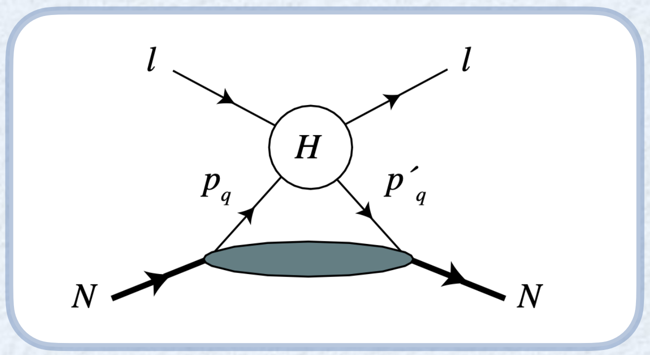


$X = p + \pi N$

Partonic approaches: large Q^2

- Handbag calculation in terms of GDPs

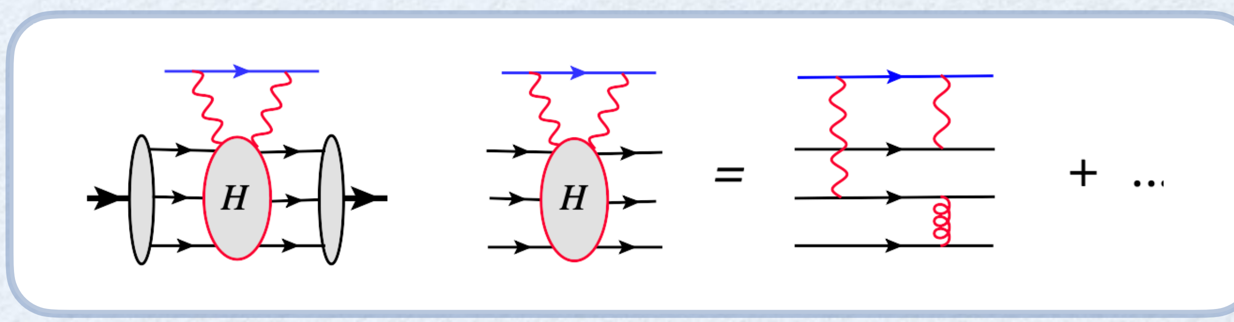
  Chen,Afanasev,Brodsky,Carlson,Vdh(2004)



- pQCD calculation

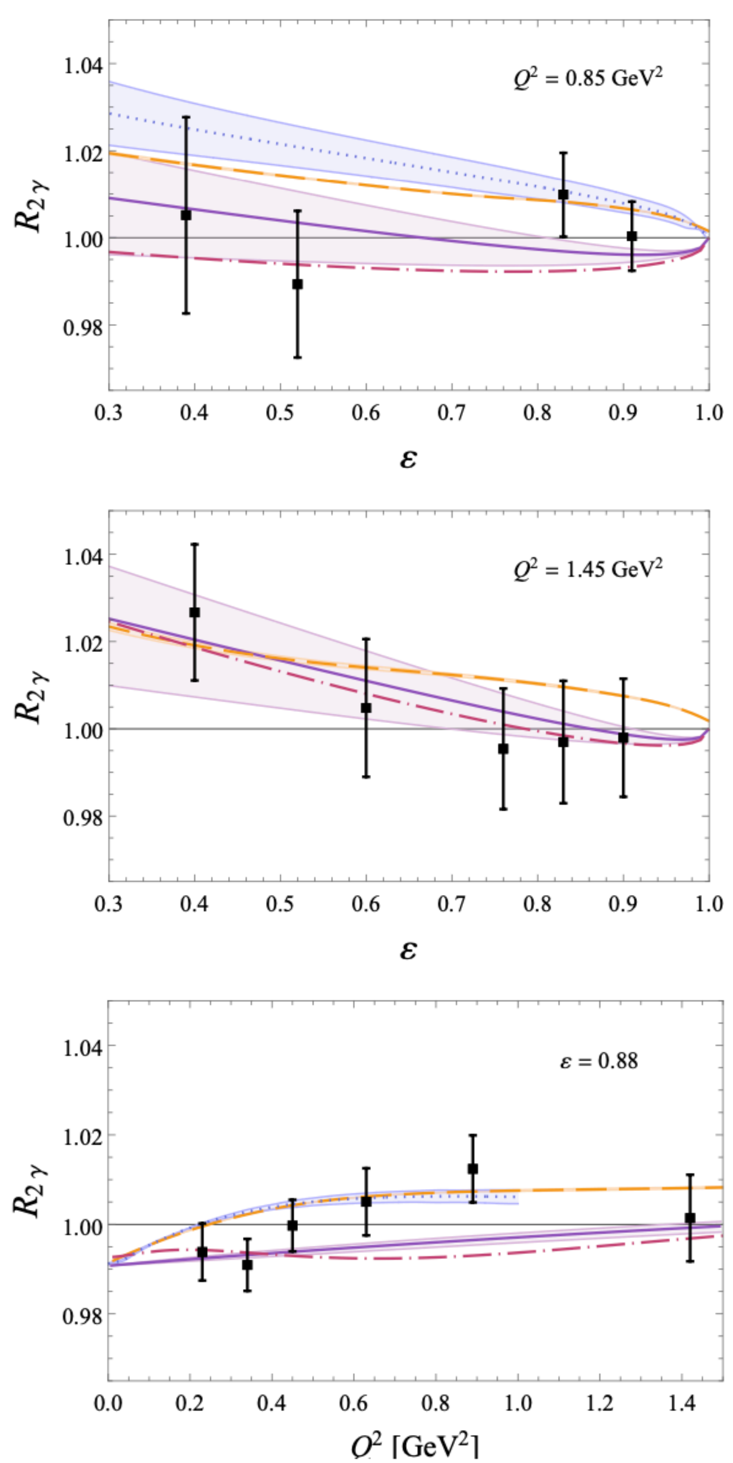
  Borisyuk,Kobushkin (2009)

  Kivel,Vdh (2009)



TPE in e+/e- proton scattering: comparison with data

JLab/CLAS data



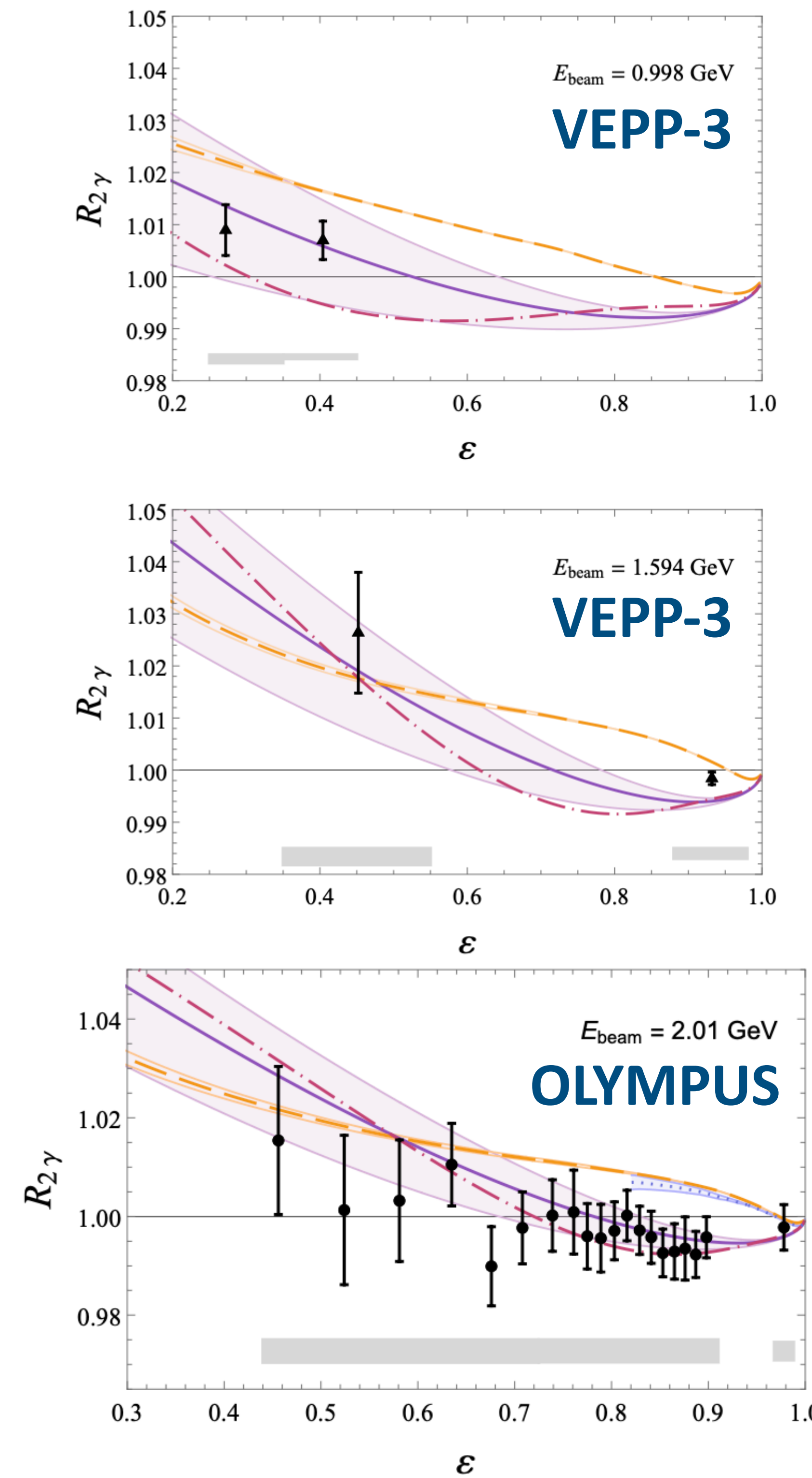
$$R_{2\gamma} = \frac{\sigma(l^+p)}{\sigma(l^-p)} \simeq 1 - 2\delta_{2\gamma}$$

- - - - - N + resonances TPE model
Ahmed et al. (2020)

······ N + πN dispersive TPE model
Tomalak et al.(2017)

——— TPE fit to σ
Bernauer et al. (2014)

— — TPE global fit
Guttman et al.(2011)



arXiv:2306.14578[hep-ph], EPJA (in press)

Observables including 2γ-exchange

$$
\begin{aligned}
\tilde{G}_M(\nu, Q^2) &= G_M(Q^2) + \delta\tilde{G}_M \\
\tilde{F}_2(\nu, Q^2) &= F_2(Q^2) + \delta\tilde{F}_2 \\
\tilde{F}_3(\nu, Q^2) &= 0 + \delta\tilde{F}_3
\end{aligned}
$$

for real part:

3 independent observables

$$
Y_{2\gamma}^M(\nu, Q^2) \equiv \mathcal{R}\left(\frac{\delta\tilde{G}_M}{G_M}\right)
$$

$$
Y_{2\gamma}^E(\nu, Q^2) \equiv \mathcal{R}\left(\frac{\delta\tilde{G}_E}{G_M}\right)
$$

$$
Y_{2\gamma}^3(\nu, Q^2) \equiv \frac{\nu}{M^2}\mathcal{R}\left(\frac{\tilde{F}_3}{G_M}\right)
$$

$$
\tilde{G}_E \equiv \tilde{G}_M - (1+\tau)\,\tilde{F}_2
$$

$$
\tilde{G}_E(\nu, Q^2) = G_E(Q^2) + \delta\tilde{G}_E
$$

$$
\begin{aligned}
\frac{\sigma_R}{G_M^2} &= 1 + \frac{\varepsilon}{\tau}\frac{G_E^2}{G_M^2} \\
&+ 2\,Y_{2\gamma}^M + 2\varepsilon\frac{G_E}{\tau G_M}\,Y_{2\gamma}^E + 2\varepsilon\left(1 + \frac{G_E}{\tau G_M}\right)Y_{2\gamma}^3 \\
&+ \mathcal{O}(e^4)
\end{aligned}
$$

$$
\begin{aligned}
-\sqrt{\frac{\tau(1+\varepsilon)}{2\varepsilon}}\frac{P_t}{P_l} &= \frac{G_E}{G_M} \\
&+ Y_{2\gamma}^E - \frac{G_E}{G_M}\,Y_{2\gamma}^M + \left(1 - \frac{2\varepsilon}{1+\varepsilon}\frac{G_E}{G_M}\right)Y_{2\gamma}^3 \\
&+ \mathcal{O}(e^4)
\end{aligned}
$$

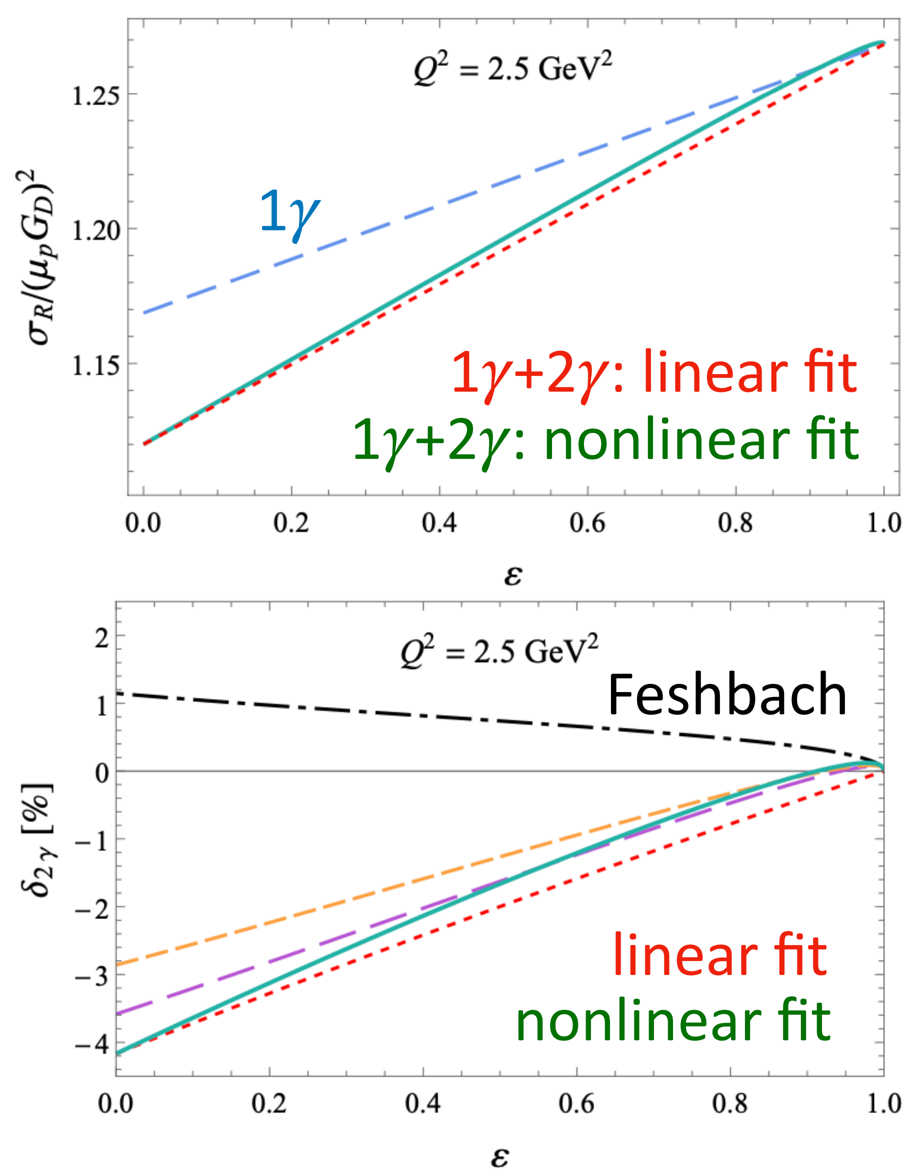
$$
\begin{aligned}
\frac{P_l}{P_l^{Born}} &= 1 \\
&-2\varepsilon\left(1 + \frac{\varepsilon}{\tau}\frac{G_E^2}{G_M^2}\right)^{-1}\left\{\left[\frac{\varepsilon}{1+\varepsilon}\left(1 - \frac{G_E^2}{\tau G_M^2}\right) + \frac{G_E}{\tau G_M}\right]Y_{2\gamma}^3 \right. \\
&\left. \qquad + \frac{G_E}{\tau G_M}\left[Y_{2\gamma}^E - \frac{G_E}{G_M}\,Y_{2\gamma}^M\right]\right\} \\
&+ \mathcal{O}(e^4)
\end{aligned}
$$

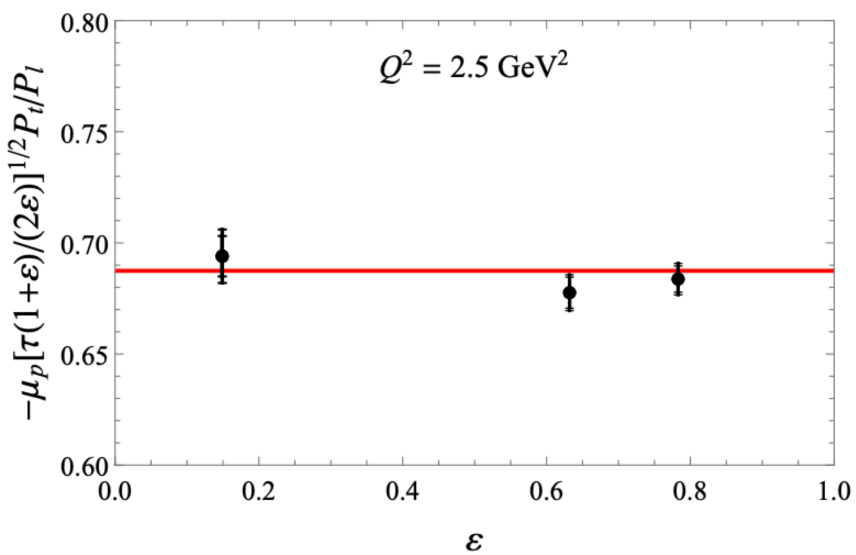
Extraction of 2γ-amplitudes from data: 3 observables

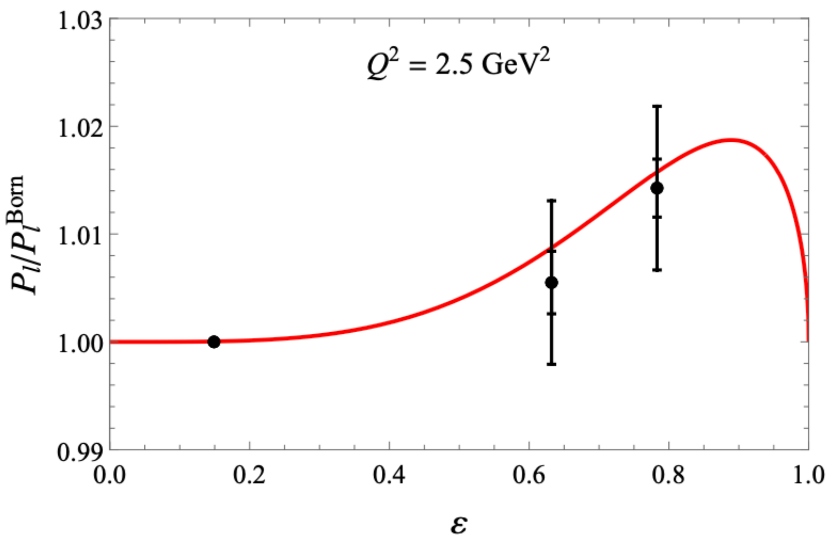
Fit to unpolarized data

$Q^2 = 2.5$ GeV^2

Polarization data: JLab (Hall C)







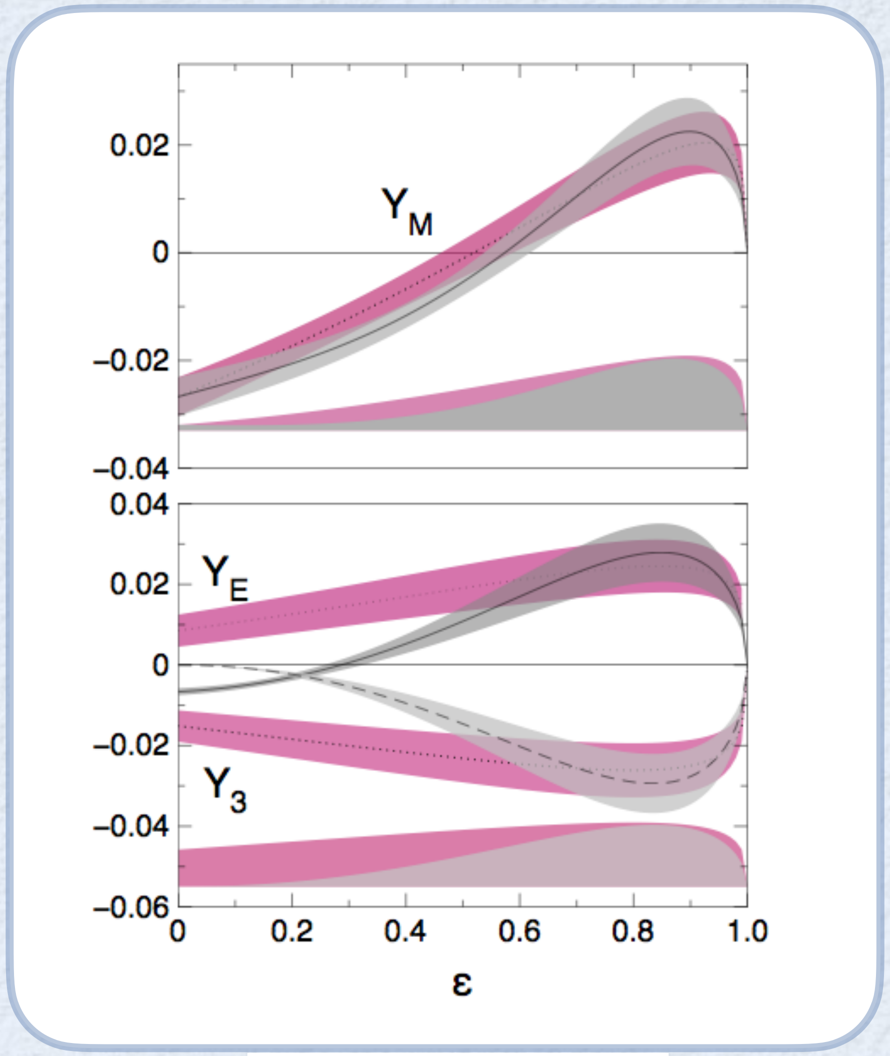
Meziane et al. (2011)

Puckett et al. (2017)

Extraction of 2γ-amplitudes from data

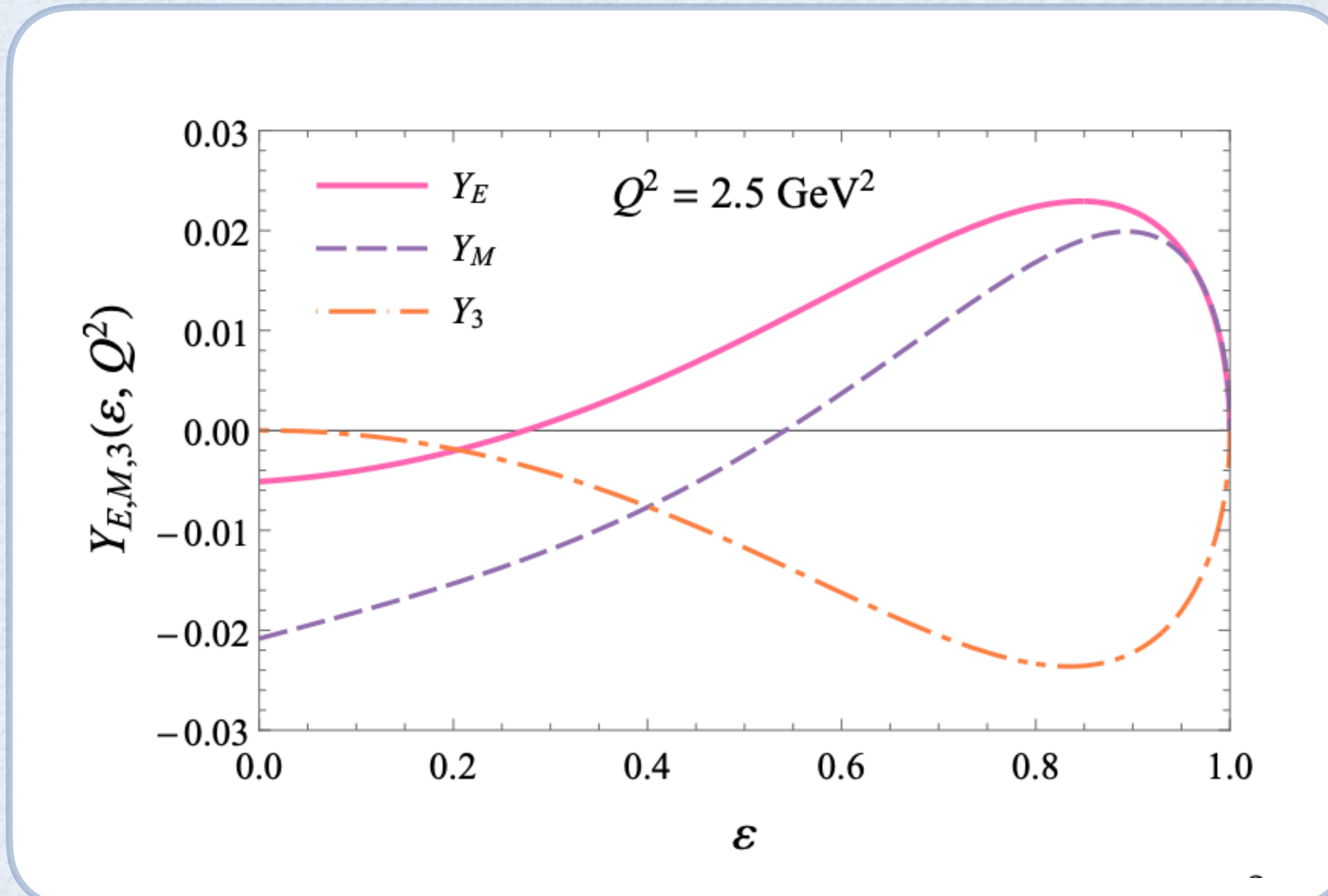
Early empirical analysis

Guttmann, Kivel, Meziane, Vdh (2011)



Recent updated analysis

arXiv:2306.14578[hep-ph], EPJA (in press)



extracted 2γ amplitudes are in the (expected) 2-3 % range for $Q^2 = 2.5$ GeV^2

μp scattering: 2γ-exchange correction

$$T^{non-flip} = \frac{e^2}{Q^2}\bar{l}(k',h')\gamma_\mu l(k,h).\bar{N}(p',\lambda')[\mathcal{G}_M(\nu,t)\gamma^\mu - \mathcal{F}_2(\nu,t)\frac{P^\mu}{M} + \mathcal{F}_3(\nu,t)\frac{\hat{K}P^\mu}{M^2}]N(p,\lambda)$$

$m_l \neq 0$ →

$$T^{flip} = \frac{e^2}{Q^2}\frac{m_l}{M}\bar{l}(k',h')l(k,h).\bar{N}(p',\lambda')[\mathcal{F}_4(\nu,t) + \mathcal{F}_5(\nu,t)\frac{\hat{K}}{M}]N(p,\lambda) + \frac{e^2}{Q^2}\frac{m_l}{M}\mathcal{F}_6(\nu,t)\bar{l}(k',h')\gamma_5 l(k,h).\bar{N}(p',\lambda')\gamma_5 N(p,\lambda)$$

Gorchtein,Guichon,Vdh (2004)   Gakh,Konchatnyi,Dbeyssi, Tomasi-Gustafsson (2014)

$$\delta_{2\gamma} = \frac{2}{G_M^2 + \frac{\epsilon}{\tau}G_E^2}\left\{G_M \Re\mathcal{G}_1 + \frac{\epsilon}{\tau}G_E\Re\mathcal{G}_2 + \frac{1-\epsilon}{1-\epsilon_0}\left(\frac{\epsilon_0}{\tau}G_E\Re\mathcal{G}_4 - G_M\Re\mathcal{G}_3\right)\right\}$$

Tomalak,Vdh (2014)

$$\mathcal{G}_1 = \mathcal{G}_M + \frac{\nu}{M^2}\mathcal{F}_3 + \frac{m_l^2}{M^2}\mathcal{F}_5$$

$$\mathcal{G}_2 = \mathcal{G}_M - (1-\tau)\mathcal{F}_2 + \frac{\nu}{M^2}\mathcal{F}_3$$

$$\mathcal{G}_3 = \frac{\nu}{M^2}\mathcal{F}_3 + \frac{m_l^2}{M^2}\mathcal{F}_5$$

$$\mathcal{G}_4 = \frac{\nu}{M^2}\mathcal{F}_4 + \frac{\nu^2}{M^4(1+\tau)}\mathcal{F}_5$$

$$\epsilon = \frac{16\nu^2 - Q^2(Q^2+4M^2)}{16\nu^2 - Q^2(Q^2+4M^2) + 2(Q^2+4M^2)(Q^2-2m_l^2)}$$

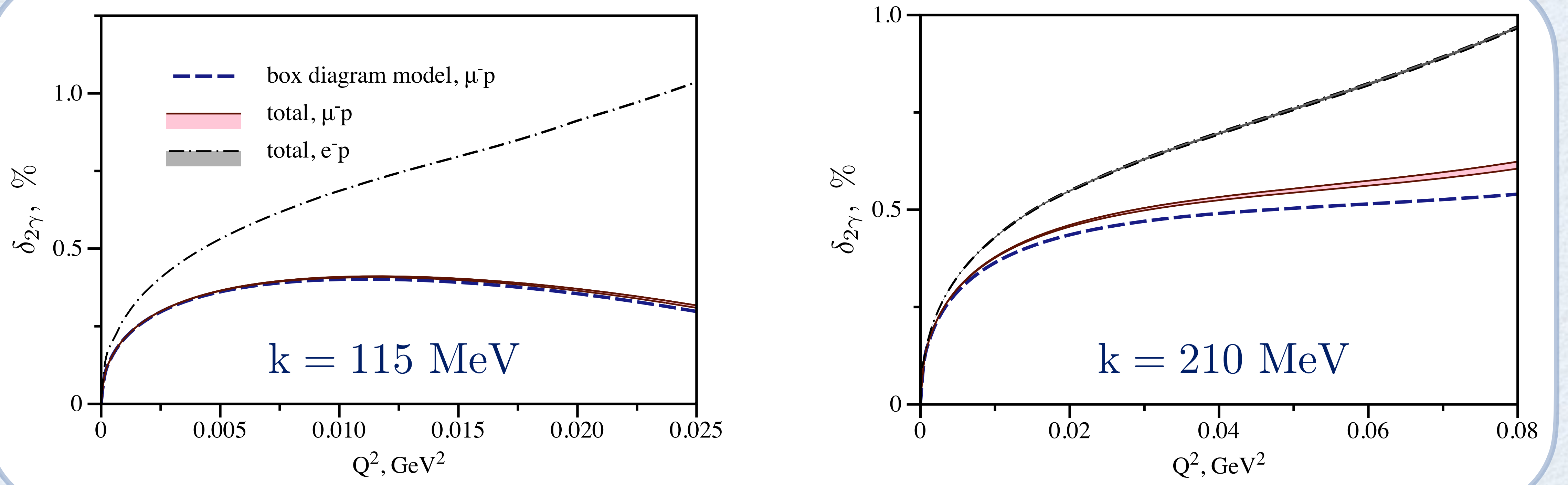
$$\epsilon_0 = \frac{2m_l^2}{Q^2}$$

For recent review, see also: arXiv:2306.14578[hep-ph], EPJA (in press)

μp estimate: MUSE kinematics

proton box diagram model

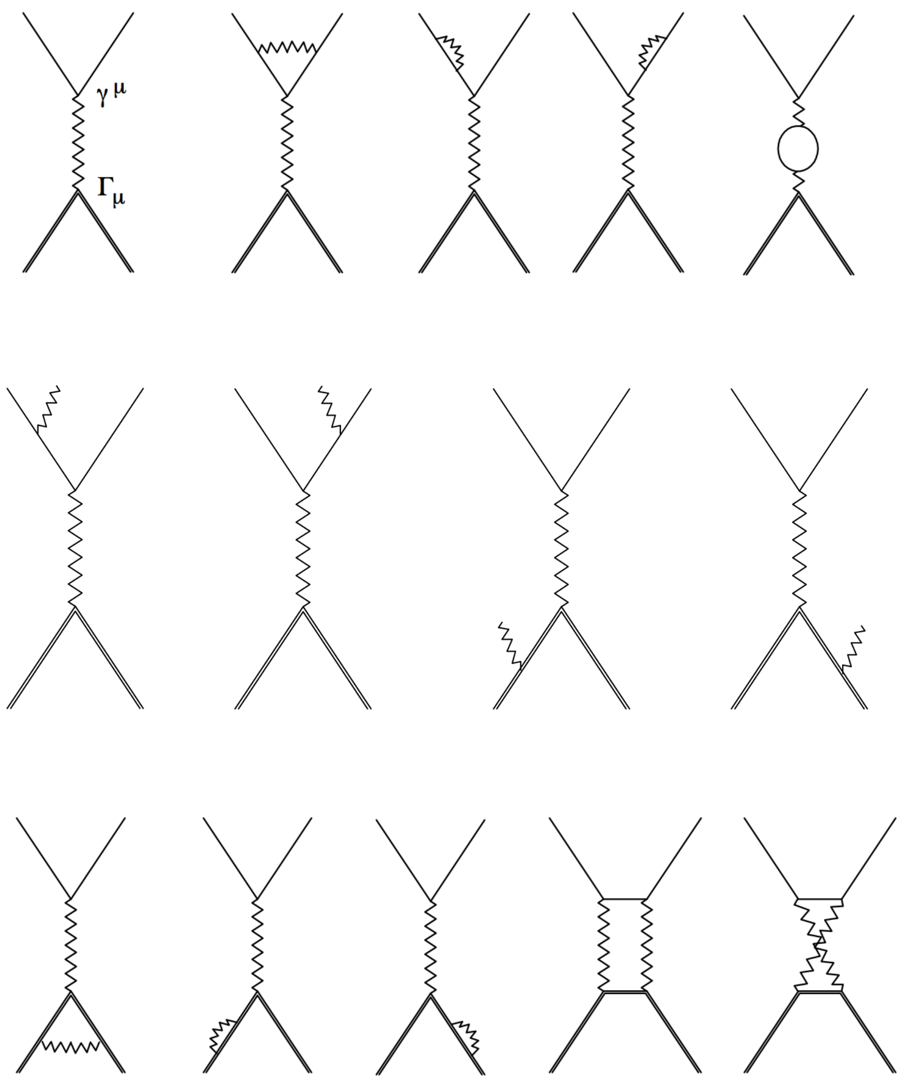
+ inelastic 2γ (near forward structure function calculation)

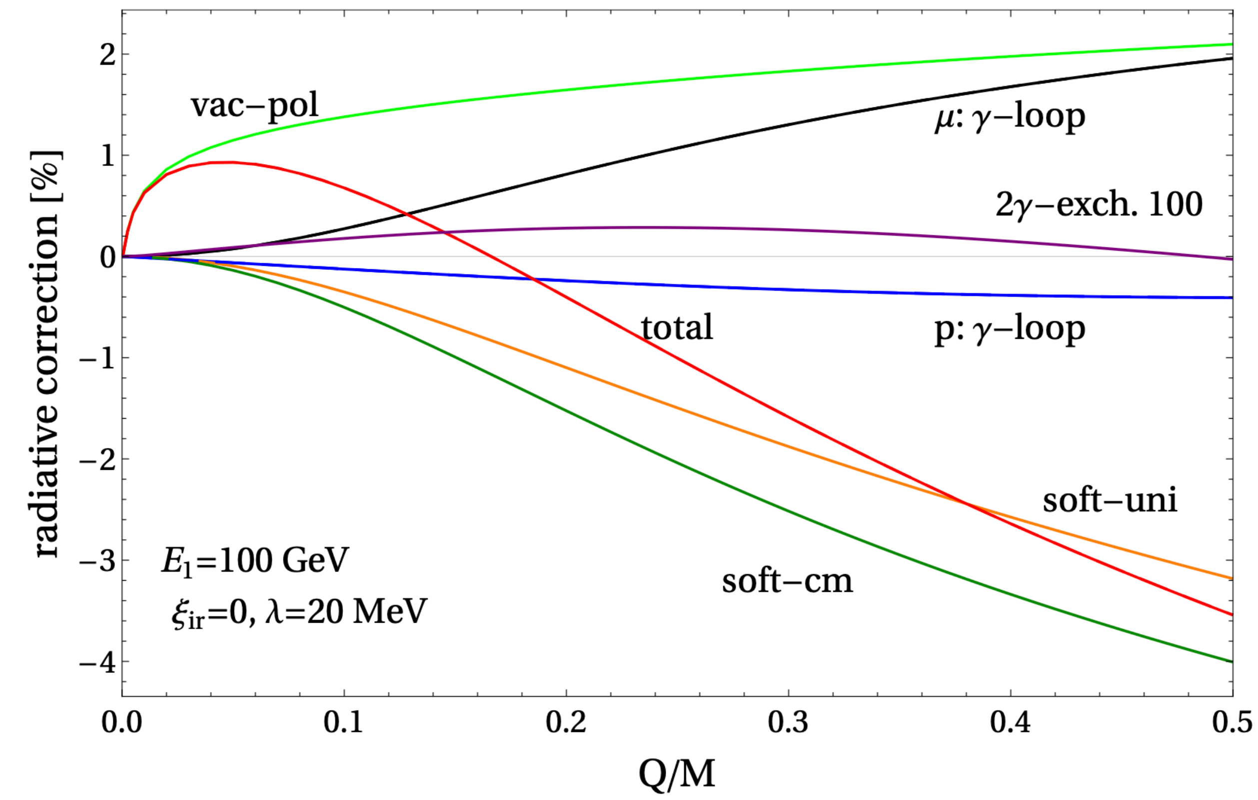


Tomalak, Vdh (2014, 2016)

In MUSE kinematics: small inelastic 2γ ⟶ small 2γ uncertainty

μp scattering: radiative corrections AMBER kinematics



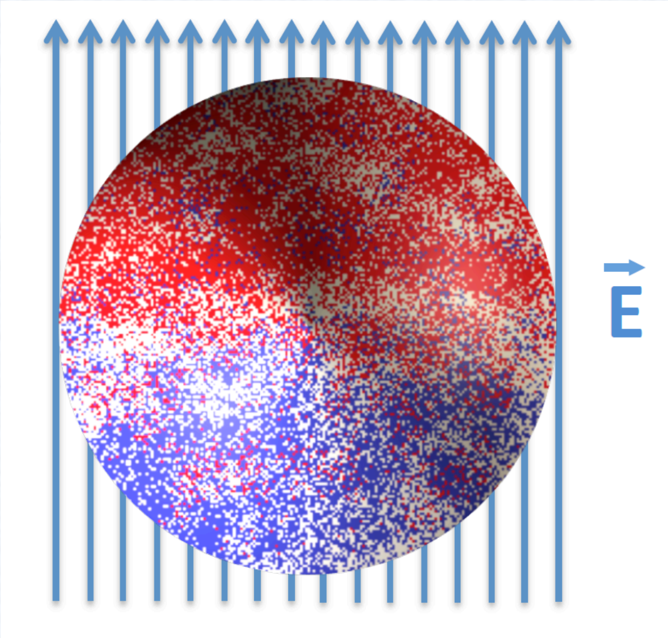


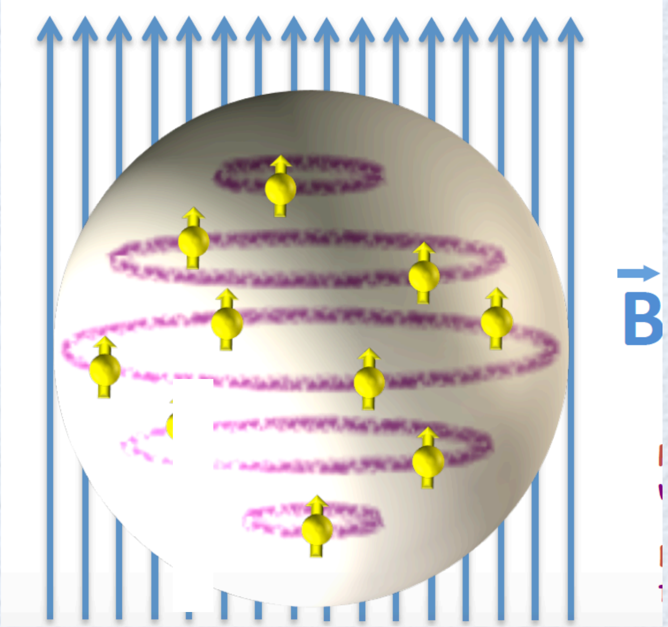
Kaiser,Lin,Meissner (2022)

For AMBER: radiative corrections dominated by vacuum polarisation (e^--loop) and soft-photon bremsstrahlung, proton structure effects negligible

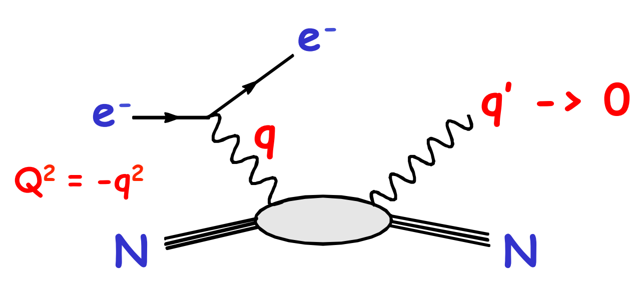
May still want to check the hard bremsstrahlung through VCS process: $\mu p \rightarrow \mu p \gamma$ for quantitatively understanding of radiative tail ⟶ McMule

Nucleon structure from Virtual Compton Scattering

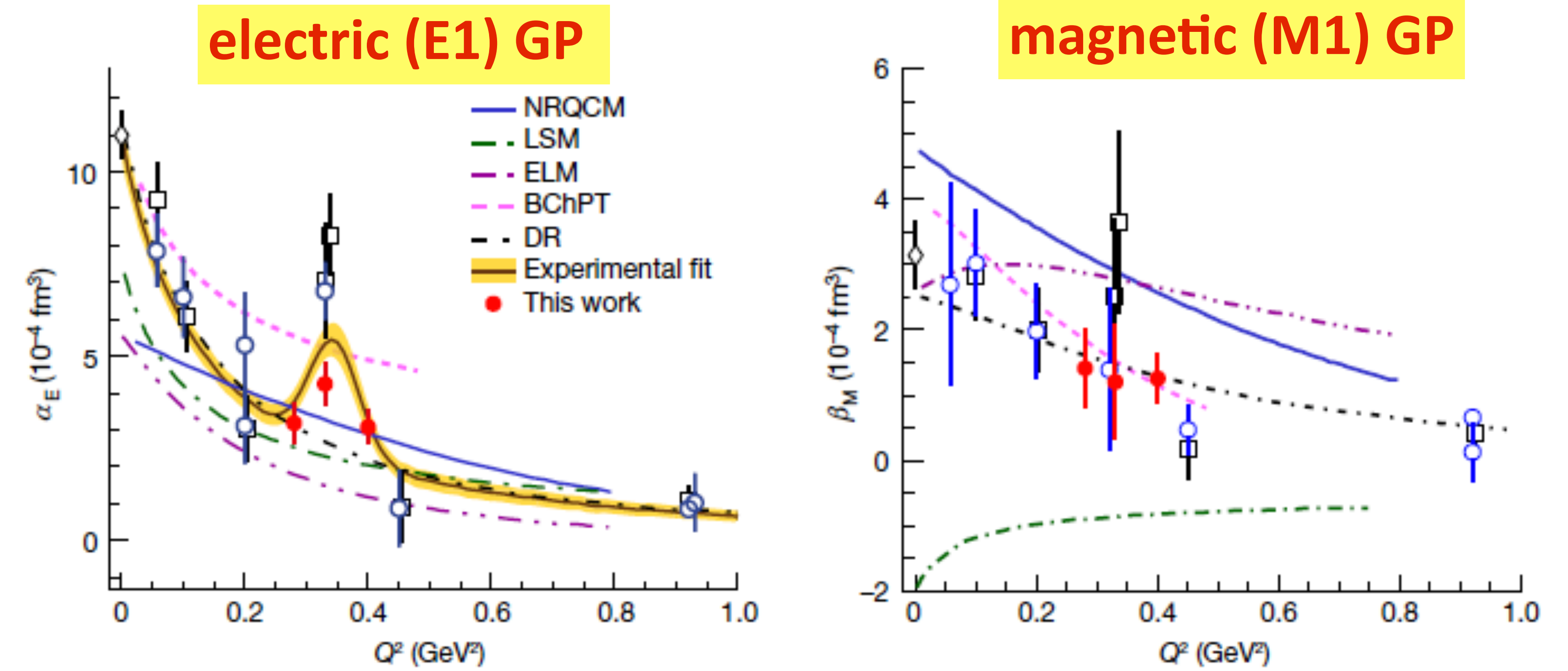




Nucleon structure at low Q with virtual photons: Generalized Polarizabilities in VCS



BATES, MAMI,
JLab data



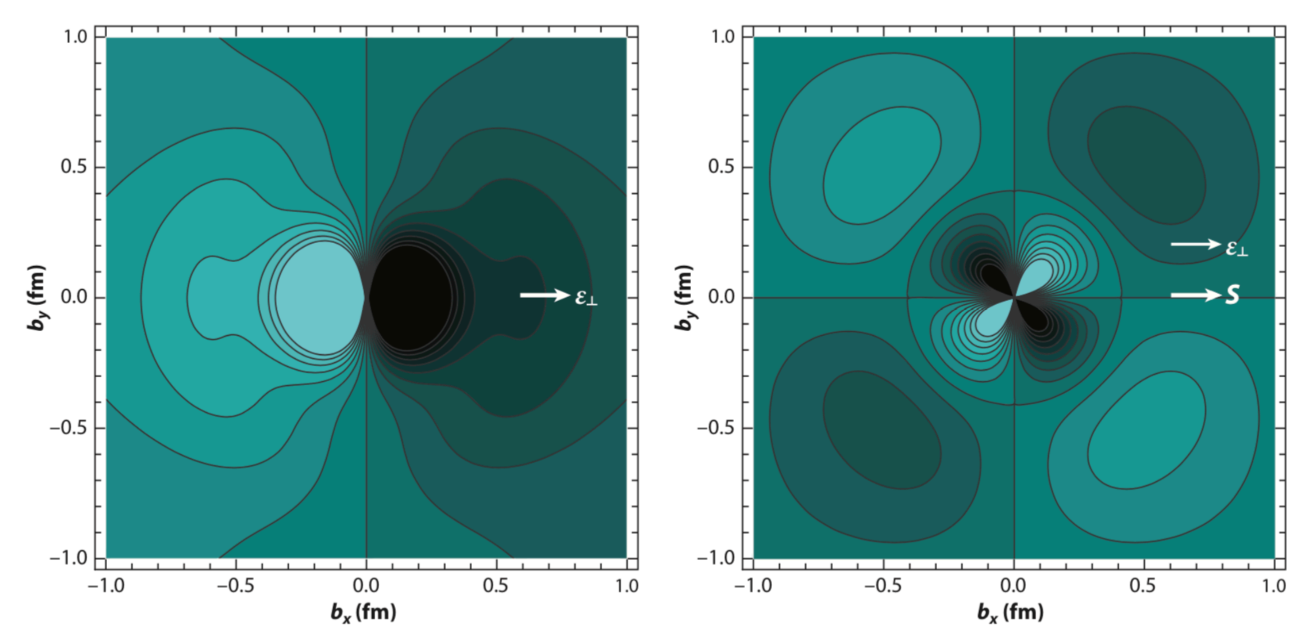
energy shift: $\delta E = -\vec{E} \cdot \vec{P}_0$

induced polarization

$$\vec{P}_0(\vec{b}) \; = \; \int \frac{d^2\vec{q}_\perp}{(2\pi)^2}\, e^{-i\vec{q}_\perp\cdot\vec{b}}\, \vec{P}_0(\vec{q}_\perp)$$

$$= \; \hat{b} \int_0^\infty \frac{dQ}{(2\pi)}\, Q\, J_1(b\,Q)\, A(Q^2)$$

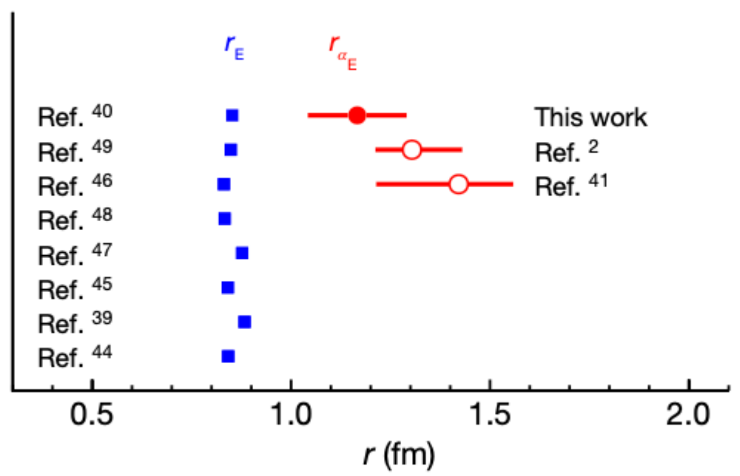
combination of nucleon
generalized polarizabilities

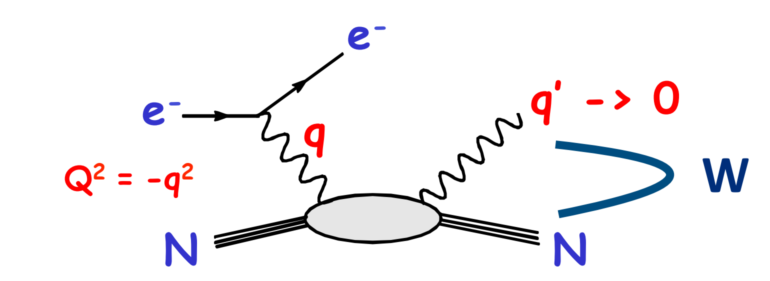


Gorchtein,Lorcé,Pasquini,Vdh(2009);
Pasquini,Vdh(2018)

Extraction of Nucleon Generalized Polarizabilities (GP) using VCS in Δ(1232) resonance region

Electric polarisability radius





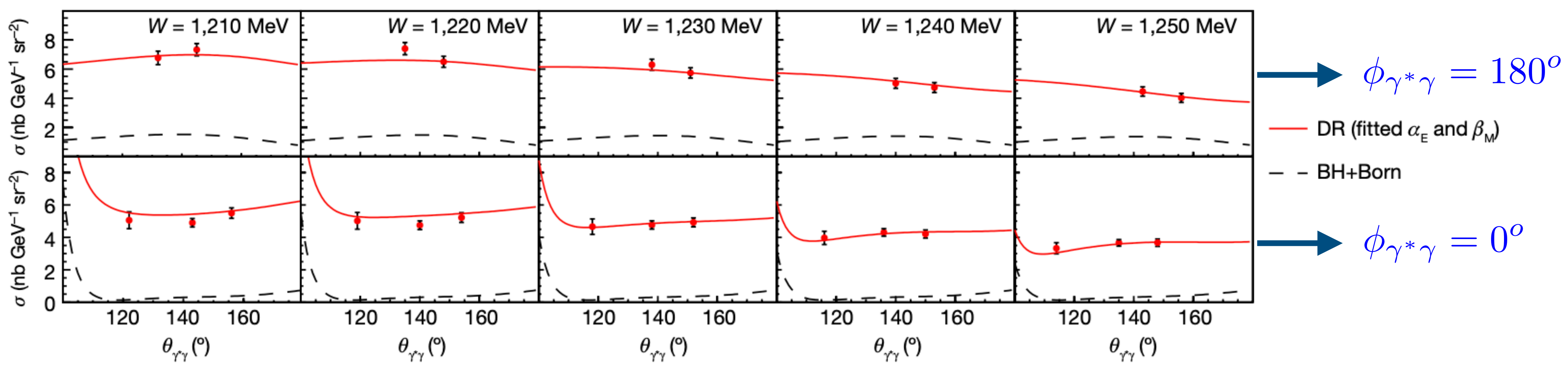
Extraction of polarizabilities using dispersion relations

Pasquini,Drechsel,Gorchtein,Metz,Vdh(2000)

$Q^2 = 0.33$ GeV^2

JLab/Hall C data

Li,Sparveris,Atac,Jones,Paolone,et al. Nature 611,265(2022)



Worthwhile to check sensitivity of AMBER to GP at very small Q^2 -> radius

Near future perspectives at low Q

- **hadronic corrections** to Lamb shift in **muonic atoms:** shift from puzzle to **precision !**
  - **µH LS**: CREMA coll.: factor 5 improvement planned
  - **µH 1S HFS**: next frontier 1ppm precision !
- **muon scattering plans:**
  - MUSE@PSI
  - AMBER@CERN
- **electron/positron scattering plans:**
  - PRad-II@JLab
  - ULQ^2@Tohoku
  - MAGIX@MESA
  - JLab, e^+ @JLab
- **VCS at low energies: polarisability radius** -> worthwhile to check feasibility at AMBER
- **Close synergy experiment <-> theory to move field forward**