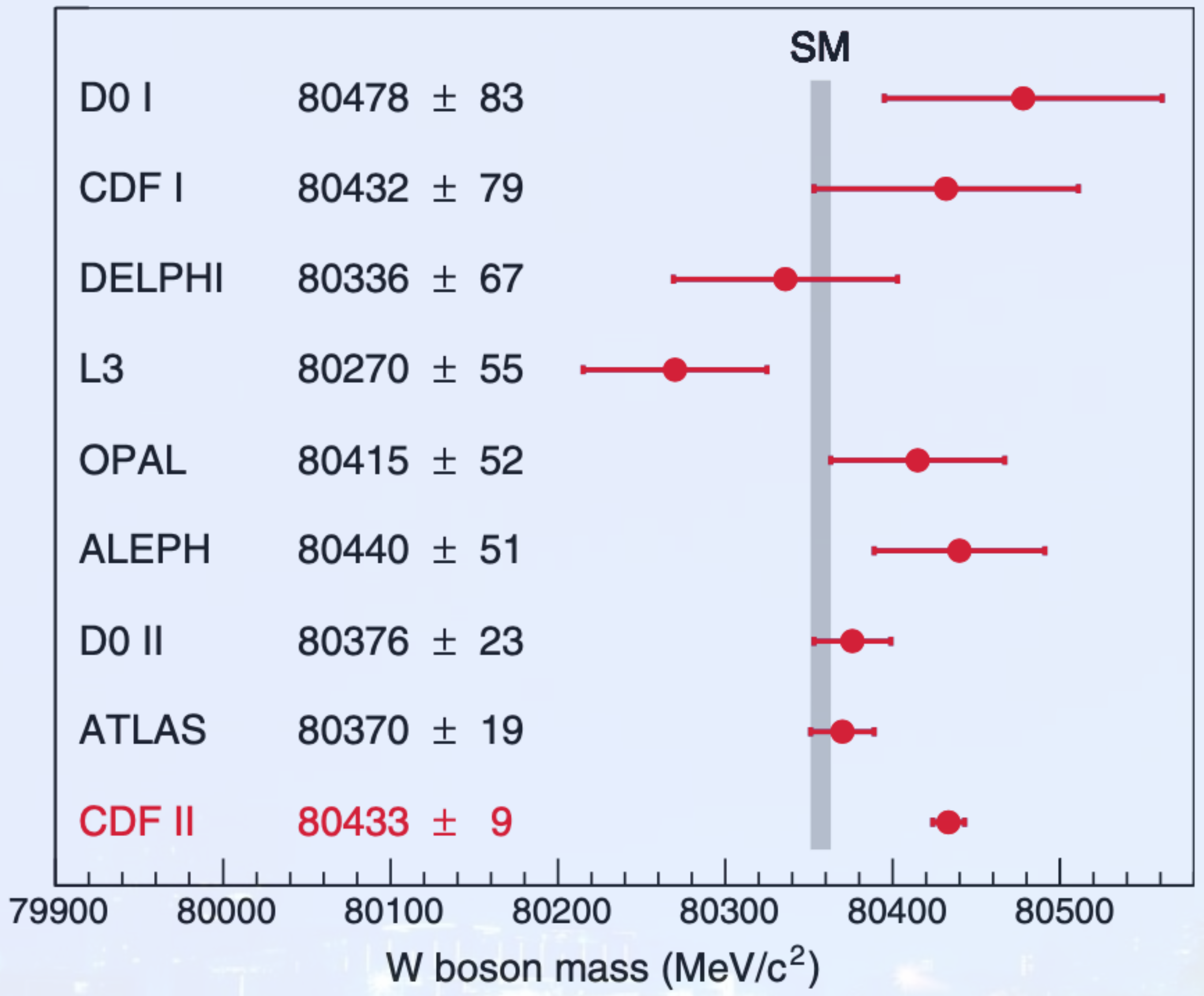
FIXED-ORDER PREDICTIONS FOR DRELL-YAN PRODUCTION

Xuan Chen

Discussion of theoretical systematics in LHC precision measurements

CERN, 26 February, 2024

W MASS IN CDFII MEASUREMENT



- PDG world average: $m_W = 80379 \pm 12\ MeV$ (PDG`20)
- CDFII latest result: $m_W = 80433 \pm 9\ MeV$ (CDF `22)
  - Indirect measurement of m_T^W, p_T^l, p_T^ν distributions

$$p_T^{l(\nu)} = \sqrt{(p_x^{l(\nu)})^2 + (p_y^{l(\nu)})^2}$$

$$E_T^{l(\nu)} = \sqrt{m^2 + (p_x^{l(\nu)})^2 + (p_y^{l(\nu)})^2} \approx p_T^{l(\nu)}$$

$$m_T^W = \sqrt{2E_T^l E_T^\nu (1 - \cos\Delta\phi)}$$

  - Template fit to the best parameter values
  - Full error = Experiment + Theory model
  - Experiment statistics: $\pm 6.4\ MeV$
  - Experiment systematic: $\pm 5.3\ MeV$
  - Theory model: $\pm 5.2\ MeV$ $\pm ??\ MeV$

ResBos, DYqT, PHOTOS, HORACE

Fixed-order predictions for Drell-Yan production

Illustrator: Gaia Fontana

Table 1. Individual fit results and uncertainties for the M_W measurements. The fit ranges are 65 to 90 GeV for the m_T fit and 32 to 48 GeV for the p_T^ℓ and p_T^ν fits. The χ^2 of the fit is computed from the expected statistical uncertainties on the data points. The bottom row shows the combination of the six fit results by means of the best linear unbiased estimator (66).

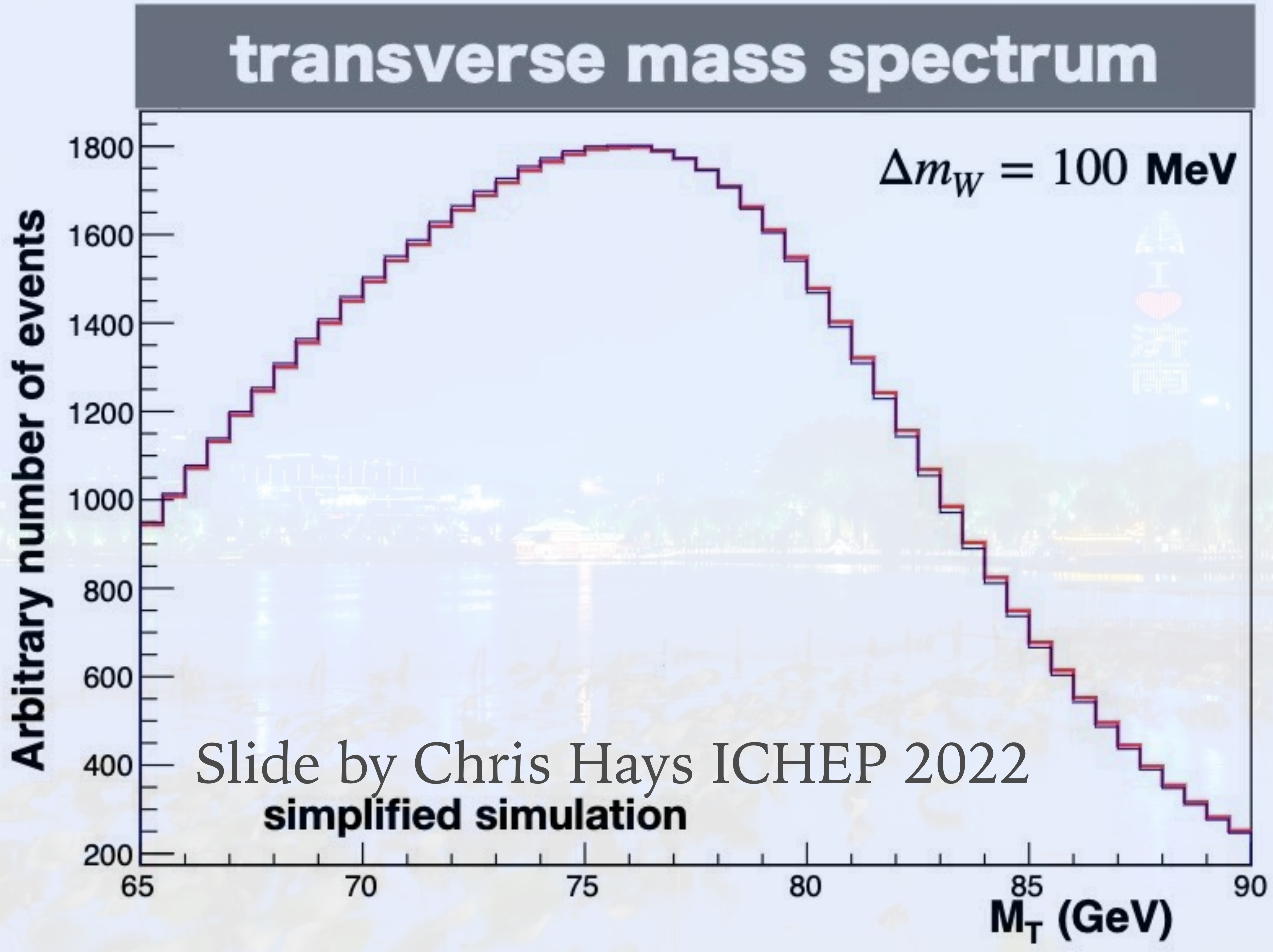
Distribution	W boson mass (MeV)	χ^2/dof
$m_T(e, \nu)$	$80{,}429.1 \pm 10.3_{stat} \pm 8.5_{syst}$	39/48
$p_T^\ell(e)$	$80{,}411.4 \pm 10.7_{stat} \pm 11.8_{syst}$	83/62
$p_T^\nu(e)$	$80{,}426.3 \pm 14.5_{stat} \pm 11.7_{syst}$	69/62
$m_T(\mu, \nu)$	$80{,}446.1 \pm 9.2_{stat} \pm 7.3_{syst}$	50/48
$p_T^\ell(\mu)$	$80{,}428.2 \pm 9.6_{stat} \pm 10.3_{syst}$	82/62
$p_T^\nu(\mu)$	$80{,}428.9 \pm 13.1_{stat} \pm 10.9_{syst}$	63/62
Combination	$80{,}433.5 \pm 6.4_{stat} \pm 6.9_{syst}$	7.4/5

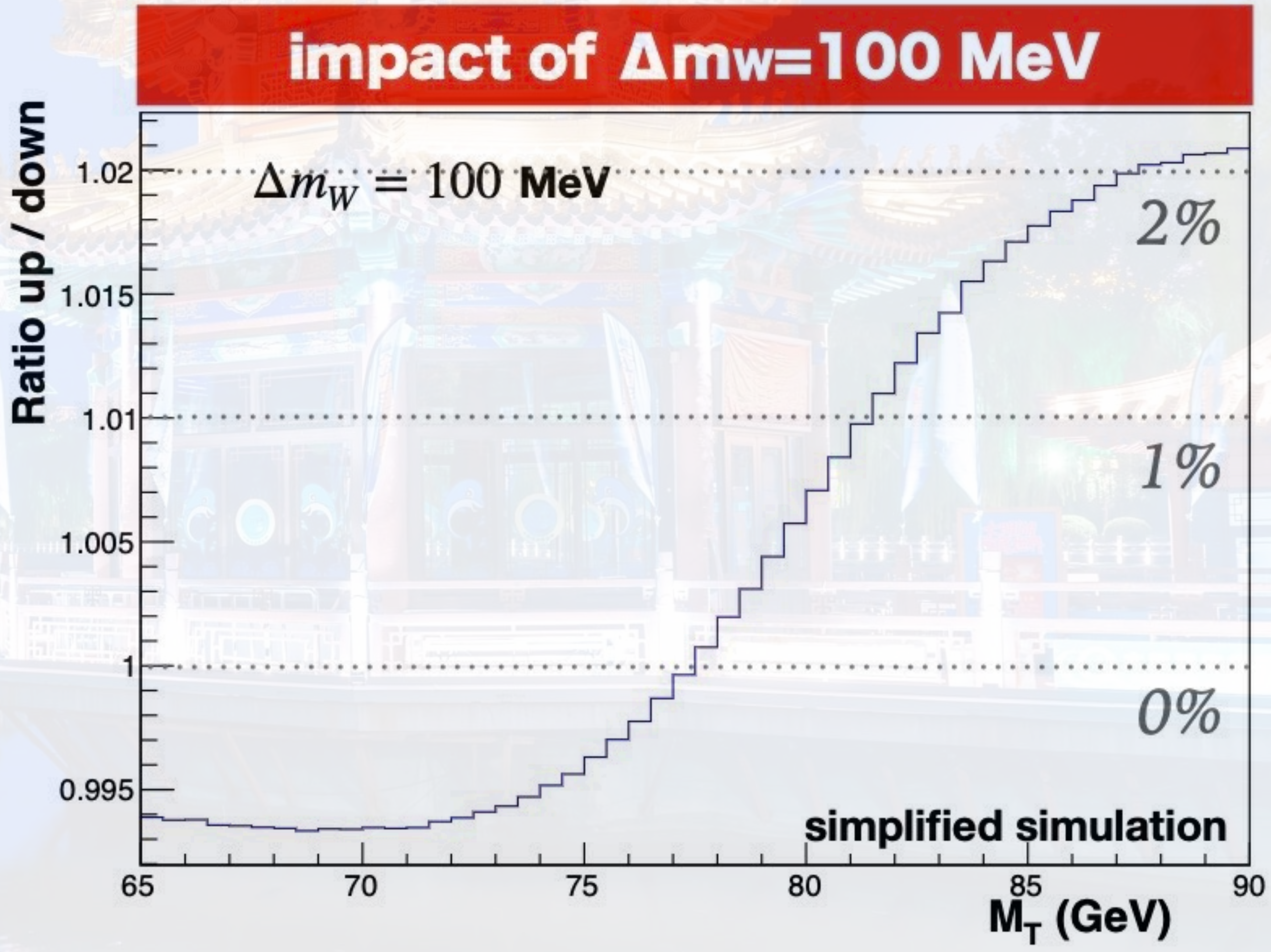
Table 2. Uncertainties on the combined M_W result.

Source	Uncertainty (MeV)
Lepton energy scale	3.0
Lepton energy resolution	1.2
Recoil energy scale	1.2
Recoil energy resolution	1.8
Lepton efficiency	0.4
Lepton removal	1.2
Backgrounds	3.3
p_T^Z model	1.8
p_T^W/p_T^Z model	1.3
Parton distributions	3.9
QED radiation	2.7
W boson statistics	6.4
Total	9.4

W MASS IN CDFII MEASUREMENT

➤$d\sigma/dm_T^W$ two templates with $\Delta m_W = 100$ MeV

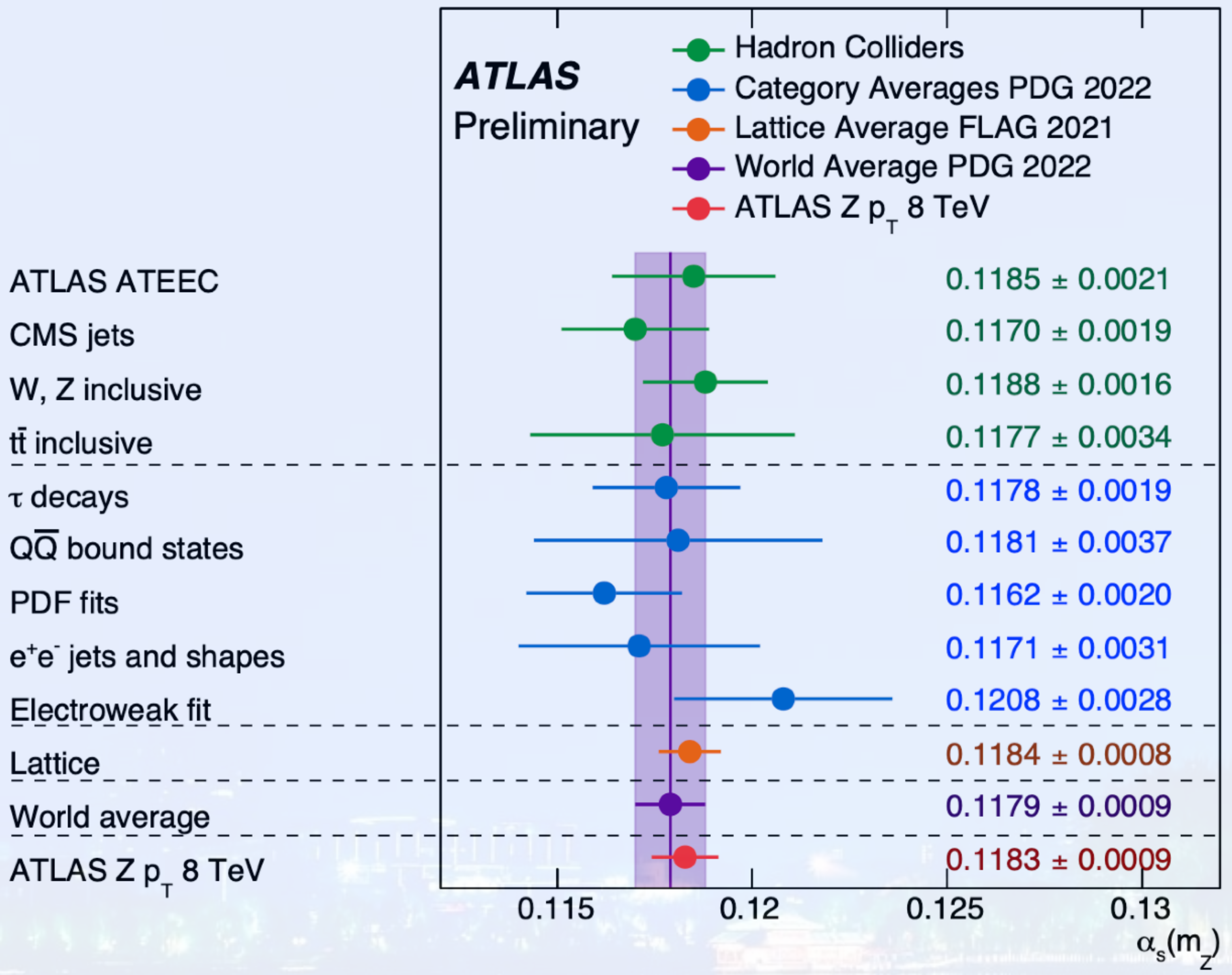


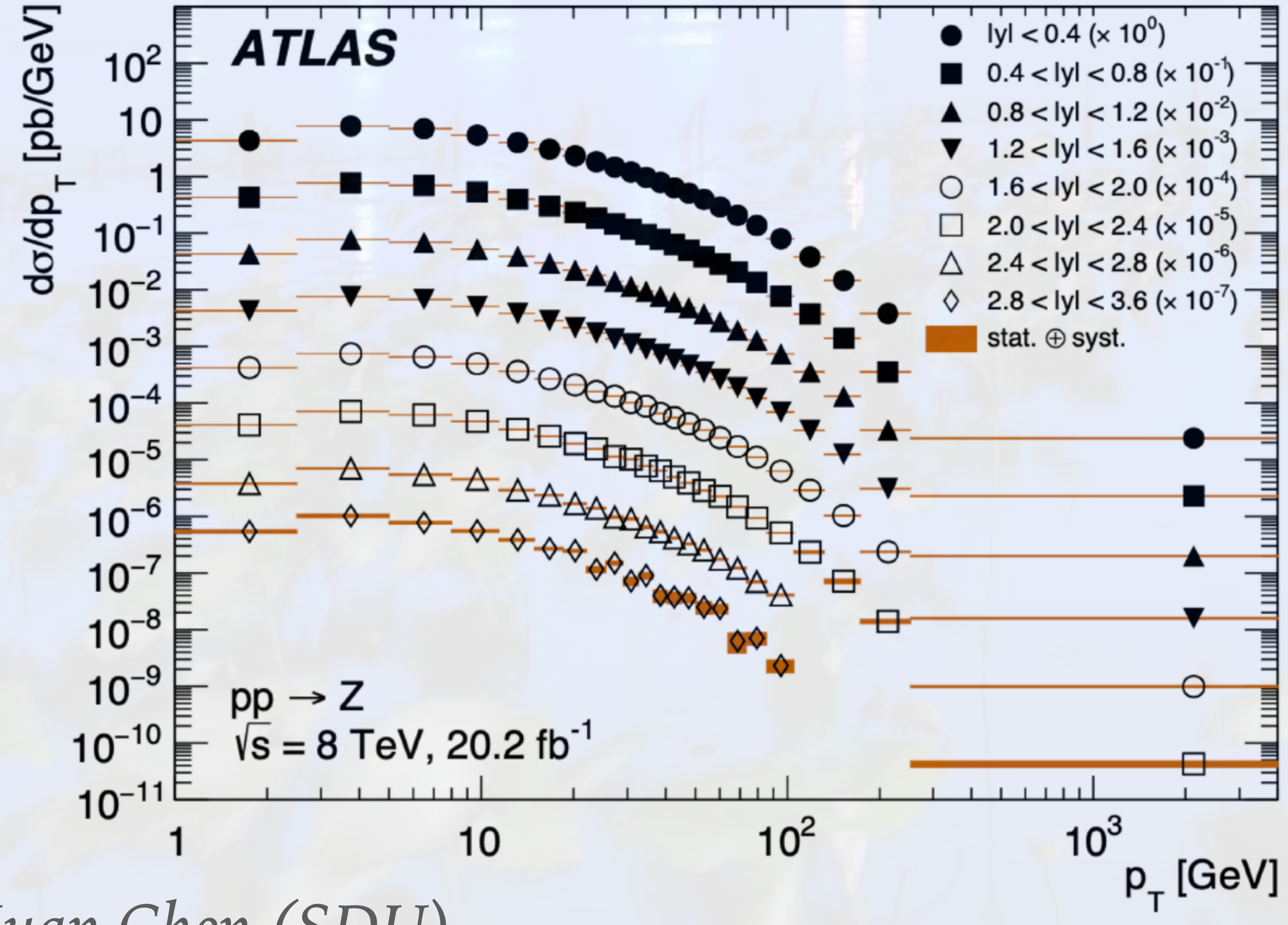


$\Delta m_W =$ 100 MeV ~ 0.5-2% change in $d\sigma/dm_T^W$ → $\Delta m_W =$ 10 MeV ~ 0.1% precision in $d\sigma/dm_T^W$

➤Binned maximum-likelihood fit among templates+ weighted average among observables

α_s MEASUREMENT BY ATLAS





- World average: $\alpha_s(m_Z) = 0.1179 \pm 0.0009$ *ATLAS 2309.12986*
- ATLAS p_T^Z 8 TeV: $\alpha_s(m_Z) = 0.1183 \pm 0.0009$ *See also ATLAS JHEP 07 (2023) 085*
  - Indirect measurement of $d\sigma/dp_T^Z/dy^Z$ distributions *ATLAS 2309.09318*
    - $80 < m_{ee(\mu\mu)} < 100$ GeV
    - $p_T^Z < 29$ GeV in 8 slices of $|y^Z| < 3.6$
    - $|\eta_{e_1}| < 2.4$, $2.5 < |\eta_{e_2}| < 4.9$ with $p_T^{e_1(e_2)} > 25$ (20) GeV
    - $|\eta_{e(\mu)}| < 2.4$ with $p_T^{e(\mu)} > 20$ GeV

8 TeV

20.2 fb^{-1}

- DYTurbo with xFitter to find the best α_s that describe the data
- Experiment unc. : ± 0.00044
- Theory model unc. : $^{+0.00072}_{-0.00076}$ ±??

Error budget of $\alpha_s(m_Z)$

Experimental uncertainty	+0.00044	-0.00044
PDF uncertainty	+0.00051	-0.00051
Scale variations uncertainties	+0.00042	-0.00042
Matching to fixed order	0	-0.00008
Non-perturbative model	+0.00012	-0.00020
Flavour model	+0.00021	-0.00029
QED ISR	+0.00014	-0.00014
N4LL approximation	+0.00004	-0.00004
Total	+0.00084	-0.00088

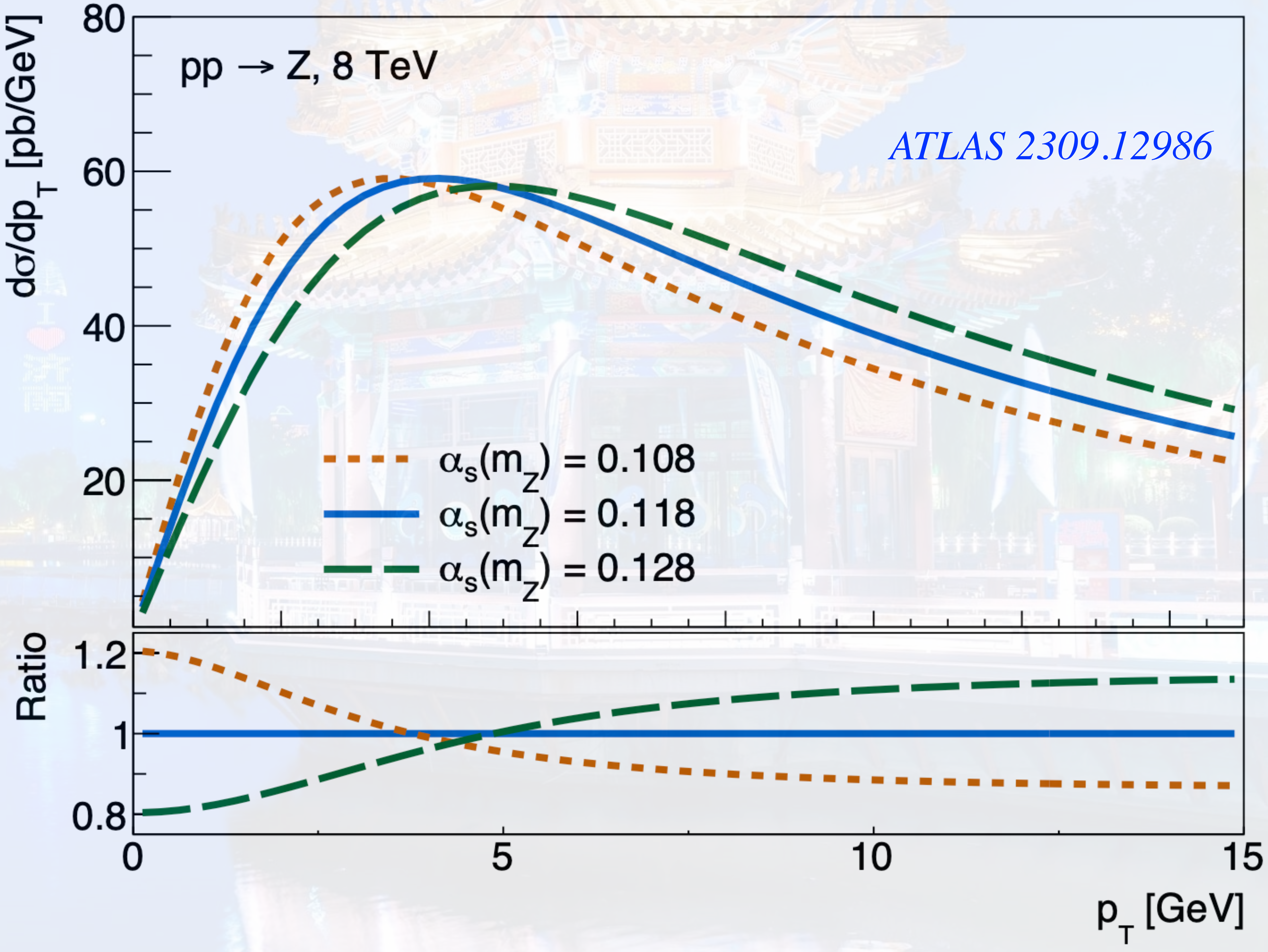
α_s MEASUREMENT BY ATLAS

- χ^2 fit in xFitter framework:

$$\chi^2(\beta_{\rm exp},\beta_{\rm th}) = \sum_{i=1}^{N_{\rm data}} \frac{\left(\sigma_i^{\rm exp} + \sum_j \Gamma_{ij}^{\rm exp}\beta_{j,\rm exp} - \sigma_i^{\rm th} - \sum_k \Gamma_{ik}^{\rm th}\beta_{k,\rm th}\right)^2}{\Delta_i^2} + \sum_j \beta_{j,\rm exp}^2 + \sum_k \beta_{k,\rm th}^2 \,.$$

Eur. Phys. J. C ***75*** *(2015) 304*

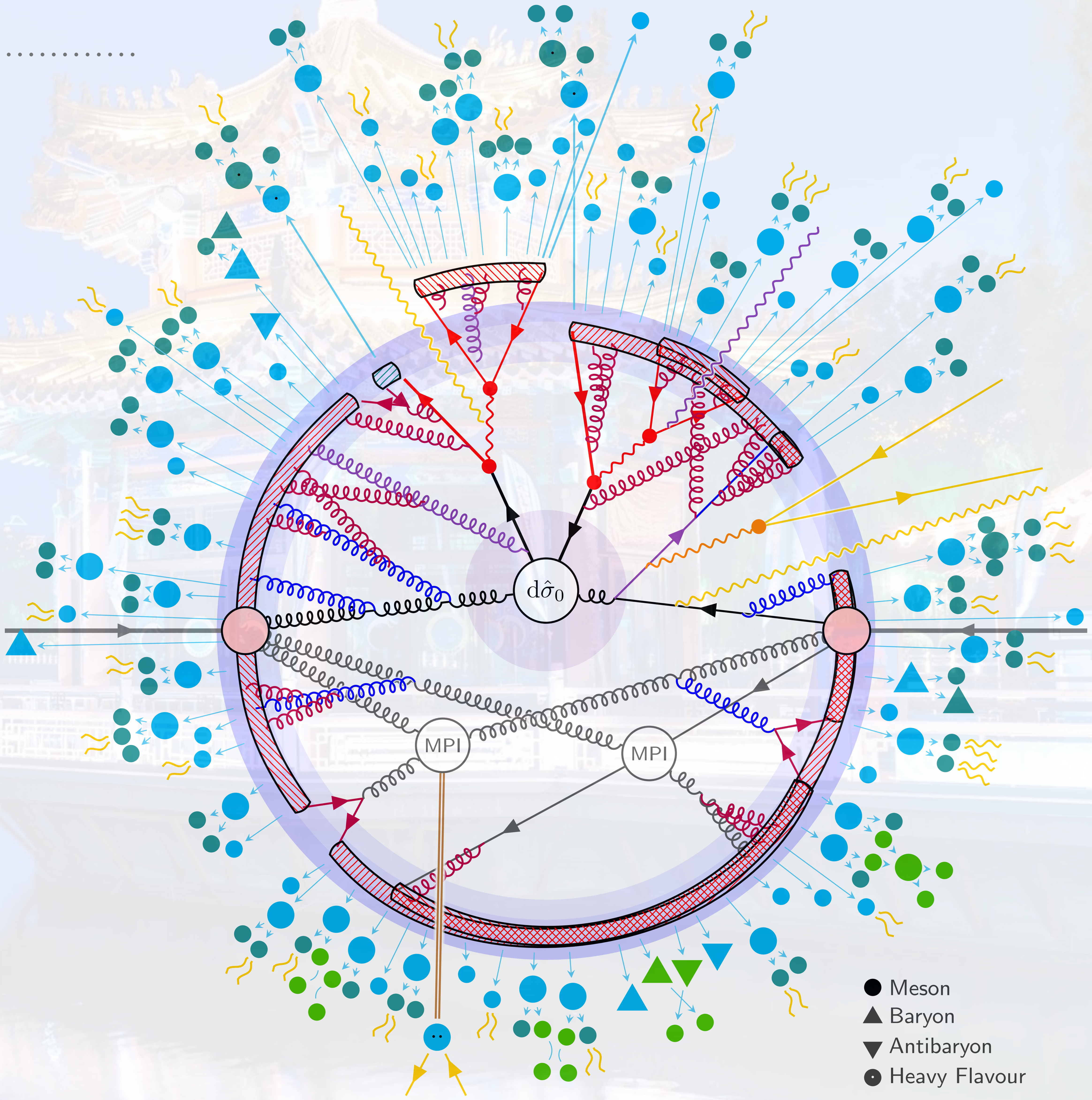
- Δ_i experimental uncertainties
- $\beta_{exp\ (th)}$ nuisance parameters
- Γ_{ik}^{th} covariant matrix covers:
  - PDF Hessian uncertainties
  - Non-perturbative form factor



$\Delta\alpha_s =$ 0.01~ 10-20% change in $d\sigma/dp_T^Z$ → $\Delta\alpha_s =$ 0.001 ~ 1-2% precision in $d\sigma/dp_T^Z$

Collider Event in Theorist's Eye

➤ For a scale of 100 GeV, the idea of factorisation in Quantum Field Theory plays important role to help understanding complex high energy processes:



PYTHIA 8.3

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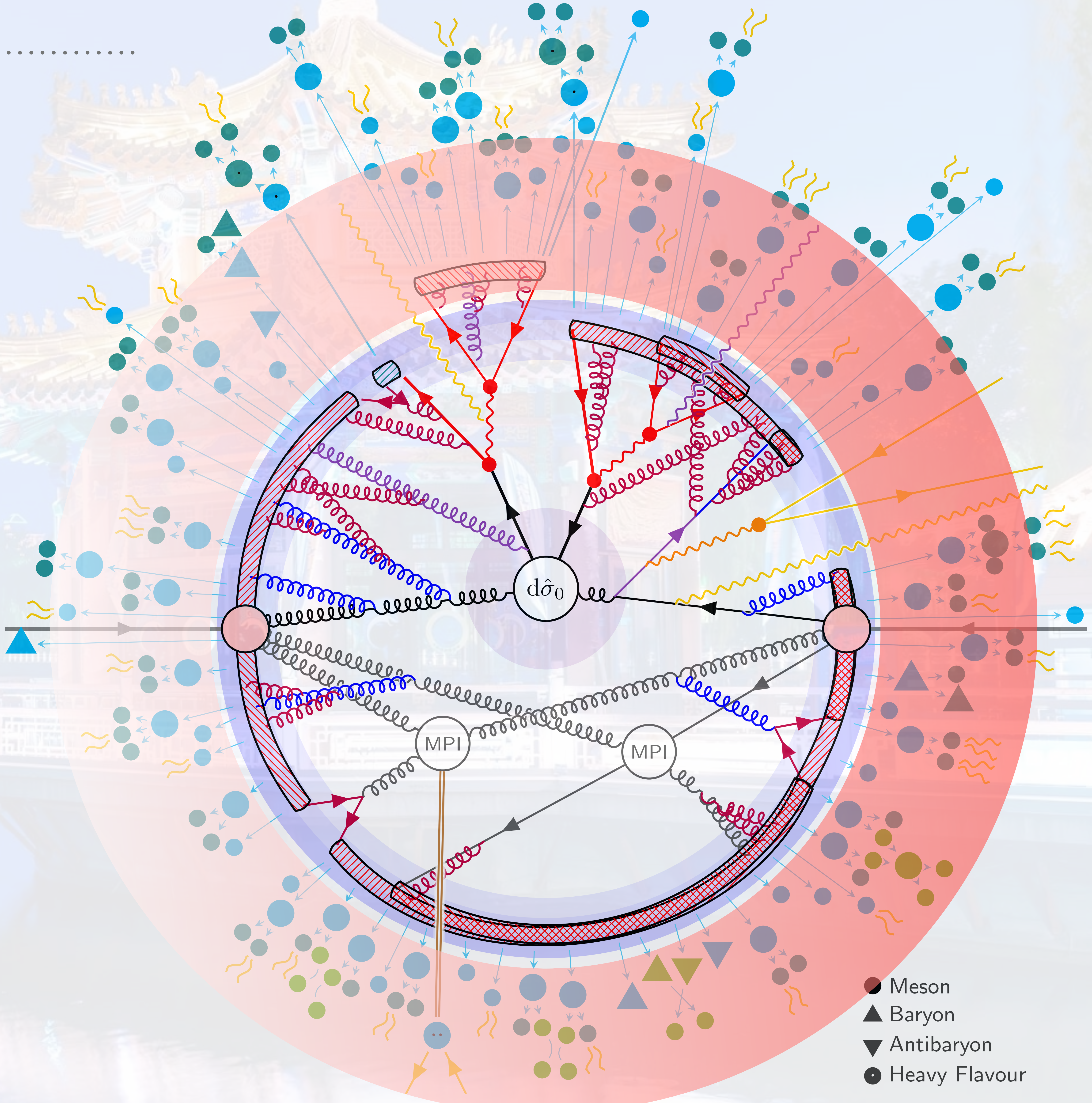
Time ordering

Hadronisation



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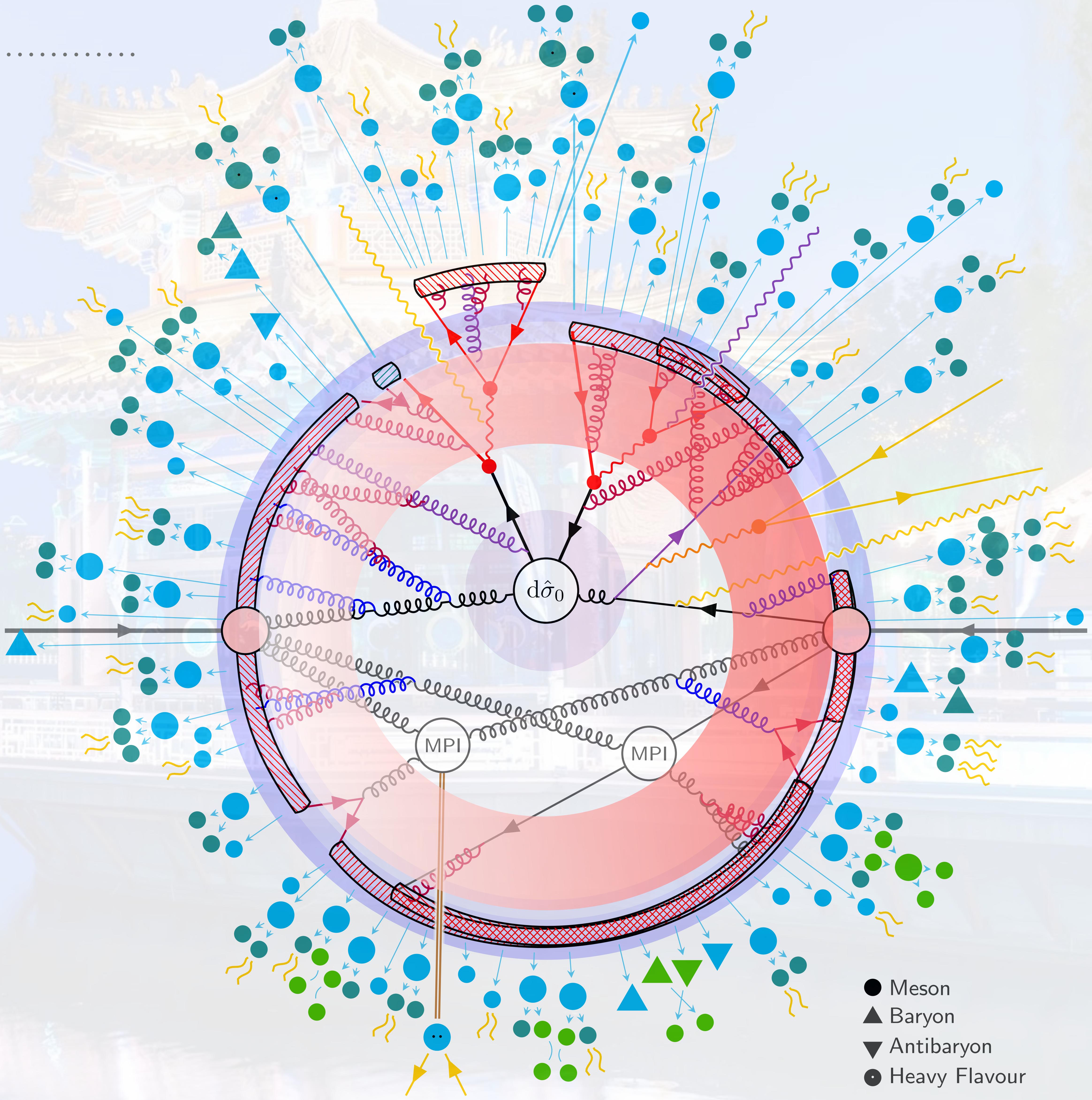
Parton Shower



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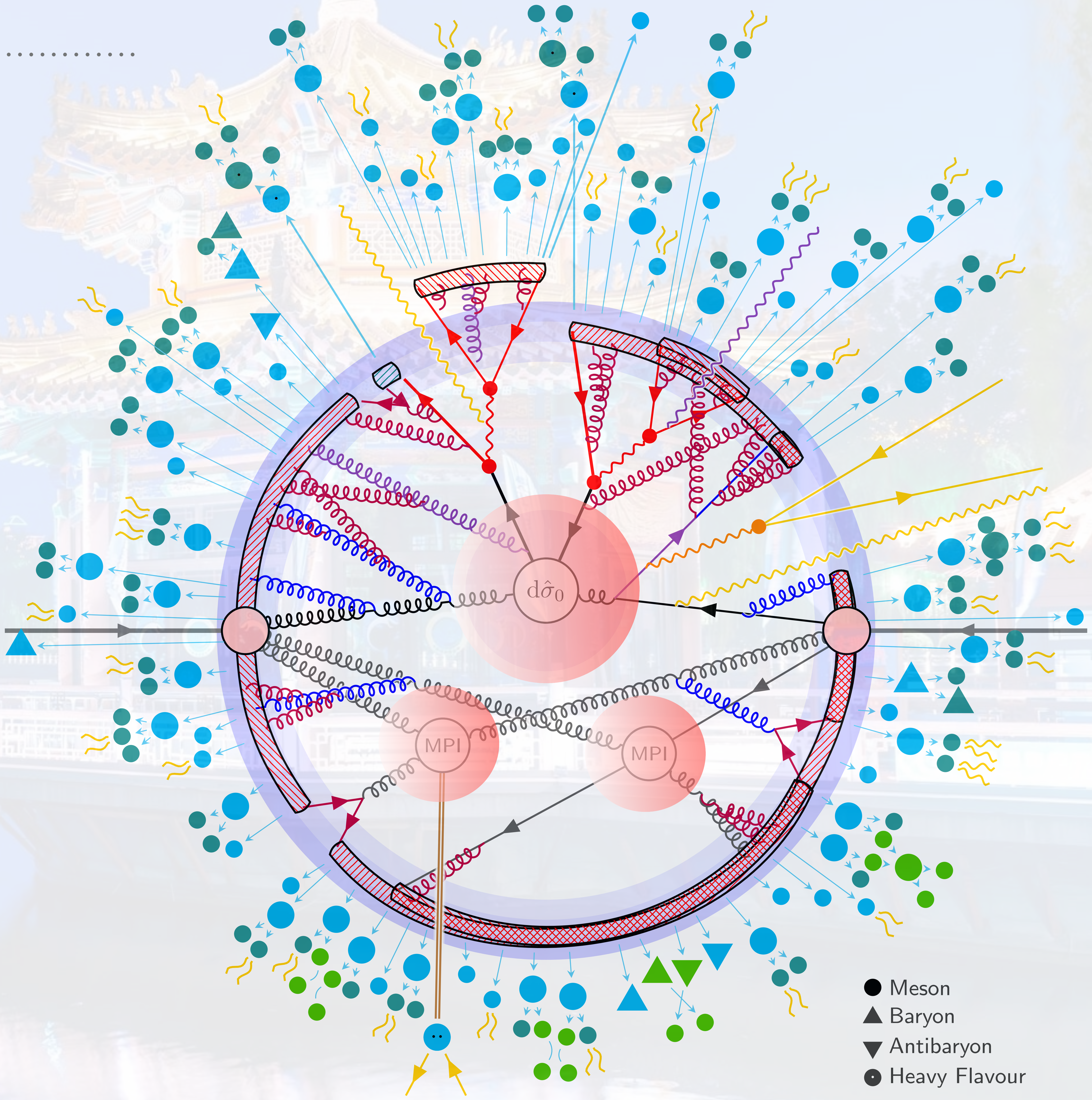
Hard Scattering



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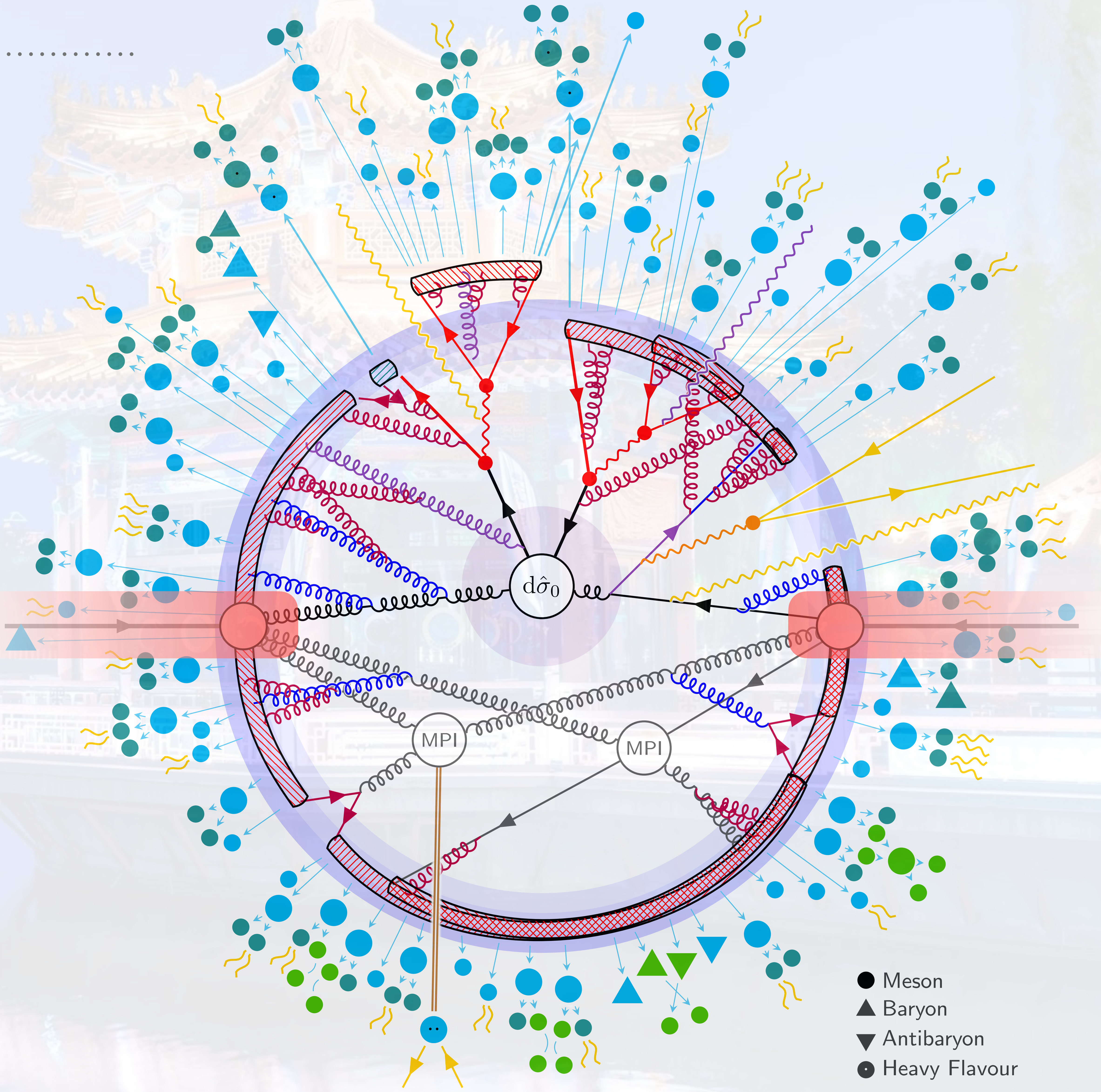
Hard Scattering

Proton → Parton



PYTHIA 8.3

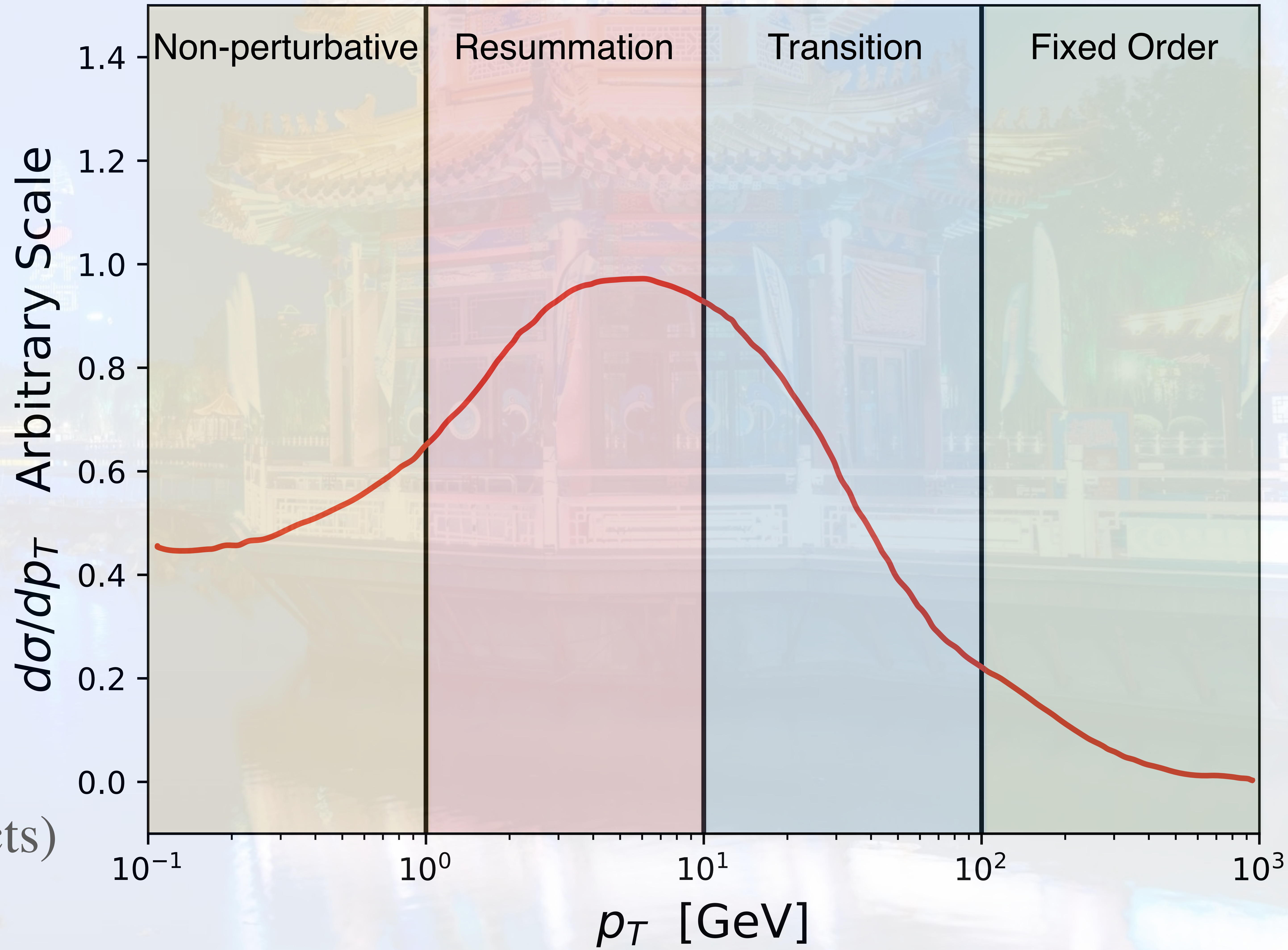
PRECISION PREDICTIONS AT HADRON COLLIDER

p_T Spectrum = multi-scale problem

- ➤Beyond QCD improved parton model
  - ➤pQCD describes the tail of spectrum
  - ➤Large logarithmic divergence

$$\ln\frac{p_T}{Q} \text{ as } p_T \to 1 \text{ GeV}$$

  - ➤Various LP resummation schemes
  - ➤Multiple solutions in transition region
  - ➤Non-perturbative effects ~ 1 GeV (Short distance and long distance effects)



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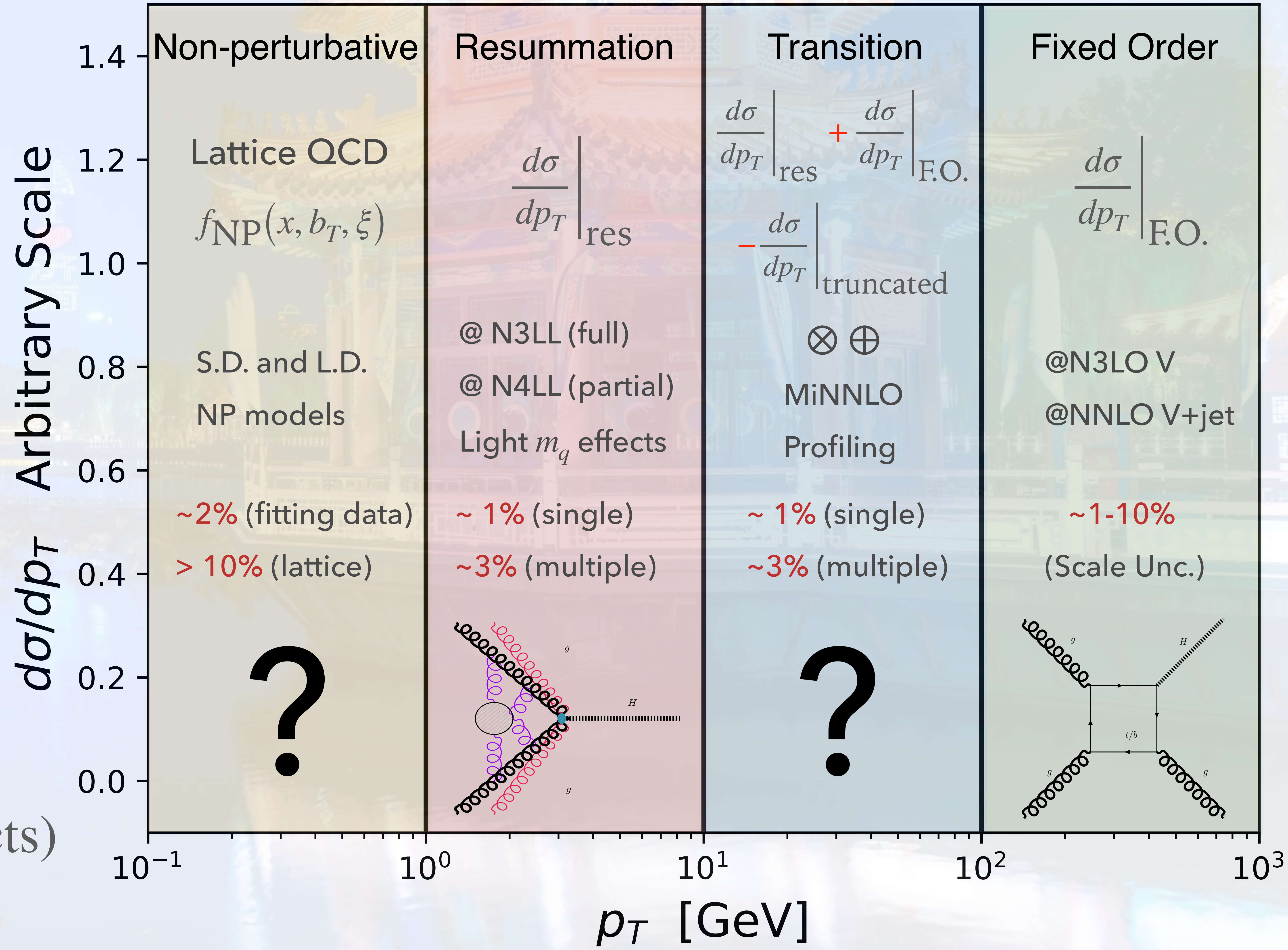
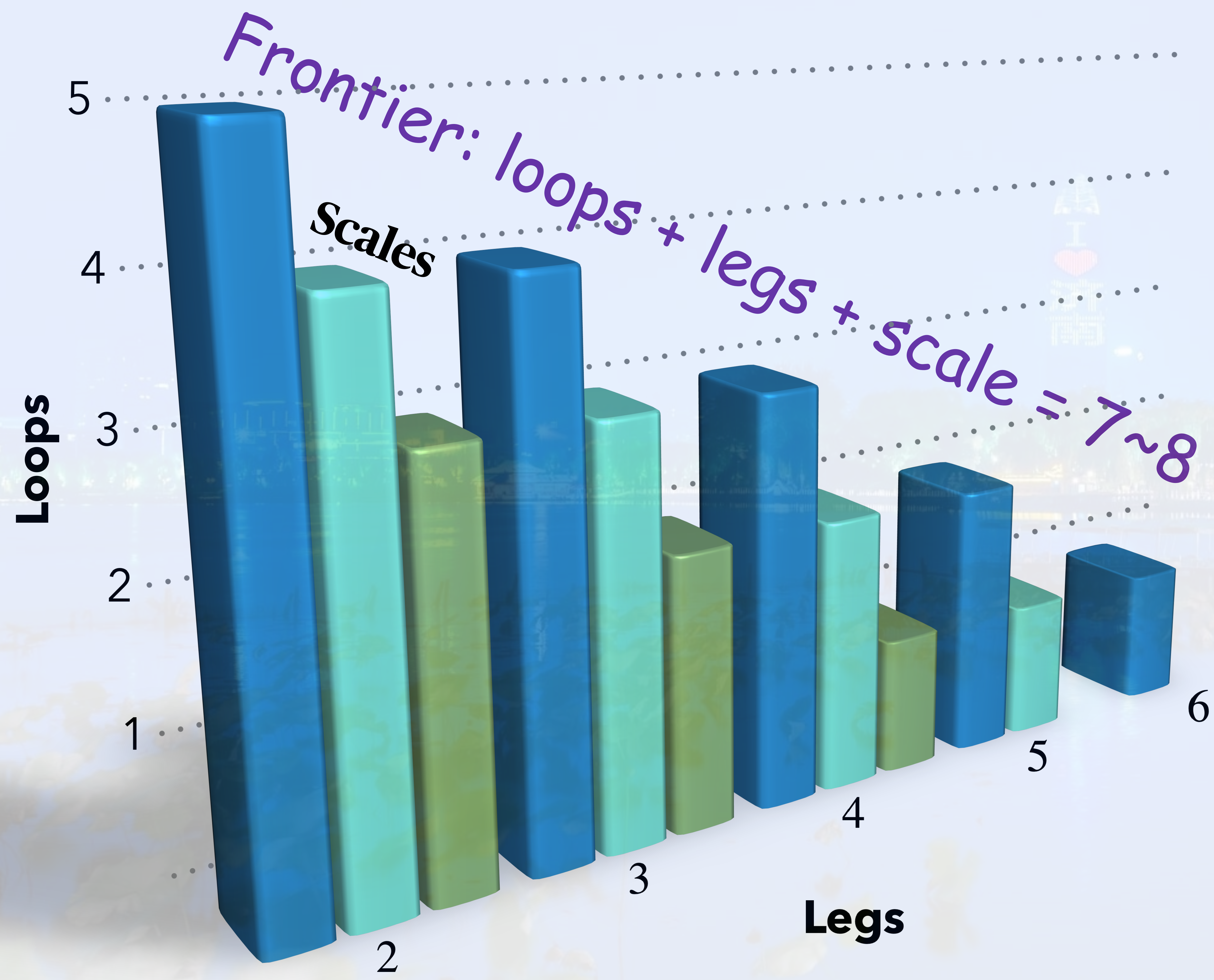
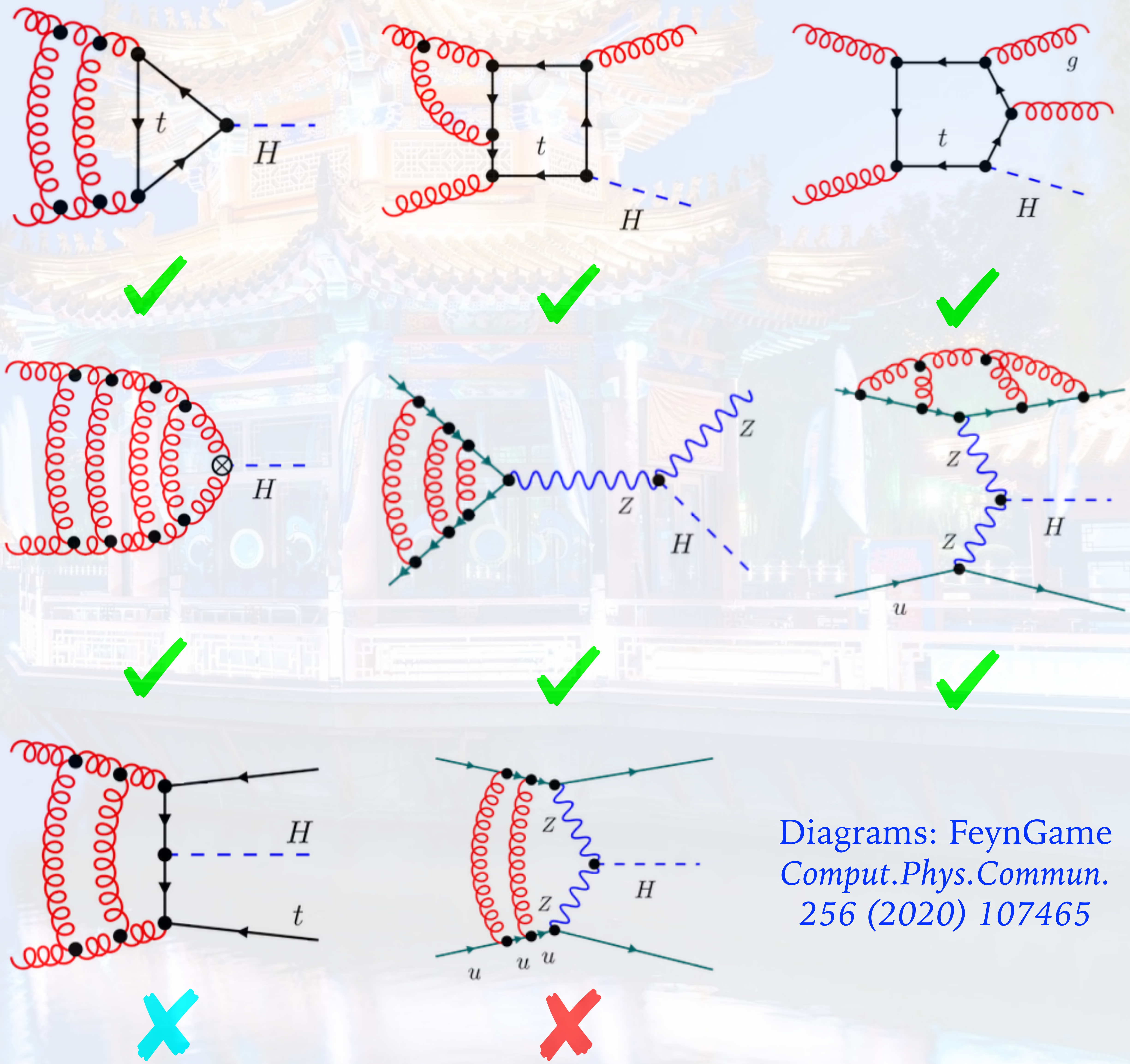


Perturbative QFT for Precision Predictions





Diagrams: FeynGame
Comput.Phys.Commun. 256 (2020) 107465

RESUMMATION FRAMEWORKS (QT FACTORISATION)

➤Resummation kernels: $d\sigma = \sigma_{LO} \otimes H \otimes B \otimes B \otimes S$

➤In SCET:
$$\frac{d\sigma}{dp_T^2} = \pi\sigma_{LO}^Z \int dx_a dx_b \delta\left(x_a x_b - \frac{m_{ll}^2}{E_{CM}^2}\right) \int \frac{d^2\vec{b}}{(2\pi)^2} e^{i\vec{p}_T\cdot\vec{b}} W(x_a, x_b, m_{ll}, \vec{b}),$$

$$W(x_a, x_b, m_{ll}, \vec{b}) = H(m_{ll}, \mu_h) U_h(m_{ll}, \mu_B, \mu_h) S_\perp(\vec{b}, \mu_s, \nu_s) U_s(b, \mu_B, \mu_s; \nu_B, \nu_s) \prod_{\gamma=a,b} B_{g/N_\gamma}^{\alpha\beta}(x_\gamma, \vec{b}, m_{ll}, \mu_B, \nu_B),$$

$$U_s(b, \mu, \mu_s; \nu, \nu_s) = \exp\left[2\int_{\mu_s}^{\mu} \frac{d\bar{\mu}}{\bar{\mu}} \left(\Gamma_{\text{cusp}}\left(\alpha_s(\bar{\mu})\right) \ln\frac{b^2\bar{\mu}^2}{b_0^2} - \gamma_s\left(\alpha_s(\bar{\mu})\right)\right)\right] \left(\frac{\nu^2}{\nu_s^2}\right)^{\int_{\mu}^{b_0/b} \frac{d\bar{\mu}}{\bar{\mu}} 2\Gamma_{\text{cusp}}\left[\alpha_s(\bar{\mu})\right] + \gamma_r\left[\alpha_s(b_0/b)\right]}.$$

➤In qT (CSS):
$$S_c(M, b) = \exp\left[-\int_{b_0^2/b^2}^{M^2} \frac{dq^2}{q^2}\left(A_c\left(\alpha_s(q^2)\right) \ln\frac{M^2}{q^2} + B_c\left(\alpha_s(q^2)\right)\right)\right]$$

$$\frac{d\sigma}{dp_T^2 dy} = \frac{m_{ll}^2}{s} \sigma_{LO}^Z \int_0^{+\infty} db \frac{b}{2} J_0(bp_T) S_c(m_{ll}, b) \sum_{a_1,a_2} \int_{x_1}^1 \frac{dz_1}{z_1} \int_{x_2}^1 \frac{dz_2}{z_2} [HC_1C_2]_{gg:a_1a_2} \prod_{i=1,2} f_{a_i/h_i}(x_i/z_i, b_0^2/b^2)$$

➤In momentum space (RadISH):
$$\sum(p_T) = \int_0^{p_T} dk_T \frac{d\sigma(k_T)}{dk_t} = \sigma_{LO}^H \int_0^\infty [dk_1] R'(m_H, k_{t,1}) \exp\left(-R(m_H, \epsilon k_{t,1})\right) \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_{\epsilon k_{t,1}}^{k_{t,1}} [dk_i] R'(m_H, k_{t,i}) \Theta\left(p_T - \left|\sum_{j=1}^{n+1} \vec{k}_{t,j}\right|\right)$$

COMPONENTS OF QT FACTORISATION (SCET)

FO	α_s^n	$H(m_V,\mu)$	$P^{(n)}_{ab}(x)$	$\ln W(x_a, x_b, m_V, \vec{b}, \mu = b_0/b) \sim \int_{\mu_h}^{\mu} d\bar{\mu}/\bar{\mu}\Big(A(\alpha_s(\bar{\mu}))\ln\frac{m_V^2}{\bar{\mu}^2} + B(\alpha_s(\bar{\mu}))\Big)$						
$\frac{d\hat{\sigma}^V_{NLO}}{dq_T}$	NLO	✓	✓	$\ln^2(b^2m_V^2)$	$\ln(b^2m_V^2)$	1				
$\frac{d\hat{\sigma}^V_{NNLO}}{dq_T}$	N2LO	✓	✓	$\ln^3(b^2m_V^2)$	$\ln^2(b^2m_V^2)$	$\ln(b^2m_V^2)$	1			
$\frac{d\hat{\sigma}^V_{N^3LO}}{dq_T}$	N3LO	✓	✓	$\ln^4(b^2m_V^2)$	$\ln^3(b^2m_V^2)$	$\ln^2(b^2m_V^2)$	$\ln(b^2m_V^2)$	1		
$\frac{d\hat{\sigma}^V_{N^4LO}}{dq_T}$	N4LO	✓	✗	$\ln^5(b^2m_V^2)$	$\ln^4(b^2m_V^2)$	$\ln^3(b^2m_V^2)$	$\ln^2(b^2m_V^2)$	$\ln(b^2m_V^2)$	1	
…	…			…	…	…	…		…	…
$\frac{d\hat{\sigma}^V_{N^kLO}}{dq_T}$	NKLO			$\ln^{k+1}(b^2m_V^2)$	$\ln^k(b^2m_V^2)$	$\ln^{k-1}(b^2m_V^2)$	$\ln^{k-2}(b^2m_V^2)$	$\ln^{k-3}(b^2m_V^2)$	…	…
…	…			…	…	…	…		…	…
Resum				LL	NLL	NNLL	N3LL	N4LL	…	$N^{k+1}LL$
A				A1 ✓	A2 ✓	A3 ✓	A4 ✓	A5 ✗	…	A_{k+2}
B					B1 ✓	B2 ✓	B3 ✓	B4 ✓	…	B_{k+1}

Parton Distributions and α_s

- PDF theory input
  - Option A: solve proton wave function with Lattice QCD *Recent progress in D. Chakrabarti, P. Choudhary et. al. 2304.09908*
  - Option B: collinear factorisation $f_a \to f_a(x,\mu)$ with p-QCD evolution of factorisation scale

$$\frac{d}{d\ln\mu^2}\begin{pmatrix} f_q \\ f_g \end{pmatrix} = \begin{pmatrix} P_{q\leftarrow q} & P_{q\leftarrow g} \\ P_{g\leftarrow q} & P_{g\leftarrow g} \end{pmatrix} \otimes \begin{pmatrix} f_q \\ f_g \end{pmatrix}$$

DGLAP evolution with

$$p_{a\leftarrow b} = \frac{\alpha_s}{\pi}P^{(0)}_{a\leftarrow b} + \frac{\alpha_s^2}{\pi^2}P^{(1)}_{a\leftarrow b} + \frac{\alpha_s^3}{\pi^3}P^{(2)}_{a\leftarrow b} + \cdots$$

1970's 1980 2004

$$\gamma^{(3)}_{q\leftarrow q}(N) = -\int_0^1 \mathrm{d}x x^{N-1} P^{(3)}_{q\leftarrow q}(x)$$ *G. Falcioni, F. Herzog et. al. Phys.Lett.B 842 (2023)*

For $N = 2,4,\cdots 20$

$$\gamma^{(3)}_{q\leftarrow g}(N) = -\int_0^1 \mathrm{d}x x^{N-1} P^{(3)}_{q\leftarrow g}(x)$$ *G. Falcioni, F. Herzog, S. Moch, A. Vogt Phys.Lett.B 846 (2023)*

See also full result of N_f^2, $N_f C_f^2$ contribution in *Gehrmann, von Manteuffel et. al. JHEP 01 (2024) 029*; *Gehrmann, von Manteuffel et. al. Phys.Lett.B 849 (2024)*

- Running of $\alpha_s(Q^2)$

$$Q^2\frac{\mathrm{d}\alpha_s}{\mathrm{d}Q^2} = \beta(\alpha_s) = -\alpha_s^2\left(b_0 + b_1\alpha_s + b_2\alpha_s^2 + b_3\alpha_s^3 + b_4\alpha_s^4 + \cdots\right)$$

1973 1979 1993 1997 2017

Generalised polylogarithms
Riemann zeta values
Elliptic functions
Unitarity
Generalised Unitarity
Recursion
Twistors
Differential equations
Integrand/Integral
Sector decomposition
Numerical unitarity
Finite field
Auxiliary mass flow
Neural network amplitude
…

Theory Tools Inside Measurements

PRECISION PREDICTIONS IN CDFII

- ➤CDF II use ResBos to generate theory templates
  - ➤NLO+NNLL QCD accuracy for W/Z production

    Balazs, Brock, Landry, Nadolsky and Yuan`97 to`03
  - ➤CSS factorisation and resummation of p_T in b space:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\,\mathrm{d}^2\vec{p}_T\,\mathrm{d}y\,\mathrm{d}\cos\theta\,\mathrm{d}\phi}=\sigma_0\int\frac{\mathrm{d}^2b}{(2\pi)^2}e^{i\vec{p}_T\cdot\vec{b}}e^{-S(b)}$$

$$\times\, C\otimes f(x_1,\mu)\, C\otimes f(x_2,\mu)+Y(Q,\vec{p}_T,x_1,x_2,\mu_R,\mu_F)$$

Collins, Soper and Sterman`85

  - ➤Non-perturbative effects at $\alpha_s(\Lambda)$ and large b:

$$S(b)=S_{\mathrm{NP}}S_{\mathrm{Pert}}\,,$$

Collins and Soper `77

$$S_{\mathrm{Pert}}(b)=\int_{C_1^2/(b^*)^2}^{C_2^2Q^2}\frac{\mathrm{d}\bar{\mu}^2}{\bar{\mu}^2}\left[\ln\left(\frac{C_2^2Q^2}{\bar{\mu}^2}\right)A\left(\bar{\mu},C_1\right)+B\left(\bar{\mu},C_1,C_2\right)\right]$$

$$S_{\mathrm{NP}}=\left[-g_1-g_2\ln\left(\frac{Q}{2Q_0}\right)-g_1g_3\ln\left(100x_1x_2\right)\right]b^2$$

S_{NP} assumes the BLNY functional form

Brock, Landry, Nadolsky and Yuan `02

- ➤LO+LL lepton EM radiation with PHOTOS and HORACE Golonka and Was `06

  Carloni Calame, Montagna et. al. `07

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Brock, Landry, Nadolsky and Yuan `02

➤LO+LL lepton EM radiation with PHOTOS and HORACE Golonka and Was `06

Carloni Calame, Montagna et. al. `07

➤Use data driven method:

Fix	g1	g2	g3	α_s
p_T^Z	Global fit `03	CDFII fit	Global fit `03	CDFII fit
p_T^Z/p_T^W			Global fit `03	

Global fit by Brock, Landry, Nadolsky and Yuan `03

$m_T^W \sim 0.7$ MeV, $p_T^l \sim 2.3$ MeV, $p_T^\nu \sim 0.9$ MeV

CDF supplementary materials `22

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S_{NP} assumes the BLNY functional form

Brock, Landry, Nadolsky and Yuan `02

➤LO+LL lepton EM radiation with PHOTOS and HORACE Golonka and Was `06

Carloni Calame, Montagna et. al. `07

➤Use data driven method:

Fix	g1	g2	g3	α_s
p_T^Z	Global fit `03	CDFII fit	Global fit `03	CDFII fit
p_T^Z/p_T^W			Global fit `03	

Global fit by Brock, Landry, Nadolsky and Yuan `03

$m_T^W \sim 0.7$ MeV, $p_T^l \sim 2.3$ MeV, $p_T^\nu \sim 0.9$ MeV

CDF supplementary materials `22

➤Scale uncertainty of p_T^Z/p_T^W by DYQT

Bozzi, Catani, Ferrera, de Florian, Grazzini `09 `11

$m_T^W \sim 3.5$ MeV, $p_T^l \sim 10.1$ MeV, $p_T^\nu \sim 3.9$ MeV

Not included in final result CDF sm`22

PRECISION PREDICTIONS IN RESBOS2

- ResBos → ResBos2
  - NNLO+N3LL accuracy for W/Z production

    Isaacson, Fu, Yuan `23
  - Upgrade CSS formalism to N3LL
  - Rescale NLO to NNLO from MCFM:

    Campbell, Ellis and Giele `15

$$\frac{d\sigma^{A_i}_{NLO}}{dp_T dy dQ} \rightarrow K^{A_i}_{\frac{NNLO}{NLO}}(p_T, y, Q)\frac{d\sigma^{A_i}_{NLO}}{dp_T dy dQ}$$

  - Dependence of angular coefficients recently included with more rescaling: $\frac{d\sigma}{d\cos\theta d\phi}$

$$= L_0(1+\cos^2\theta) + A_0(1-3\cos^2\theta) + A_1\sin 2\theta\cos\phi + A_2\sin^2\theta\cos 2\phi + A_3\sin\theta\cos\phi + A_4\cos\theta + A_5\sin^2\theta\sin 2\phi + A_6\sin 2\theta\sin\phi + A_7\sin\theta\sin\phi$$

Isaacson, Fu and Yuan`22 `23

- A_i at each fixed order:
  - LO: L_0, A_4
  - NLO: L_0, $A_0 = A_2$, A_1, A_3, A_4
  - NNLO: L_0, $A_0 \neq A_2$, A_1, A_3, A_4, A_5, A_6, A_7
- Resummation choices for only L_0, A_4 or all A_i
- New non-perturbative functional form.

We determine that the data-driven techniques used by CDF capture most of the higher order corrections, and using higher order corrections would result in a decrease in the value reported by CDF by at most 10 MeV

PRECISION PREDICTIONS IN ATLAS α_s DETERMINATION

➤ATLAS use DYTurbo as theory input
Camarda, Boonekamp et. al. `20

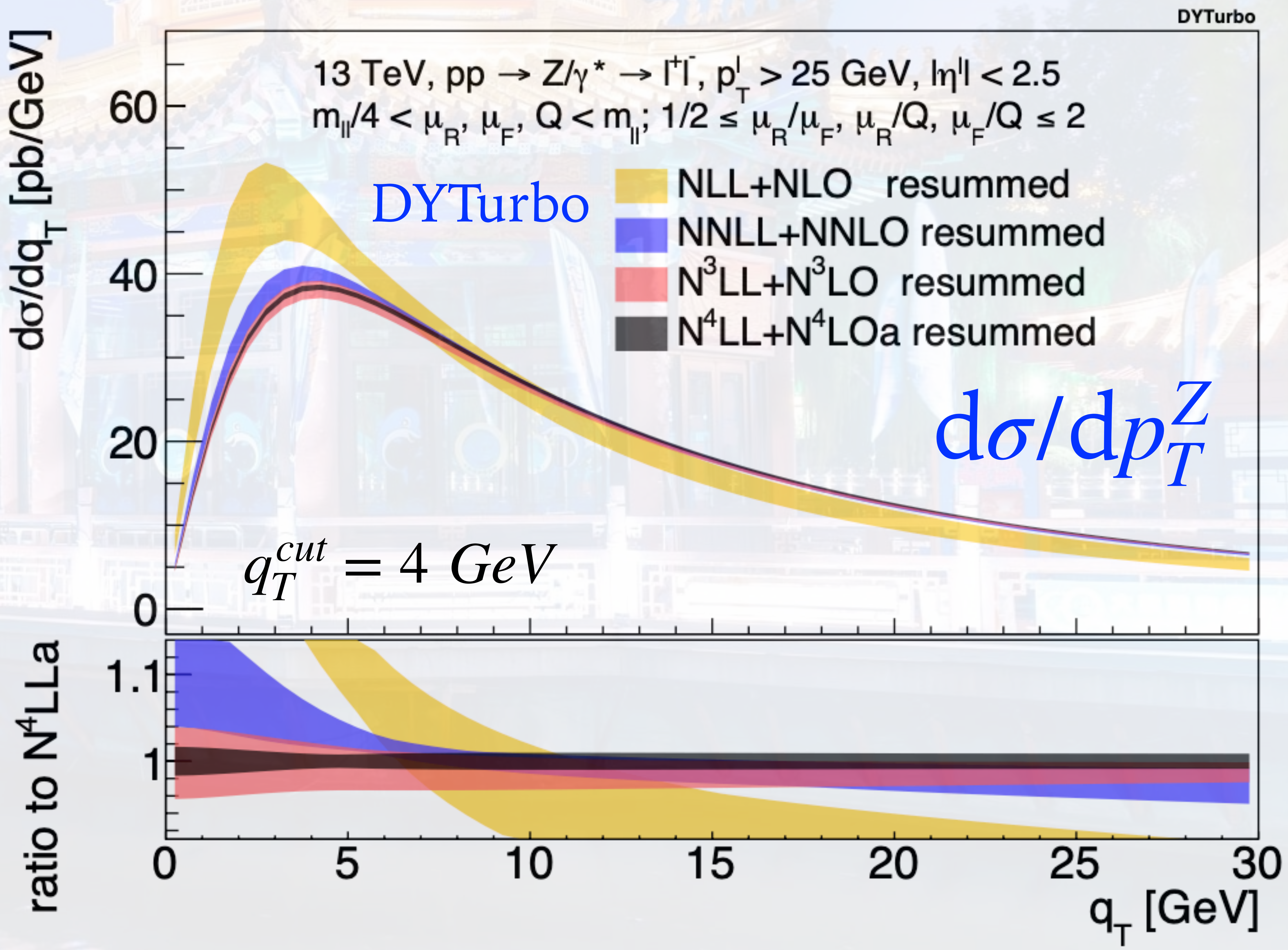
- ➤aN4LO + aN4LL accuracy for DY production
  Camarda, Cieri, Ferrera `23
- ➤FO: NNLO qT slicing from DYqT + $\mathcal{O}(\alpha_S^3)$ for $\delta(qT)$ + MCFM @ $\mathcal{O}(\alpha_S^3)$ for qT > 5 GeV. Neumann, Campbell `22
- ➤CSS resummation of p_T in b space:
  - ➤ Expansion up to $\mathcal{O}(\alpha_S^4)$ for small qT (approx.)
  - ➤ Exact B4 coefficient with all other N4LL components approx. (A5, H4, DGLAP etc.) Moult, Zhu, Zhu `22
- ➤aN3LO PDF MSHT20: approx. in DGLAP, TH input
  McGowan, Cridge, Larland-Lang, Thorne `22
- ➤Non-perturbative effects at $\alpha_s(\Lambda)$ and large b:

$$S_{NP}(b) = \exp\{-g_1 b^2 - g_K(b)\,\ln(M^2/Q_0^2)\}$$

$$g_K(b) = g_0\left(1 - \exp\left[-\frac{C_F\alpha_S((b_0/b_\star)^2)b^2}{\pi g_0 b_{\text{lim}}^2}\right]\right)$$

Collins, Rogers `14

➤LL ISR photons radiation + normalisation to NLO QED and virtual EW cor. in ReneSANCe
Bondarenko, Dydyshka et. al. `22



S. Camarda, L. Cieri, G. Ferrera Phys. Lett. B 845 (2023)

➤See also DYTurbo in α_s fitting with CDF data
Camarda , Ferrera, Schott `23

State-of-the-Art Phenomenology Progress

Parton Distribution Functions

- Approximated N3LO PDF available:

  MSHT20aN3LO *Eur.Phys.J.C 83 (2023) 4*

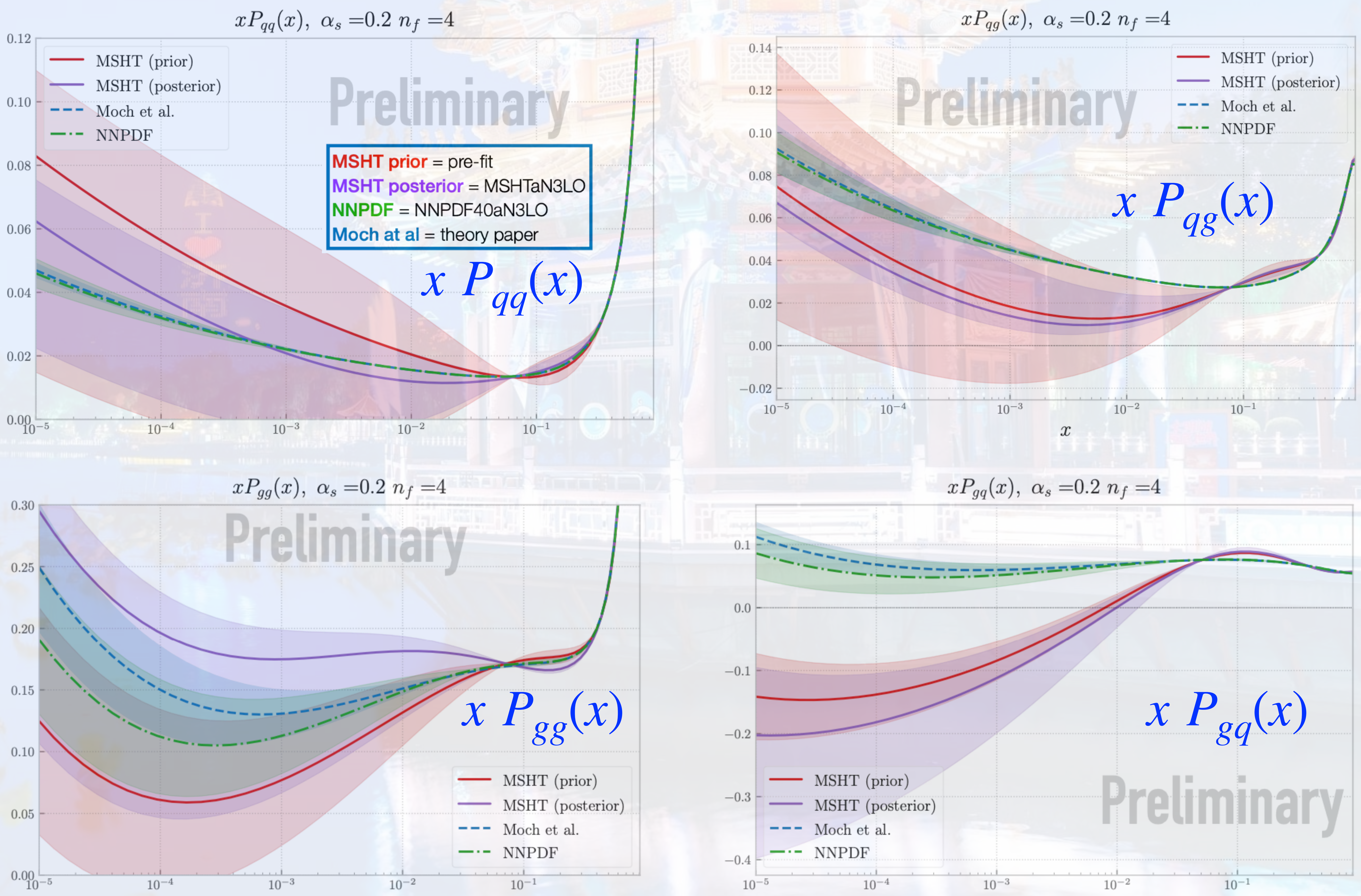
  NNPDFaN3LO *NNPDF preliminary*

- More precise 4-loop splitting functions affect small x region:

  4 → 10 Mellin Moments

- Large correction at aN3LO at small x region outside 68% c.l. region.
- Missing Higher Order Uncertainty (MHOU) not included in standard NNLO PDF.
- Crucial to consider MHOU and IHOU to understand consistency between NNLO and N3LO PDF.

M. Ubiali (NNPDF) @LHCHWG `23



Parton Distribution Functions

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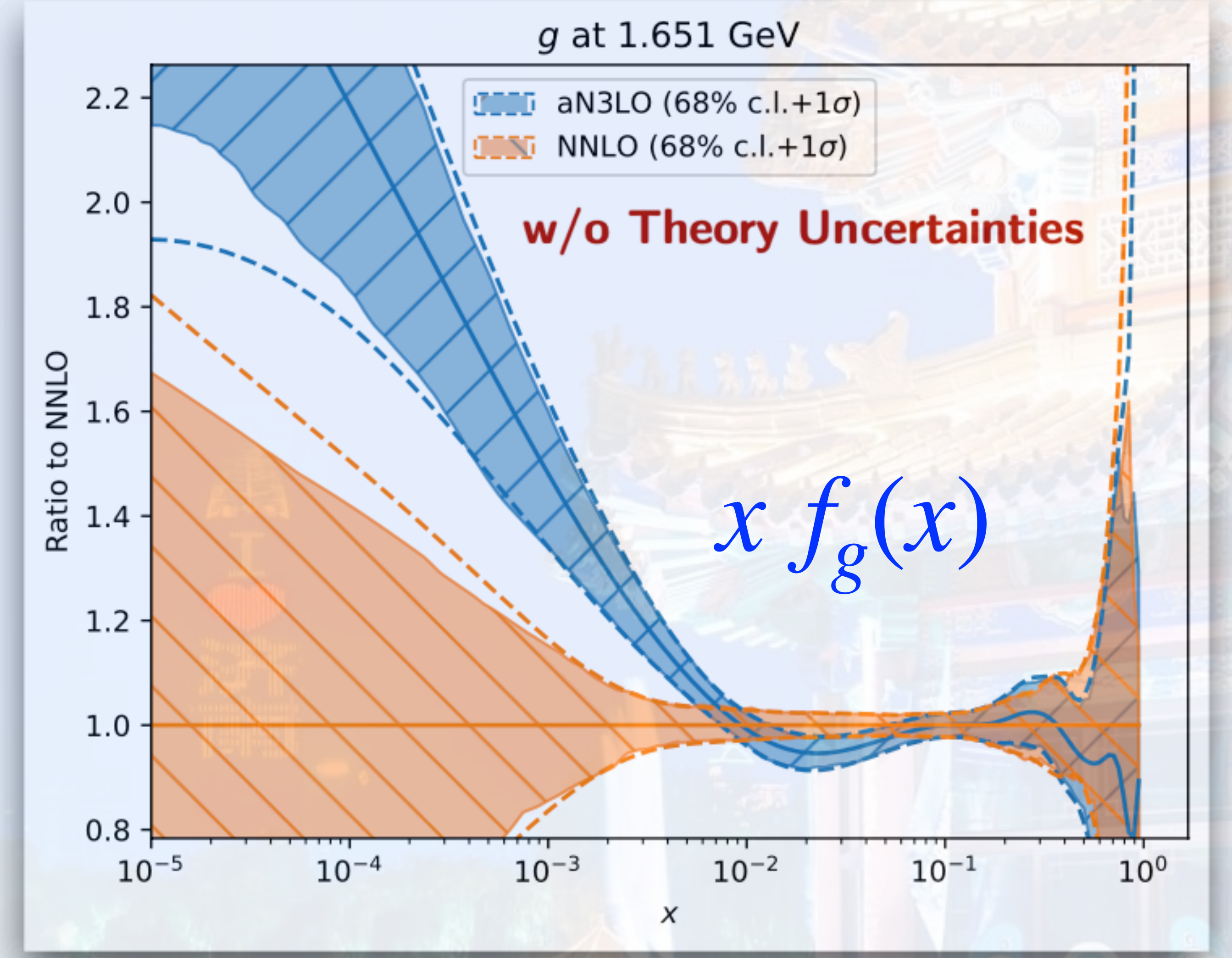
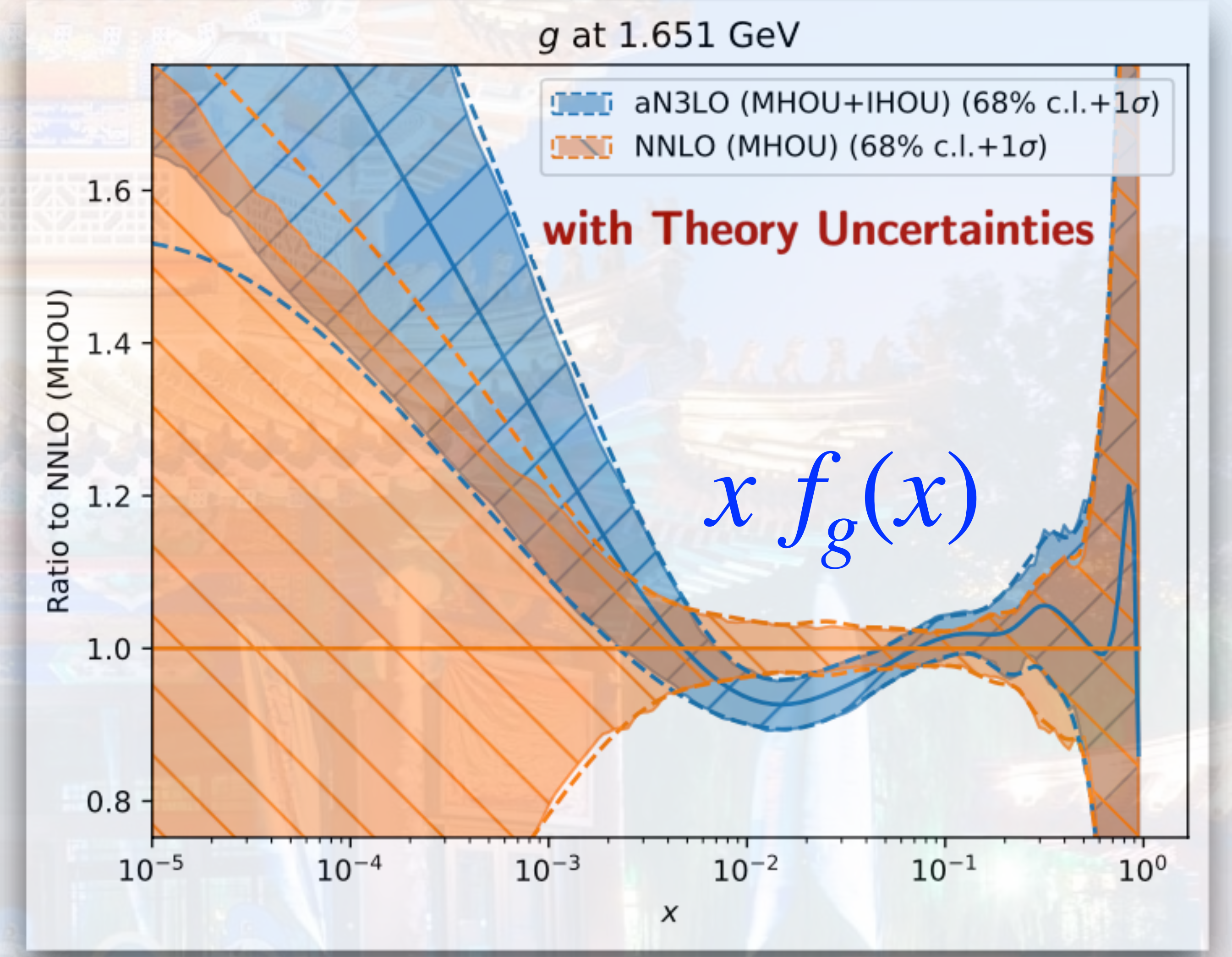
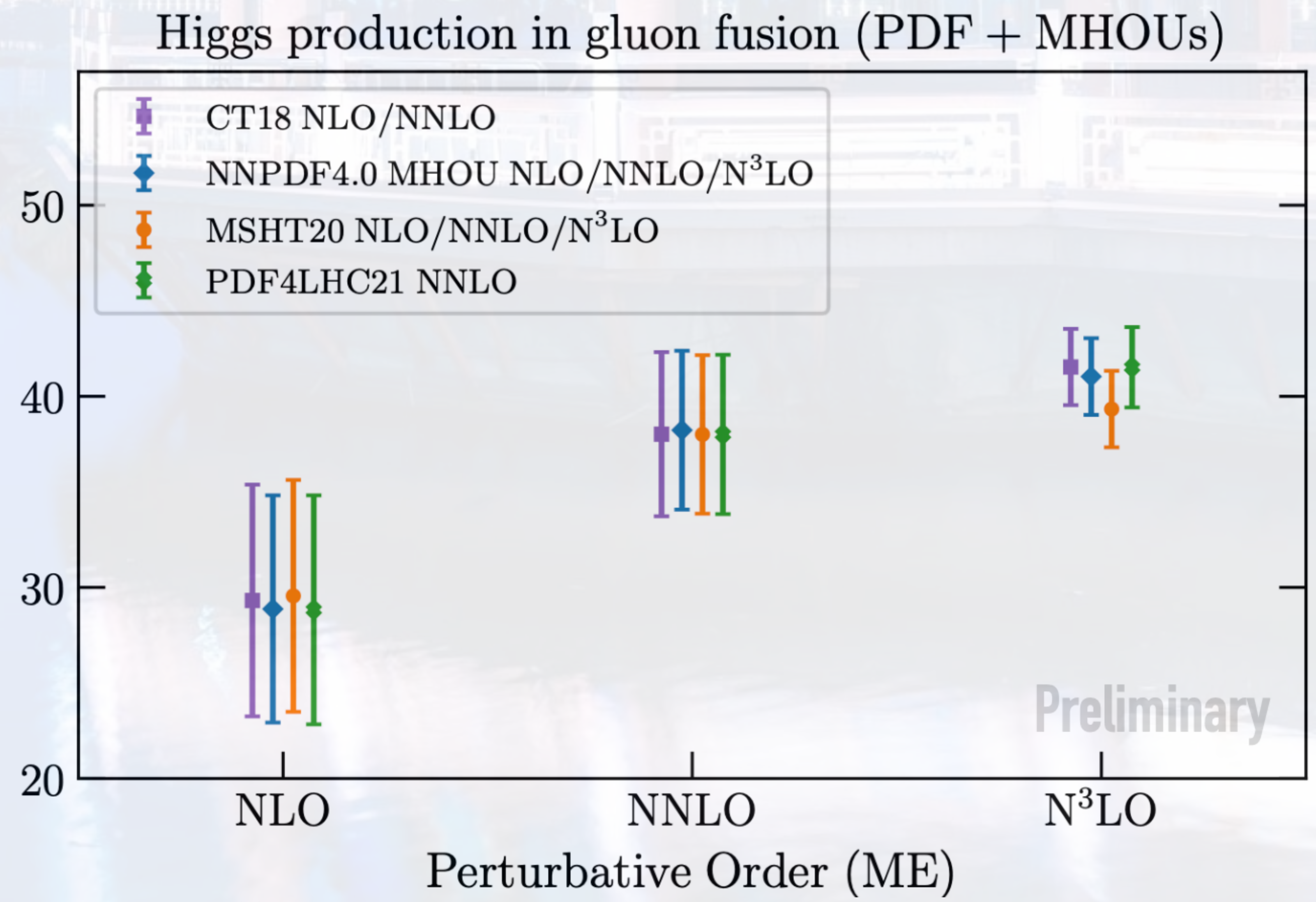
G. Magni (NNPDF) @ Les Houches 23







M. Ubiali (NNPDF) @LHCHWG `23

STATE-OF-THE-ART PREDICTIONS: $\alpha_{EW} \times \alpha_s$

➤NNLO QCD-EW mixed corrections

$pp \to l\nu$

$$\hat{\sigma}_{ab} = \hat{\sigma}_{ab}^{(0,0)} + \hat{\sigma}_{ab}^{(1,0)} + \hat{\sigma}_{ab}^{(2,0)} + \hat{\sigma}_{ab}^{(3,0)} + \ldots \quad \text{QCD}$$

$$+ \hat{\sigma}_{ab}^{(0,1)} + \ldots \quad \text{EW}$$

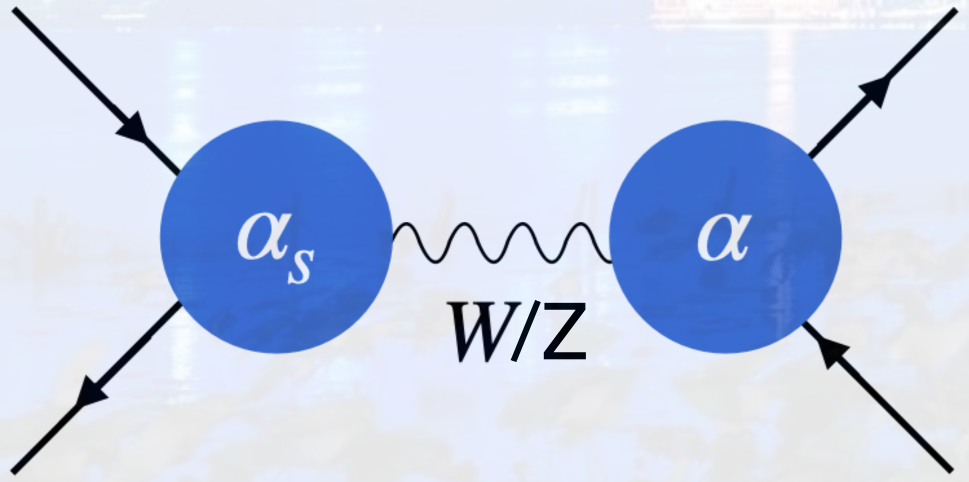
$$+ \hat{\sigma}_{ab}^{(1,1)} + \ldots \quad \text{QCD-EW}$$

σ [pb]	$\sigma_{\rm LO}$	$\sigma^{(1,0)}$	$\sigma^{(0,1)}$	$\sigma^{(2,0)}$	$\sigma^{(1,1)}$	
$q\bar{q}$	5029.2	970.5(3)	−143.61(15)	251(4)	− 7.0(1.2)	
qg	—	−1079.86(12)	—	−377(3)	39.0(4)	
$q(g)\gamma$	—	—	2.823(1)	—	0.055(5)	
$q(\bar{q})q'$	—	—	—	44.2(7)	1.2382(3)	
gg	—	—	—	100.8(8)	—	
tot	5029.2	−109.4(4)	−140.8(2)	19(5)	33.3(1.3)	
$\sigma^{(m,n)}/\sigma_{\rm LO}$		−2.2 %	−2.8 %	+0.4 %	+0.6 %	$\mu = m_W$
$\sigma^{(m,n)}/\sigma_{\rm LO}$		+10 %	−2.9 %	+4.2 %	+0.8 %	$\mu = \frac{m_W}{2}$

Buonocore, Grazzini, Kallweit, Savoini, Tramontano `21

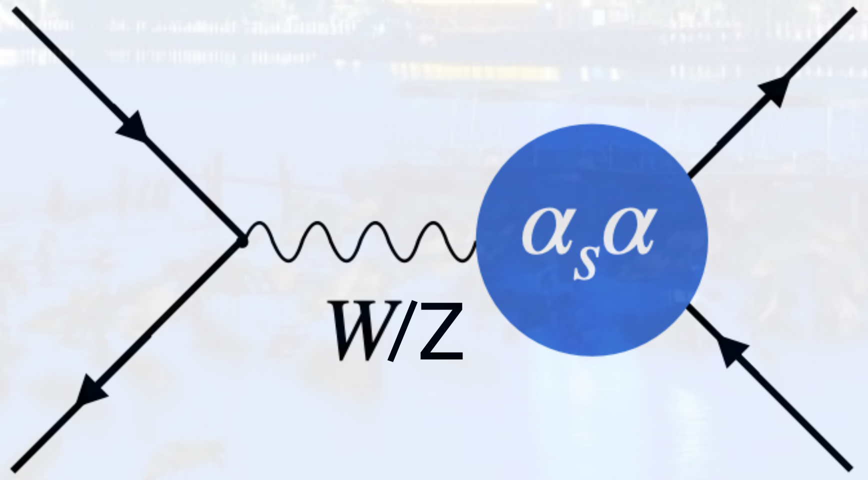
➤NLO EW correction from W decay is large

➤Scale variation from EW power counting is small



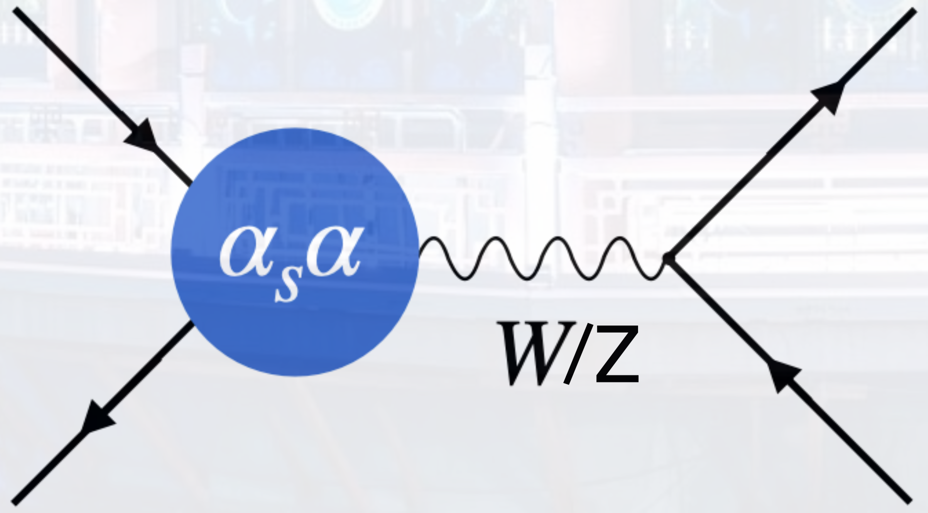
Initial-Final contribution from automation tools

Carloni Calame, Chiesa, Martinez, Montagna, Nicrosini, Piccinini, Vicini `16



Final-Final finite renormalisation constant

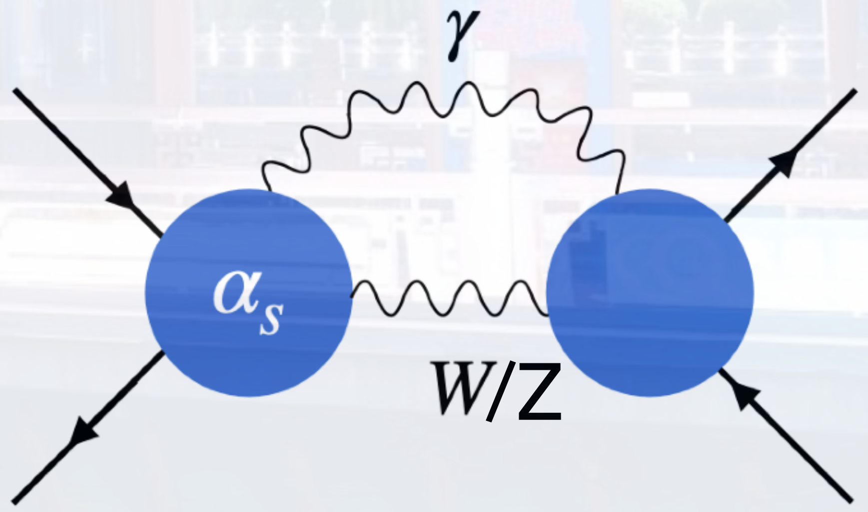
Dittmaier, Huss, Schwinn `15



Initial-Initial QCD-EW 2-loop form factors

Bonciani, Buccioni, Rana, Vicini `20 `21

Behring, Buccioni, Caola, Delto, Jaquier, et. al. `20 `21



Non-factorizable contribution

Dittmaier, Huss, Schwinn `14 (PA)
Dittmaier, Huss, Schwarz `24 (PA)

Bonciani, Buonocore et. al. `21

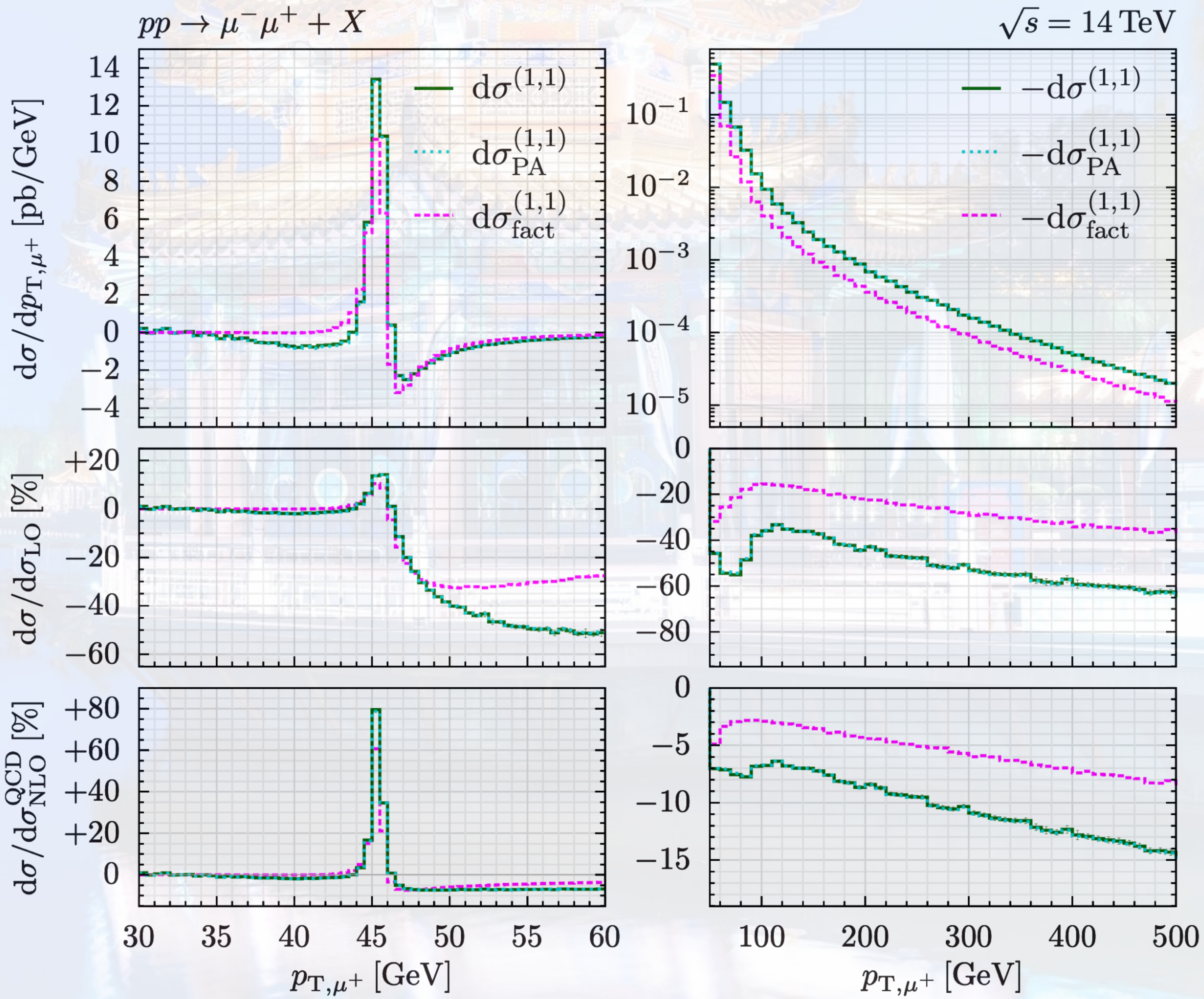
Buccioni, Caola et. al. `22

STATE-OF-THE-ART PREDICTIONS: $\alpha_{EW} \times \alpha_s$

➤ $pp \to \mu^+\mu^-$@ NNLO $\alpha_{EW} \times \alpha_s$

σ [pb]	$\sigma_{\rm LO}$	$\sigma^{(1,0)}$	$\sigma^{(0,1)}$	$\sigma^{(2,0)}$	$\sigma^{(1,1)}$
$q\bar{q}$	809.56(1)	191.85(1)	−33.76(1)	49.9(7)	−4.8(3)
qg	—	−158.08(2)	—	−74.8(5)	8.6(1)
$q(g)\gamma$	—	—	−0.839(2)	—	0.084(3)
$q(\bar{q})q'$	—	—	—	6.3(1)	0.19(0)
gg	—	—	—	18.1(2)	—
$\gamma\gamma$	1.42(0)	—	−0.0117(4)	—	—
tot	810.98(1)	33.77(2)	−34.61(1)	−0.5(9)	4.0(3)

Buonocore, Grazzini, Kallweit, Savoini, Tramontano `21



- ➤ 14 TeV with NNPDF31_nnlo_luxqed
- ➤ $p_T^\mu > 25$ GeV, $|y_\mu| < 2.5, m_{\mu\mu} > 50$ GeV
- ➤ Bare muons in final states
- ➤ Large cancellation between NLO QCD and EW
- ➤ Large QCDxEW compare to NNLO QCD
- ➤ Diff. Comparisons: full vs. PA vs. Factorizable

STATE-OF-THE-ART PREDICTIONS: $\alpha_{EW} \times \alpha_s$

[Slide from Gavin Salam @ ICHEP 2022]

- ➤Full study of fit to distribution is not easy at fixed order
- ➤Instead study **mass determination from mean lepton pT**, inclusive or fiducial

  (Here just the **production corrections**; decay corrections should factorise)

$$\frac{\delta m_W^{\rm meas}}{m_W^{\rm meas}} = \frac{\delta C_{\rm th}}{C_{\rm th}} = \frac{\delta \langle p_\perp^{l,Z} \rangle^{\rm th}}{\langle p_\perp^{l,Z} \rangle^{\rm th}} - \frac{\delta \langle p_\perp^{l,W} \rangle^{\rm th}}{\langle p_\perp^{l,W} \rangle^{\rm th}}$$

Behring, Buccioni, Caola, Delto et. al. `21

	δm_Z (scaled by m_W/m_Z)	δm_W	difference
inclusive $\langle p_{t\ell} \rangle$ @ α_{EW}	–32 MeV	–32 MeV	0.3 MeV
inclusive $\langle p_{t\ell} \rangle$ @ $\alpha_{EW}\,\alpha_s$	+62 MeV	+55 MeV	–7 MeV
fiducial $\langle p_{t\ell} \rangle$ @ $\alpha_{EW}\,\alpha_s$	[ATLAS cuts]		–17±2 MeV

- ➤Relevant for both Z-calibrated methods & standalone W methods. (Impact by fiducial cuts)
- ➤Combining QED and QCD qT resummation (factorizable) Cieri, Ferrera, Sborlini `18 Autieri et. al. `23

STATE-OF-THE-ART PREDICTIONS: $d\sigma_{N^3LO}$

- Differential N3LO accuracy
  - Projection to Born

$$\frac{d\sigma^F_{N^kLO}}{d\mathcal{O}} = \left(\frac{d\sigma^{F+jet}_{N^{k-1}LO}}{d\mathcal{O}} - \frac{d\sigma^{F+jet}_{N^{k-1}LO}}{d\tilde{\mathcal{O}}}\right) + \frac{d\sigma^F_{N^kLO}}{d\tilde{\mathcal{O}}}$$

    - Jet production in DIS (NNLOJET) Currie, Gehrmann, Glover, Huss, Niehues `18
    - Higgs decay to $b\bar{b}$ (MCFM) Mondini, Schiavi, Williams `19
    - Higgs production via ggF (RapidiX+NNLOJET) XC, Gehrmann, Glover, Huss, Mistlberger, Pelloni `21
  - qT slicing

$$d\sigma^F_{N^kLO} = \mathcal{H}^F_{N^kLO} \otimes d\sigma^F_{LO}\Big|_{\delta(\tau)} + \left[d\sigma^{F+jet}_{N^{k-1}LO} - d\sigma^{F\ CT}_{N^kLO}\right]_{\tau>\tau_{cut}} + \mathcal{O}(\tau^2_{cut}/Q^2)$$

    - Higgs production via ggF (HN3LO+NNLOJET) Cieri, XC, Gehrmann, Glover, Huss `18
    - Higgs pair production via ggF (with modified iHixs2) Chen, Li, Shuo, Wang `19
    - Drell-Yan production (NNLOJET) XC, Gehrmann, Glover, Huss, Yang, Zhu `21 `22 (MCFM) Neumann and Campbell `22 `23
- Combined with resummation (N3LL/aN4LL at small qT)
  - Drell-Yan production Ju and Schönherr `21 (DYTurbo) Camarda, Cieri, Ferrera `21 `23 (RadISH(N3LL)+NNLOJET) XC, Gehrmann, Glover, Huss, Monni, Re, et. al. `18 `19 `22 (CuTe-MCFM) Neumann and Campbell `22 `23
  - Higgs production via ggF (SCET+NNLOJET) XC, Gehrmann et. al.`18 (SCETlib) Billis, Dehnadi, et. al.`21

STATE-OF-THE-ART PREDICTIONS: $d\sigma_{N^3LO}$

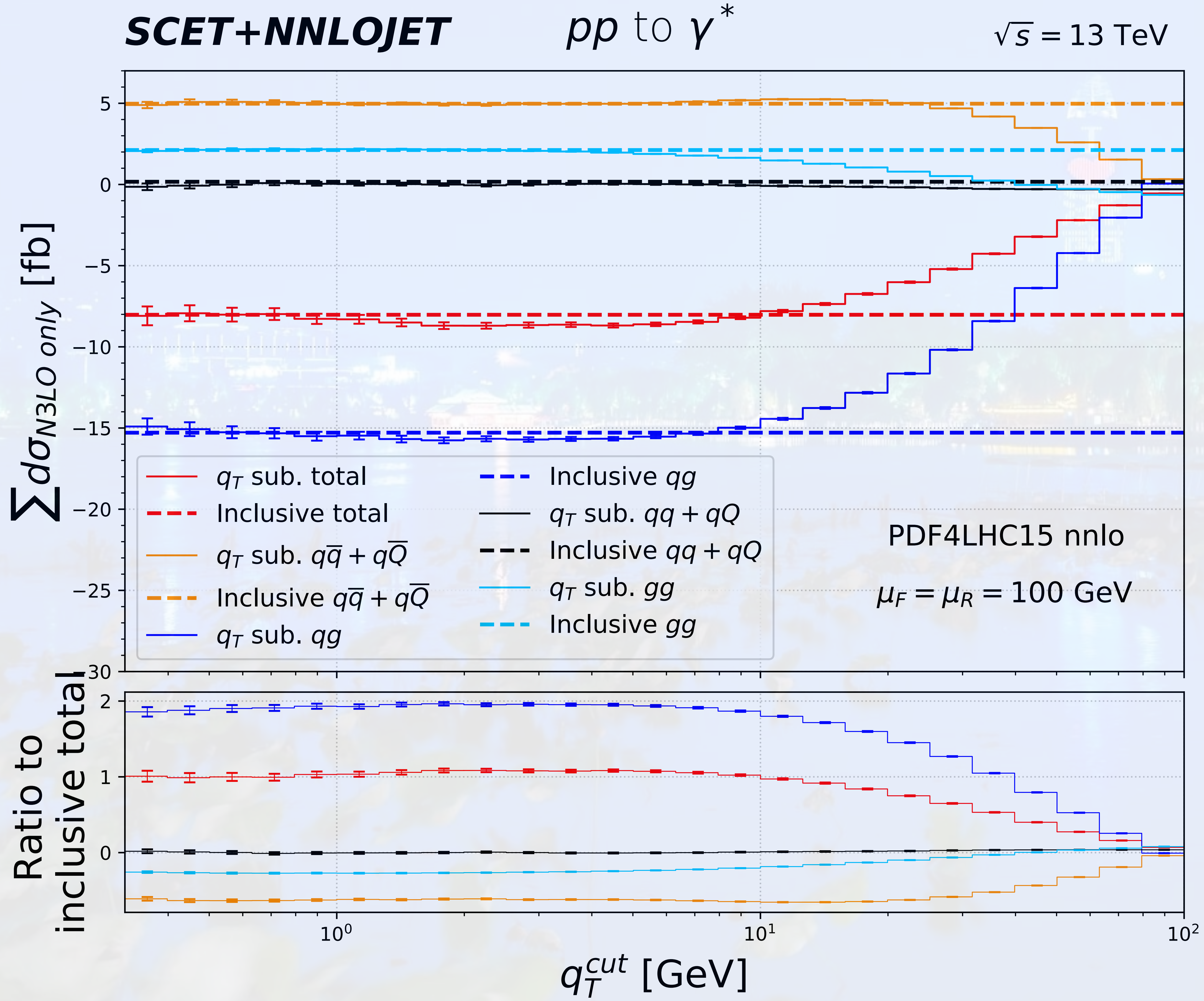
➤qT slicing at N3LO for neutral and charged current production (NNLOJET)

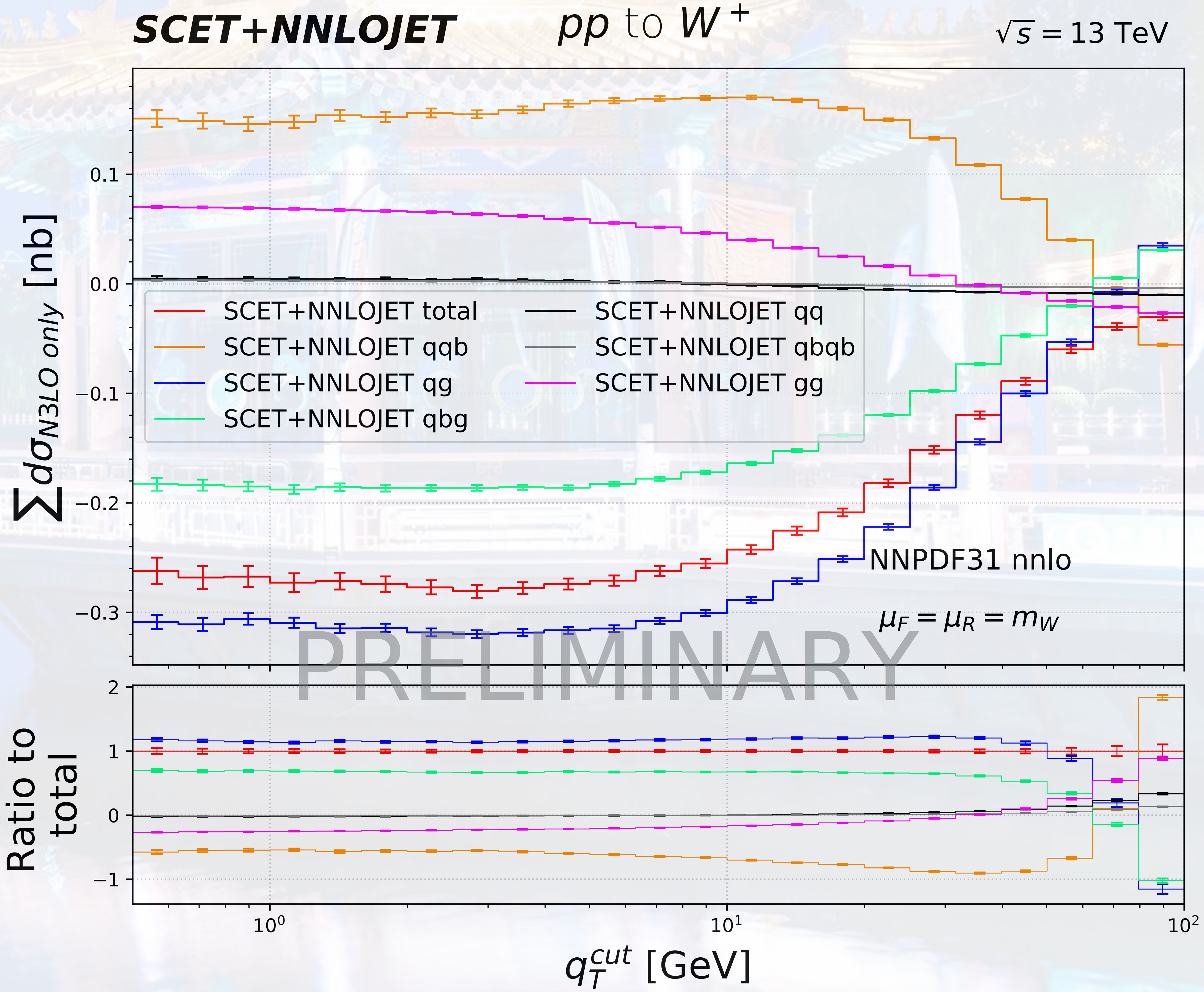
$$\sum d\sigma^{V}_{N3LO} \equiv \sum_{dp_{T,V}} d\sigma^{V+jet}_{NNLO}/dp_{T,V}|_{p_{T,V}>q_T^{cut}} + \sum_{dp_{T,V}} d\sigma^{V\,SCET}_{N^3LO}/dp_{T,V}|_{p_{T,V}\in[0,q_T^{cut}]}$$

NC and CC Validated against inclusive XS within ± 5% uncertainty

$$\Delta\sigma^{\gamma^*}_{N^3LO} = -7.98 \pm 0.36\, fb \quad vs. \quad -8.03\, fb$$

Duhr, Dulat, Mistlberger Phys.Rev.Lett. 125 (2020)





XC, Gehrmann, Glover, Huss, Yang, Zhu Phys.Rev.Lett. 128 (2022) 5

XC, Gehrmann, Glover, Huss, Yang, Zhu Phys.Lett.B 840 (2023)

STATE-OF-THE-ART PREDICTIONS: $d\sigma_{N^3LO}$

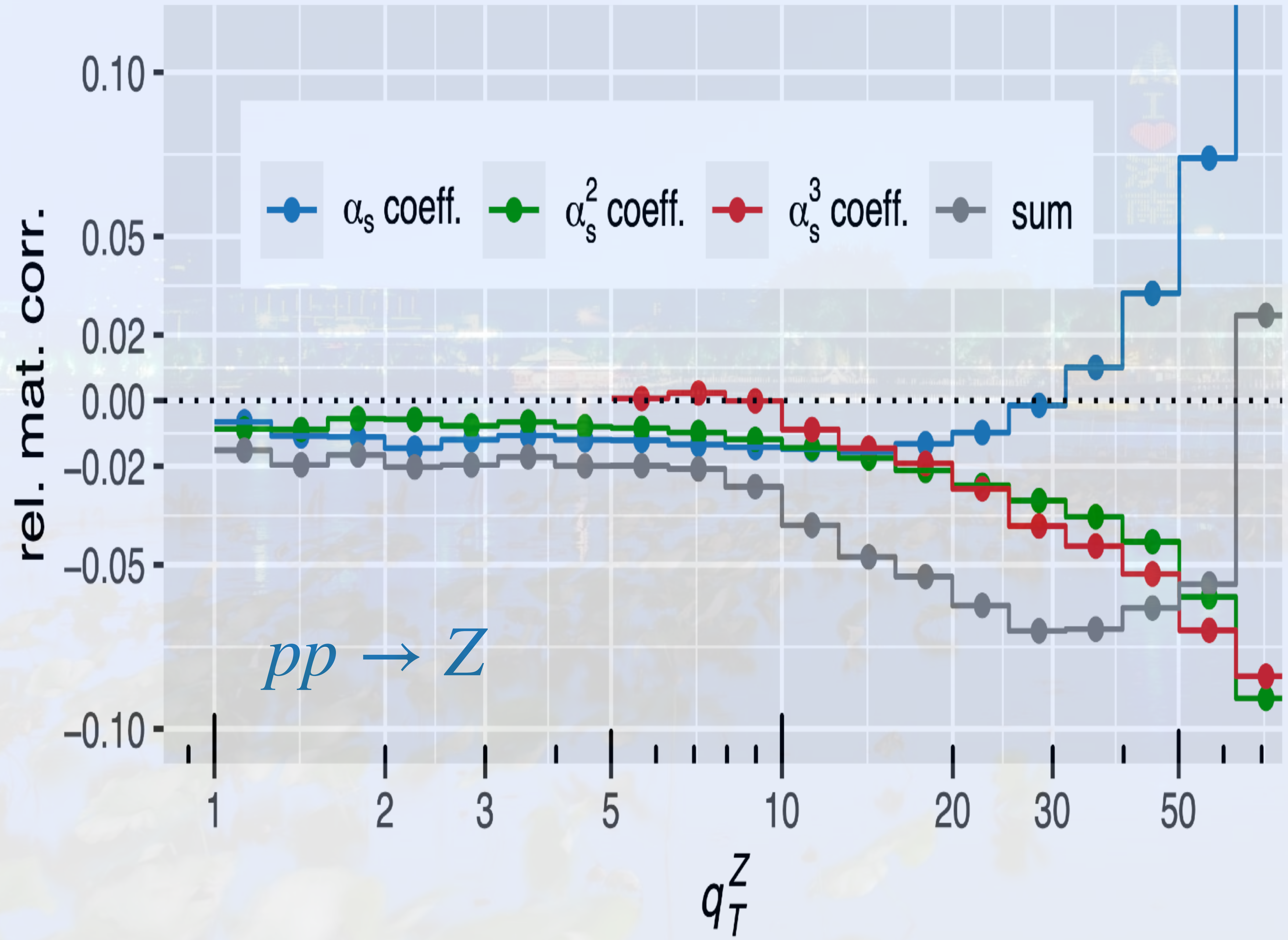
➤qT slicing at N3LO for neutral and charged current production (MCFM)

$$\sum_{dp_{T,V}} d\sigma^{V}_{N3LO} \equiv \sum_{dp_{T,V}} d\sigma^{V+jet}_{NNLO}/dp_{T,V}|_{p_{T,V}>q_T^{cut}} + \sum_{dp_{T,V}} d\sigma^{V\,SCET}_{N^3LO}/dp_{T,V}|_{p_{T,V}\in[0,q_T^{cut}]}$$

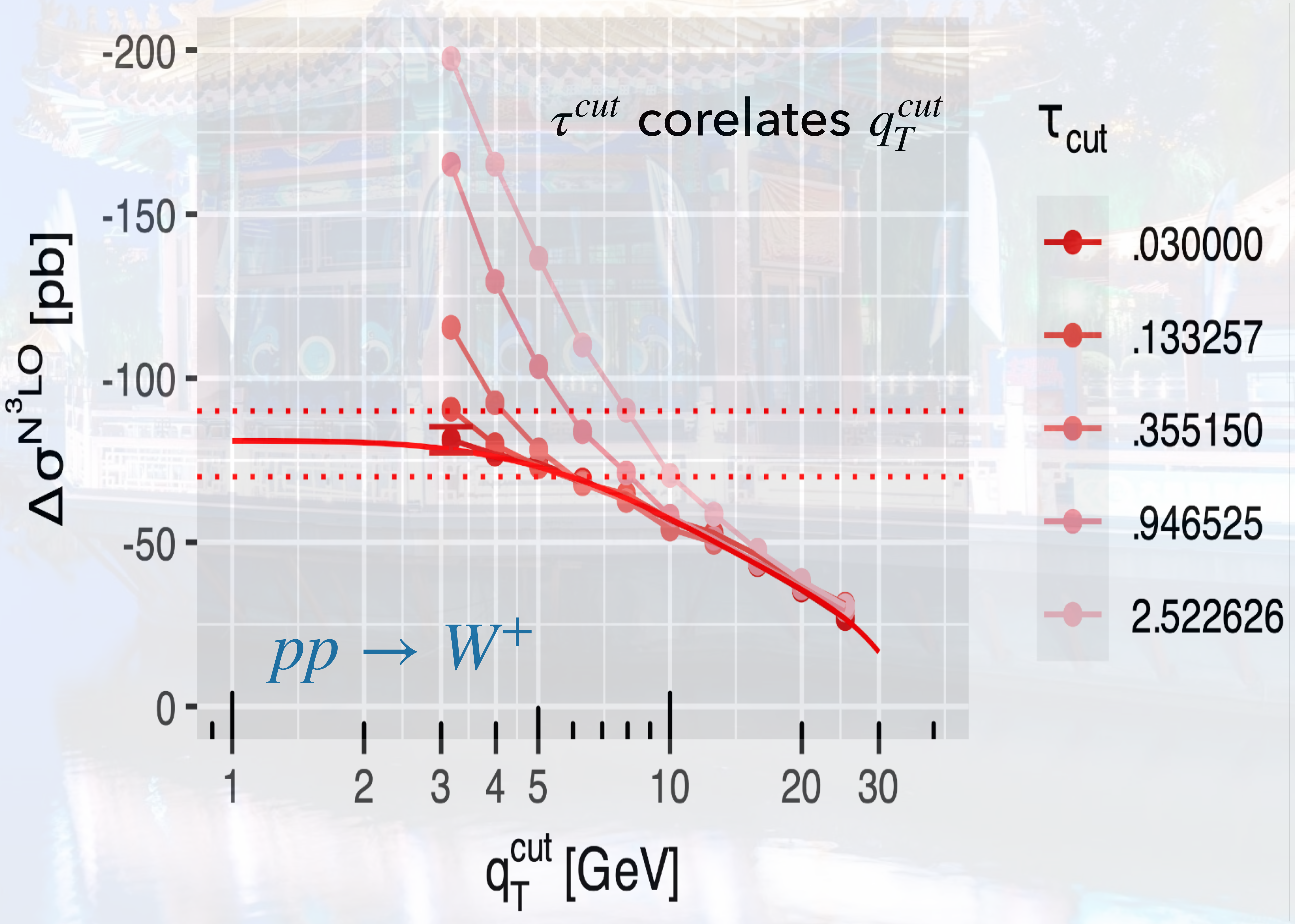
NC MCFM: $-22.6\,\text{pb} \pm 1.4\,\text{pb}\,(\text{num.}) \pm 1\,\text{pb}\,(\text{slicing})$

NC NNLOJET: $-18.7\,\text{pb} \pm 1.1\,\text{pb}\,(\text{num.}) \pm 0.9\,\text{pb}\,(\text{slicing})$

CC agree to inclusive XS within ± 60% uncertainty of $\Delta(\alpha_s^3)$



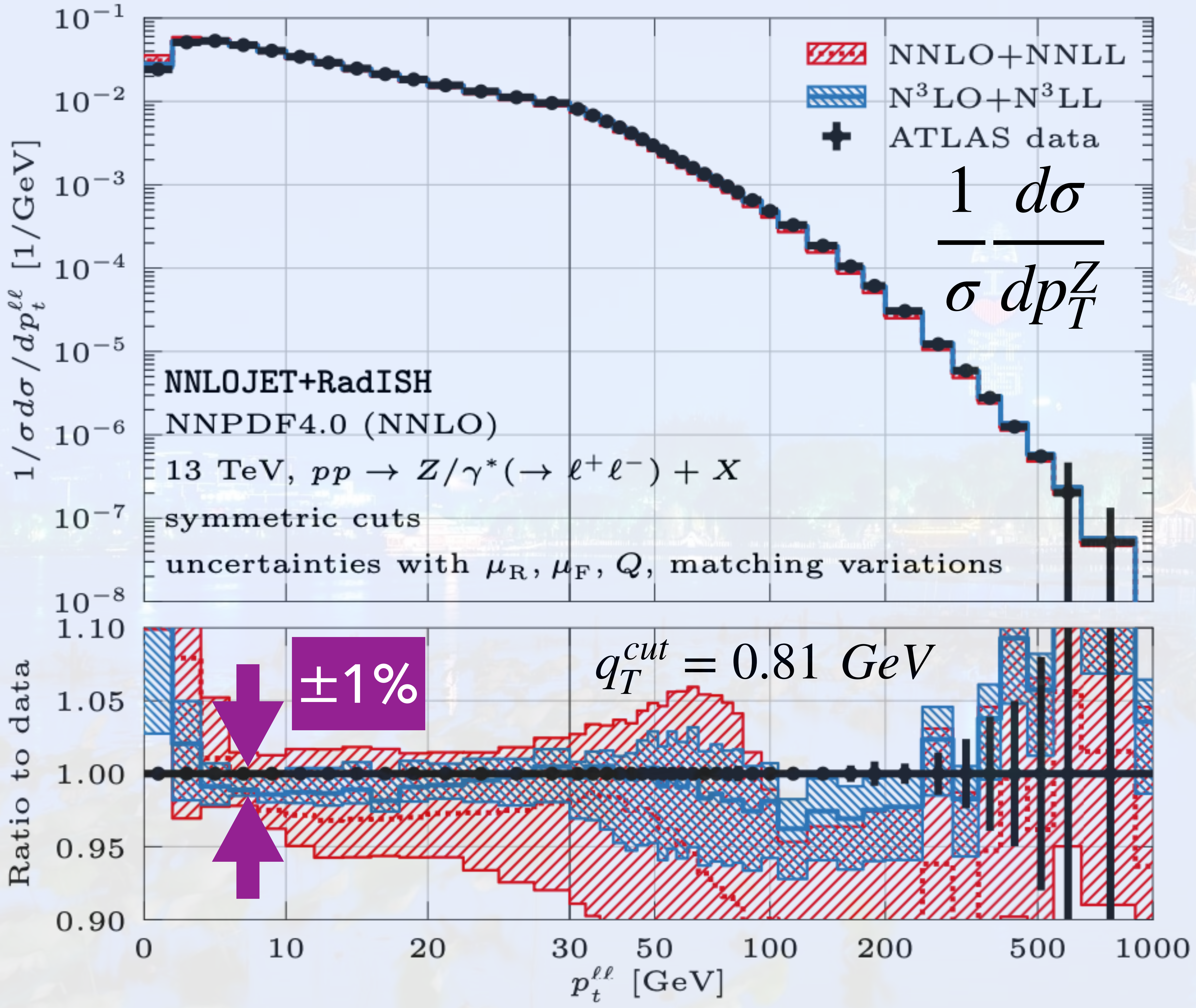
Neumann and Campbell *Phys.Rev.D* 107 (2023) 1



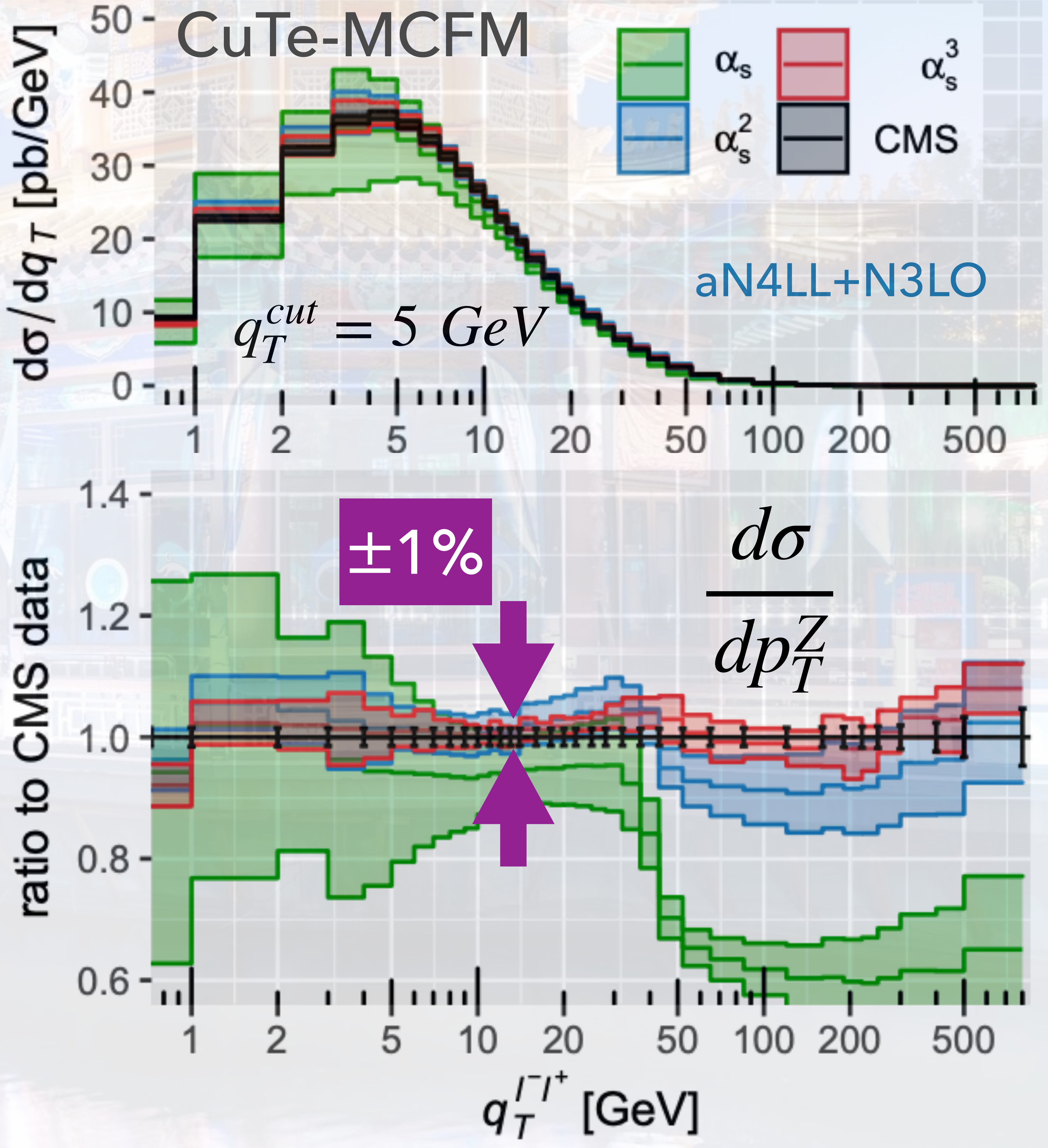
Neumann and Campbell *JHEP 11 (2023) 127*

STATE-OF-THE-ART PREDICTIONS: $d\sigma_{N^3LO}$

➤Differential N3LO predictions for neutral current production



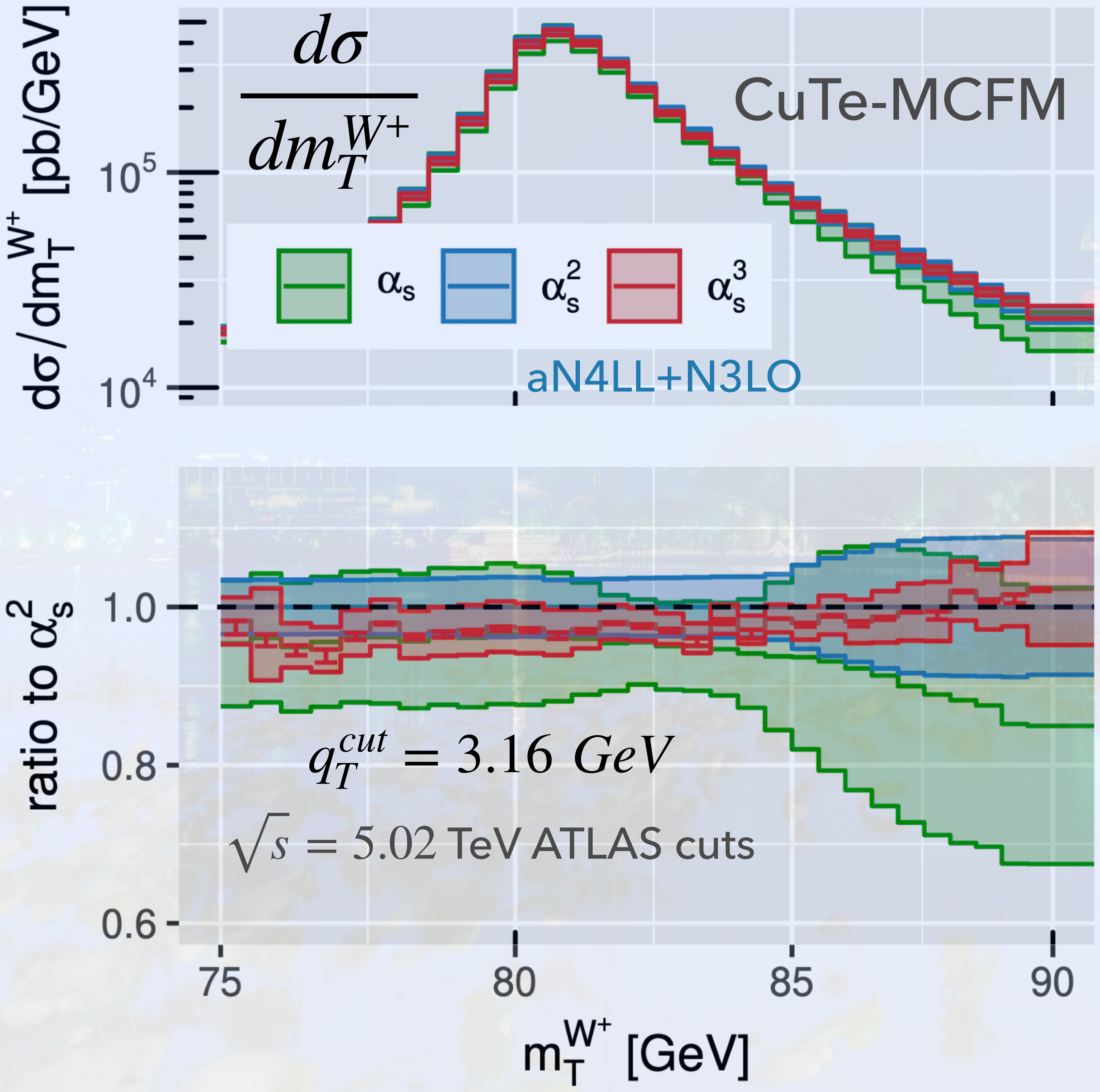
XC, Gehrmann, Glover, Huss, Monni, Re, Rottoli, Torrielli
Phys.Rev.Lett. 128 (2022) 25



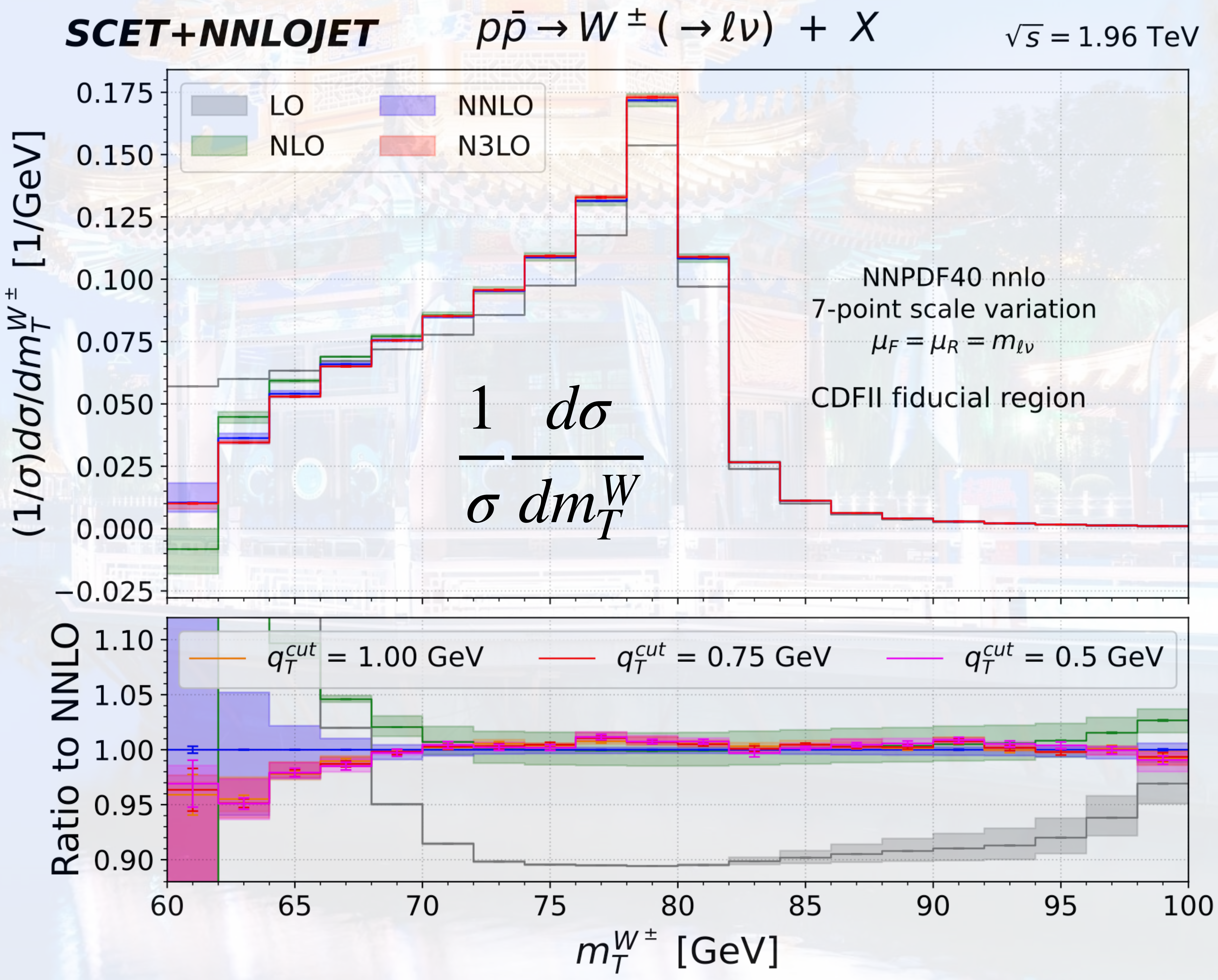
Neumann and Campbell *Phys.Rev.D* 107 (2023) 1

STATE-OF-THE-ART PREDICTIONS: $d\sigma_{N^3LO}$

➤Differential N3LO predictions for charged current production



Neumann and Campbell *JHEP 11 (2023) 127*



XC, Gehrmann, Glover, Huss, Yang, Zhu *Phys.Lett.B 840 (2023)*

STATE-OF-THE-ART PREDICTIONS: $d\sigma_{N^3LO}$

➤Numerical error during template generation

Plan B: grid with all D.O.F.: $$\frac{d\sigma}{dm_{l\nu}dp_T dy}\left[(1+\cos^2\theta)+\sum_{i=0}^{7}A_i f_i(\theta,\phi)\right]$$

ResBos2 approach for N3LL+NNLO

- ➤ Numerically challenging for D.O.F = 11 (may drop $A_{5,6,7}$ for being very small)
- ➤ MC error of each grid bin + interpolation error cross bins (prefer fine granularity)
- ➤ Once $A_i(p_T, y, m_{l\nu})$ available, no new calculation is needed for different fiducial cuts

Z+J @ NNLO

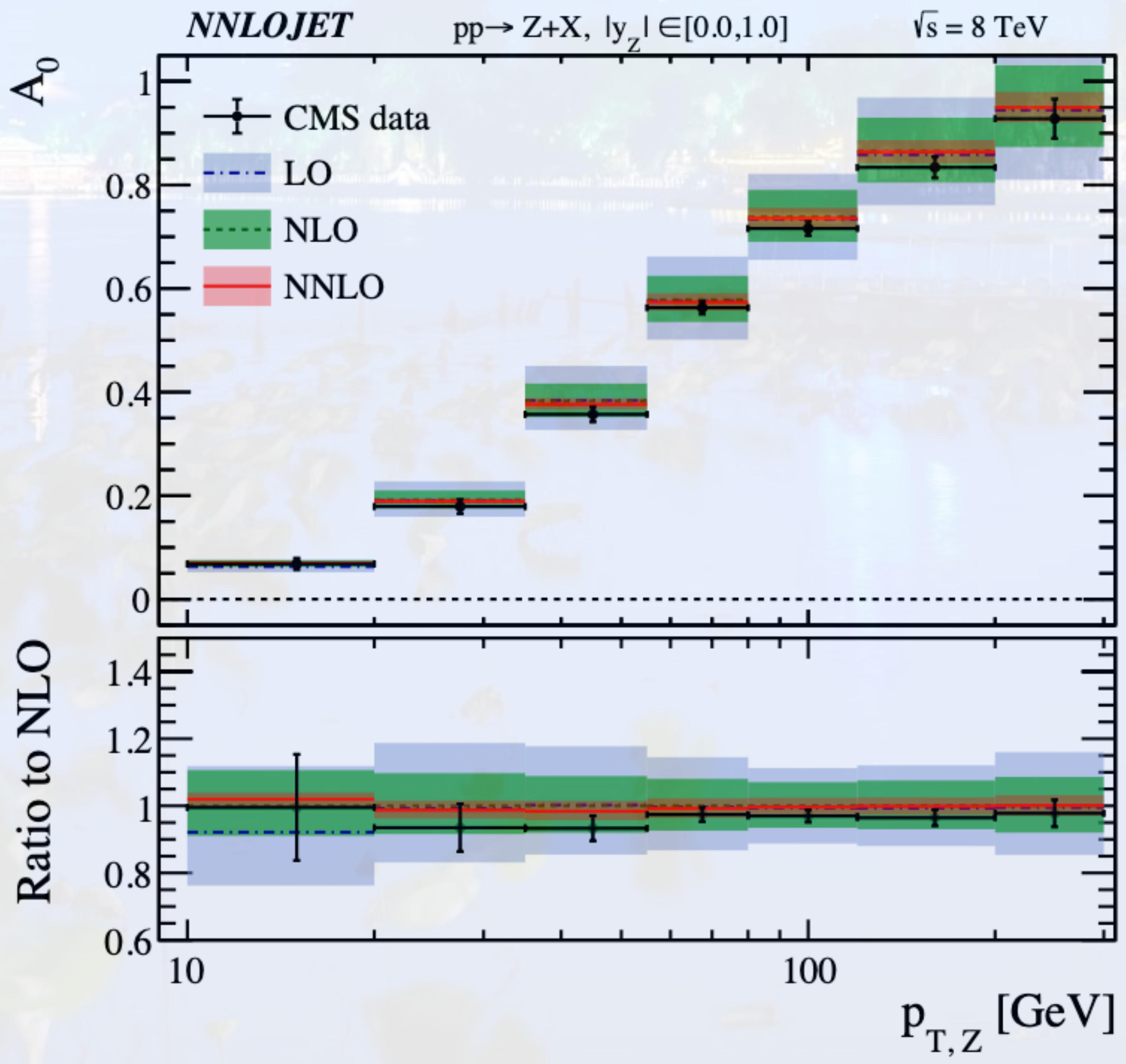
$A_0(p_T, y)$

$|y| < 1$

Inclusive in $m_{l\nu}$

Smallest bin @ 10 GeV

Gauld, Gehrmann-De Ridder, Gehrmann, Glover, Huss `17



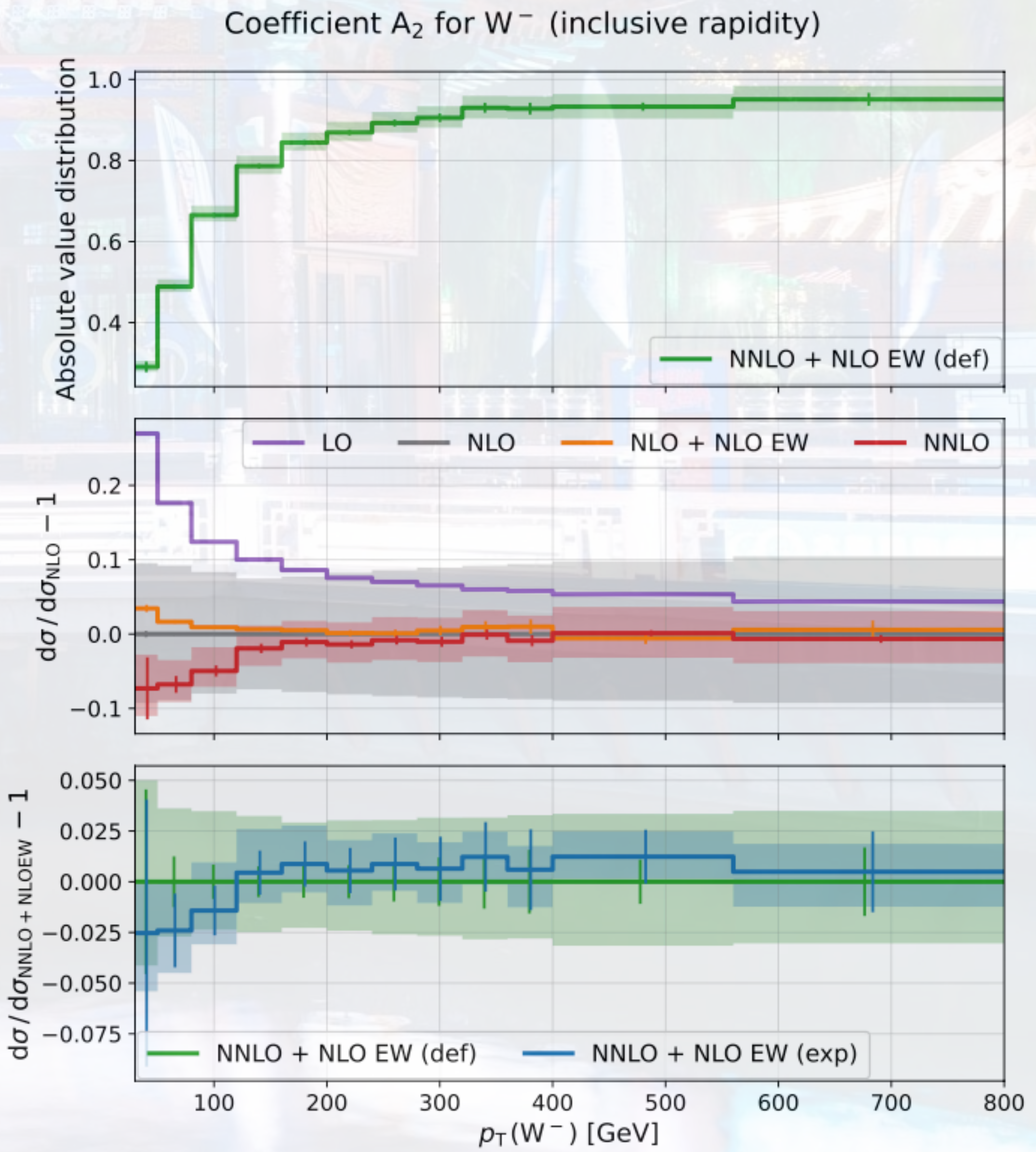
W+J @ NNLO

$A_2(p_T)$

Inclusive in y, $m_{l\nu}$

Smallest bin ~ 20 GeV

Pellen, Poncelet, Popescu, Vitos `22



CONCLUSION AND OUTLOOK

- The determination of m_W and α_s requires delicate treatment and thorough understanding of experiment and theory uncertainties.
- Theoretical uncertainties at 0.1% level is required to achieve 10 MeV accuracy in m_W
- Theoretical uncertainties at 1% level is required to reach ± 0.001 accuracy in α_s
- Best predictions for NC and CC DY production at N3LO QCD achieves 1% accuracy.
- Thorough study of resummation uncertainties (matching and scheme choice), mixed QCD-EW, approximated PDFs, indicate corrections and irreducible errors at % level.
- Both CDFII m_W and recent ATALS α_s measurement could be improved with more precise theory tools and more suitable error estimation.
- Require collective efforts to turn controversial results to convincing results: most of the controversial approximations are expected to be replaced during LHC Run 3.

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Thank You for Your Attention

Illustrator: Gaia Fontana

LIGHT QUARK MASS EFFECT AT SMALL TRANSVERSE MOMENTUM

➤At few GeV b and c quark mass are comparable to the resummation and factorisation scales

➤Retain full quark mass dependence in FO, PDF and resummation: GM-VFN scheme
Collins `98

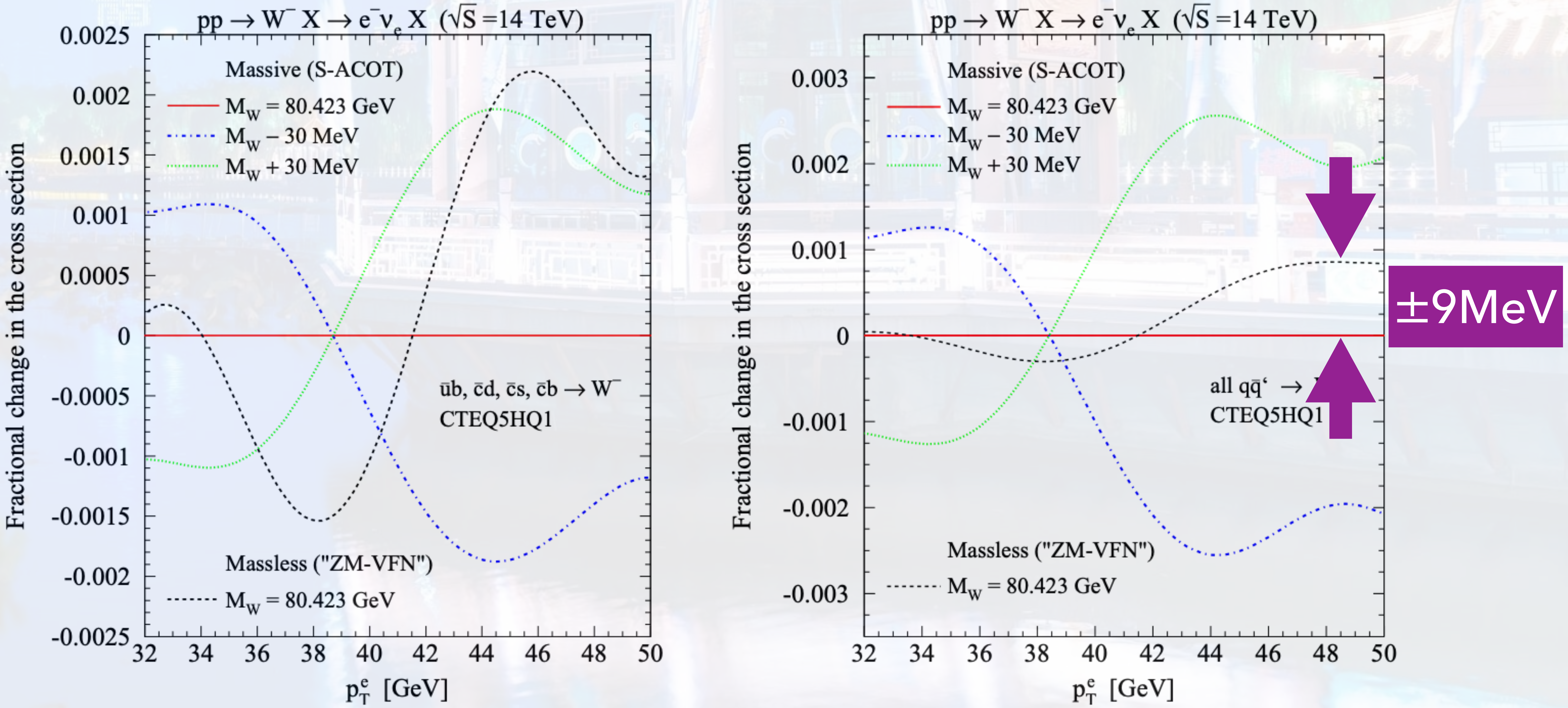
➤Reasonably good approximation in S-ACOT scheme (ignore quark mass from initial states)
Kramer, Olness and Soper `00
Nadolsky, Kidonakis, Olness, Yuan `03

➤NLO+NLL indicate 9 MeV (LHC) and 3 MeV (Tevatron) shift of mW.
Berge, Nadolsky, Olness `05

➤Extension to NNLL' is available
Pietrulewicz, Samitz, Spiering, Tackmann `17

➤Revisit m_q uncertainty in m_W is needed with modern tools! (FO, resummation scheme, PDF)

	W^+					W^-					Z^0				
Subprocesses	$u\bar{d}$	$u\bar{s}$	$c\bar{d}$	$c\bar{s}$	$c\bar{b}$	$d\bar{u}$	$s\bar{u}$	$d\bar{c}$	$s\bar{c}$	$b\bar{c}$	$u\bar{u}$	$d\bar{d}$	$s\bar{s}$	$c\bar{c}$	$b\bar{b}$
Tevatron Run-2	90	2	1	7	0	90	2	1	7	0	57	35	5	2	1
LHC	74	4	1	21	0	67	2	3	28	0	36	34	15	9	6



Fixed-order predictions for Drell-Yan production

Berge, Nadolsky, Olness `05

W MASS IN CDFII MEASUREMENT

- $d\sigma/dm_T^W$ Template fit to best best parameter values:
  - Relativistic Breit-Wigner form: $(s^2 - m_W^2 + is^2\Gamma_W/m_W)^{-1}$ with fixed Γ_W
  - Binned maximum-likelihood fit: (Poisson distribution cross bins)

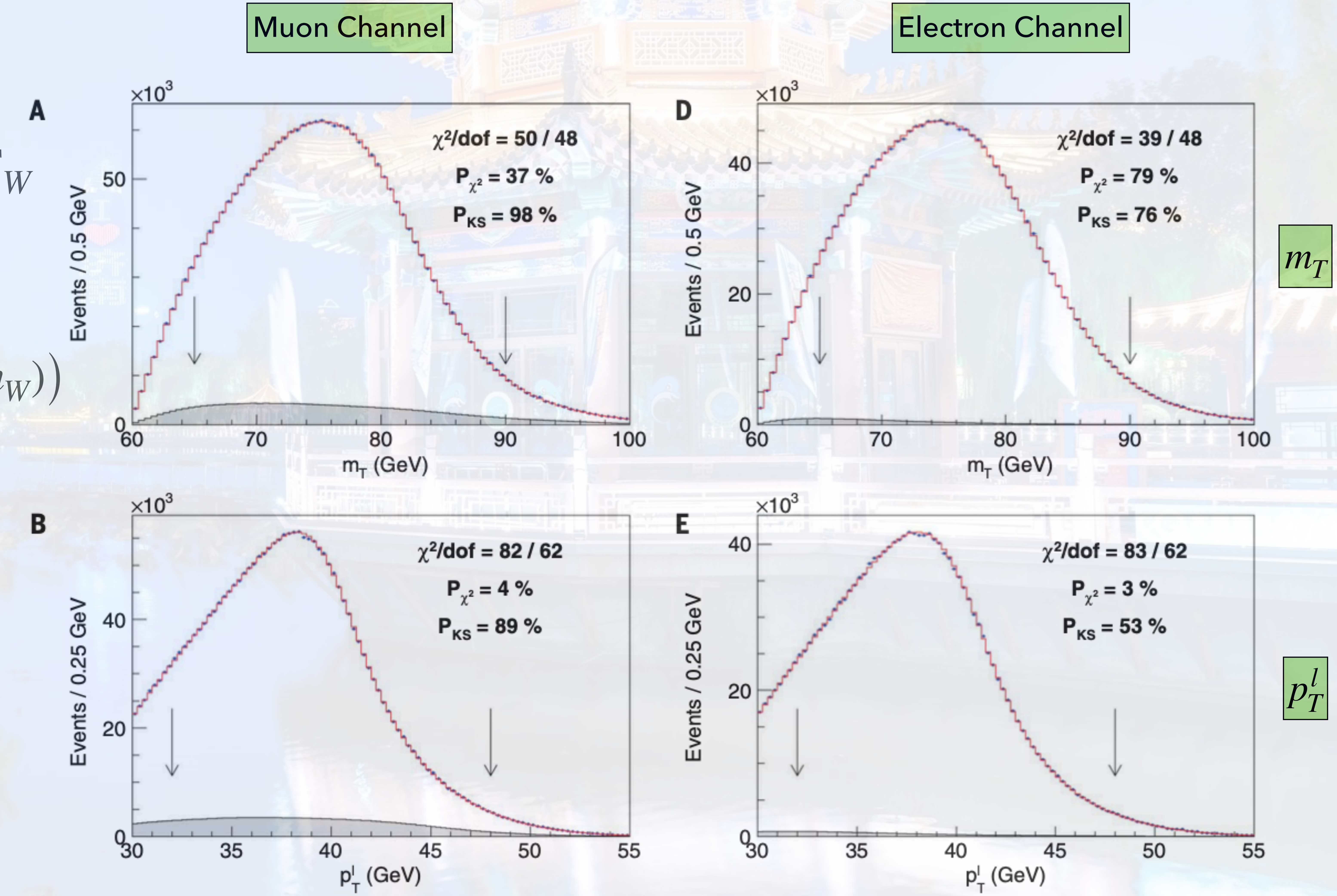
$$-\ln \mathscr{L}_b(m_W) = -\sum_b \left(n_b \ln\left(\Delta\sigma_b(m_W)\right) - \Delta\sigma_b(m_W)\right)$$

  n_b: observed event, $\Delta\sigma_b(m_W)$: predicted

  - The best linear unbiased estimator to combine each observable:
    - $\chi^2/\text{dof} = 7.4/5 \to$ p-value $= 20\%$
    - Weight distribution:

$$m_T^W \sim 64.2\,\%\,,\ p_T^l \sim 25.4\,\%\,,\ p_T^\nu \sim 10.4\,\%$$

CDFII: Best fitted results for m_T^W, p_T^l



PRECISION PREDICTIONS IN RESBOS2

- ResBos → ResBos2
  - NNLO+N3LL accuracy for W/Z production
    Isaacson, Fu, Yuan `23
  - Upgrade CSS formalism to N3LL
  - Rescale NLO to NNLO from MCFM:
    Campbell, Ellis and Giele `15

$$\frac{d\sigma^{A_i}_{NLO}}{dp_T dy dQ} \rightarrow K^{A_i}_{\frac{NNLO}{NLO}}(p_T, y, Q)\frac{d\sigma^{A_i}_{NLO}}{dp_T dy dQ}$$

  - Dependence of angular coefficients recently included with more rescaling: $\frac{d\sigma}{d\cos\theta d\phi}$

$$= L_0(1+\cos^2\theta) + A_0(1-3\cos^2\theta) + A_1\sin 2\theta\cos\phi$$
$$+A_2\sin^2\theta\cos 2\phi + A_3\sin\theta\cos\phi + A_4\cos\theta$$
$$+A_5\sin^2\theta\sin 2\phi + A_6\sin 2\theta\sin\phi + A_7\sin\theta\sin\phi$$

Isaacson, Fu and Yuan`22 `23

- Alternative tools on the market
  - NNLO QCD
    - DYNNLO Catani, Cieri, Ferrera, Florian Grazzini `09
    - FEWZ Gavin, Li, Petriello, Quackenbush `13
    - MATRIX Grazzini, Kallweit, Wiesemann `17
    - MCFM Boughezal et. al. `16 Campbell, Neumann `19
    - Cross check Alekhin, Kardos, Moch, Trocsanyi `21
  - NNLO + PS
    - MiNNLO_{PS} in POWHEG-BOX
      Monni, Nason, Re, Wiesemann, Zanderighi `20

PRECISION PREDICTIONS IN RESBOS2

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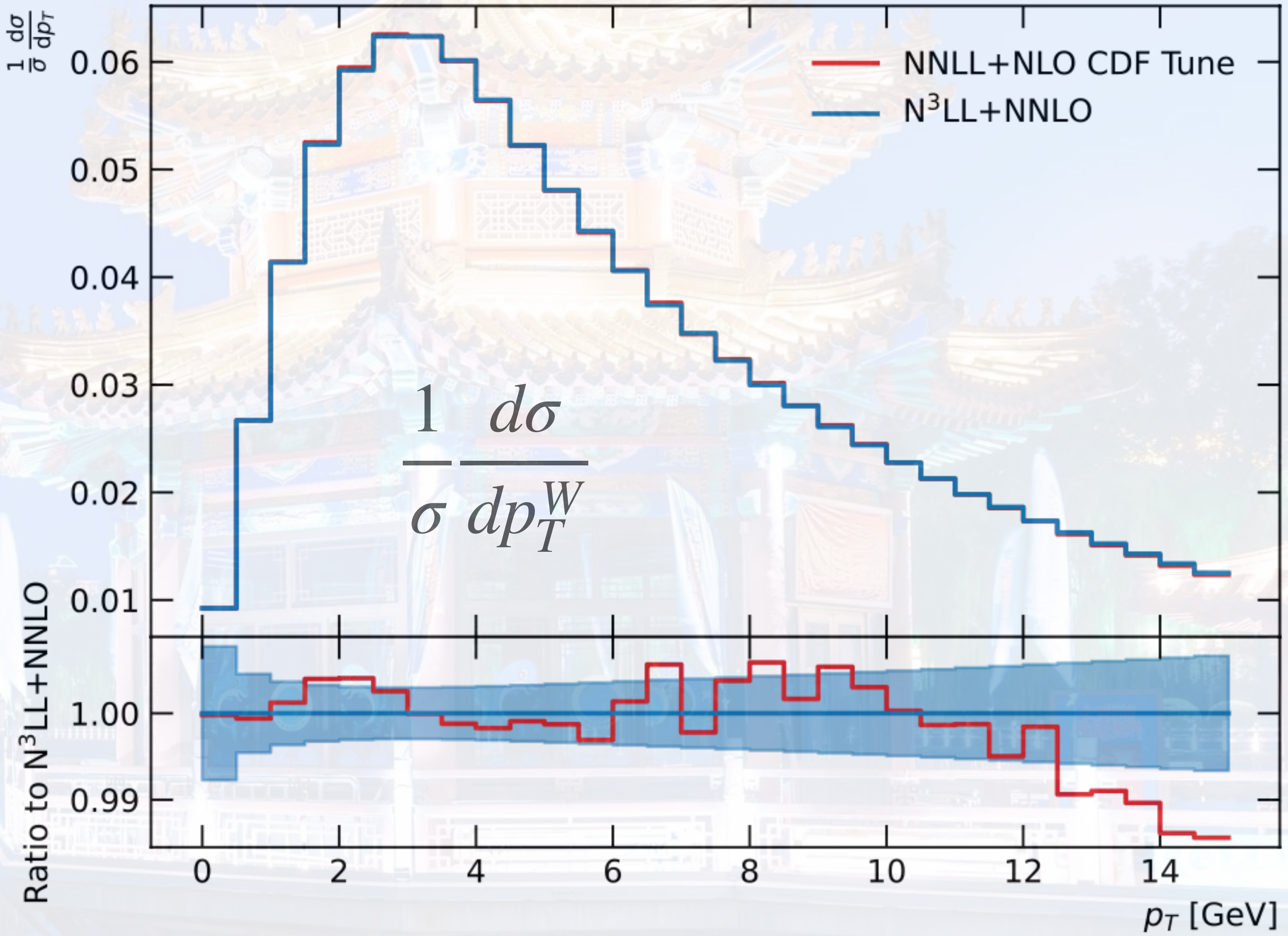
Isaacson, Fu and Yuan`22 `23



- Pseudo data: NNLO+N3LL p_T^Z with global fit
- Fit g2, α_s in NLO+NNLL p_T^Z to pseudo data
- Use fitted g2, α_s in NLO+NNLL W templates

PRECISION PREDICTIONS IN RESBOS2

Width	Mass Shift [MeV]
2.0475 GeV	2.0 ± 0.5
2.1315 GeV	0.3 ± 0.5
NLO	1.2 ± 0.5

➤W mass details by ResBos2 Isaacson, Fu and Yuan`22 `23

	m_T		$p_T(\ell)$		$p_T(\nu)$	
PDF Set	NNLO	NLO	NNLO	NLO	NNLO	NLO
CT18	0.0 ± 1.3	1.8 ± 1.2	0.0 ± 15.9	2.0 ± 14.3	0.0 ± 15.5	2.9 ± 14.2
MMHT2014	1.0 ± 0.6	2.6 ± 0.6	6.2 ± 7.8	36.7 ± 7.0	3.9 ± 7.5	36.0 ± 6.7
NNPDF3.1	1.1 ± 0.3	2.1 ± 0.4	2.1 ± 3.8	13.5 ± 4.9	5.4 ± 3.7	10.0 ± 4.9
CTEQ6M	N/A	2.8 ± 0.9	N/A	19.0 ± 10.4	N/A	20.9 ± 10.2

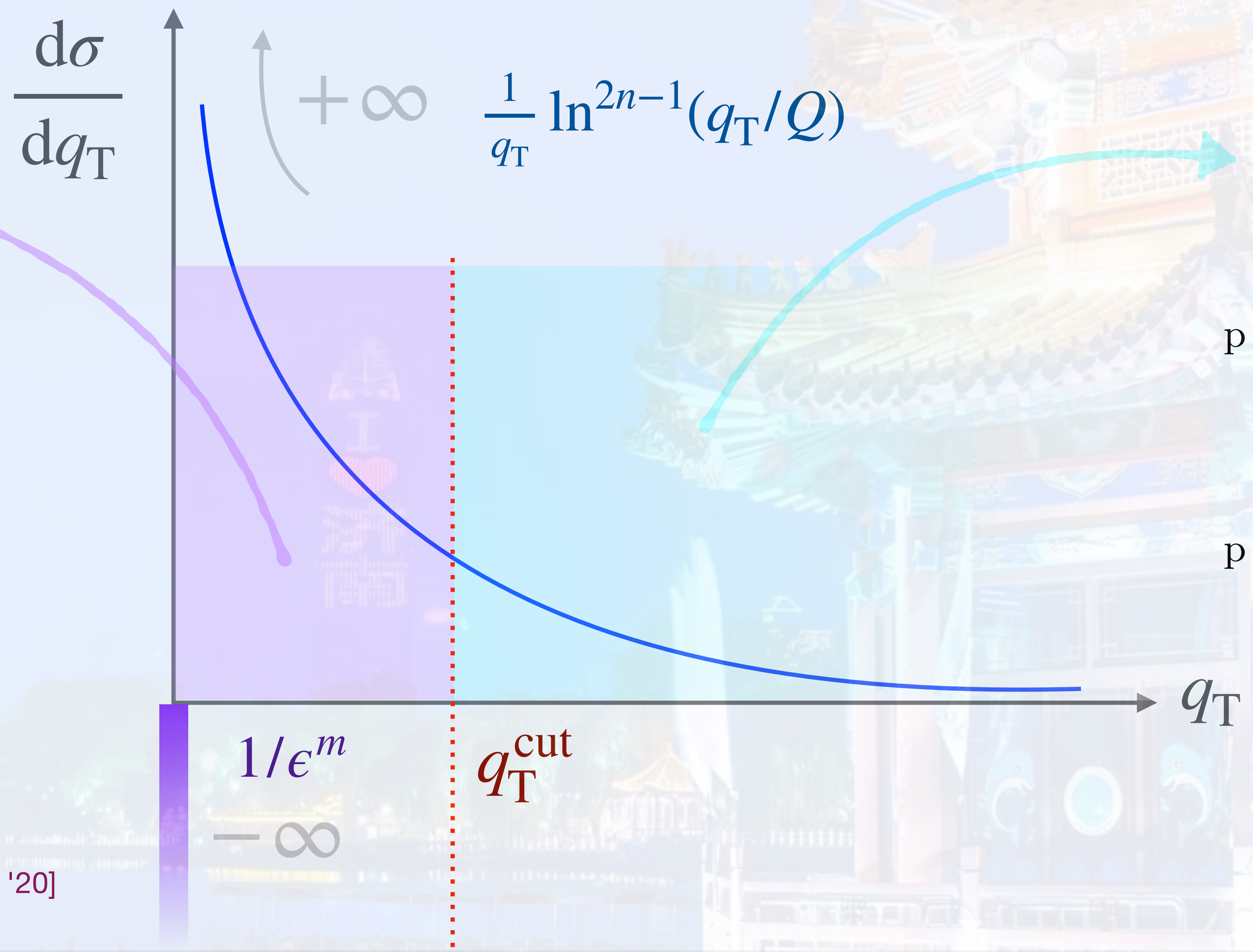
	Mass Shift [MeV]	
Observable	Smearing 1	Smearing 2
m_T	$0.2 \pm 1.8 \pm 1.0$	$1.0 \pm 2.1 \pm 1.3$
$p_T(\ell)$	$4.3 \pm 2.7 \pm 1.3$	$4.5 \pm 2.6 \pm 1.4$
$p_T(\nu)$	$3.0 \pm 3.4 \pm 2.2$	$3.8 \pm 4 \pm 2.7$

	Mass Shift [MeV]					
	m_T		$p_T(\ell)$		$p_T(\nu)$	
Scale	ResBos2	+Detector Effect+FSR	ResBos2	+Detector Effect+FSR	ResBos2	+Detector Effect+FSR
Upper	1.2 ± 0.5	$0.8 \pm 1.8 \pm 1.1$	3.1 ± 2.1	$-6.5 \pm 2.7 \pm 1.3$	1.4 ± 2.1	$-4.9 \pm 3.4 \pm 2.0$
Lower	1.2 ± 0.5	$-0.7 \pm 1.8 \pm 01.$	1.8 ± 2.1	$9.4 \pm 2.6 \pm 1.2$	0.0 ± 2.1	$4.8 \pm 3.4 \pm 1.9$

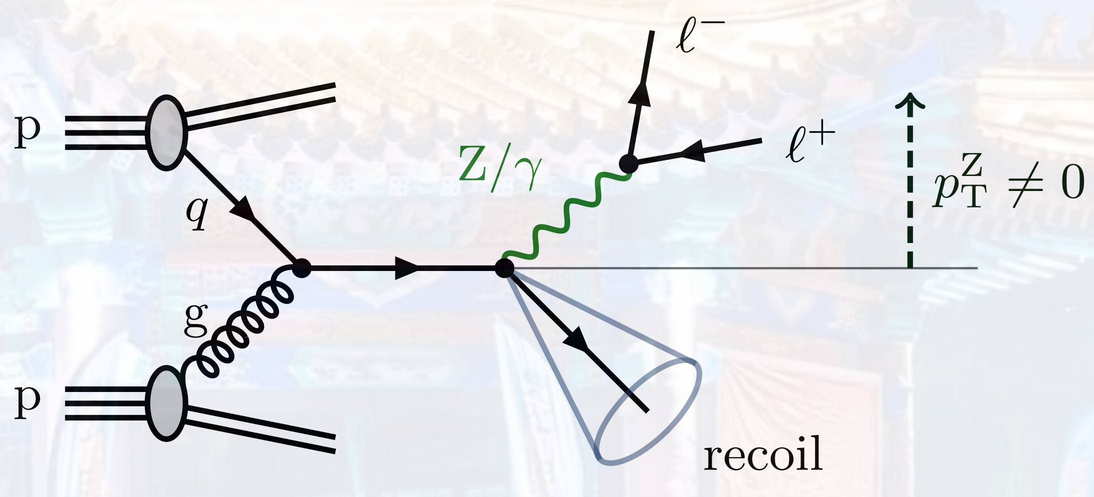
q_T Subtraction @ N³LO

q_T resummation

- expand to fixed order
- $\mathcal{O}(\alpha_s^3)$ ingredients:
  - hard function $H_{q\bar{q}}$ [Gehrmann, Glover, Huber, Ikizlerli, Studerus '10]
  - soft function $S(\mathbf{b}_\perp)$ [Li, Zhu '16]
  - beam function $B_q(\mathbf{b}_\perp)$ [Luo, Yang, Zhu, Zhu '19] [Ebert, Mistlberger, Vita '20]



V+jet @ NNLO



[Catani, Grazzini '07]

$$d\sigma^V_{N^3LO} = d\sigma^V_{N^3LO}\Big|_{q_T<q_T^{cut}} + d\sigma^V_{N^3LO}\Big|_{q_T>q_T^{cut}}$$

$$= \mathcal{H}^V_{N^3LO} \otimes d\sigma^V_{LO} + \left[d\sigma^{V+jet}_{NNLO} - d\sigma^{V,CT}_{N^3LO}\right]_{q_T>q_T^{cut}} + \mathcal{O}\left((q_T^{cut}/Q)^n\right)$$

Competing interests: q_T^{cut} as small as possible ⇝ q_T^{cut} as large as possible

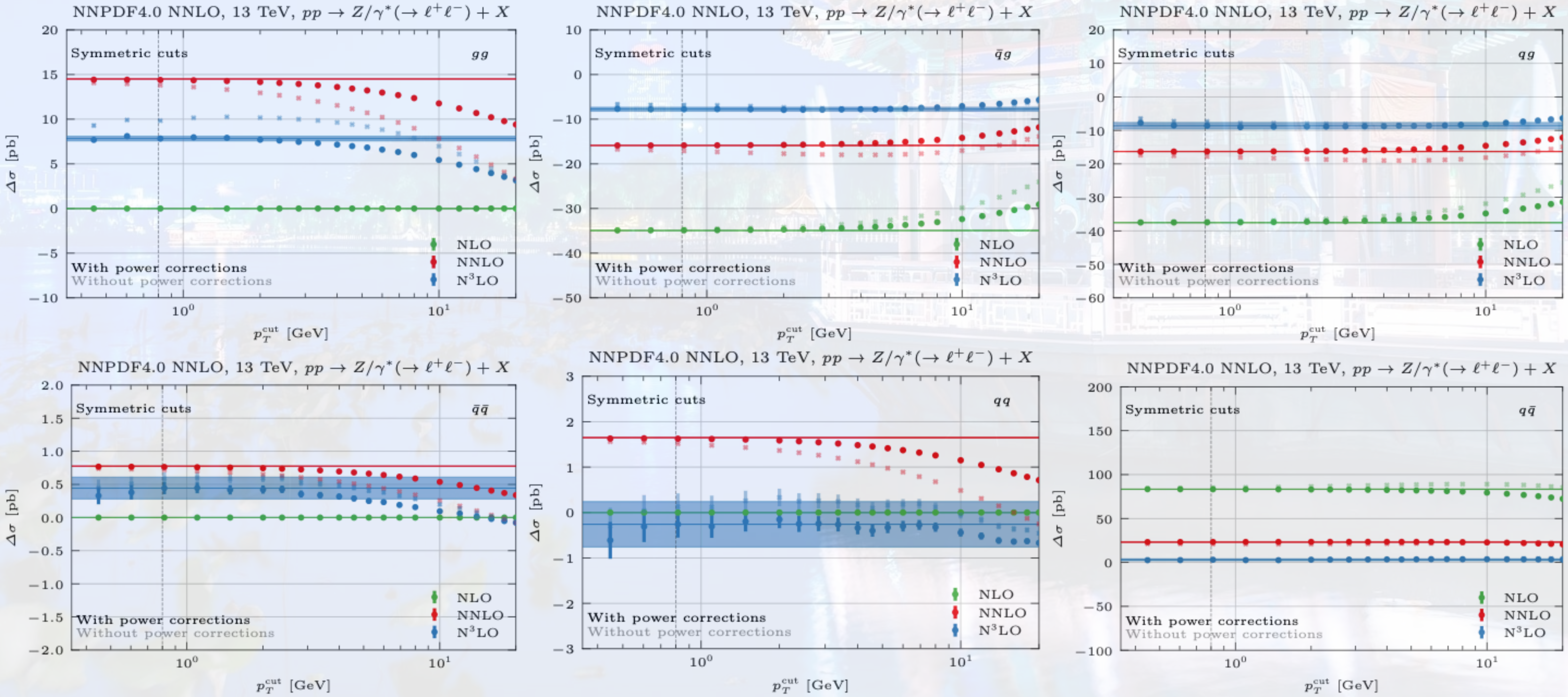
↪ suppress power corrections ↪ numerical stability & efficiency

BACKUP SLIDES

- Differential N3LO predictions for neutral current production with fiducial cuts
  - Resum all order contributions at N3LL using RadISH and matched to N3LO

Order	σ [pb] Symmetric cuts		σ [pb] Product cuts	
k	N^{k}LO	N^{k}LO+N^{k}LL	N^{k}LO	N^{k}LO+N^{k}LL
0	$721.16^{+12.2\%}_{-13.2\%}$	—	$721.16^{+12.2\%}_{-13.2\%}$	—
1	$742.80(1)^{+2.7\%}_{-3.9\%}$	$748.58(3)^{+3.1\%}_{-10.2\%}$	$832.22(1)^{+2.7\%}_{-4.5\%}$	$831.91(2)^{+2.7\%}_{-10.4\%}$
2	$741.59(8)^{+0.42\%}_{-0.71\%}$	$740.75(5)^{+1.15\%}_{-2.66\%}$	$831.32(3)^{+0.59\%}_{-0.96\%}$	$830.98(4)^{+0.74\%}_{-2.73\%}$
3	$722.9(1.1)^{+0.68\%}_{-1.09\%} \pm 0.9$	$726.2(1.1)^{+1.07\%}_{-0.77\%}$	$816.8(1.1)^{+0.45\%}_{-0.73\%} \pm 0.8$	$816.6(1.1)^{+0.87\%}_{-0.69\%}$

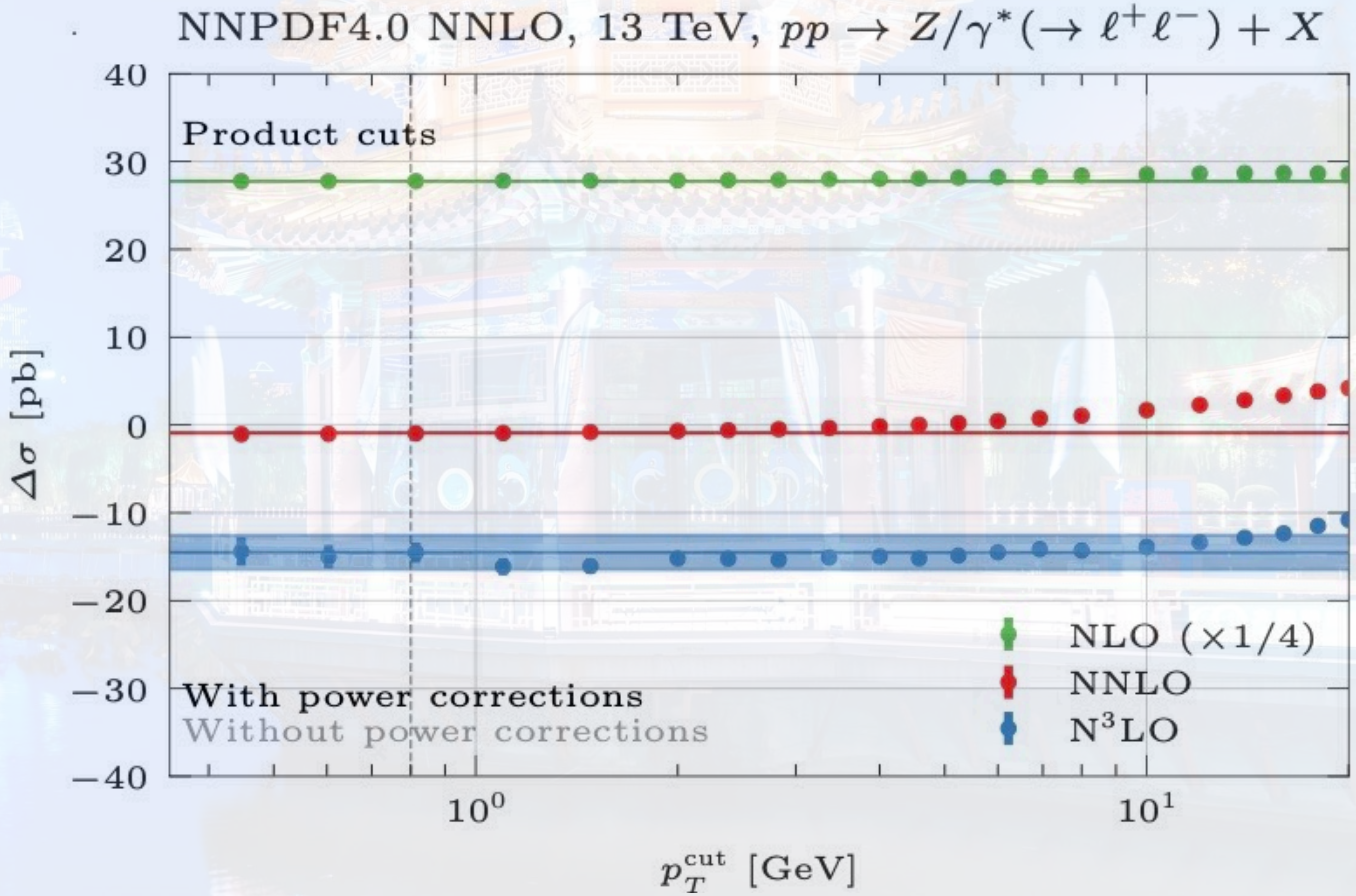
XC, Gehrmann, Glover, Huss, Monni, Rottoli, Re, Torrielli `22



BACKUP SLIDES

➤ Differential N3LO predictions for neutral current production with fiducial cuts

- Apply ATLAS fiducial cuts at 13 TeV
  - Dynamical scale $\mu_F = \mu_R = \sqrt{m_{ll}^2 + p_T^{ll2}}$
  - $m_{ll} \in [66,116]$ GeV, $|\eta^{l^\pm}| < 2.5$
  - Symmetric cuts: $|p_T^{l^\pm}| > 27$ GeV
    Introduce power correction at $\mathcal{O}(q_T^{cut}/m_{ll})$
  - Solution:
    - Apply Lorentz Boost below q_T^{cut}
      Buonocore, Rottoli, Kallweit, Wiesemann `21
      Camarda, Cieri, Ferrera `21
    - Product cuts: $\sqrt{p_T^{l^+} p_T^{l^-}} > 27$ GeV
      Salam, Slade `21
      $\min\{p_T^{l^+}, p_T^{l^-}\} > 20$ GeV

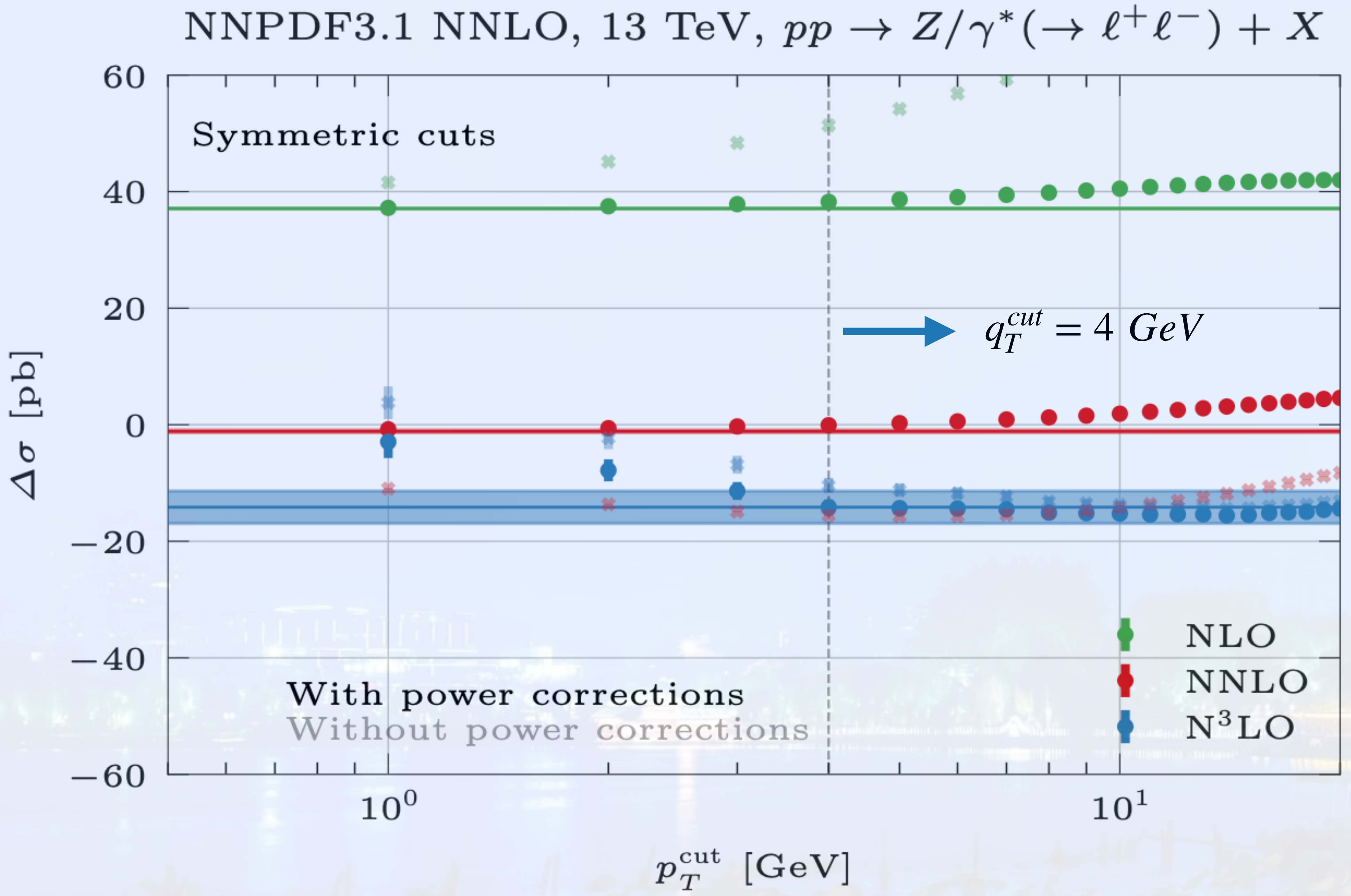


XC, Gehrmann, Glover, Huss, Monni, Rottoli, Re, Torrielli `22

- Typical fiducial cuts for m_T^V, p_T^V in DY production
  - Large log terms appear in $p_T^l \sim m_V/2$, $m_T^V \sim 2 \times \min[p_T^l]$, $p_T^V \sim 0$

Precision Predictions at Hadron Collider

$2 \to 1$ @ N3LO (+ N3LL) QCD



XC, T. Gehrmann, N. Glover, et. al. PRL 128, 252001 (2022)

- Solid horizontal lines: NLO, NNLO at 1 GeV, N3LO at 4 GeV with MC error.
  - N3LO shows no plateau in 1905.05171
- Pale dots are values used by DYTurbo in 2103.04974 and 2303.12781 (taken from 1905.05171).
  - Fiducial power corrections are not included.
  - Leads to 30% difference of N3LO coefficients at $q_T^{cut} = 4\ GeV$.
- Solid dots are corrected values with fiducial power correction.
  - Central value shifts 2 pb starting from NLO (the dominant error).
  - ± 2.1 pb uncertainty from MC and q_T^{cut} (estimated from [3,5] GeV region).
  - Not consistent with DYTurbo update result of ± 0.7 pb uncertainty.

DYTurbo result with fiducial power correction

Order	N^3LO
q_T subtr. ($q_T^{cut} = 4\,\text{GeV}$)	$747.1 \pm 0.7\,\text{pb}$
recoil q_T subtr.	$745.7 \pm 0.7\,\text{pb}$

S. Camarda, L. Cieri, G. Ferrera Eur.Phys.J.C 82 (2022) 6

DYTurbo result without fiducial power correction cited in ATLAS α_s fitting

Order	NLO	NNLO	N^3LO
$\sigma(pp \to Z/\gamma^* \to l^+l^-)$ [pb]	766.3 ± 1	757.4 ± 2	746.1 ± 2.5
Order	NLL+NLO	NNLL+NNLO	N^3LL+N^3LO
$\sigma(pp \to Z/\gamma^* \to l^+l^-)$ [pb]	773.7 ± 1	759.8 ± 2	749.6 ± 2.5

*S. Camarda, L. Cieri, G. Ferrera Phys. Rev. D **104**, L111503 (2021)*