

Theory uncertainties in the α_s measurement from $p_T Z$

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**Discussion of theoretical systematics in LHC
precision measurements
26 Feb 2024**

- Introduction
- Quick recap of the Z p_T cross section measurement
- Theory uncertainties
- Conclusions

Measure $\alpha_s(m_Z)$ from semi-inclusive DY

QCD COHERENT BRANCHING AND SEMI-INCLUSIVE PROCESSES AT LARGE x^*

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- Measuring $\Lambda_{\overline{MS}}$ from semi-inclusive (radiation inhibited) DY cross sections was first proposed in [Nucl. Phys. B 349 \(1991\) 635-654](#)
- Use Monte Carlo parton showers to determine Λ_{MC} and convert to $\Lambda_{\overline{MS}}$

Resummation arguments show that a set of universal QCD corrections can be absorbed in coherent parton showers by applying the Catani-Marchesini-Webber (CMW) rescaling of the \overline{MS} value of Λ_{QCD}

$$P(\alpha_s, z) = \frac{\alpha_s}{2\pi} C_F \frac{1+z^2}{1-z} + \left(\frac{\alpha_s}{\pi}\right)^2 \frac{A^{(2)}}{1-z}$$



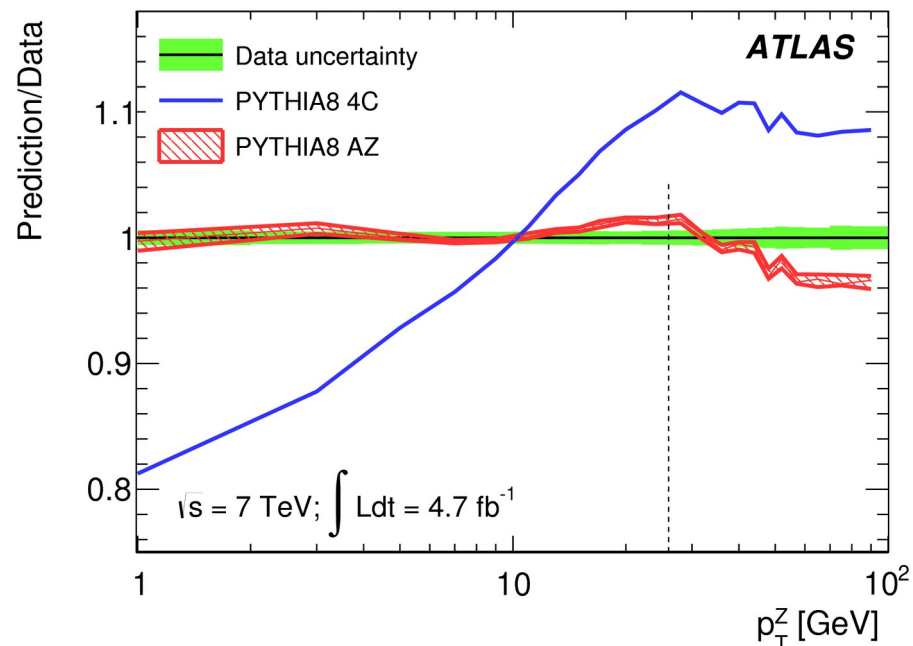
$$\alpha_s^{(MC)} = \alpha_s^{(\overline{MS})} \left(1 + K \frac{\alpha_s^{(\overline{MS})}}{2\pi} \right)$$

- The Z p_T distribution at small transverse momentum is one of such semi-inclusive observables

Measure $\alpha_s(m_Z)$ with MC parton showers

- ATLAS 7 TeV result on Pythia 8 MC tuning to the Z p_T distribution can be seen as a first step towards the measurement of $\alpha_s(m_Z)$

JHEP 09 (2014) 145



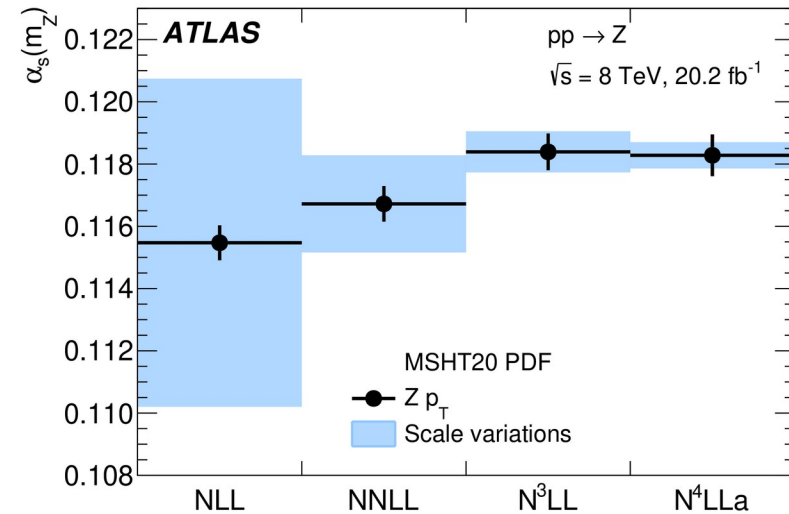
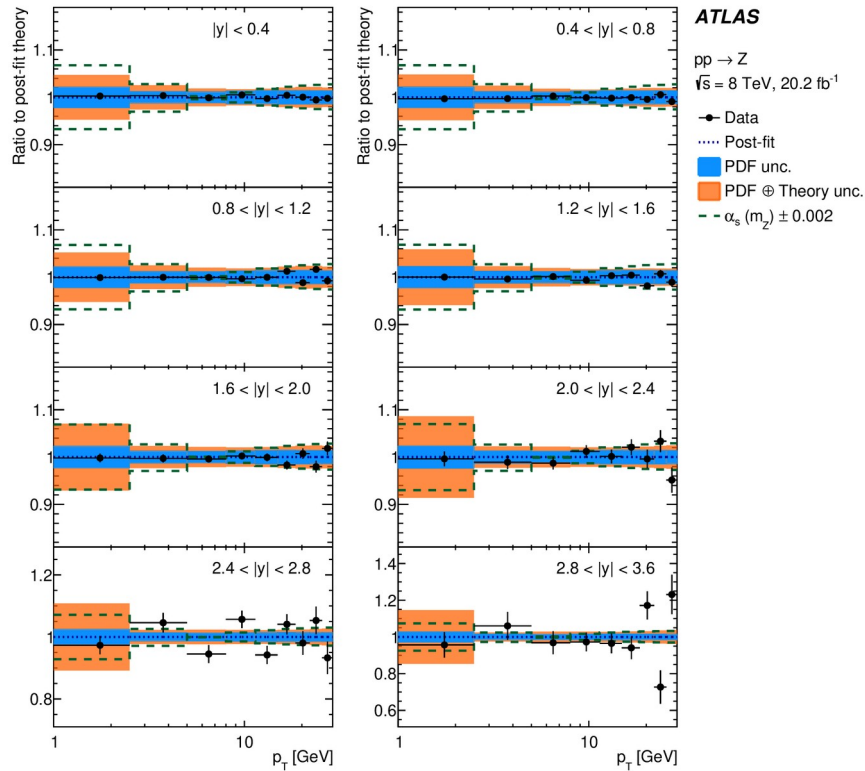
PYTHIA8	
Tune Name	AZ
Primordial k_T [GeV]	1.71 ± 0.03
ISR $\alpha_s^{\text{ISR}}(m_Z)$	0.1237 ± 0.0002
ISR cut-off [GeV]	0.59 ± 0.08
$\chi_{\text{min}}^2/\text{dof}$	45.4/32

$$a_s^{\text{CMW}}(m_Z) = 0.124 \rightarrow a_s^{\overline{\text{MS}}}(m_Z) = 0.116$$

- Naive result missing important theory uncertainties as PDFs and missing higher order corrections
- However this simple exercise already shows:
 - Great experimental sensitivity (0.2%)
 - Possible to control non-perturbative QCD uncertainties, insofar primordial k_T and shower cut-off are fitted simultaneously with α_s

Measure $\alpha_s(m_Z)$ with analytic resummation

arXiv:2309.12986



$$\alpha_s = 0.1183 \pm 0.0009$$

- In 2023 the first ATLAS determination of $\alpha_s(m_Z)$ from $p_T Z$, after a decade of experimental and theoretical developments
- The $Z p_T$ spectrum is measured with 0.2% uncertainty, the total uncertainty is dominated by theory uncertainties, which are the focus of today's talk

Experimental uncertainty	± 0.44
PDF uncertainty	± 0.51
Scale variation uncertainties	± 0.42
Matching to fixed order	0 -0.08
Non-perturbative model	$+0.12$ -0.20
Flavour model	$+0.40$ -0.29
QED ISR	± 0.14
N^4LL approximation	± 0.04
Total	$+0.91$ -0.88

Theoretical inputs

- Many theory results are directly or indirectly used for this measurement

→ 4/5-loop results

- P. A. Baikov, K. G. Chetyrkin and J. H. Kühn, Five-Loop Running of the QCD coupling constant, Phys. Rev. Lett. 118 (2017) 082002
- F. Herzog et al., Five-loop contributions to low- N non-singlet anomalous dimensions in QCD, Phys. Lett. B 790 (2019) 436
- C. Duhr, B. Mistlberger and G. Vita, Four-Loop Rapidity Anomalous Dimension and Event Shapes to Fourth Logarithmic Order, Phys. Rev. Lett. 129 (2022) 162001
- I. Moul, H. X. Zhu and Y. J. Zhu, The four loop QCD rapidity anomalous dimension, JHEP 08 (2022) 280
- A. Chakraborty et al., Hbb vertex at four loops and hard matching coefficients in SCET for various currents, Phys. Rev. D 106 (2022) 074009
- J. McGowan, T. Cridge, L. A. Harland-Lang and R. S. Thorne, Approximate N3LO Parton Distribution Functions with Theoretical Uncertainties: MSHT20aN3LO PDFs, Eur. Phys. J. C 83 (2023)

→ V+jet at NNLO

- T. Neumann and J. Campbell, Fiducial Drell–Yan production at the LHC improved by transverse-momentum resummation at N4LLp + N3LO, Phys. Rev. D 107 (2023) L011506
- R. Boughezal et al., Z-boson Production in Association with a Jet at Next-to-Next-To-Leading Order in Perturbative QCD, Phys. Rev. Lett. 116 (2016) 152001
- A. G.-D. Ridder, T. Gehrmann, E. Glover, A. Huss and T. Morgan, Precise QCD Predictions for the Production of a Z Boson in Association with a Hadronic Jet, Phys. Rev. Lett. 117 (2016)

→ CdFG formalism

- G. Bozzi, S. Catani, D. de Florian and M. Grazzini, Transverse-momentum resummation and the spectrum of the Higgs boson at the LHC, Nucl. Phys. B 737 (2006) 73
- S. Catani, D. de Florian, G. Ferrera and M. Grazzini, Vector boson production at hadron colliders: transverse-momentum resummation and leptonic decay, JHEP 12 (2015) 047

→ DYTurbo

- S. Camarda et al., DYTurbo: Fast predictions for Drell–Yan processes, Eur. Phys. J. C 80 (2020) 251
- S. Camarda, L. Cieri and G. Ferrera, Drell–Yan lepton-pair production: qT resummation at N3LL accuracy and fiducial cross sections at N3LO, Phys. Rev. D 104 (2021) L111503
- S. Camarda, L. Cieri and G. Ferrera, Drell–Yan lepton-pair production: qT resummation at N4LL accuracy, Physics Letters B 845 (2023) 138125

→ QED resummation

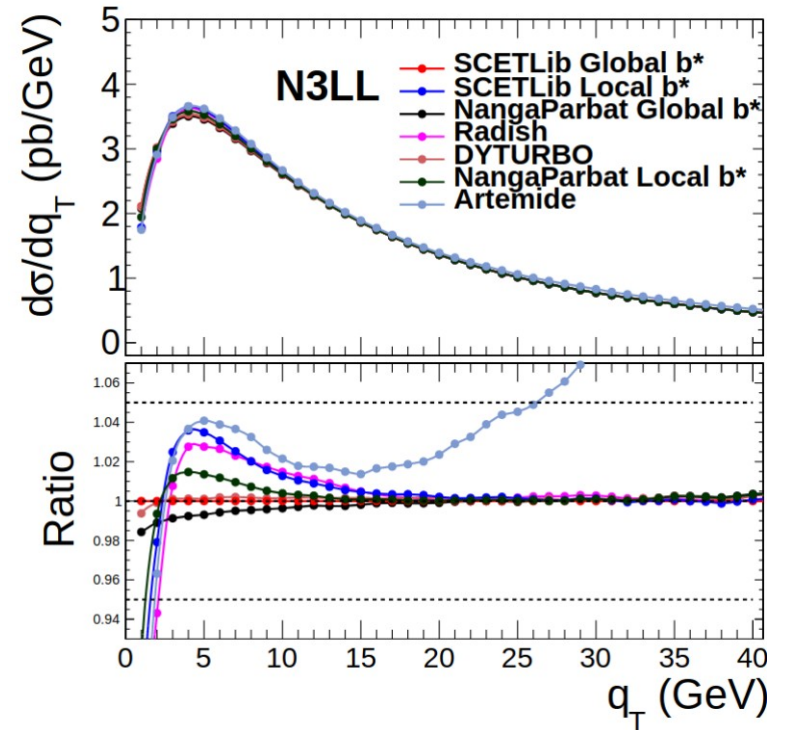
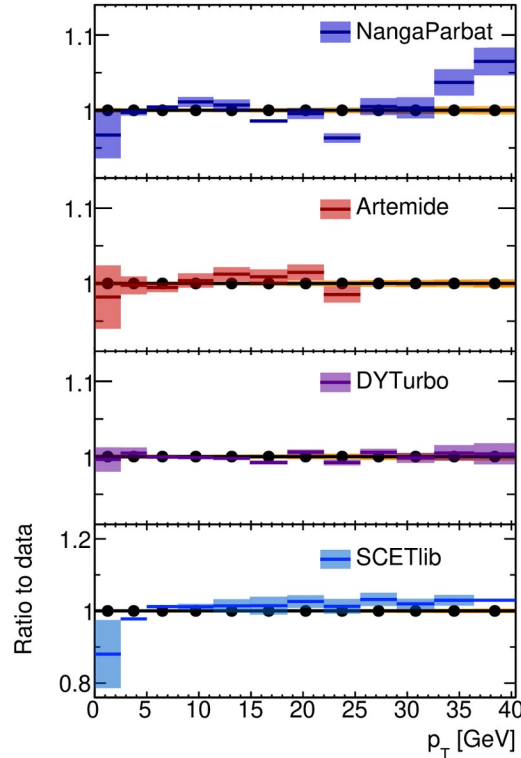
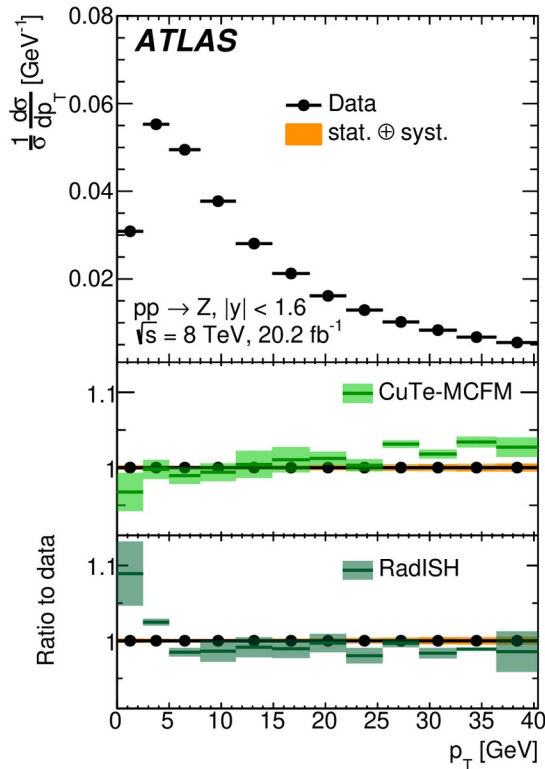
- L. Cieri, G. Ferrera and G. F. R. Sborlini, Combining QED and QCD transverse-momentum resummation for Z boson production at hadron colliders, JHEP 08 (2018) 165

→ Non pQCD

- J. Collins and T. Rogers, Understanding the large-distance behavior of transverse-momentum-dependent parton densities and the Collins-Soper evolution kernel, Phys. Rev. D 91 (2015) 074020

→ [...]

W/Z p_T benchmark



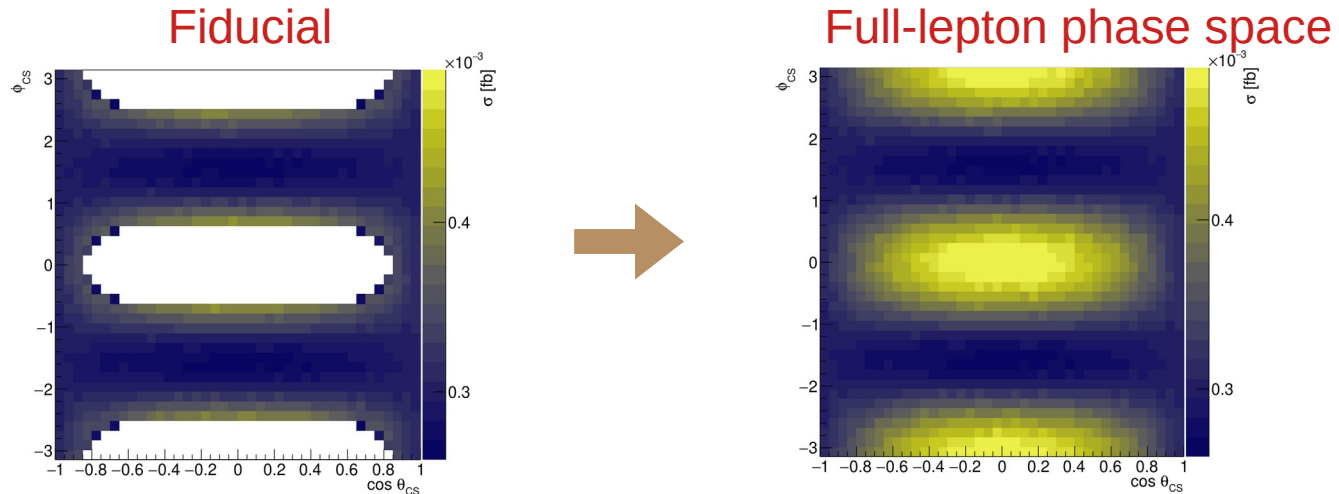
Program	Formalism	Type	Reference
DYTURBO	CdFG	b-space	arXiv:1910.07049
CuTe+MCFM	SCET	p_T -space	arXiv:2207.07056
NangaParbat	TMD	b-space	arXiv:1912.07550
Artemide	TMD	b-space	arXiv:1706.01473
Radish	PS-like	p_T -space	arXiv:2104.07509
SCETlib	SCET	b-space	arXiv:2102.08039

- The benchmark work in the EW precision WG was also an important input for this measurement
- Achieved an excellent agreement between q_T -resummation approaches, and with the measured cross sections, and an improved understanding of many physics and technical details of q_T resummation
- $O(\alpha_s^3)$ matching corrections from MCFM/NNLOjet

Cross section measurement, quick recap

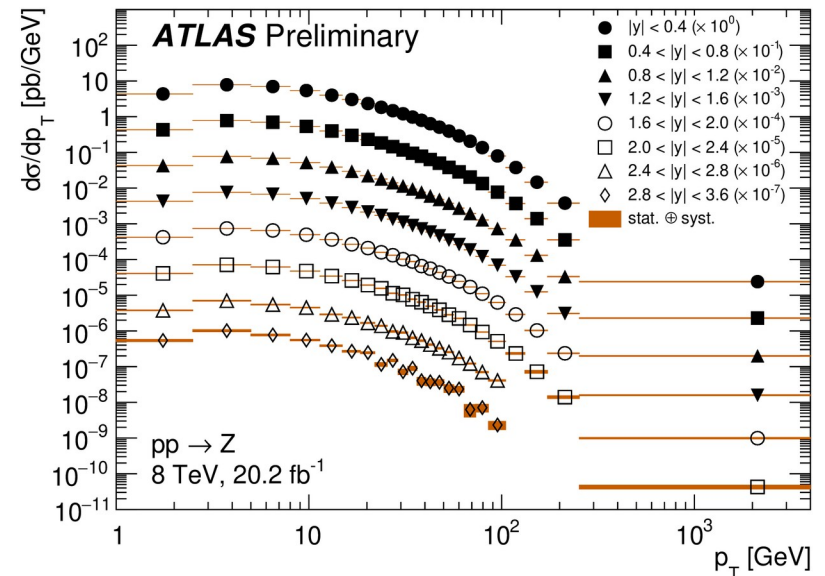
- Cross section measurement based on the angular coefficients decomposition
- Measurement defined in full-lepton phase without relying on theory extrapolation

$$\frac{d\sigma}{dpdq} = \frac{d^3\sigma^{U+L}}{dp_T dy dm} \left(1 + \cos^2 \theta + \sum_{i=0}^7 A_i(y, p_T, m) P_i(\cos \theta, \phi) \right)$$



- $80 < m_{ll} < 100$ GeV
- Double differential p_T, y cross section
 - 8 y bins over $|y| < 3.6$
 - 23 p_T bins

- The measurement methodology already addresses and removes the A_i uncertainty
- Predictions in full-lepton phase space are not affected by polarization uncertainties, q_T -recoil ambiguities, or fiducial power corrections
- Example of using large statistics to reduce theory uncertainties



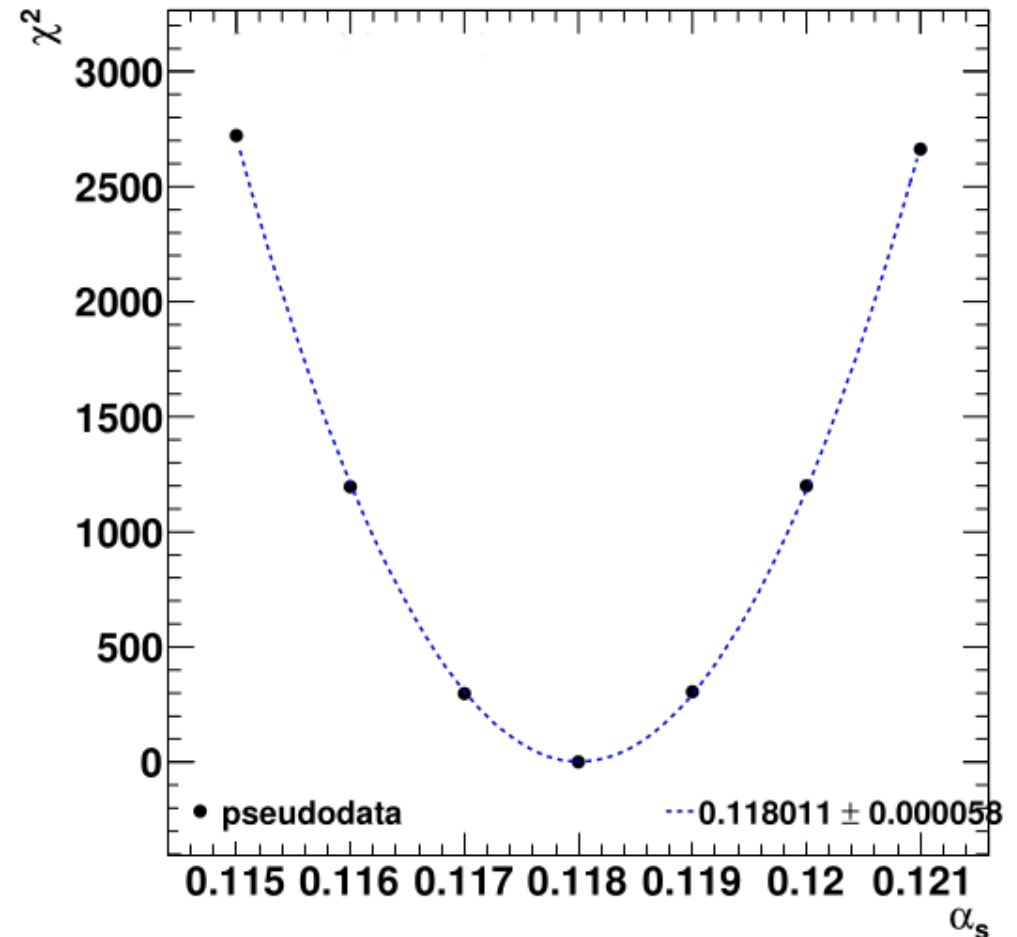
Experimental sensitivity

- Scanning the $\alpha_s(m_Z)$ dependence of the χ^2 with α_s -series of PDFs, to account for correlation with the gluon PDF
- No additional theory uncertainties, no free theory parameters in the fit
- Methodological closure and experimental sensitivity evaluated with Asimov dataset:

$$\alpha_s(m_Z) = 0.11801 \pm 0.00006$$

$$\Delta\alpha_s/\alpha_s = 0.05\%$$

- Pure experimental uncertainties are 20 times smaller than theory uncertainties
- Great potential to use the data to constrain theory uncertainties



Theory uncertainties

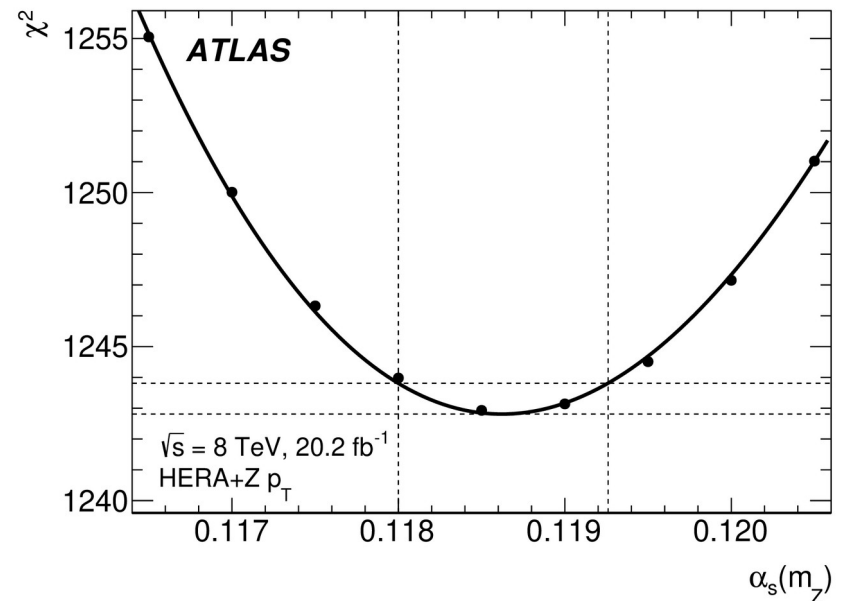
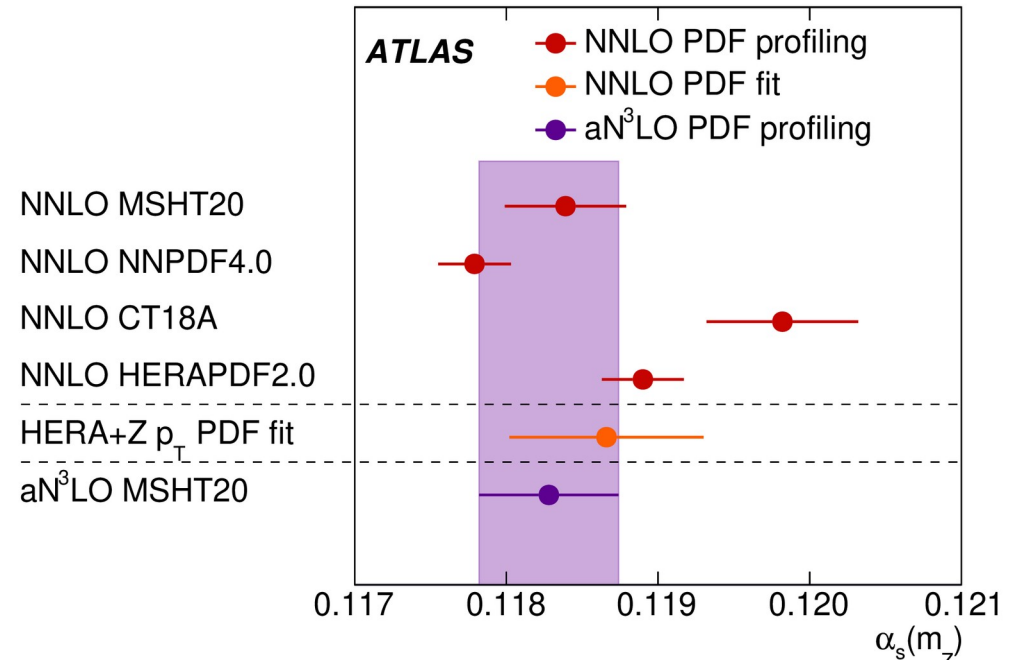
- PDFs
- Missing higher orders
- Matching to fixed order
- Non pQCD
- Heavy flavours
- QED/EW
- 4-loop approximations

Experimental uncertainty	± 0.44	
PDF uncertainty	± 0.51	
Scale variation uncertainties	± 0.42	
Matching to fixed order	0	-0.08
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Total	+0.91	-0.88

- Only PDFs and two non perturbative parameters are included in the fit. The NP parameters are included with no prior (conservative)
- All the rest of theory uncertainties are treated with the offset conservative method, i.e. the precision of the data is not used to reduce them

PDF uncertainties

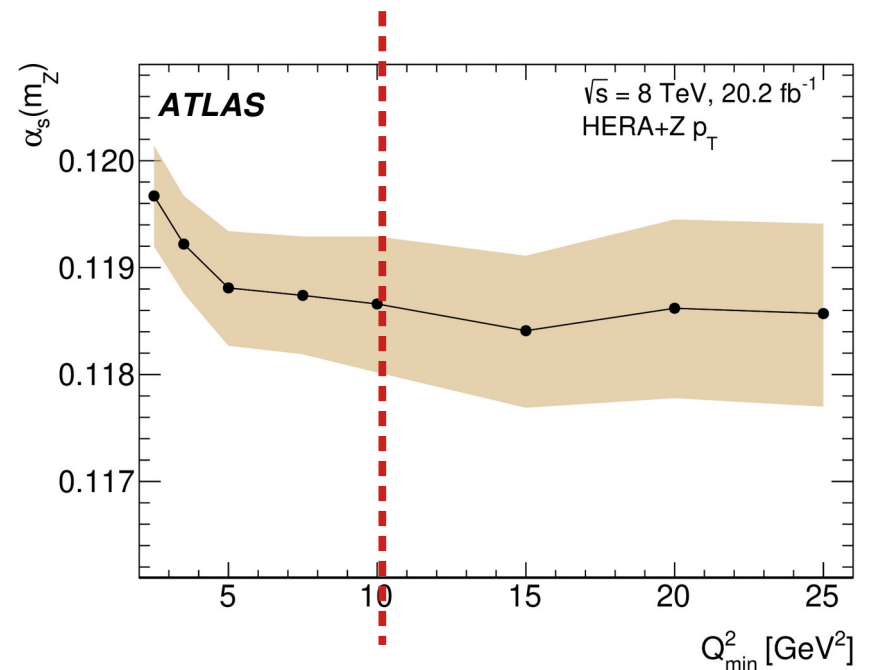
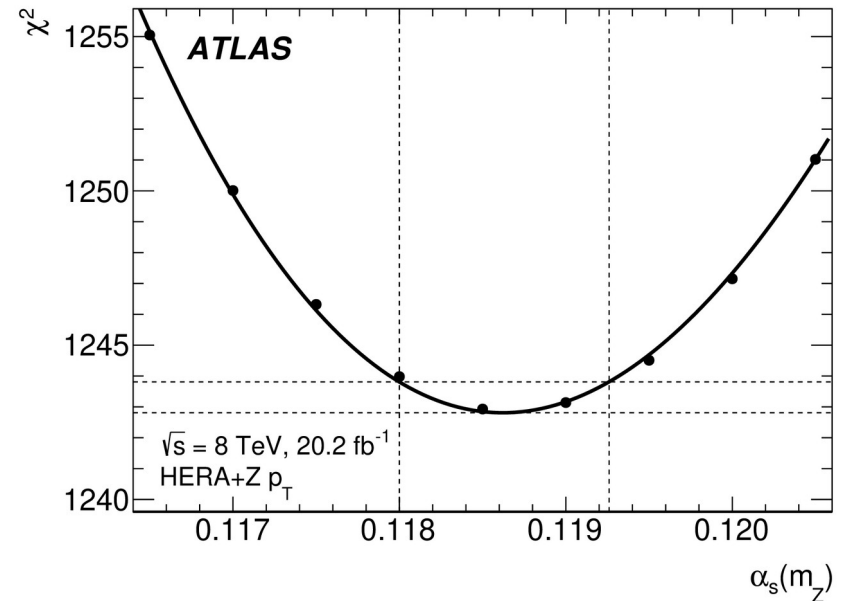
- The MSHT20 aN3LO PDF set is used for the nominal result
- Four other PDF sets at NNLO are considered, they all agree with the nominal result within PDF uncertainties, except CT18
- A simultaneous fit of PDFs, non perturbative QCD parameters, and $\alpha_s(m_Z)$ to HERA+Z p_T , agrees with global PDF sets at NNLO and aN3LO (except CT18) within fit uncertainties



- In order to understand the tension of CT18 with the other PDF sets, a test is performed in which the inclusive DIS HERA data is added to the PDF profiling
 - ➔ The value of $\alpha_s(m_Z)$ with CT18 is shifted by -1.7×10^{-3} , compared to a CT18 PDF uncertainty of 0.5×10^{-3}
 - ➔ Values extracted with other NNLO PDF sets are shifted by less than 0.5×10^{-3}
 - ➔ After reinclusion of the HERA data in the PDF profiling, all $\alpha_s(m_Z)$ values extracted with NNLO PDF sets are within $\pm 0.2 \times 10^{-3}$
- Older CT14 PDF set is also in good agreement with other NNLO PDF sets, and in tension with CT18
- Older versions of MSHT and NNPDF are in agreement with newer versions
- All flavours of CT18 (CT18, CT18A, CT18Z) give similar results
- Indication that in the CT18 PDF set the gluon PDF is pulled away from what is preferred by DIS data, to accommodate tensions with other datasets sensitive to the gluon PDF

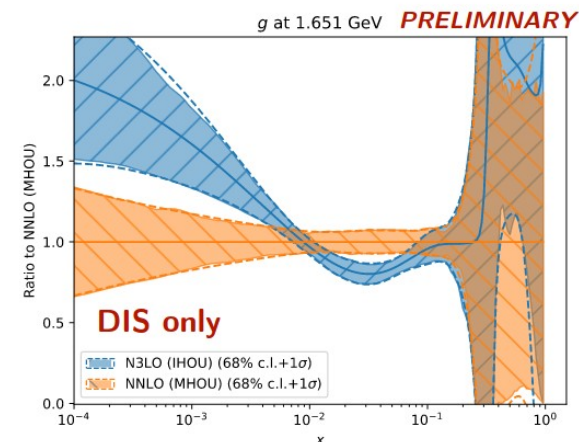
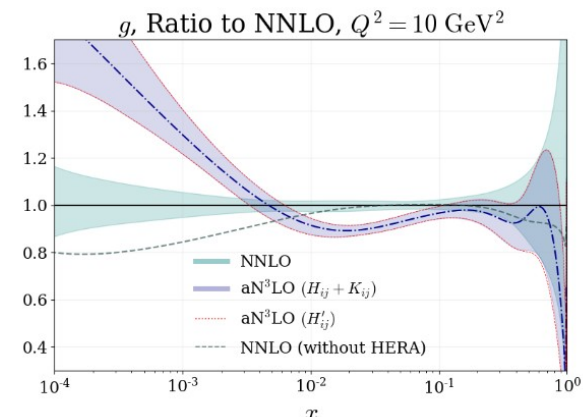
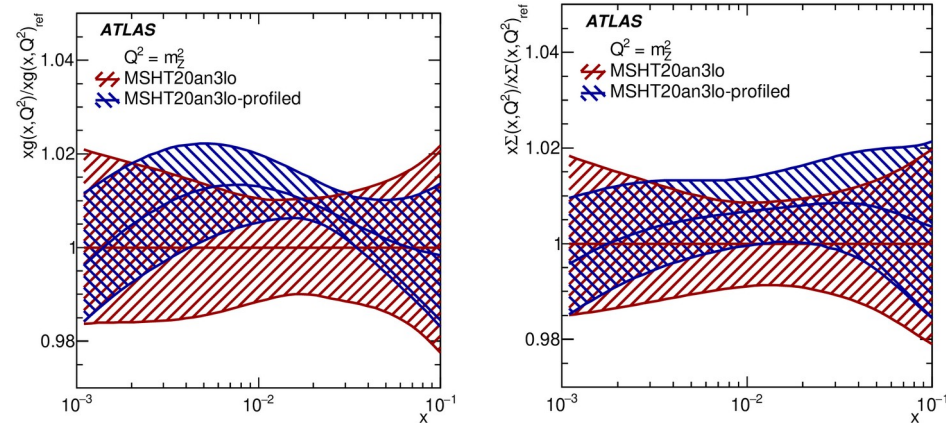
Simultaneous PDF fit

- Simultaneous fit of PDF, non perturbative QCD parameters, and $\alpha_s(m_Z)$ to HERA+Z p_T , confirms the result of the PDF profiling
- Study of the dependence of $\alpha_s(m_Z)$ on the Q^2_{\min} cut of the HERA data confirms small effect of low-x resummation in the region considered for the fit ($Q^2_{\min} > 10 \text{ GeV}^2$)



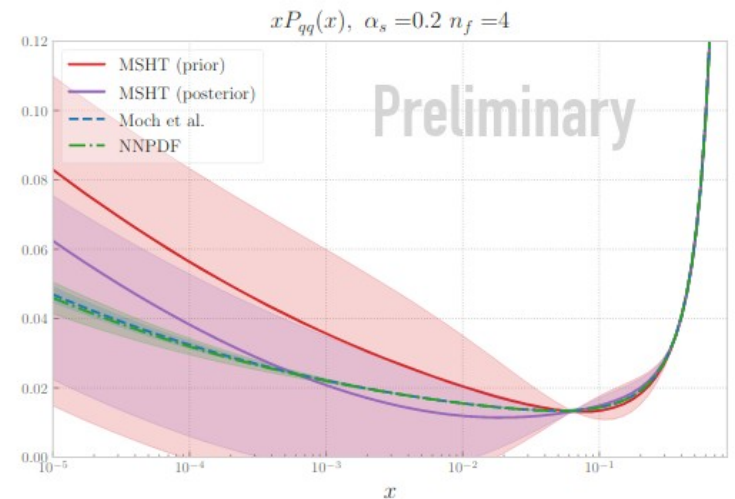
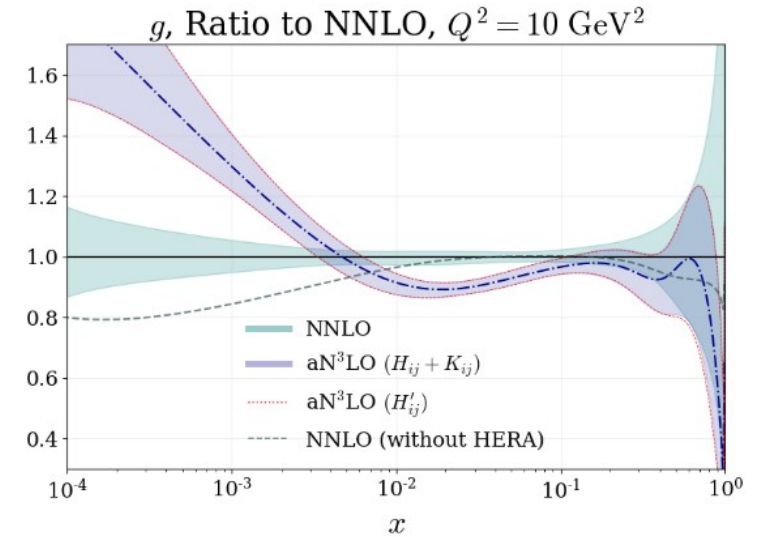
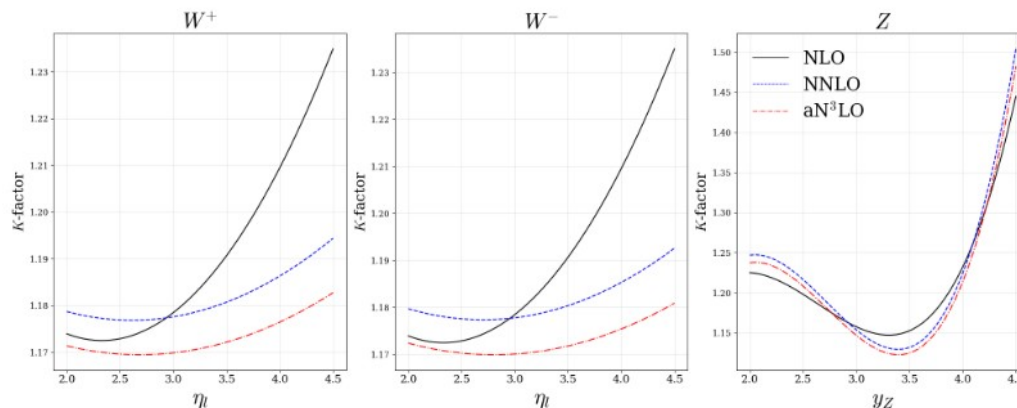
PDF uncertainties

- Profiled MSHT20 an3lo gluon PDF is in fair agreement with prefit gluon PDF (bands overlap)
- The NNLO PDF sets generally show a stronger pull of the gluon PDF, with non overlapping band of pre- and post-fit gluon PDFs
- MSHT20 gluon PDF is significantly different at aN3LO, as also confirmed by preliminary studies of NNPDF
- Smaller pull of the gluon PDF means that the p_T Z data at low p_T also confirms a preference for the the gluon PDF of the aN3LO PDF fit.
- Despite stronger PDF pulls at NNLO, $\alpha_s(m_Z)$ extracted with NNLO PDF sets is generally in agreement with the aN3LO result (except for CT18)



PDF uncertainties

- The MSHT20 an3lo PDF fit is not a complete N3LO PDFs. However, it includes some important N3LO ingredients:
 - ➔ N3LO DIS coefficient functions
 - ➔ Numerically approximate 4-loop splitting functions. In particular, more recent precise numerical calculation of 4-loop pure-singlet Pqq is in agreement with MSHTaN3LO estimate. Pqq is the most relevant splitting function approximation for N4LL accuracy in the Z p_T spectrum, because the 4-loop Pqg Pqg Pgg are formally of higher order for the Z p_T
 - ➔ Drell Yan N3LO k-factors are not included, but the postfit k-factors are in agreement with expectations from N3LO predictions fo negative k-factors



PDF uncertainties - summary

- Loosely speaking there are four possible choices for PDF uncertainties
 - A) Take only one PDF set and its uncertainties
 - B) Take one PDF set and its uncertainties, but add the half envelope to all other PDF sets
 - C) Take the unweighted average for the central value and a simple average for the PDF uncertainties of various different PDF sets
 - D) Perform a full PDF+ $\alpha_s(m_Z)$ fit
- We choose A) and take MSHT20an3lo, which is the single PDF set with largest uncertainties of 0.51×10^{-3} , and with the least pulled gluon PDF after profiling
- Option B) was not considered, because other PDF sets are either in agreement with MSHTan3lo, or fails the test of reinclusion of HERA data in the PDF profiling (CT18)

PDF set	$\alpha_s(m_Z)$	PDF uncertainty	$g [GeV^2]$	$q [GeV^4]$
MSHT20 [37]	0.11839	0.00040	0.44	-0.07
NNPDF4.0 [84]	0.11779	0.00024	0.50	-0.08
CT18A [29]	0.11982	0.00050	0.36	-0.03
HERAPDF2.0 [65]	0.11890	0.00027	0.40	-0.04

- Option D), the simultaneous fit of PDFs and $\alpha_s(m_Z)$ confirms this choice and the results obtained with PDF profiling

Non perturbative QCD model

- The NP model is characterized by a non-perturbative Sudakov form factor and a prescription for regularizing the Landau pole of the α_s running

$$S_{\text{NP}}(b) = \exp \left[-g_j(b) - g_K(b) \log \frac{m_{\ell\ell}^2}{Q_0^2} \right] \quad b_{\star}^2 = \frac{b^2}{1 + b^2/b_{\text{lim}}^2}$$

- The non perturbative model includes a total of 6 parameters which are either fitted to the data or varied to assess an uncertainty

Non perturbative QCD model

- NP model is generally determined from the data, parameters values depend on the chosen prescription to avoid the Landau pole in b-space

$$b_{\star} = \frac{b}{1 + b^2/b_{\text{lim}}^2}$$

$$S_{\text{NP}}(b) = \exp \left[-g_j(b) - g_K(b) \log \frac{m_{\ell\ell}^2}{Q_0^2} \right] \left\{ \begin{array}{l} g_j(b) = \frac{g b^2}{\sqrt{1 + \lambda b^2}} + \text{sign}(q) \left(1 - \exp[-q b^4] \right) \\ g_K(b) = g_0 \left(1 - \exp \left[-\frac{C_F \alpha_s (b_0/b_{\star}) b^2}{\pi g_0 b_{\text{lim}}^2} \right] \right) \end{array} \right.$$

- g_j functions include a quadratic and a quartic term, with g and q free parameters of the fit
- The theory should not depend on b_{lim} (freezing scale) and Q_0 (starting scale of the TMD evolution), provided S_{NP} is flexible enough. Q_0 and b_{lim} are varied to assess a parameterisation uncertainty
- g_0 controls the very high b (very small p_T) behaviour, should be fitted to data, but there is no sensitivity to it, so it is varied
- λ controls the transition from Gaussian (quadratic) to exponential (linear), set to 1 GeV^{-2} and varied by factor of 2 up and down
- Total of 6 NPQCD parameters which are either fitted to the data or varied to assess an uncertainty

Remarks on the generality of the NP model

- Tafat, renormalon analysis (hep-ph/0102237):

- Small b behavior should be Gaussian $W_0 = \exp\left(-\ln\frac{Q^2}{Q_0^2}\left[c_1 b^2 + c_2 b^4 + c_3 b^6 + \mathcal{O}(b^8)\right]\right),$

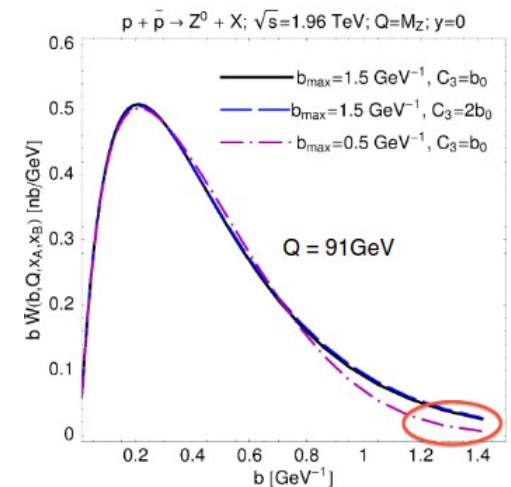
- Large b behavior should be exponential

- Quadratic and quartic terms are the first two terms in the perturbative expansion at small b

- Collins and Rogers (arxiv:1507.05542)

- At large $Q=m_l$ the cross section is eventually dominated by perturbative effects, even at $q_T = 0$

- Z production is dominated by small b (peak at $b = 0.2$, negligible contribution for $b > 1.5$)



$$Q = m_Z, \sqrt{s} = 1800 \text{ GeV}$$

Gaussian behavior of primordial k_T

- The non perturbative model is based on the crucial ansatz of absence of linear corrections above a given scale of the order of 1 to 2 GeV. An exponential primordial k_T up to 20-30 GeV would prevent a simultaneous fit of primordial k_T and $\alpha_s(m_Z)$ due to the large correlation, and spoil the sensitivity to $\alpha_s(m_Z)$.

- Ferrario-Ravasio, Limatola, Nason (arxiv:2011.14114):

“The absence of linear corrections in this context has also a rather simple intuitive explanation. The primordial transverse momentum smearing gives a transverse kick, of the order of typical hadronic scales, to the perturbative distribution. However, it is azimuthally symmetric. Thus, its first-order effects cancel out, leaving only quadratic corrections”

- Schweitzer, Strikman and Weiss (arXiv:1210.1267)

- Exponential behavior driven by a chiral scale of 0.3 fm = 1.5 GeV⁻¹ and a confinement scale of 1 fm = 5 GeV⁻¹

- TMD fits confirm an exponential scale of order 1 GeV: b_0

$$\sqrt{\lambda_3} x^{\lambda_4} \sim 0.2 \text{ GeV}$$

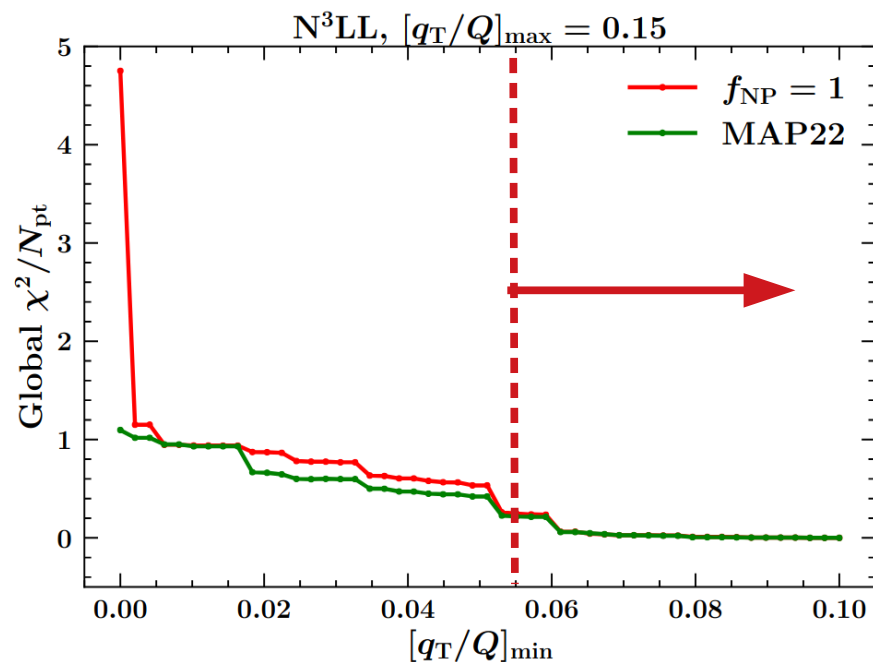
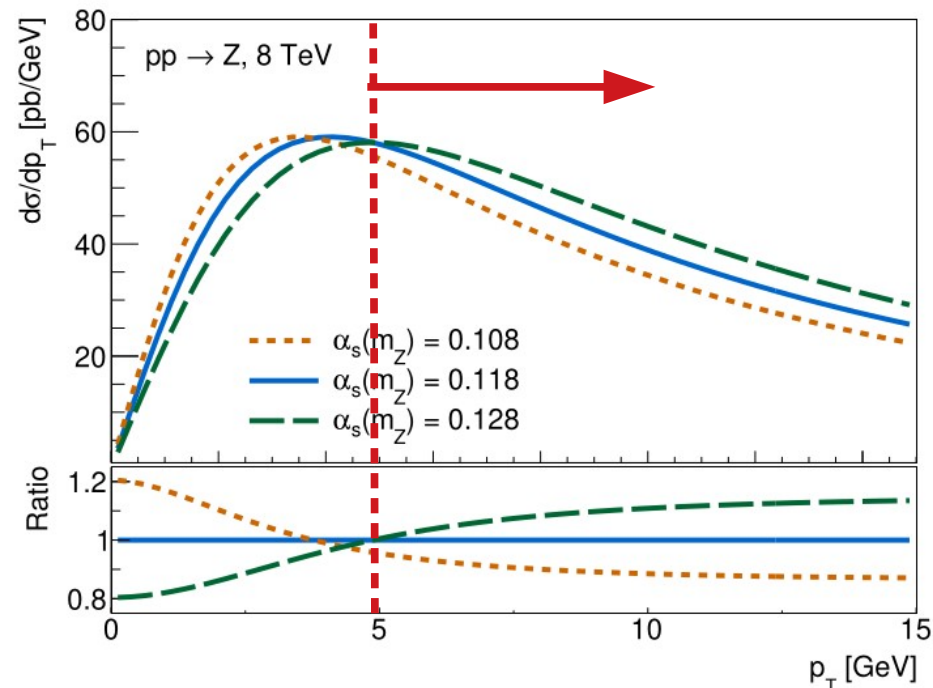
arXiv:1912.06532

$$f_{NP}(x, b) = \exp\left(-\frac{\lambda_1(1-x) + \lambda_2 x + x(1-x)\lambda_5}{\sqrt{1 + \lambda_3 x^{\lambda_4} b^2}} b^2\right)$$

TMDPDF	$\lambda_1 = 0.224 \pm 0.029$	$\lambda_2 = 9.24 \pm 0.46$	$\lambda_3 = 375. \pm 89.$
	$\lambda_4 = 2.15 \pm 0.19$	$\lambda_5 = -4.97 \pm 1.37$	

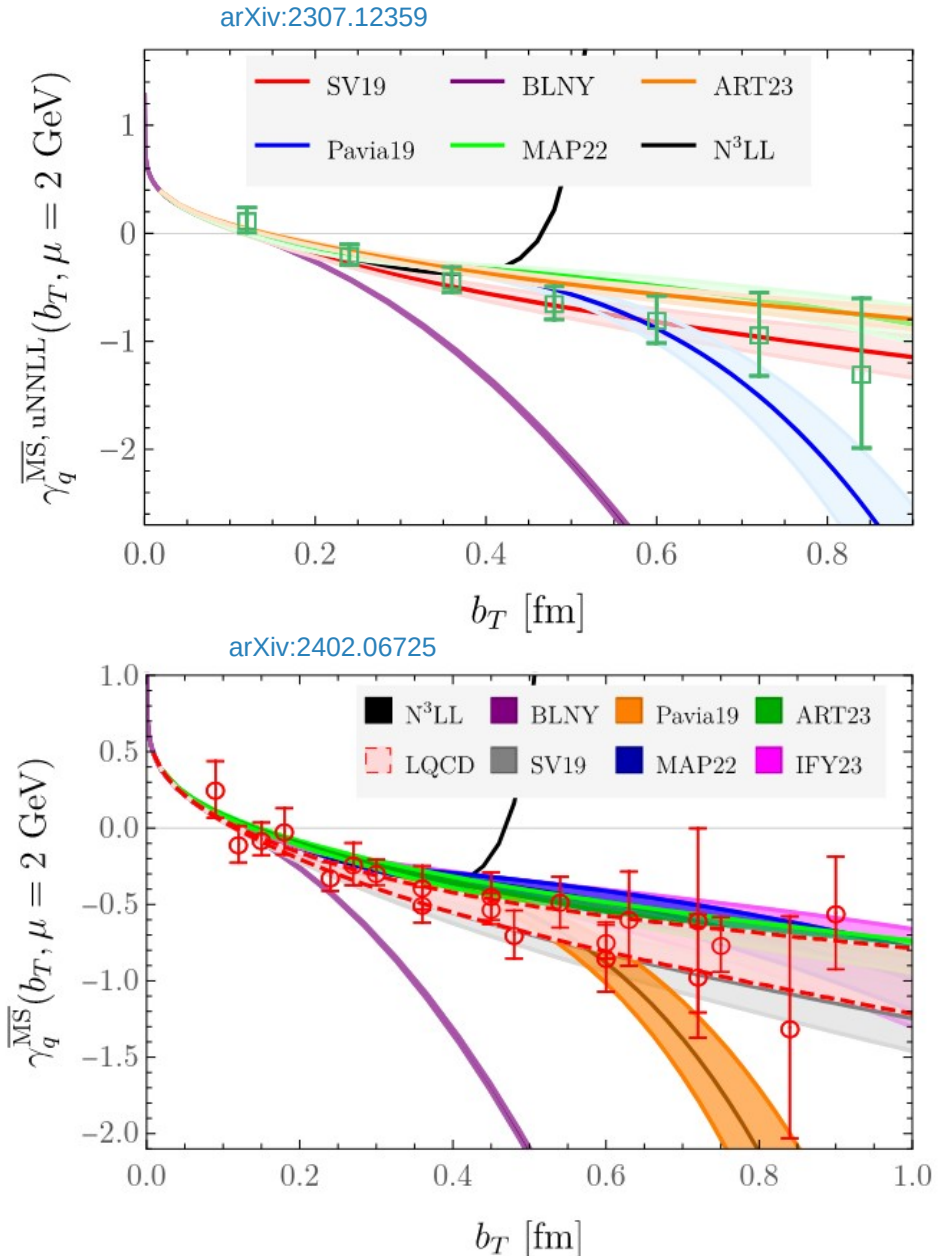
Non perturbative QCD model

- Fits excluding the region 0-5 GeV yields $\alpha_s(m_Z)$ with a spread of $\pm 0.2 \times 10^{-3}$, and fit uncertainty increased from 0.67×10^{-3} to 0.71×10^{-3}
- Correlation between $\alpha_s(m_Z)$ and g largely reduced
- Demonstrates independence of the result from NP effects and good modelling of NP effects
- For $p_T > 5$ GeV f_{NP} is negligible



Input from lattice QCD

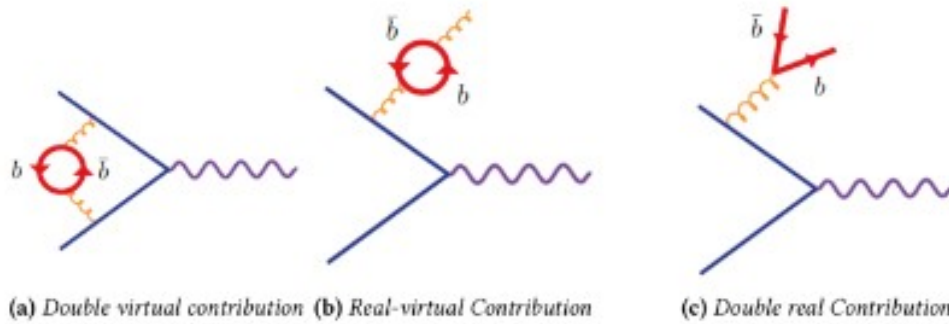
- Recent determinations of the Collins Soper kernel with lattice QCD calculations can provide input on the non pQCD effects
- Very appealing to have a first-principle understanding/determination of these effects
- Particularly useful to set limits on the exponential term at high b_T



Heavy flavour thresholds

- Nominal results with fixed flavour number scheme and $n_f=5$ active flavours
- Backward PDF evolution with FFN $n_f = 5$, charm and bottom PDF switched off at their threshold with a b^* prescription
- Estimate uncertainties related to heavy flavour thresholds with envelope of variations:
 - VFN forward PDF evolution (-0.29×10^{-3})
 - VFN running of α_s in the Sudakov form factor ($+0.21 \times 10^{-3}$)
 - Variations of charm threshold, μ_c ($+0.07 \times 10^{-3}$)
 - Variations of bottom threshold, μ_b (-0.29×10^{-3})
- Inclusion of VFN effects in PDFs and α_s would partially cancel

Heavy flavour mass effects

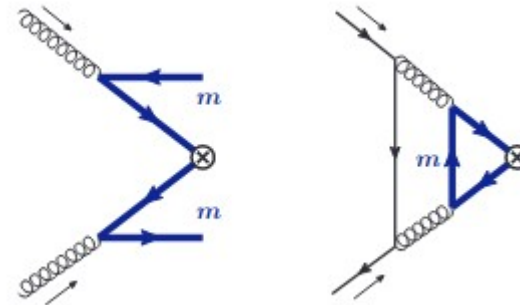
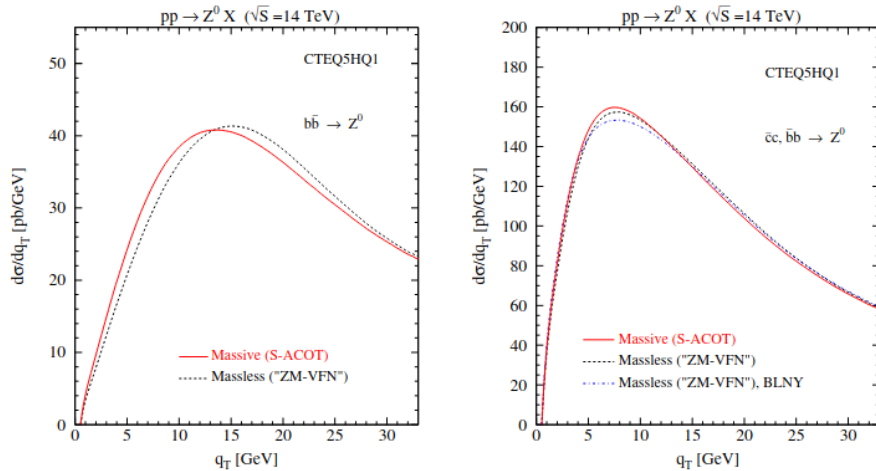


- Secondary or final state HF mass effects: softer p_T spectrum, estimated $\delta\alpha_s$ with positive sign:

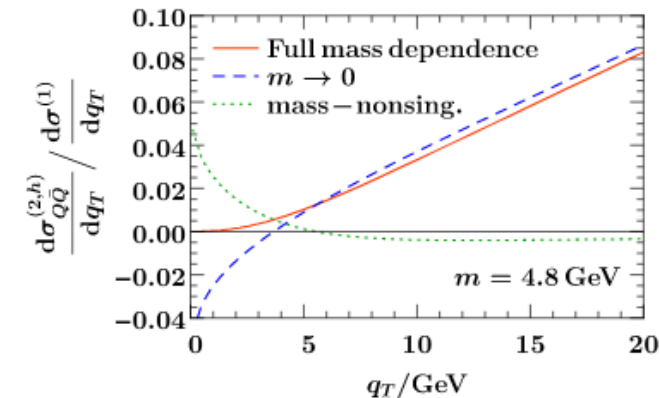
- $g \rightarrow bb$: $+0.40 \times 10^{-3}$
- $g \rightarrow cc$: $+0.01 \times 10^{-3}$

- Included as uncertainty

hep-ph/0509023



arXiv:1703.09702



- Primary or initial state HF mass effects: softer p_T spectrum in the $bb \rightarrow Z$, $cc \rightarrow Z$ channels
- Expected to be negligible for α_s due to the small contributions of $bb \rightarrow Z$ and $cc \rightarrow Z$ (but important for the W/Z p_T ratio, and for m_W)

- 2% bb
- 6% cc

- SCET-based approach for q_T -resummation with massive quark effects

- Corrections evaluated with ReneSANCe

- Normalisation correction of -0.8% from pure weak NLO corrections, flat in rapidity, in agreement with LPCC EW benchmark (SANC, TauSpinner+Dizet, Rady, Powheg_ew, WZGRAD2)

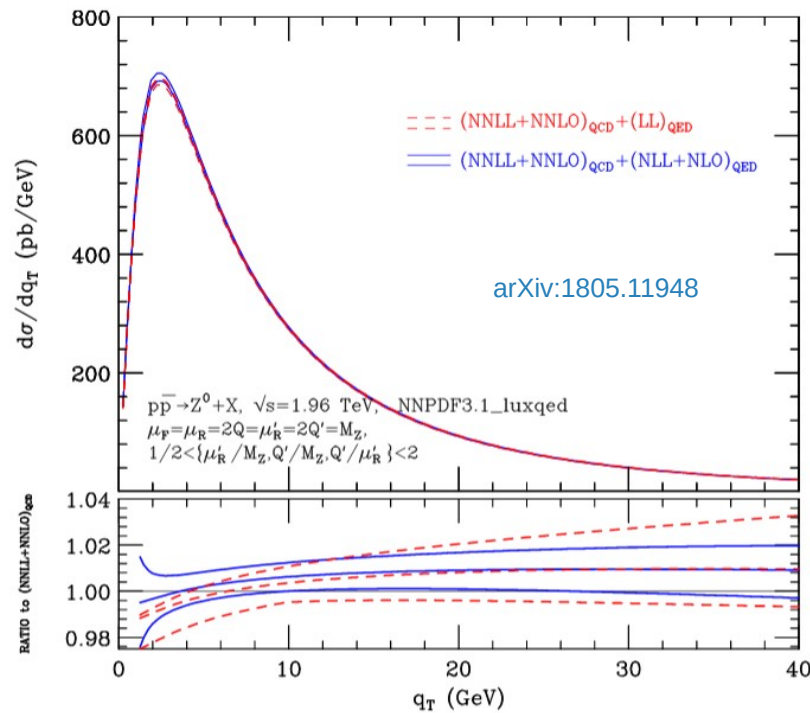
- Normalisation correction of +0.4% from NLO QED ISR flat in rapidity (benchmarked with MCSANC)

- Overall -0.4% correction, with effect on $\alpha_s(m_Z)$ of $+0.06 \times 10^{-3}$. Uncertainties due to missing higher orders are neglected

- Small effect on $\alpha_s(m_Z)$ because these are normalization corrections, not affecting the p_T shape

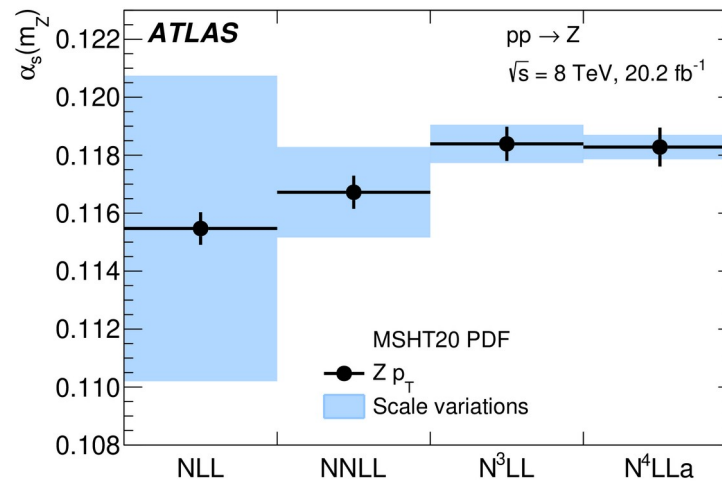
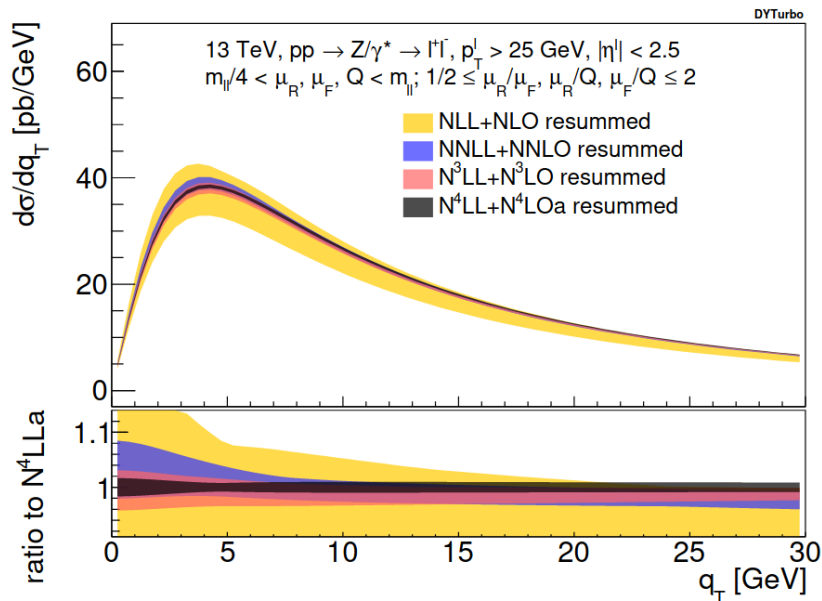
NLO weak corrs: $(G_\mu, M_W, M_Z), d\sigma/dM(ll)$				
Code:	$89 < M_{\ell\bar{\ell}}[\text{GeV}] < 93$	$60 < M_{\ell\bar{\ell}}[\text{GeV}] < 81$	$81 < M_{\ell\bar{\ell}}[\text{GeV}] < 101$	$101 < M_{\ell\bar{\ell}}[\text{GeV}] < 150$
$\sigma(\text{LO})$ (pb)				
MCSANC	612.531(5)	46.870(2)	880.527(6)	-
POWHEG _{ew} (FS)	612.529(8)	46.8697(8)	880.513(9)	30.8686(5)
RADY (FS)	612.526(1)	46.8708(1)	880.520(2)	30.86835(6)
WZGRAD2	612.521(7)	46.868(4)	880.520(10)	-
$\sigma(\text{NLO})/\sigma(\text{LO})$				
MCSANC	0.99167(2)	1.02865(7)	0.99206(1)	-
POWHEG _{ew} (FS)	no α resc.	0.99121(3)	1.02972(4)	0.99163(2)
	α resc.	0.99150(3)	1.02871(4)	0.99191(2)
RADY (FS)	no α resc.	0.99118(1)	1.02965(1)	0.99160(1)
	α resc.	0.99148(1)	1.02863(1)	0.99189(1)
WZGRAD2	0.99198(1)	1.02913(4)	0.99239(1)	-
$\sigma(\text{NLO} + \text{HO})/\sigma(\text{LO})$				
MCSANC	0.99232(2)	1.02614(7)	0.99268(1)	-
POWHEG _{ew} (FS)	α resc.	0.99216(3)	1.02603(4)	0.99253(2)
	no α resc.	0.99181(3)	1.02577(4)	0.99218(1)
RADY (FS) α no resc.	0.99179(1)	1.02589(1)	0.99216(1)	0.98915(3)
TauSpinner+DIZET (estimated)	0.99211(0)	1.02321(0)	0.99264(0)	0.98884(0)

- Effect of LL QED ISR evaluated with Pythia and validated with analytic resummation at NLL QED
- Overall correction at LL is 0.28×10^{-3} , half value of the correction is taken as estimate of the missing higher order uncertainties

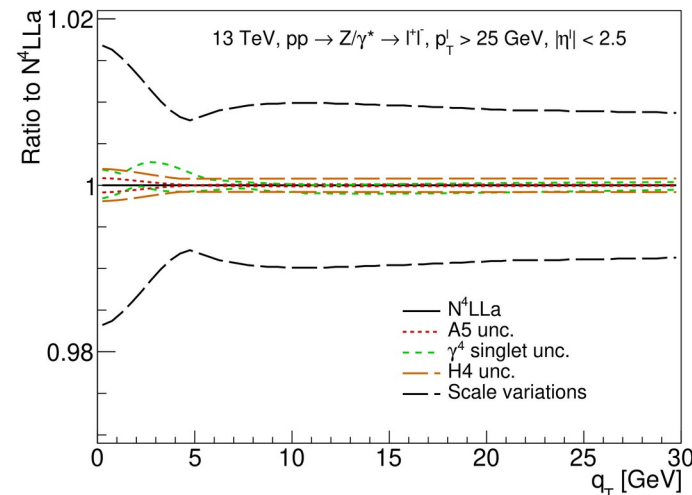


Missing higher order and N4LL approximation

- Missing higher order uncertainties estimated with scale variations (μ_R, μ_F, Q)
- Standard approach, would be interesting to explore other more sophisticated approaches
- Good overlap of scale variations bands between orders for the Z p_T distribution and for the $\alpha_s(m_Z)$ determination confirms the validity of this approach

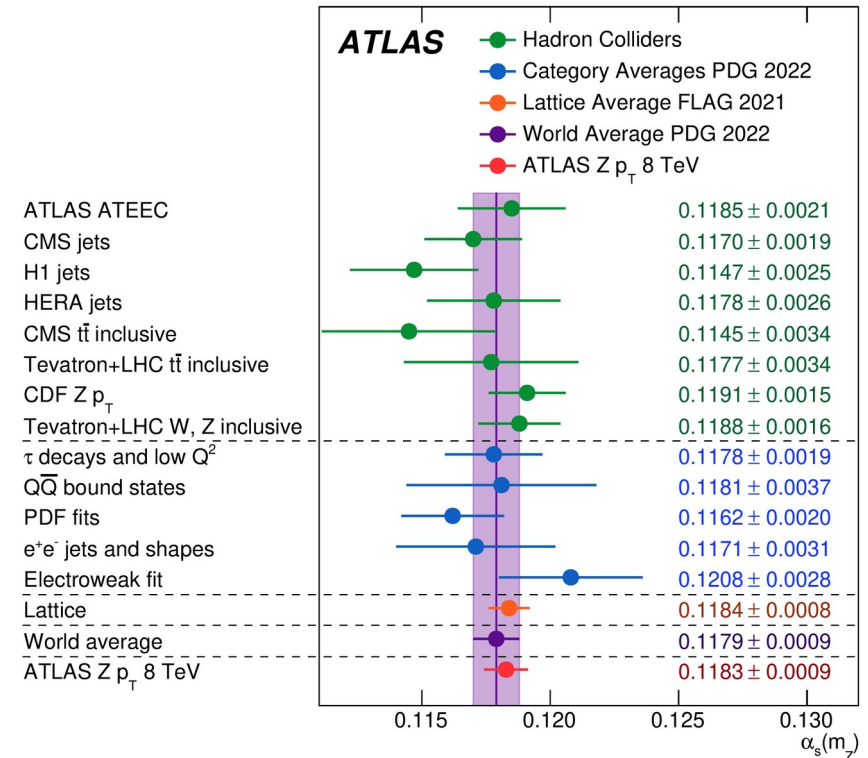


- N4LL approximations are much smaller than missing higher order uncertainties (and even smaller with recent results on 4-loop singlet splitting functions)
- Work towards the completion of the N4LO would be extremely valuable



Summary

- Precise measurements of $p_T Z$ at the LHC at the permille level have opened the possibility of performing precision measurements of $\alpha_s(m_Z)$
- Measuring the strong coupling constant from $p_T Z$ is a new field, and as such, some time is needed to fully understand its potential and the related theory uncertainties
- There are a few issues were theoretical studies and advancements are highly desirable: understanding of non pQCD effects, gluon PDF, Heavy flavour initiated production, completion of higher orders
- The same studies which are aimed at better understanding some of the theory uncertainties, could also lead to significant reduction of these uncertainties
- The $p_T Z$ measurement at the LHC is extremely precise, and is likely to become even more precise in the future, or at least more differential. Such precision can be used to control and reduce theory uncertainties, provided that there is a good a priori understanding and/or parametrization of such theory uncertainties



Public information and reproducibility

- Publicly available data and tools
 - HEPdata: full 2d p_T - y info, and simpler 1d $|y| < 1.6$ for quick studies: <https://www.hepdata.net/record/ins2698794>
 - DYTurbo code, v1.3.2: <https://dyturbo.hepforge.org/>
 - MCFM v10.3 <https://mcfm.fnal.gov/>
 - Xfitter interfaced to DYTurbo: <https://www.xfitter.org/xFitter/>
- Useful starting point to reproduce the analysis
- Differently from m_W which is a detector level analysis, $Z p_T$ is an unfolded particle level cross section measurement → many more possibilities for the theory community to perform studies

Open questions

- Is it possible to demonstrate the quadratic ansatz for the primordial k_T ? Can we have further theoretical insights into the expected form and properties of the non perturbative power corrections? Is it possible to use inputs from lattice QCD to reduce non pQCD uncertainties?
- Is it possible to derive a GM VFN scheme for DY coherent with the DIS scheme used in a given PDF fit?
- Which datasets or which methodological procedures drive the gluon PDF in the CT18 fit?
- How can we use the data to validate and eventually further constraints theoretical uncertainties in $\alpha_s(m_Z)$ determination from $Z p_T$?

BACKUP

Setup of the QCD analysis

- NNLO PDF evolution (QCDnum)
- Bonvini-Giuli PDF parametrisation at $Q_0^2 = 1.9$ [arXiv:1902.11125](https://arxiv.org/abs/1902.11125)
- RT OPT VFN scheme
- $fs = 0.4$
- $Q_{min} = 10$ GeV
- Google CERES as minimizer

$$xg(x, \mu_0^2) = A_g x^{B_g} (1-x)^{C_g} \left[1 + F_g \log x + G_g \log^2 x \right] \quad (2.5a)$$

$$xu_v(x, \mu_0^2) = A_{u_v} x^{B_{u_v}} (1-x)^{C_{u_v}} \left[1 + E_{u_v} x^2 + F_{u_v} \log x + G_{u_v} \log^2 x \right] \quad (2.5b)$$

$$xd_v(x, \mu_0^2) = A_{d_v} x^{B_{d_v}} (1-x)^{C_{d_v}} \quad (2.5c)$$

$$x\bar{u}(x, \mu_0^2) = A_{\bar{u}} x^{B_{\bar{u}}} (1-x)^{C_{\bar{u}}} \left[1 + D_{\bar{u}} x + F_{\bar{u}} \log x \right] \quad (2.5d)$$

$$x\bar{d}(x, \mu_0^2) = A_{\bar{d}} x^{B_{\bar{d}}} (1-x)^{C_{\bar{d}}} \left[1 + D_{\bar{d}} x + F_{\bar{d}} \log x \right], \quad (2.5e)$$

Orders

	Virtual		Sudakov			Real
	H[$\delta(1-z)$]	H[z]	Cusp AD	Collinear, RAD	PDF	CT,V+jet
LL+LO	1	1	1-loop	0	const.	1
NLL+NLO	α_s	C1	2-loop	1-loop	LO	α_s
NLL*+NLO	α_s	C1	2-loop	1-loop	NLO	α_s
NNLL+NNLO	α_s^2	C2	3-loop	2-loop	NLO	α_s^2
N3LL+N3LO	α_s^3	C3	4-loop	3-loop	NNLO	α_s^3
N4LLa+N3LO	α_s^4	C4	5-loop	4-loop	N3LO	α_s^4

Known analytically

Approximated numerically

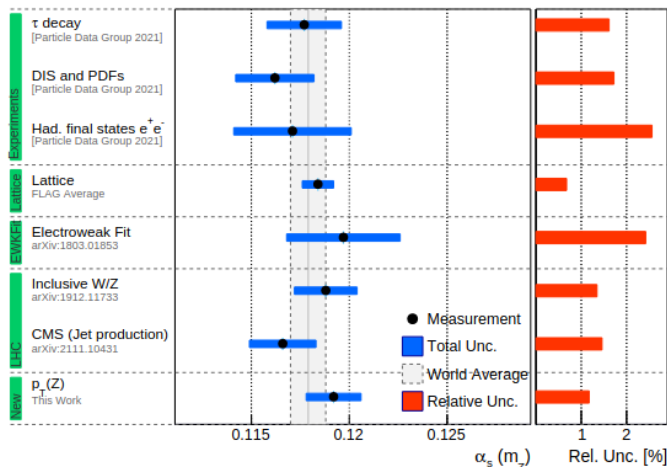
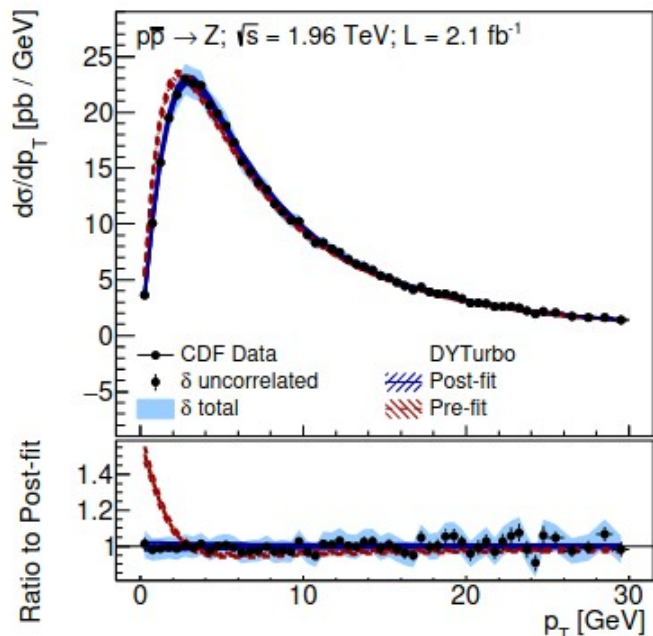
Unknown, estimated with series acceleration

Not included

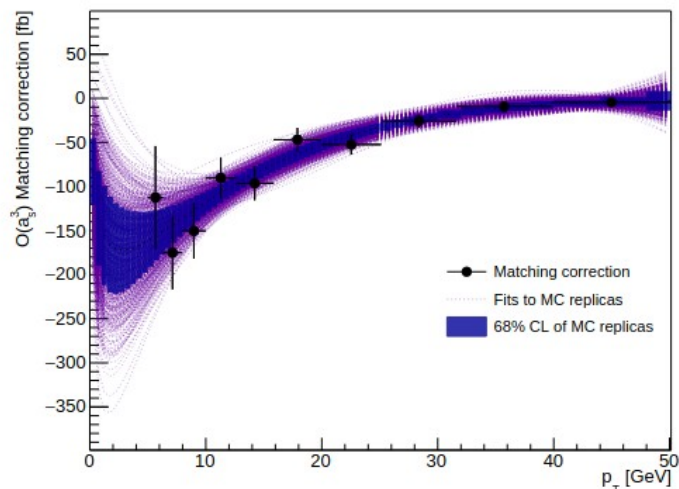
Result at the Tevatron

arXiv:2203.05394

$$\alpha_s = 0.1191 +0.0013 -0.0016$$



Statistical uncertainty	± 0.7
Experimental systematic uncertainty	± 0.1
PDF uncertainty (NNPDF4.0)	± 0.4
PDF uncertainty (envelope of PDFs)	± 0.7
Scale variations uncertainties	$+0.4 -0.9$
Matching at $\mathcal{O}(\alpha_s^3)$	± 0.1
Non-perturbative model	± 0.7
Flavour model	$0 -0.3$
QED ISR	$< \pm 0.1$
Lower limit of fit range	± 0.2
Total	$+1.3 -1.6$



	$\alpha_s(m_Z)$	$g [\text{GeV}^2]$	χ^2/dof
NNPDF4.0	0.1192 ± 0.0008	0.66 ± 0.05	41/53
CT18	0.1189 ± 0.0010	0.67 ± 0.05	40/53
CT18Z	0.1198 ± 0.0009	0.62 ± 0.05	41/53
MSHT20	0.1185 ± 0.0009	0.72 ± 0.05	40/53
HERAPDF2.0	0.1188 ± 0.0008	0.69 ± 0.05	40/53
ABMP16	0.1185 ± 0.0007	0.62 ± 0.05	42/53
MSHT20an3lo ($N^4\text{LL}$)	0.1184 ± 0.0009	0.73 ± 0.05	40/53
PDF fit	0.1184 ± 0.0006	0.71 ± 0.05	1405/1184