Higher Spin Theories and Holography

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Course Contents

- Three Motivations
- A Little Bit About Higher Spin Theories
- AdS_4/CFT_3 dualities for Vector Models
- AdS_3/CFT_2 dualities for Vector-like Models
- Checks (and generalizations) of the Dualities
- Exotic Black holes and Conical Defects in AdS_3

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Three Motivations

- Understand (classical) string theory on strongly curved AdS backgrounds. Can we reproduce features of weakly coupled (perturbative) gauge theories from the dual string theory?
- Are there simpler (non-supersymmetric) examples of AdS/CFT than gauge-string dualities? Yes - for vector-like large N theories. Potentially more tractable.
- Can tractable holographic examples teach us about stringy geometry? - black holes and their thermodynamics in a theory with much larger gauge invariances. Resolution of singularities?

- We will focus on some of the recent progress in answering the second and third questions. I find the first question very interesting but not much progress has been made on this front and in the rest of the introduction will make some remarks about this.

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- Theories of gravity on *AdS* are dual to CFTs on the boundary
- Classical limit $G_N \to 0 \leftrightarrow N \to \infty$
- Conventional Einstein theories (with small higher derivative corrections) are dual to large N CFTs with $\lambda \to \infty$.
- Most bulk calculations in AdS/CFT are in this regime - ultra strong coupling in the CFT.
- What if we are interested in the CFT with $\lambda \sim \mathcal{O}(1)$?
- We need to quantize string theory on *AdS* with $\frac{R_{AdS}}{\ell_s} \sim \lambda^{\frac{1}{4}}$.
- Currently outside analytic control even for SUSY theories.
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- Consider the free field point $\lambda = 0$.
- This has a much larger set of global symmetries than generic interacting theory.
- An infinite number of conserved currents of arbitrary spin.

$$J_{(\mu_1\dots\mu_s)}(x) = \sum_{k=0}^{s} c_k^{(s)} \mathrm{Tr}[\partial_{(\mu_1}\dots\partial_{\mu_k}\Phi^\dagger(x)\partial_{\mu_{k+1}}\dots\partial_{\mu_s)}\Phi(x)] - (Traces)$$

- $\Phi(x)$ is an adjoint scalar for instance. Therefore $\Delta(J^{(s)}) = s+2$ - twist two currents. (in $d=3$, $\Delta = s+1$.)
- $c_k^{(s)}$ are some combinatorial coefficients.
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- The bulk gravitational dual should have gauge fields corresponding to these global symmetries in the boundary theory.

$$\phi_{(\alpha_1 \ldots \alpha_s)} \sim \phi_{(\alpha_1 \ldots \alpha_s)} + \nabla_{(\alpha_1}\xi_{\alpha_2 \ldots \alpha_s)}.$$

- We need a generalization of Einstein's theory with the above (linearised) gauge invariances and therefore "massless" gauge fields of all spin (i.e. symmetric tensors of rank) $s = 2 \ldots \infty$.

- These fields believed to lie on the leading Regge trajectory (which contains the graviton) of the string spectrum on AdS (with $\lambda = 0$).

- Analogue of $\alpha^{\mu_1}_{-1} \ldots \alpha^{\mu_s}_{-1} \tilde{\alpha}^{\mu_1}_{-1} \ldots \tilde{\alpha}^{\mu_s}_{-1} |p\rangle$ in flat space.

- Prediction for the tensionless limit ($\frac{R_{AdS}}{\ell_s} \sim \lambda^{\frac{1}{4}} \to 0$) of the AdS string theory (Sundborg, Witten, Sezgin-Sundell).

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- There are many more states in the Yang-Mills theory (therefore the dual *AdS* string theory) than these twist two operators.
- A Hagedorn density of stringy states as opposed to a single Regge trajectory with a single field for a given spins s.
- Nevertheless, the sector of twist two operators in the free theory are closed amongst themselves under the OPE.
- This should therefore describe a closed subsector of the dynamics of the full theory.

- Reasonable to expect that there is a closed subsector for the dynamics of the dual higher spin gauge fields.
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- Might expect dynamics of this subsector to be simpler compared to the full "string field theory" of highly curved AdS.
- In fact, dynamics highly constrained by the higher spin gauge symmetries - an alternative to the power of supersymmetry?
- An almost unique, consistent, classical theory of interacting higher spin gauge fields in AdS_D exists $(D = 3, 4, 5)$ - constructed by Vasiliev.
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- Is the full spectrum of the Free Yang-Mills theory organized by the higher spin symmetry of the twist two operators? Some evidence (Bianchi et.al.)

- What about going away from $\lambda = 0$? Difficult to have higher spin symmetry exactly preserved in an interacting CFT in $d > 2$.

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General Features of the Vasiliev Theory

- Very non-linear realization of the higher spin symmetry - vast generalization of diffeomorphism invariant theories.
- Necessarily contains an infinite tower of higher spin fields (excepting for special cases in $D = 3$).
- Does not appear to reduce (in any limit) to Einstein's equations for $D > 3$.
- Appears to contain an infinite number of derivatives - non-local on the scale of the AdS radius.
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Free Higher Spin Theory

- Start with non-interacting theory of massless higher spin fields $\phi_{(\alpha_1\ldots\alpha_s)}$ in a curved background (Fronsdal).

$$\phi^{\beta\gamma}_{\beta\gamma\alpha_1\ldots\alpha_{s-4}} = 0 \qquad \phi_{\alpha_1\ldots\alpha_s} \sim \phi_{\alpha_1\ldots\alpha_s} + \nabla_{\alpha_1}\xi_{\alpha_2\ldots\alpha_s}$$

- Gauge parameter is traceless $\xi^{\alpha}_{\alpha\alpha_3\ldots\alpha_{s-1}} = 0$.
- Linearised equation of motion consistent with gauge invariance

$$\hat{\mathcal{F}}_{\alpha_1\ldots\alpha_s} \equiv \nabla^2_{(s)}\phi_{\alpha_1\ldots\alpha_s} - \nabla_{\alpha_1}\nabla^{\lambda}\phi_{\lambda\alpha_2\ldots\alpha_s} + \nabla_{\alpha_1}\nabla_{\alpha_2}\phi^{\lambda}_{\lambda\alpha_3\ldots\alpha_s} - \frac{1}{R^2_{AdS}}\left(a_{s,D}\phi_{\alpha_1\ldots\alpha_s} + 2g_{\alpha_1,\alpha_2}\phi^{\lambda}_{\lambda\alpha_3\ldots\alpha_s}\right) = 0.$$

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- Challenge is to generalize action/equations of motion to the interacting theory preserving ("non-abelian") gauge invariance.
- First recast Fronsdal (linearised) theory by moving to a frame like formulation : generalization of vielbein and connection

$$e^a_\alpha, \omega^{ab}_\alpha \rightarrow e^{a_1 \dots a_{s-1}}_\alpha, \omega^{a_1 \dots a_{s-1}, b}_\alpha.$$

- Enlarged gauge invariance - generalized local lorentz rotations → more gauge fields.

$$\delta_\xi e^{a_1 \dots a_{s-1}}_\alpha = \partial_\alpha \xi^{a_1 \dots a_{s-1}}$$

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- This is captured by the "unfolded formalism" - express e.o.m. in terms of constraints on an infinite number of auxiliary fields.
- E.g. for massless scalar fields ($s = 0$) satisfying $\partial^2 C = 0$, define a tower of symmetric traceless zero form fields $C^{(a_1 \dots a_n)}$ and the chain of equations between them

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- For gravity, Bianchi identity $DR^{ab} = \bar{e}_c \wedge \bar{e}_d DC^{ac,bd} = 0$.
- An analysis of the symmetries of the RHS implies $DC^{ac,bd} = \bar{e}_f(2C^{acf,bd} + C^{acb,df} + C^{acd,bf})$.
- The new auxiliary fields $C^{acf,bd}$ parametrise the non-vanishing first derivatives of the Weyl tensor. But they are not completely arbitrary.
- Repeating the Bianchi identity (essentially $d^2 = 0$) on the first derivatives gives a constraint on the non-vanishing second derivatives and so on

$$DC^{a(2+k),b(2)} = \bar{e}_f\Big((k+2)C^{a(k+2)f,bd} + C^{a(k+2)b,df} + C^{a(k+2)d,bf}\Big)$$

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- More generally, for the higher spin s fields we have the generalized Weyl tensor and derivatives consisting of two row Young tableaux with $(k+s)$ and s boxes respectively - $C^{a(k+s),b(s)}$.
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- Thus the content of the linearised Fronsdal equations are contained in the two infinite (for each s) set of equations which are for a) generalized torsion b) covariant constancy of generalized Weyl tensor.

$$\begin{aligned} R^{a(s-1),b(t)}(\omega) &= 0, \quad (t < s-1) \\ R^{a(s-1),b(s-1)}(\omega) &= \bar{e}_c \wedge \bar{e}_d C^{a(s-1)c,b(s-1)d} \\ DC^{a(s+k),b(s)} &= \bar{e}_f\Big((k+2)C^{a(s+k)f,b(s)} + sC^{a(s+k)(b_1,b(s-1))f}\Big) \end{aligned}$$

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- Then we can go between the bases $A_{\beta(n)\dot{\alpha}(m)} \oplus c.c. \leftrightarrow A_{a(p)b(q)}$ with $p = \frac{1}{2}|n+m|$ and $q = \frac{1}{2}|n-m|$.
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- We now package together all these individual fields of fixed s into a "superfield" using (grassmann even) spinor oscillators $Y^{\beta}, \bar{Y}^{\dot{\delta}}$.
- Thus we have a "connection superfield" (1-form):

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- We similarly have another "Weyl tensor superfield" (0-form):

$$C(Y|X) = \sum_s \sum_{n,m;n-m=2s} C^{(s)}_{\alpha}(X)_{\beta_1\ldots\beta_n;\dot{\delta}_1\ldots\dot{\delta}_m} \times Y^{\beta_1}\ldots Y^{\beta_n}\bar{Y}^{\dot{\delta}_1}\ldots\bar{Y}^{\dot{\delta}_m}$$

- We define

$$D^{ad} = D + \bar{e}^{\beta\dot{\delta}}\left(Y_\beta \frac{\partial}{\partial \bar{Y}^{\dot{\delta}}} + \bar{Y}_{\dot{\delta}}\frac{\partial}{\bar{Y}^{\beta}}\right)$$

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- Then we can re-express the linearised equations in the unfolded formalism as

$$R(Y,\bar{Y}|X) = \bar{e}^{\beta\dot{\delta}}\bar{e}_{\beta}^{\dot{\gamma}}\frac{\partial^2}{\partial\bar{Y}^{\dot{\delta}}\partial\bar{Y}^{\dot{\gamma}}}C(0,\bar{Y}|X) + c.c.$$

$$\tilde{D}C(Y,\bar{Y}|X) = 0$$

- But we want to go beyond the linearised equations and write down nonlinear equations for these fields.
- For that we use the higher spin algebra as captured by the algebra of spinor oscillators.

$$[Y^{\beta}, Y^{\gamma}] = 2i\epsilon^{\beta\gamma};\ [\bar{Y}^{\dot{\delta}}, \bar{Y}^{\dot{\gamma}}] = 2i\epsilon^{\dot{\delta}\dot{\gamma}};\ [Y^{\beta}, \bar{Y}^{\dot{\delta}}] = 0.$$

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- The AdS_4 isometries $O(3,2)$ captured by this oscillator construction.

$$M_{\beta\gamma} = \frac{1}{2i}\{Y_\beta, Y_\gamma\}; \quad P_{\beta\dot{\delta}} = \frac{1}{i} Y_\beta \bar{Y}_{\dot{\delta}}$$

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- More generally, the elements $T^{(n,m)} = Y^{\beta(n)}\bar{Y}^{\dot{\delta}(m)}$ form a basis for the higher spin algebra in $D = 4$ - note that $n + m = 2(s-1)$ (in $D = 3$, only one set of oscillators).

- They generate the algebra which is schematically

$$[T^{s_1}, T^{s_2}] = \sum_{l=1}^{min(s_1,s_2)-1} T^{s_1+s_2-2l}.$$

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- It turns out that to do this consistently requires another set of oscillators $Z^{\beta}, \bar{Z}^{\dot{\delta}}$ and another auxiliary superfield $S(Y,Z|X) = S_{\beta}dZ^{\beta} + S_{\dot{\delta}}d\bar{Z}^{\dot{\delta}}$

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- The generalized gauge symmetry acts (linearly) as

$$\begin{aligned}
\delta_{\epsilon}W &= d_X\epsilon + \epsilon \star W - W \star \epsilon \\
\delta_{\epsilon}S &= d_Z\epsilon + \epsilon \star S - S \star \epsilon \\
\delta_{\epsilon}B &= \epsilon \star B - B \star \pi(\epsilon)
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$$\begin{aligned}
\delta_\epsilon W &= d_X \epsilon + \epsilon \star W - W \star \epsilon \\
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\end{aligned}$$

- For realizing the symmetry non-linearly we need to construct the nonabelian field strength $R(Y,\bar{Y}|X) = d\Omega(Y,\bar{Y}|X) + \Omega(Y,\bar{Y}|X) \star \Omega(Y,\bar{Y}|X)$ and write equations in terms of this field and the superfield $C(Y,\bar{Y}|X)$.
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- The non-linear Vasiliev equations take the form

$$\begin{aligned} d\mathcal{A} + \mathcal{A} \star \mathcal{A} &= B \star K \; dZ^{\alpha} d\bar{Z}_{\alpha} + c.c. \\ dB + \mathcal{A} \star B - B \star \pi(\mathcal{A}) &= 0 \end{aligned}$$

- $\mathcal{A} = W + S$, $K = e^{Z^{\alpha} Y_{\alpha}}$ ("Kleinian") and $\pi(f(Y, \bar{Y}, Z, \bar{Z}|X)) = f(-Y, \bar{Y}, -Z, \bar{Z}|X)$.
- Actually, the RHS of the first equation can in general be $f(B \star K)$ which by field redefinition can be put in the form $f(w) = w \exp i\theta(w)$.
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Vasiliev theories in AdS_3

- In AdS_3, gravity does not have propagating d.o.f. Neither do the higher spin fields.
- Nevertheless, a rich classical (and quantum) theory which includes black holes and other solitonic solutions.
- Family of Vasiliev theories with inequivalent symmetry algebras $hs(\lambda)$ - one (real) parameter deformation of oscillator algebra.
- The Vasiliev equations of motion (with $B = 0$) reduce to $F(A) = 0$ for gauge fields $A, \tilde{A} \in hs(\lambda)$. Scalars are optional (with mass $M^2 = -1 + \lambda^2$).
- Hence the action (for $A, \tilde{A}$, $B = 0$) is a sum of Chern-Simons terms with gauge group $hs(\lambda)$. (Blencowe; Blencowe-Bergshoeff-Stelle)
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- $S(A, \tilde{A}) = S_{\rm CS}[A] - S_{\rm CS}[\tilde{A}]$ with level $k_{CS} = \frac{R_{AdS}}{4G_N}$.
- Recognise as a generalization of formulation of classical 3d gravity in terms of $SL(2,R) \times SL(2,R)$ Chern-Simons theory.
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Asymptotic Symmetries in $3d$

- AdS_3 gravity has an asymptotic symmetry algebra larger than the isometries $SL(2,R) \times SL(2,R)$ - full Virasoro (two copies) with $c = \bar{c} = \frac{3\ell}{2G_N}$ -Brown-Henneaux.
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  Result: W_N extended symmetry algebra - containing holomorphic currents $W^{(s)}(z)$ of spins $s = 2 \dots N$. $(W^{(2)}(z) = T(z))$
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Klebanov-Polyakov Duality for 3d Vector Models

- Dual to a Vasiliev theory needs a much smaller infinity of single particle operators compared to a gauge theory. Not a hagedorn density of states.
- Vector like models have far fewer degrees of freedom $\propto N$, rather than gauge theories $\propto N^2$.
- The *only* single particle operators are the symmetric bilinears $\sum_{k=0}^{s} c_k^{(s)}[\partial_{(\mu_1} \ldots \partial_{\mu_k}\phi_i(x)\partial_{\mu_{k+1}} \ldots \partial_{\mu_s)}\phi_i(x)] - (Traces)$.
- Therefore dual bulk fields are only the Vasiliev gauge fields (together with the scalar). (Klebanov-Polyakov, Sezgin-Sundell)
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- Vector like models have far fewer degrees of freedom $\propto N$, rather than gauge theories $\propto N^2$.
- The *only* single particle operators are the symmetric bilinears $\sum_{k=0}^{s} c_k^{(s)}[\partial_{(\mu_1} \ldots \partial_{\mu_k}\phi_i(x)\partial_{\mu_{k+1}} \ldots \partial_{\mu_s})\phi_i(x)] - (Traces)$.
- Therefore dual bulk fields are only the Vasiliev gauge fields (together with the scalar). (Klebanov-Polyakov, Sezgin-Sundell)
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- In $O(N)$ vector models can add to the free action $S_0 = \int d^3x \partial_\mu \phi_i(x) \partial_\mu \phi_i(x)$ an interaction ("double trace") term $S_1 = \lambda \int d^3x (\phi_i(x)\phi_i(x))^2$.
- There is a nontrivial fixed point ("Wilson-Fisher") of the RG in the infrared. Can be analyzed exactly in the large N limit.
- The scalar bilinear $\phi_i(x)\phi_i(x)$ has dimension $\Delta = 2 + O(\frac{1}{N})$ instead of the canonical $\Delta = 1$ at the free (UV) fixed point.
- Proposal (KP): The free and interacting CFTs are both dual to the type A Vasiliev theory with spins $s = 0, 2, 4 \ldots$ on AdS_4 but with the bulk scalar $(m^2 = -\frac{2}{R^2_{AdS}})$ quantized in two inequivalent ways.
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Minimal Model Holography

- Can sidestep MZ theorem in 2d CFTs. Hence proposal for a Vasiliev dual to a class of interacting CFTs with higher spin (i.e. W_N) symmetries. (Gaberdiel-R.G.)
- The CFT: a coset WZW model. $\frac{SU(N)_k \times SU(N)_1}{SU(N)_{k+1}}$ - W_N minimal models.
- Take the 't Hooft large N limit, keeping $0 \leq \lambda = \frac{N}{N+k} \leq 1$ fixed. A line of CFTs with central charge $c_N(\lambda) = N(1-\lambda^2)$ - vector like model.
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Coset Models

- A G/H coset theory is defined in terms of a G WZW theory in which a subgroup H is gauged (without kinetic term).
- Therefore $T_{G/H}(z) = T_G(z) - T_H(z)$ and $c_{G/H} = c_G - c_H$
- Building block for rational CFTs for different G and H.
- Basic case: $G = SU(N)_k \times SU(N)_l$ and $H = SU(N)_{k+l}$ (diagonal).
- We will consider the case $l = 1$ (in the large N limit, additional l is like adding flavour)
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- For this family,

$$c_N(k) = (N-1)[1 - \frac{N(N+1)}{p(p+1)}] \leq (N-1)$$

where $p = k + N$. i.e. $(p = N+1, N+2, \ldots)$.

- In the case $N = 2$, this is the coset construction of the unitary Virasoro discrete series (GKO).

$$c_2(k) = 1 - \frac{6}{p(p+1)} \leq 1$$

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- Special cases - $k = 1$:Z_N parafermion theory.
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- Branching functions also known explicitly:

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- $\hat{W}$ is the affine Weyl group (affine translations and usual Weyl reflections).
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Symmetries

- The $SU(N)$ cosets have an extended $\mathcal{W}_N$ symmetry. In addition to $T(z)$, higher spin conserved currents $W^{(3)}(z), \ldots W^{(N)}(z)$.
- Constructed using higher order Casimir invariants. For Instance:

$$W^{(3)}(z) \propto d^{abc}[a_1(J^a_{(1)}J^b_{(1)}J^c_{(1)})(z) + a_2(J^a_{(2)}J^b_{(1)}J^c_{(1)})(z) + a_3(J^a_{(2)}J^b_{(2)}J^c_{(1)})(z) + a_4(J^a_{(2)}J^b_{(2)}J^a_{(2)})(z).$$

- Similarly, $W^{(m)}(z)$ from mth order Casimir combinations of the currents $J^a_{(1,2)}(z)$ of $SU(N)_k$ and $SU(N)_1$ respectively.
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RG Flows

- One can flow between the minimal models with different k or p (for fixed N).
- The relevant operator of the pth minimal model, $(0; \mathrm{adj})$, induces the RG flow. The IR fixed point is the $p-1$th model.
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$$(0; \mathrm{adj})_p \xrightarrow{\text{RG-flow by } (0;\mathrm{adj})} (\mathrm{adj}; 0)_{p-1}.$$
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- The 'tHooft limit: $N, k \to \infty$ with $0 \le \lambda = \frac{N}{k+N} \le 1$ fixed.
- In this limit, the central charge $c_N(\lambda) \simeq N(1-\lambda^2) \to \infty$.
- Dimensions of operators simplify remarkably:

$$\begin{aligned} h(0;\mathrm{f}) &= \frac{(N-1)}{2N}\left(1-\frac{N+1}{N+k+1}\right) \to \frac{1}{2}(1-\lambda) \\ h(\mathrm{f};0) &= \frac{(N-1)}{2N}\left(1+\frac{N+1}{N+k}\right) \to \frac{1}{2}(1+\lambda) \\ h(0;\mathrm{adj}) &= 1-\frac{N}{N+k+1} \to 1-\lambda \\ h(\mathrm{adj};0) &= 1+\frac{N}{N+k} \to 1+\lambda. \end{aligned}$$

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- In general, representations which are finite tensor powers of the fund./anti-fund. have finite scaling dimensions in the 'tHooft limit.

- However, there is a large (exponential) degeneracy in this limit. Many operators with almost the same dimension.
- E.g. the $(\Lambda;\Lambda)$ primaries are almost degenerate with the vacuum state $h(\Lambda;\Lambda) = \frac{C_2(\Lambda)}{(N+k)(N+k+1)} \rightarrow \frac{B(\Lambda)}{2} \times \frac{\lambda^2}{N} \rightarrow 0$-"light states". ($B(\Lambda)$ is the number of boxes in Λ.)
- Does a good large N limit exist then?
- Nevertheless, correlation functions of the $\mathcal{W}_N$ minimal model seem to behave well at large N - expected factorization behavior in the four point function and $\frac{1}{N}$ suppression of interactions. (Papadodimas-Raju, Chang-Yin)
- The large degeneracy does not spoil the large N behaviour because thefusion rules between the states are very special. Strong selection rules (at finite N) lead to most of the light states exactly decoupling in any k-point correlation function.

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Checks of the Proposal

Now try to check various aspects of the proposed duality between the 'tHooft limit of the $\mathcal{W}_N$ minimal models and the $hs[\lambda]$ higher spin theory on AdS_3:

- Symmetries and Spectrum
- Correlation Functions
- Properties of Black Holes (and other solitons)

Symmetries

- The bulk $hs[\lambda]$ theory has an asymptotic $W_\infty[\lambda]$ symmetry. Naively, seems different from the large N limit of the $\mathcal{W}_N$ algebra.

- However, there is now a lot of evidence that the two are the same - from matching of representations. (Gaberdiel-Hartman; Gaberdiel-R.G-Hartman-Raju)

- Motivated by a generalized (to non integer) level-rank duality:

$$\frac{SU(N)_k \times SU(N)_1}{SU(N)_{k+1}} \equiv \frac{SU(\lambda)_l \times SU(\lambda)_1}{SU(\lambda)_{l+1}}$$

  where $\lambda = \frac{N}{N+k}$ and $l = \frac{\lambda}{N} - \lambda$. (Kuniba et.al.)

- The symmetry group of the RHS is the extension of the wedge algebra $sl(\lambda) = hs[\lambda]$ while that that of the LHS is the $\mathcal{W}_N$. Indeed, there is evidence for this equality at finite N, k as well.

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Spectrum (Bulk)

Can the linearised fluctuations of the higher spin gauge fields (and two scalars) account for all the states in the CFT (to leading order in large N)?

Perturbative bulk spectrum given by

$$Z_{\rm bulk} = Z_{class} Z_{1-loop} = (q\bar{q})^{-c/24} Z_{\rm HS} Z_{\rm scal}(h_+)^2 Z_{\rm scal}(h_-)^2.$$

where Z_{HS}, Z_{scal} are the bulk one loop determinants from the higher spin fields ($s = 2, 3 \ldots, \infty$) and scalars resp.

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$$Z_{HS} = \prod_{s=2}^{\infty}\prod_{n=s}^{\infty}\frac{1}{|1-q^n|^2} = \prod_{n=1}^{\infty}|1-q^n|^2 \times \prod_{n=1}^{\infty}\frac{1}{|(1-q^n)^n|^2} \equiv |\tilde{M}(q)|^2.$$

Gaberdiel-R. G.-Saha

$$\begin{aligned} Z_{scal}(h) &= \prod_{l=0,l'=0}^{\infty}\frac{1}{(1-q^{h+l}\bar{q}^{h+l'})} \\ &= \exp\left[\sum_{n=1}^{\infty}\frac{Z_{\text{sing par}}(h,q^n,\bar{q}^n)}{n}\right] \\ &= \sum_R \chi_R^{u(\infty)}(z_i)\ \chi_R^{u(\infty)}(\bar{z}_i) \qquad (z_i = q^{i+h-1}). \end{aligned}$$

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Putting it all together:

$$Z_{\rm bulk} = (q\bar{q})^{-c/24}|\tilde{M}(q)|^2 \sum_{R_\pm, S_\pm} |\chi_{R_+}(z_i^+)\,\chi_{S_+}(z_i^+)\,\chi_{R_-}(z_i^-)\,\chi_{S_-}(z_i^-)|^2.$$

$R_\pm, S_\pm$ are representations of $U(\infty)$ with a finite number of boxes in the Young Tableaux. $(z_i^\pm = q^{i+h_\pm-1})$.

View this as the combined contribution from (weakly coupled) multi-particle states of the complex scalar with dimension h_+ (the pieces R_+, S_+), and that of the scalar with dimension h_- (the pieces R_-, S_-) all dressed with the boundary graviton excitations in $\tilde{M}(q)$.

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The branching functions simplify considerably in the 't Hooft limit (Gaberdiel-R.G.-Hartman-Raju)

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using the Verlinde formula. ($B_\pm = B(\Lambda_\pm) \equiv B(R_\pm) + B(S_\pm)$).

Further simplifying the RHS

$$\begin{aligned} b_{(\Lambda;0)}(q) &\cong q^{-\frac{N-1}{24}(1-\lambda^2)}\cdot\tilde{M}(q)\cdot q^{\frac{\lambda}{2}B(\Lambda)}\,q^{C_2(\Lambda)}\cdot \dim_q(\Lambda) \\ &\cong q^{-\frac{N-1}{24}(1-\lambda^2)}\cdot\tilde{M}(q)\cdot\chi_{R^T}(z_i^+)\,\chi_{S^T}(z_i^+) \end{aligned}$$

Spectrum (CFT)

The branching functions simplify considerably in the 't Hooft limit (Gaberdiel-R.G.-Hartman-Raju)

$$\begin{aligned} b_{(\Lambda_+;\Lambda_-)}(q) &\cong q^{-\frac{c}{24}}\, \tilde{M}(q)\, q^{\frac{\lambda}{2}(B_+-B_-)}\, q^{C_2(\Lambda_+)+C_2(\Lambda_-)}\, \frac{S_{\Lambda_+\Lambda_-}}{S_{00}} \\ &\cong q^{\frac{\lambda}{2}(B_+-B_-)} \sum_{\Lambda} N^{\Lambda}_{\Lambda_+\bar{\Lambda}_-}\, q^{-\frac{\lambda}{2}B(\Lambda)}\, b_{(\Lambda;0)}(q)\ , \end{aligned}$$

using the Verlinde formula. $(B_\pm = B(\Lambda_\pm) \equiv B(R_\pm) + B(S_\pm))$.

Further simplifying the RHS

$$\begin{aligned} b_{(\Lambda;0)}(q) &\cong q^{-\frac{N-1}{24}(1-\lambda^2)} \cdot \tilde{M}(q) \cdot q^{\frac{\lambda}{2}B(\Lambda)}\, q^{C_2(\Lambda)} \cdot \dim_q(\Lambda) \\ &\cong q^{-\frac{N-1}{24}(1-\lambda^2)} \cdot \tilde{M}(q) \cdot \chi_{R^T}(z_i^+)\, \chi_{S^T}(z_i^+) \end{aligned}$$

If we drop the contribution from the extra light (degenerate) states in the branching functions then it turns out that the modified CFT character is

$$\mathrm{ch}^{\mathrm{cft}}_{R_+S_+R_-S_-}(q) = q^{-\frac{c}{24}} \cdot \tilde{M}(q) \cdot \chi_{R_+^T}(z_i^+)\chi_{S_+^T}(z_i^+)\chi_{R_-^T}(z_i^-)\chi_{S_-^T}(z_i^-) \ .$$

so that the modified CFT partition function is

$$\tilde{Z}_{\mathrm{CFT}}(\lambda) = \sum_{R_+S_+R_-S_-} |\mathrm{ch}^{\mathrm{cft}}_{R_+S_+R_-S_-}(q)|^2 \ .$$

This agrees with the contribution from the perturbative excitations in the bulk

$$Z_{\mathrm{bulk}}(\lambda) = (q\bar{q})^{-c/24}|\tilde{M}(q)|^2 \sum_{R_\pm,S_\pm} |\chi_{R_+}(z_i^+)\,\chi_{S_+}(z_i^+)\,\chi_{R_-}(z_i^-)\,\chi_{S_-}(z_i^-)|^2.$$

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- Actually, both sides are expressed in terms of characters of $hs[\lambda]$ - indicates that the $\mathcal{W}_{N,k}$ models have $hs[\lambda]$ symmetry in the 'tHooft limit.
- But we need the additional light states at finite N for a modular invariant CFT.
- Where are they in the bulk $hs[\lambda]$ theory?
- A complete accounting of all the additional states has not yet been done. But strong indications that these are related to *light non-perturbative* states in the bulk theory.
- Special feature of the Vasiliev theory - can have smooth conical defect geometries. These states form a discretuum stretching all the way to the vacuum! (Castro-R.G.-Gutperle-Raeymakers)
- For instance, the (Λ, Λ) primaries are perfectly accounted for by these novel solutions (after an analytic continuation).

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Correlation Functions

- Compare CFT three point function of two scalar primaries and one spin s current $J^{(s)}$ $\langle \mathcal{O}_\pm \bar{\mathcal{O}}_\pm J^{(s)} \rangle$ with bulk three point function of two scalars and one spin s gauge field.
  (Chang-Yin, Ahn, Ammon-Kraus-Perlmutter)
- This has now been matched for any value of the spin s and parameter λ.

$$\langle \mathcal{O}_\pm(z_1)\bar{\mathcal{O}}_\pm(z_2)J^{(s)}(z_3)\rangle = \frac{(-1)^{s-1}}{2\pi}\frac{\Gamma(s)^2}{\Gamma(2s-1)}\frac{\Gamma(s\pm\lambda)}{\Gamma(1\pm\lambda)} \times \left(\frac{z_{12}}{z_{23}z_{13}}\right)^s \langle \mathcal{O}_\pm(z_1)\bar{\mathcal{O}}_\pm(z_2)\rangle$$

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Black Holes

- 3d Vasiliev theories have a novel set of black holes - generalizations of BTZ black holes - which carry higher spin charges. (Gutperle-Kraus; Ammon et.al.)
- Original construction was in $SL(N)$ Vasiliev theory (i.e. $\lambda = N$), in particular $N = 3$.
- The notion of singularity is now a gauge dependent concept. Since curvature tensor is not gauge covariant under higher spin gauge transformations.
- Thus a solution that has a singularity maybe smooth after a gauge transformation. Singularities are gauge artifacts! (GK, Castro, Hirano et.al.)
- Gauge invariant quantity in $SL(N)$ CS theory are holonomies - $P \exp \int A$ along some cycle.

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- A black hole in Euclidean AdS_3 can be taken to be one which has a contractible loop in the time direction. Smoothness demands this must be trivial.
- Prescription (GK): Take the holonomies in the time direction to be the same as for the BTZ black hole. ($(0, \pm 2\pi i)$ eigenvalues for $N = 3$)
- Gives two relations amongst four quantities (Mass, Temperature, W_3 charge, μ). Analogue of smoothness at horizon determining one relation between M and β.
- First law of thermodynamics then follows! Non geometric way of obtaining black hole entropy.
- Construction generalized to higher spins (Tan) especially to $W_\infty(\lambda)$ (Kraus-Perlmutter). Partition function (in a series exp. in μ) agrees with CFT answer for $\lambda = 0, 1$. Appears to now also agree for arbitrary λ (Gaberdiel-Hartman-Jin).

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Where to?

- Understand completely the role of HS symmetry in organizing the spectrum of free Yang-Mills theory - i.e. all the higher twist operators. (Bianchi et.al). What is the role of massive higher spin theories in string theory (Sagnotti et.al.)
- Understand better the role of the higher spin algebra in Yang-Mills theory for $\lambda \neq 0$. (Porrati et.al.) How exactly does the higgsing of the gauge invariance in the bulk take place? What constraints does it place on the theory?
- Can we use MZ techniques to see how "softly broken" higher spin symmetry might still be usefully studied?
- Use these insights to develop systematic methods of expansion about $\lambda = 0$ in the bulk and learn something about the string theory on AdS_5?

- Under what conditions are Vasiliev theories dual to CFTs? Do they have to be embeddable in a string theory? How do vector model dualities fit into the general class of AdS/CFT examples?
- Can we construct more non-SUSY examples of AdS/CFT using Vasiliev(-like) theories? (GMPTYW, Aharony et.al. examples?) Are there generically new qualitative features in non-SUSY AdS/CFT examples (like light states)? What can vector dualities teach us about non-SUSY gauge theories in 4d?
- Generalizations to other 2d cosets (Ahn, Gaberdiel-Vollenweider). Other RCFTs (Kiritsis). Supersymmetric CFTs (Creutzig-Hikida-Ronne).
- Can we generalize the dualities to massive theories? A large space of 2d integrable QFTs related by RG flows.

- Study other classical solutions of higher spin e.o.m. More black holes? With scalar Hair?
- Role of integrability in black hole dynamics. Short Poincare recurrence time in these 2d CFTs (Chang-Yin). Understand black hole puzzles in a toy model.
- de Sitter Holography? dS_4/CFT_3 (Anninos-Hartman-Strominger). dS_3/CFT_2 (Ouyang).
- Can we prove these vector model dualities? Might be the simplest examples of holography. (Douglas-Mazzucato-Razamat).