Tensions and anomalies in cosmology

Javier de Cruz Pérez

Departamento de Física Teórica, Universidad Complutense de Madrid.

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Tensions and anomalies affecting the ΛCDM model

- H_0-**tension**: Mismatch between the CMB observations (based on the ΛCDM) and the direct local measurements of the H_0 parameter.
- σ_8-**tension**: Some large scale structure (LSS) measurements indicate matter clustering weaker than that of the standard model fitted with CMB data.
- **Lensing anomaly**: The observed discrepancy between the prediction of the amount of weak lensing within the standard model and the observed one in the CMB spectra.

Running vacuum in the universe

We consider the following dynamical structure for the running vacuum energy density[1] ($\dot{} \equiv d/dt$)

$$\rho_{\rm vac}(H) = \frac{3}{8\pi G_N}\left(c_0 + \nu H^2 + \tilde{\nu}\dot{H}\right) + \mathcal{O}(H^4)$$

which, assuming $\tilde{\nu} = \nu/2$, can be rewritten as

$$\rho_{\rm vac}(H) = \frac{3}{8\pi G_N}\left(c_0 + \frac{\nu}{12}\mathcal{R}\right) \equiv \rho_{\rm vac}(\mathcal{R})$$

with $\mathcal{R} = 12H^2 + 6\dot{H}$. The local covariant conservation law in a FLRW metric takes the form ($G = G(t)$)

$$\frac{d}{dt}\left[G(\rho_t + \rho_{\rm vac})\right] + 3GH(\rho_t + p_t) = 0.$$

[1] J. Solà, A. Gómez-Valent, JdCP and C. Moreno-Pulido Universe 9 (2023) 6, 262.

Running vacuum interacting with dark matter (Type-I)

In this scenario the conservation equation is

$$\dot{\rho}_{\mathsf{cdm}} + 3H\rho_{\mathsf{cdm}} = -\dot{\rho}_{\rm vac}\,,$$

and the expressions for the energy densities

$$\begin{aligned}
\rho_{\mathsf{cdm}}(a) &= \rho_m^0 a^{-3\xi} - \rho_{\rm b}^0 a^{-3} \\
\rho_{\mathsf{vac}}(a) &= \rho_{\mathsf{vac}}^0 + \left(\frac{1}{\xi} - 1\right)\rho_m^0\left(a^{-3\xi} - 1\right)
\end{aligned}$$

with

$$\xi = \frac{1-\nu}{1-\frac{3}{4}\nu} \simeq 1 - \frac{\nu}{4} + \mathcal{O}\left(\nu^2\right) \equiv 1 - \nu_{\rm eff} + \mathcal{O}\left(\nu_{\rm eff}^2\right)\,.$$

Type-I models with threshold

We switch off the interaction between vacuum and the cold dark matter, except when we approach the usual epoch of vacuum domination ($z_* \simeq 1$)

$\underline{a < a_*}$ **($z > z_*$)**

$$\rho_{\text{cdm}}(a) = \rho_{\text{cdm}}(a_*) \left(\frac{a}{a_*}\right)^{-3}$$

$$\rho^*_{\text{vac}} = \rho^0_{\text{vac}} + \left(\frac{1}{\xi} - 1\right) \rho^0_m \left(a_*^{-3\xi} - 1\right) = \text{const.}$$

$\underline{a > a_*}$ ($z < z_*$)

$$\rho_{\text{cdm}}(a) = \rho^0_m a^{-3\xi} - \rho^0_{\rm b} a^{-3}$$

$$\rho_{\text{vac}}(a) = \rho^0_{\text{vac}} + \left(\frac{1}{\xi} - 1\right) \rho^0_m \left(a^{-3\xi} - 1\right).$$

Running vacuum with running G (Type-II)

The conservation equation that characterizes this scenario is

$$\dot{G}\,(\rho_t + \rho_{\rm vac}) + G\,\dot{\rho}_{\rm vac} = 0\,.$$

In this case it is not possible to find analytical solutions for the energy densities

$$3H^2 = \frac{8\pi G_N}{\varphi}\left[\rho_t + C_0 + \frac{3\nu}{16\pi G_N}(2H^2 + \dot{H})\right]\,,$$

$$-(3H^2 + 2\dot{H}) = \frac{8\pi G_N}{\varphi}\left[p_t - C_0 - \frac{3\nu}{16\pi G_N}(2H^2 + \dot{H})\right]\,,$$

$$\frac{\dot{\varphi}}{\varphi} = \frac{\dot{\rho}_{\rm vac}}{\rho_t + \rho}\,,$$

with $\varphi(t) \equiv G_N/G(t)$.

Results obtained with the Baseline dataset[3]

Parameter	ΛCDM	Type-I RRVM	Type-I RRVM$_{\rm thr.}$	Type-II RRVM
Ω_m	0.3029 ± 0.0045	0.3036 ± 0.0056	0.3235 ± 0.0071	0.3032 ± 0.0089
H_0[km/s/Mpc]	68.27 ± 0.35	68.22 ± 0.47	67.65 ± 0.38	68.12 ± 0.97
$\nu_{\rm eff}$	-	0.00006 ± 0.00030	0.0227 ± 0.0055	-0.00008 ± 0.00035
$\varphi(0)$	-	-	-	1.008 ± 0.028
$\sigma_8(0)$	0.8003 ± 0.0064	0.799 ± 0.011	0.7733 ± 0.0092	0.801 ± 0.010
ΔDIC[2]	-	-2.04	+15.34	-4.18

- Without forcing the interaction to only take place in the late universe we do not observe evidence in favor of a dynamical vacuum.
- For all the models we observe $H_0 \sim 68$ km/s/Mpc and only the type-I RRVM$_{\rm thr}$ is capable of lowering the value of $\sigma_8(0)$.

[2] $\Delta{\rm DIC} = {\rm DIC}_{\Lambda{\rm CDM}} - {\rm DIC}_{\rm X}$

[3] This dataset includes SNIa+BAO+$H(z)$+LSS+CMB but not the data from SH0ES.

Results obtained with the Baseline+SH0ES dataset

Parameter	ΛCDM	Type-I RRVM	Type-I $\text{RRVM}_{\text{thr.}}$	Type-II RRVM
Ω_m	0.2961 ± 0.0041	0.2928 ± 0.0049	0.3128 ± 0.0064	0.2808 ± 0.0058
H_0[km/s/Mpc]	68.82 ± 0.33	69.17 ± 0.43	68.33 ± 0.35	70.79 ± 0.69
ν_{eff}	-	-0.00037 ± 0.00029	0.0197 ± 0.0055	-0.00003 ± 0.00033
$\varphi(0)$	-	-	-	0.950 ± 0.021
$\sigma_8(0)$	0.7978 ± 0.0064	0.808 ± 0.011	0.7747 ± 0.0093	0.807 ± 0.010
ΔDIC	-	-0.64	+10.94	+6.58

- The running vacuum does not help to alleviate the H_0-tension.
- A variable Newtonian coupling $G(t)$ allows to increase the value of the Hubble parameter $H_0 \sim 71$ km/s/Mpc.

Baseline+SH0ES contour plots

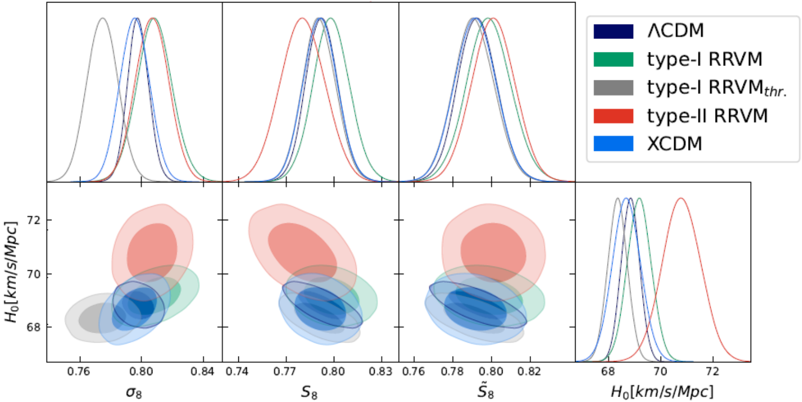

Figure: With $S_8 = \sigma_8\sqrt{\Omega_m^0/0.5}$ and $\tilde{S} = S_8/\sqrt{\varphi(0)}$

The lensing anomaly

Incorrect prediction of the amount of weak lensing in the CMB temperature and polarizaiton power spectra in the context of the ΛCDM.

Some of the possible solutions:

- Non-flat models $\Omega_k \neq 0$ ($(\Omega_m + \Omega_k)h^2$).
- A phenomenological approach $A_L \neq 1$ ($C_l^{\Psi} \Rightarrow A_L C_l^{\Psi}$).

Different primordial power spectra

To analyze the CMB anisotropy data one must assume a form for the primordial power spectrum (PS).

- $P_\delta(k) = A_s \left(\frac{k}{k_0}\right)^{n_s}$ (Tilted flat PS)
- $P_\delta(q) \propto \frac{(q^2-4K^2)^2}{q(q^2-K)} \left(\frac{k}{k_0}\right)^{n_s-1}$ (Planck PS)
- $P_\delta(q) \propto (q^2 - 4K)^2 |P_\zeta(A)|$ (new PS) with $A = \frac{q}{\sqrt{|K|}} - 1$ (closed) and $A = \frac{q}{\sqrt{|K|}}$ (open). [4]

where

$$k_0 = 0.05\mathrm{Mpc}^{-1} \quad q = \sqrt{k^2 + K^2} \quad K = -(H_0^2/c^2)\Omega_k$$

[4] For the details see B. Ratra PRD 106 (2022) 12, 123524 and JdCP, C-G. Park and B. Ratra PRD 107 (2023) 6, 063522.

Analytical expression for the closed new P(q)

$$\sqrt{|P_\zeta(A)|} = \left(\frac{16\pi}{m_p^2}\right)^{1/2} Q^{1/p} \frac{(2+q_s)p}{\sqrt{\pi q_s}} F(A) G(A) H(A)$$

$$F(A) = \left| -1 + \frac{W(A)}{p} \right|$$

$$G(A) = \frac{2^{-(6-4q_s+2A-W(A))/p}}{\sqrt{A}(A-1)(A+3)}$$

$$H(A) = \left| \frac{\Gamma\left(1+W(A)/p\right)\Gamma\left((2+q_s)/(2p)\right)}{\Gamma\left((2+W(A))/p\right)} \right|$$

with

$$W(A) = \sqrt{-8-4q_s+q_s^2+4A(A+2)} \quad q_s = \frac{2-2n_s}{3-n_s} \quad p = 2-q_s$$

Results obtained with Planck 2018 TT,TE,EE+lowE (PR3)

Parameter	ΛCDM	ΛCDM+A_L	non-flat Planck $P(q)$	non-flat new $P(q)$
Ω_m	0.3165 ± 0.0084	0.3029 ± 0.0093	0.481 ± 0.062	0.444 ± 0.055
H_0[km/s/Mpc]	67.28 ± 0.61	68.31 ± 0.71	$54.5^{+3.1}_{-3.9}$	56.9 ± 3.6
A_L	-	1.181 ± 0.067	-	-
Ω_k	-	-	$-0.043^{+0.018}_{-0.015}$	$-0.033^{+0.017}_{-0.011}$
$\sigma_8(0)$	0.8118 ± 0.0074	0.7997 ± 0.0088	0.775 ± 0.015	0.786 ± 0.014
ΔDIC	-	$+5.52$	$+7.34$	$+6.39$

- Evidence in favor of closed universe $\sim 2.4\sigma$[5] and 2.7σ evidence in favor of $A_L > 1$.
- The non-flat models are strongly favored over the flat ΛCDM

[5] W. Handley PRD 103, (2021) L041301, E. Di Valentino, A. Melchiorri and J. Silk Nature. Astron. 4, (2019) 196.

The inclusion of the lensing data (PR3+lensing)

The inclusion of the lensing data changes significantly the results

- It breaks partially the Ω_m-Ω_k-H_0-A_L degeneracy.
- The evidence in favor of closed hypersurfaces decreases until $\sim 1.5\sigma$.
- For the flat ΛCDM+A_L model $A_L = 1.073 \pm 0.041$ (1.78σ).
- None of the models is strongly favoured over the flat ΛCDM.

Results obtained with PR3+non-CMB[6] data

Parameter	flat ΛCDM	flat ΛCDM+A_L	non-flat Planck $P(q)$	non-flat new $P(q)$
Ω_m	0.3054 ± 0.0051	0.2997 ± 0.0054	0.3043 ± 0.0054	0.3045 ± 0.0053
H_0[km/s/Mpc]	68.08 ± 0.39	68.55 ± 0.42	68.31 ± 0.56	68.29 ± 0.55
A_L	-	1.198 ± 0.060	-	-
Ω_k	-	-	0.0009 ± 0.0017	0.0008 ± 0.0017
$\sigma_8(0)$	0.8052 ± 0.0067	0.7964 ± 0.0075	0.8070 ± 0.0077	0.8071 ± 0.0075
ΔDIC	-	+8.47	-1.45	-1.17

- The evidence in favor of $\Omega_k \neq 0$ has almost subsided.
- There is still evidence in favor of $A_L > 1$ at $\sim 3.3\sigma$.

[6]Here we call non-CMB to the data set composed by SNIa+BAO+$H(z)$+LSS JdCP, C.G. Park and B. Ratra arXiv:2404.19194

Statistical estimators for the tensions

We can quantify the tension between two given data sets within a particular cosmological model

$$\mathcal{I}(D_1, D_2) \equiv \exp\left(-\frac{\mathcal{G}(D_1, D_2)}{2}\right)$$ [7]

where

$$\mathcal{G}(D_1, D_2) = \mathrm{DIC}(D_1 \cup D_2) - \mathrm{DIC}(D_1) - \mathrm{DIC}(D_2)$$

Therefore $\log_{10}\mathcal{I} > 0$ when the two data sets are mutually consistent and when $\log_{10}\mathcal{I} < 0$ the two data sets are inconsistent. Applying Jeffreys' scale the level of consistency or inconsistency between the two data sets is

- substantial if $|\log_{10}\mathcal{I}| > 0.5$.
- strong if $|\log_{10}\mathcal{I}| > 1$.
- decisive if $|\log_{10}\mathcal{I}| > 2$.

[7] S. Joudaki et al. MNRAS 465 (2017) 2033.

Statistical estimators for the tensions

We can compute the probability p of two data sets being inconsistent by chance [8]

$$p = \int_{d-2log(S_D)}^{\infty} \chi_d^2(x)dx = \int_{d-2log(S_D)}^{\infty} \frac{x^{d/2-1}e^{-x/2}}{2^{d/2}\Gamma(d/2)}dx,$$

being d the Bayesian model dimensionality and S_D the suspiciousness parameter.

Considering a Gaussian analogy the value of p can be converted into a "sigma value" using

$$N_\sigma = \sqrt{2}\mathrm{Erfc}^{-1}(1-p)$$

- $p \lesssim 0.05$ ($N_\sigma = 2$) the results obtained with the two data sets are in moderate tension.
- $p \lesssim 0.003$ ($N_\sigma = 3$) the results obtained with the two data sets are in strong tension.

[8] W. Handley and P. Lemos PRD 100 (2019) 043504.

Data set tensions

PR3 vs. non-CMB

	$\log_{10} \mathcal{I}$	N_σ
Flat ΛCDM	0.805	1.152
Flat ΛCDM+A_L	1.446	0.164
Non-flat Planck $P(q)$	-0.796	2.704
Non-flat new $P(q)$	-0.391	2.308

New PR4 results

Recently a new release of the Planck likelihood (PR4) has been made public [9] and its analysis changes some of the results obtained with the former release PR3.

Parameter	ΛCDM	ΛCDM+A_L	non-flat Planck $P(q)$	XCDM	CPL
Ω_m	0.3100 ± 0.0077	$0.3070^{+0.0081}_{-0.0094}$	0.354 ± 0.033	$0.215^{+0.022}_{-0.070}$	$0.218^{+0.028}_{-0.074}$
H_0[km/s/Mpc]	67.63 ± 0.57	67.86 ± 0.65	$63.4^{+2.5}_{-3.1}$	83^{+10}_{-8}	83^{+10}_{-9}
A_L	-	1.035 ± 0.055	-	-	-
Ω_k	-	-	-0.011 ± 0.009	-	-
ω_0	-	-	-	$-1.46^{+0.21}_{-0.38}$	$-1.26 \pm 0.42 (\omega_a = -0.9^{+1.1}_{-1.4})$
$\sigma_8(0)$	0.8062 ± 0.0067	0.8036 ± 0.0077	0.7964 ± 0.0097	$0.933^{+0.10}_{-0.058}$	$0.930^{+0.10}_{-0.068}$

[9] M. Tristram et al. Astron.Astrophys 682 (2024) A37.

PR3 results vs. PR4 results

PR3 vs. PR4

	PR3	PR4	σ
ΛCDM+A_L	$A_L = 1.181 \pm 0.067$	$A_L = 1.035 \pm 0.055$	1.68σ
Planck $P(q)$	$\Omega_k = -0.043^{+0.018}_{-0.015}$	$\Omega_k = -0.011 \pm 0.009$	1.59σ
XCDM	$\omega_0 = -1.59^{+0.15}_{-0.34}$	$\omega_0 = -1.46^{+0.21}_{-0.38}$	0.32σ
CPL	$\omega_0 = -1.25^{+0.43}_{-0.56}$ ($\omega_a = -1.3 \pm 1.2$)	$\omega_0 = -1.26 \pm 0.42$ ($\omega_a = -0.9^{+1.1}_{-1.4}$)	0.01σ (0.22σ)

- We observe a decrease in the evidence of $\Omega_k < 0$ and $A_L > 1$ when we move from PR3 to PR4.
- The cosmological parameter constraints of the dynamical dark energy density seem to be less affected by the change of CMB data.

PR4+BAO[10]+SNIa

Parameter	ΛCDM	ΛCDM+A_L	non-flat Planck $P(q)$	XCDM	CPL
Ω_m	0.3043 ± 0.0052	0.3025 ± 0.0054	0.3028 ± 0.0054	0.3081 ± 0.0068	0.3073 ± 0.0068
H_0[km/s/Mpc]	68.05 ± 0.39	68.19 ± 0.41	68.45 ± 0.55	67.54 ± 0.71	67.85 ± 0.72
A_L	-	1.049 ± 0.050	-	-	-
Ω_k	-	-	0.0018 ± 0.0016	-	-
ω_0	-	-	-	-0.978 ± 0.026	$-0.858 \pm 0.067 (\omega_a = -0.56 \pm 0.29)$
$\sigma_8(0)$	0.8039 ± 0.0063	0.8013 ± 0.0068	0.8078 ± 0.0071	0.797 ± 0.011	0.809 ± 0.012

[10] A.G. Adame et al. arXiv:2404.03002.

Comparing contour plots for the non-flat Planck $P(q)$ model

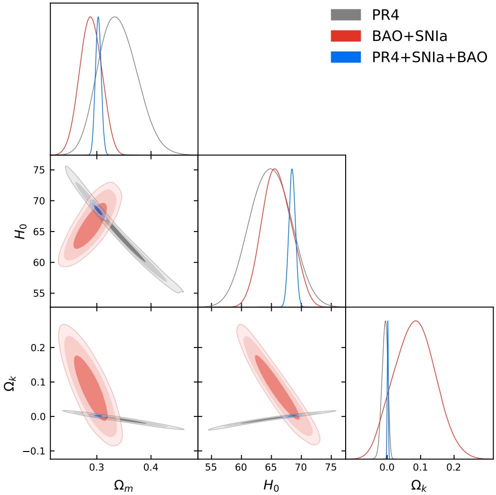

Figure: H_0 is expressed in km/s/Mpc.

Comparing contour plots for the XCDM model

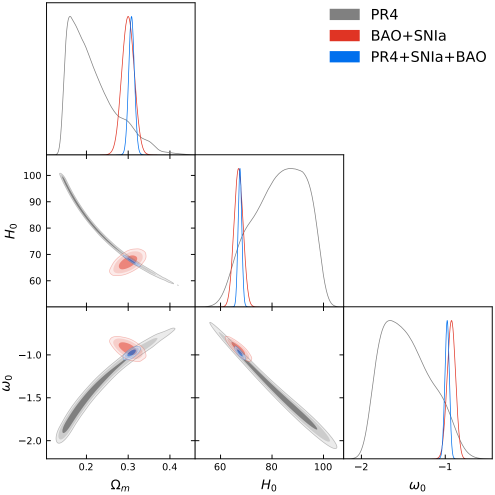

Figure: H_0 is expressed in km/s/Mpc.

Conclusions

- A dynamical vacuum component helps to deal with σ_8-tension but cannot alleviate the H_0-tension.
- Values $H_0 \sim 71$ km/s/Mpc are favored when in addition to a running vacuum we allow the variation of the gravitational Newtonian constant $G(t)$.
- Both $\Omega_k < 0$ and $A_L > 1$ are capable of ease the lensing anomaly observed with PR3.
- There is a tension between PR3 and non-CMB cosmological parameter constraints within the context of the non-flat models.
- The PR4 likelihood reduces the evidence of both $\Omega_k < 0$ and $A_L > 1$.

Thank you for your attention!