ATHEXIS

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Non-adiabatic Inflation

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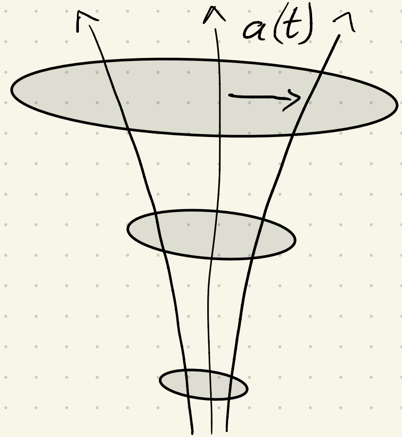
arXiv: 2311.03281 Jackson, Assadullahi, Gow, Koyama, Vennin & Wands

(and astro-ph/0101406 Leach, Sasaki, Wands & Liddle)

classical inflation:



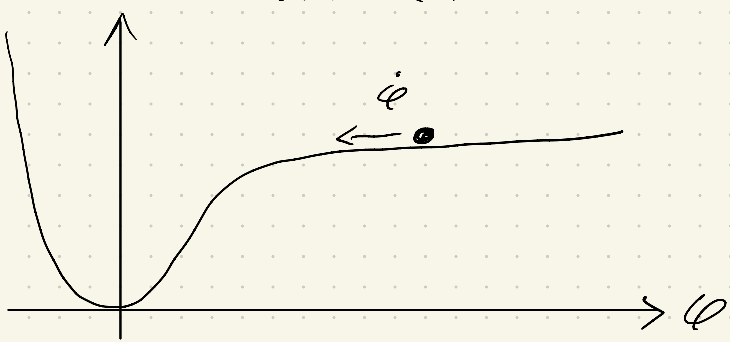
FLRW spacetime:

adiabatic Hubble expansion

($\Delta Q = 0$) $\quad H = \dot{a}/a$

single scalar field, $\varphi(t)$

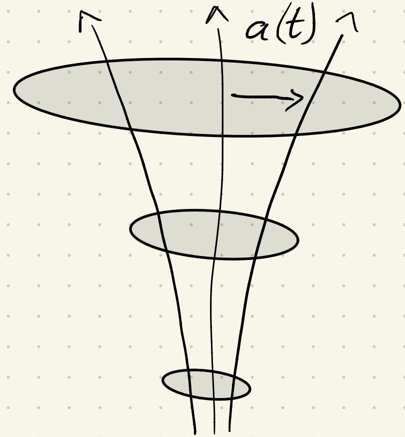
potential energy, $V(\varphi)$



$$\left.\begin{aligned} H^2 &= \frac{8\pi G}{3}\left(V + \frac{1}{2}\dot{\varphi}^2\right) \\ \ddot{\varphi} &+ 3H\dot{\varphi} + V'(\varphi) = 0 \end{aligned}\right\}$$

classical inflation:

FLRW spacetime:



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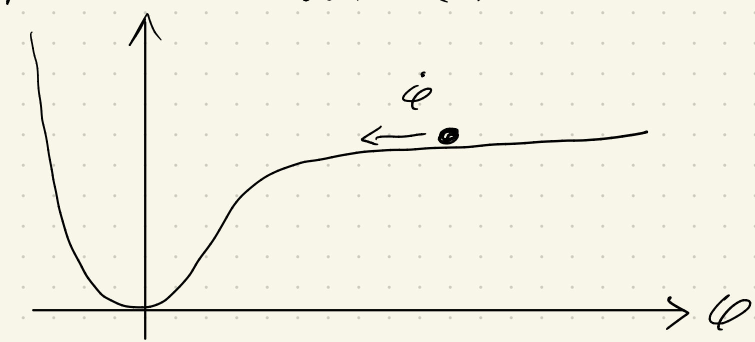


inflation:

$$\ddot{a} = \frac{d}{dt}(aH) > 0$$

when $V > \dot{\varphi}^2$

quantum fluctuations

comoving waves $\propto e^{i\vec{k}\cdot\vec{x}}$

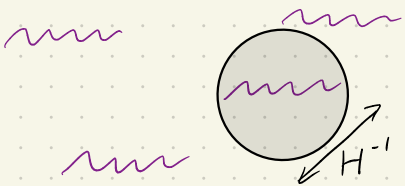
sub-Hubble

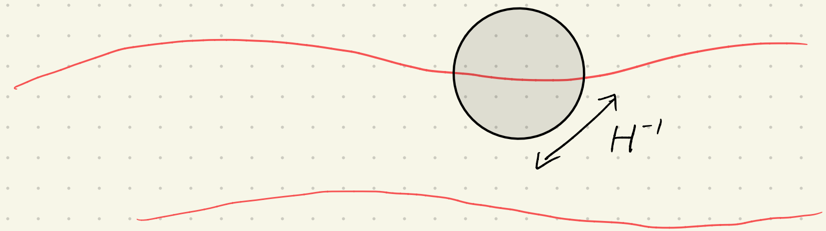
UV : $k > aH$ $\xrightarrow{\ddot{a}>0}$ super-Hubble

IR : $k < aH$

underdamped oscillations of quantum field, $\hat{\delta\varphi}$

overdamped squeezed state $\approx$ classical random field, φ





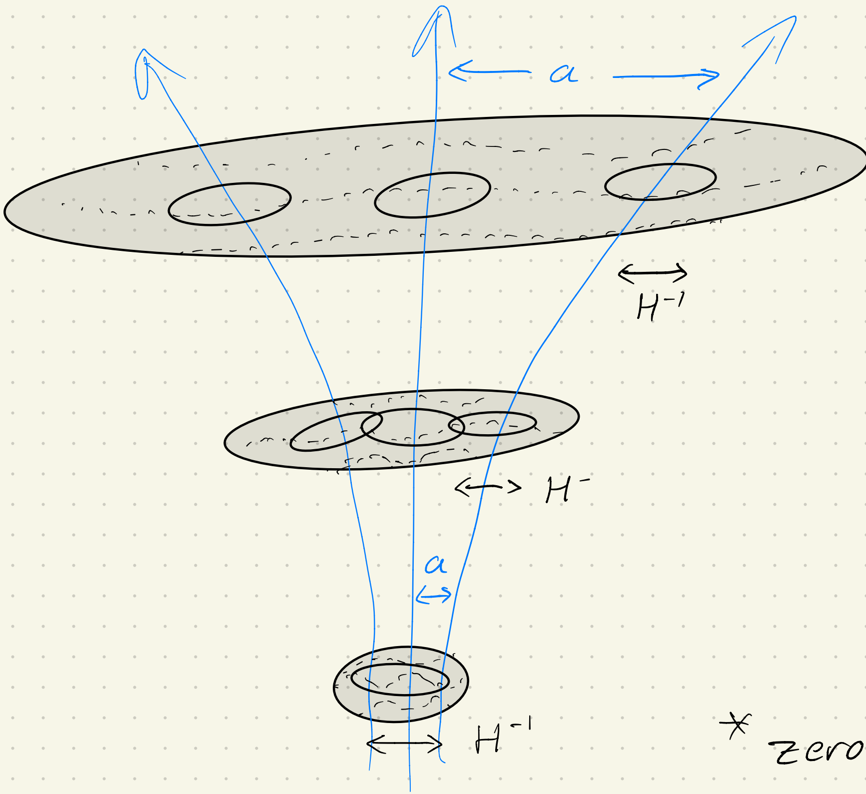
perturbative QFT

separate universe approach

non-perturbative

(stochastic, δN, etc...)

Separate universes



coarse-grained fields ($k < aH$)

follow same evolution locally as homogeneous FLRW cosmology

$\varphi(\vec{x})$ and $H(\vec{x})$

if spatial gradients* and anisotropy negligible

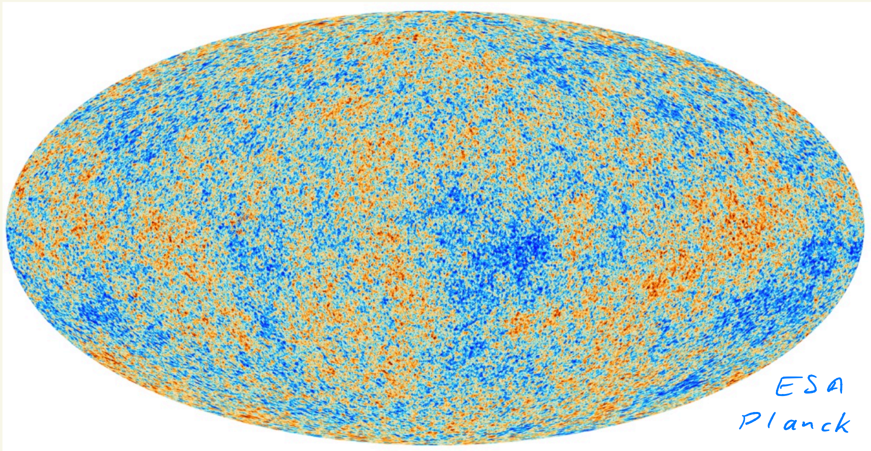
* zeroth-order in gradient expansion

Salopek & Bond (1990)

slow-roll inflation:

- **adiabatic** quantum vacuum $\rightarrow$ **adiabatic** density pertbns

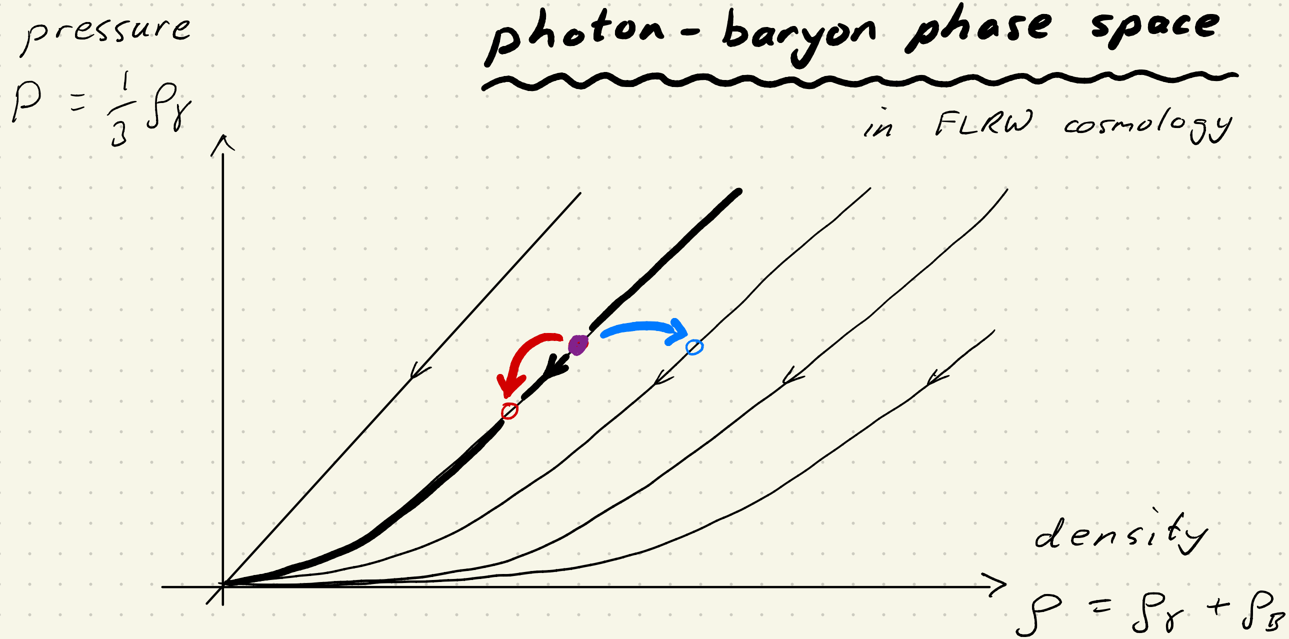
$$\langle R^2 \rangle = \left(\frac{H}{\dot{\varphi}}\right)^2 \langle 0 | \delta\hat{\varphi}^2 | 0 \rangle_{BD} \longrightarrow \langle R^2 \rangle = \left(\frac{H}{\dot{\rho}}\right)^2 \langle \delta\rho^2 \rangle$$



$$\frac{\Delta T}{T} \sim \frac{2}{5} R$$

photon-baryon phase space

in FLRW cosmology



adiabatic perturbation

$$\mathcal{R} = H\frac{\delta\rho}{\dot{\rho}} = H\frac{\delta P}{\dot{P}}$$

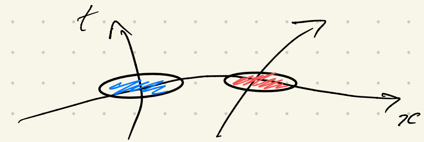
$$\Rightarrow \delta\left(n_\gamma/n_B\right) = 0$$

non-adiabatic perturbation

$$S = 3H\left(\frac{\delta P}{\dot{P}} - \frac{\delta\rho}{\dot{\rho}}\right)$$

a.k.a. isocurvature,
entropy perturbn.

scalar field phase space



$$\ddot{\varphi} + 3H\dot{\varphi} + \frac{dV}{d\varphi} = 0$$

two regimes

(i) quasi-equilibrium : $\dot{\varphi} \simeq -\frac{V'}{3H} \longrightarrow$ **slow roll**

$|\ddot{\varphi}| \ll |3H\dot{\varphi}|$ attractor, $\dot{\varphi}(\varphi)$

(ii) free evolution : $\ddot{\varphi} \simeq -3H\dot{\varphi} \longrightarrow$ **ultra-slow roll**

$|V'| \ll |3H\dot{\varphi}|$

$$\varphi \simeq D\int \frac{dt}{a^3}$$

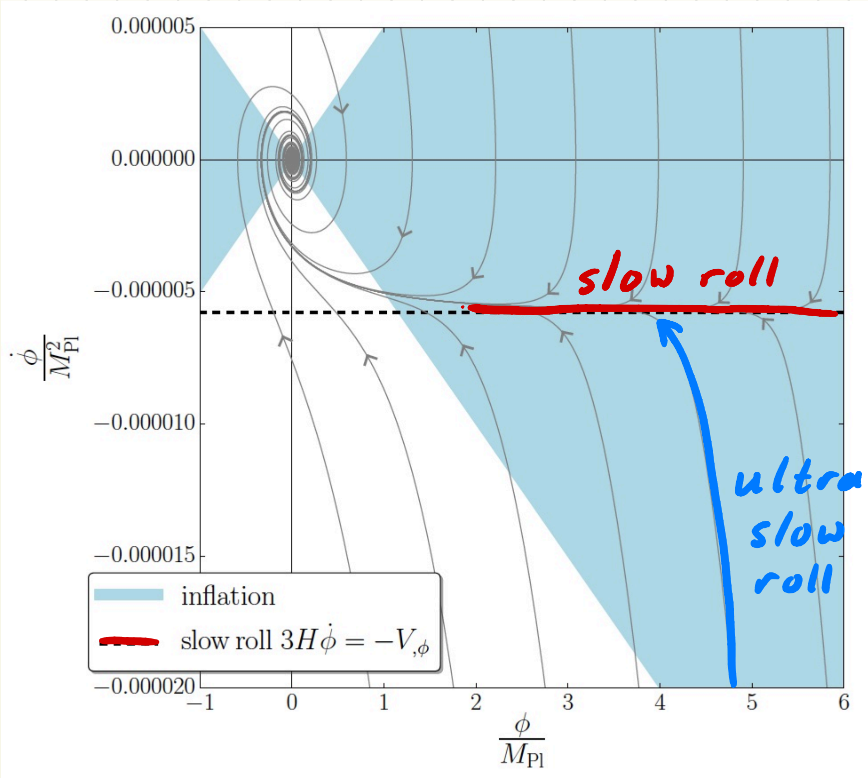
transient / decaying solution

as $a \longrightarrow \infty$

single-field inflation phase space

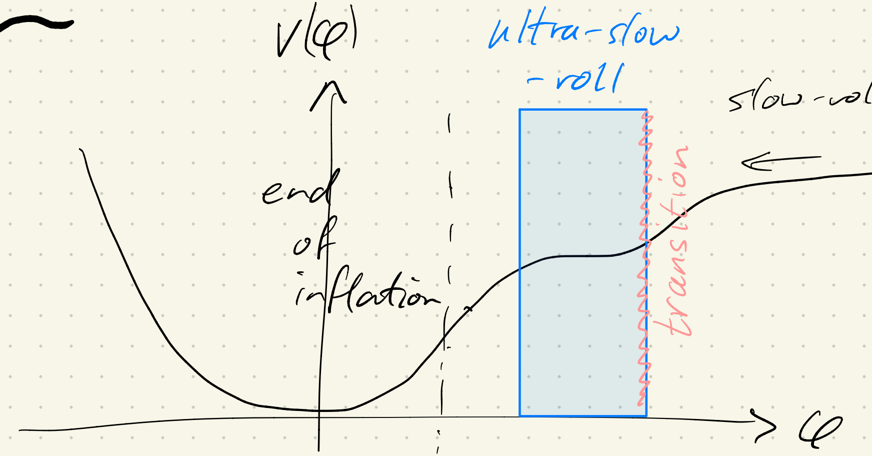
Grain & Vennin (2017)

$V = \frac{1}{2} m^2 \varphi^2$



Ultra-slow-roll inflation

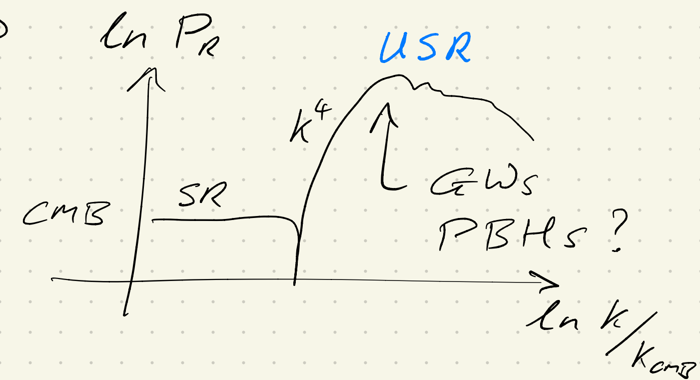
e.g. inflection point



primordial curvature power spectrum

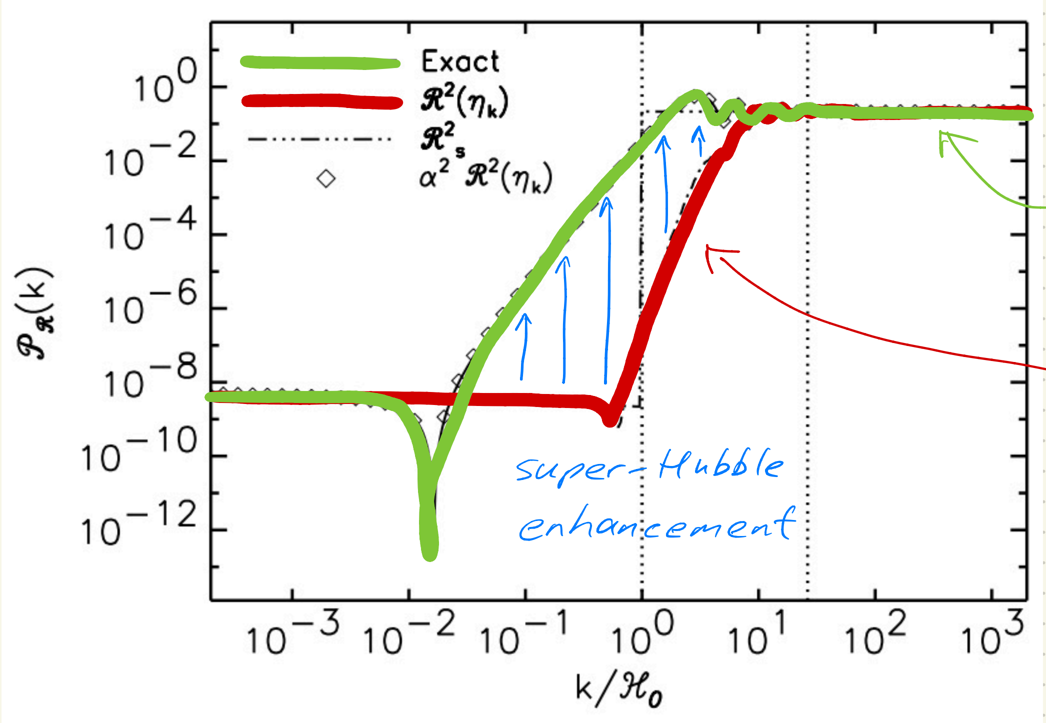
$$P_R(k) = \left(\frac{H}{\dot{\varphi}}\right)^2 P_{\delta\varphi}(k)$$

boosted as $\left(\frac{\dot{\varphi}}{H}\right)^2 \to 0$



Enhancement of superhorizon scale inflationary curvature perturbations *

Samuel M. Leach[1], Misao Sasaki[2], David Wands[3] and Andrew R. Liddle[1]



* linear perturbations

arXiv:astro-ph/0101406v2 21 Feb 2001

non-adiabatic pressure in single-field inflation

curvature evolution : $\dot{\mathcal{R}} = -\dfrac{3H^2}{2\dot{V}}\,\delta P_{nad}$

FLRW cosmology : $\rho = \frac{1}{2}\dot{\varphi}^2 + V(\varphi)$, $P = \frac{1}{2}\dot{\varphi}^2 - V(\varphi)$

non-adiabatic perturbn : $\delta P_{nad} = \delta P - (\dot{P}/\dot{\rho})\,\delta\rho$

$= 0$ if $\exists$ attractor $\dot{\varphi} = \dot{\varphi}(\varphi)$

Hamilton - Jacobi approach

Salopek + Bond (1990)

$\dot{\mathcal{R}} \propto k^2 \Phi \longrightarrow 0$ as $k \to 0$

finite Bardeen potential $\Rightarrow$ attractor $H(\varphi)$ on large scales

but $\delta P_{nad} \neq 0$ in USR !

gauge dependence of large-scale limit

Artigas et al, in preparation

in longitudinal gauge

Bardeen potential $\Phi \to \Phi_0$ as $k \to 0$

$\Rightarrow$ $\delta P_{nad} \to 0$ "

note: δP_{nad} is gauge invariant

so this is a physical restriction

in spatially flat gauge

allows $\delta P_{nad} \neq 0$ as $k \to 0$

necessary for stochastic ultra-slow roll

validity of separate universe approach?

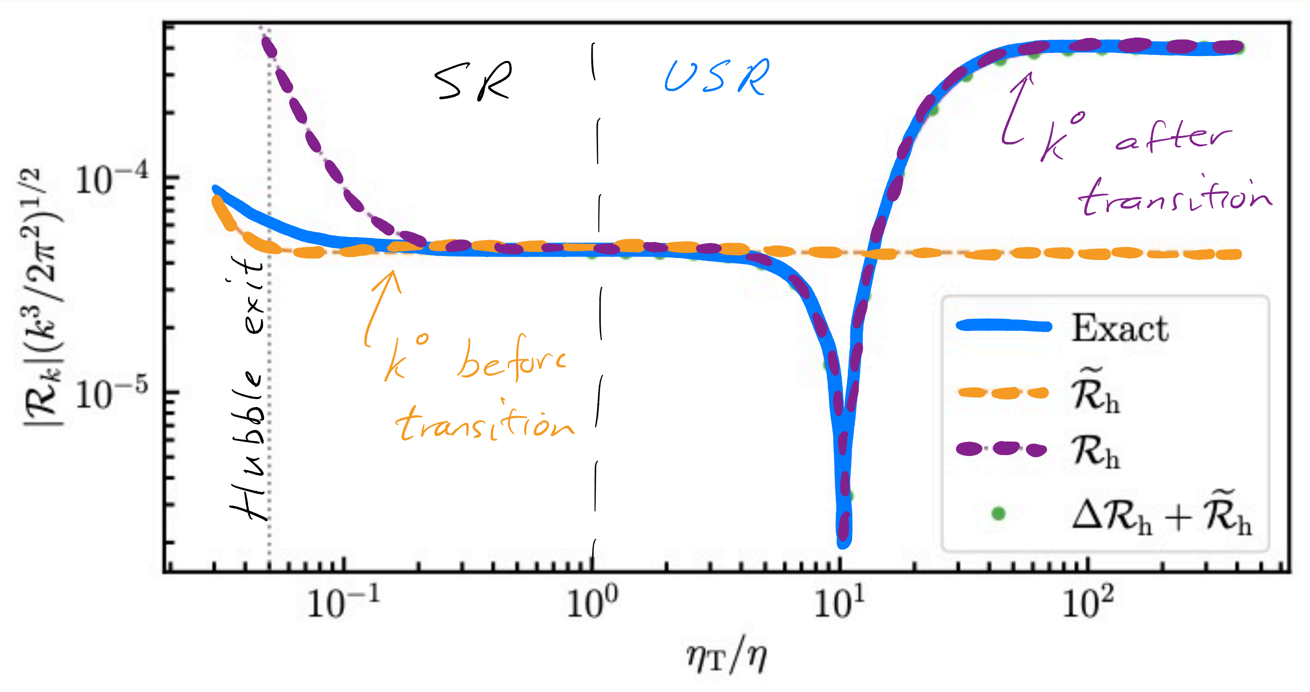
stochastic inflation uses local FRW eqns on super-H scales

✓ **slow roll** 0th-order gradient expansion sufficient
only **adiabatic** noise

✓ **ultra-slow roll** 0th-order gradient expansion sufficient
with **non-adiabatic** noise

The separate-universe approach and sudden transitions during inflation

Joseph H. P. Jackson,[a] Hooshyar Assadullahi,[a,b] Andrew D. Gow.[a] Kazuya Koyama,[a] Vincent Vennin,[c,a] David Wands[a]



arXiv:2311.03281v2

* linear perturbations

validity of separate universe approach?

stochastic inflation uses local FRW eqns on super-H scales

✓ **slow roll** 0^{th}-order gradient expansion sufficient

only adiabatic noise

! SR → USR transition requires 0^{th} + 2^{nd} order gradients

can only apply separate universes **piecewise**

✓ **ultra-slow roll** 0^{th}-order gradient expansion sufficient

with non-adiabatic noise

challenges for stochastic USR inflation

- Bunch-Davies vacuum not valid after transition (Pattison et al 2021)
- Hamilton-Jacobi, $H(\varphi)$ attractor, (Rigopoulos & Wilkins, 2022)

  cannot describe non-adiabatic modes on super-Hubble scales
- waiting until transients decay ("frozen noise", Tomberg 2023)

  cannot model quantum diffusion during USR phase

- allow **non-adiabatic noise** (non BD vacuum) **in USR**

  + **discrete stochastic kick at transition** (non-Markovian?)

  Jackson et al, in progress

- full numerical GR? Launay et al 2024

conclusions:

sudden transitions → non-adiabatic inflation

(diabatic?)

- particle production on sub-Hubble scales
- non-adiabatic perturbations on super-Hubble scales

→ enhanced density perturbations

challenging to model non-perturbative fluctuations

most nonlinear studies invoke large scale limit

but gradient terms important at transition ...

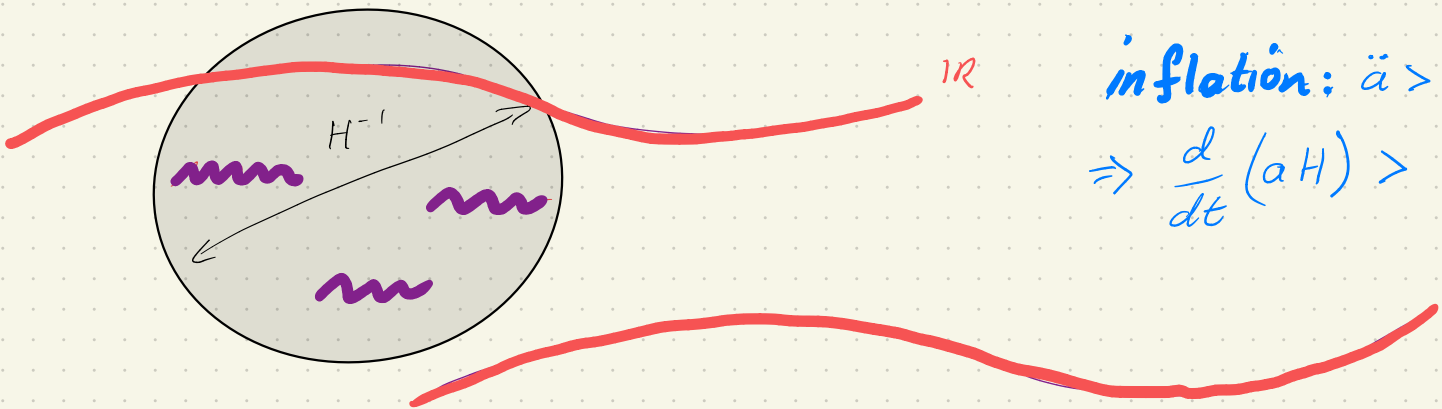
Stochastic inflation

- effective theory for quantum fields in inflating spacetime

- stochastic behaviour emerges from coarse-graining at Hubble scale
  $H = \dot{a}/a$

inflation: $\ddot{a} > 0$

$$\Rightarrow \frac{d}{dt}(aH) > 0$$



noise : $\langle \hat{\xi}_\varphi^{\,2} \rangle \sim \frac{d \ln \tilde{k}}{d N} \times P_{\delta\varphi}(\tilde{k})$, $\tilde{k} \approx aH$

Stochastic Ultra-Slow-Roll Inflation

Pattison et al (2021)

UV : linear mode functions on sub-Hubble scales

stochastic noise* as modes cross coarse-graining scale

IR : coarse-grained field obeys Langevin equations for φ and $\pi = a^3\dot{\varphi}$

$$\left.\begin{aligned} \frac{d\varphi}{dN} &= \frac{\pi}{a^3 H} + \hat{\xi}_\varphi \\ \frac{d\pi}{dN} &= -\frac{a^3 V'}{H} + \hat{\xi}_\pi \end{aligned}\right\}$$

* squeezed state

→ correlated noise

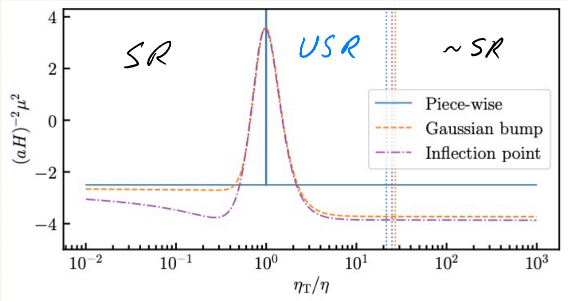
but not adiabatic in USR

non-adiabatic transition SR → USR

Jackson et al (2023)

mode equation $u_k'' + \left(k^2 - \frac{z''}{z}\right) u_k = 0$

$u_k = z R_k$



general solution in SR / USR limit

$$\Rightarrow \quad u_k \simeq \frac{1}{\sqrt{2k}} \left\{ \alpha_k \left(1 - \frac{i}{k\eta}\right) e^{-ik\eta} + \beta_k \left(1 + \frac{i}{k\eta}\right) e^{ik\eta} \right\}$$

before transition: BD vacuum, $\alpha_k = 1$, $\beta_k = 0$

after transition: particle production on sub-Hubble scales

non-adiabatic pertbns on super-Hubble scales

$\alpha_k \neq 0$, $\beta_k \propto \left(k_T/k\right)^3$ for $k \ll k_T$

Classical δN

- $\mathcal{R} = \delta N = N(\varphi + \delta\varphi) - N(\varphi)$ single kick about classical

  $N(\varphi) = \int_{\varphi_{end}}^{\varphi} H \, dt$

- linear: $\delta N = \frac{H}{\dot{\varphi}} \delta\varphi$

$$\Rightarrow \quad P_{\mathcal{R}}(k) = \left(\frac{H}{\dot{\varphi}}\right)^2 P_{\delta\varphi}(k)$$

$$= \left(\frac{H^2}{2\pi\dot{\varphi}}\right)^2_{k=aH} \quad \text{in slow-roll}$$

$\longrightarrow$ nearly scale invariant primordial density pertbns seen in CMB

- non-linear expansion $N = \bar{N} + N'\delta\varphi + \frac{1}{2} N'' \delta\varphi^2 + \ldots$

  gives perturbative non-Gaussianity

Classical δN

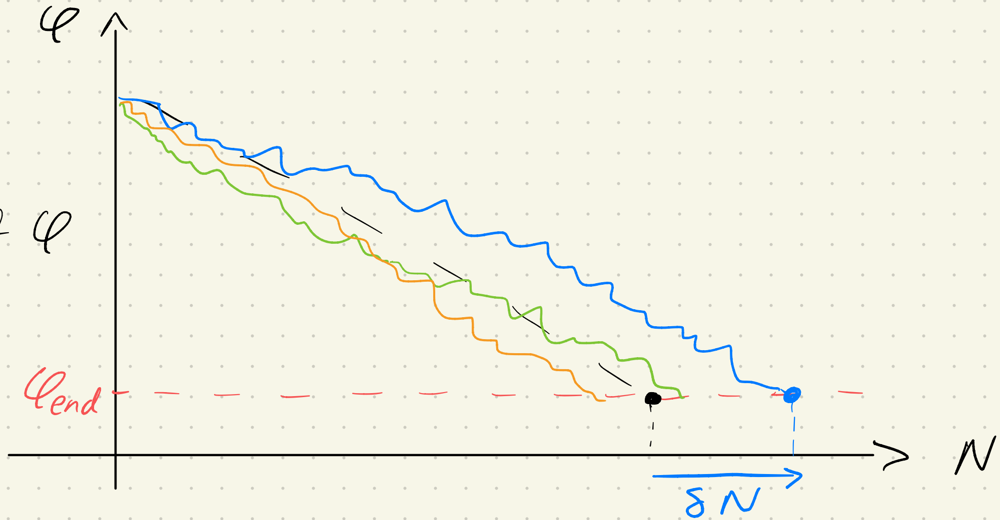
– $\mathcal{R} = \delta N = N(\varphi + \delta\varphi) - N(\varphi)$ single kick about classical

$$N(\varphi) = \int_{\varphi_{end}}^{\varphi} H \, dt$$

Stochastic δN

stochastic evolution of φ

calculate first passage time to cross φ_{end}



→ non-perturbative $\delta N = N - \langle N \rangle$

– large excursions in δN give rise to PBHs

e.g. Gow et al, arXiv:2211.08348