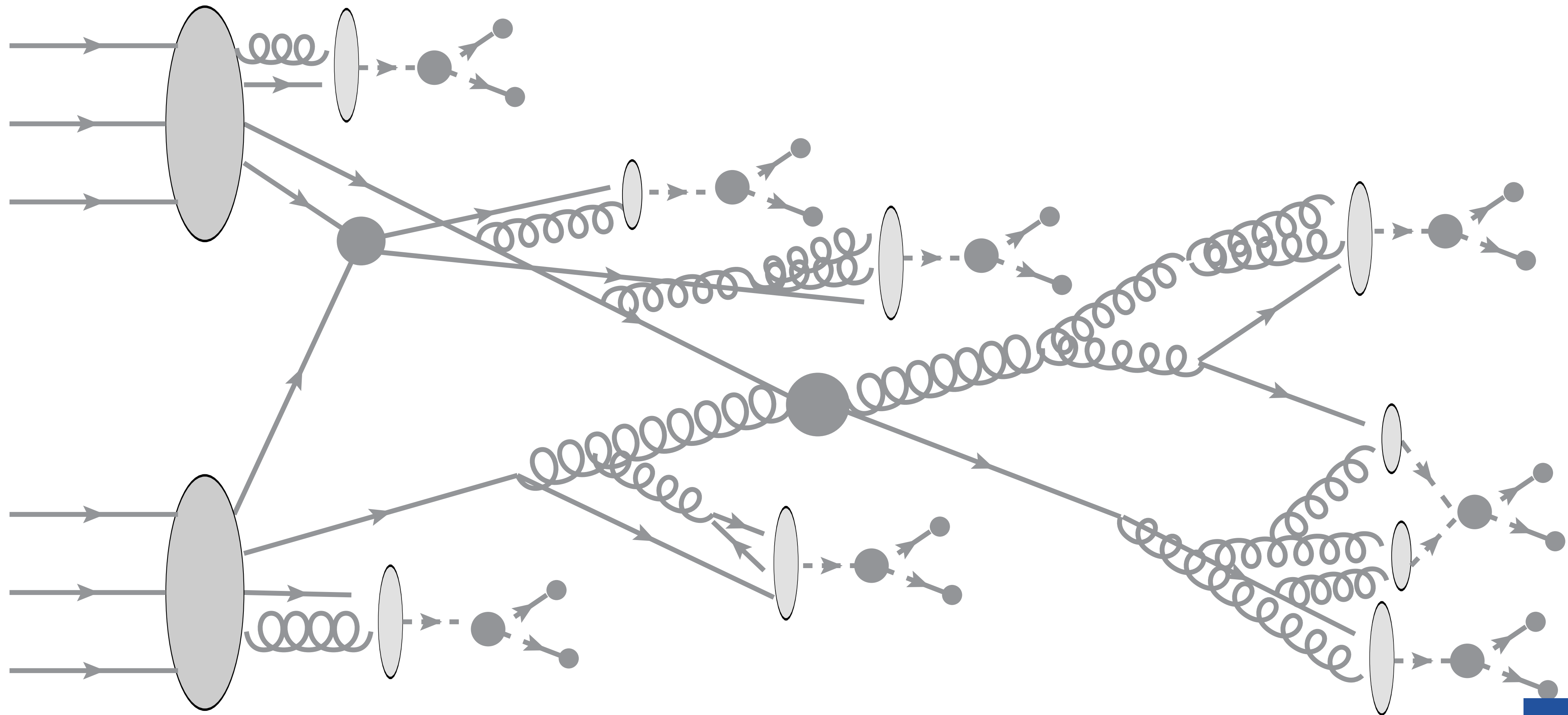


Overview on Parton Shower Developments



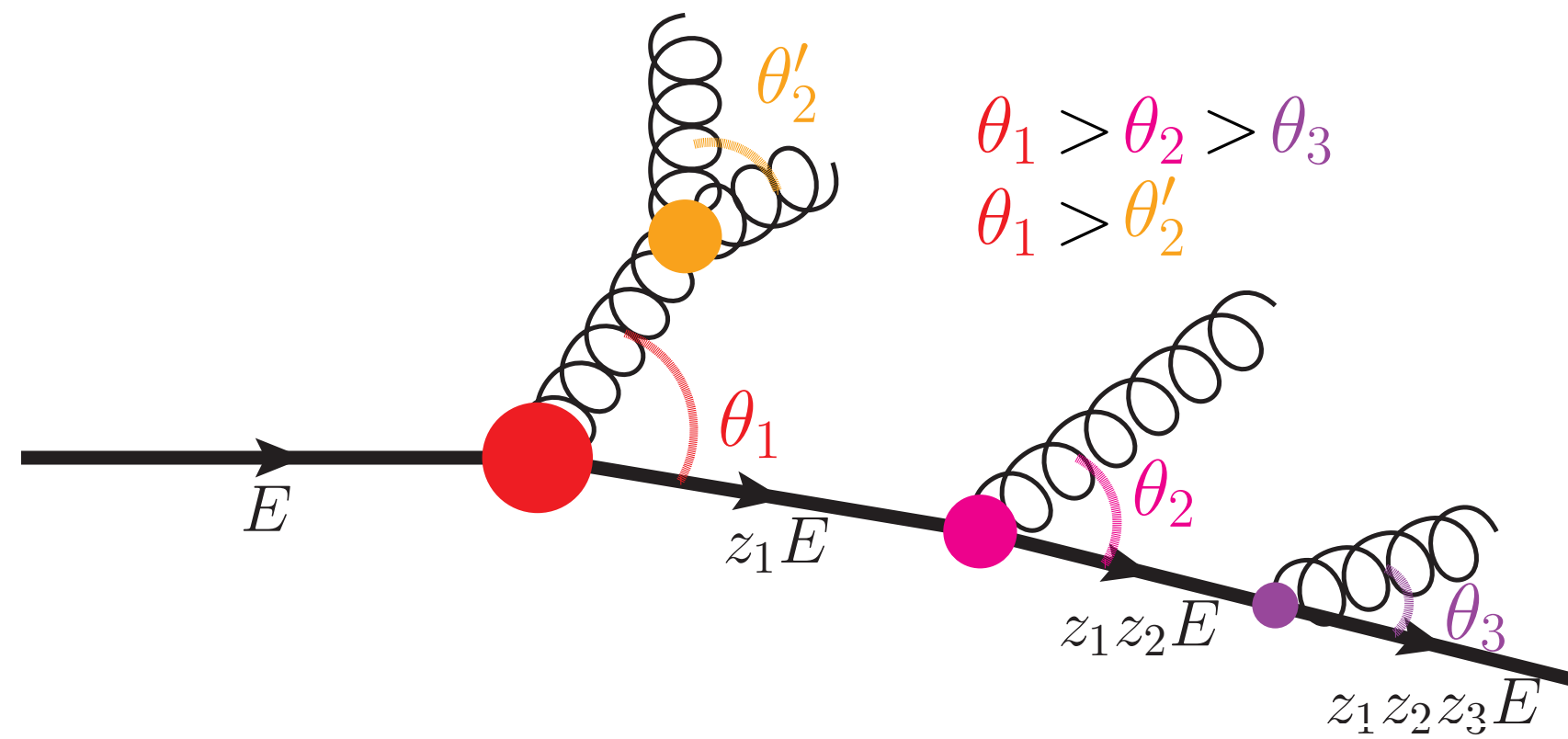
Silvia Ferrario Ravasio

Parton Showers and Resummation 2024

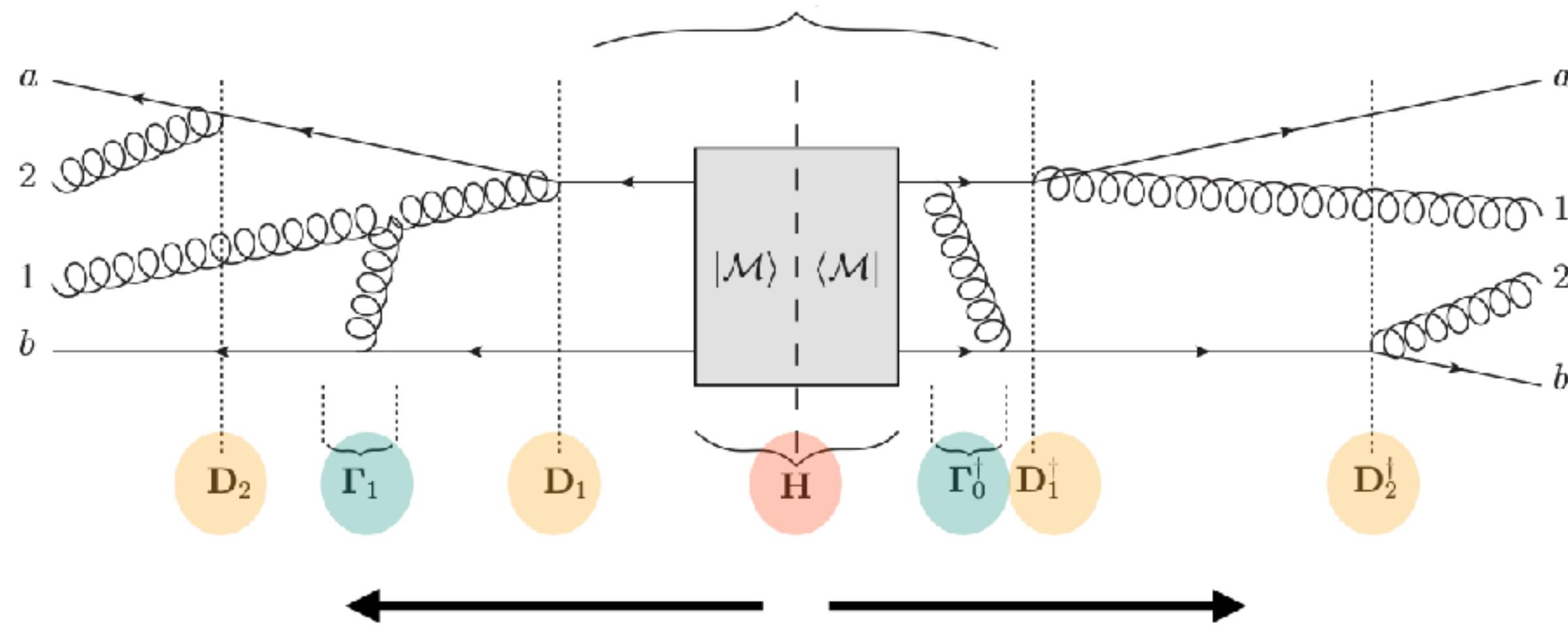
2nd July 2024, **Universität Graz**



What is a parton shower?



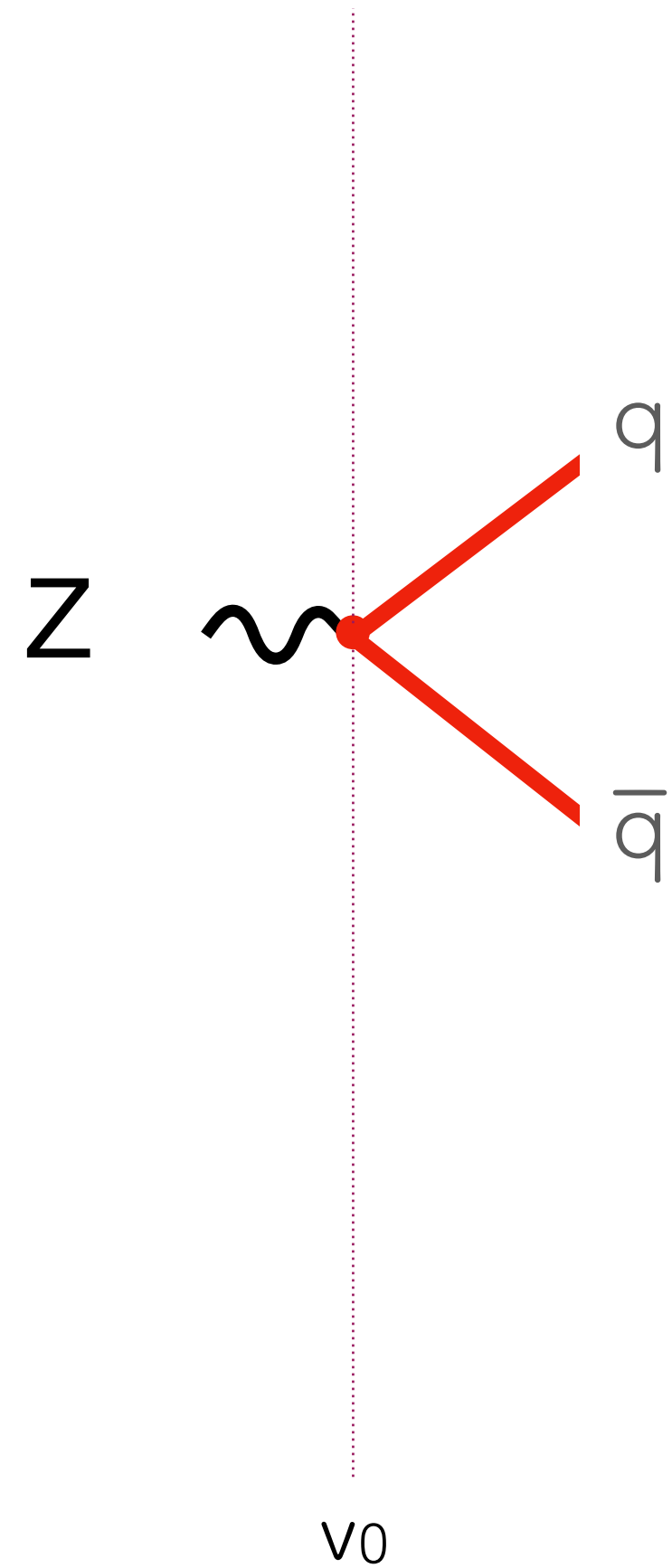
$Z \sim$



Dipole Showers in a nutshell

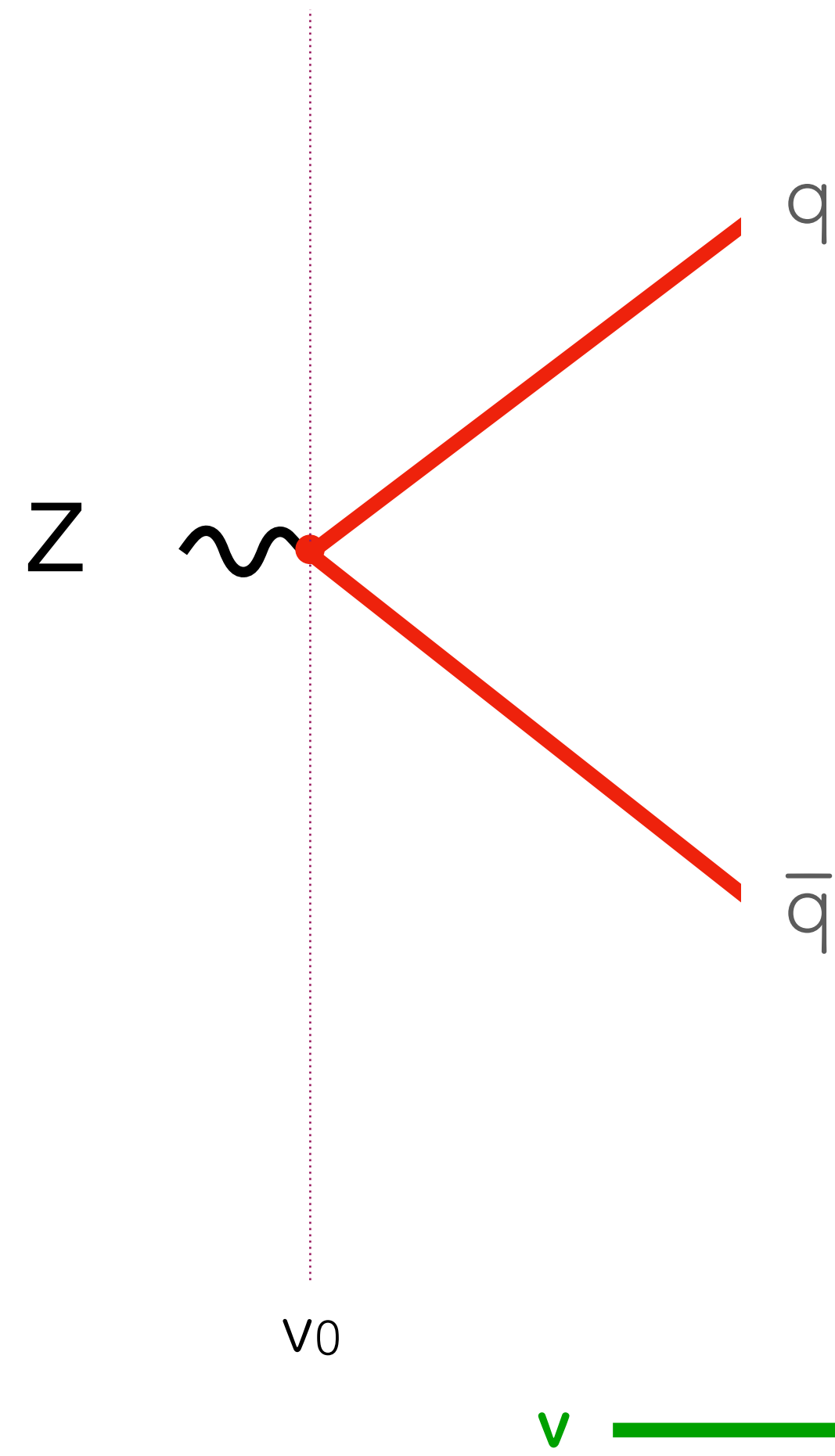
Dipole showers [Gustafson, Pettersson, '88] are the most used shower paradigm

Start with $q\bar{q}$ state produced at a hard scale v_0 .



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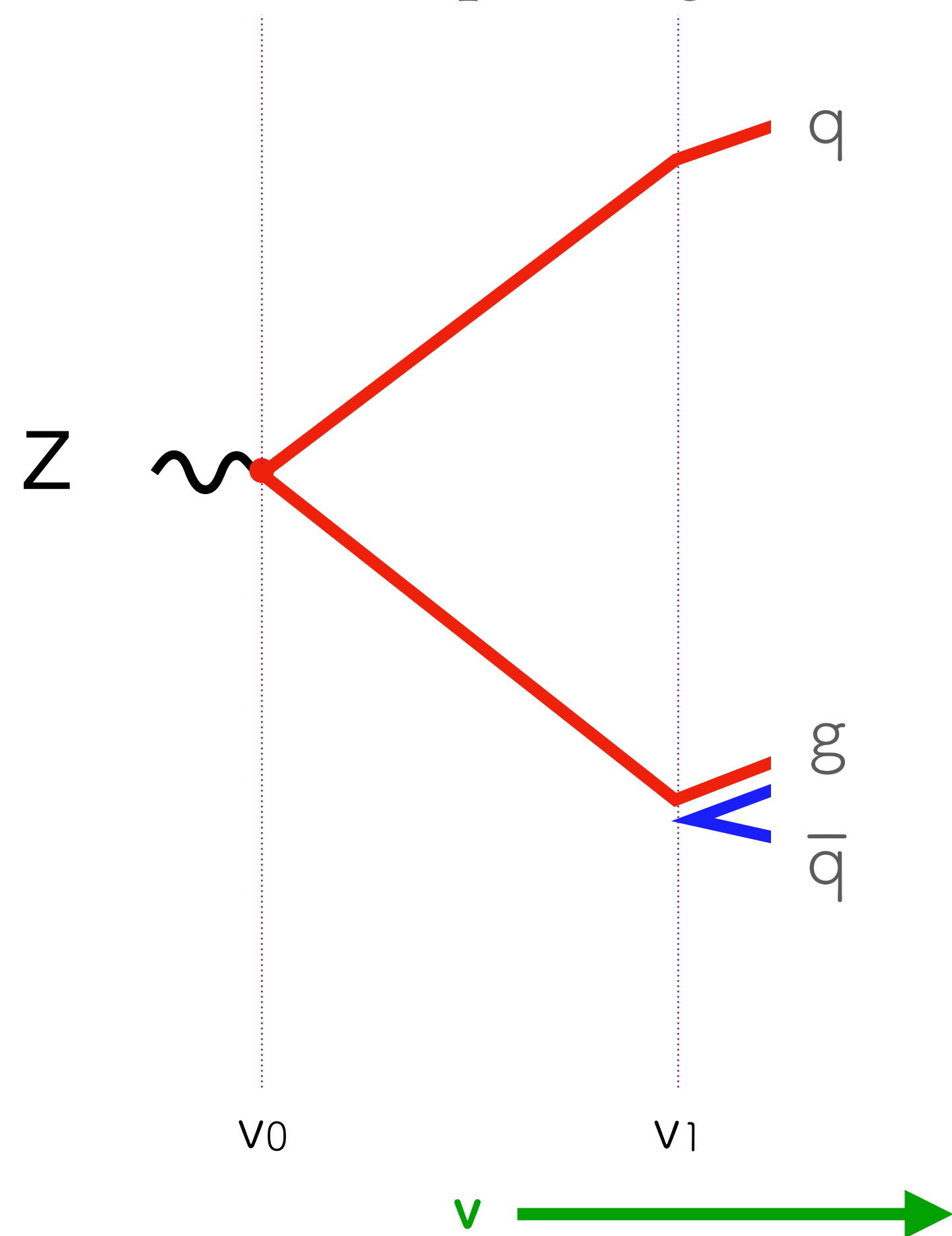
Start with $q\bar{q}$ state produced at a hard scale v_0 .

Throw a random number to determine down to what **scale** state persists unchanged

$$\Delta(v_0, v) = \exp \left(- \int_v^{v_0} dP_{q\bar{q}}(\Phi) \right)$$

Dipole Showers in a nutshell

Dipole showers [Gustafson, Pettersson, '88] are the most used shower paradigm



Start with $q\bar{q}$ state produced at a hard scale v_0 .

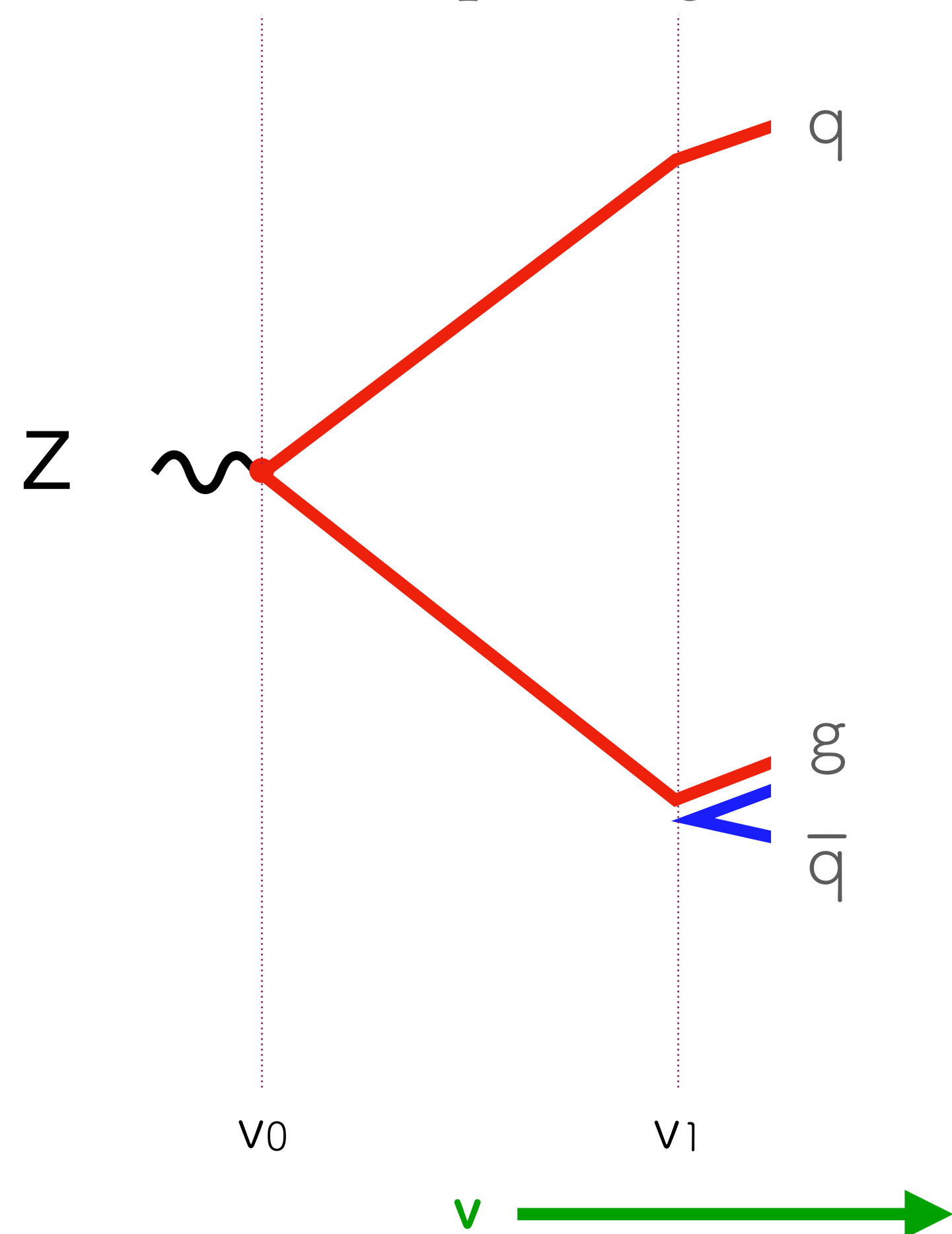
Throw a random number to determine down to what **scale** state persists unchanged

At some point, **state splits** ($2 \rightarrow 3$, i.e. emits gluon) at a scale $v_1 < v_0$. The kinematic (rapidity and azimuth) of the gluon is chosen according to

$$dP_{q\bar{q}}(\Phi(v_1)) \quad \Phi = \{v, \eta, \varphi\}$$

Dipole Showers in a nutshell

Dipole showers [Gustafson, Pettersson, '88] are the most used shower paradigm



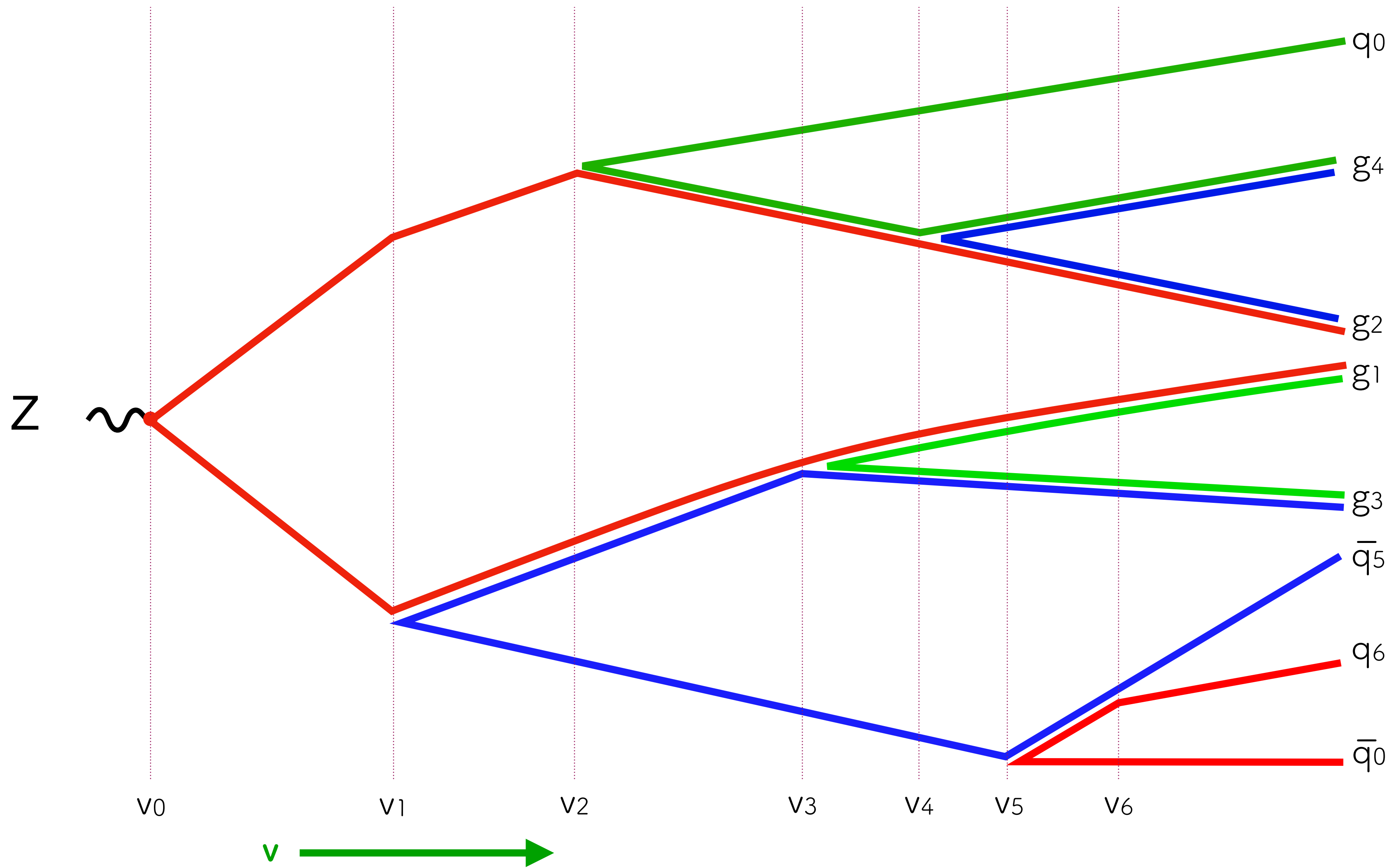
Start with $q\bar{q}$ state produced at a hard scale ν_0 .

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At some point, **state splits** ($2 \rightarrow 3$, i.e. emits gluon) at a scale $\nu_1 < \nu_0$.

The gluon is part of two dipoles (qg), ($g\bar{q}$).

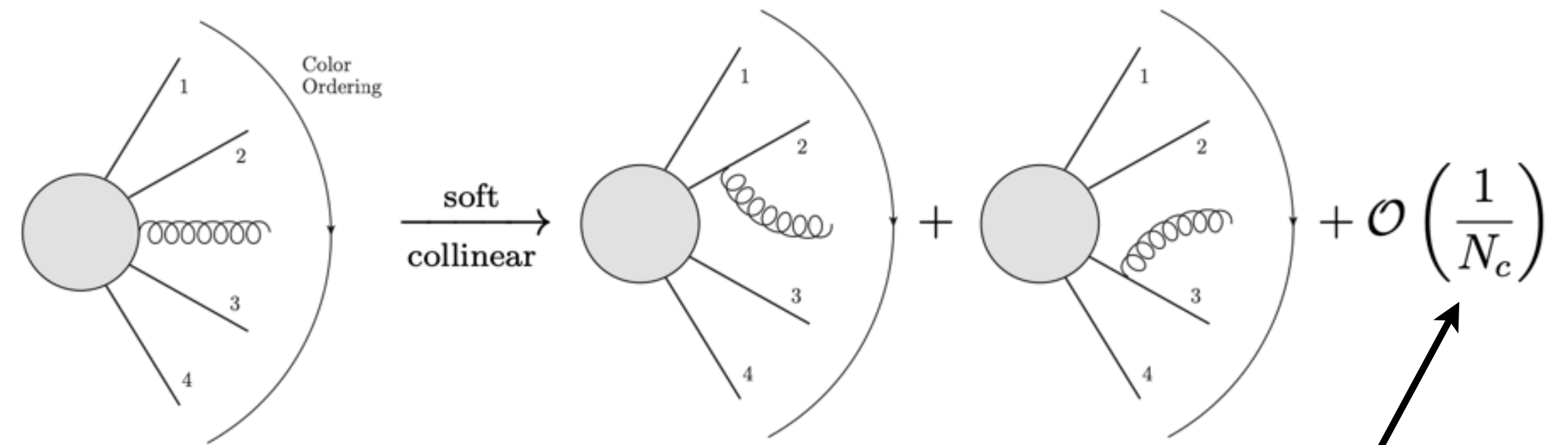
Iterate the above procedure for both dipoles independently, using ν_1 as starting scale.



self-similar
 evolution
 continues until it
 reaches a non-
 perturbative
 scale

Adding QED and EW interactions in dipole showers: the Vincia solution

- Dipole showers reproduce the soft QCD radiation pattern, exploiting the large number of colour approximation

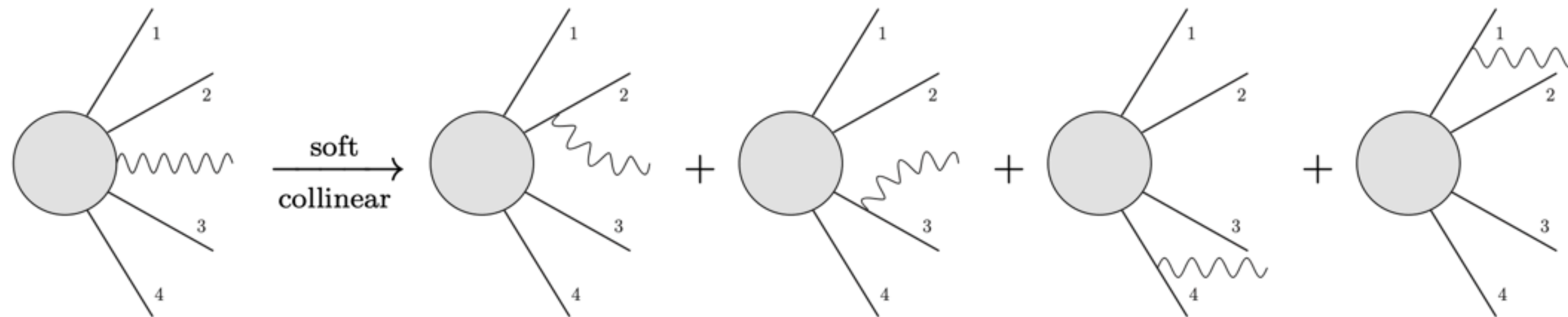
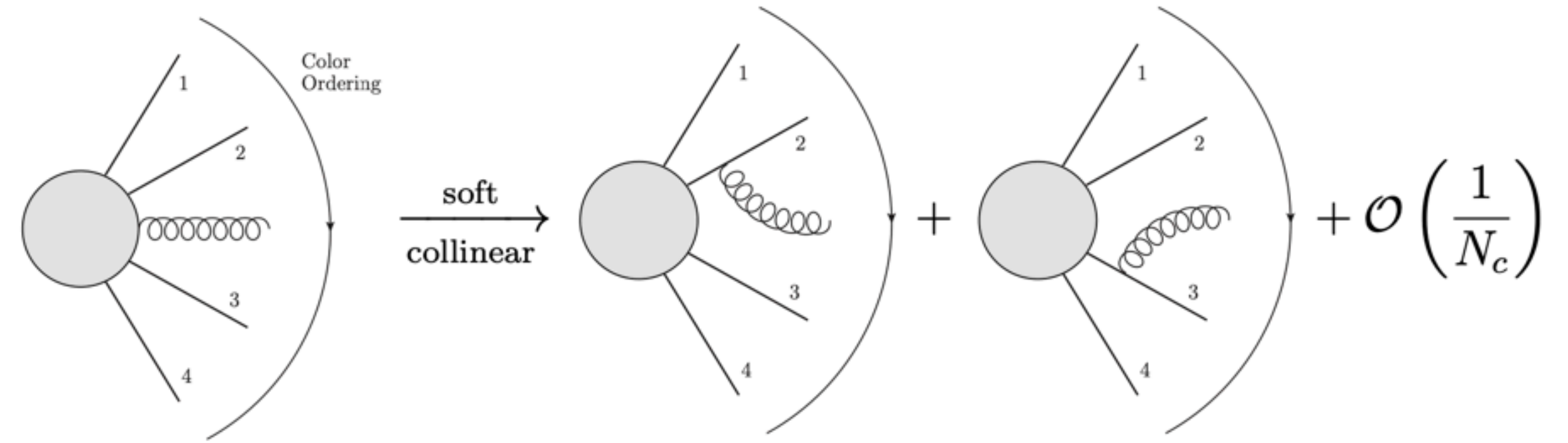


For the inclusion of **subleading N_c corrections** in a dipole shower see 1202.4496 + 1501.00778 (**Deductor**), 2011.10054 (**PanScales**), 2011.15087 (**FHP**)

Adding QED and EW interactions in dipole showers: the Vincia solution

Plots from Verheyen PhD thesis

- Dipole showers reproduce the soft QCD radiation pattern, exploiting the large number of colour approximation

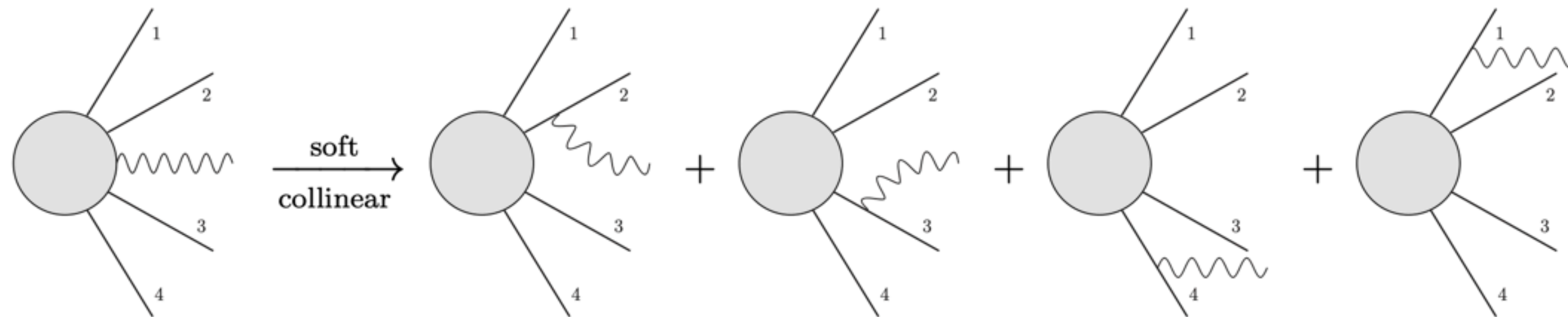
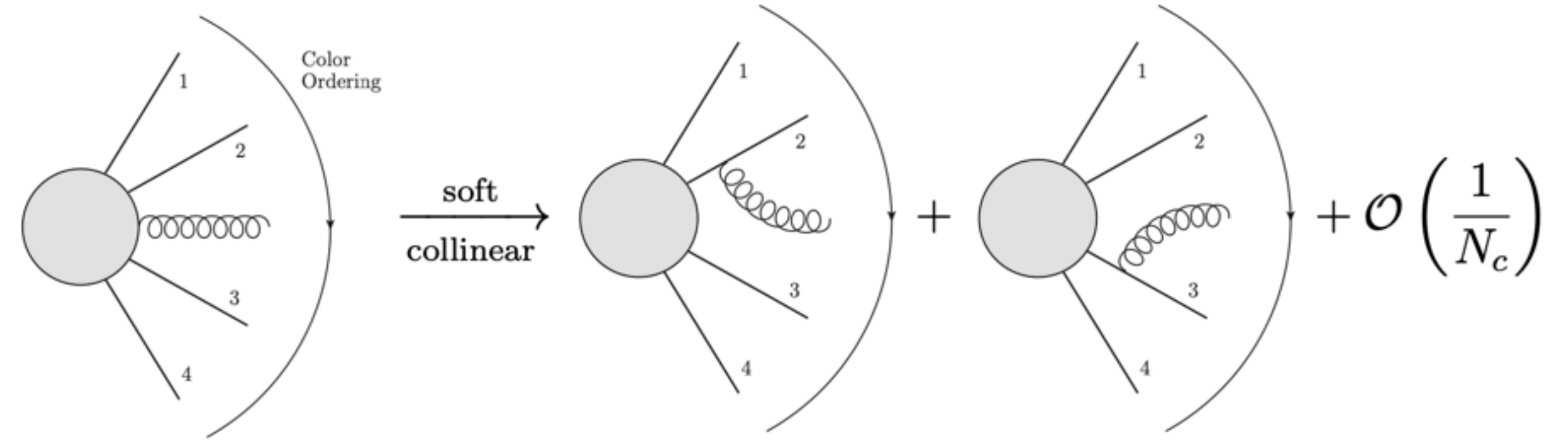


- For photon emissions, all charged particles contribute equally → **multipole**

Adding QED and EW interactions in dipole showers: the Vincia solution

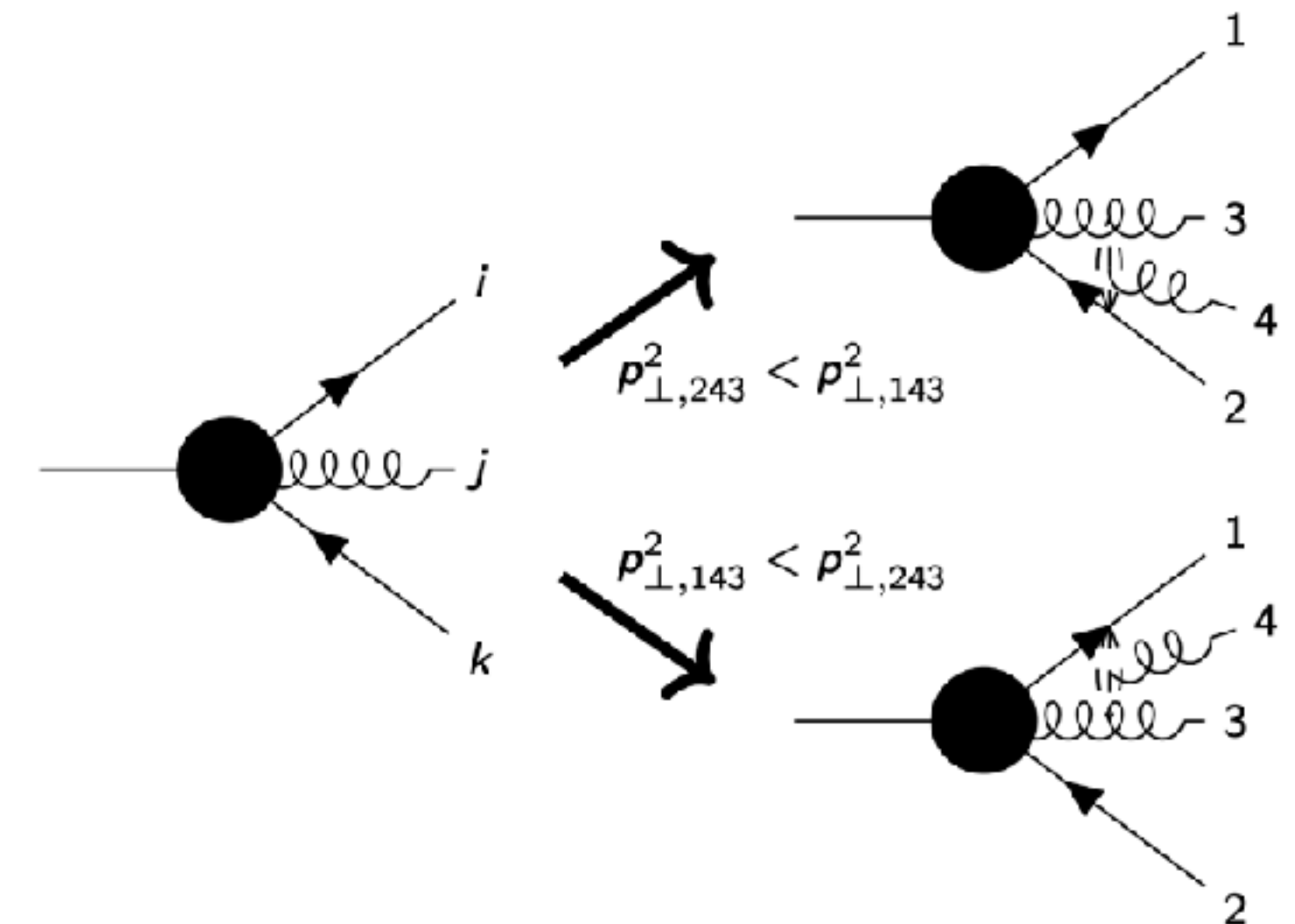
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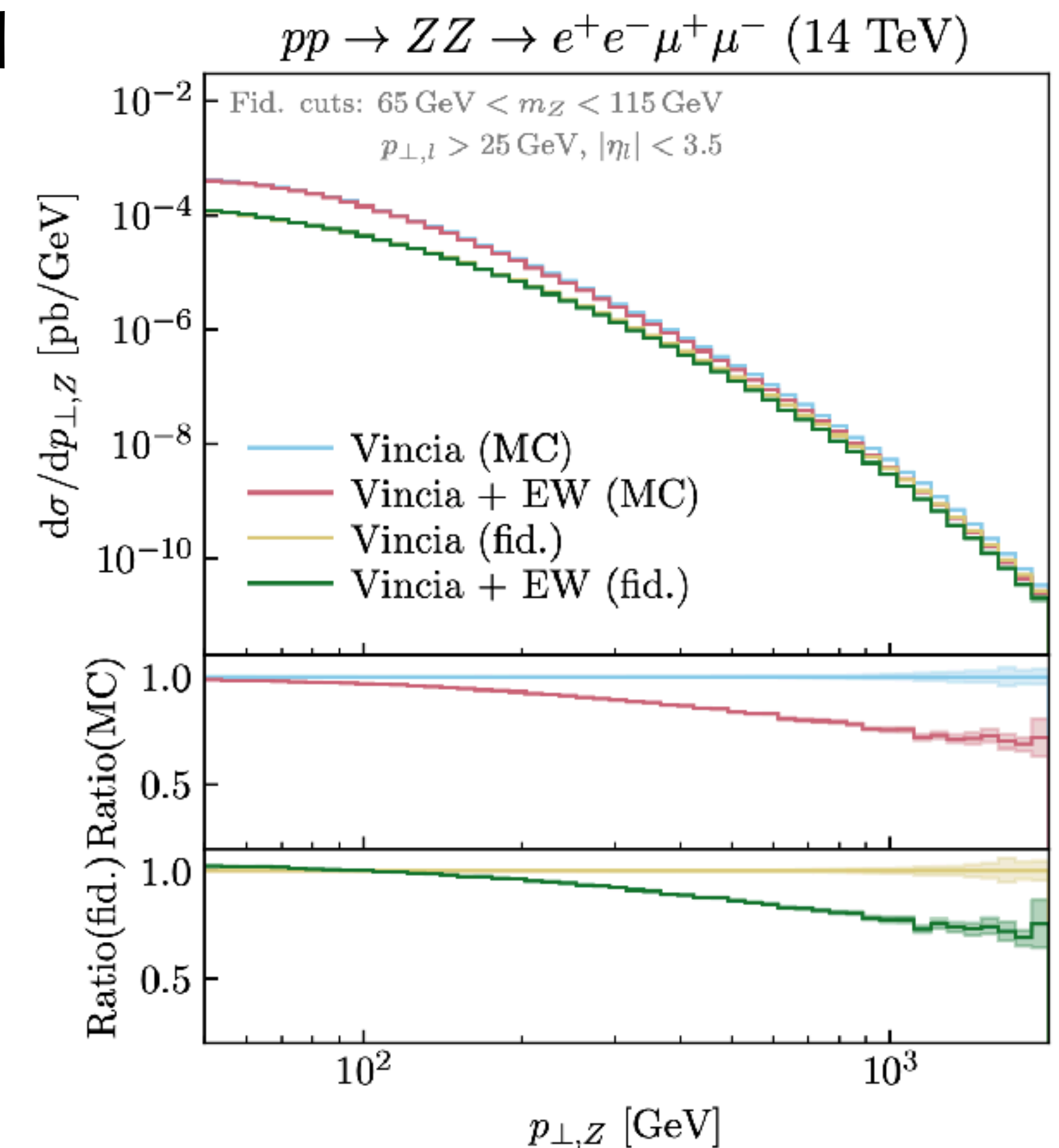
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- **VINCIA** is a **sector shower**: phase space available for an emission **sectorised** with \ominus mimicking jet clustering [Brooks, Preuss, Skands 2003.00702; Lopez-Villarejo, Skands 1109.3608]
- Apply a similar factorisation to break the giant QED multipole [Verheyen, Skands 2002.04939]



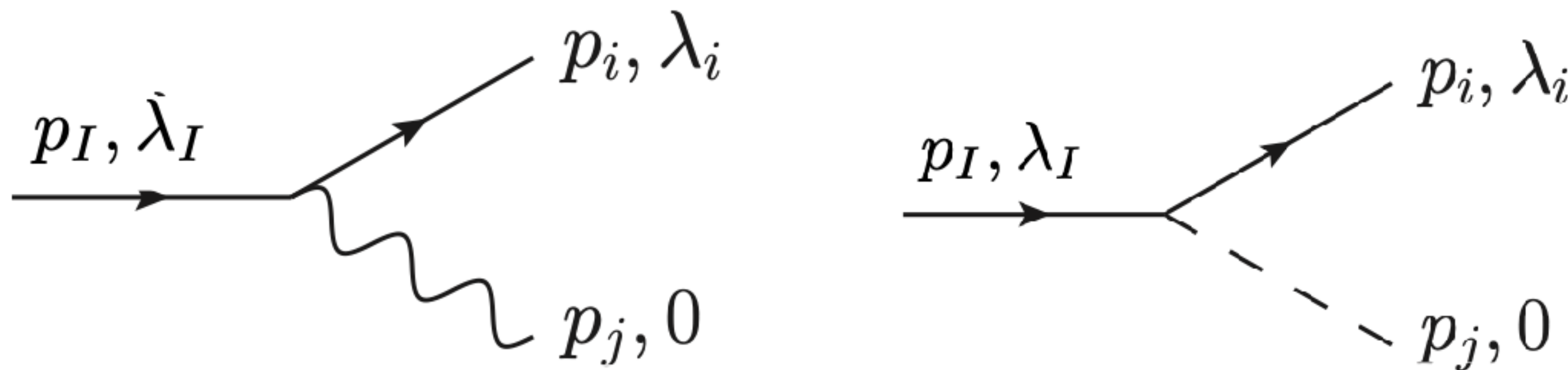
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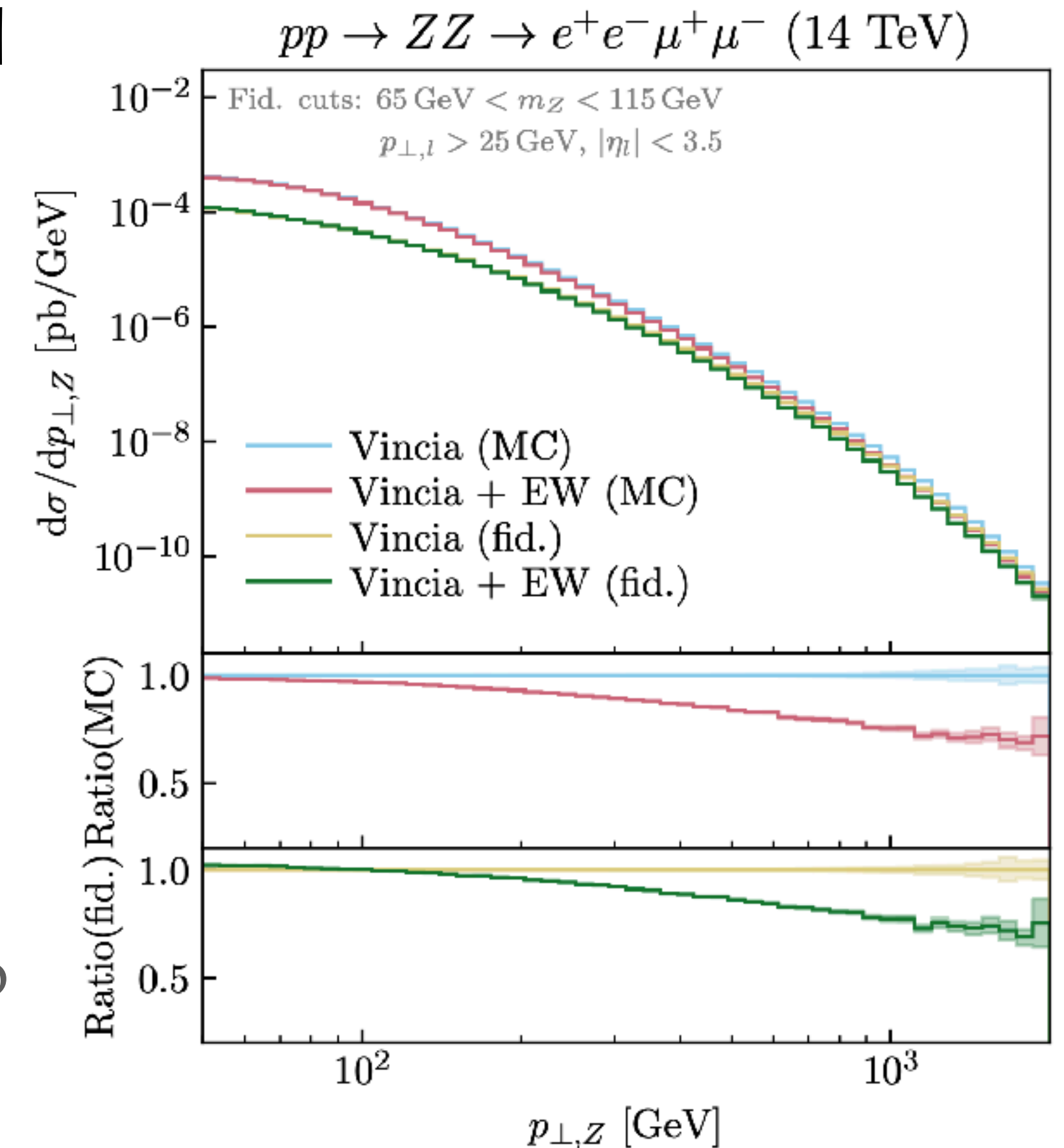


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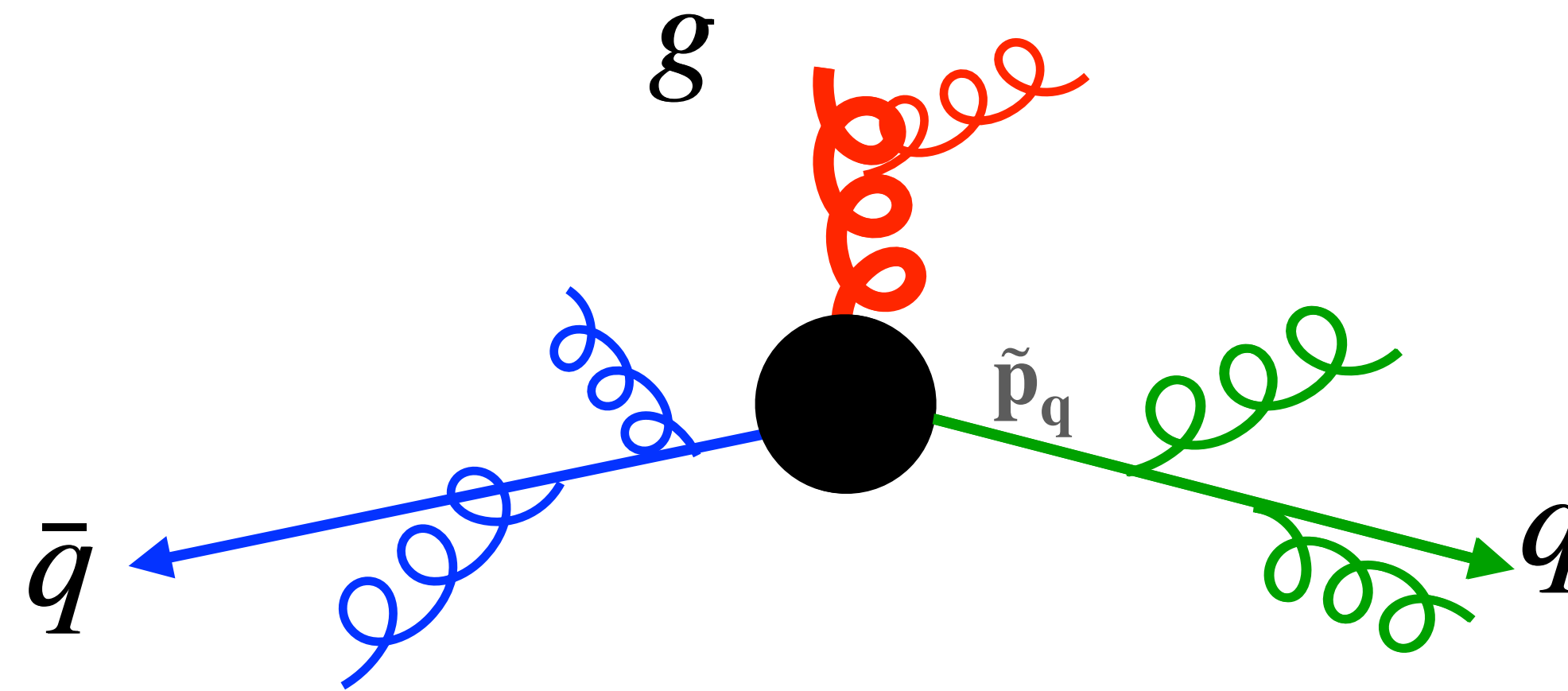
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- EW showers critically are **helicity showers** [1301.0933, Larkoski, Lopez-Villarejo, Skands]



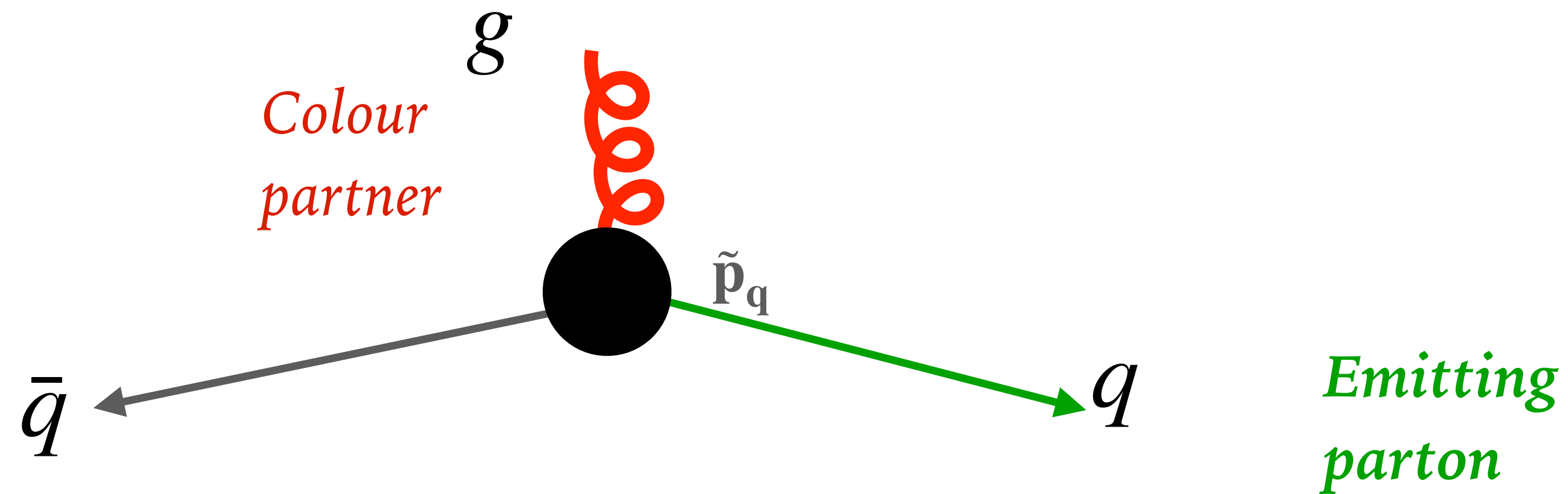
For handling of **spin correlations** in (QCD) dipole showers, see also 1807.01955 (**Herwig**) and 2103.16526, 2111.01161 (**PanScales**)



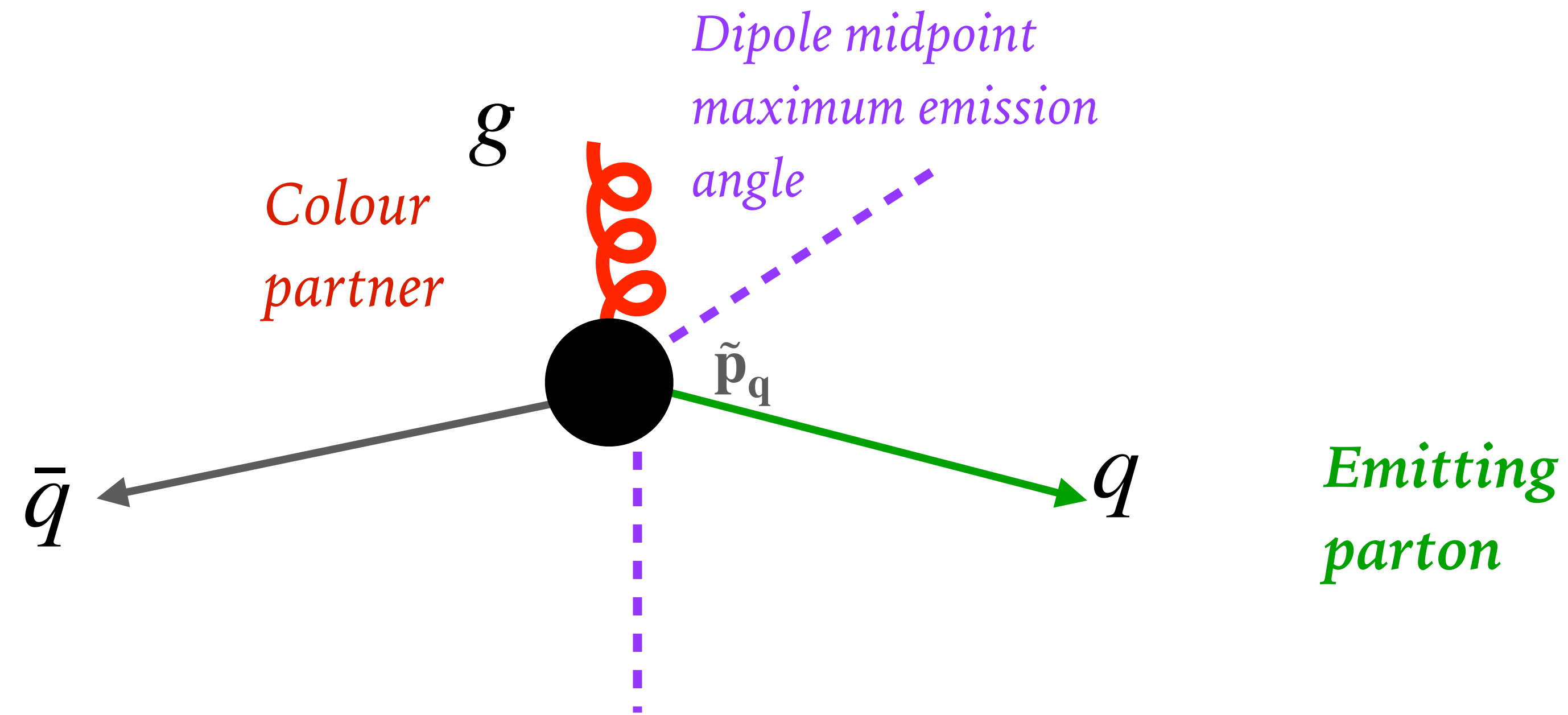
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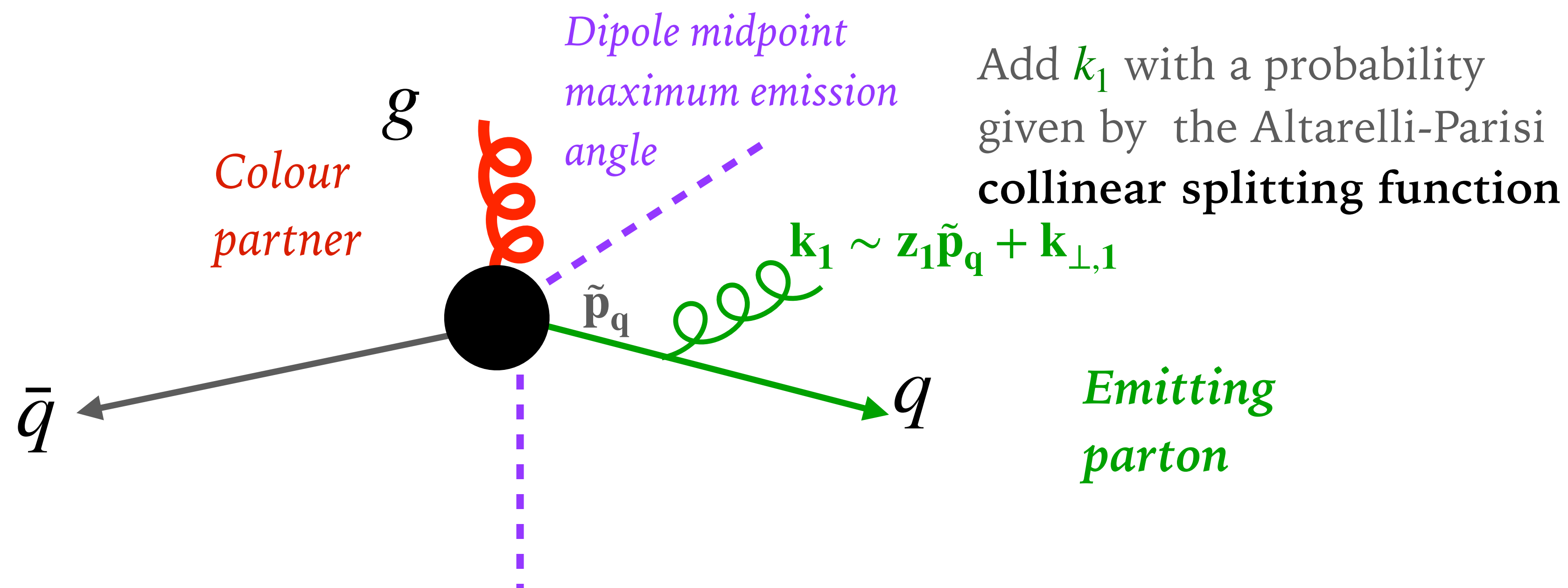
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Herwig7 Angular-Orderd **parton** shower

[Marchesini, Webber '88;
Gieseke, Stephens, Webber [hep-ph/0310083](https://arxiv.org/abs/hep-ph/0310083)]

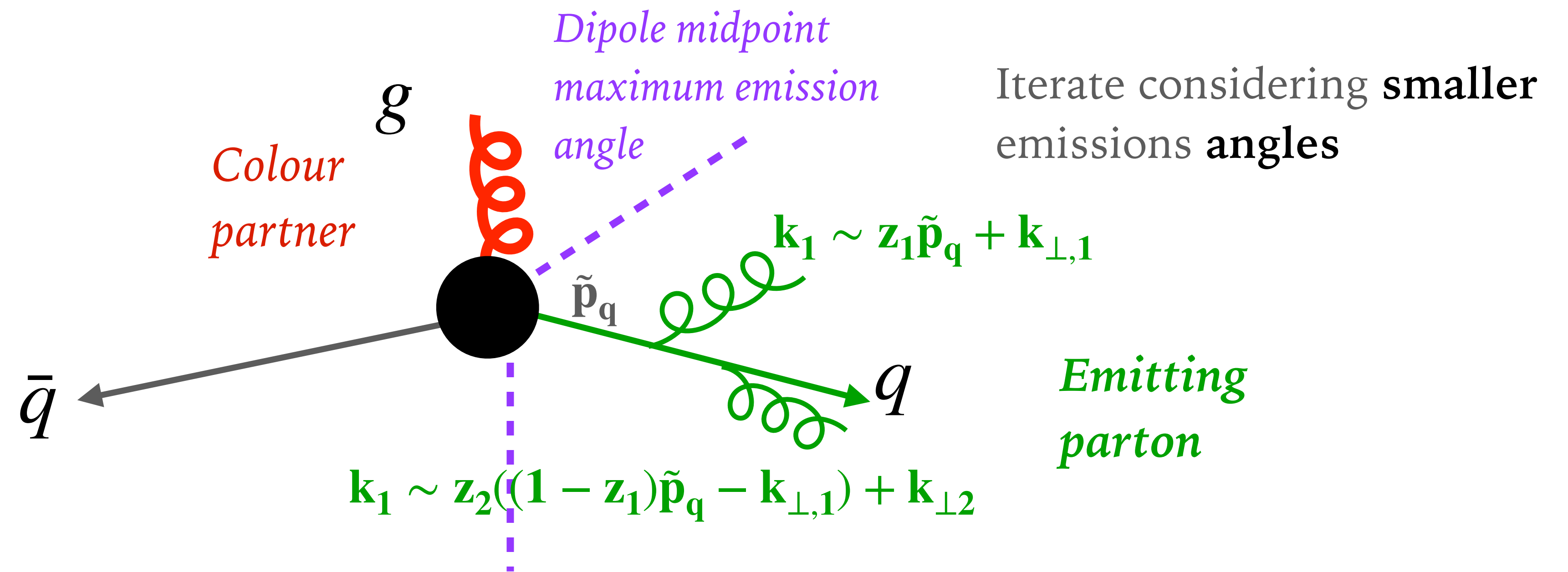
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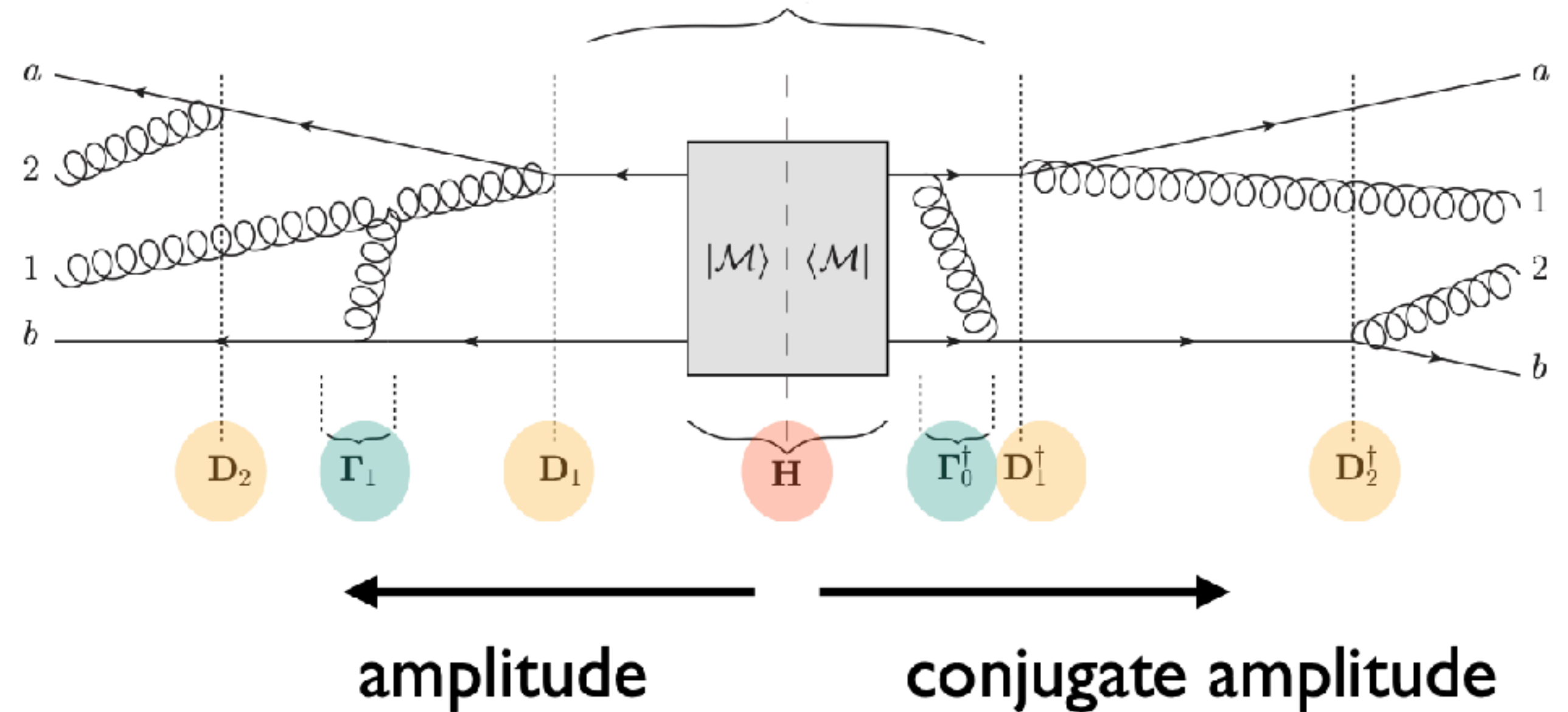
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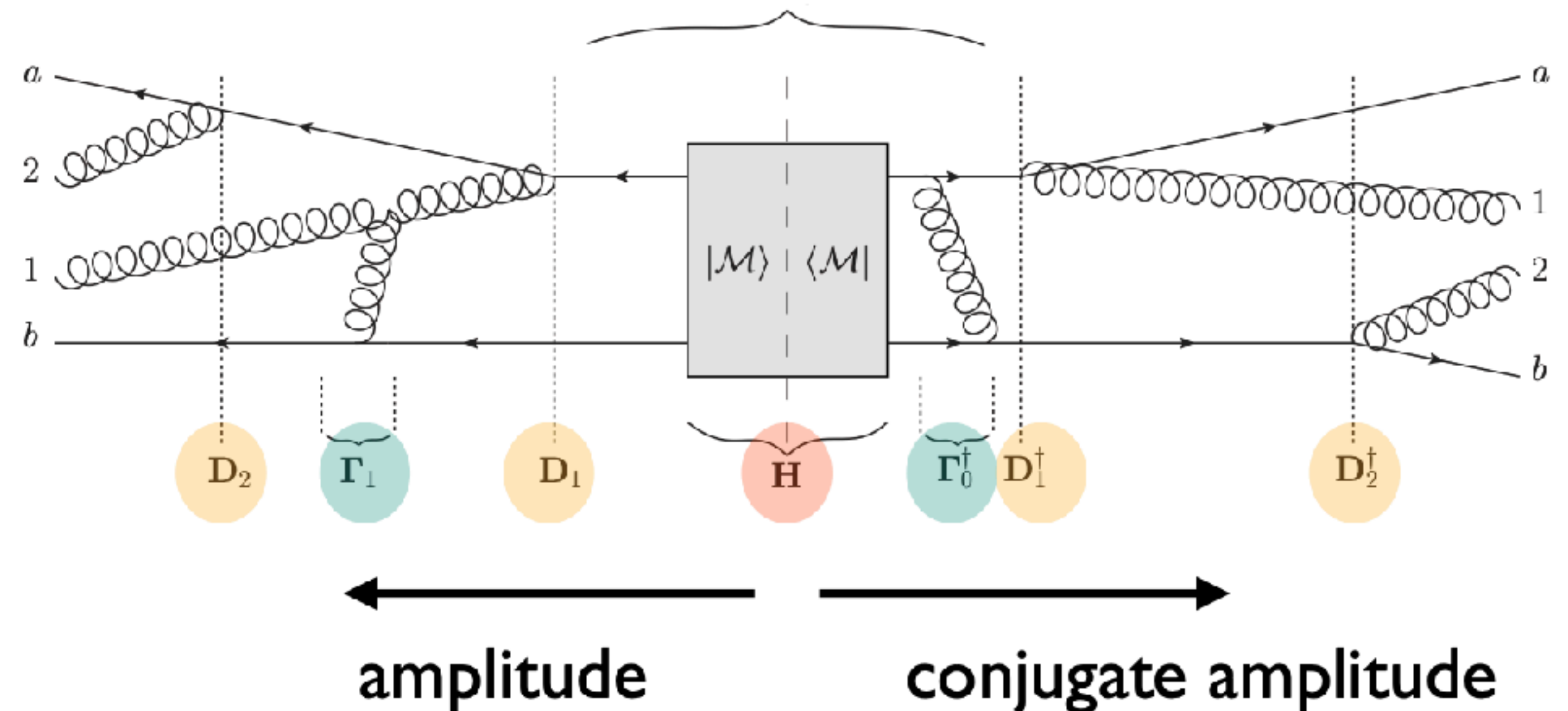
Amplitude-level evolution

- **CVolver** = evolve separately **amplitude** and **conjugate amplitude**: subleading **colour**, **spin** correlations, **Glauber** phases taken into account
[Plätzer, Sjudahl, De Angelis, Forshaw, Holguin, see [2210.09178](#) and refs therein]
+ **EW** interactions [Plätzer, Sjudahl [2204.03258](#)]



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+ **EW** interactions [Plätzer, Sjodahl [2204.03258](#)]



- **Deductor** = density matrix in **colour** [Nagy, Soper [0805.0216](#), [1202.4496](#), [1401.6364](#), [1501.00778](#), [1902.02105](#), [1905.07176](#), [1908.11420](#)] to resum “**nasty soft & Glauber contributions**” [and more]



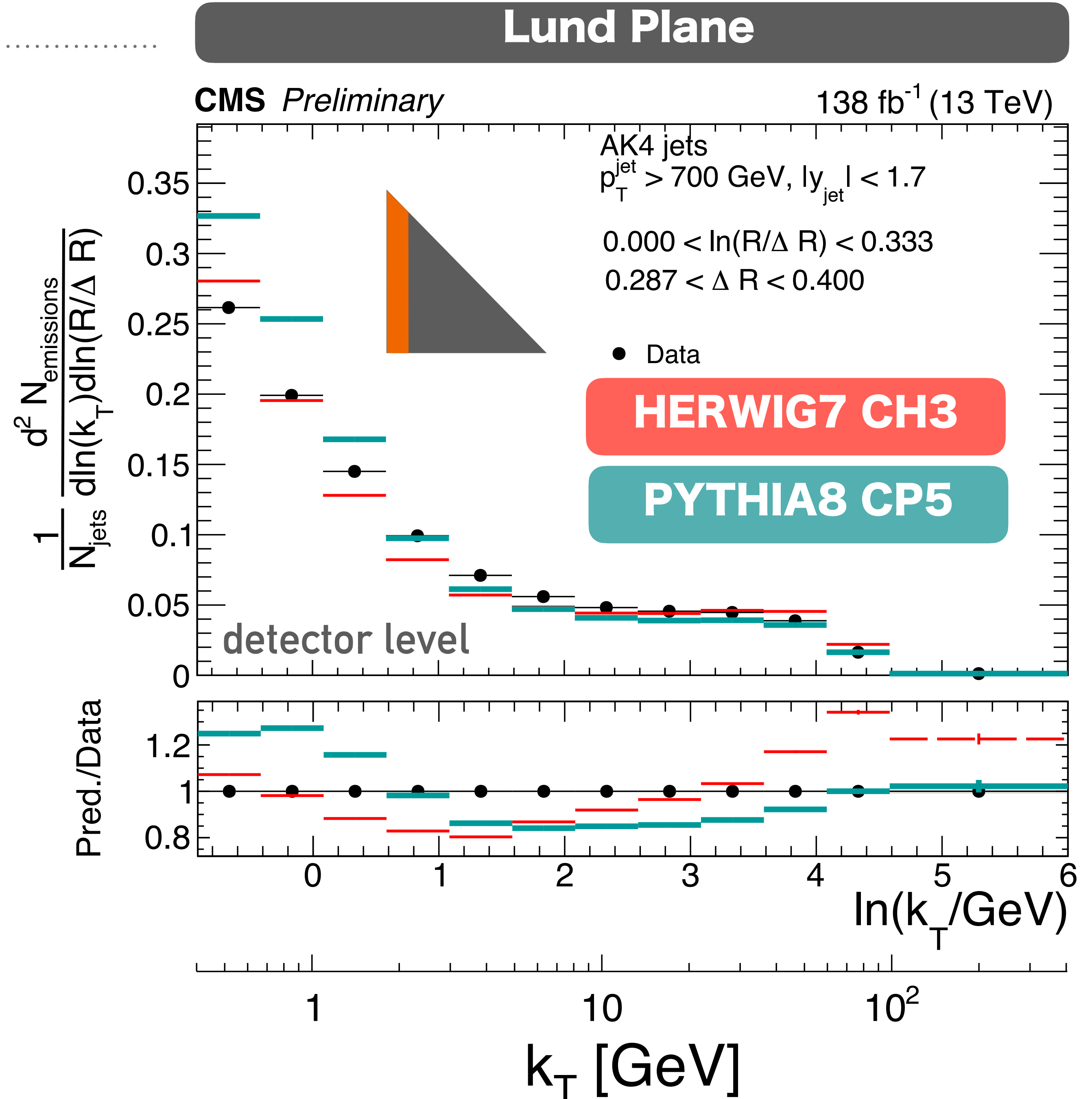
Are current showers **good enough?**

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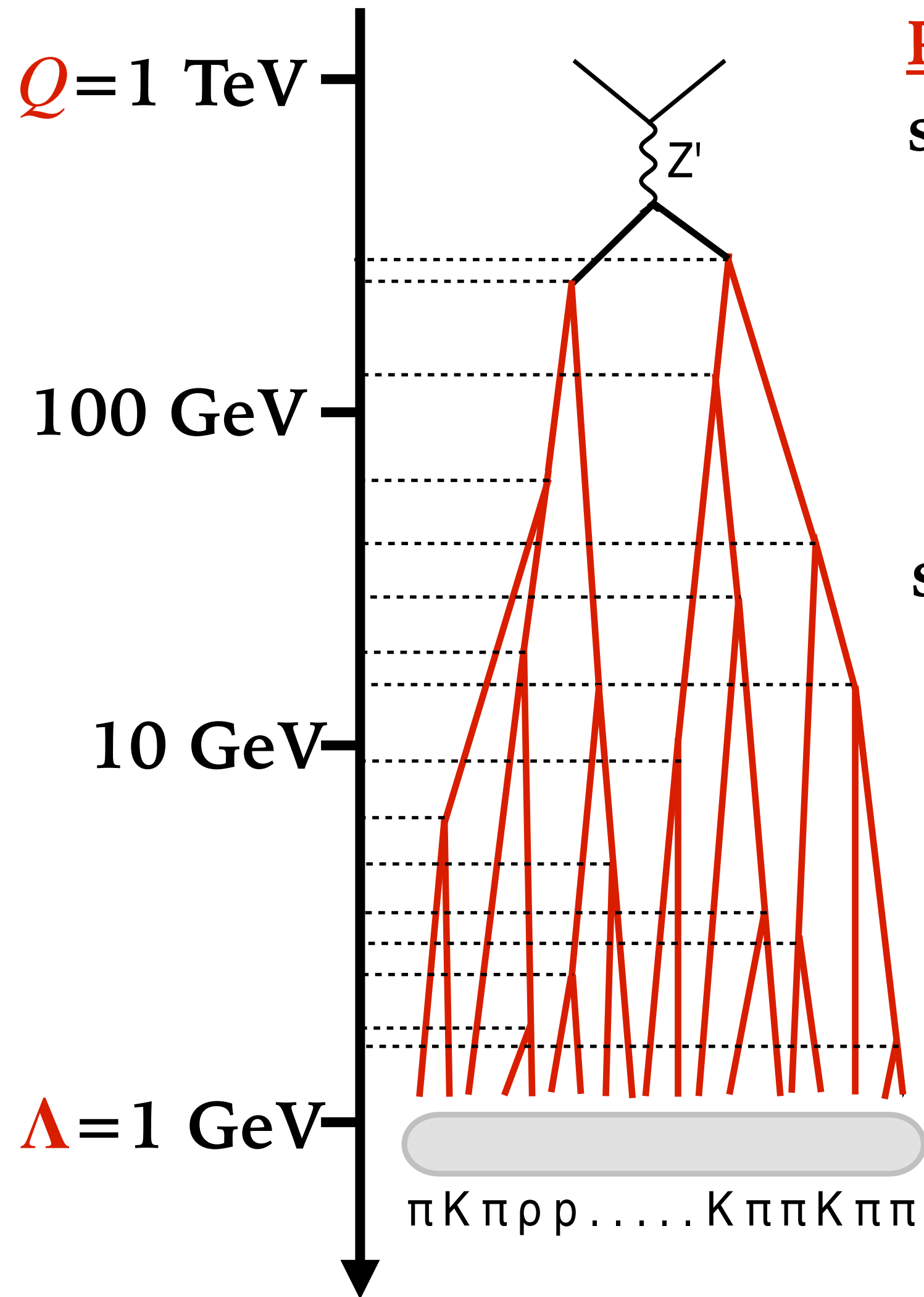
What does **good** shower mean?

Are current showers good enough?

- showers do an amazing job on many observables for **LHC**
- various places see **10–30% discrepancies** between showers and data
- A lot of work is required to reach the **percent precision target!**



Logarithmically-accurate Parton Showers



PARTON SHOWERS = energy degradation via an iterated sequence of softer and softer emissions

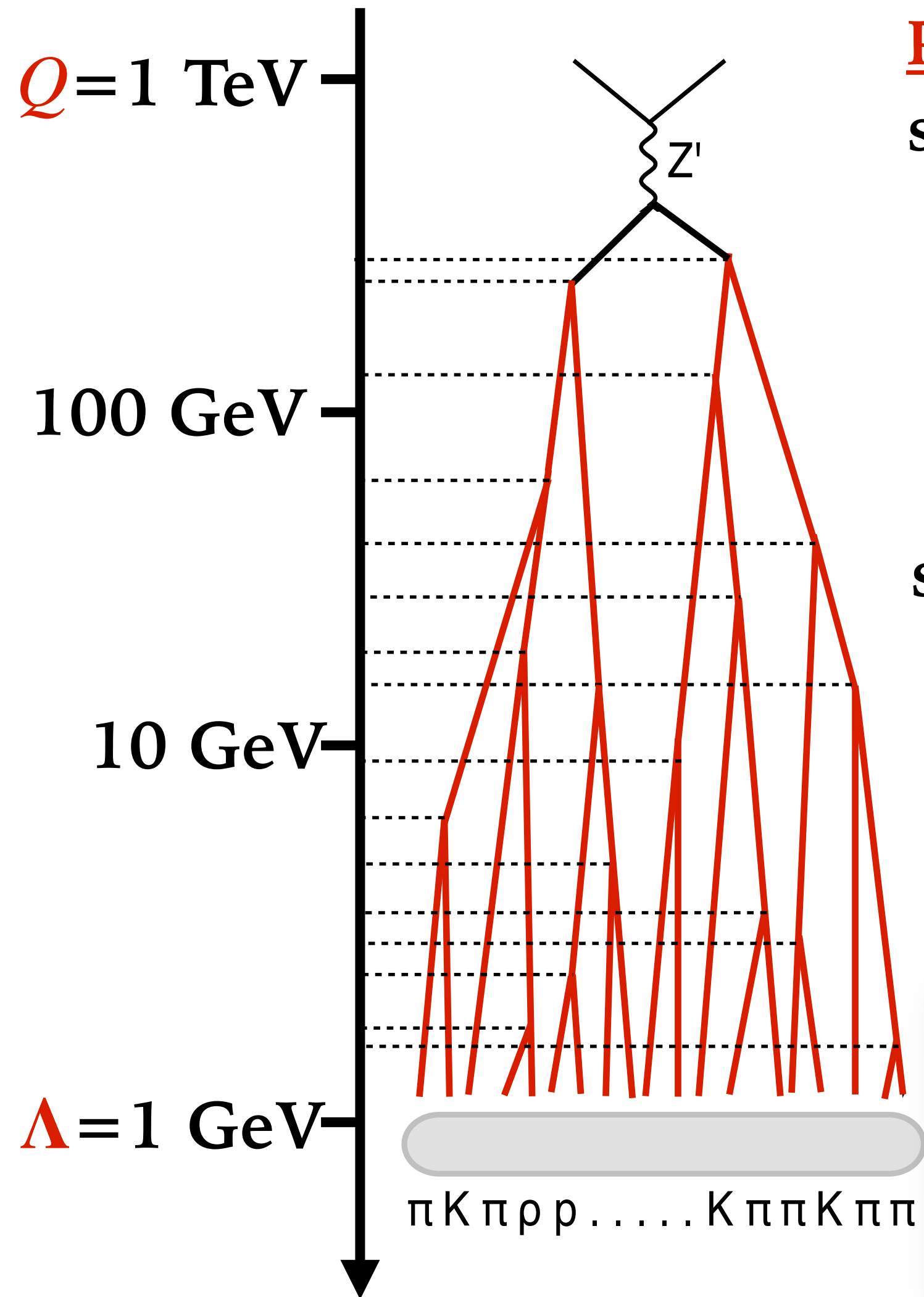
$$L = \ln \frac{Q}{\Lambda} \gg 1$$

simple algorithm to include the **dominant radiative corrections** at all orders for **any observable!**

$$\Sigma(O < e^{-L}) = \exp \left(-L g_{LL}(\beta_0 \alpha_s L) + \dots \right)$$

LL = leading logs

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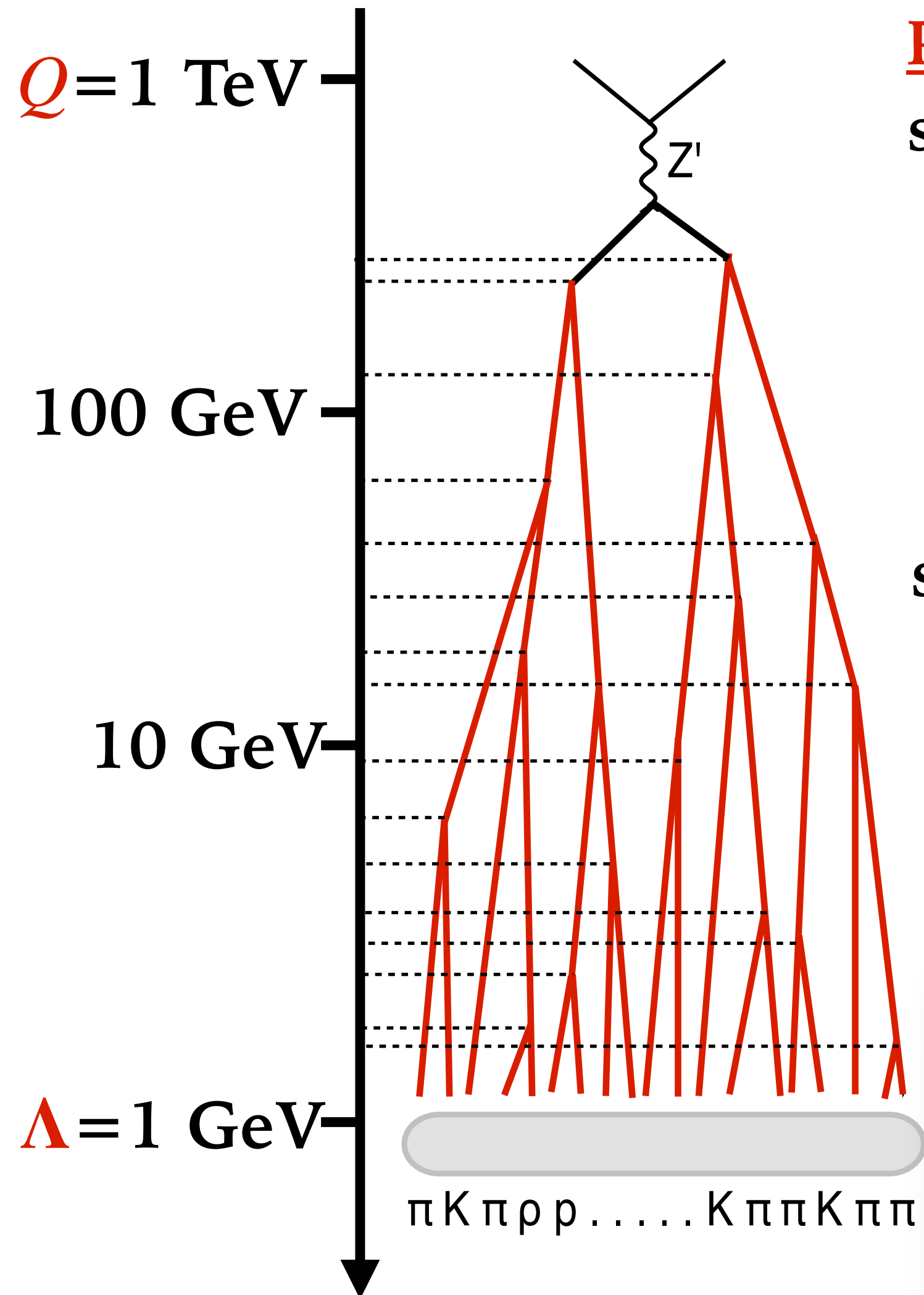
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For $Q \sim 50 - 10000 \text{ GeV}$, $\beta_0 \alpha_s L \sim 0.3 - 0.5$:
Next-to-Leading Logarithms needed for quantitative predictions!

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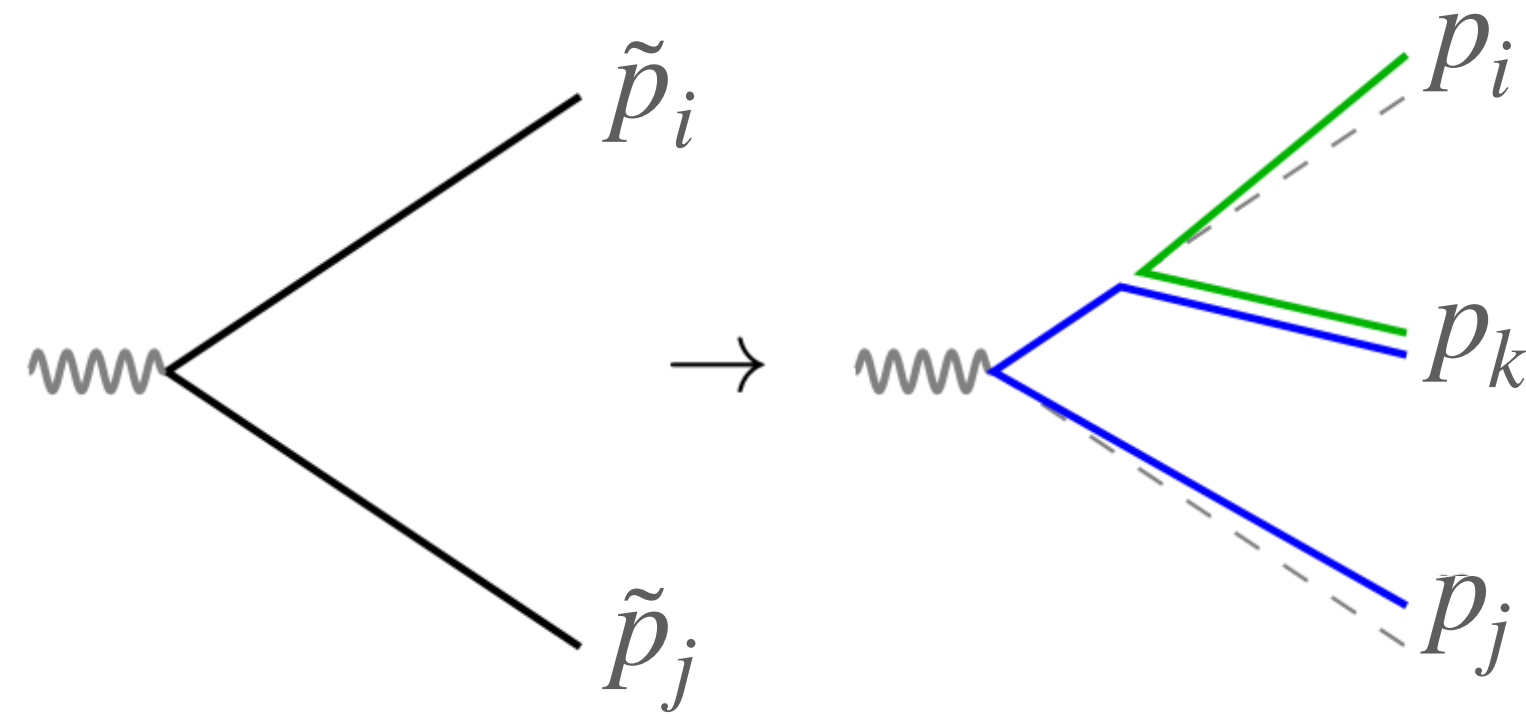
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Dissecting the parton shower emission probability

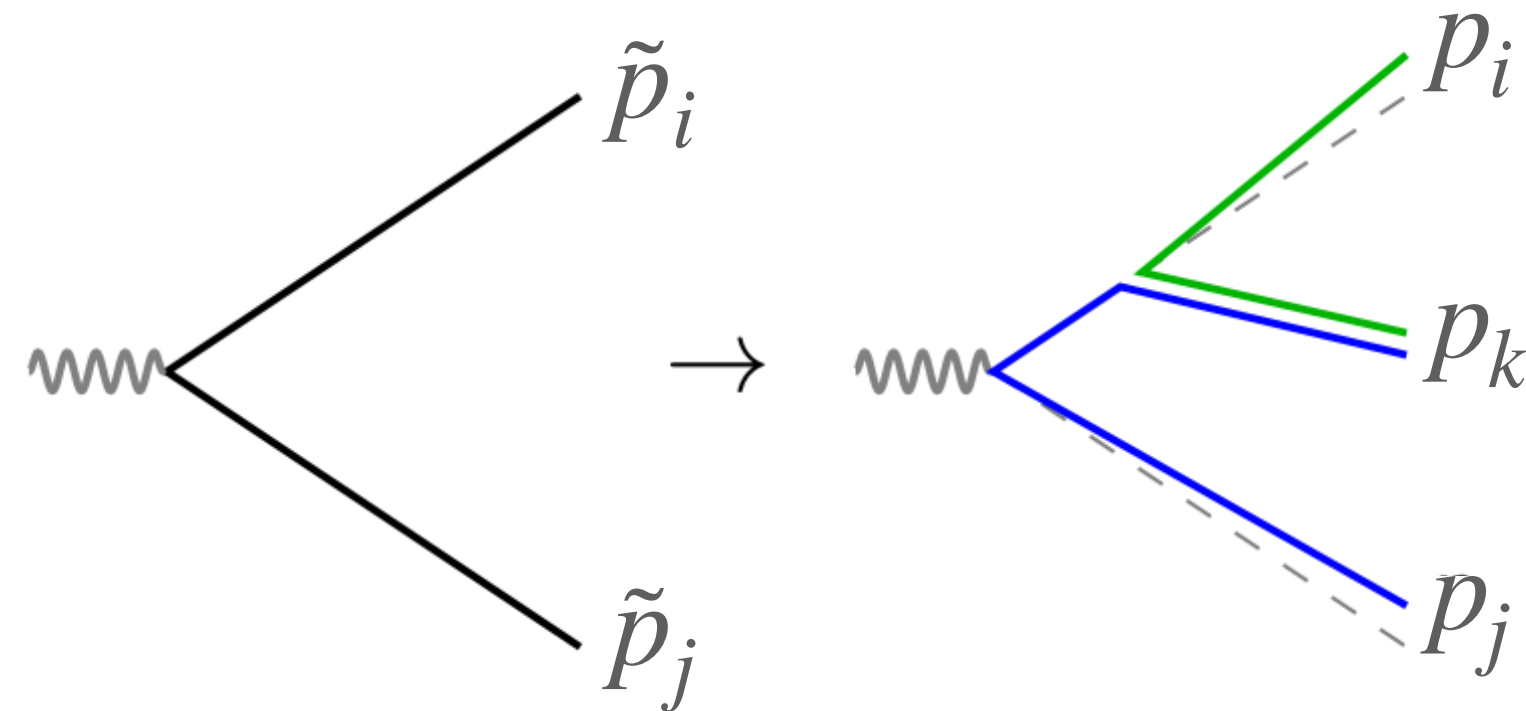
Starting from a $e^+e^- \rightarrow Z^* \rightarrow q\bar{q}$ system, what is the splitting probability?



$$d\mathcal{P}_{\tilde{i}\tilde{j} \rightarrow ijk} \sim \frac{dv^2}{v^2} d\bar{\eta} \frac{d\varphi}{2\pi} P_{\tilde{i},\tilde{j} \rightarrow i,j,k}(v, \bar{\eta}, \varphi)$$

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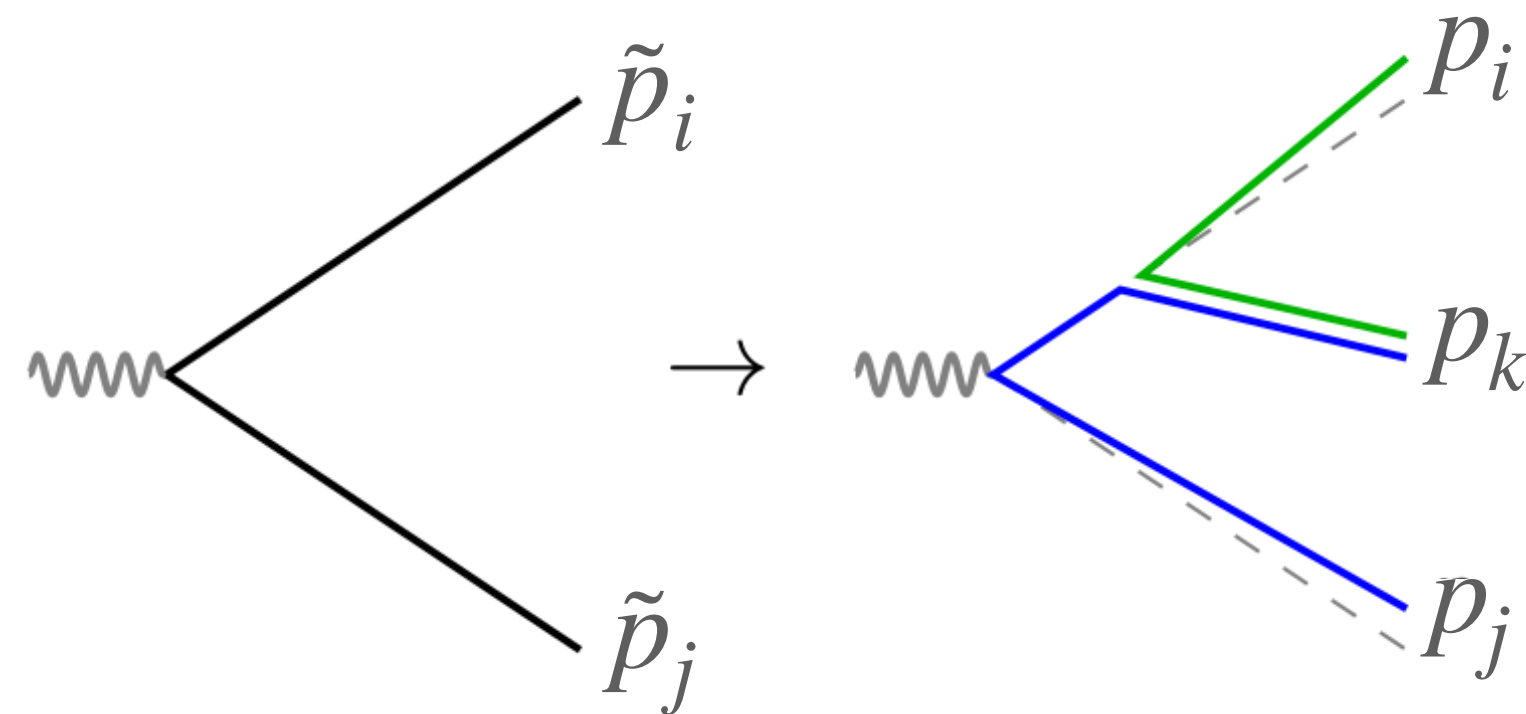


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Matrix element for emitting a parton k from a parton i (or j)

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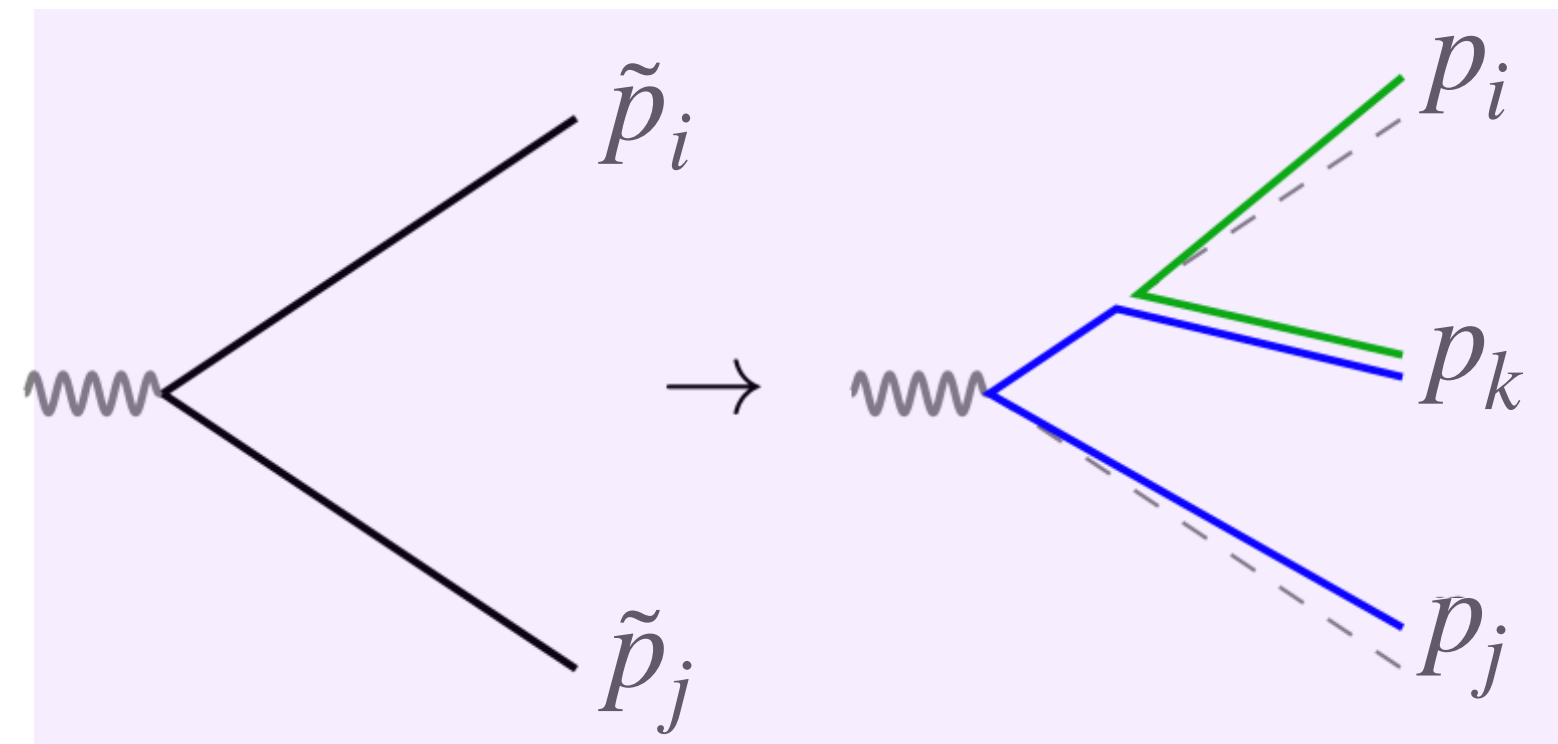
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Kinematic mapping:
how to reshuffle the momenta of i and j after the emission takes place

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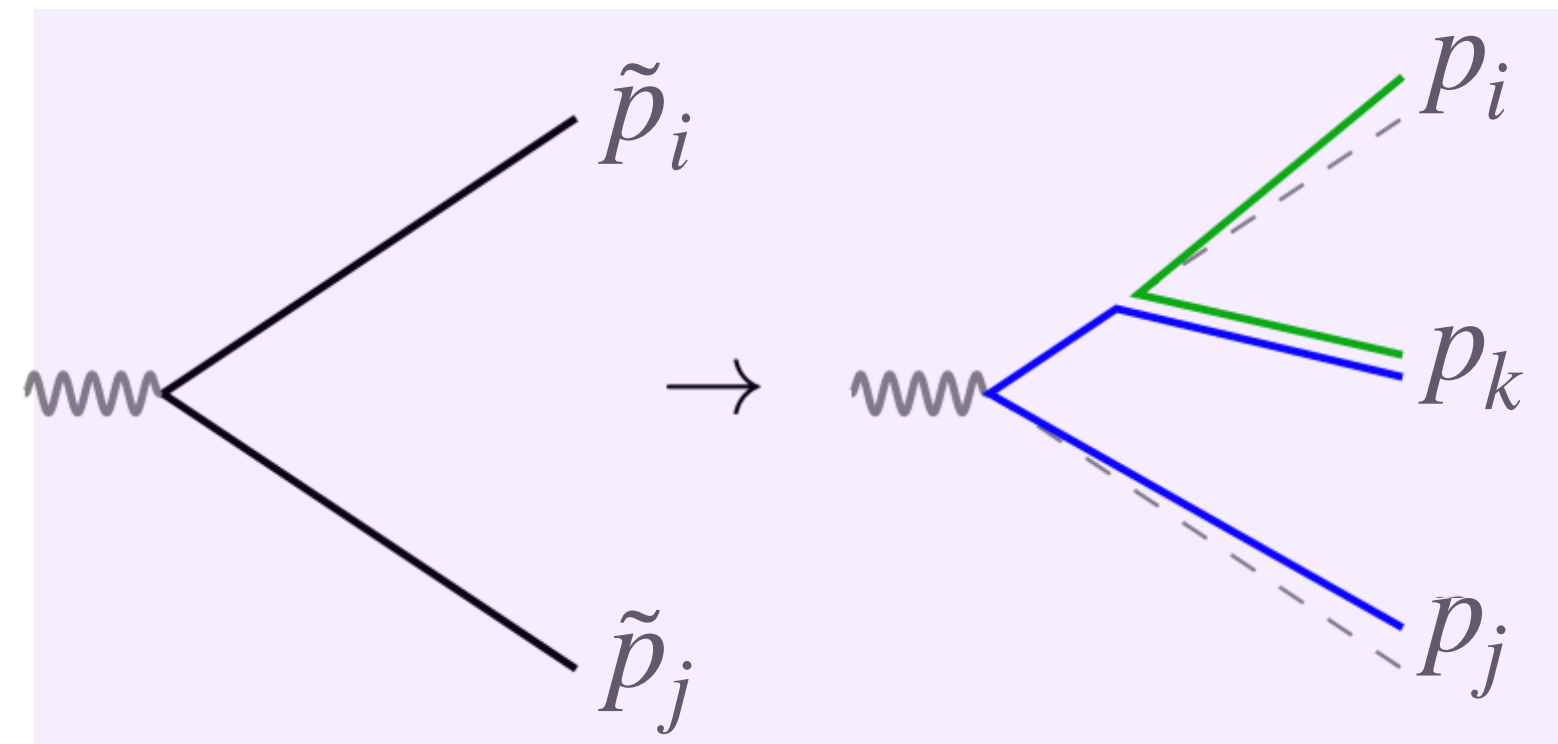
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Their interplay determines the shower **logarithmic accuracy**



Kinematic mapping: how to reshuffle the momenta of i and j after the emission takes place

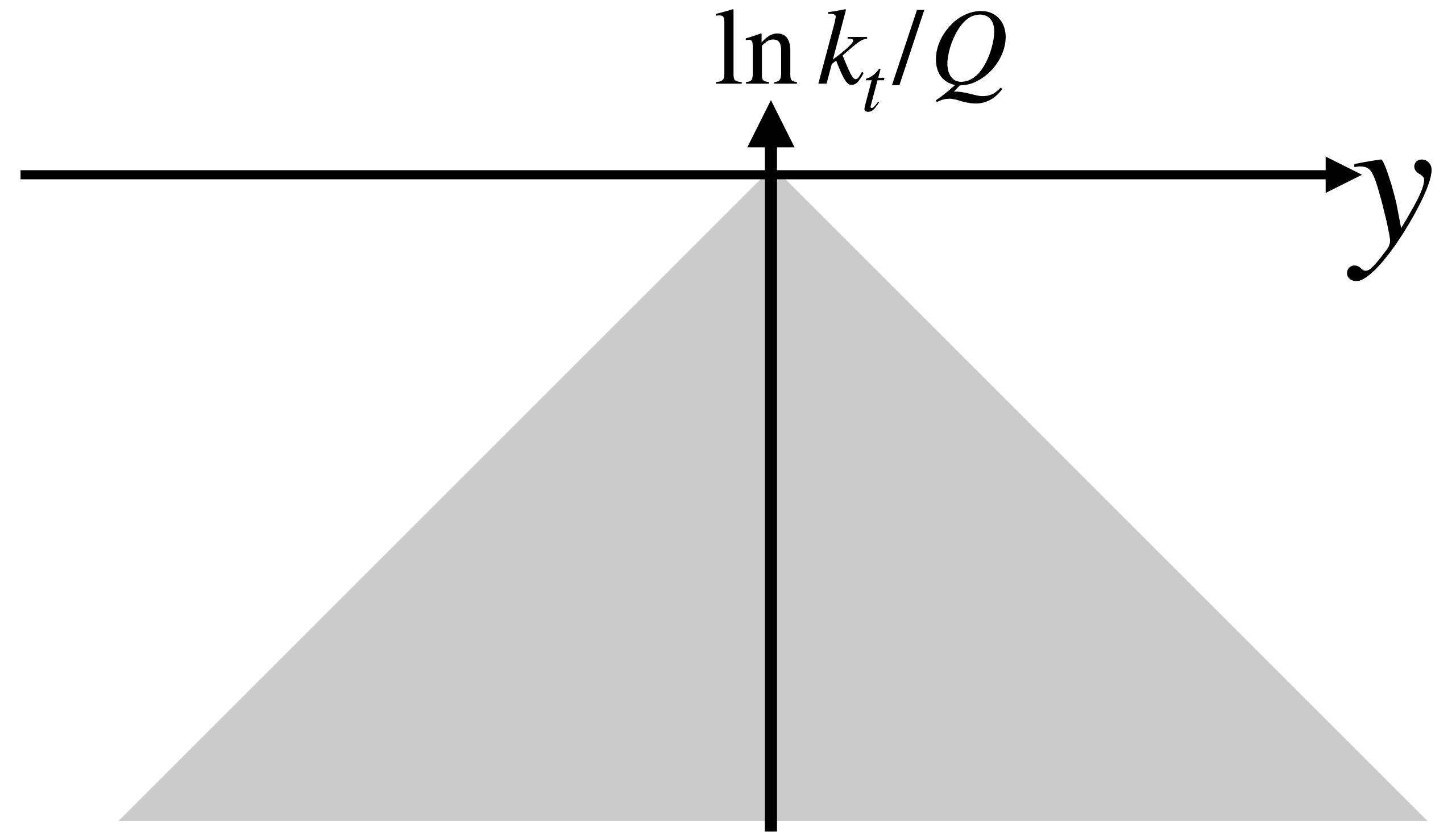
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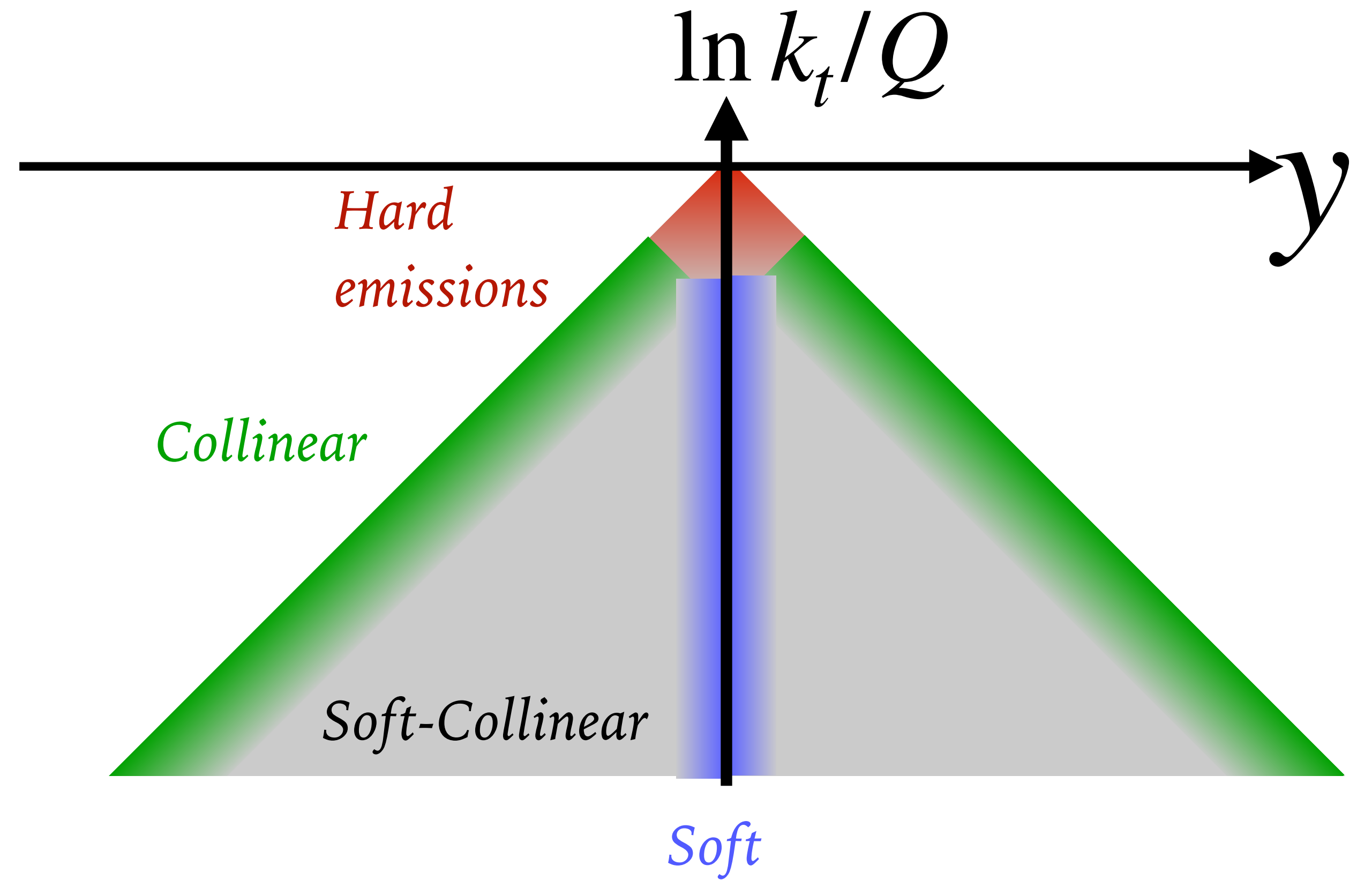
How to build a logarithmically-accurate parton shower?

- **The Lund plane:** diagnostic tools for resummation and parton showers



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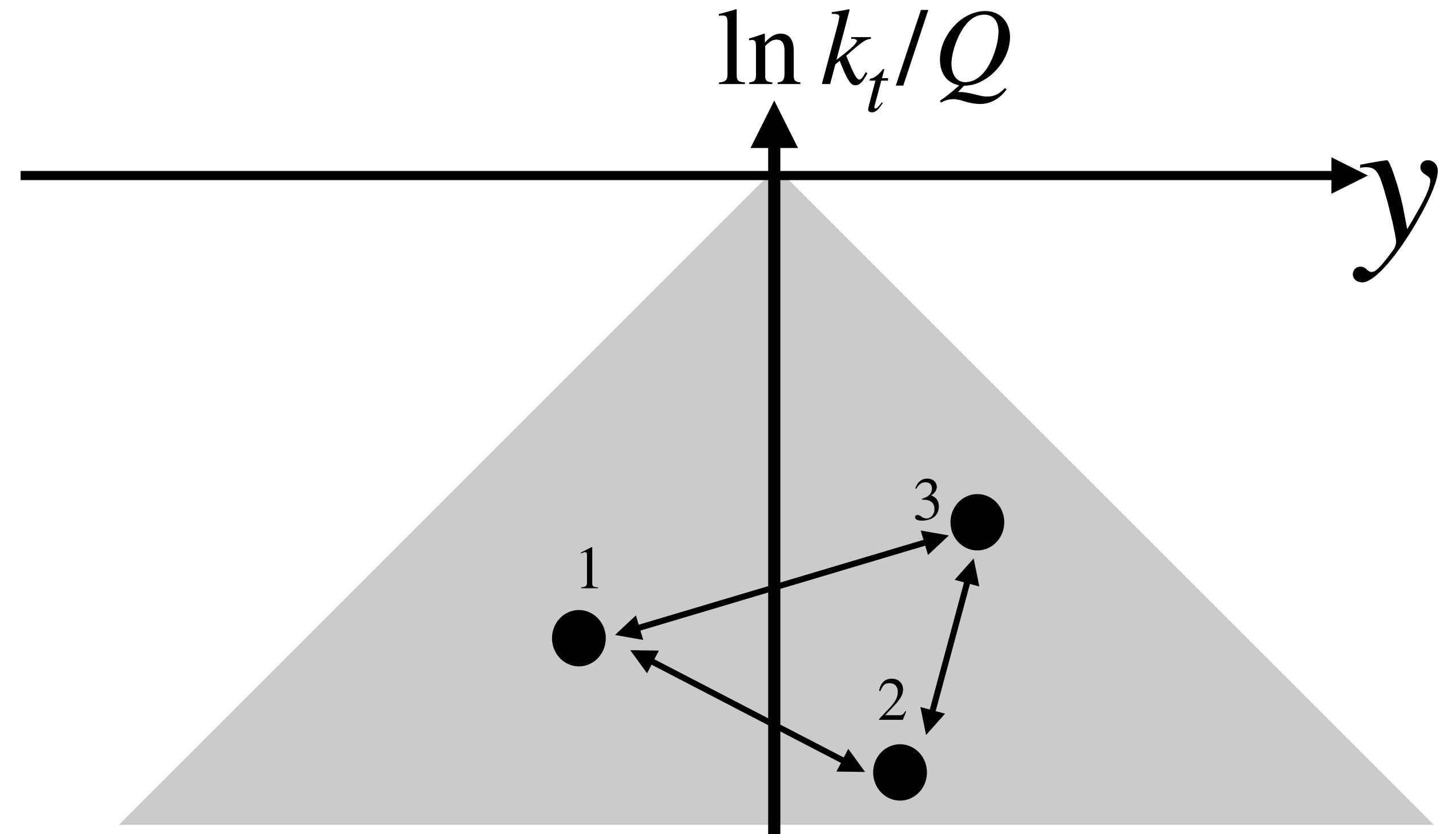
How to build a LL parton shower?

- The Lund plane: diagnostic tools for resummation and parton showers
- At Leading Logarithmic accuracy we only care about **soft-collinear emissions** very separated between each others

$$dP_i = \frac{\alpha_s(k_t)}{\pi} \frac{2C_F}{z} dz d \ln k_t$$

One-loop QCD coupling constant at $\mu_R = k_t$

LO soft splitting function



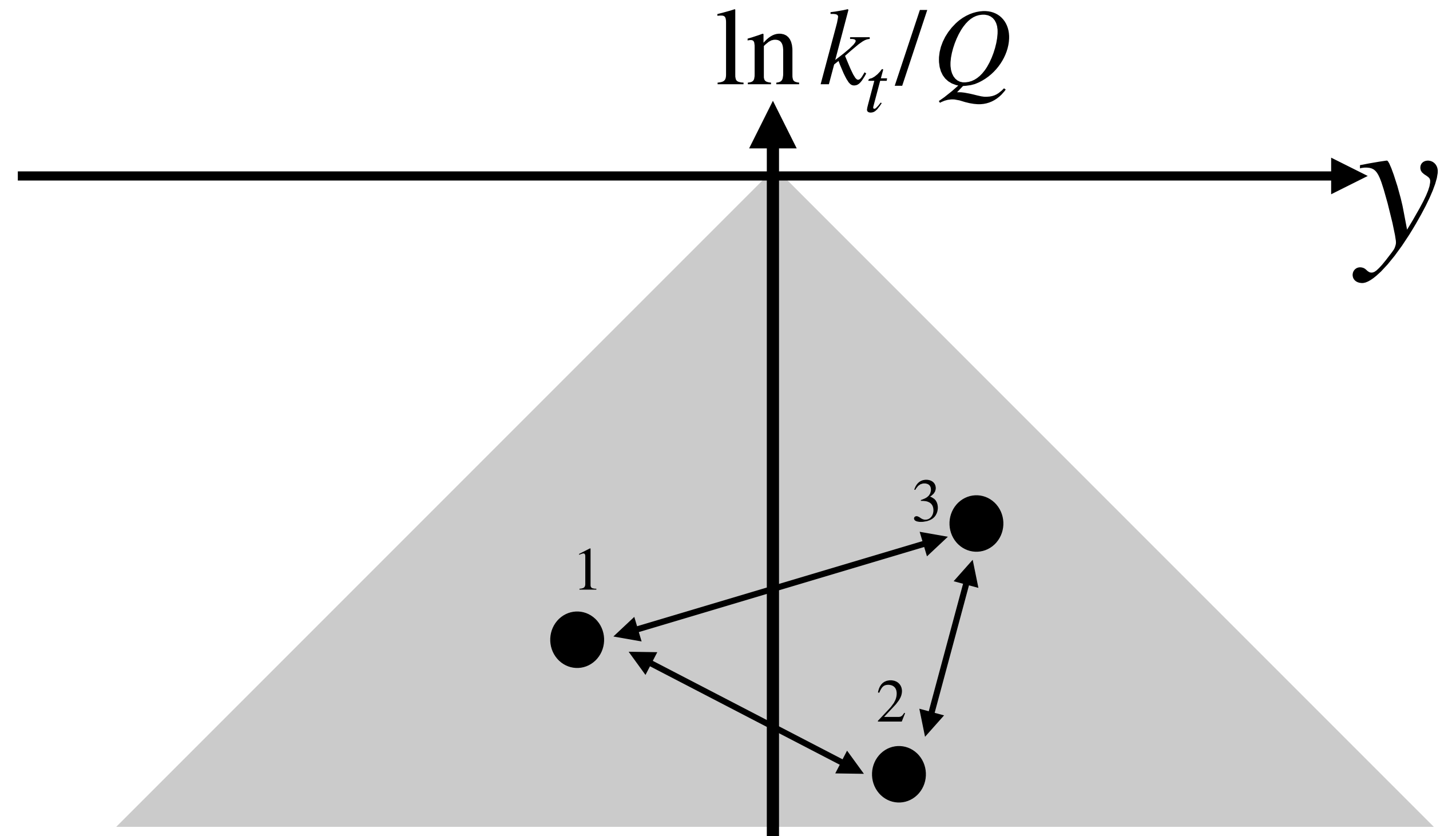
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This tells us what **matrix element** should we use to generate a new emission

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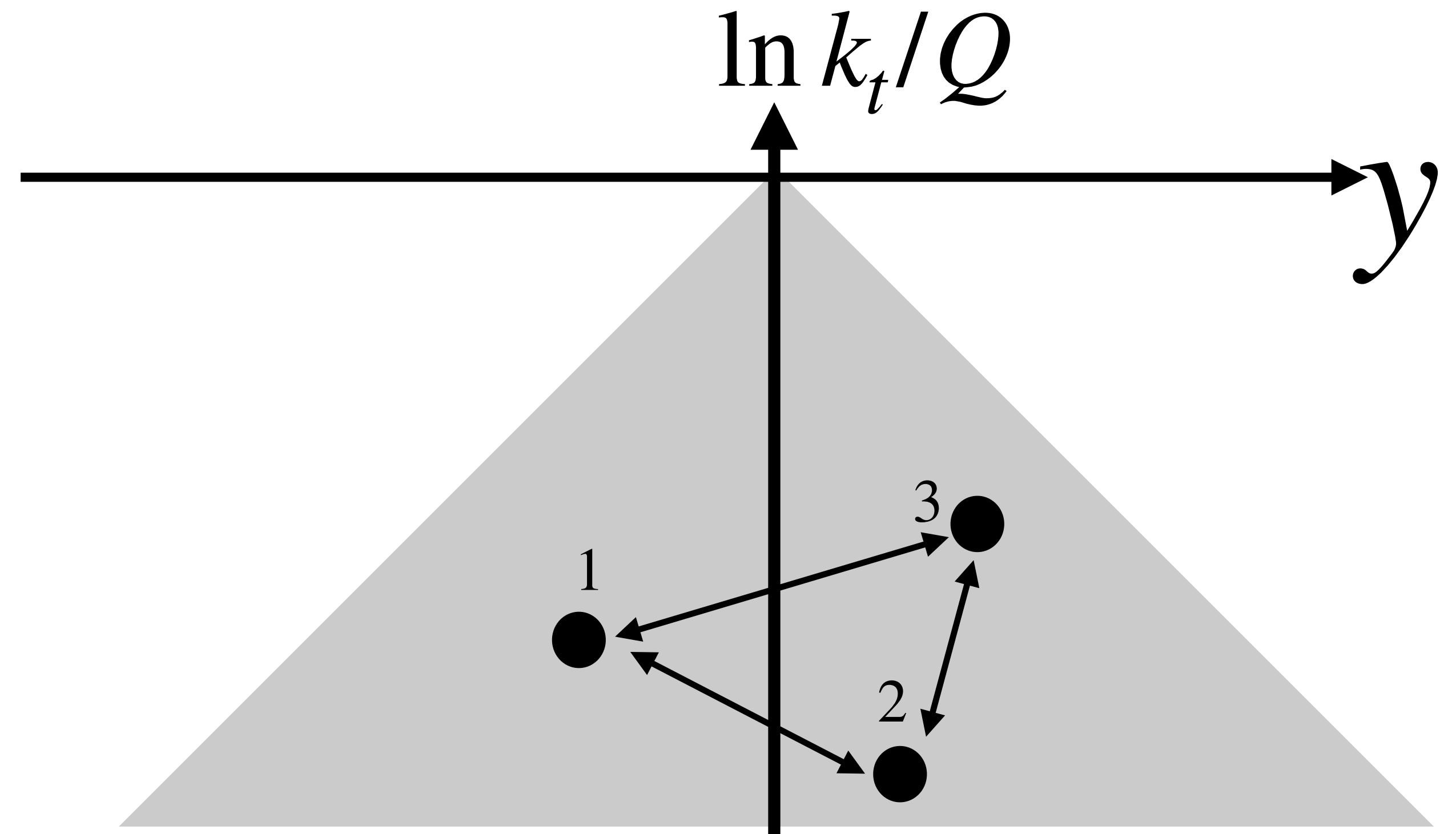
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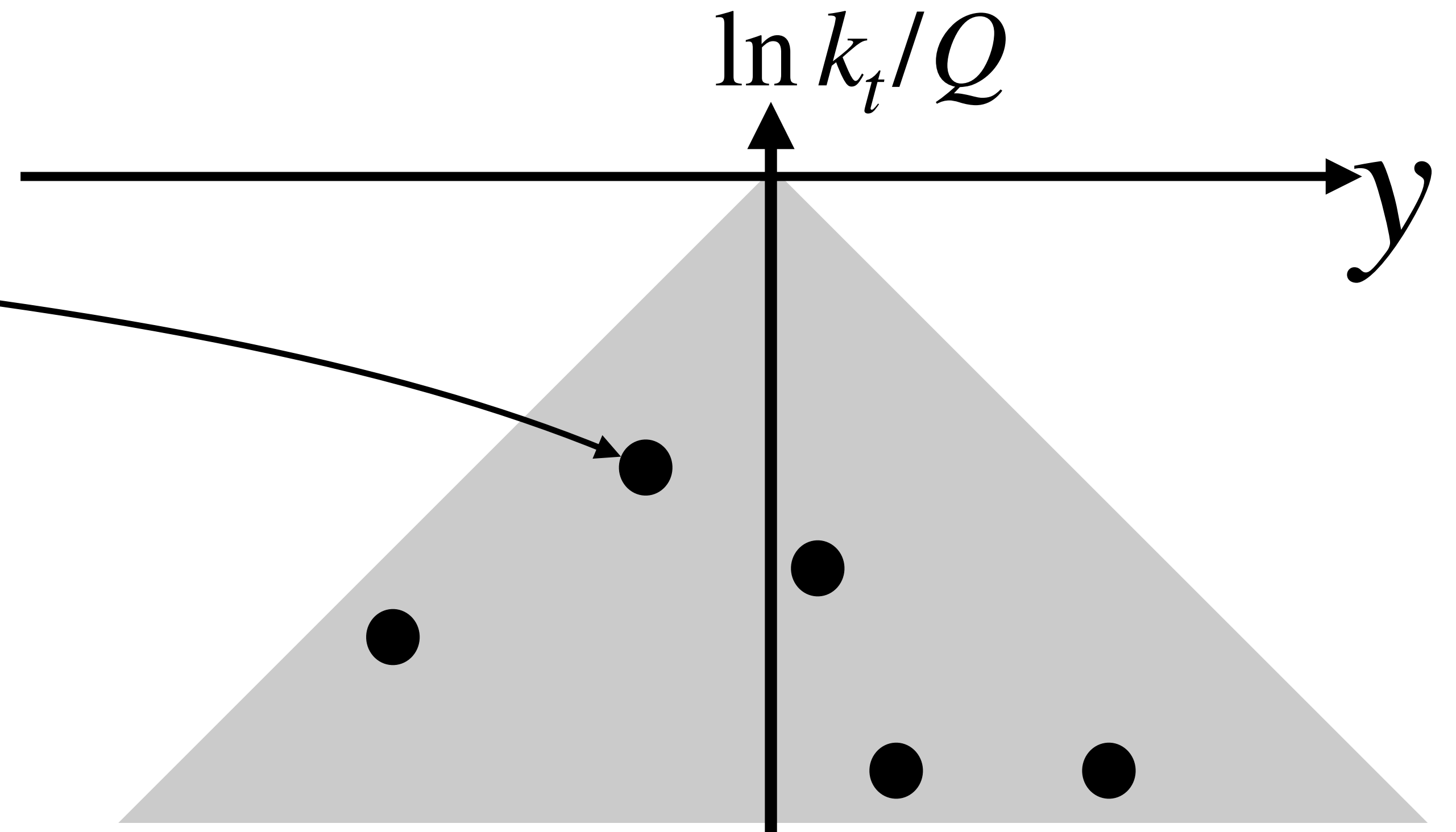
This constrains the **kinematic mapping** $\Phi_n \rightarrow \Phi_{n+1}$ and the **ordering variable** choice: emissions well separated in rapidity and transverse momentum are independent from each other

How to build a NLL parton shower?

At NLL accuracy:

- The rate for soft-collinear emissions must be correct at NLO

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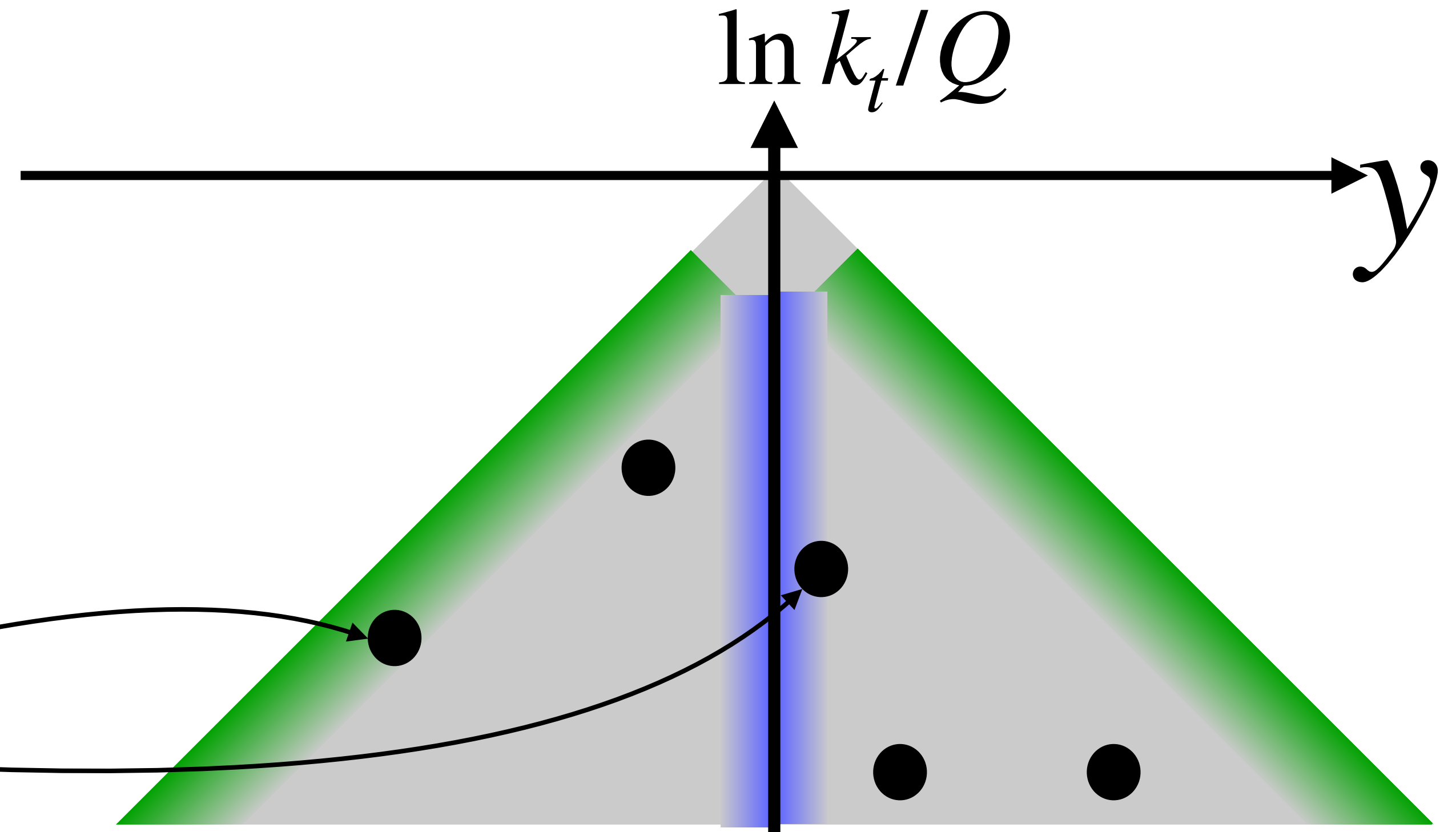
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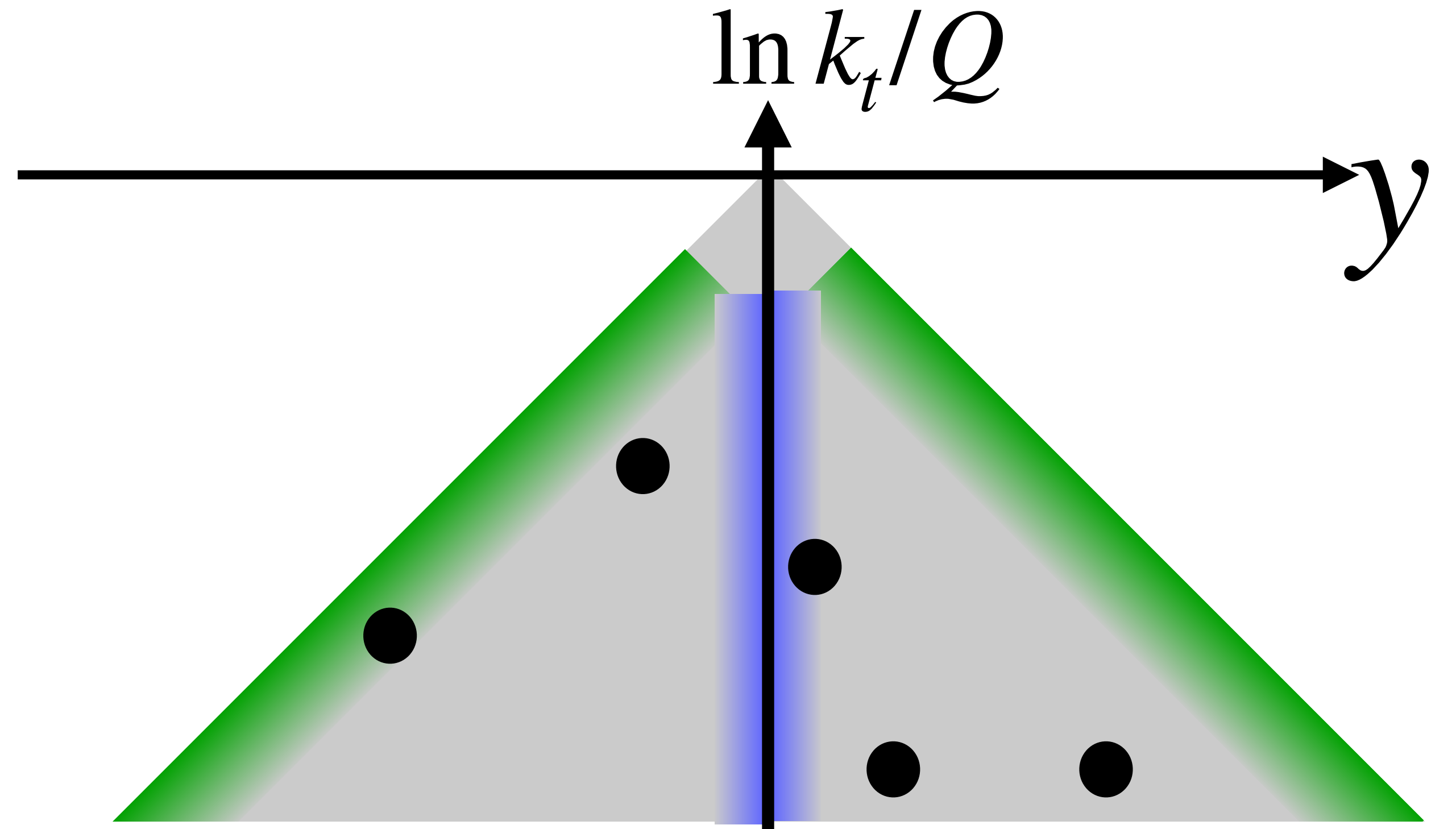
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Catani, Marchesini, Webber '91

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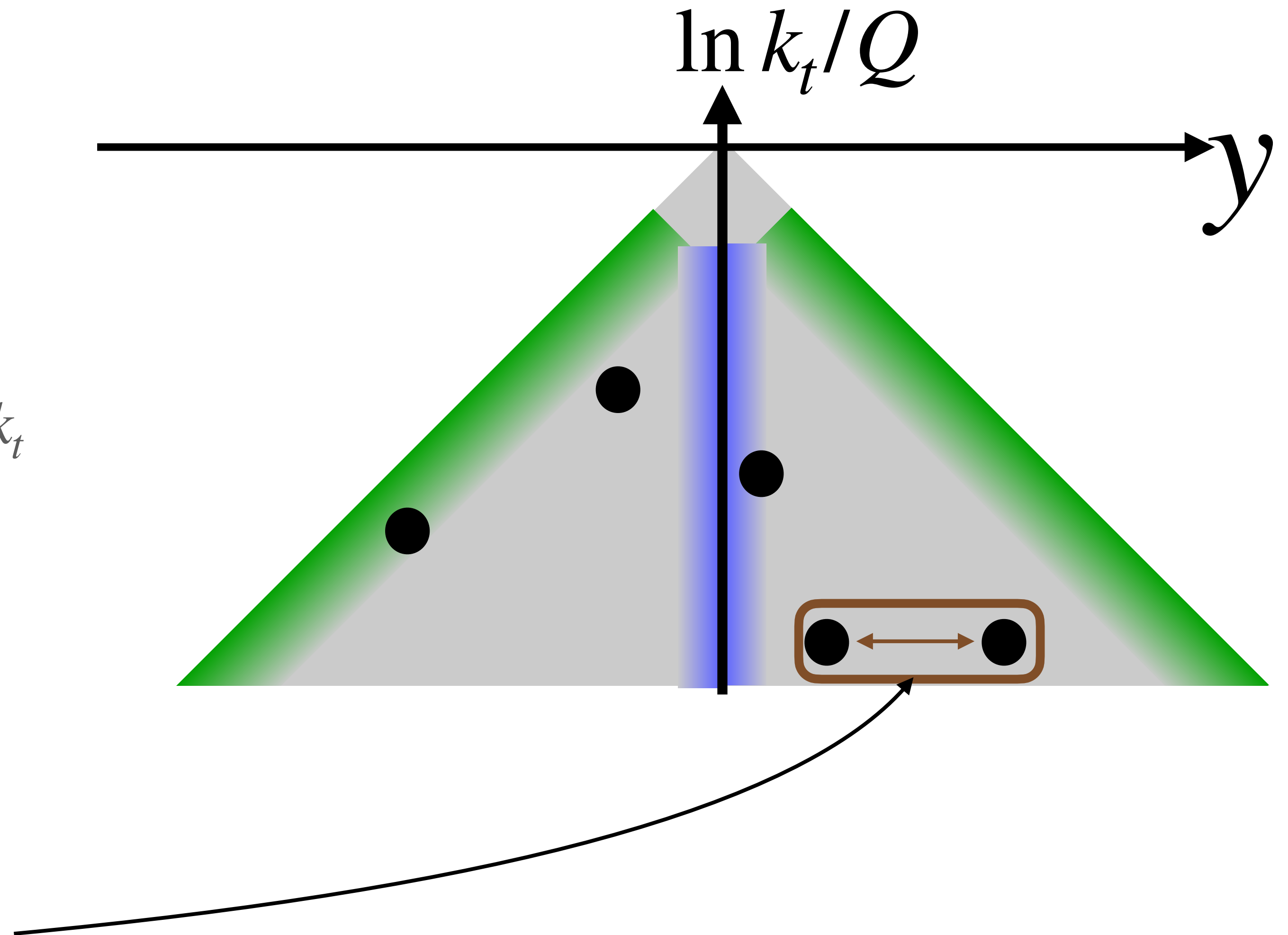
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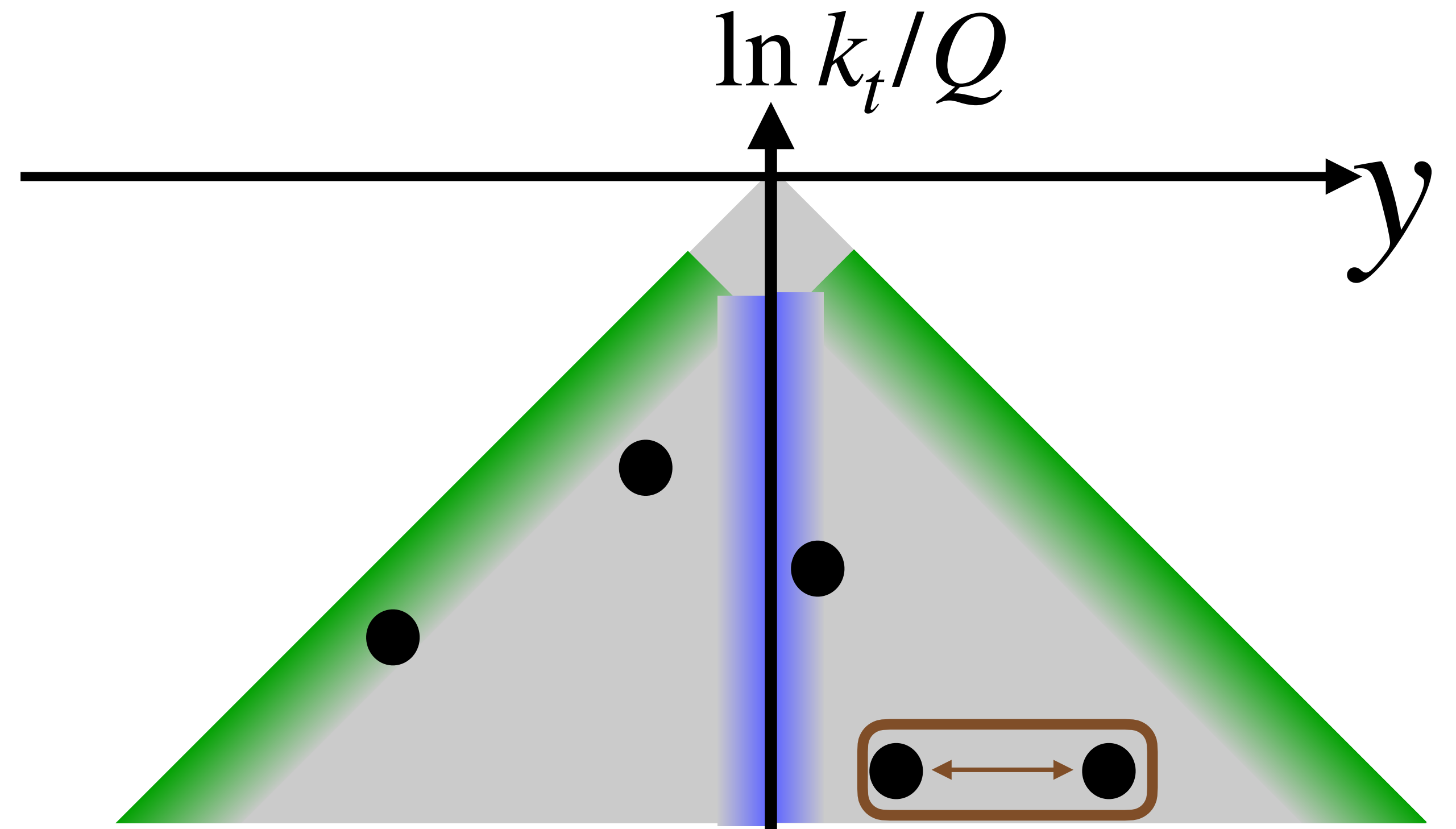
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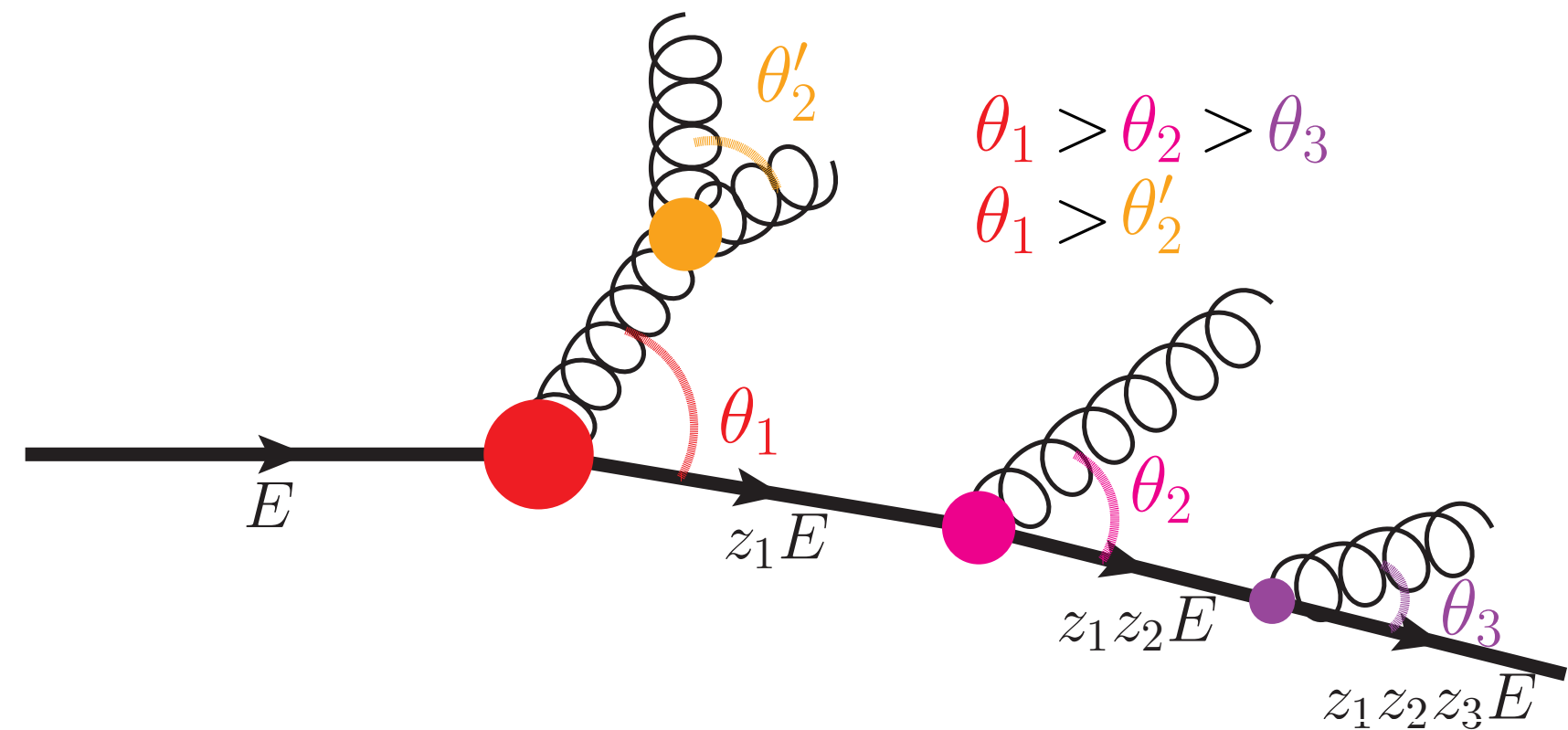
- Emissions separated in **just one direction** in the Lund plane enter at this order



Constrains **kinematic mapping** $\Phi_n \rightarrow \Phi_{n+1}$ and **ordering variable**: emissions well separated in rapidity are independent from each other, even if they have similar transverse momentum

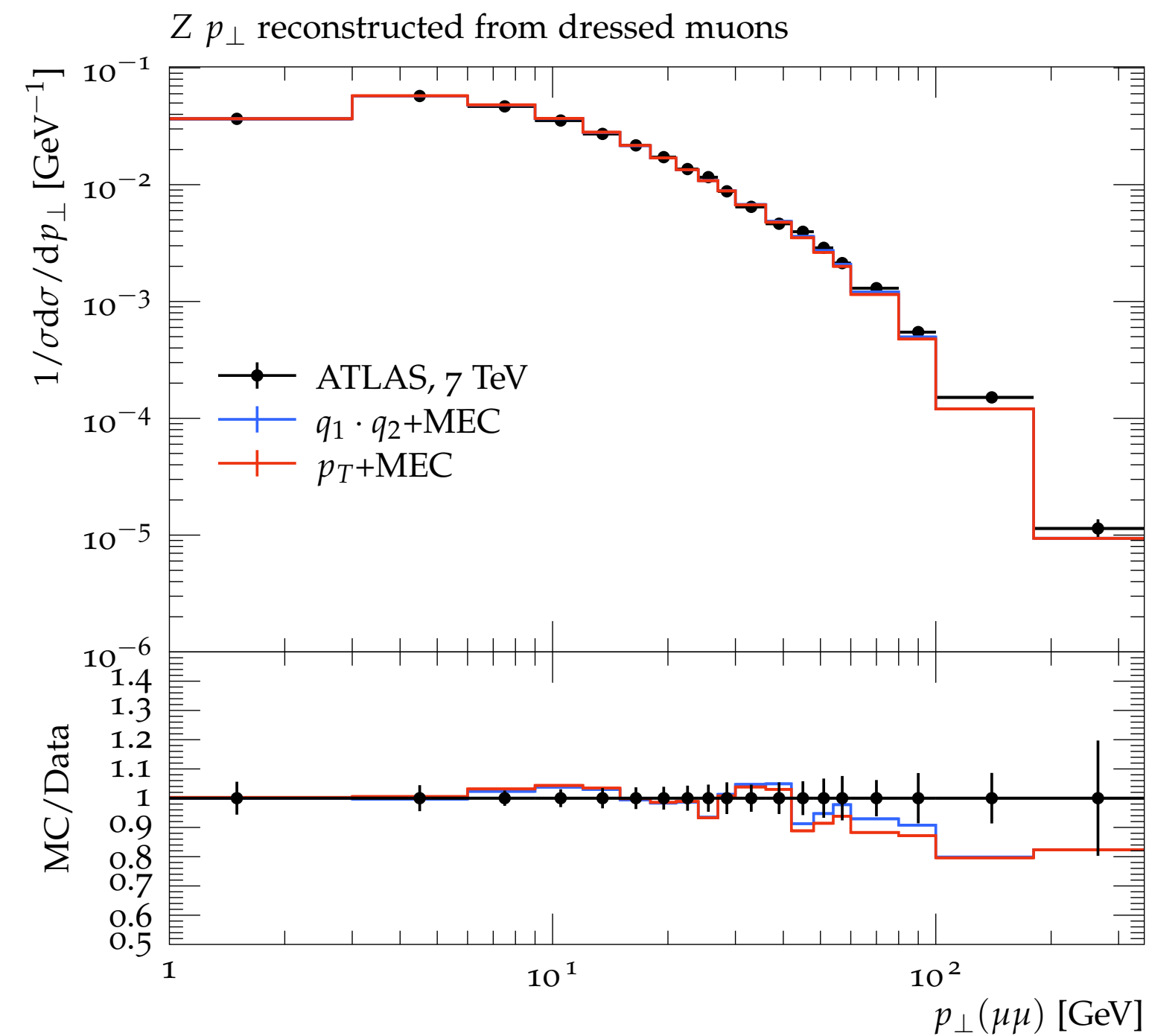
Parton Showers in a nutshell

Angular-ordered shower (Herwig)



- Designed to achieve **NLL** for many observables [Marchesini, Webber '88]

p_{\perp} of the Z at LHC

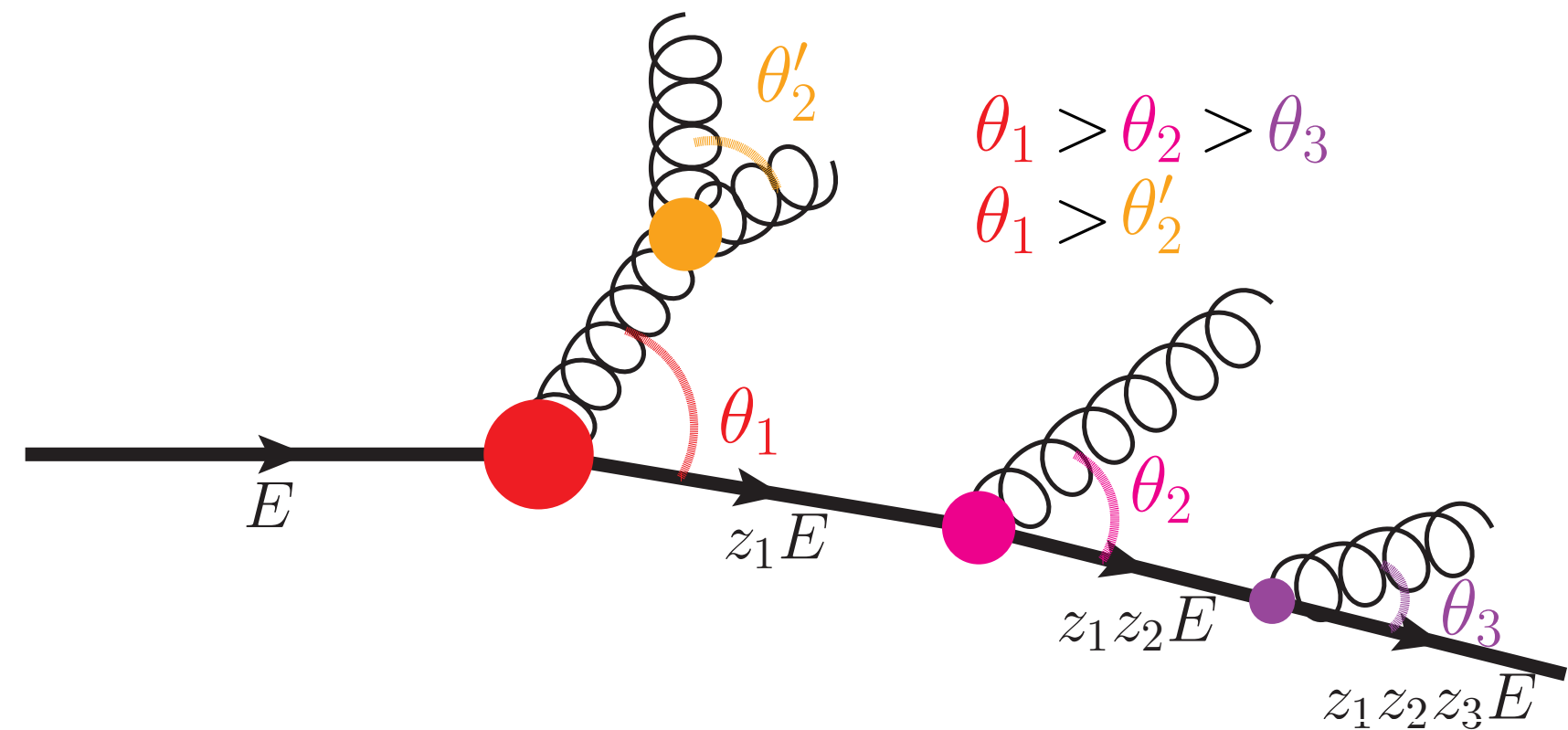


2 recoil schemes that achieve **NLL accuracy** for global event shapes
(difference can be used to estimate shower uncertainties)

[Bewick, SFR, Richardson, Seymour; 1904.11866, 2107.04051]

Parton Showers in a nutshell

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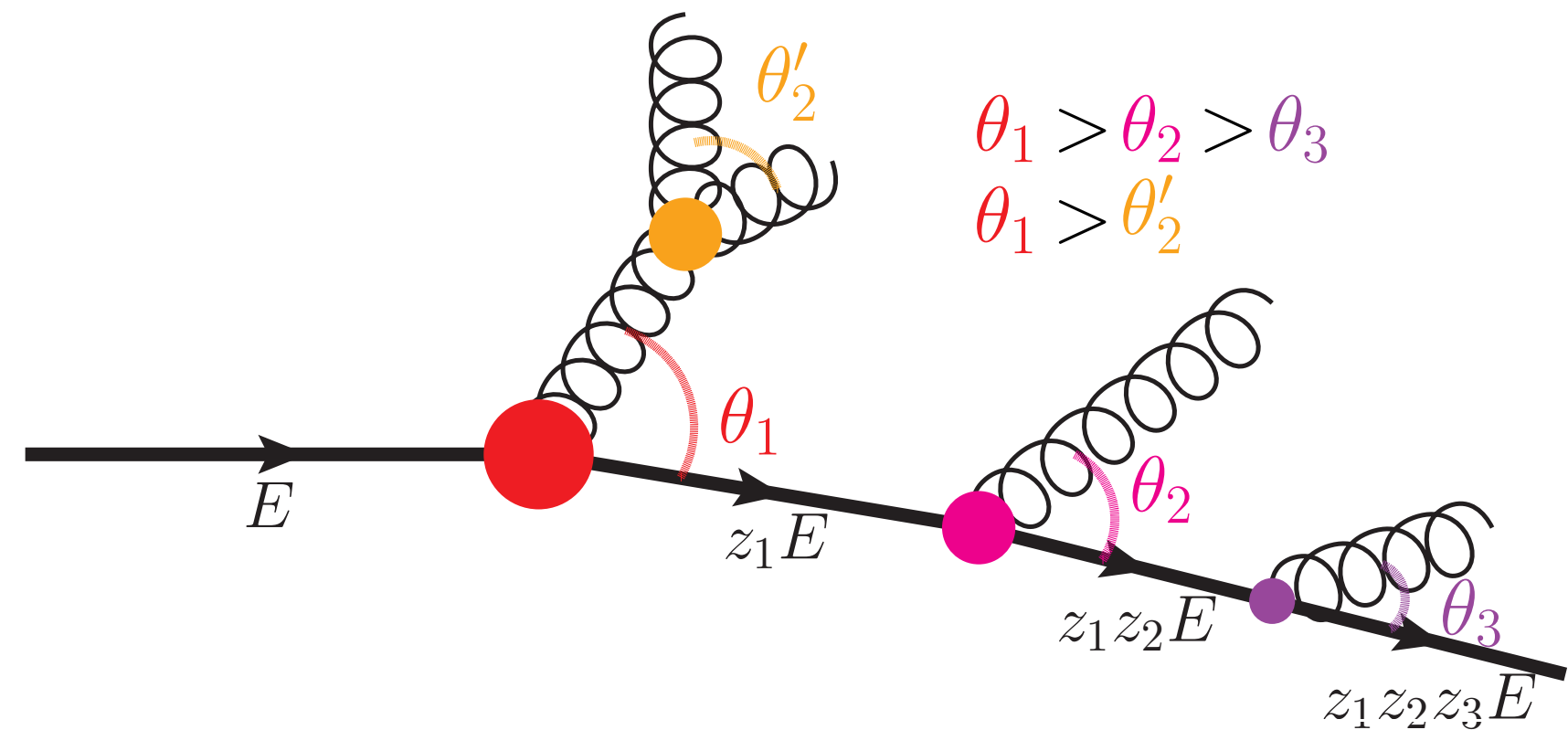
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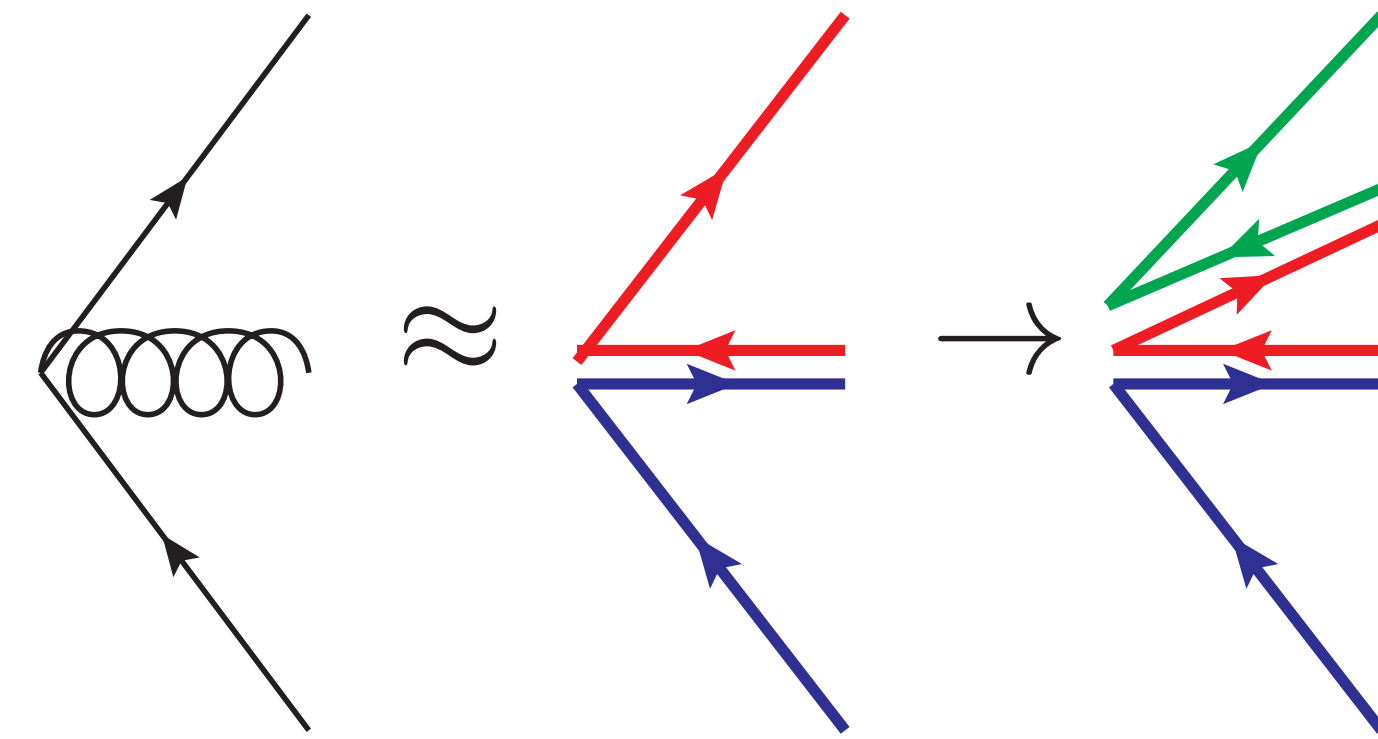


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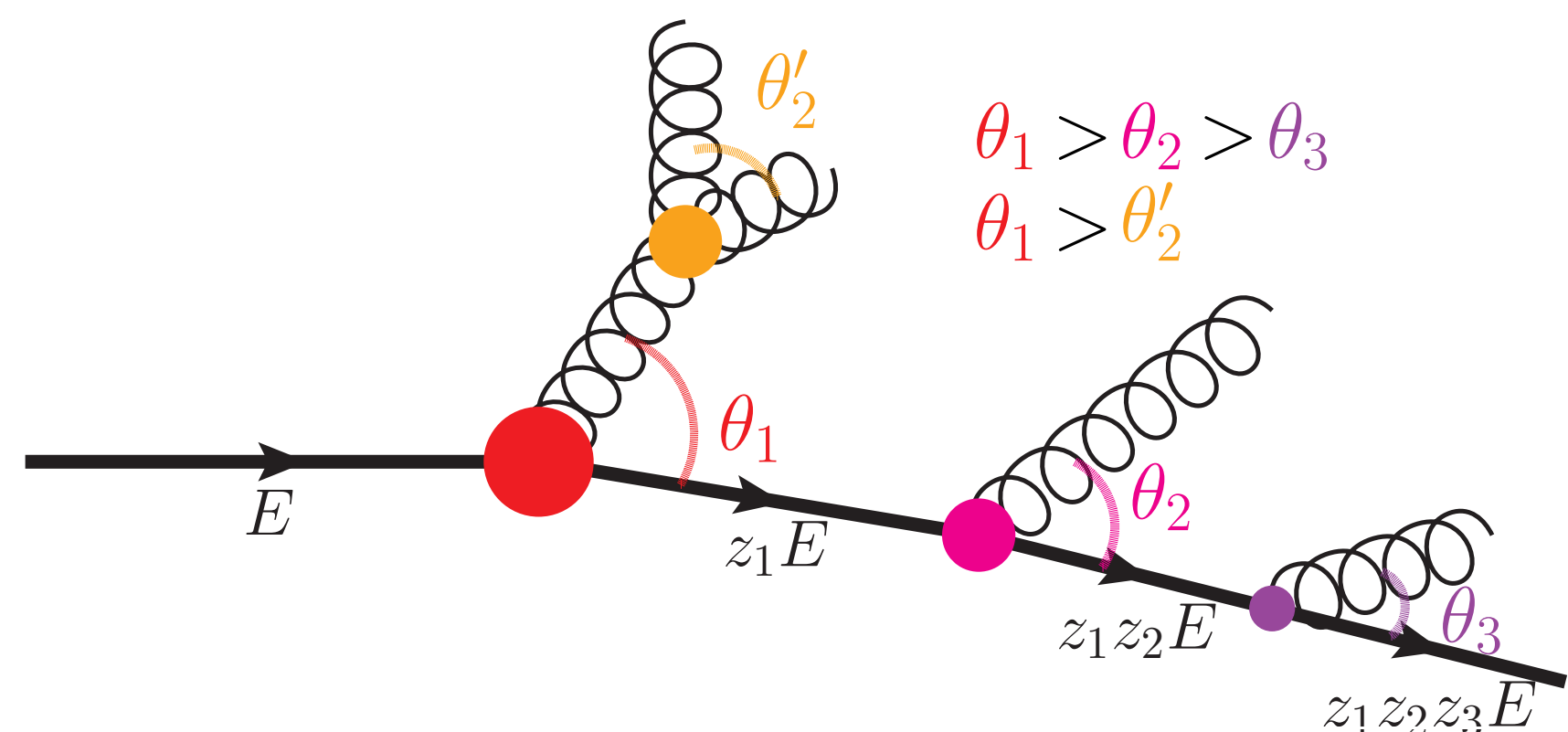
- Dipole showers are the more popular alternative to angular-ordered showers
[Gustafson, Pettersson '88]

- **Matching beyond NLO and multi-jet merging** much simpler as hardest emissions come first
- Azimuthal dependence of soft emission necessary for **non-global logs**

BUT THEY ARE NOT YET (N)NLL!

Parton Showers in a nutshell

Angular-ordered shower (Herwig)

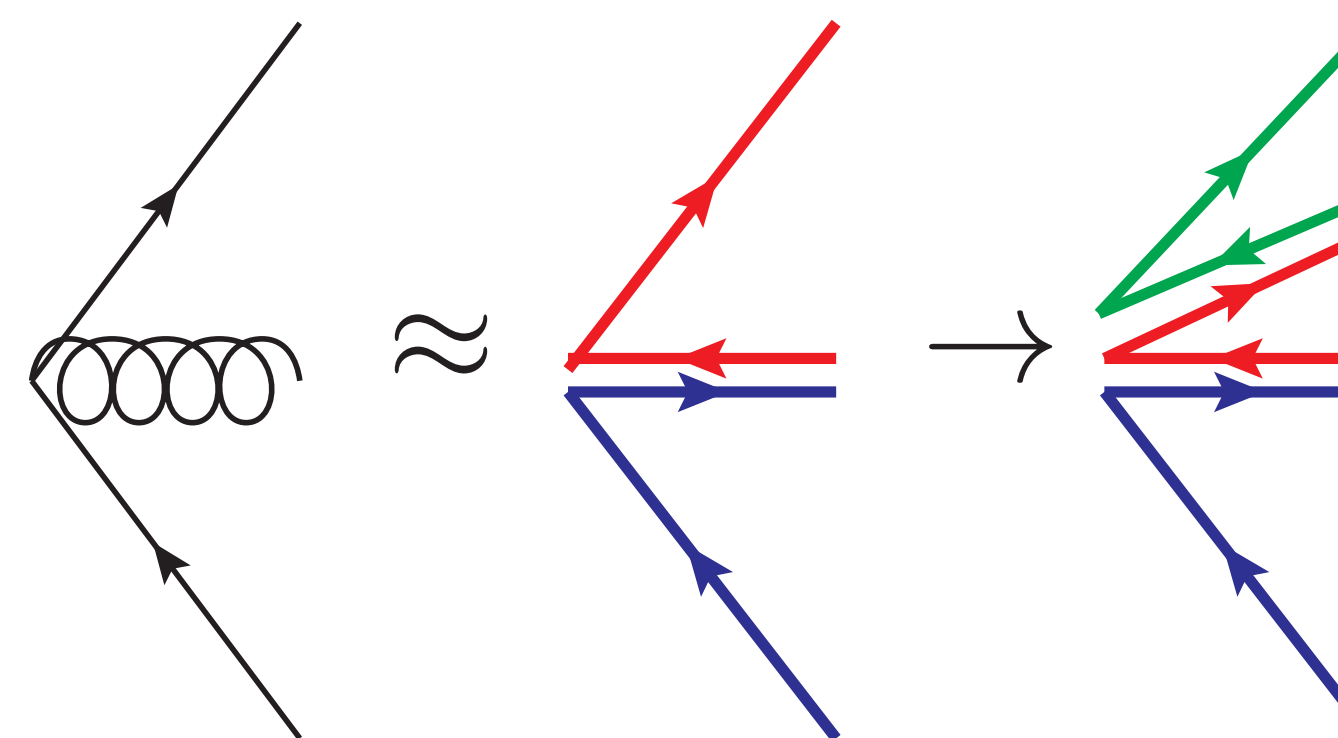


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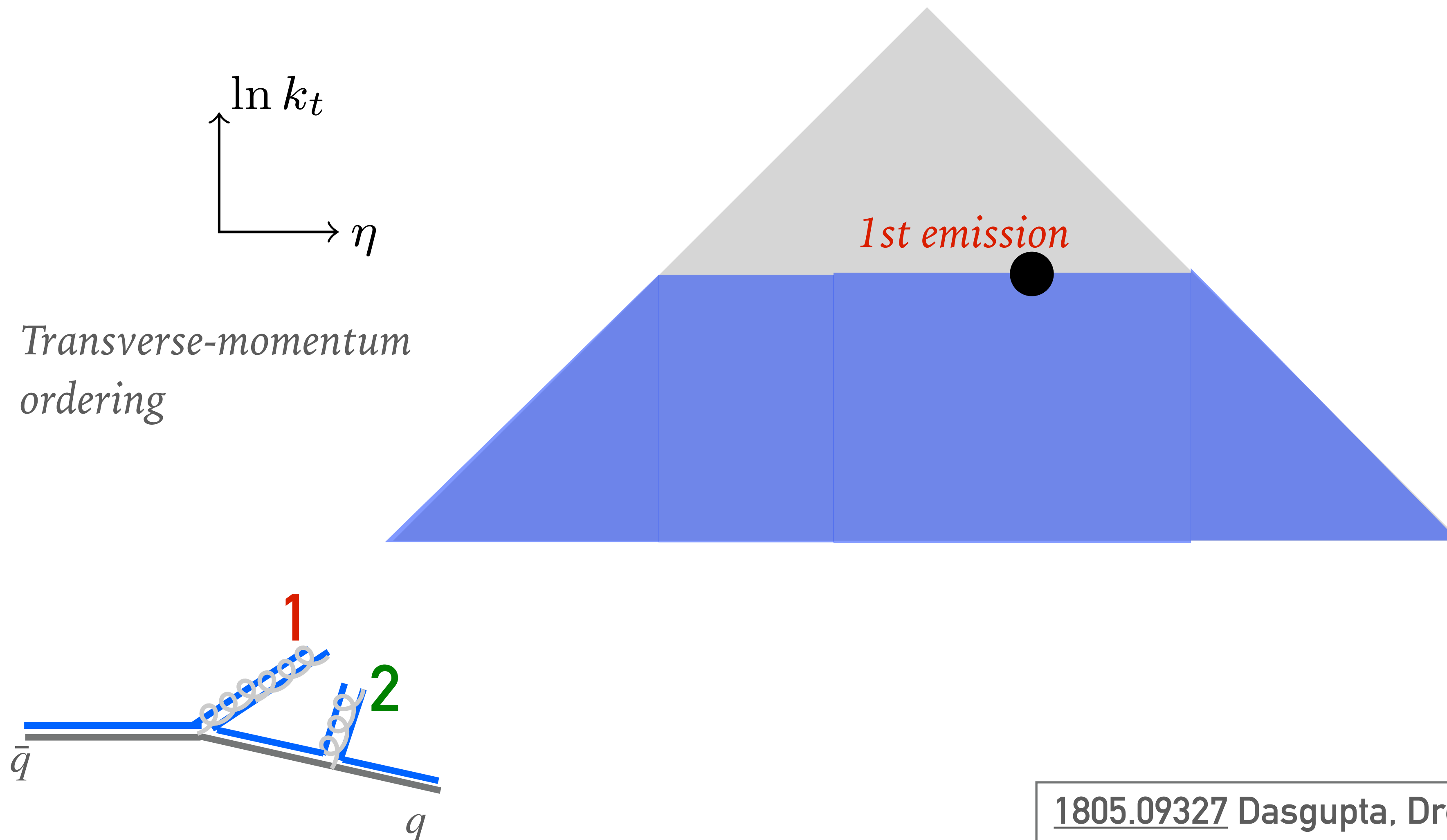
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Steady progresses in building (N)NLL

Why are “standard” dipole showers not NLL?

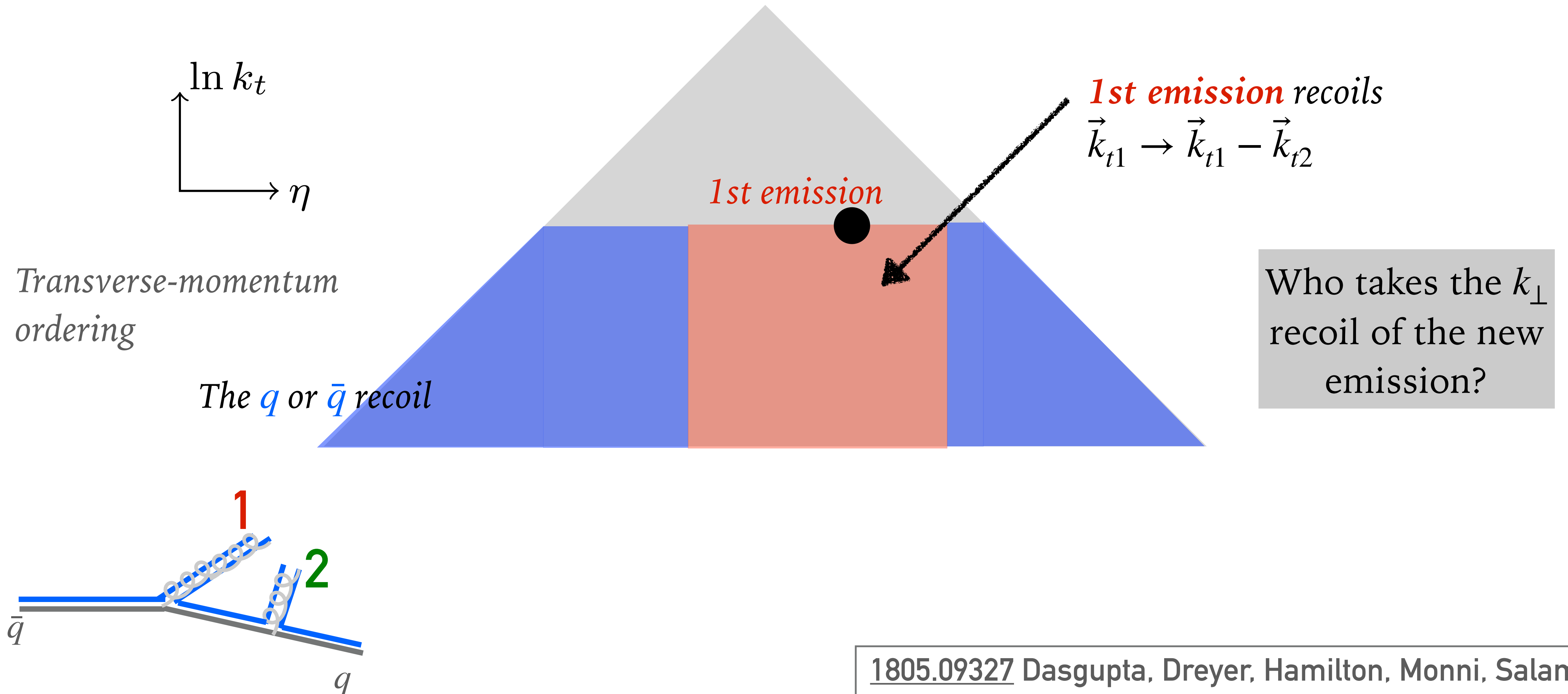
Emission of a **soft-collinear gluon** g_2 , from a $q\bar{q}g_1$ final-state, where g_1 is soft-collinear as well



1805.09327 Dasgupta, Dreyer, Hamilton, Monni, Salam

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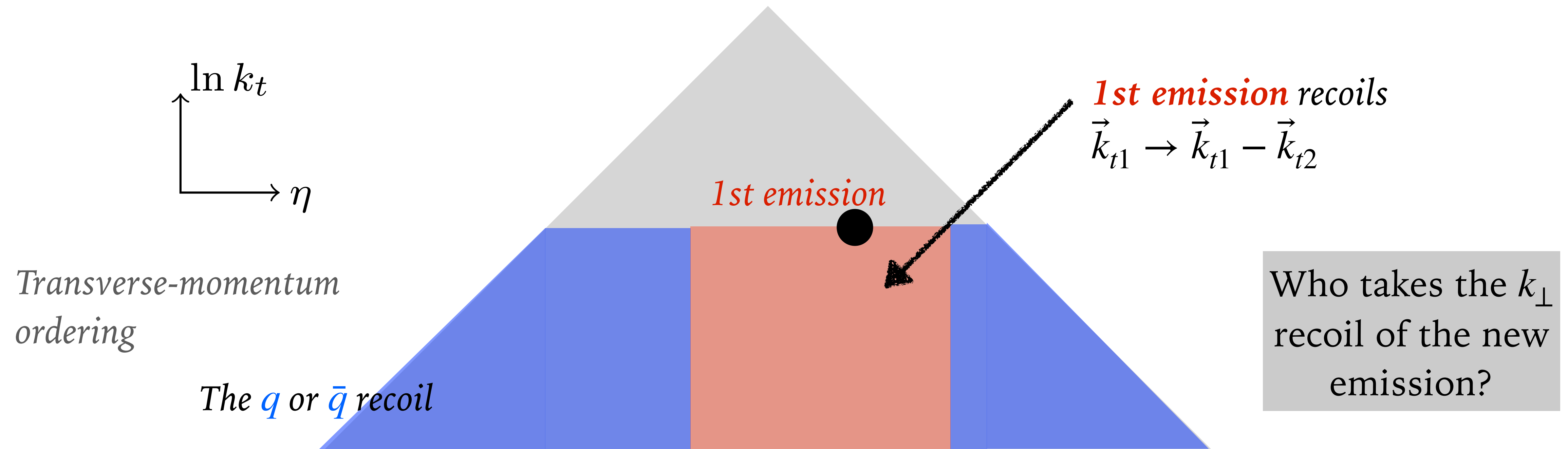
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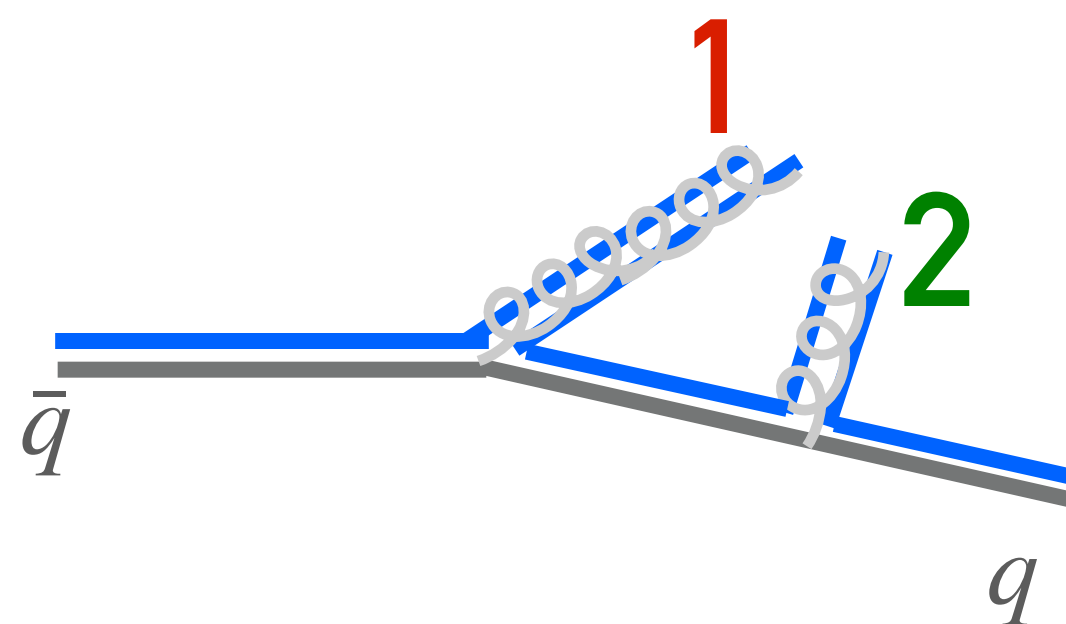
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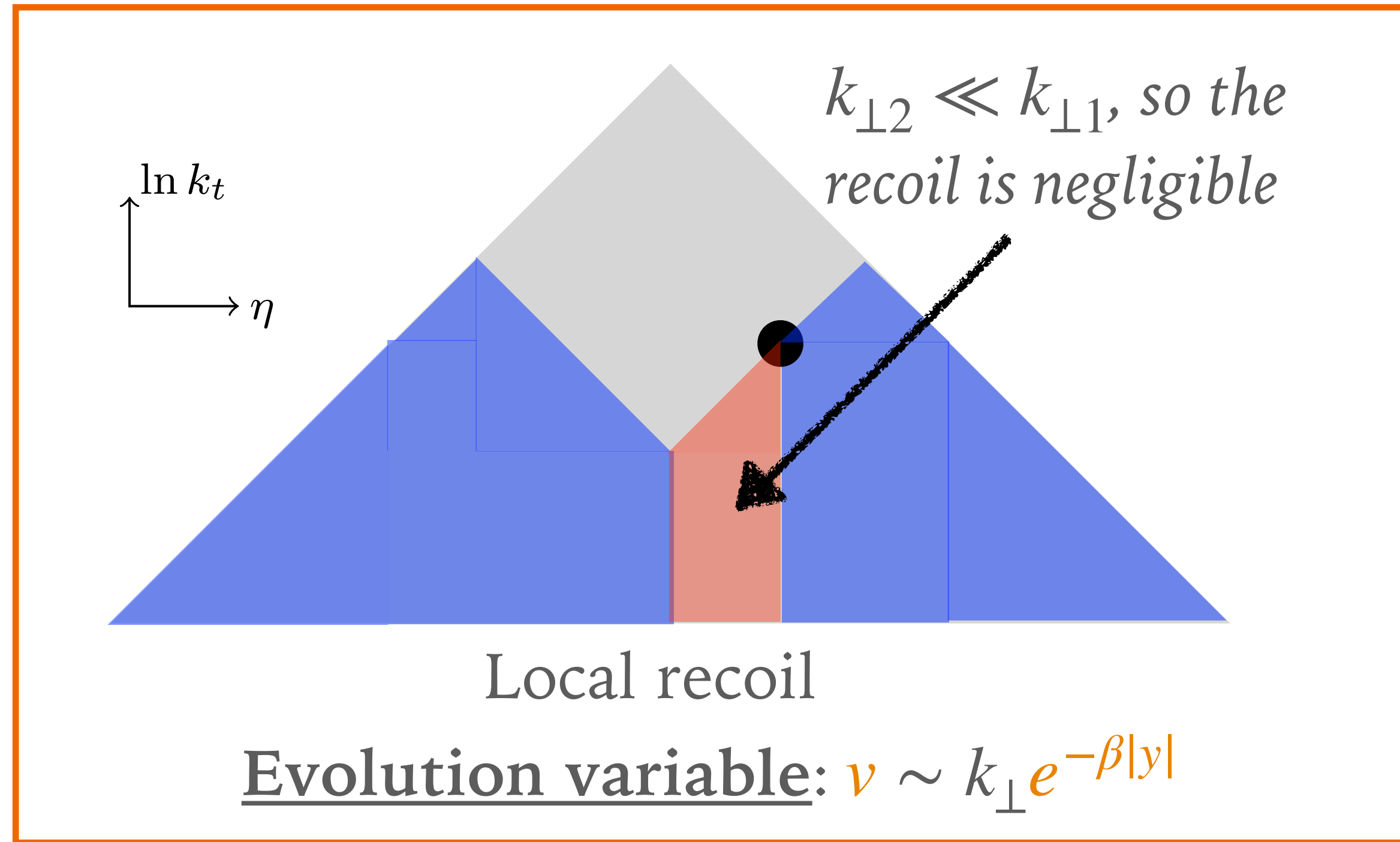


For $|\eta_1 - \eta_2| \gg 1$ but $k_{t,1} \sim k_{t,2}$, the 1st emission is affected by the 2nd: **NLL is not OK!**



1805.09327 Dasgupta, Dreyer, Hamilton, Monni, Salam

Building a NLL shower

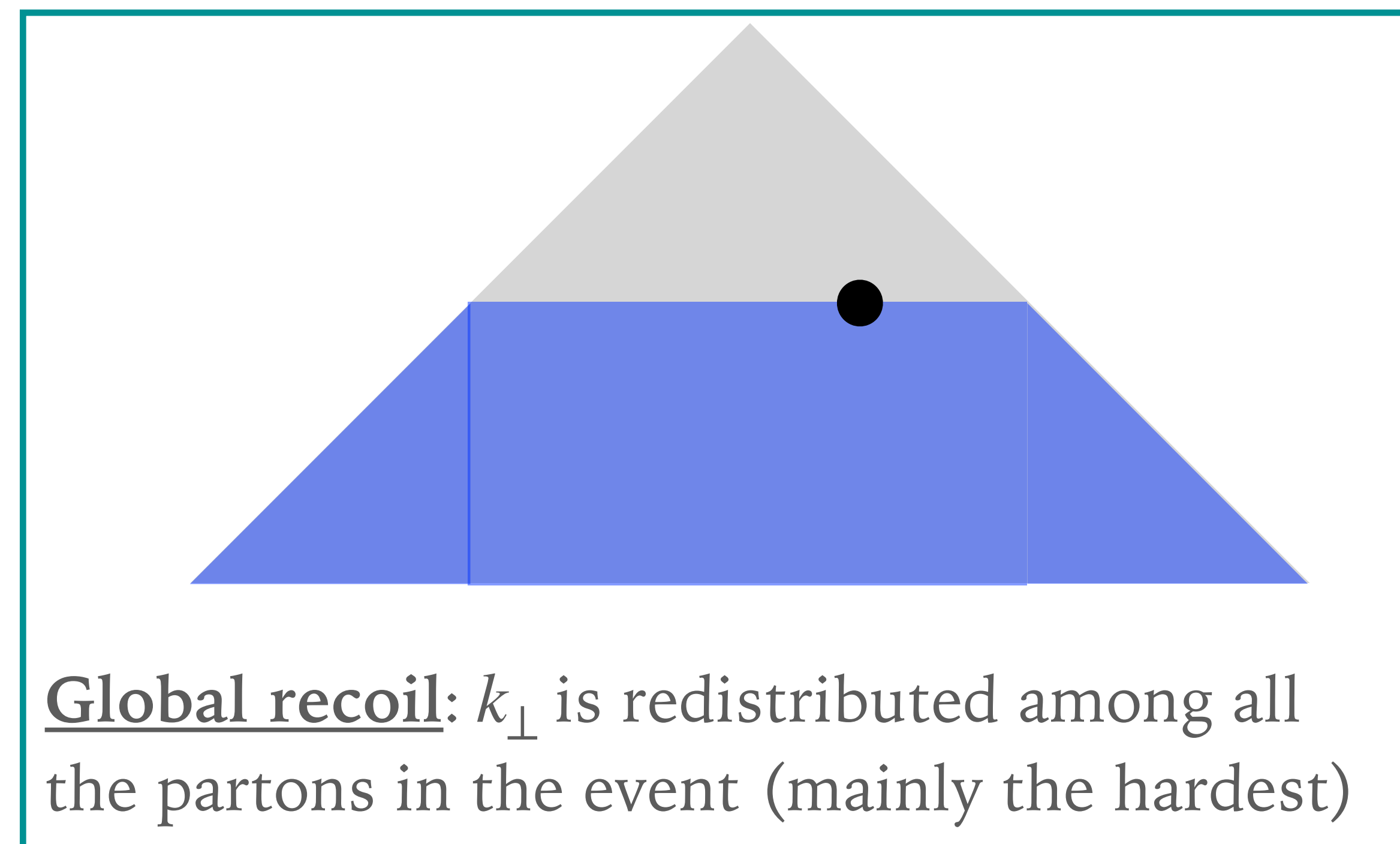
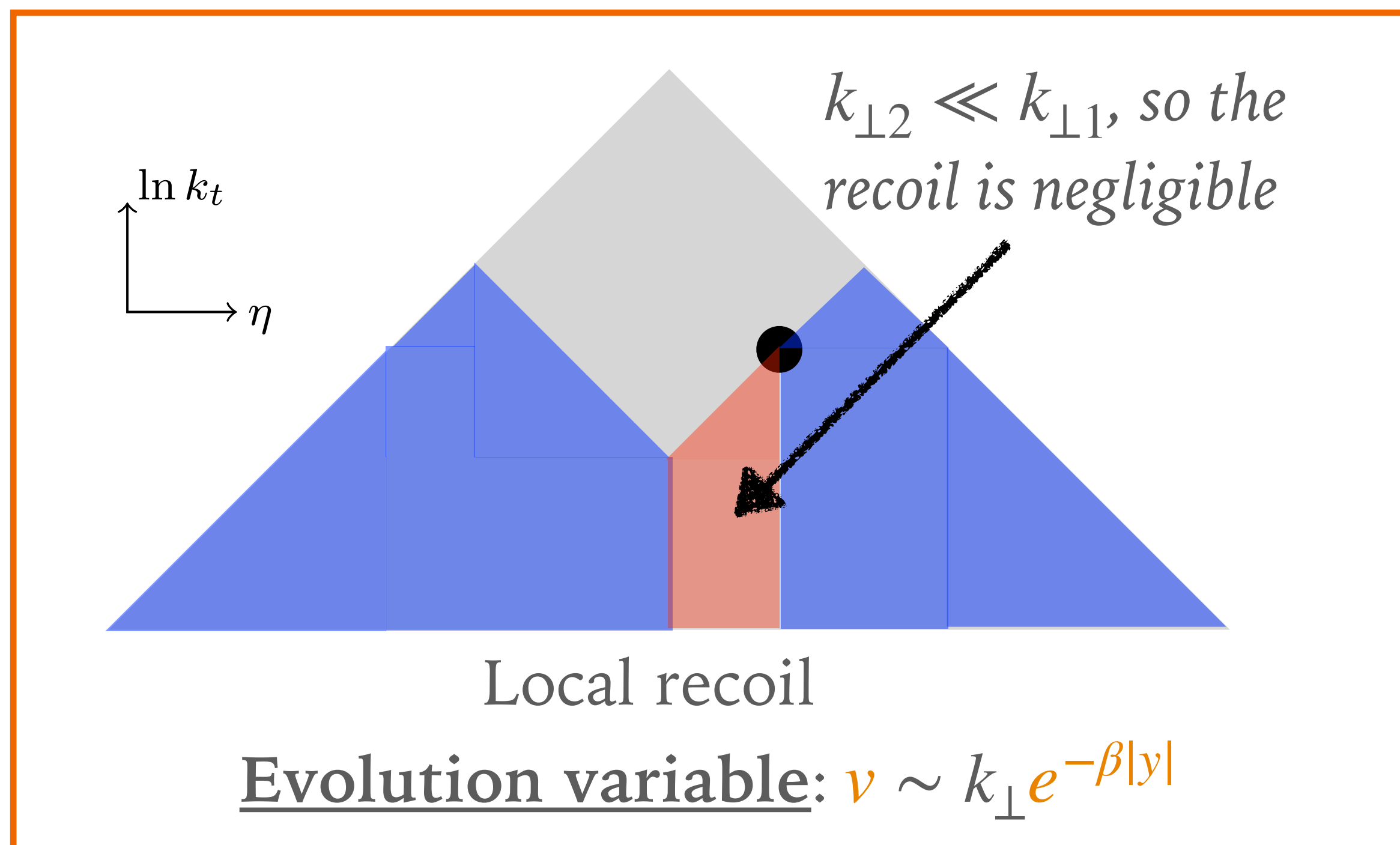


Deductor by Nagy & Soper 0912.4534

PanScales (local variant), Dasgupta et al.
2002.11114

+ angles in the event frame

Building a NLL shower



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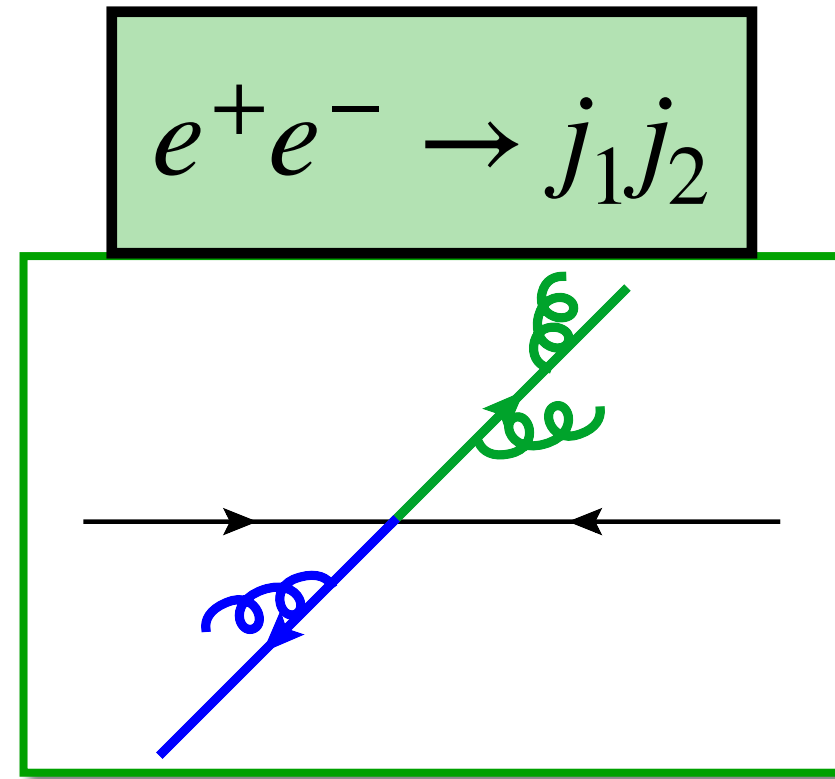
Forshaw, Holguin, and Plätzer [2003.06400](#)

Alaric by Herren et al. [2208.06057](#)

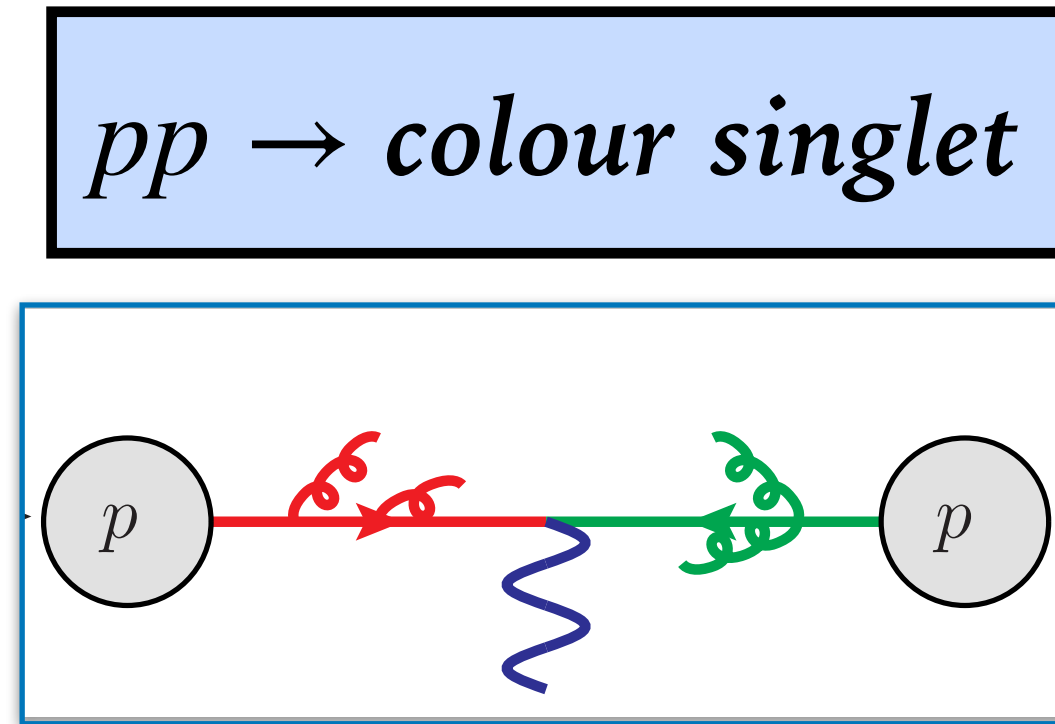
Apollo by Preuss [2403.19452](#)

Status of NLL PanScales showers

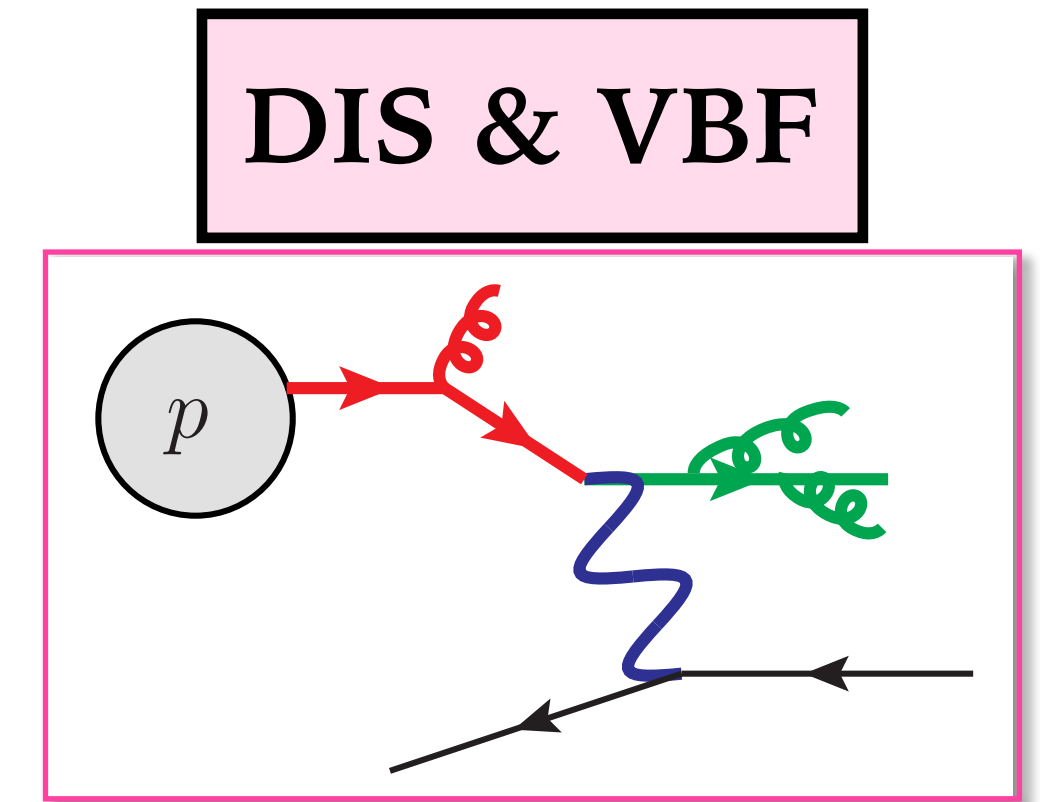
- PanScales is the first shower with **general** NLL accuracy for



Dasgupta, Dreyer, Hamilton,
Monni, Salam, Soyez,
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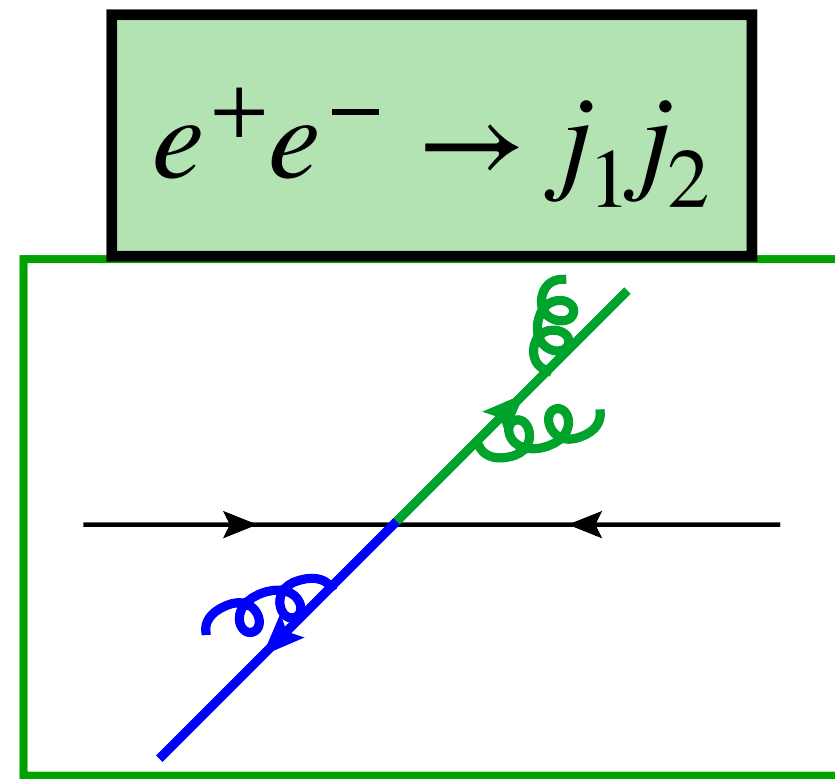
van Beekveld, SFR, Soto-Ontoso,
Salam, Soyez, Verheyen, 2205.02237,
+ Hamilton 2207.09467



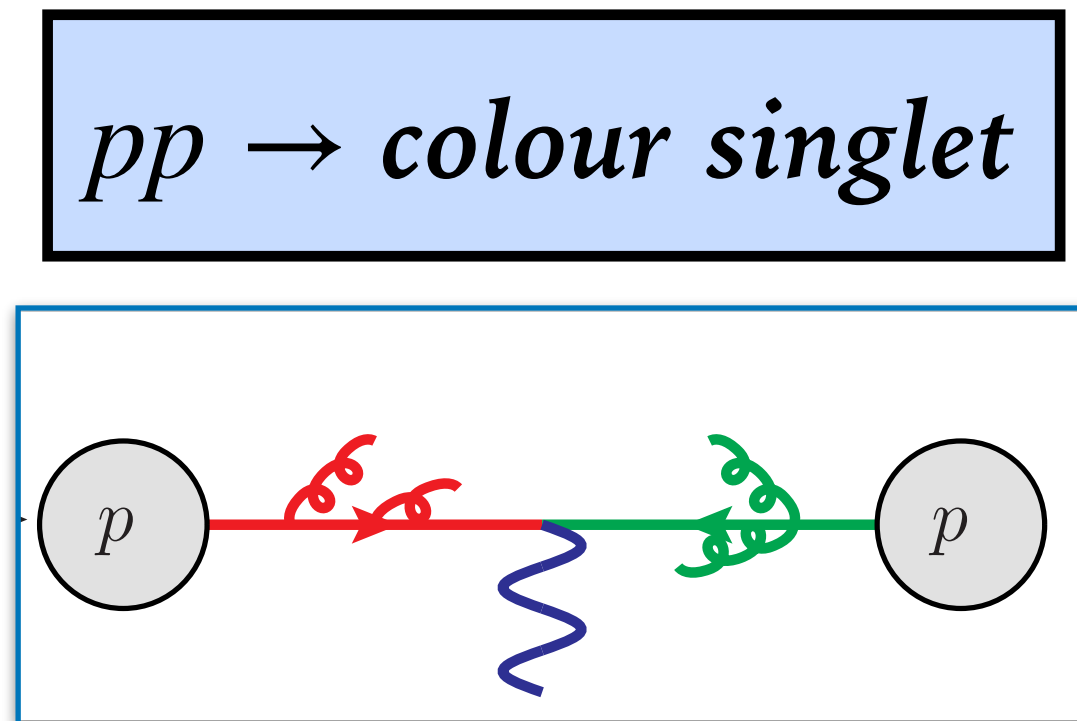
van Beekveld, SFR,
2305.08645

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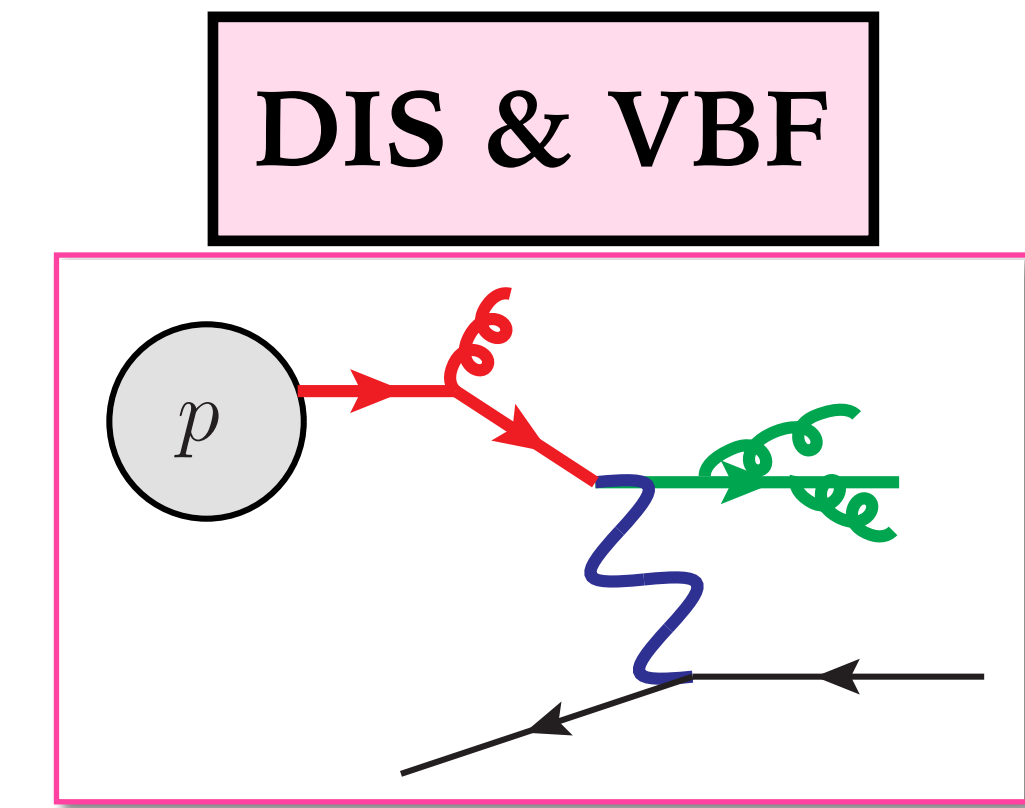
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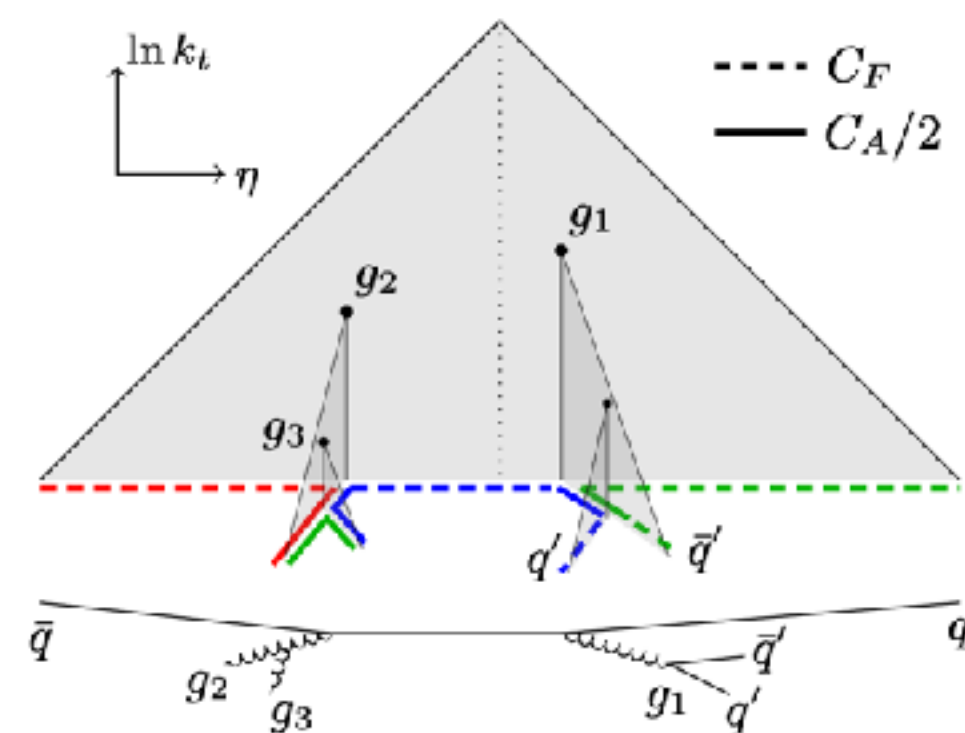
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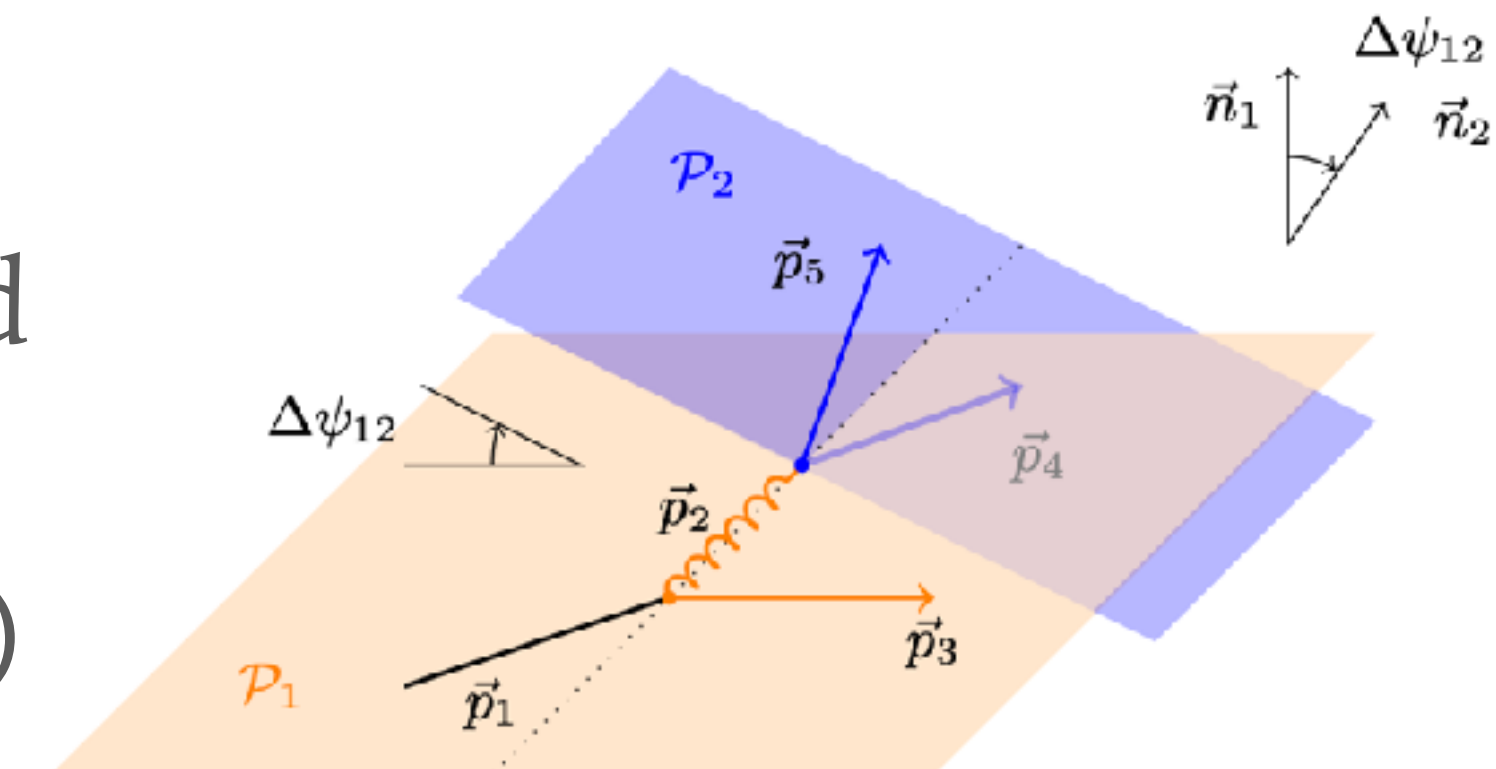
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van Beekveld, SFR,
2305.08645



...with **subleading colour** (Hamilton,
Medves, Salam, Scyboz, Soyez 2011.10054) and
spin correlations (Karlberg, Salam, Scyboz,
Verheyen 2103.16526, + Hamilton 2111.01161)



Alaric highlights

NLL shower for e^+e^- in [2208.06057](#)

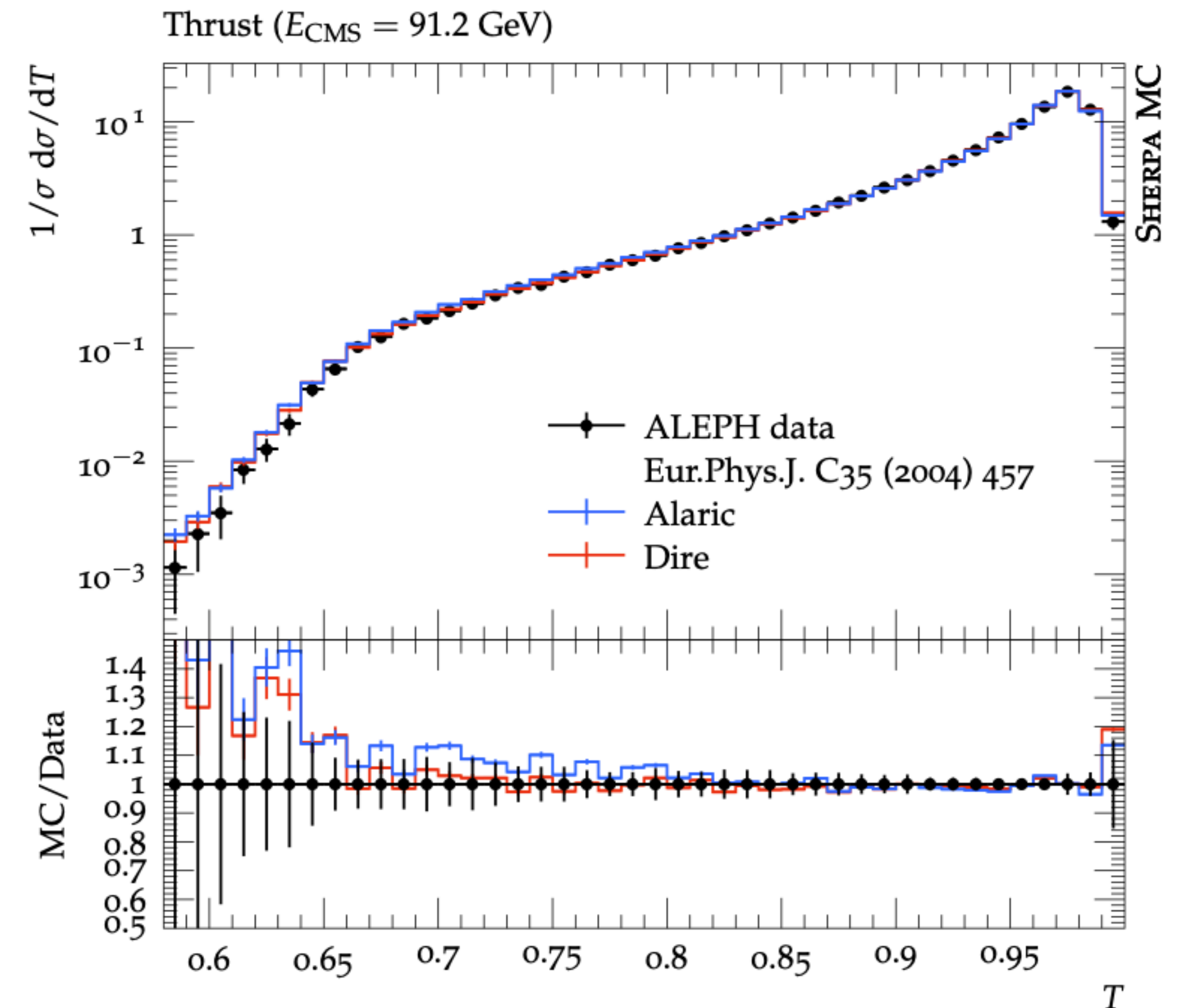
[Herren, Höche, Krauss, Reichelt, Schönherr]

- The **anti-collinear** component is conserved globally in the map: the colour partner is not used as recoiler (as in **Deductor** and **FHP**)

- Separate **soft** from **collinear** evolution e.g.

$$\frac{P_{q \rightarrow qg}}{C_F} = \frac{2z}{1-z} + 1 - z$$

(similar proposal by Nagy and Soper in [2204.05631](#))



NLL shower similar to LL

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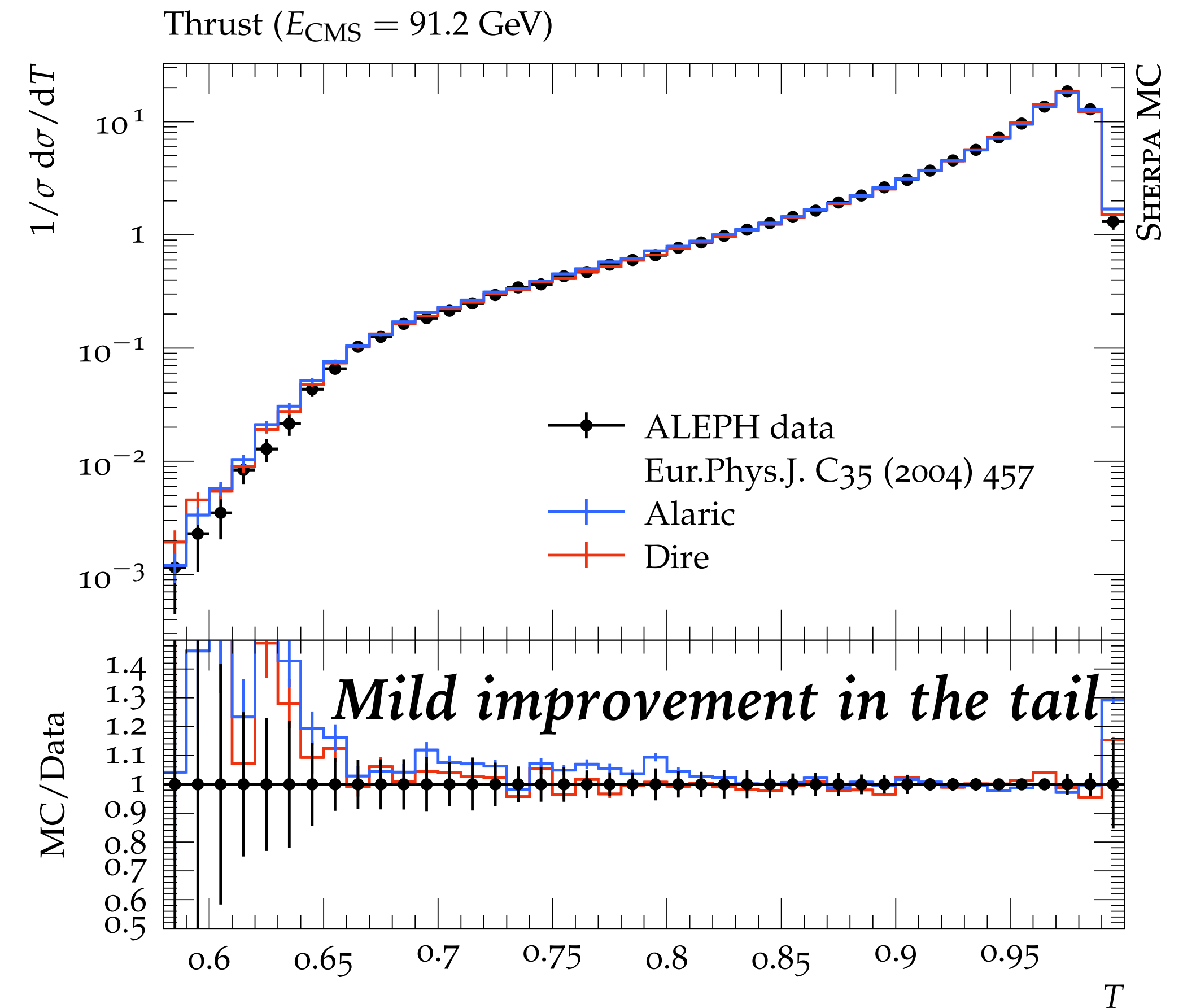
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With mass effects [2307.00728](#) [Assi, Höche]

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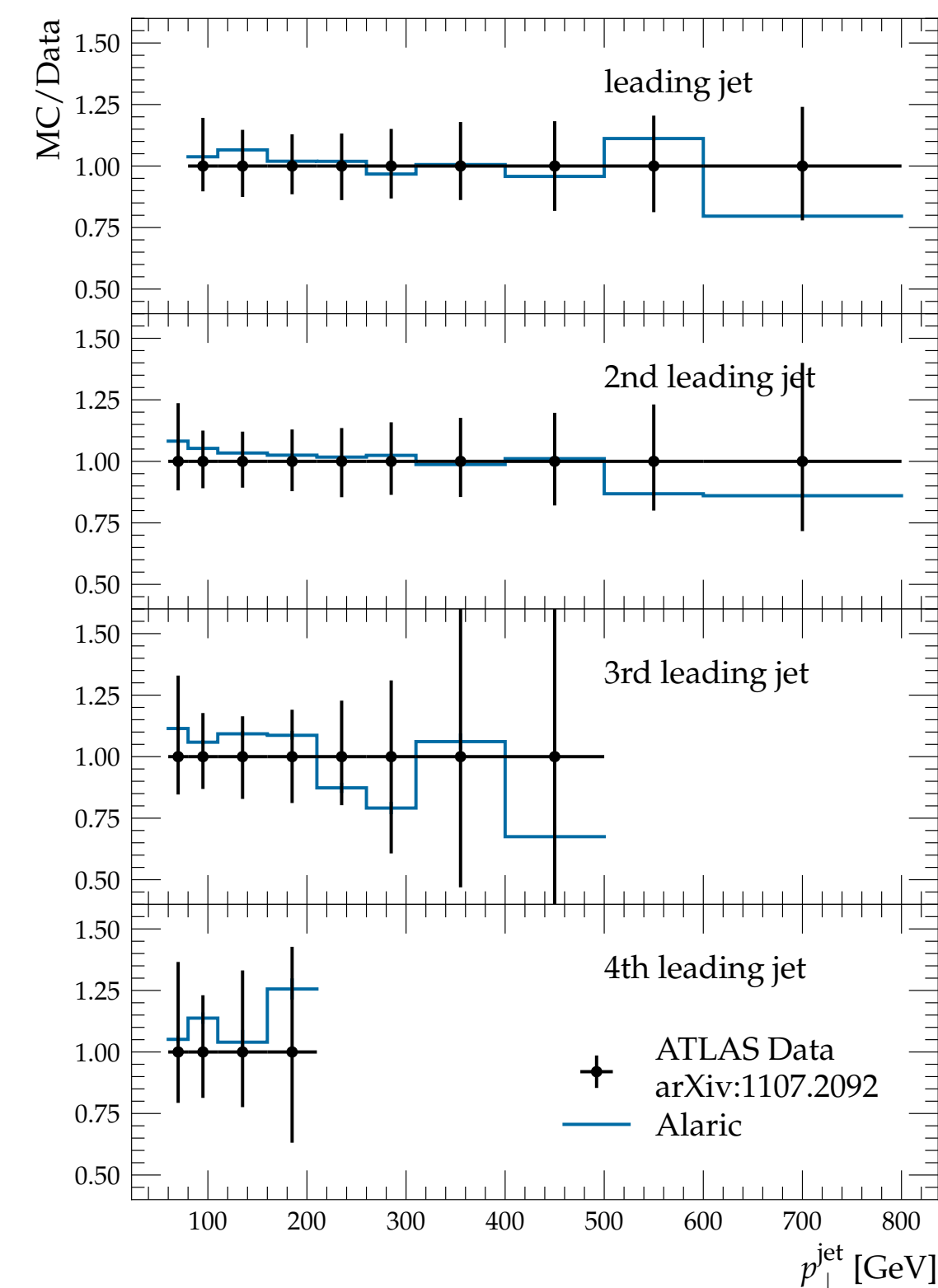
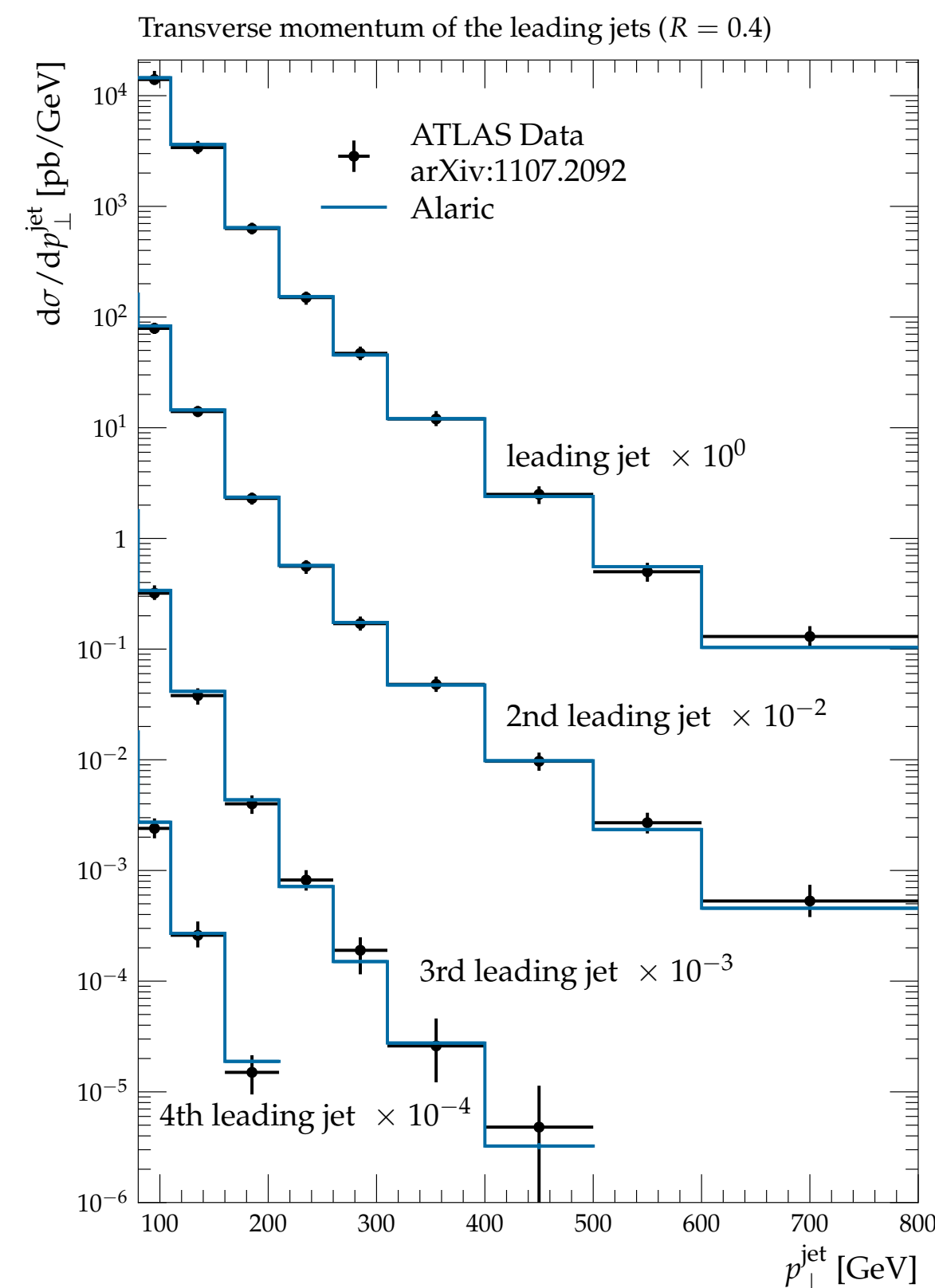
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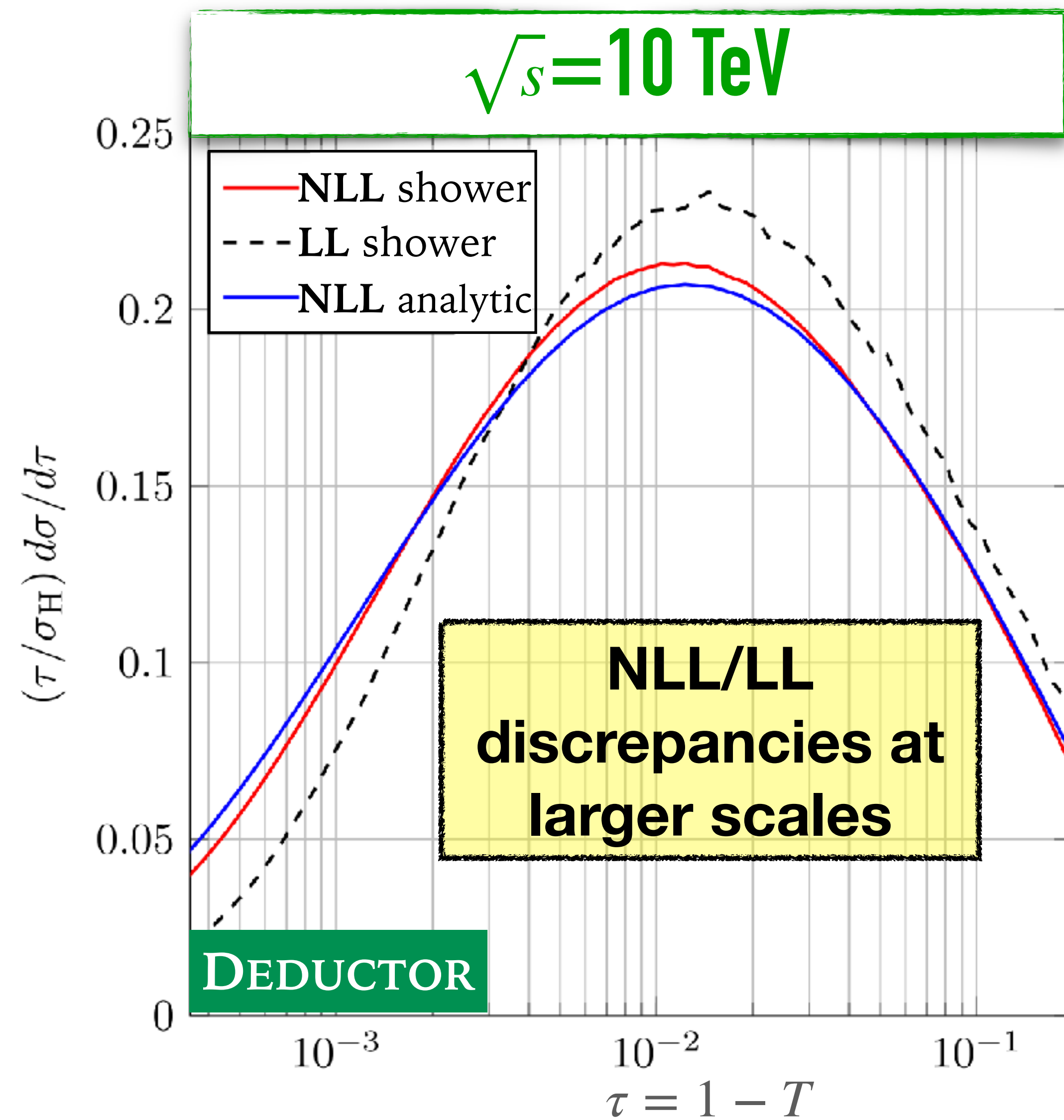
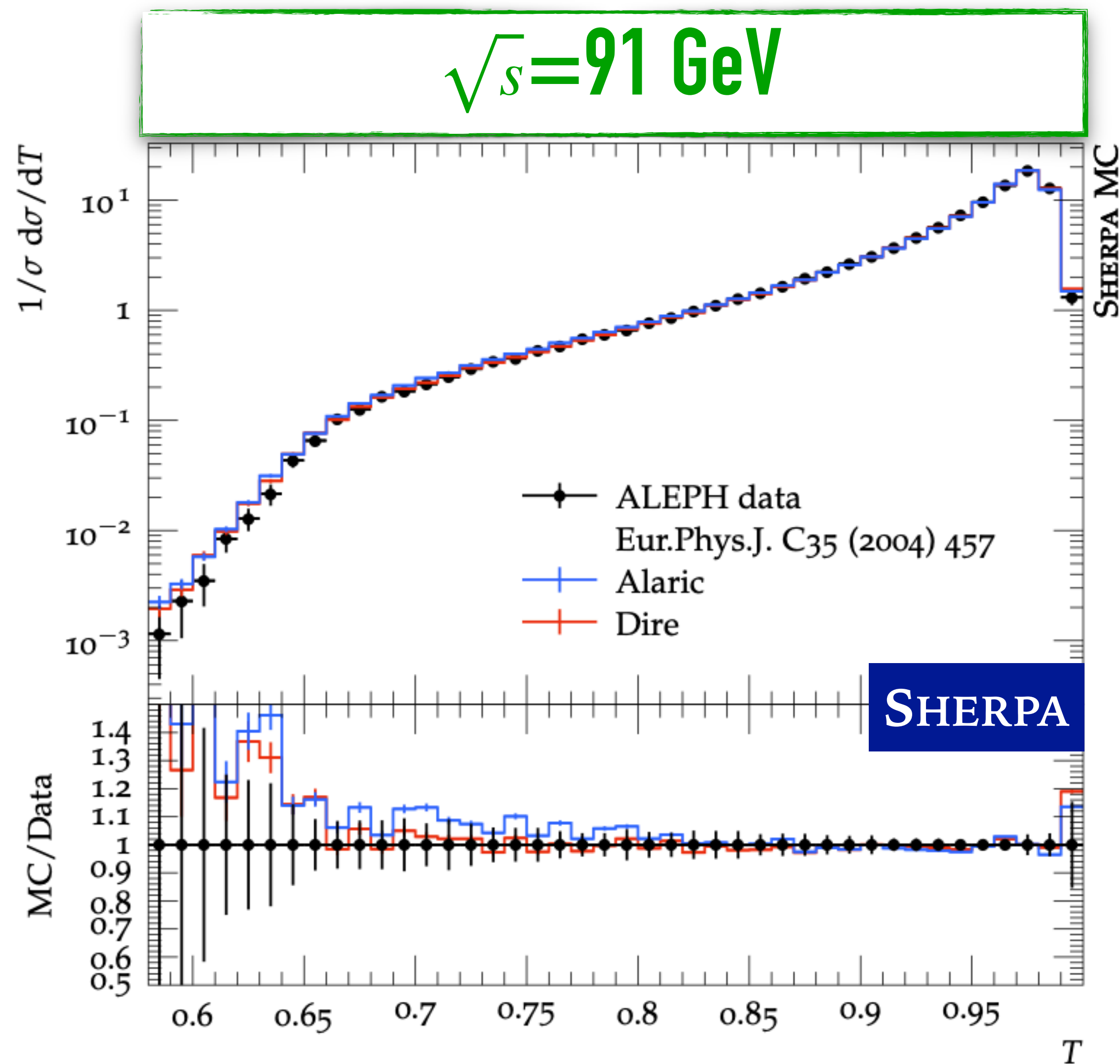
- Local k_{\perp} conservation for **collinear** evolution

**pp colliders and multi-jet merging [2404.14360](#)
[Höche, Krauss, Reichelt]**

- Global k_{\perp} conservation for **ISR**



LEP: when is NLL important?



The
(Panscales)
route to
NLL

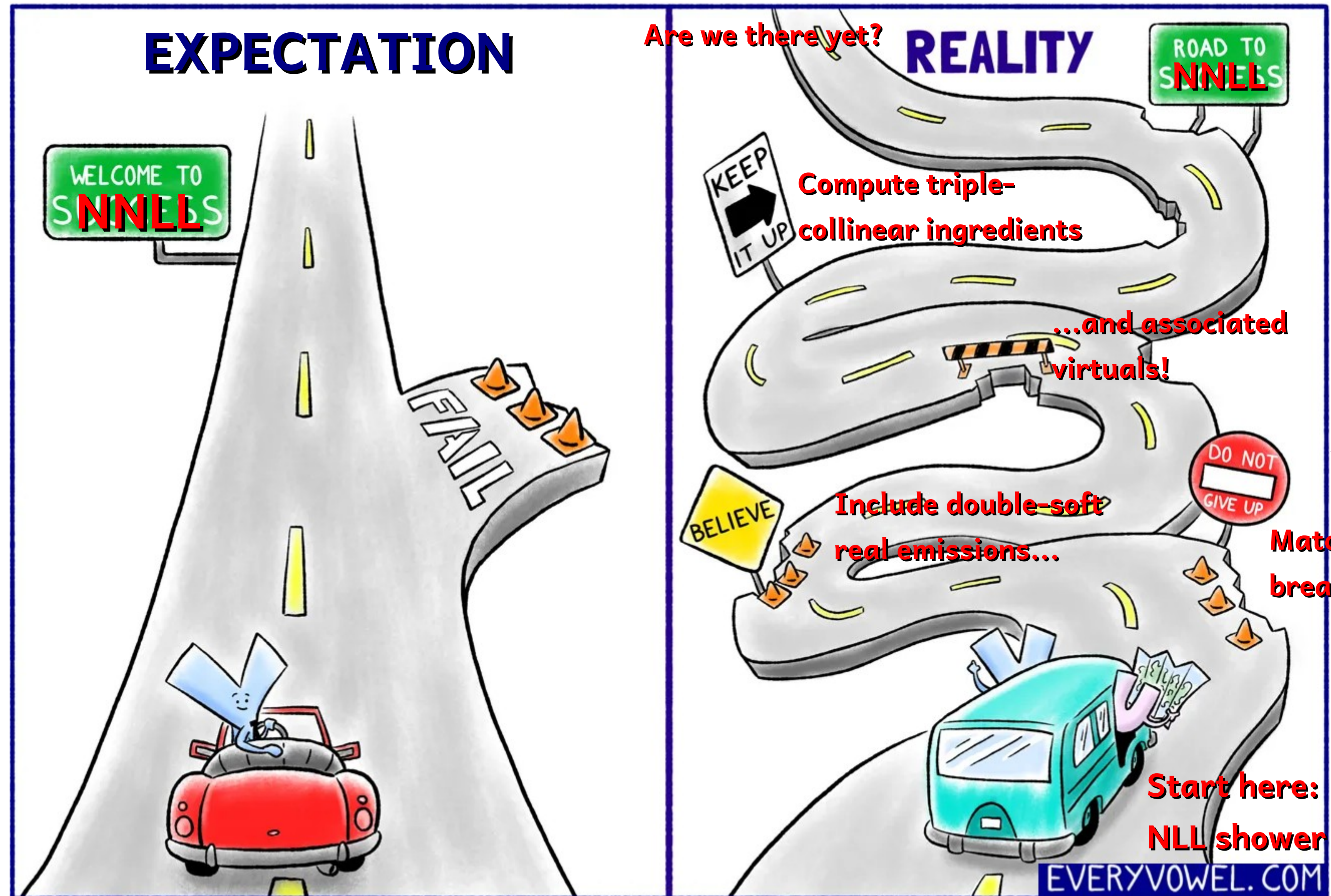


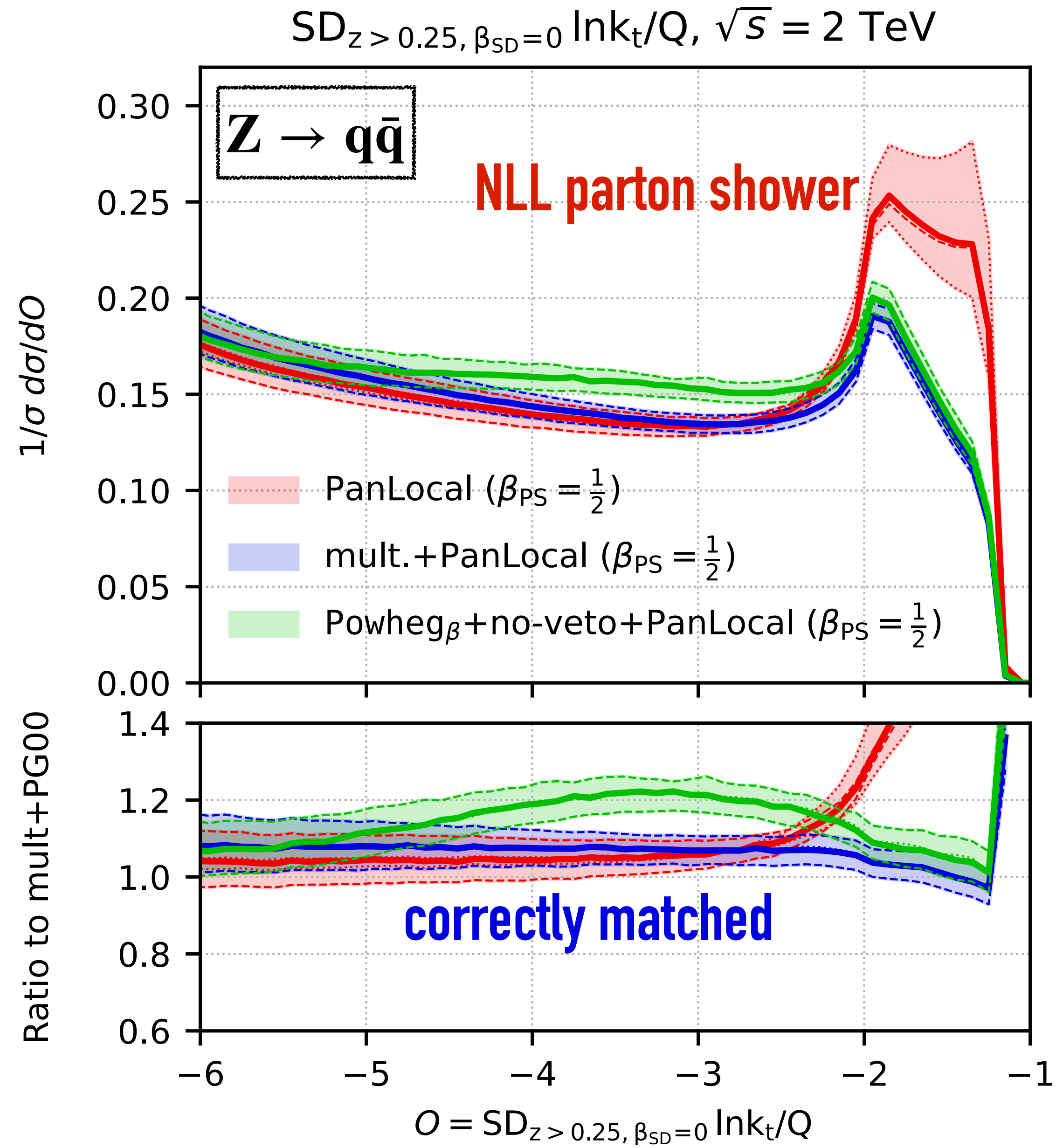
Figure from Alexander Karlberg

Matching and Logarithmic Accuracy

► **NLO matching** is necessary for NNLL accuracy

$$\Sigma_{\text{NLO}} = \Sigma_{\text{LO}}(1 + \alpha_s \Delta_{\text{NLO}} + \dots)$$

$$\Sigma_{\text{NNLL}} = \exp(Lg_{\text{LL}}(\alpha_s L) + g_{\text{NLL}}(\alpha_s L) + \alpha_s g_{\text{NNLL}}(\alpha_s L))$$



[Hamilton, Karlberg, Salam, Scyboz, Verheyen, 2301.09645]

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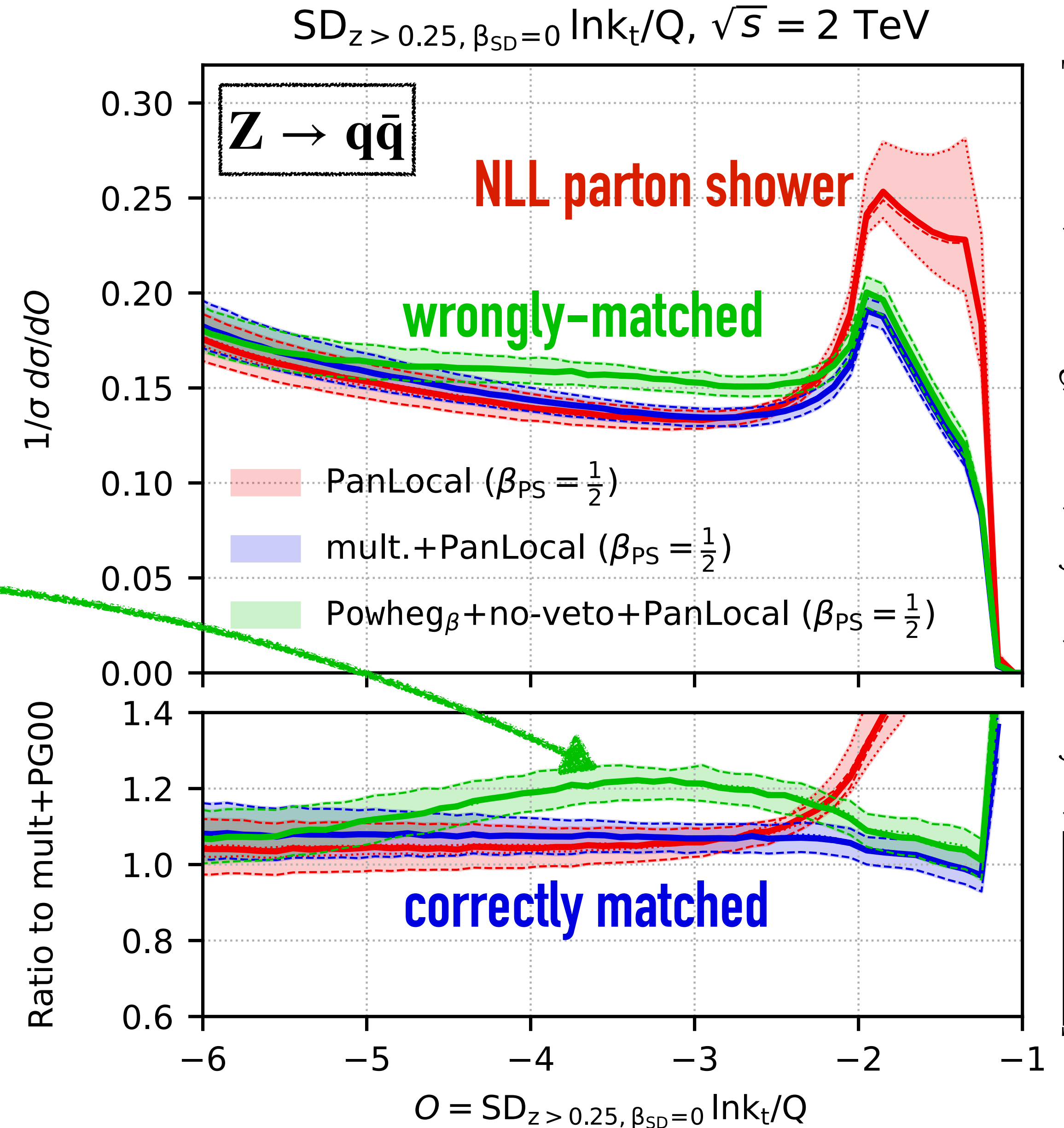
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- But it could instead **downgrade the NLL accuracy** of the shower

e.g. Soft Drop

$$\partial_L \Sigma_{\text{SD}}(L) = \bar{\alpha} c e^{\bar{\alpha} c L - \bar{\alpha} \Delta} - 2\bar{\alpha} L e^{-\bar{\alpha} L^2} (1 - e^{-\bar{\alpha} \Delta})$$



[Hamilton, Karlberg, Salam, Scyboz, Verheyen, 2301.09645]

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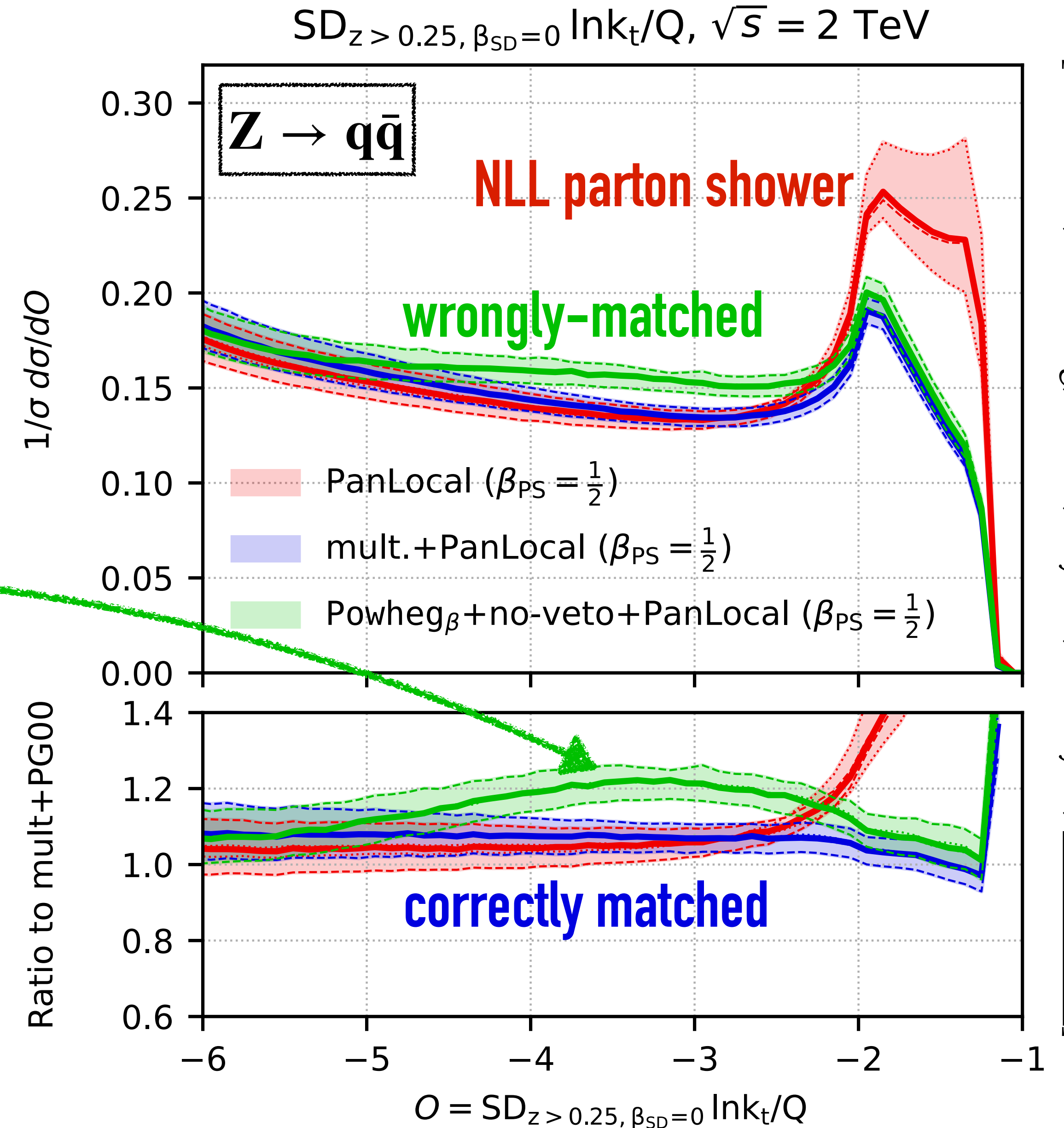
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General strategy for NLO matching with NLL showers for generic processes still missing!



[Hamilton, Karlberg, Salam, Scyboz, Verheyen, 2301.09645]

How to go beyond NLL in a parton shower?

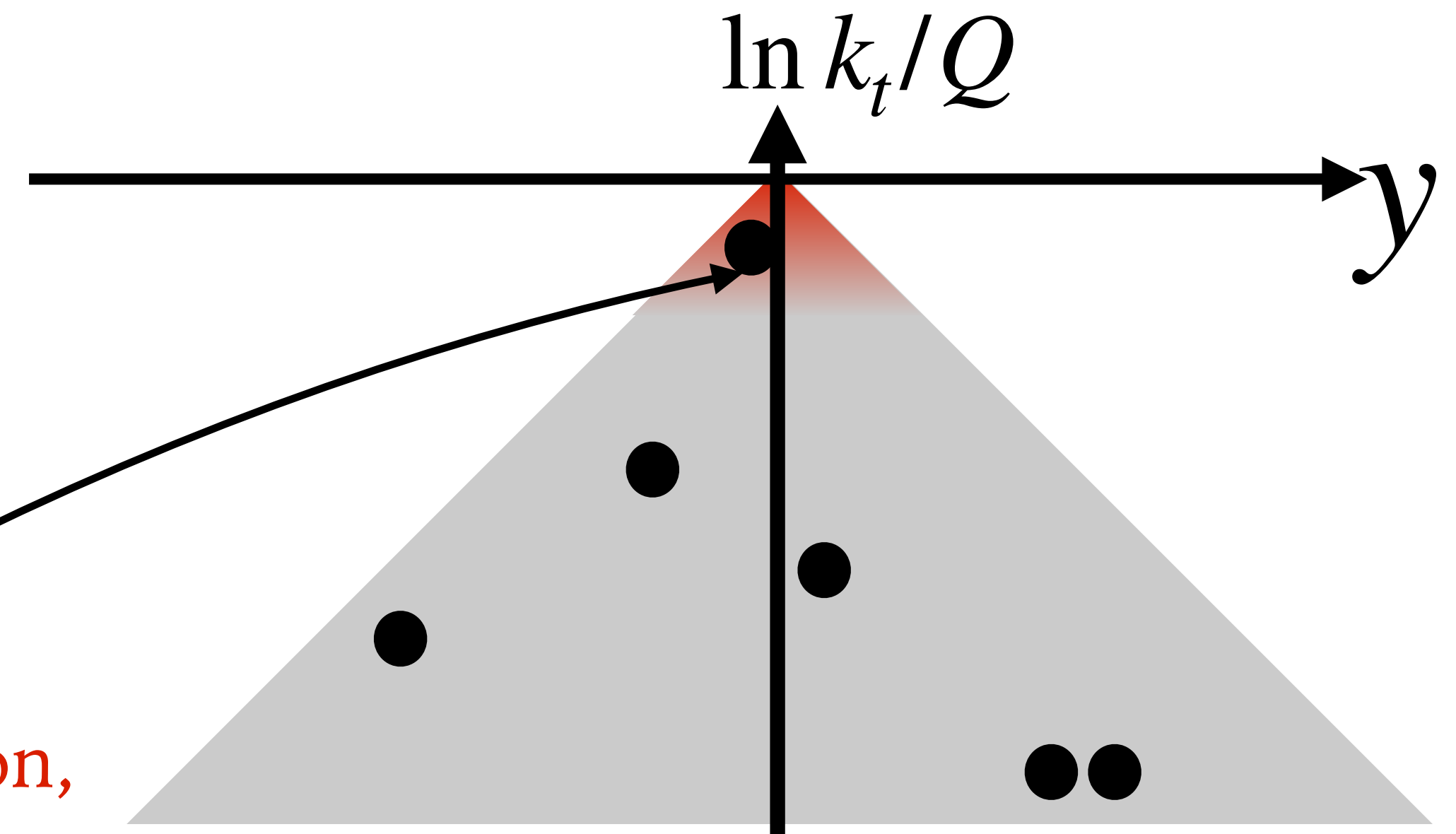
[SFR, Hamilton, Karlberg, Salam, Scyboz, Soyez [2307.11142](#)]

Focus on soft emissions

- ✓ Soft-collinear emsns at NLO
- ✓ Soft (large angle) emsns at LO
- ✓ Correct rate for pair of emsns separated only in one Lund coordinate

- ✓ **Hard** emissions at **LO**

[Hamilton, Karlberg, Scyboz, Salam, Verheyen, [2301.09645](#)]



NLL

NNLL

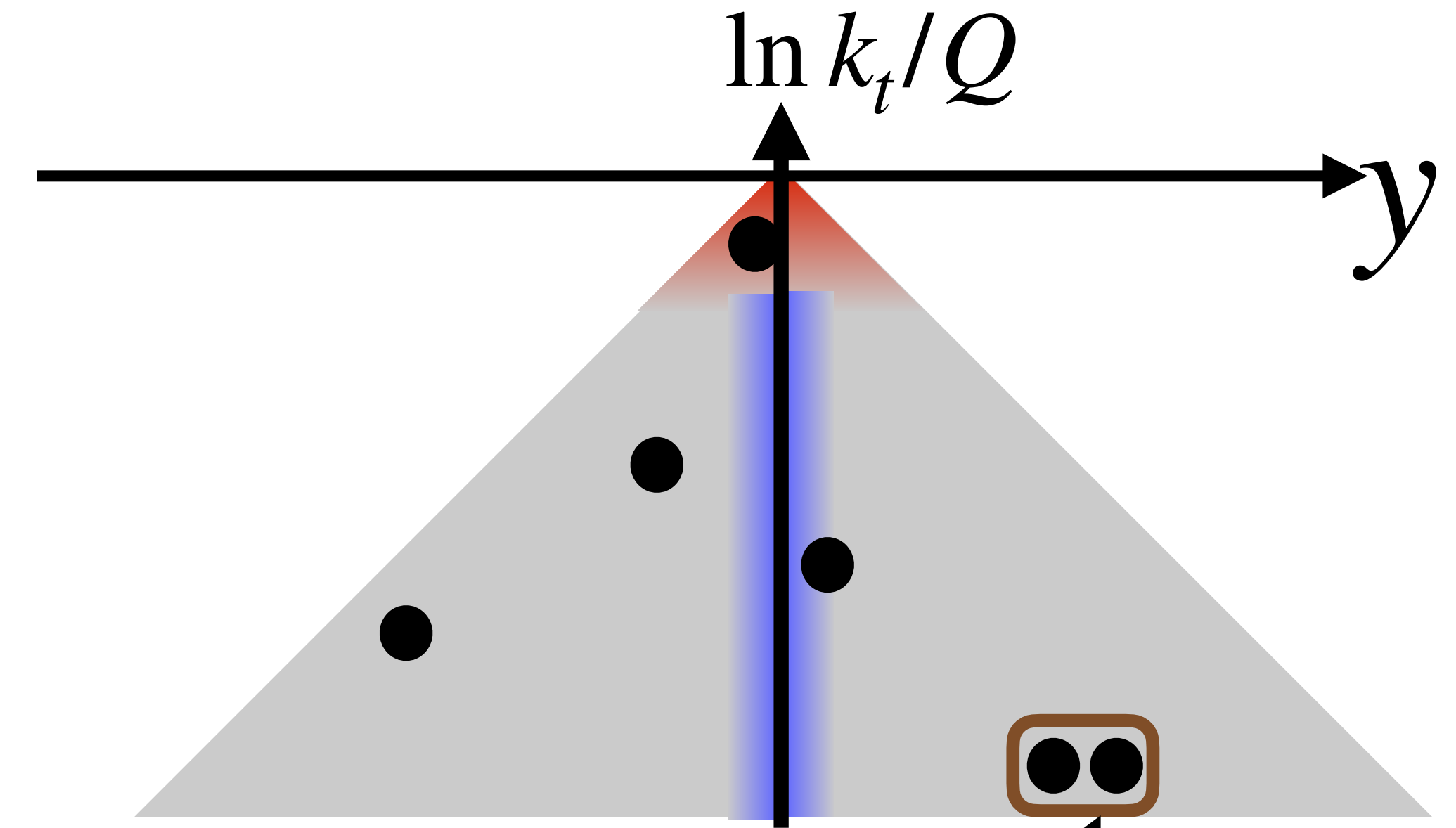
How to go beyond NLL in a parton shower?

[SFR, Hamilton, Karlberg, Salam, Scyboz, Soyez [2307.11142](#)]

Focus on soft emissions

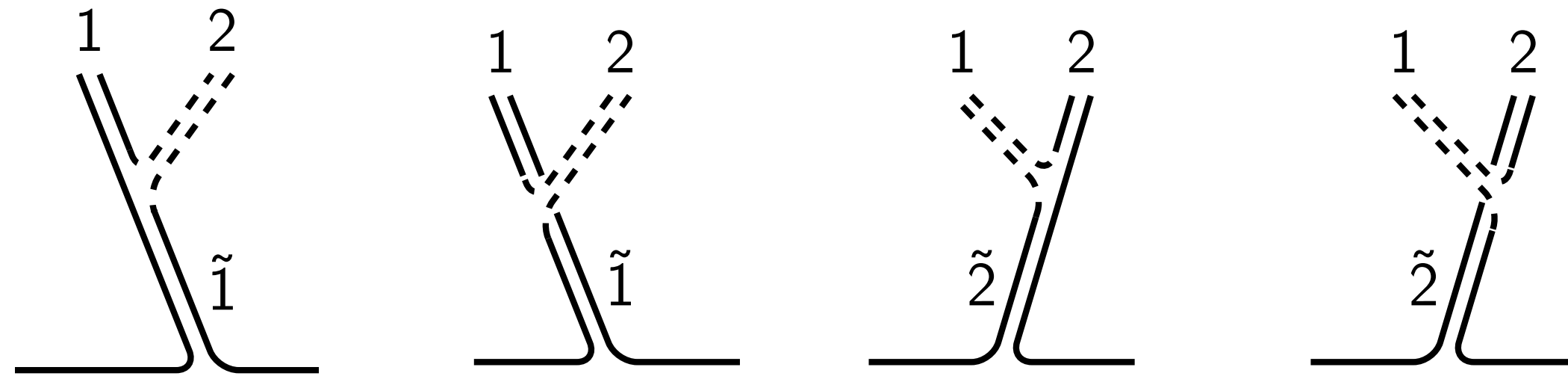
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- ✓ **Hard** emissions at LO
- ✓ Soft (large angle) emsns at NLO
- ✓ Correct rate for pair of emsns **close in the Lund plane**



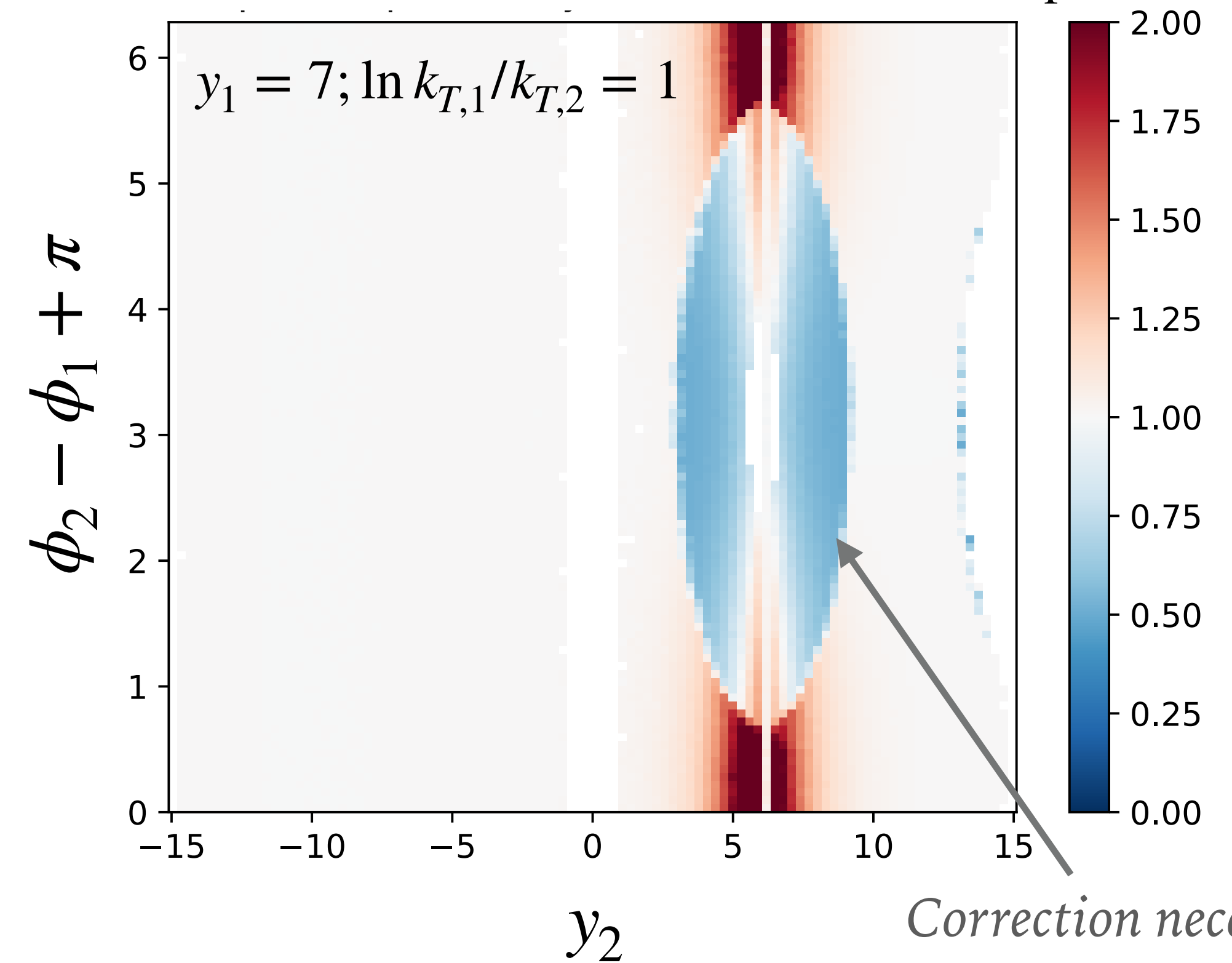
NB: Vincia and Sherpa groups have also explored inclusion of the double-soft current; novelty here is doing so to reach NNLL

Correct rate for pairs or soft emissions



- a given two-emission configuration can come from several shower histories
- **accept a given emission with exact double-soft $M_{\text{exact}}^{(\text{DS})}$ divided by shower's effective double-soft matrix element summed over the histories h that could have produced that configuration**

Double-soft acceptance P_{accept}



Correction necessary only for neighbouring emsn as the shower is already NLL

$$P_{\text{accept}} = \frac{M_{\text{exact}}^{(\text{DS})}}{\sum_h M_{h,\text{PS}}^{(\text{DS})}}$$

How to go beyond NLL in a parton shower?

[SFR, Hamilton, Karlberg, Salam, Scyboz, Soyez [2307.11142](#)]

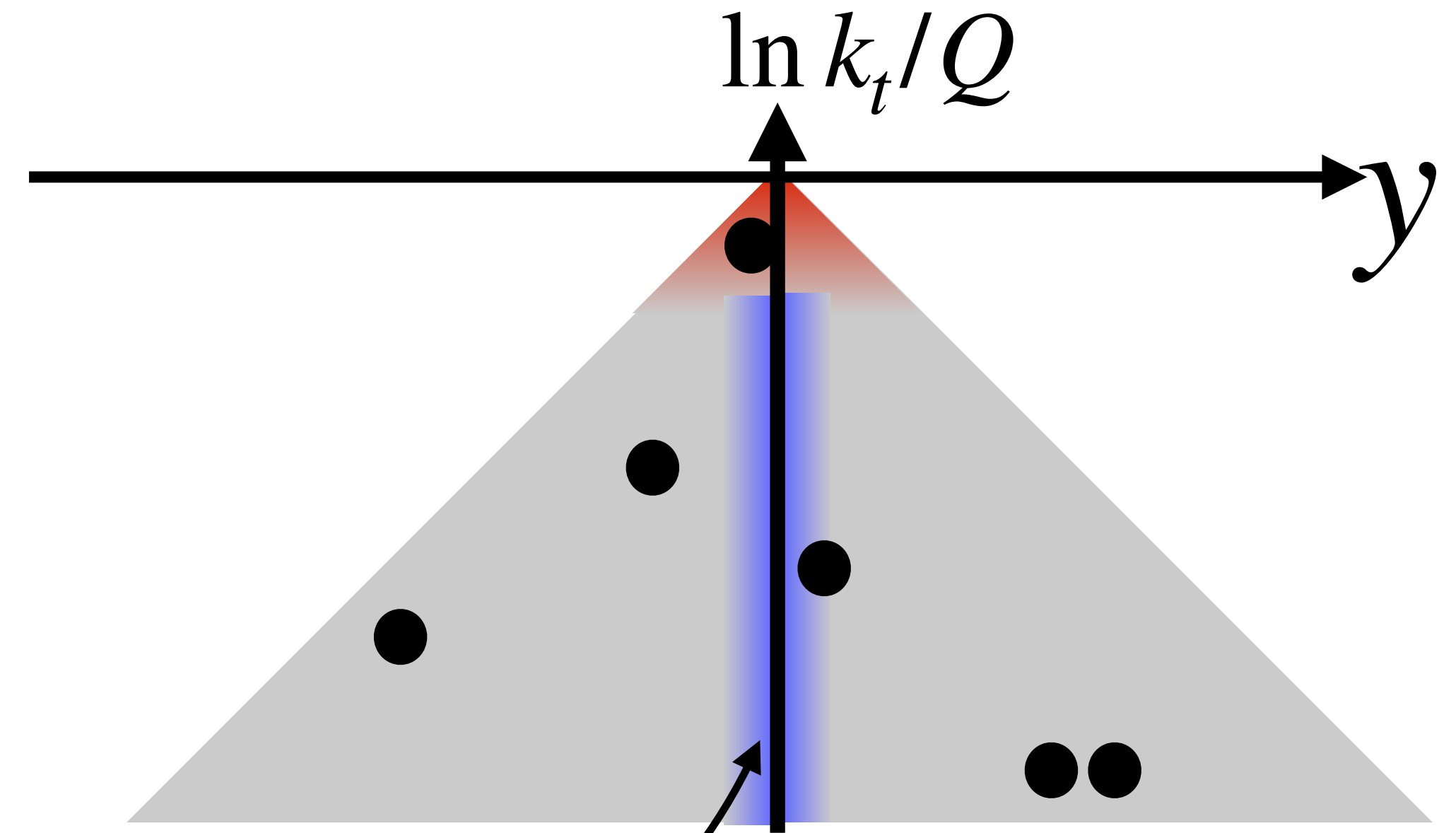
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NNLL

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Virtual corrections for soft emissions

Parton shower unitarity ensures

$$\text{V}_{\text{PS}} = \frac{\alpha_s}{2\pi} K_1 - \int \text{R} \quad \text{fixed "shower variables"}$$

but analytic resummation tells we need the integral at **fixed rapidity and k_t** !

Virtual corrections for soft emissions

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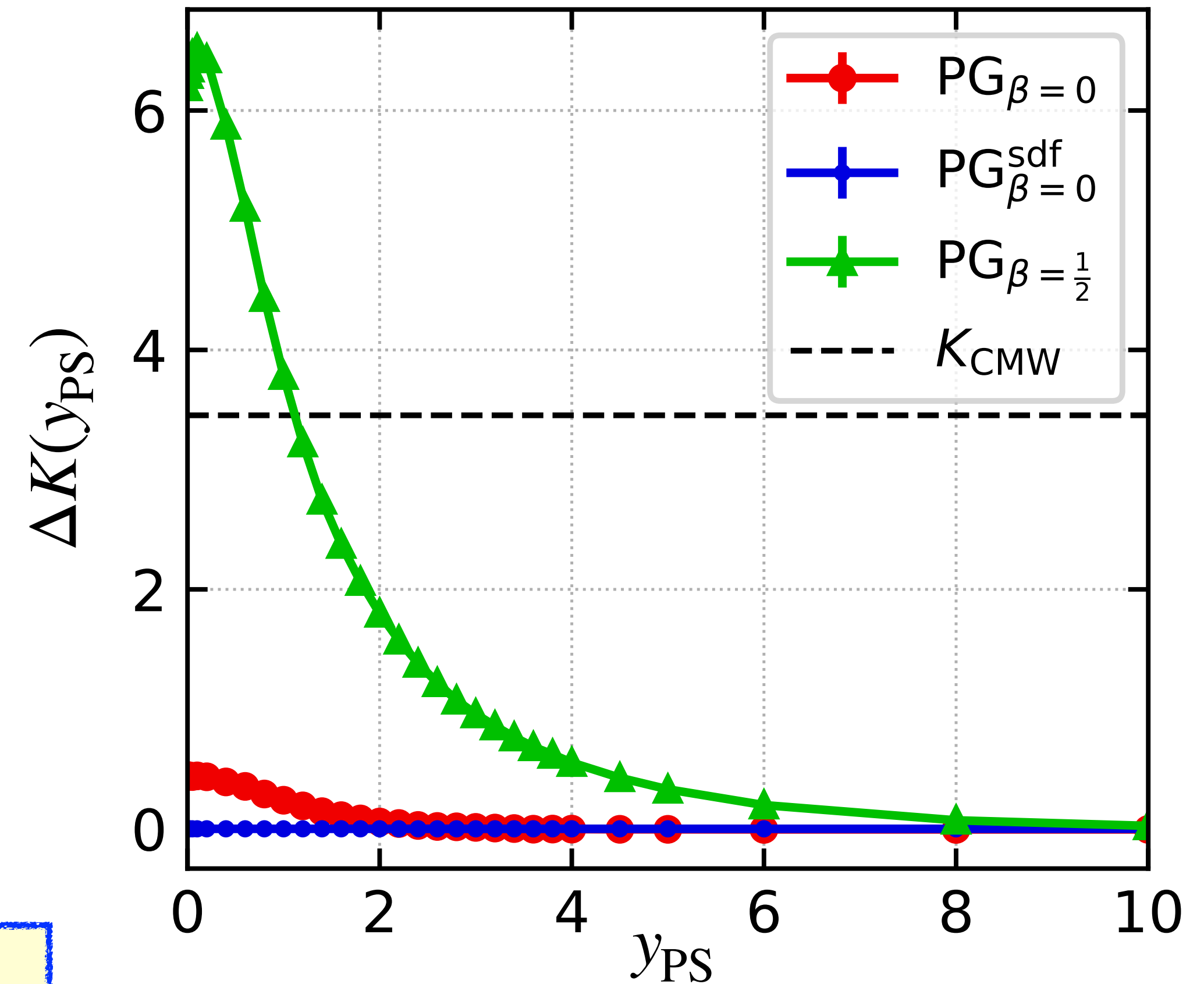
$$\underbrace{\text{Diagram with } V_{PS}}_{\text{orange cone}} = \frac{\alpha_s}{2\pi} K_1 - \int \underbrace{\text{Diagram with } R}_{\text{pink cone}} \text{ fixed "shower variables"}$$

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$$K_1^{PS} = K_1 + \Delta K(\Phi_{PS}^{(1)})$$

$$\frac{\alpha_s}{2\pi} \Delta K(\Phi_{PS}^{(1)}) = \int \underbrace{\text{Diagram with } R}_{\text{pink cone}} \text{ fixed "shower variables"} - \int \underbrace{\text{Diagram with } R}_{\text{orange cone}} \text{ } y, p_{\perp} \text{ fixed}$$

example ΔK correction



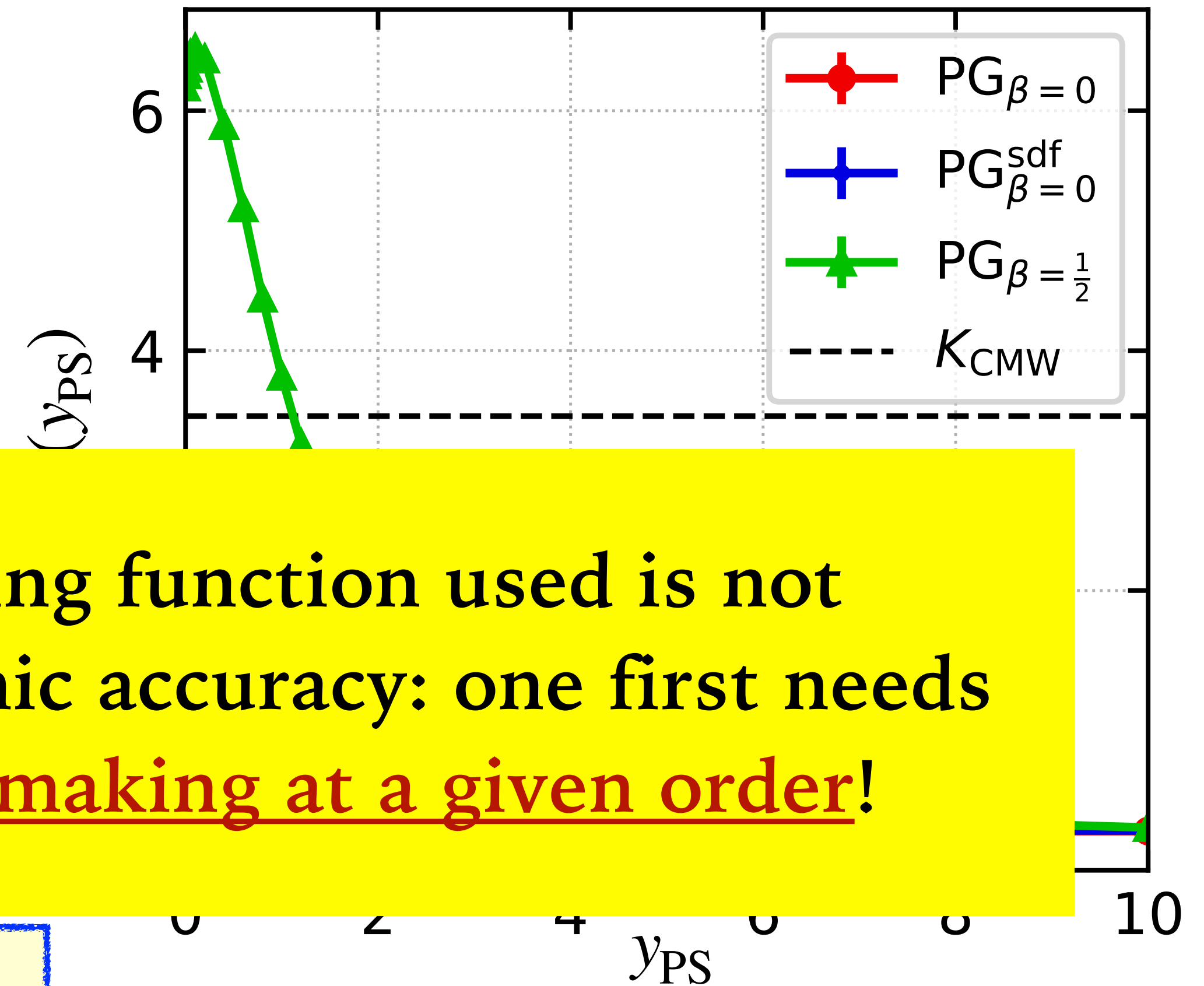
ΔK vanishes for large rapidities since virtual corrections to soft-collinear emissions are OK for NLL showers

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example ΔK correction



Augmenting the order of the splitting function used is not sufficient to achieve superior logarithmic accuracy: one first needs to remove the mistakes a shower is making at a given order!

$$\frac{\alpha_s}{2\pi} \Delta K(\Phi_{PS}^{(1)}) = \int \mathcal{R} - \int \mathcal{R} \quad \text{fixed "shower variables"}$$

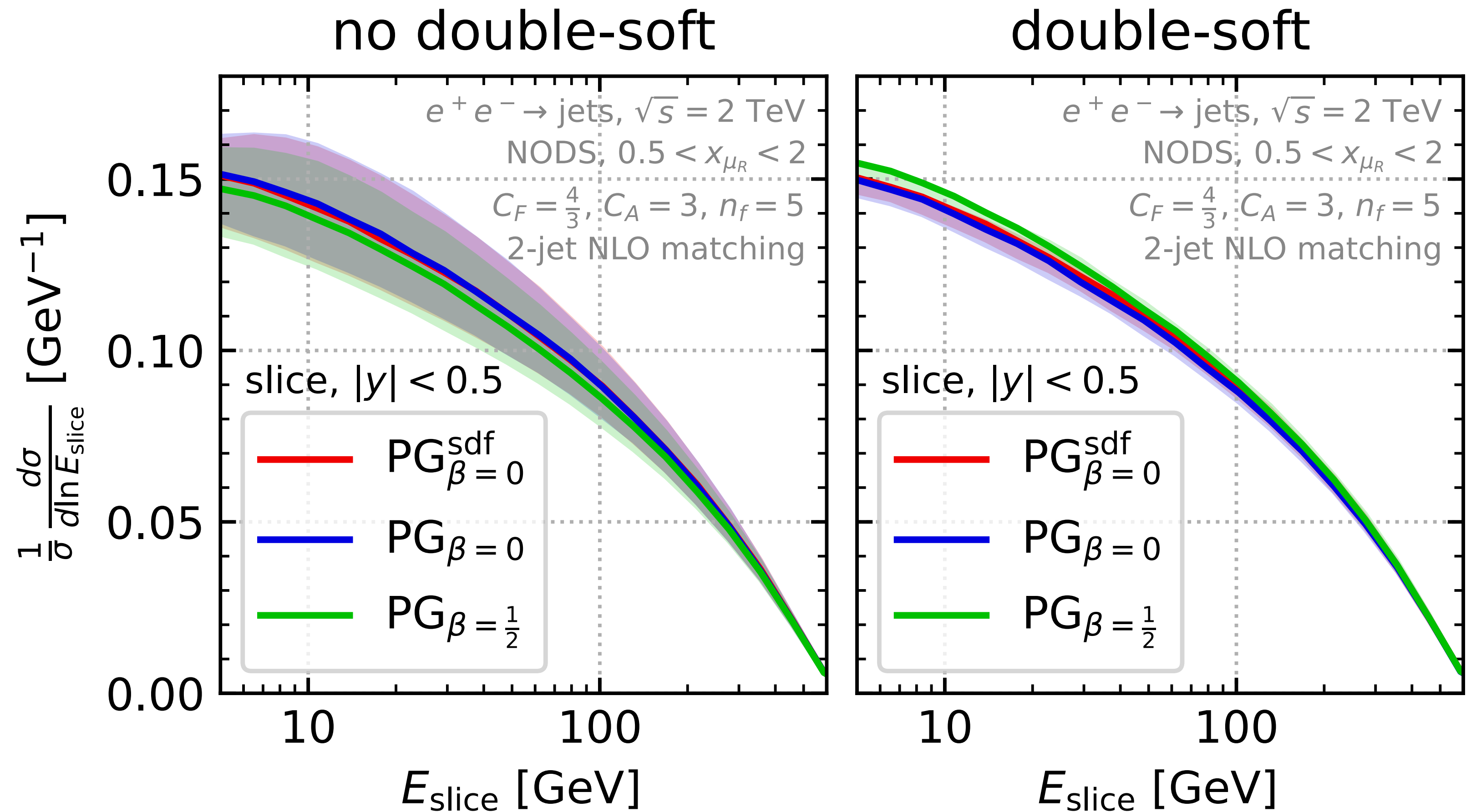
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Non-global observables at “NNLL”

S.F.R., Hamilton, Karlberg,
Salam, Scyboz, Soyez
[2307.11142](#)

- Energy flow in slice between two 1 TeV jets
- First time non-global obs is known at **Next-to-Single Logs** (at leading N_c) including the full n_f dependence
- **Double-soft reduces uncertainty band**

Uncertainty here is estimated varying the renormalisation scale



$$\alpha_s^{\text{CMW}}(k_t; x_R) = \alpha_s(x_R k_t) \left(1 + \frac{\alpha_s(x_R k_t)}{2\pi} (K_{\text{CMW}} + \Delta K(\Phi)) + 2\alpha_s(x_R k_t) b_0 (1-z) \ln x_R \right)$$

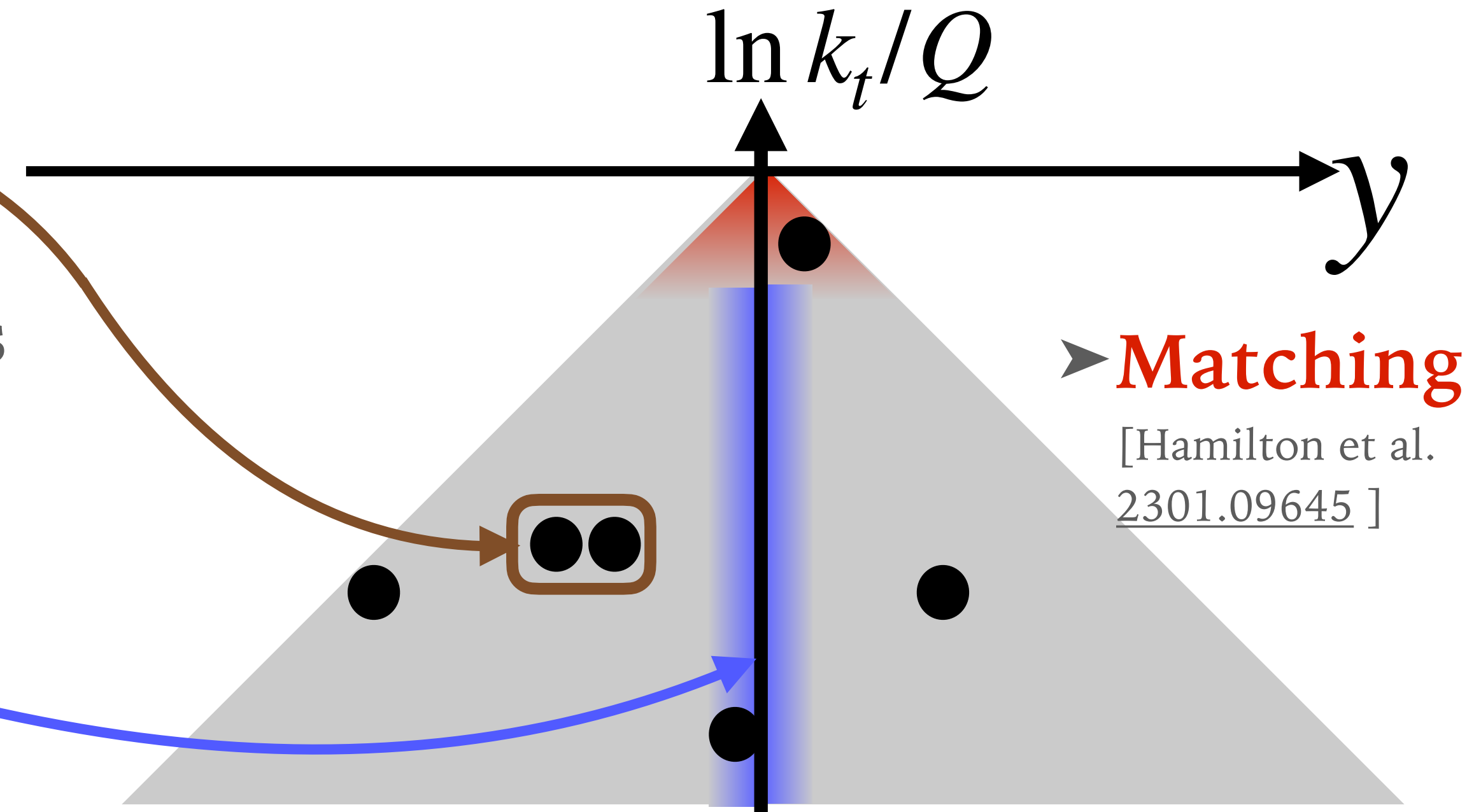
Building a NNLL shower

SFR et al, 2307.11142

- **Double-soft “reweighting”** for neighbouring soft-collinear emsns
- NLO corrections for soft, large-angle emissions

$$\alpha_s^{\text{eff}}(k_t) = \alpha_s(k_t) \left(1 + \frac{\alpha_s(k_t)}{2\pi} (K_1 + \Delta K_1) \right)$$

*Catani, Marchesini,
Webber, '91*



➤ **Matching**
[Hamilton et al.
2301.09645]

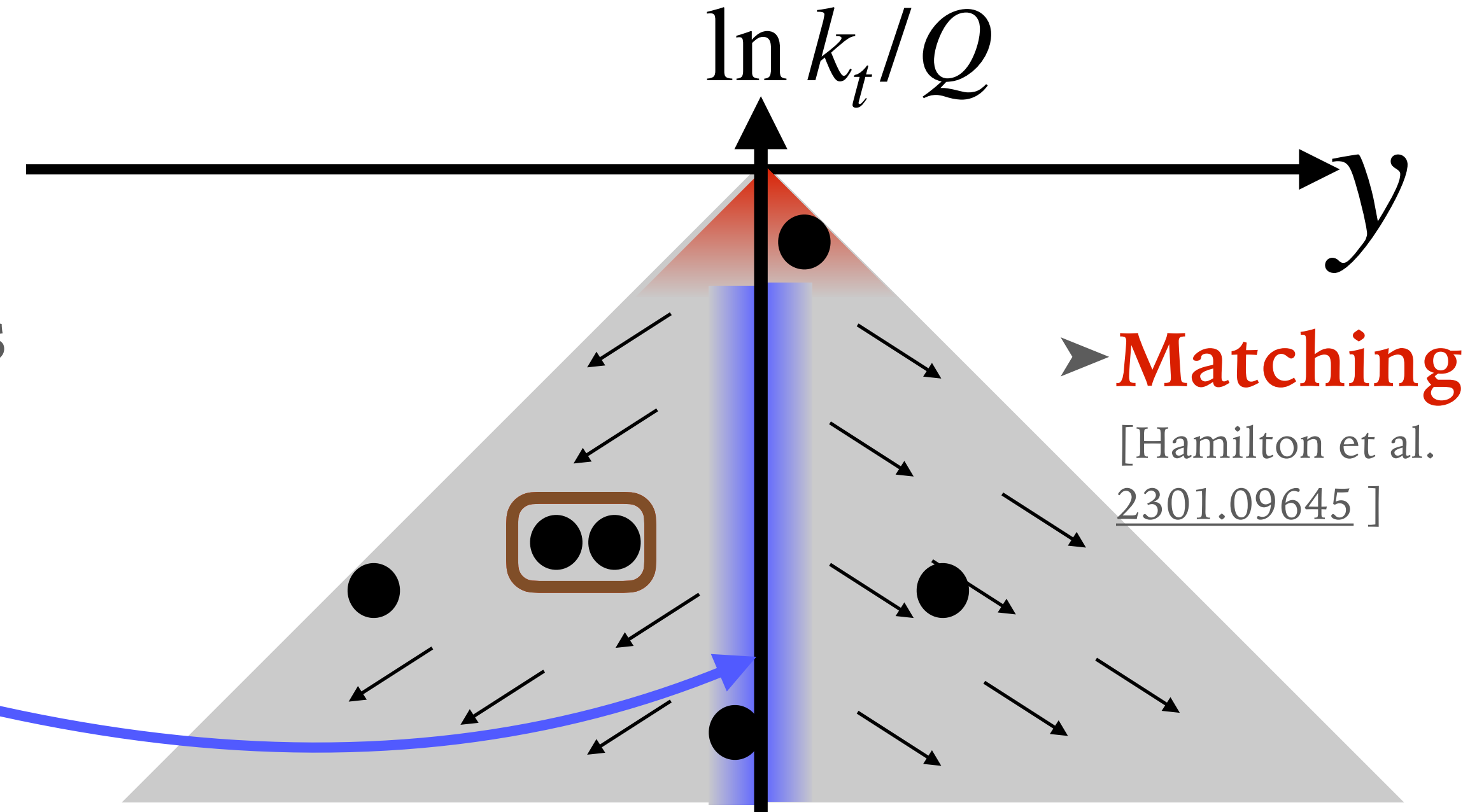
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➤ **Matching**
[Hamilton et al. 2301.09645]

Drift in rapidity of an emission when it further branches

$$\int 2C_F d\eta \Delta K_1(\eta) \propto \langle \Delta y \rangle$$

⇒ *correct the shower mistake*

Building a NNLL shower

SFR et al, 2307.11142

- ▶ **Double-soft “reweighting”** for neighbouring soft-collinear emsns

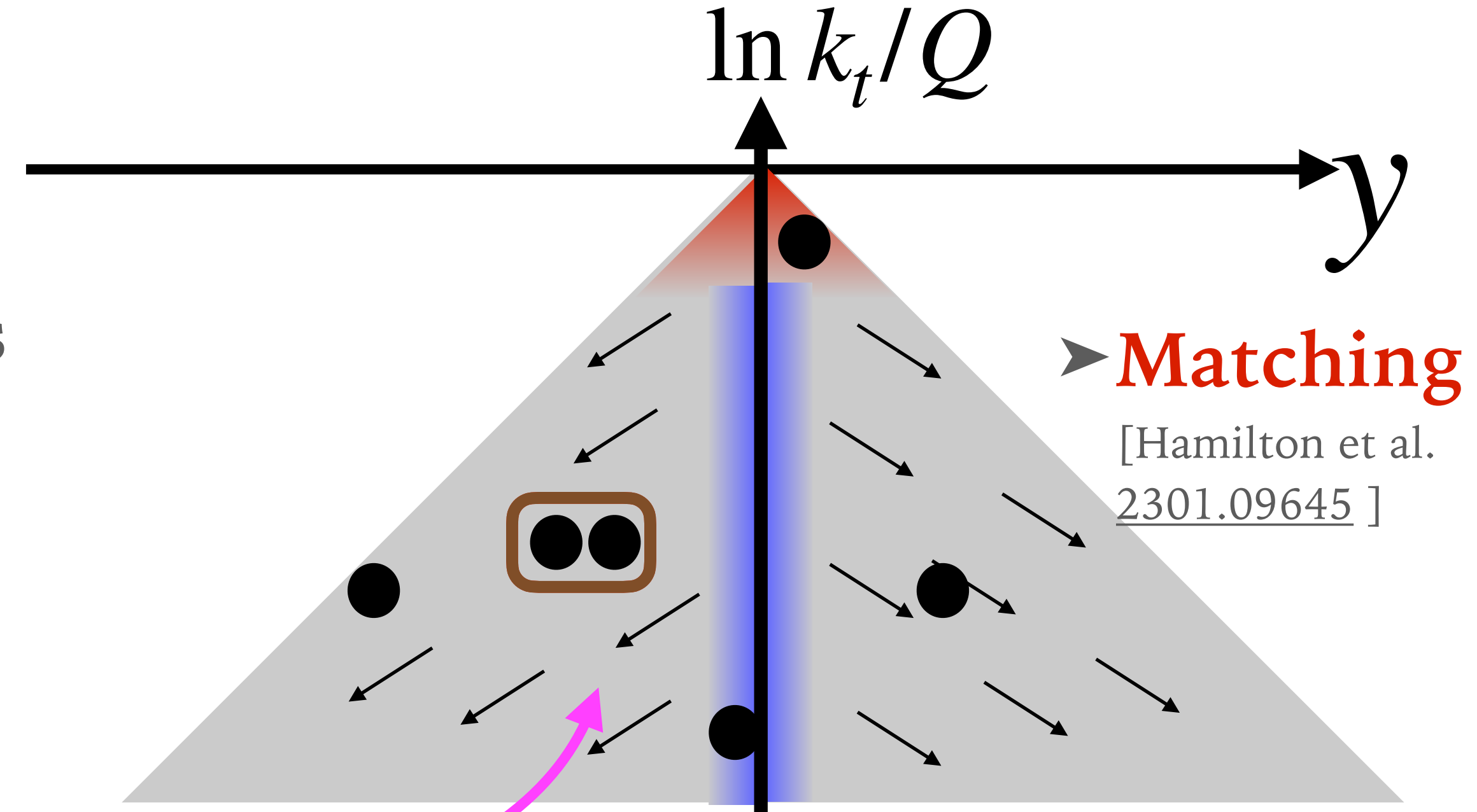
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- ▶ **NNLO corrections** for soft-collinear emsns

$$\alpha_s^{\text{eff}}(k_t) = \alpha_s(k_t) \left(\dots + \frac{\alpha_s^2(k_t)}{4\pi^2} (K_2 + \Delta K_2) \right)$$

Banfi, El-Menoufi,
Monni, 1807.11487



▶ **Matching**

[Hamilton et al.
2301.09645]

Drift in $\ln k_t$ of an emission when it further branches

$$\Delta K_2 \propto \beta_0 \langle \Delta \ln k_t \rangle$$

⇒ **correct the shower mistake**

At this accuracy, it is sufficient to get the average

Building a NNLL shower

SFR et al, 2307.11142

- **Double-soft “reweighting”** for neighbouring soft-collinear emsns

- NLO corrections for soft, large-angle emissions

$$\alpha_s^{\text{eff}}(k_t) = \alpha_s(k_t) \left(1 + \frac{\alpha_s(k_t)}{2\pi} (K_1 + \Delta K_1) \right)$$

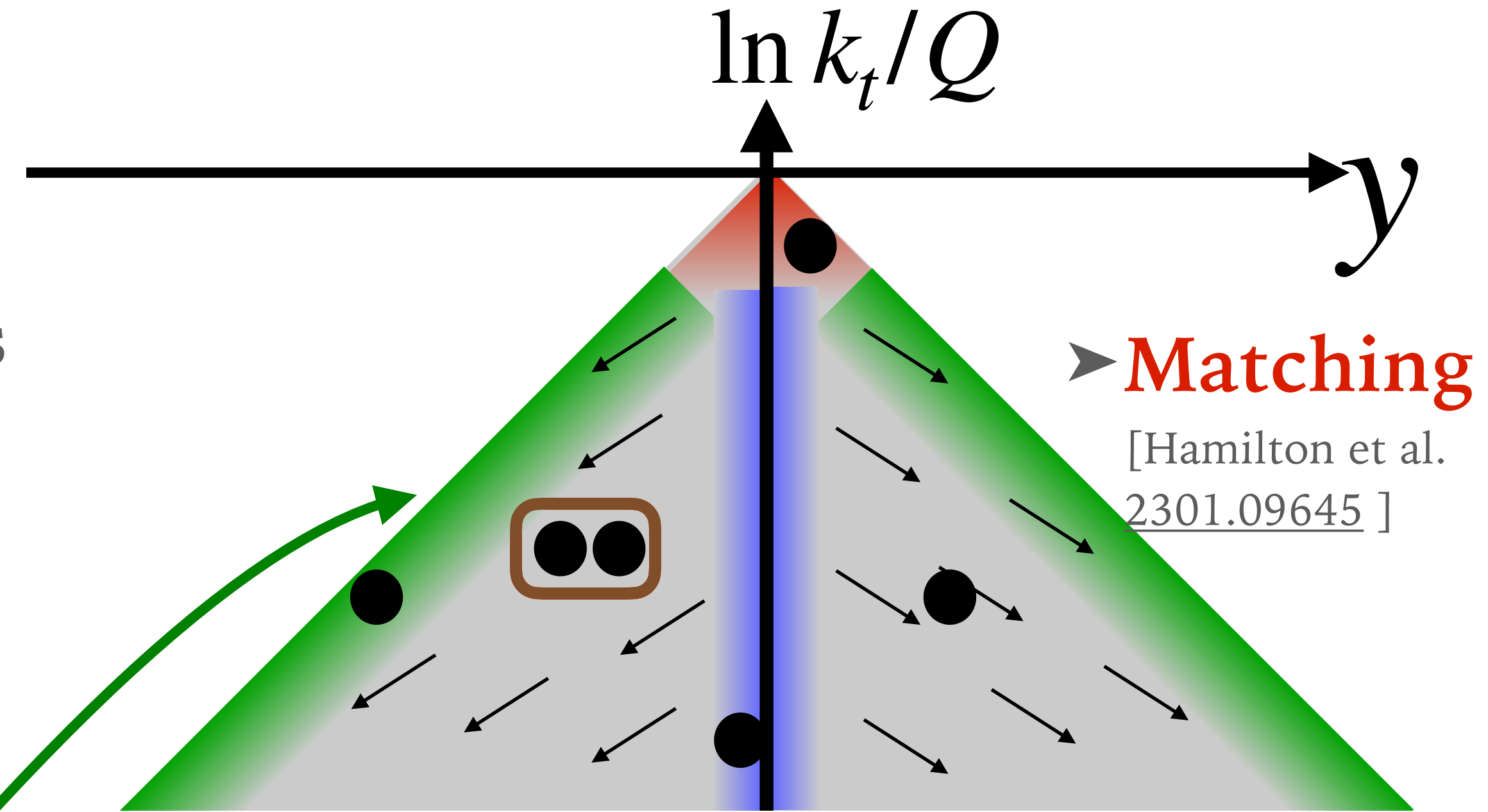
- **NNLO corrections** for soft-collinear emsns

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- **NLO corrections** for collinear emsns

$$d\mathcal{P}_{\text{coll}} \propto P(z) \left(1 + \frac{\alpha_s}{2\pi} (B_2(z) + \Delta B_2(z)) \right)$$

Dasgupta, El-Menoufi 2109.07496,
+ van Beekveld, Helliwell, Monni 2307.15734,
++ Karlberg 2402.05170



Drift in $\ln z = \ln k_t + y$ of an emission when it further branches

$$\int P(z) dz \Delta B_2(z) \propto - \langle \Delta z \rangle$$

⇒ **correct the shower mistake**

At this accuracy, it is sufficient to get the integral right, not the functional form of $\Delta B_2(z)$

A new standard for the logarithmic accuracy of parton showers

Melissa van Beekveld,¹ Mrinal Dasgupta,² Basem Kamal El-Menoufi,³ Silvia Ferrario Ravasio,⁴ Keith Hamilton,⁵ Jack Helliwell,⁶ Alexander Karlberg,⁴ Pier Francesco Monni,⁴ Gavin P. Salam,^{6,7} Ludovic Scyboz,³ Alba Soto-Ontoso,⁴ and Gregory Soyez⁸

We report on a major milestone in the construction of logarithmically accurate final-state parton showers, achieving next-to-next-to-leading-logarithmic (NNLL) accuracy for the wide class of observables known as event shapes. The key to this advance lies in the identification of the relation between critical NNLL analytic resummation ingredients and their parton-shower counterparts. Our analytic discussion is supplemented with numerical tests of the logarithmic accuracy of three shower variants for more than a dozen distinct event-shape observables in two final states. The NNLL terms are phenomenologically sizeable, as illustrated in comparisons to data.

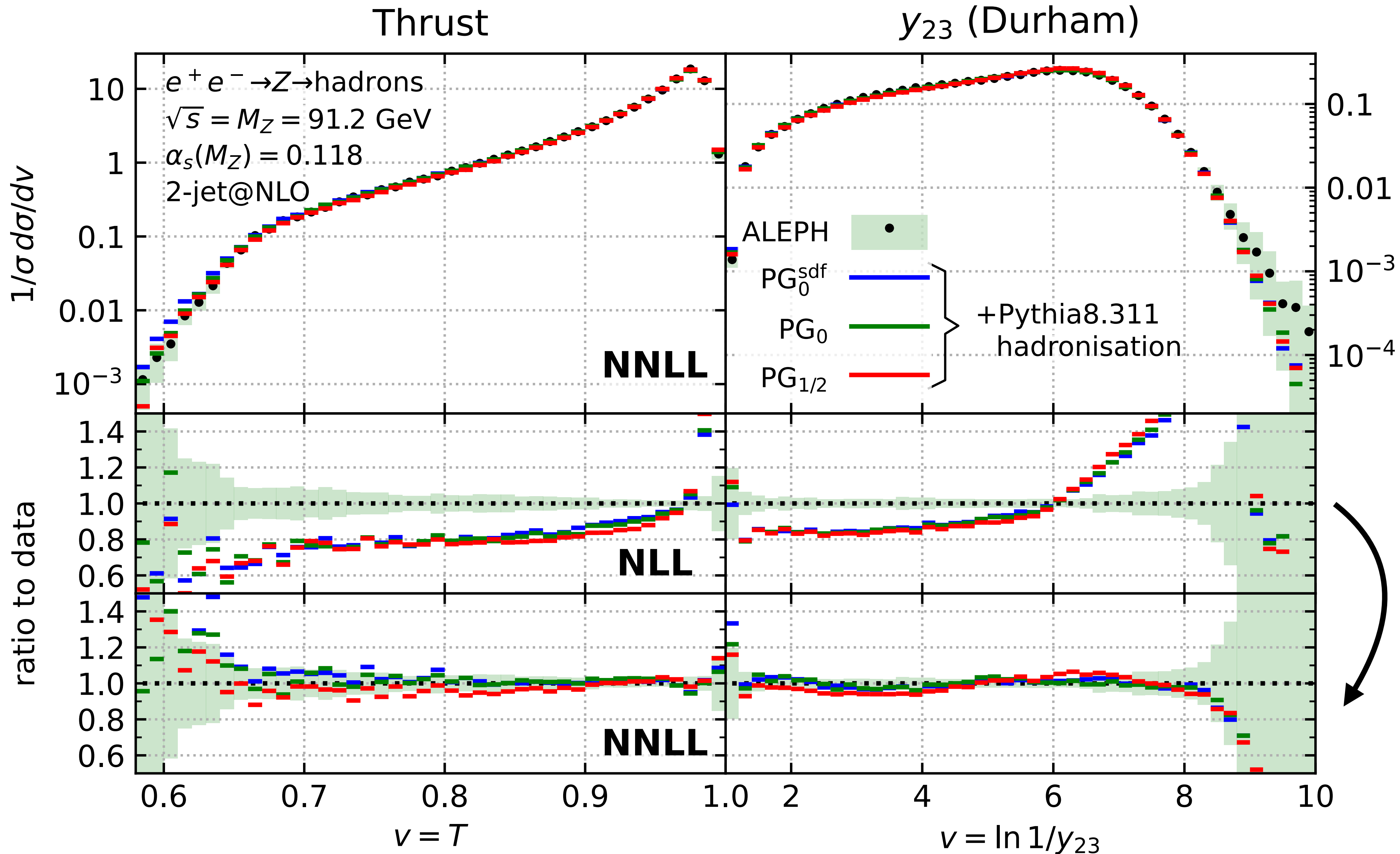
Dasgupta, El-Menoufi 2109.07496,
+ van Beekveld, Helliwell, Monni 2307.15734,
++ Karlberg 2402.05170

2406.02661

to get the integral
the junctional form of $\Delta B_2(z)$

NNLL showers vs NLL showers: pheno outlook

*The PanScales
collaboration,
[2406.02661](https://arxiv.org/abs/2406.02661)*



Agreement to
data
substantially
better when
 using **NNLL**
 showers

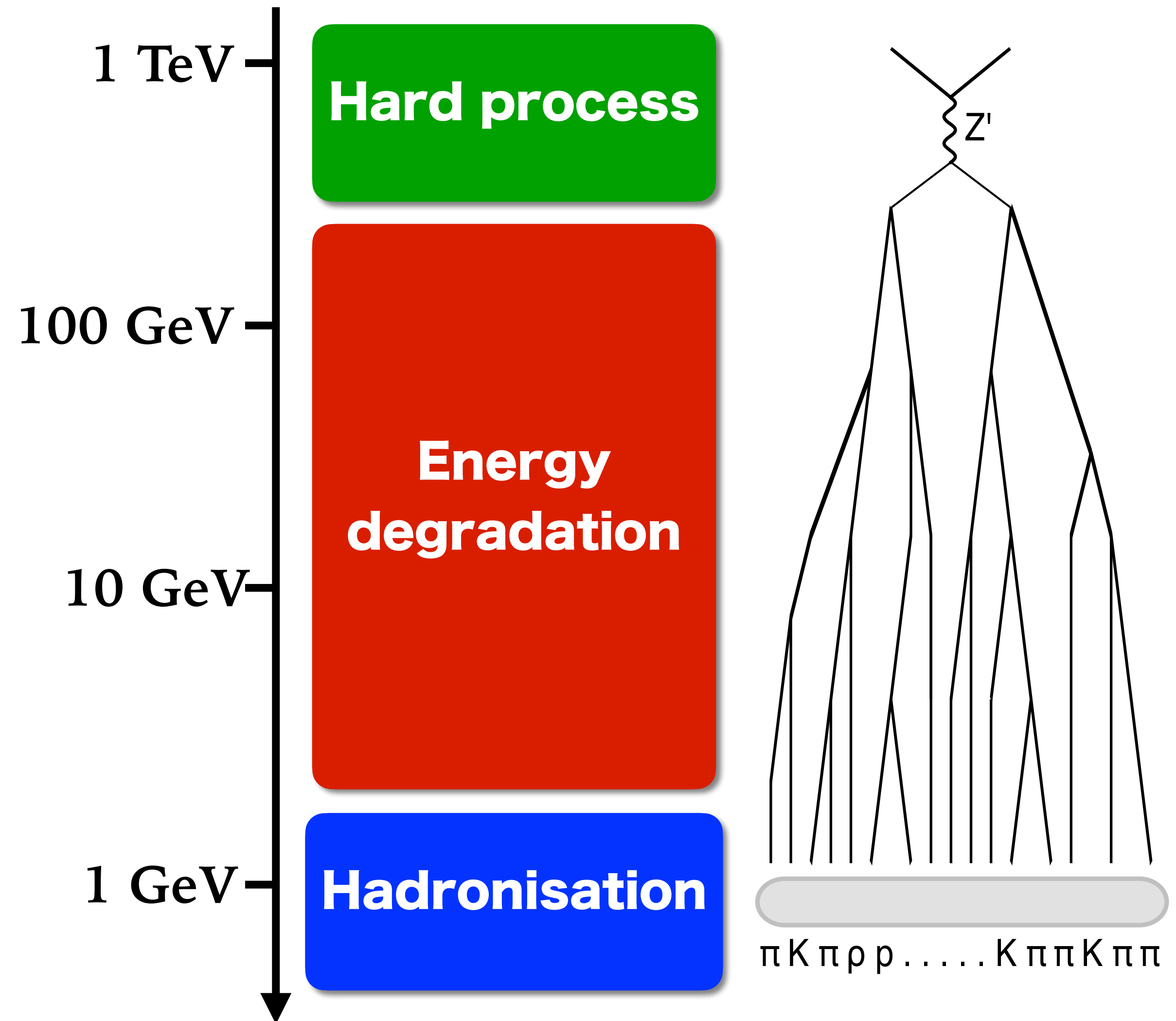
Conclusions

- Collider phenomenology critically relies on **Parton Showers**
- Parton Showers can be considered highly flexible **resummation tools**
- The logarithmic accuracy of **analytic resummations** is (almost always) ahead of the parton shower one: precious input to boost current showers
- Recent **paradigm shift** in the shower community with many **NLL showers** appearing
- Current focus: getting NLL showers ready for **phenomenology** (masses, matching, generic processes), as well as going **beyond NLL**
- Combining **fixed order and logarithmic accuracy** is still an open issue
- QCD showers the current bottleneck at the LHC, but let's not forget about **QED and EW**, relevant for precise EW measurements and the FCC
- In this talk I focused on the semi-classical approach, but **amplitude-level evolution** (CVolver [Forshaw, Plätzer, Sjö Dahl, Holguin + ...], and Deductor [Nagy, Soper]) will certainly offer advantages in terms of colour and spin handling, so stay tuned!

Backup

Shower Monte Carlo event generators

SHOWER MONTE CARLO EVENT GENERATORS = default tool for interpreting collider data

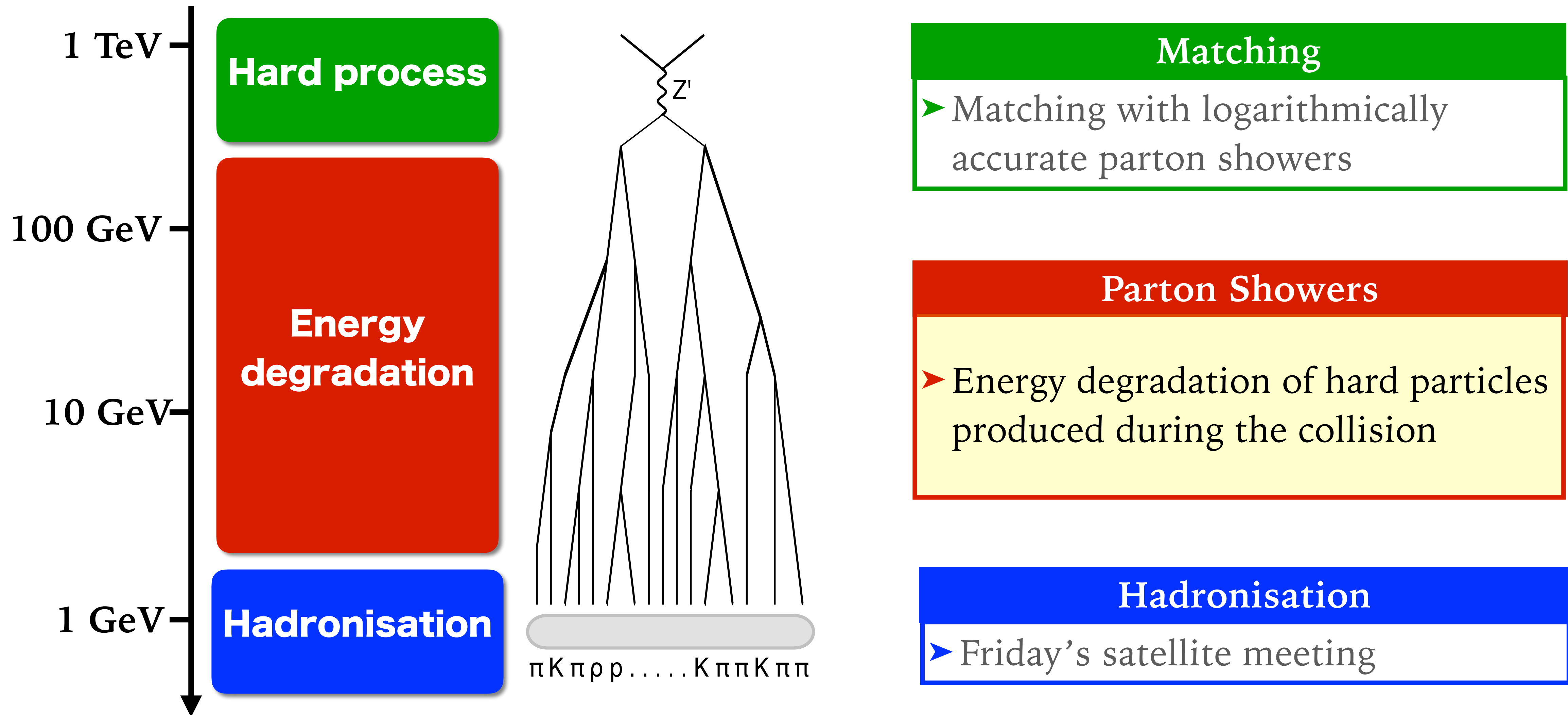


Parton Showers

► Energy degradation of hard particles produced during the collision

Shower Monte Carlo event generators

SHOWER MONTE CARLO EVENT GENERATORS = default tool for interpreting collider data



Logarithmic accuracy beyond QCD

- ▶ The **angular-ordering of QCD emissions** ensures that also the **soft** limit is correct, and hence NLL accuracy is achieved
- ▶ For **QED** and **EW**, the parton branching formalism ensures **only collinear** (and soft-collinear) logs are resummed: only **LL accuracy** is expected

- ◉ **QCD**: $\alpha_s \sim 0.1$, $\alpha_s L = \mathcal{O}(1)$ $\Sigma = \exp(Lg_{LL}(\alpha_s L) + g_{NLL}(\alpha_s L) + \dots)$

- ◉ **QED**: $\alpha_{em} \sim 0.01$, $\alpha_{em} L^2 = \mathcal{O}(1)$ $\Sigma = f_{DL}(\alpha_{em} L^2) + \sqrt{\alpha_{em}} f_{NDL}(\alpha_{em} L^2) + \dots$ (DL = double logs)

Only collinear ones are included, not soft ones: few % mistake for processes without QCD; necessary (but not sufficient) e.g. for the FCC-ee

Log Accuracy of the Angular-Ordered **parton** shower

- **Angular-ordering** = algorithmic implementation of the **QCD** coherent branching formalism, used for **NLL** calculations for **global observables** (event shapes, many kinematic distributions e.g. $p_{\perp,Z}$) [Marchesini, Webber '88; Gieseke, Stephens, Webber [hep-ph/0310083](https://arxiv.org/abs/hep-ph/0310083)]

Logarithmic accuracy beyond QCD

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 - ◉ **QED**: $\alpha_{em} \sim 0.01$, $\alpha_{em} L^2 = \mathcal{O}(1)$ $\Sigma = f_{DL}(\alpha_{em} L^2) + \sqrt{\alpha_{em}} f_{NDL}(\alpha_{em} L^2) + \dots$ (DL = double logs)

QED and EW logs in other SMC tools

- **SHERPA**: soft QED logs implemented with the **YFS formalism** [Krauss, Price, Schönherr, [2203.10948](#)]; one-loop virtual **EW Sudakov Logs** [Bothmann, Napoletano [2006.14635](#)]
- **PHOTOS**: [Barberio, Was '94] default tool used in experiments, based on YFS, runned after the SMC simulation for QED FSR off leptons