SCET AND THE GLAUBER SERIES

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European Research Council

AdG EFT4jets

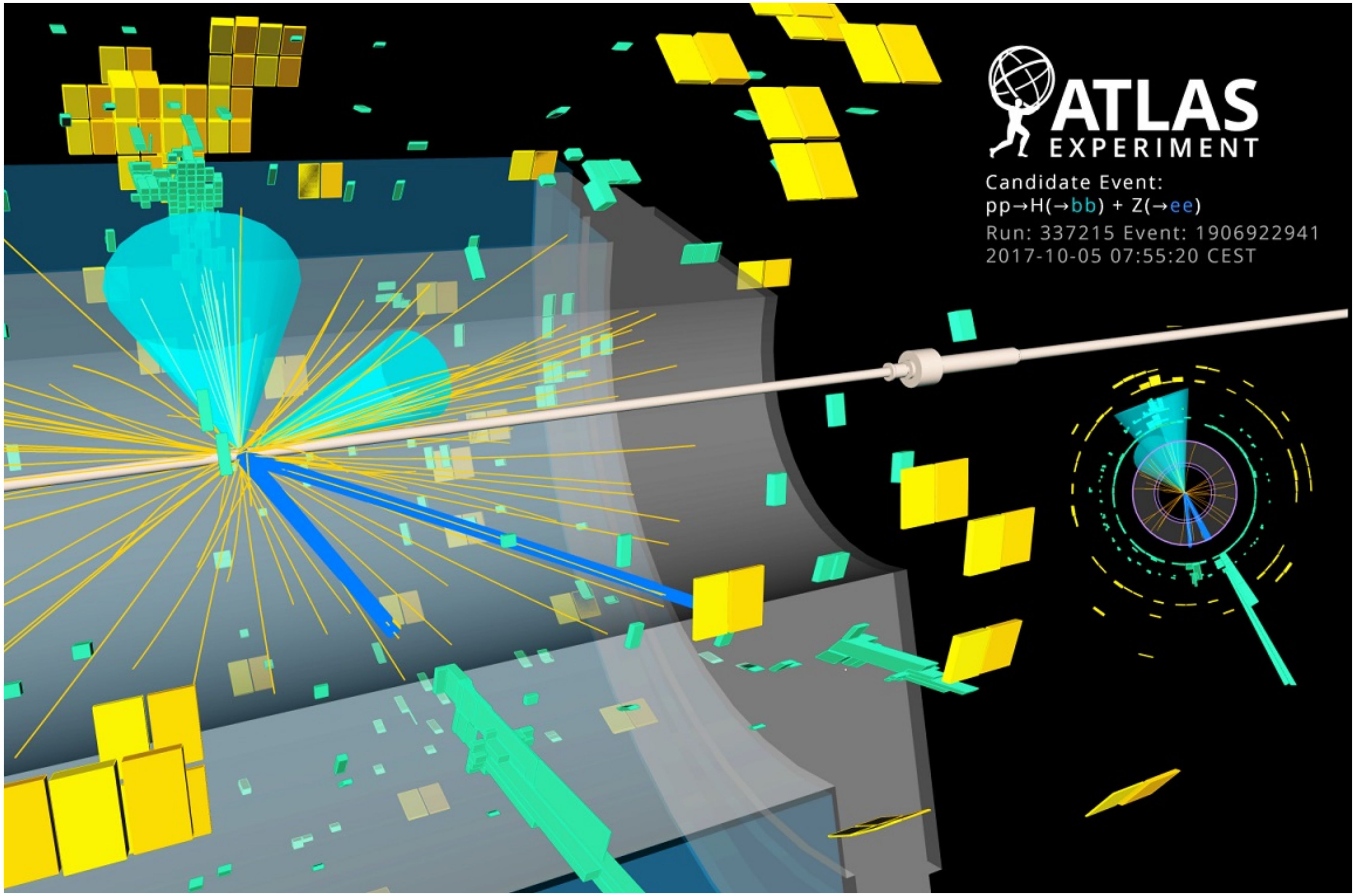
PARTON SHOWER RESUMMATION WORKSHOP | UNIVERSITÄT GRAZ — 3 JULY 2024

based on:

T. Becher, MN, D. Shao [2107.01212]; T. Becher, MN, D. Shao, M. Stillger [2307.06359]

P. Böer, MN, M. Stillger [2307.11089]; P. Böer, P. Hager, MN, M. Stillger, X. Xu [2311.18811, 2405.05305, 2407.01691]

LARGE LOGARITHMS IN LHC JET PROCESSES



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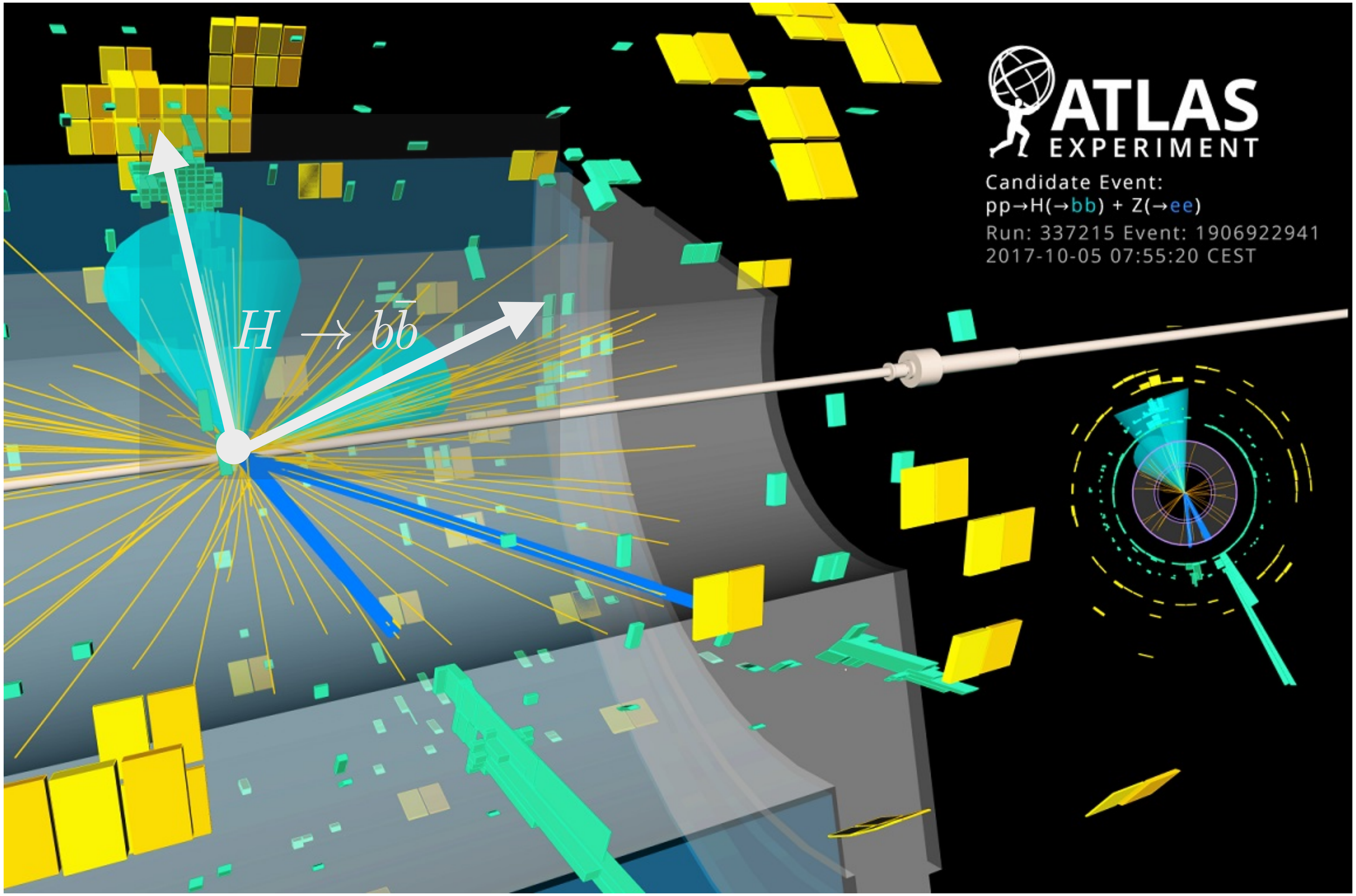
LARGE LOGARITHMS IN LHC JET PROCESSES



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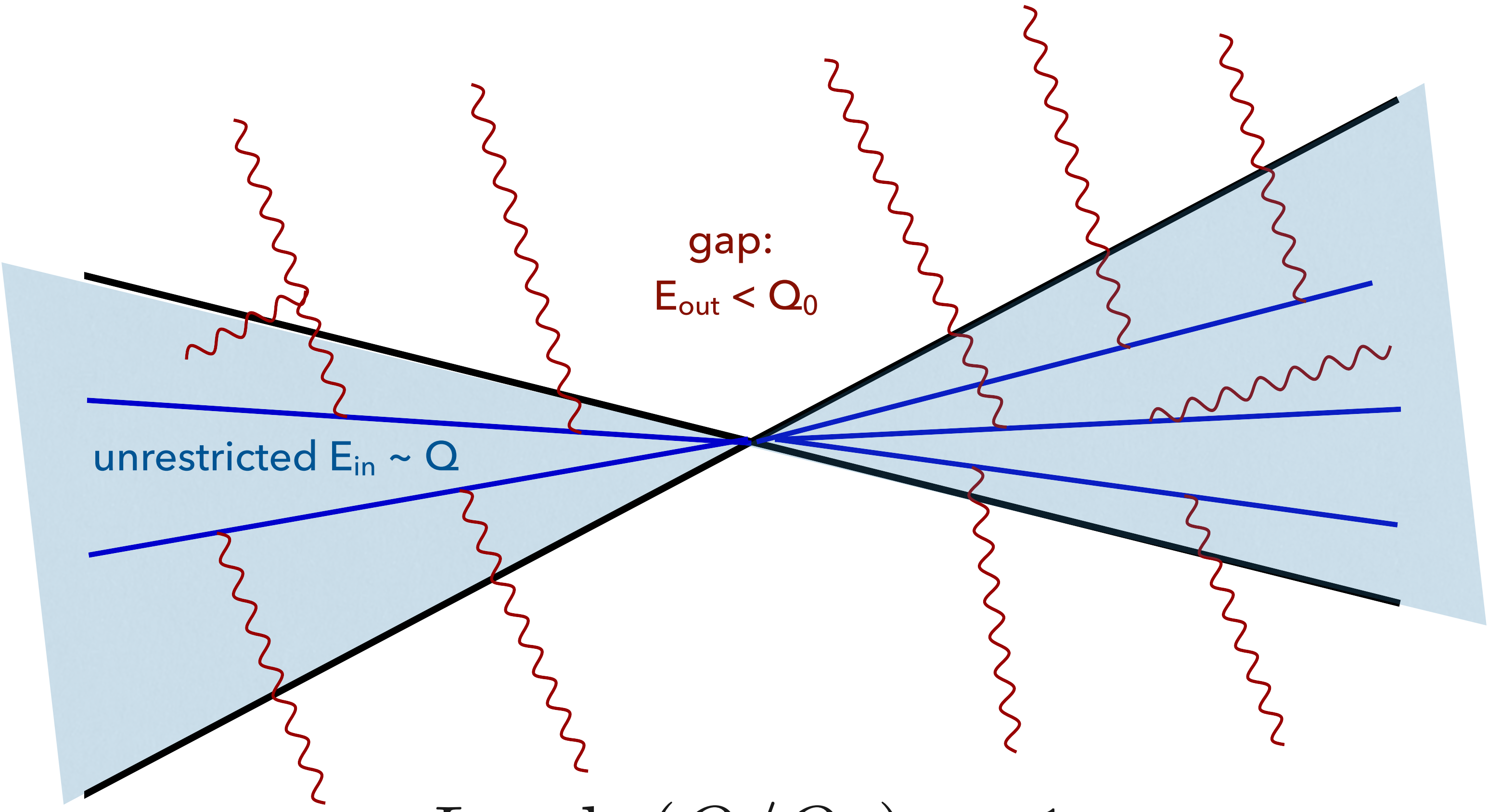
LARGE LOGARITHMS IN LHC JET PROCESSES



$$L = \ln(Q/Q_0) \gg 1$$

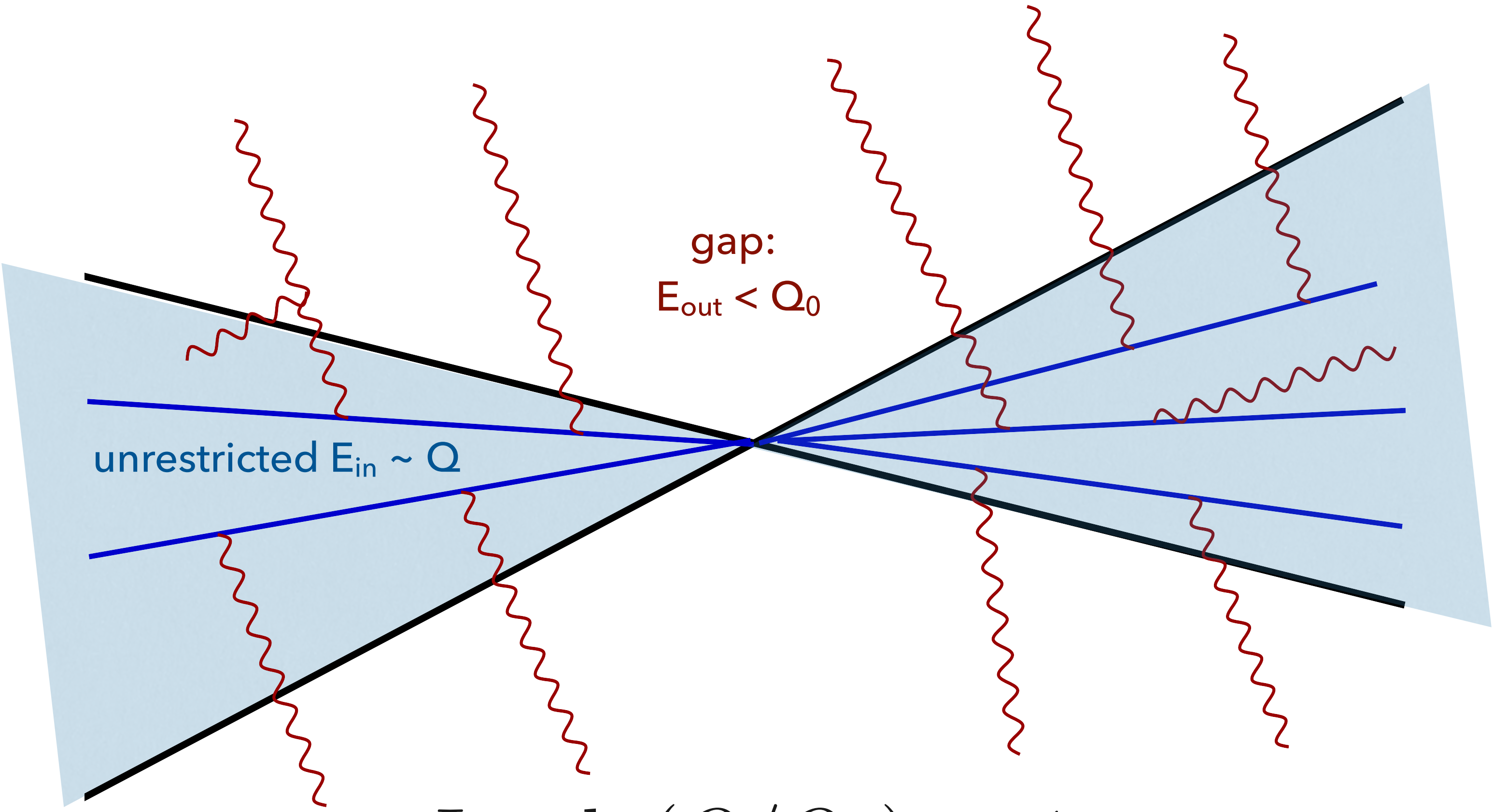
Perturbative expansion includes "super-leading" logarithms:

$$\sigma \sim \sigma_{\mathrm{Born}} \times \{1 + \alpha_s L + \underset{\uparrow}{\alpha_s^2} L^2 + \alpha_s^3 L^3 + \underbrace{\alpha_s^4 \, L^5 + \alpha_s^5 \, L^7 + \dots}_{\text{formally larger than } O(1)} \}$$

state-of-the-art

J. R. Forshaw, A. Kyrieleis, M. H. Seymour (2006)

LARGE LOGARITHMS IN LHC JET PROCESSES



$$L = \ln(Q/Q_0) \gg 1$$

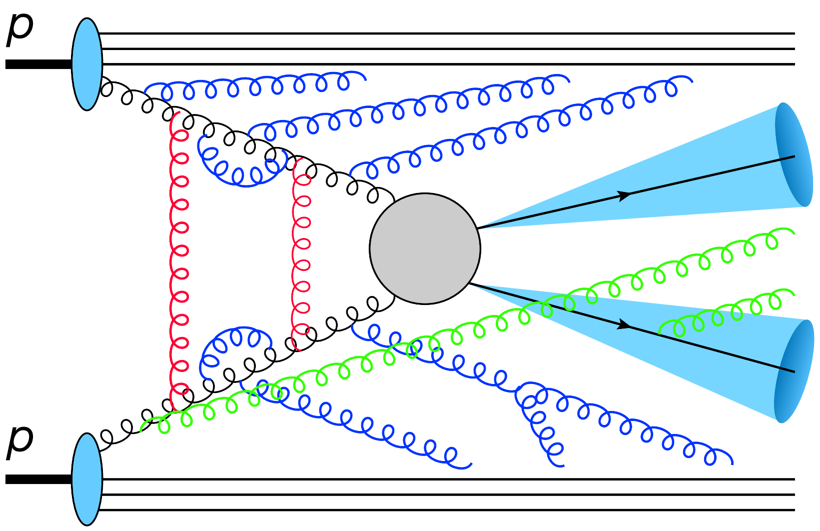
Really, a double logarithmic series starting at 3-loop order:

$$\sigma \sim \sigma_{\text{Born}} \times \left\{1 + \alpha_s L + \alpha_s^2 L^2 + \boxed{(\alpha_s \underset{\uparrow}{\pi^2}) \underbrace{\left[\alpha_s^2 L^3 + \alpha_s^3 L^5 + \dots\right]}_{\text{formally larger than } O(1)}}\right\}$$

$$(\Im\text{m}\, L)^2$$

GLAUBER PHASES BREAK COLOR COHERENCE

Super-leading logarithms



- Breakdown of color coherence due to initial-state soft gluon (Glauber) exchange

J. R. Forshaw, A. Kyrieleis, M. H. Seymour (2006)
S. Catani, D. de Florian, G. Rodrigo (2011); J. R. Forshaw, M. H. Seymour , A. Siodmok (2012)

- Soft anomalous dimension:

$$\boldsymbol{\Gamma}(\{\underline{p}\},\mu) = \sum_{(ij)} \frac{\boldsymbol{T}_i \cdot \boldsymbol{T}_j}{2}\, \gamma_{\rm cusp}(\alpha_s)\, \ln\frac{\mu^2}{-s_{ij}} + \sum_i \gamma^i(\alpha_s) + \mathcal{O}(\alpha_s^3)$$

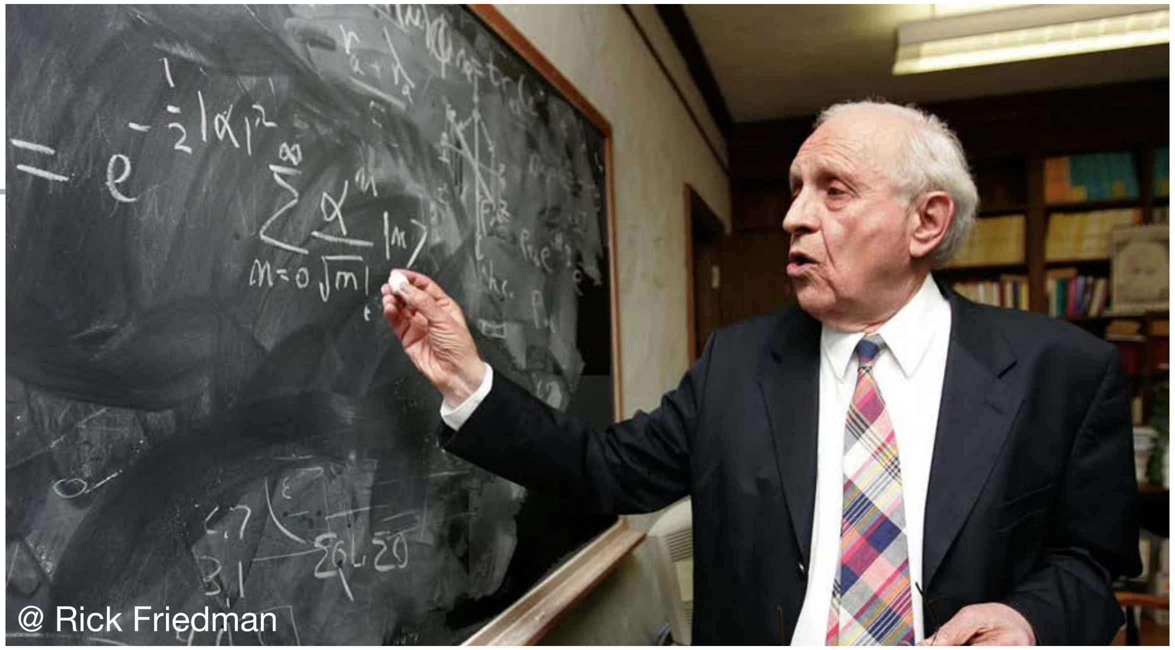
T. Becher, M. Neubert (2009)

where $s_{ij} > 0$ if particles i and j are both in initial or final state

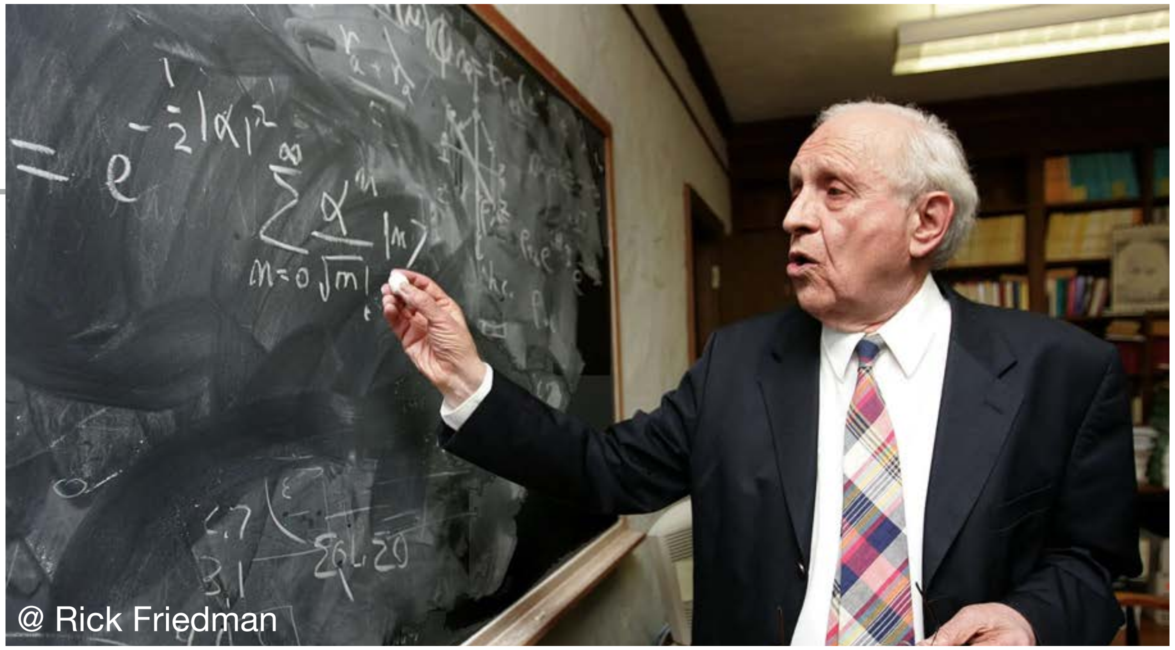
- Imaginary part (only at hadron colliders):

$$\boxed{\mathrm{Im}\,\boldsymbol{\Gamma}(\{\underline{p}\},\mu) = +2\pi\,\gamma_{\rm cusp}(\alpha_s)\; \boldsymbol{T}_1 \cdot \boldsymbol{T}_2} + (\dots)\,\underset{\substack{\uparrow \\ \text{irrelevant}}}{\mathbf{1}}$$

THE GLAUBER SERIES


@ Rick Friedman

Roy J. Glauber (Nobel Prize in Physics 2005)



Roy J. Glauber (Nobel Prize in Physics 2005)

THE GLAUBER SERIES

Structure of the cross section

- Super-leading logarithms (SLLs):

$$\sigma \sim \sum_{n=0}^{\infty} \left[c_{0,n} \left(\frac{\alpha_s}{\pi} L \right)^n + c_{1,n} \underbrace{\left(\frac{\alpha_s}{\pi} L \right) \left(\frac{\alpha_s}{\pi} i\pi L \right)^2 \left(\frac{\alpha_s}{\pi} L^2 \right)^n}_{\alpha_s^{n+3} L^{2n+3}} + \dots \right]$$

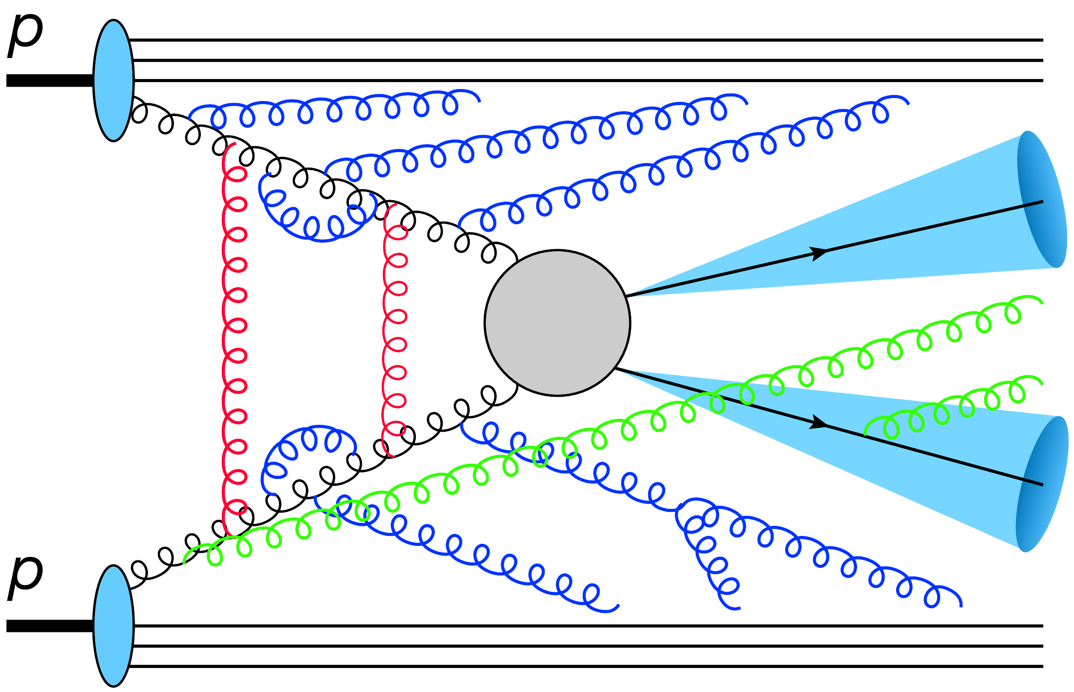
- Introduce two parameters, numerically O(1):

$$w = \frac{N_c \alpha_s(\bar{\mu})}{\pi} L^2 \,, \qquad w_\pi = \frac{N_c \alpha_s(\bar{\mu})}{\pi} \pi^2$$

- Including multiple Glauber insertions:

$$\sigma^{\text{SLL+G}} \sim \frac{\alpha_s L}{\pi N_c} \sum_{\ell=1}^{\infty} \sum_{n=0}^{\infty} c_{\ell,n} \, w_\pi^\ell \, w^{n+\ell}$$

THEORY OF JET PROCESSES AT LHC



red: Glauber (Coulomb)

blue: gluons emitted along beams

green: soft gluons between jets

Loss of color coherence from initial-state Coulomb interactions

↓

- Weird "super-leading logarithms"
- Breakdown of collinear factorization?
- Phenomenological consequences?

erc AdG **EFT4jets**

Today: Exact (semi-analytic) results for all double-logarithmic and π^2-enhanced contributions to the cross section in RG-improved perturbation theory!

Gap-between-jets cross sections

THEORY OF NON-GLOBAL LHC OBSERVABLES

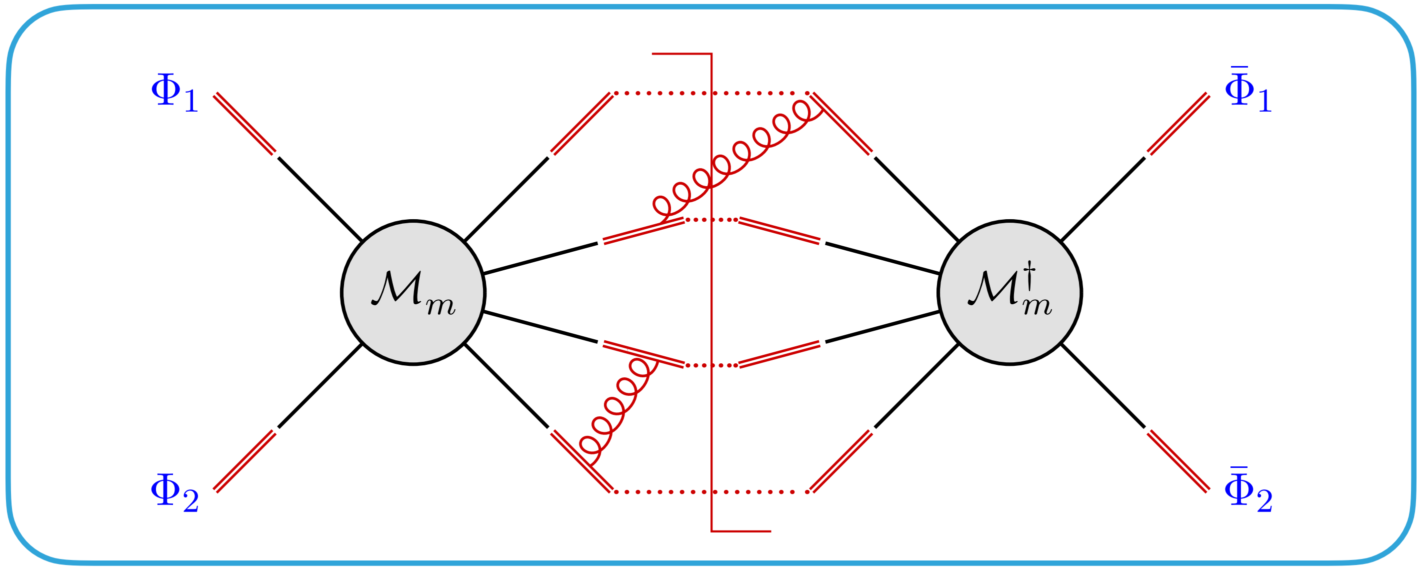
SCET factorization theorem

$$\sigma_{2\to M}(Q,Q_0) = \sum_{a,b=q,\bar{q},g} \int dx_1 dx_2 \sum_{m=2+M}^{\infty} \langle \boldsymbol{\mathcal{H}}_m^{ab}(\{\underline{n}\},Q,\mu) \otimes \boldsymbol{\mathcal{W}}_m^{ab}(\{\underline{n}\},Q_0,x_1,x_2,\mu)\rangle$$

high scale

low scale

T. Becher, MN, D. Shao (2021); T. Becher, MN, L. Rothen, D. Shao (2015, 2016)
[see also: Z. Nagy, D.E. Soper (2007, 2008, 2012, …)
R.A. Martínez, M. De Angelis, J.R. Forshaw, S. Plätzer, M.H. Seymour (2018)
J.R. Forshaw, J. Holguin, S. Plätzer (2019–2022)]



⇒ new perspective to think about non-global observables!

THEORY OF NON-GLOBAL LHC OBSERVABLES

SCET factorization theorem

$$\sigma_{2\to M}(Q,Q_0) = \sum_{a,b=q,\bar{q},g} \int dx_1 dx_2 \sum_{m=2+M}^{\infty} \big\langle \boldsymbol{\mathcal{H}}_m^{ab}(\{\underline{n}\},Q,\mu) \otimes \boldsymbol{\mathcal{W}}_m^{ab}(\{\underline{n}\},Q_0,x_1,x_2,\mu) \big\rangle$$

high scale

low scale

Renormalization-group evolution equation:

$$\mu \frac{d}{d\mu} \boldsymbol{\mathcal{H}}_l^{ab}(\{\underline{n}\},Q,\mu) = -\sum_{m\le l} \boldsymbol{\mathcal{H}}_m^{ab}(\{\underline{n}\},Q,\mu)\, \boldsymbol{\Gamma}_{ml}^{H}(\{\underline{n}\},Q,\mu)$$

operator in color space and in the infinite space of parton multiplicities

All-order summation of large logarithmic corrections, including the super-leading logarithms!

RESUMMATION OF SUPER-LEADING LOGARITHMS

Evaluate factorization theorem at low scale $\mu_s \sim Q_0$

- Low-energy matrix element:

$$\boldsymbol{\mathcal{W}}_m^{ab}(\{\underline{n}\}, Q_0, x_1, x_2, \mu_s) = f_{a/p}(x_1)\, f_{b/p}(x_2)\, \mathbf{1} + \mathcal{O}(\alpha_{\mathbf{s}})$$

- Hard-scattering functions:

$$\boldsymbol{\mathcal{H}}_m^{ab}(\{\underline{n}\}, Q, \mu_s) = \sum_{l\le m} \boldsymbol{\mathcal{H}}_l^{ab}(\{\underline{n}\}, Q, Q)\, \mathbf{P} \exp\left[\int_{\mu_s}^{Q} \frac{d\mu}{\mu}\, \boldsymbol{\Gamma}^H(\{\underline{n}\}, Q, \mu)\right]_{lm}$$

- Expanding the solution in a power series generates arbitrarily high parton multiplicities starting from the $2 \to M$ Born process

RESUMMATION OF SUPER-LEADING LOGARITHMS

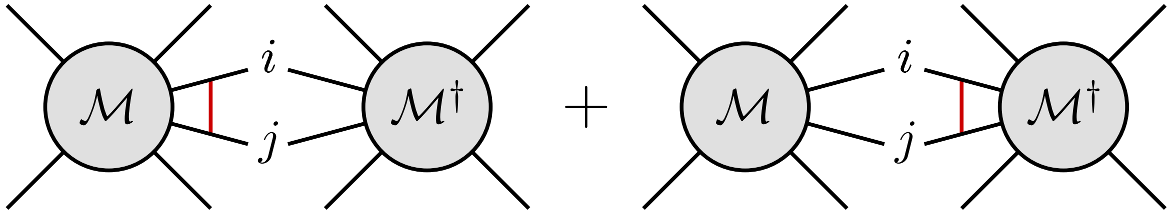
Evaluate factorization theorem at low scale $\mu_s \sim Q_0$

- Anomalous-dimension matrix:

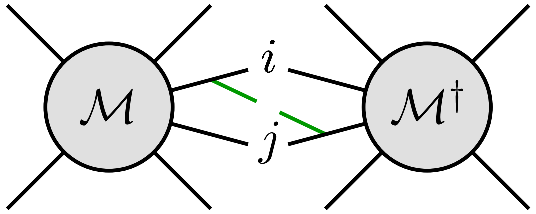
$$\boldsymbol{\Gamma}^H = \frac{\alpha_s}{4\pi} \begin{pmatrix} \boldsymbol{V}_{2+M} & \boldsymbol{R}_{2+M} & 0 & 0 & \dots \\ 0 & \boldsymbol{V}_{2+M+1} & \boldsymbol{R}_{2+M+1} & 0 & \dots \\ 0 & 0 & \boldsymbol{V}_{2+M+2} & \boldsymbol{R}_{2+M+2} & \dots \\ 0 & 0 & 0 & \boldsymbol{V}_{2+M+3} & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} + \mathcal{O}(\alpha_s^2)$$

- Action on hard functions:

$$\boldsymbol{\mathcal{H}}_m \, \boldsymbol{V}_m = \sum_{(ij)}$$



$$\boldsymbol{\mathcal{H}}_m \, \boldsymbol{R}_m = \sum_{(ij)}$$



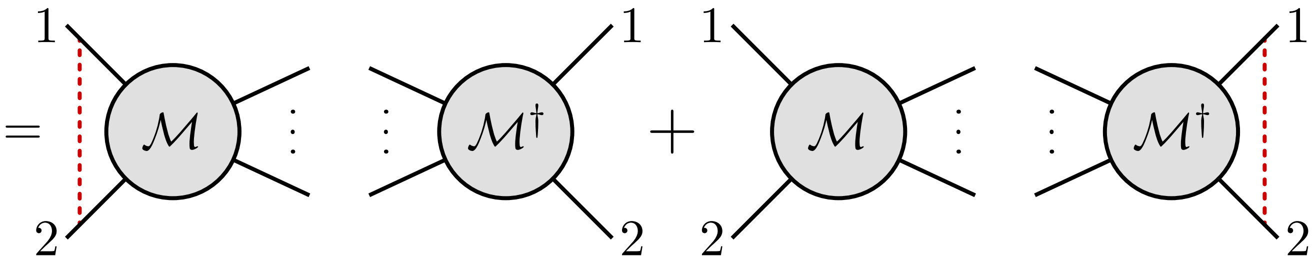
RESUMMATION OF SUPER-LEADING LOGARITHMS

Detailed structure of the anomalous-dimension coefficients

$$\left.\begin{aligned} \boldsymbol{V}_m &= \overline{\boldsymbol{V}}_m + \boldsymbol{V}^G + \sum_{i=1,2} \boldsymbol{V}_i^c \ln\frac{\mu^2}{\hat{s}} \\ \boldsymbol{R}_m &= \overline{\boldsymbol{R}}_m + \sum_{i=1,2} \boldsymbol{R}_i^c \ln\frac{\mu^2}{\hat{s}} \end{aligned}\right\} \quad \boldsymbol{\Gamma} = \underset{\substack{\text{soft emission}\\ \text{(collinear div. subtracted)}}}{\overline{\boldsymbol{\Gamma}}} + \boxed{\overset{\text{Glauber phase}}{\boldsymbol{V}^G} + \underset{\text{collinear emission}}{\boldsymbol{\Gamma}^c}} \ln\frac{\mu^2}{\hat{s}}$$

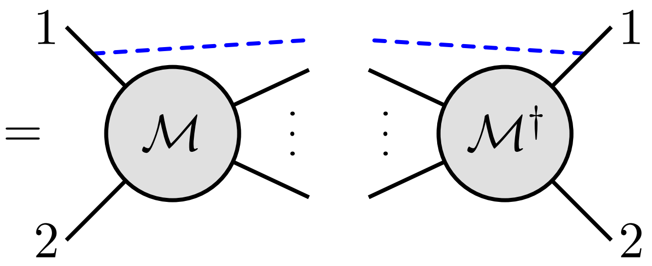
where:

$\boldsymbol{\mathcal{H}}_m\, \boldsymbol{V}^G =$



$$\boldsymbol{V}^G = -2i\pi\left(\boldsymbol{T}_{1,L}\cdot\boldsymbol{T}_{2,L} - \boldsymbol{T}_{1,R}\cdot\boldsymbol{T}_{2,R}\right)$$

$\boldsymbol{\mathcal{H}}_m\, \boldsymbol{R}_1^c =$



new color space of emitted gluon

$$\boldsymbol{\Gamma}^c = \sum_{i=1,2}\left[C_i\,\boldsymbol{1} - \boldsymbol{T}_{i,L}\circ\boldsymbol{T}_{i,R}\,\delta(n_k - n_i)\right]$$

RESUMMATION OF SUPER-LEADING LOGARITHMS

Detailed structure of the anomalous-dimension coefficients

Glauber phase

$$\left.\begin{aligned} \boldsymbol{V}_m &= \overline{\boldsymbol{V}}_m + \boldsymbol{V}^G + \sum_{i=1,2} \boldsymbol{V}_i^c \ln\frac{\mu^2}{\hat{s}} \\ \boldsymbol{R}_m &= \overline{\boldsymbol{R}}_m + \sum_{i=1,2} \boldsymbol{R}_i^c \ln\frac{\mu^2}{\hat{s}} \end{aligned}\right\} \quad \boldsymbol{\Gamma} = \overline{\boldsymbol{\Gamma}} + \boldsymbol{V}^G + \boldsymbol{\Gamma}^c \ln\frac{\mu^2}{\hat{s}}$$

soft emission
(collinear div. subtracted)

collinear emission

Properties:

- color coherence without Glauber phases: $\boldsymbol{\mathcal{H}}_m\,\boldsymbol{\Gamma}^c\,\overline{\boldsymbol{\Gamma}} = \boldsymbol{\mathcal{H}}_m\,\overline{\boldsymbol{\Gamma}}\,\boldsymbol{\Gamma}^c$
- collinear safety: $\langle \boldsymbol{\mathcal{H}}_m\,\boldsymbol{\Gamma}^c \otimes \boldsymbol{1} \rangle = 0$
- cyclicity of the trace: $\langle \boldsymbol{\mathcal{H}}_m\,\boldsymbol{V}^G \otimes \boldsymbol{1} \rangle = 0$

RESUMMATION OF SUPER-LEADING LOGARITHMS

SLLs arise from the terms in $\mathbf{P}\exp\left[\int_{\mu_s}^{Q}\frac{d\mu}{\mu}\,\mathbf{\Gamma}^H(\{\underline{n}\},Q,\mu)\right]_{lm}$ with the highest number of insertions of Γᶜ

- Expand out all terms except the log-enhanced soft-collinear piece:

$$\boldsymbol{U}_{\text{SLL}}(\{\underline{n}\},\mu_h,\mu_s) = \int_{\mu_s}^{\mu_h}\frac{d\mu_1}{\mu_1}\int_{\mu_s}^{\mu_1}\frac{d\mu_2}{\mu_2}\int_{\mu_s}^{\mu_2}\frac{d\mu_3}{\mu_3}$$

cusp anomalous dimension

$$\times\,\boldsymbol{U}_c(\mu_h,\mu_1)\,\gamma_{\text{cusp}}\big(\alpha_s(\mu_1)\big)\,\boldsymbol{V}^G\,\boldsymbol{U}_c(\mu_1,\mu_2)\,\gamma_{\text{cusp}}\big(\alpha_s(\mu_2)\big)\,\boldsymbol{V}^G\,\frac{\alpha_s(\mu_3)}{4\pi}\,\overline{\mathbf{\Gamma}}$$

P. Böer, P. Hager, MN, M. Stillger, X. Xu (2024)

where we define the Sudakov operator:

$$\boldsymbol{U}_c(\mu_i,\mu_j) = \exp\left[\mathbf{\Gamma}^c\int_{\mu_j}^{\mu_i}\frac{d\mu}{\mu}\,\gamma_{\text{cusp}}\big(\alpha_s(\mu)\big)\,\ln\frac{\mu^2}{\mu_h^2}\right]$$

$\mu_h \simeq Q$

$\mu_s \simeq Q_0$

matrix in the color & multiplicity space

resums all double-logarithmic terms

RESUMMATION OF SUPER-LEADING LOGARITHMS

SLLs arise from the terms in $\mathbf{P}\exp\left[\int_{\mu_s}^{Q}\frac{d\mu}{\mu}\,\mathbf{\Gamma}^H(\{\underline{n}\},Q,\mu)\right]_{lm}$ **with the highest number of insertions of** $\mathbf{\Gamma}^c$

- Expand out all terms except the log-enhanced soft-collinear piece:

$$\boldsymbol{U}_{\text{SLL}}(\{\underline{n}\},\mu_h,\mu_s)=\int_{\mu_s}^{\mu_h}\frac{d\mu_1}{\mu_1}\int_{\mu_s}^{\mu_1}\frac{d\mu_2}{\mu_2}\int_{\mu_s}^{\mu_2}\frac{d\mu_3}{\mu_3}$$

cusp anomalous dimension

$$\times\,\boldsymbol{U}_c(\mu_h,\mu_1)\,\gamma_{\text{cusp}}\big(\alpha_s(\mu_1)\big)\,\boldsymbol{V}^G\,\boldsymbol{U}_c(\mu_1,\mu_2)\,\gamma_{\text{cusp}}\big(\alpha_s(\mu_2)\big)\,\boldsymbol{V}^G\,\frac{\alpha_s(\mu_3)}{4\pi}\,\overline{\mathbf{\Gamma}}$$

P. Böer, P. Hager, MN, M. Stillger, X. Xu (2024)

- All double-logarithmic terms are exponentiated!
- One scale integral for each insertion of $\boldsymbol{V}^G$ and $\overline{\mathbf{\Gamma}}$
- Easy to include running-coupling effects

RESUMMATION OF SUPER-LEADING LOGARITHMS

Rewrite the evolution kernel for the Glauber series

- Analogous relation holds for higher-order terms in the Glauber series (more $\boldsymbol{V}^G$ factors and additional integrals):

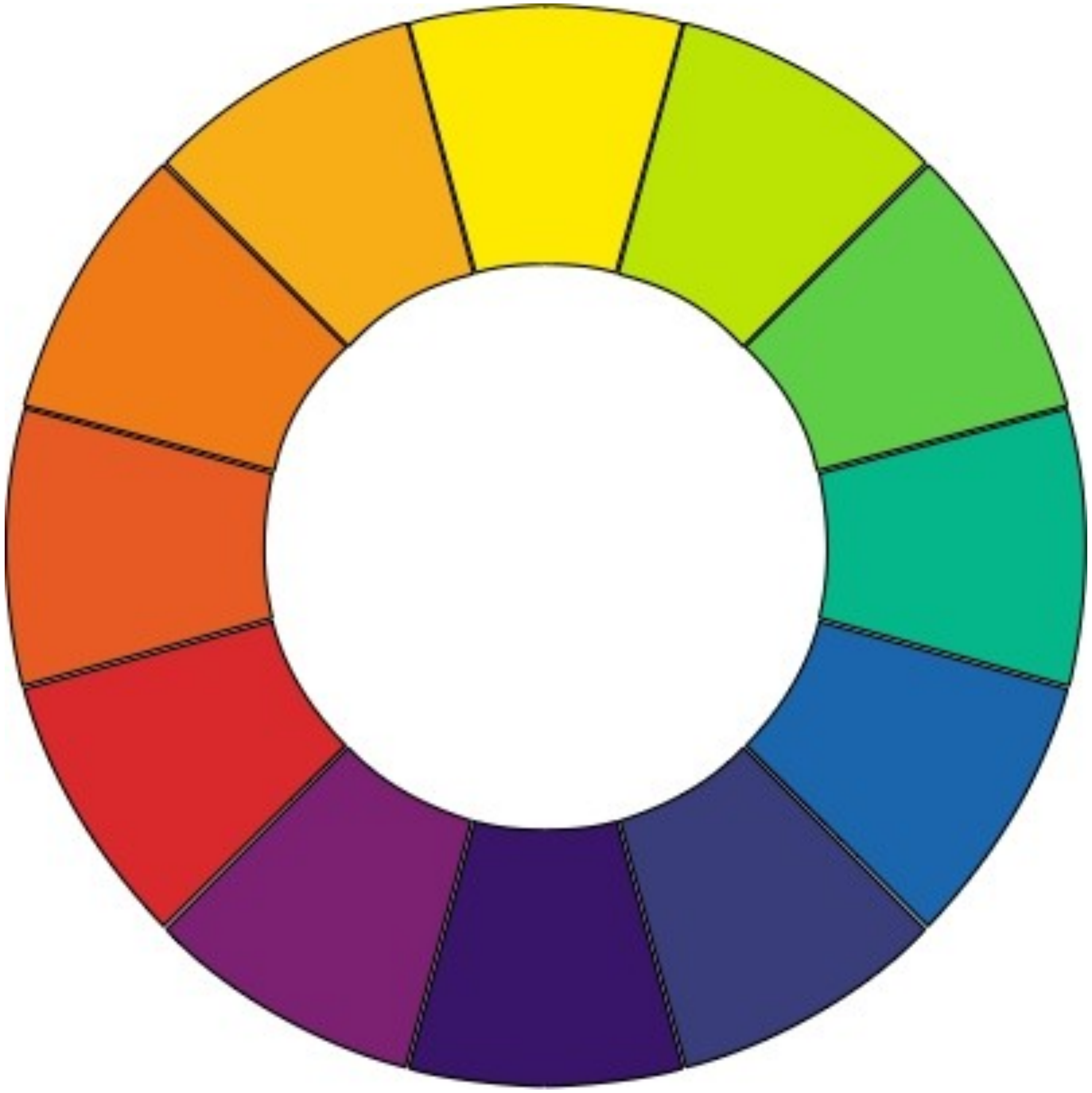
$$\boldsymbol{U}_{\rm SLL}^{(l)}(\{\underline{n}\},\mu_h,\mu_s) = \int_{\mu_s}^{\mu_h}\frac{d\mu_1}{\mu_1}\dots\int_{\mu_s}^{\mu_l}\frac{d\mu_{l+1}}{\mu_{l+1}}\left[\prod_{i=1}^{l}\boldsymbol{U}_c(\mu_{i-1},\mu_i)\,\gamma_{\rm cusp}\big(\alpha_s(\mu_i)\big)\,\boldsymbol{V}^G\right]\frac{\alpha_s(\mu_{l+1})}{4\pi}\,\overline{\boldsymbol{\Gamma}}$$

$$\boldsymbol{U}_{\rm SLL+G}(\{\underline{n}\},\mu_h,\mu_s) = \sum_{l=1}^{\infty}\boldsymbol{U}_{\rm SLL}^{(l)}(\{\underline{n}\},\mu_h,\mu_s)$$

P. Böer, P. Hager, MN, M. Stillger, X. Xu (2024)

- Structure share similarities with a parton shower, but the Sudakov operator and Glauber phases imply a non-trivial operator mixing in color space and involve both real and virtual emissions

Introducing a color basis



MANAGING THE COLOR ALGEBRA

Introduce a color basis (closed under applications of $\boldsymbol{\Gamma}^c$ and $\boldsymbol{V}^G$)

- Simplest case of (anti-)quark-initiated scattering processes:

$$\boldsymbol{X}_1 = \sum_{j>2}^{2+M} J_j \, i f^{abc} \, \boldsymbol{T}_1^a \boldsymbol{T}_2^b \boldsymbol{T}_j^c \,, \qquad \boldsymbol{X}_4 = \frac{1}{N_c} \, J_{12} \, \boldsymbol{T}_1 \cdot \boldsymbol{T}_2 \,,$$

$$\boldsymbol{X}_2 = \sum_{j>2}^{2+M} J_j \, (\sigma_1 - \sigma_2) \, d^{abc} \, \boldsymbol{T}_1^a \boldsymbol{T}_2^b \boldsymbol{T}_j^c \,, \qquad \boldsymbol{X}_5 = J_{12} \, \boldsymbol{1} \,,$$

$$\boldsymbol{X}_3 = \frac{1}{N_c} \sum_{j>2}^{2+M} J_j \, (\boldsymbol{T}_1 - \boldsymbol{T}_2) \cdot \boldsymbol{T}_j \,,$$

P. Böer, MN, M. Stillger (2023)

where $\sigma_i = -1 \; (+1)$ for an initial-state quark (anti-quark), and all structures are normalized such that their trace with a hard function is at most of $O(N_c^0)$ in the large-N_c limit

MANAGING THE COLOR ALGEBRA

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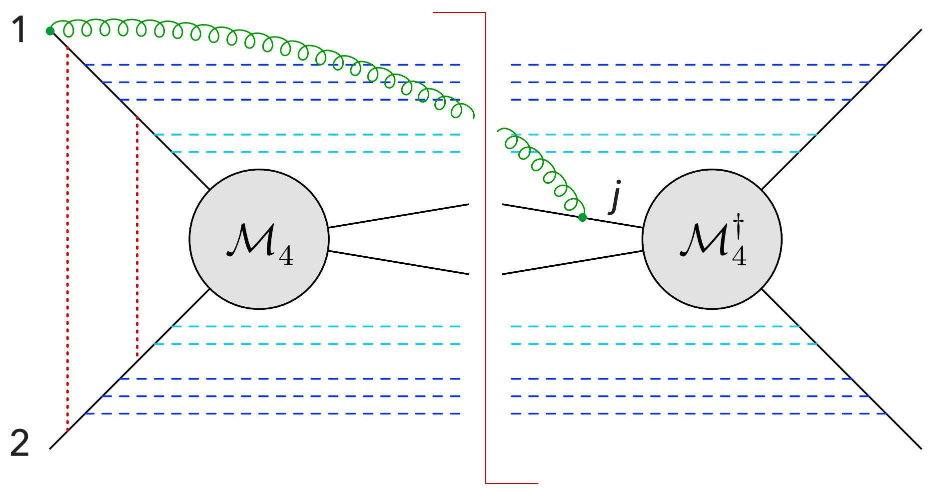
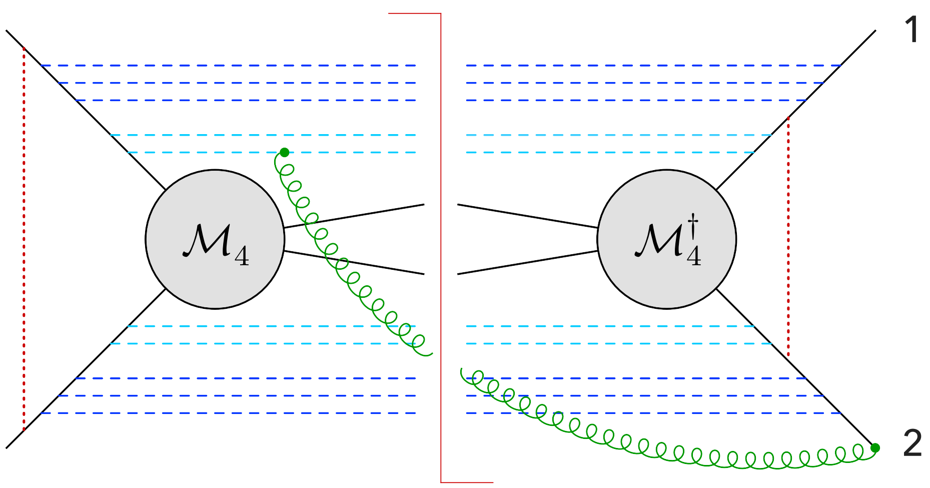
$$\boldsymbol{X}_5 = J_{12}\, \boldsymbol{1} \,,$$

P. Böer, MN, M. Stillger (2023)





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P. Böer, MN, M. Stillger (2023)

- Kinematic information contained in $(M+1)$ angular integrals from $\overline{\Gamma}$:

$$J_j = \int \frac{d\Omega(n_k)}{4\pi} \left(W_{1j}^k - W_{2j}^k \right) \Theta_{\text{veto}}(n_k) \,; \quad \text{with} \quad W_{ij}^k = \frac{n_i \cdot n_j}{n_i \cdot n_k \, n_j \cdot n_k}$$

MANAGING THE COLOR ALGEBRA

Introduce a color basis (closed under applications of $\boldsymbol{\Gamma}^c$ and $\boldsymbol{V}^G$)

- Simplest case of (anti-)quark-initiated scattering processes:

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P. Böer, MN, M. Stillger (2023)

- Extension to processes with initial-state gluons requires an enlarged operator basis containing **20** (gg scattering) and **14** ($qg, \bar{q}g$ scattering) operators, respectively

P. Böer, P. Hager, M. Neubert, M. Stillger, X. Xu (2023)

MANAGING THE COLOR ALGEBRA

Introduce a color basis (closed under applications of $\boldsymbol{\Gamma}^c$ and $\boldsymbol{V}^G$)

- Represent $\boldsymbol{\Gamma}^c$, $\boldsymbol{V}^G$ and $\boldsymbol{V}^G\,\overline{\boldsymbol{\Gamma}}$ as objects acting in that basis:

$$\boldsymbol{\Gamma}^c \to N_c\,\mathbb{I}\Gamma^c \qquad \text{with} \qquad \mathbb{I}\Gamma^c = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & -\frac{C_F}{N_c} & 0 & 0 \end{pmatrix}$$

Positive eigenvalues: {0, 1/2, 1}
(additional ones for initial-state gluons)

Recall:

$$\boldsymbol{U}_{\mathrm{SLL}}(\{\underline{n}\},\mu_h,\mu_s) = \int_{\mu_s}^{\mu_h}\frac{d\mu_1}{\mu_1}\int_{\mu_s}^{\mu_1}\frac{d\mu_2}{\mu_2}\int_{\mu_s}^{\mu_2}\frac{d\mu_3}{\mu_3}$$
$$\times\,\boldsymbol{U}_c(\mu_h,\mu_1)\,\gamma_{\mathrm{cusp}}\big(\alpha_s(\mu_1)\big)\,\boldsymbol{V}^G\,\boldsymbol{U}_c(\mu_1,\mu_2)\,\gamma_{\mathrm{cusp}}\big(\alpha_s(\mu_2)\big)\,\boldsymbol{V}^G\,\frac{\alpha_s(\mu_3)}{4\pi}\,\overline{\boldsymbol{\Gamma}}$$

MANAGING THE COLOR ALGEBRA

Introduce a color basis (closed under applications of $\boldsymbol{\Gamma}^c$ and $\boldsymbol{V}^G$)

- Represent $\boldsymbol{\Gamma}^c$, $\boldsymbol{V}^G$ and $\boldsymbol{V}^G\,\overline{\boldsymbol{\Gamma}}$ as objects acting in that basis:

$$\boldsymbol{U}_c(\mu_i,\mu_j) \;\to\; \mathbb{U}_c(\mu_i,\mu_j) = \begin{pmatrix} U_c(1;\mu_i,\mu_j) & 0 & 0 & 0 & 0 \\ 0 & U_c(1;\mu_i,\mu_j) & 0 & 0 & 0 \\ 0 & 0 & U_c(\frac12;\mu_i,\mu_j) & 0 & 0 \\ 0 & 0 & 2\left[U_c(\frac12;\mu_i,\mu_j) - U_c(1;\mu_i,\mu_j)\right] & U_c(1;\mu_i,\mu_j) & 0 \\ 0 & 0 & \frac{2C_F}{N_c}\left[1 - U_c(\frac12;\mu_i,\mu_j)\right] & 0 & 1 \end{pmatrix}$$

Generalized Sudakov factors: $U_c(v;\mu_i,\mu_j) = \exp\left[v\,N_c \int_{\mu_j}^{\mu_i} \frac{d\mu}{\mu}\,\gamma_{\text{cusp}}\big(\alpha_s(\mu)\big)\ln\frac{\mu^2}{\mu_h^2}\right] \le 1$

Recall:

$$\boldsymbol{U}_{\text{SLL}}(\{\underline{n}\},\mu_h,\mu_s) = \int_{\mu_s}^{\mu_h}\frac{d\mu_1}{\mu_1}\int_{\mu_s}^{\mu_1}\frac{d\mu_2}{\mu_2}\int_{\mu_s}^{\mu_2}\frac{d\mu_3}{\mu_3}$$

$$\times\,\boldsymbol{U}_c(\mu_h,\mu_1)\,\gamma_{\text{cusp}}\big(\alpha_s(\mu_1)\big)\,\boldsymbol{V}^G\,\boldsymbol{U}_c(\mu_1,\mu_2)\,\gamma_{\text{cusp}}\big(\alpha_s(\mu_2)\big)\,\boldsymbol{V}^G\,\frac{\alpha_s(\mu_3)}{4\pi}\,\overline{\boldsymbol{\Gamma}}$$

double-log terms (SLLs)
always lead to suppression!

MANAGING THE COLOR ALGEBRA

Introduce a color basis (closed under applications of $\boldsymbol{\Gamma}^c$ and $\boldsymbol{V}^G$)

- Represent $\boldsymbol{\Gamma}^c$, $\boldsymbol{V}^G$ and $\boldsymbol{V}^G\,\overline{\boldsymbol{\Gamma}}$ as objects acting in that basis:

$$\boldsymbol{V}^G \to i\pi N_c \mathbb{V}^G \quad \text{with} \quad \mathbb{V}^G = \begin{pmatrix} 0 & -2\delta_{q\bar{q}}\,\frac{N_c^2-4}{N_c^2} & \frac{4}{N_c^2} & 0 & 0 \\ -\frac{1}{2} & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Recall:

$$\boldsymbol{U}_{\text{SLL}}(\{\underline{n}\},\mu_h,\mu_s) = \int_{\mu_s}^{\mu_h}\frac{d\mu_1}{\mu_1}\int_{\mu_s}^{\mu_1}\frac{d\mu_2}{\mu_2}\int_{\mu_s}^{\mu_2}\frac{d\mu_3}{\mu_3}$$

$$\times\,\boldsymbol{U}_c(\mu_h,\mu_1)\,\gamma_{\text{cusp}}\big(\alpha_s(\mu_1)\big)\,\boldsymbol{V}^G\,\boldsymbol{U}_c(\mu_1,\mu_2)\,\gamma_{\text{cusp}}\big(\alpha_s(\mu_2)\big)\,\boldsymbol{V}^G\,\frac{\alpha_s(\mu_3)}{4\pi}\,\overline{\boldsymbol{\Gamma}}$$

MANAGING THE COLOR ALGEBRA

Introduce a color basis (closed under applications of $\boldsymbol{\Gamma}^c$ and $\boldsymbol{V}^G$)

- Represent $\boldsymbol{\Gamma}^c, \boldsymbol{V}^G$ and $\boldsymbol{V}^G\,\overline{\boldsymbol{\Gamma}}$ as objects acting in that basis:

$$\boldsymbol{V}^G\,\overline{\boldsymbol{\Gamma}} \to 16i\pi X_1 \equiv 16i\pi X^T\varsigma \qquad \text{with} \qquad \varsigma = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Recall:

$$\boldsymbol{U}_{\rm SLL}(\{\underline{n}\},\mu_h,\mu_s) = \int_{\mu_s}^{\mu_h}\frac{d\mu_1}{\mu_1}\int_{\mu_s}^{\mu_1}\frac{d\mu_2}{\mu_2}\int_{\mu_s}^{\mu_2}\frac{d\mu_3}{\mu_3}$$

$$\times\,\boldsymbol{U}_c(\mu_h,\mu_1)\,\gamma_{\rm cusp}\big(\alpha_s(\mu_1)\big)\,\boldsymbol{V}^G\,\boldsymbol{U}_c(\mu_1,\mu_2)\,\gamma_{\rm cusp}\big(\alpha_s(\mu_2)\big)\,\boldsymbol{V}^G\,\frac{\alpha_s(\mu_3)}{4\pi}\,\overline{\boldsymbol{\Gamma}}$$

MANAGING THE COLOR ALGEBRA

Introduce a color basis (closed under applications of $\boldsymbol{\Gamma}^c$ and $\boldsymbol{V}^G$)

- This yields:

$$\sigma_{2\to M}^{\text{SLL+G}}(Q_0) = \sum_{\text{partonic channels}} \int d\xi_1 \int d\xi_2\, f_1(\xi_1,\mu_s)\, f_2(\xi_2,\mu_s)$$

$$\times \sum_{l=1}^{\infty} \big\langle \boldsymbol{\mathcal{H}}_{2\to M}(\mu_h)\, \boldsymbol{X}^T \big\rangle\, \mathbb{U}_{\text{SLL}}^{(l)}(\mu_h,\mu_s)\, \varsigma\,,$$

5 process-dependent color traces

with:

$$\mathbb{U}_{\text{SLL}}^{(l)}(\mu_h,\mu_s) = 16\,(i\pi)^l\, N_c^{l-1} \int_{\mu_s}^{\mu_h} \frac{d\mu_1}{\mu_1} \dots \int_{\mu_s}^{\mu_l} \frac{d\mu_{l+1}}{\mu_{l+1}}\, \mathbb{U}_c(\mu_h,\mu_1)$$

$$\times \left[\prod_{i=1}^{l-1} \gamma_{\text{cusp}}\big(\alpha_s(\mu_i)\big)\, \mathbb{V}^G\, \mathbb{U}_c(\mu_i,\mu_{i+1}) \right] \gamma_{\text{cusp}}\big(\alpha_s(\mu_l)\big)\, \frac{\alpha_s(\mu_{l+1})}{4\pi}$$

RG–IMPROVED PERTURBATION THEORY

Perform the scale integrals in terms of the running coupling

- Generalized Sudakov factors in RG-improved perturbation theory:

$$U_c(v;\mu_i,\mu_j) = \exp\left[v\,N_c \int_{\mu_j}^{\mu_i} \frac{d\mu}{\mu}\,\gamma_{\rm cusp}\big(\alpha_s(\mu)\big)\ln\frac{\mu^2}{\mu_h^2}\right]$$

2-loop cusp anomalous dimension
and β-function

$$= \exp\left\{ \frac{\gamma_0\,v N_c}{2\beta_0^2}\left[\frac{4\pi}{\alpha_s(\mu_h)}\left(\frac{1}{x_i}-\frac{1}{x_j}-\ln\frac{x_j}{x_i}\right) + \left(\frac{\gamma_1}{\gamma_0}-\frac{\beta_1}{\beta_0}\right)\left(x_i-x_j+\ln\frac{x_j}{x_i}\right) + \frac{\beta_1}{2\beta_0}\left(\ln^2 x_j-\ln^2 x_i\right)\right]\right\}$$

P. Böer, P. Hager, MN, M. Stillger, X. Xu (2024)

with $x_i \equiv \alpha_s(\mu_i)/\alpha_s(\mu_h)$ and:

$$U_c(v;\mu_i,\mu_j)\,U_c(v;\mu_j,\mu_k) = U_c(v;\mu_i,\mu_k)\,, \qquad U_c(0;\mu_i,\mu_j) = 1$$

- Encounter products of Sudakov factors:

$$U_c(v_1,\dots,v_l;\mu_h,\mu_1,\dots,\mu_l) \equiv U_c(v_1;\mu_h,\mu_1)\,U_c(v_2;\mu_1,\mu_2)\dots U_c(v_l;\mu_{l-1},\mu_l)$$

RG-IMPROVED RESUMMATION OF HIGHER GLAUBER TERMS

Evolution functions with two and four Glauber insertions

P. Böer, P. Hager, MN, M. Stillger, X. Xu (2024)

- *l*=2:

$$\mathbb{U}^{(2)}_{\text{SLL}}(\mu_h,\mu_s)\,\varsigma = -\frac{32\pi^2}{\beta_0^3}\,N_c\int_1^{x_s}\frac{dx_2}{x_2}\,\ln\frac{x_s}{x_2}\int_1^{x_2}\frac{dx_1}{x_1}\begin{pmatrix} 0 \\ -\frac{1}{2}\,U_c(1;\mu_h,\mu_2) \\ U_c(\frac{1}{2},1;\mu_h,\mu_1,\mu_2) \\ 2\left[U_c(\frac{1}{2},1;\mu_h,\mu_1,\mu_2)-U_c(1;\mu_h,\mu_2)\right] \\ \frac{2C_F}{N_c}\left[U_c(1;\mu_1,\mu_2)-U_c(\frac{1}{2},1;\mu_h,\mu_1,\mu_2)\right] \end{pmatrix}$$

- *l*=4

$$\mathbb{U}^{(4)}_{\text{SLL}}(\mu_h,\mu_s)\,\varsigma = \frac{128\pi^4}{\beta_0^5}\,N_c^3\int_1^{x_s}\frac{dx_4}{x_4}\,\ln\frac{x_s}{x_4}\int_1^{x_4}\frac{dx_3}{x_3}\int_1^{x_3}\frac{dx_2}{x_2}\int_1^{x_2}\frac{dx_1}{x_1}\begin{pmatrix} 0 \\ -\frac{1}{2}\left[K_{12}\,U_c(1;\mu_h,\mu_4)+\frac{4}{N_c^2}\,U_c(1,\frac{1}{2},1;\mu_h,\mu_2,\mu_3,\mu_4)\right] \\ K_{12}\,U_c(\frac{1}{2},1;\mu_h,\mu_1,\mu_4)+\frac{4}{N_c^2}\,U_c(\frac{1}{2},1,\frac{1}{2},1;\mu_h,\mu_1,\mu_2,\mu_3,\mu_4) \\ \begin{gathered} 2\left[K_{12}\,U_c(\frac{1}{2},1;\mu_h,\mu_1,\mu_4)+\frac{4}{N_c^2}\,U_c(\frac{1}{2},1,\frac{1}{2},1;\mu_h,\mu_1,\mu_2,\mu_3,\mu_4)\right] \\ -2\left[K_{12}\,U_c(1;\mu_h,\mu_4)+\frac{4}{N_c^2}\,U_c(1,\frac{1}{2},1;\mu_h,\mu_2,\mu_3,\mu_4)\right] \end{gathered} \\ \begin{gathered} \frac{2C_F}{N_c}\left[K_{12}\,U_c(1;\mu_1,\mu_4)+\frac{4}{N_c^2}\,U_c(1,\frac{1}{2},1;\mu_1,\mu_2,\mu_3,\mu_4)\right] \\ -\frac{2C_F}{N_c}\left[K_{12}\,U_c(\frac{1}{2},1;\mu_h,\mu_1,\mu_4)+\frac{4}{N_c^2}\,U_c(\frac{1}{2},1,\frac{1}{2},1;\mu_h,\mu_1,\mu_2,\mu_3,\mu_4)\right] \end{gathered} \end{pmatrix}$$

$$K_{12}\equiv(\sigma_1-\sigma_2)^2\,\frac{N_c^2-4}{4N_c^2}=\frac{N_c^2-4}{N_c^2}\,\delta_{q\bar{q}}$$

RG-IMPROVED RESUMMATION OF THE GLAUBER SERIES

Resummation of the Glauber series in the limit of large *N*$_c$

- Closed analytic expression in terms of a double integral:

$$\sum_{l=2,4,6,\dots} \mathbb{U}^{(l)}_{\rm SLL}(\mu_h,\mu_s)\,\varsigma = -\frac{32\pi^2 N_c}{\beta_0^3}\int_1^{x_s}\frac{dx}{x}\,\ln\frac{x_s}{x}\int_1^{x}\frac{dx_1}{x_1}\left[1-2\delta_{q\bar{q}}\,\sin^2\!\left(\frac{\pi N_c}{\beta_0}\,\ln\frac{x}{x_1}\right)\right]\begin{pmatrix} 0 \\ -\frac{1}{2}\,U_c(1;\mu_h,\mu_x) \\ U_c(\frac{1}{2},1;\mu_h,\mu_1,\mu_x) \\ 2\left[U_c(\frac{1}{2},1;\mu_h,\mu_1,\mu_x)-U_c(1;\mu_h,\mu_x)\right] \\ \frac{2C_F}{N_c}\left[U_c(1;\mu_1,\mu_x)-U_c(\frac{1}{2},1;\mu_h,\mu_1,\mu_x)\right] \end{pmatrix}$$

- Super-leading logarithms (*l*=2 term) are exact
- First RG-improved resummation of the Glauber series!
- Analogous results can be derived for processes with initial-state gluons, involving eigenvalues $\{0,\frac{1}{2},1,\frac{3}{2}\}$ for *qg* scattering and $\{0,\frac{1}{2},1,\frac{3}{2},2\}$ for *gg* scattering – **mysterious spin-color connection**

P. Böer, P. Hager, MN, M. Stillger, X. Xu: arXiv:2407.01691

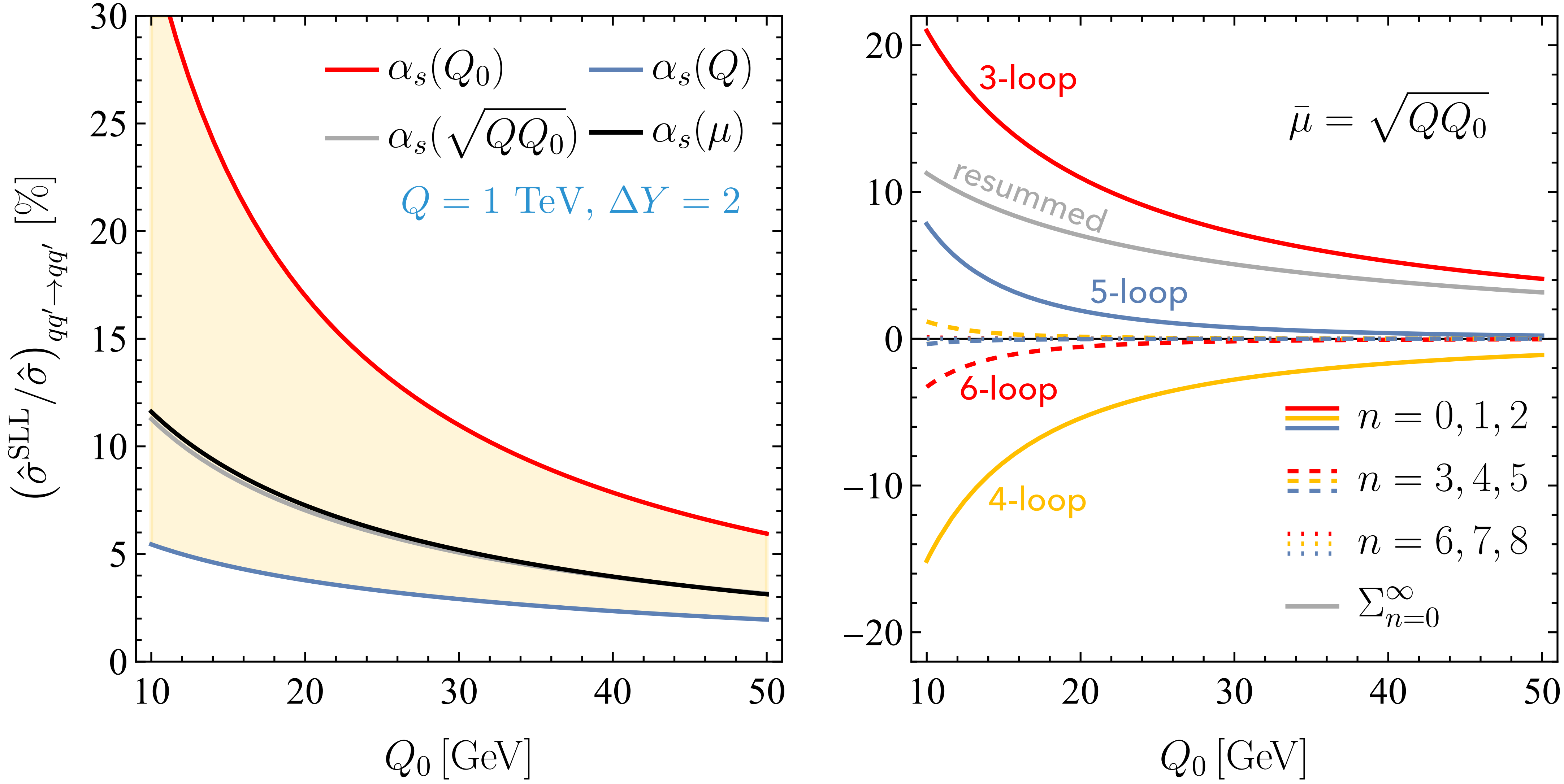
Phenomenology:

$$qq' \to qq'$$

PHENOMENOLOGICAL IMPACT (PARTON LEVEL)

Partonic channels contributing to $pp \to$ 2 jets (gap between jets)

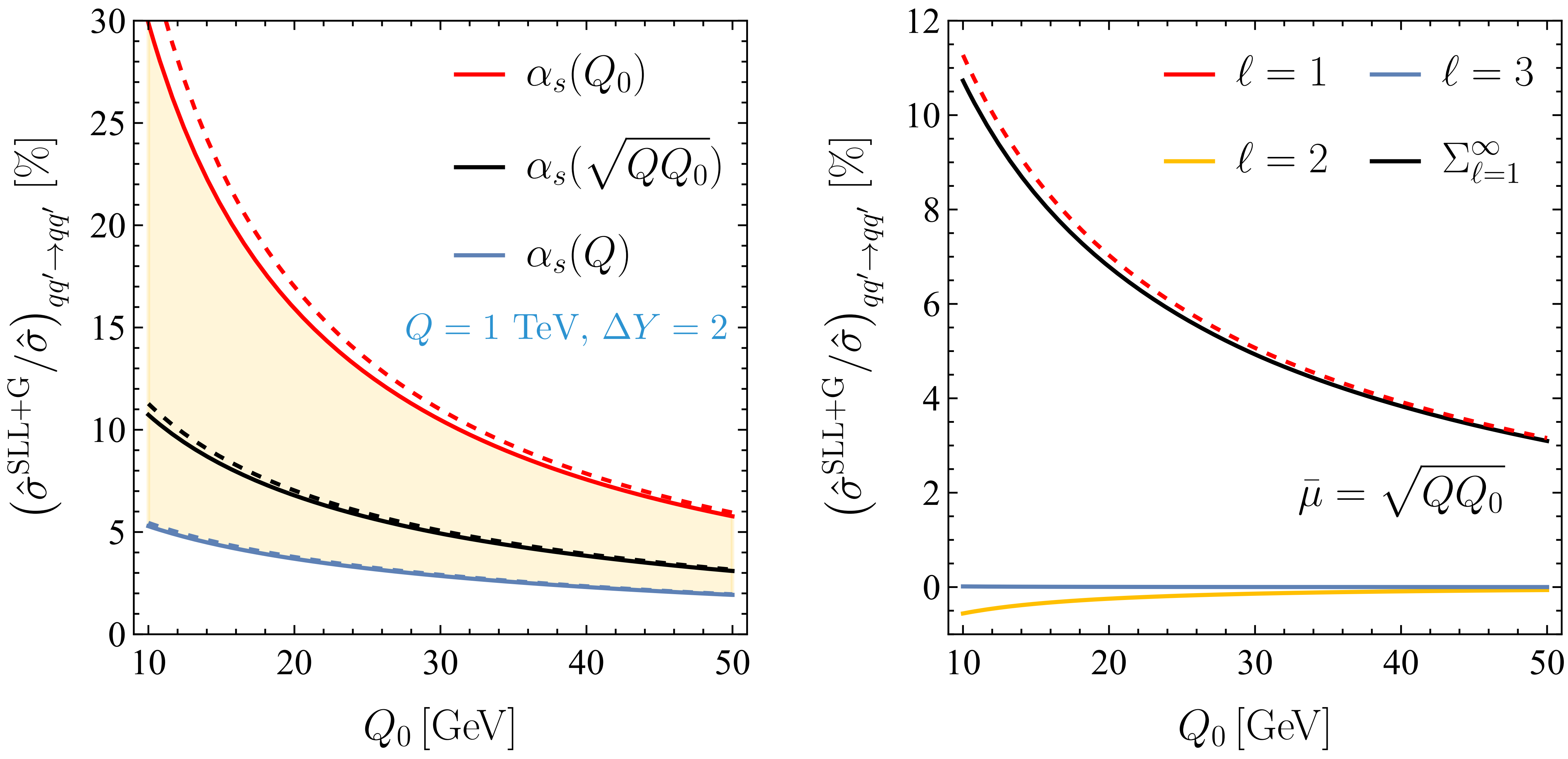
T. Becher, MN, D. Shao, M. Stillger (2023)



PHENOMENOLOGICAL IMPACT (PARTON LEVEL)

Surprising suppression of higher Glauber contributions

P. Böer, MN, M. Stillger (2023)



Phenomenology:

$$gg \to q\bar{q}$$

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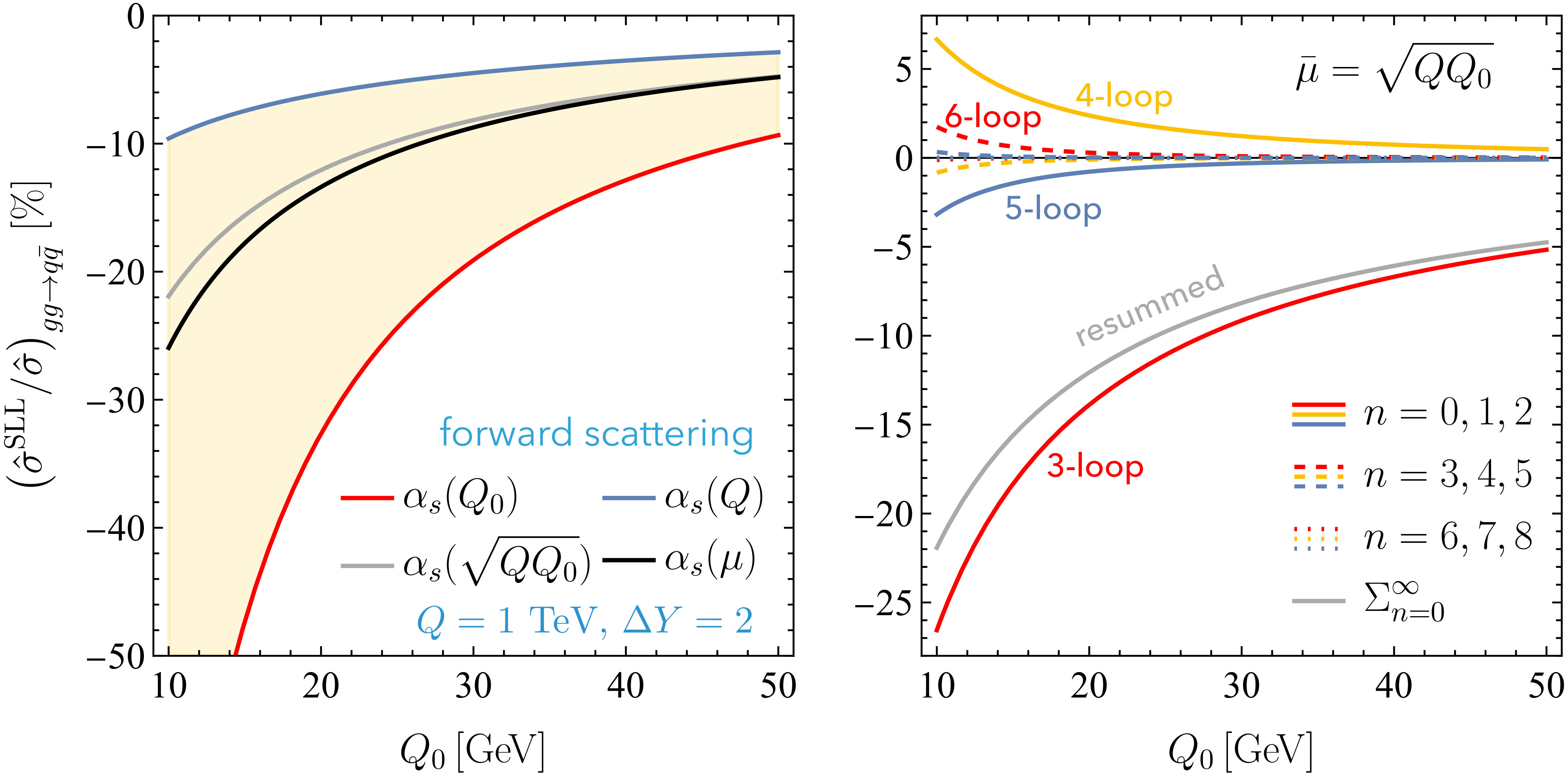

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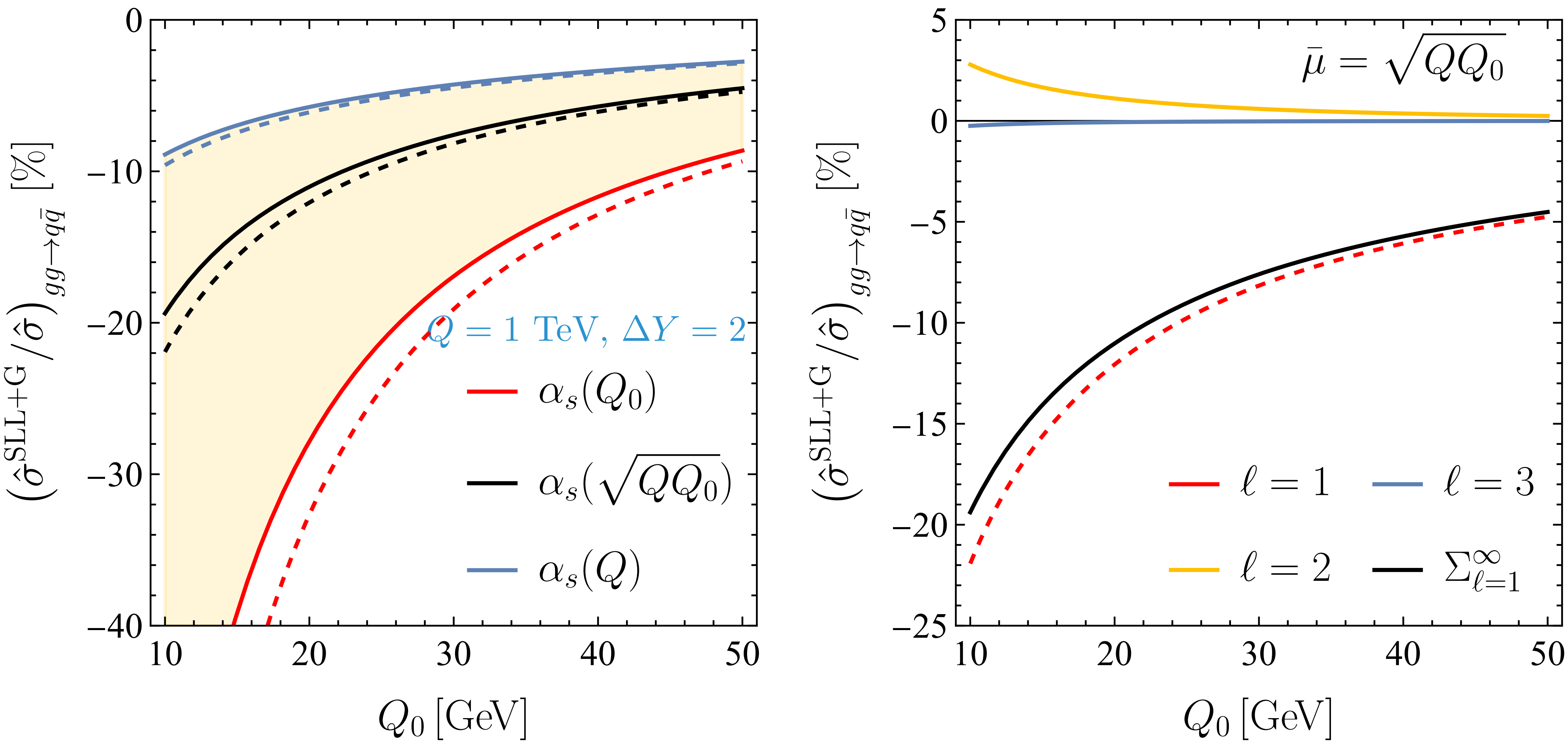


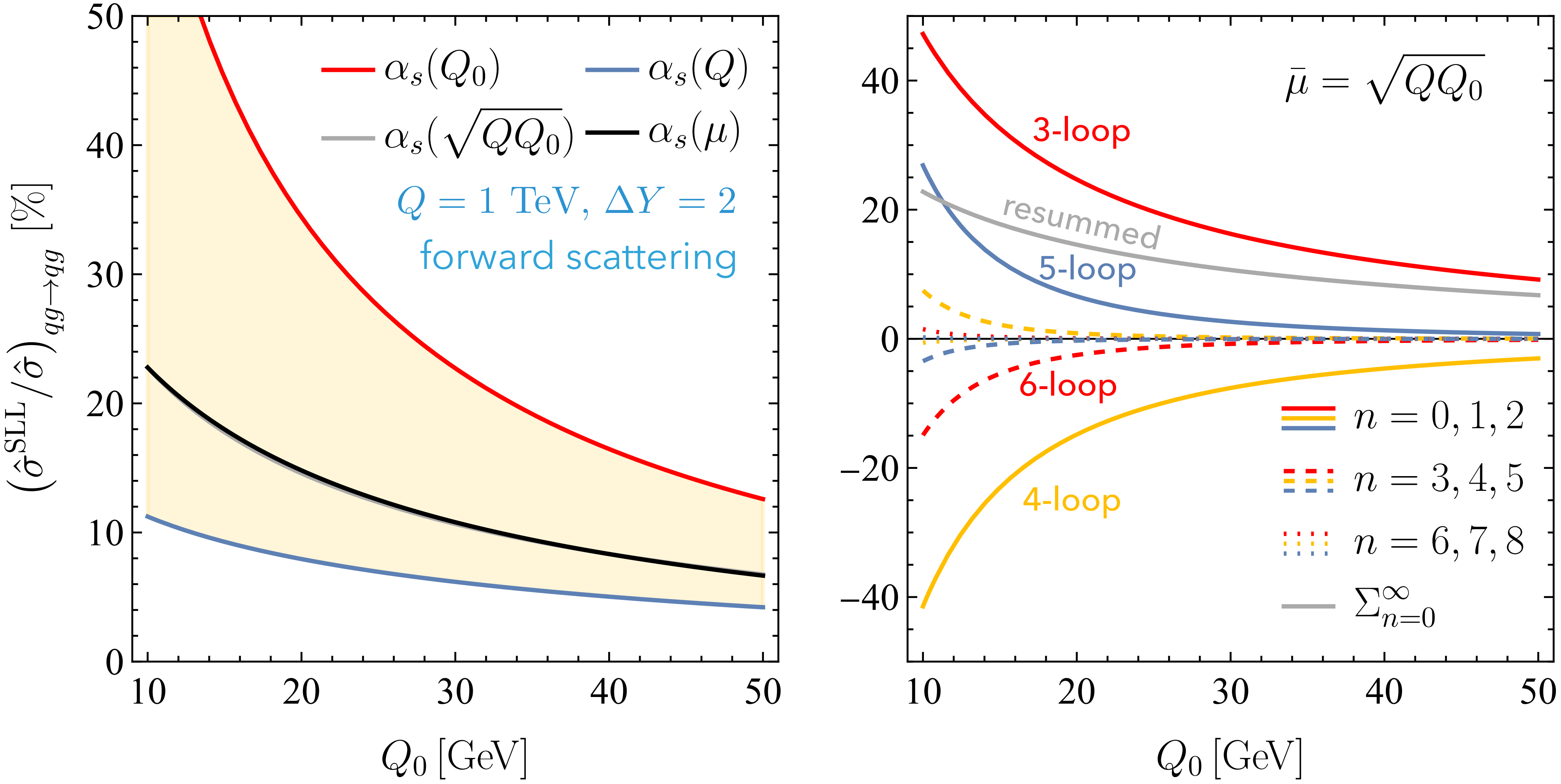
Phenomenology:

$$qg \to qg$$

PHENOMENOLOGICAL IMPACT (PARTON LEVEL)

Partonic channels contributing to $pp \to$ 2 jets (gap between jets)

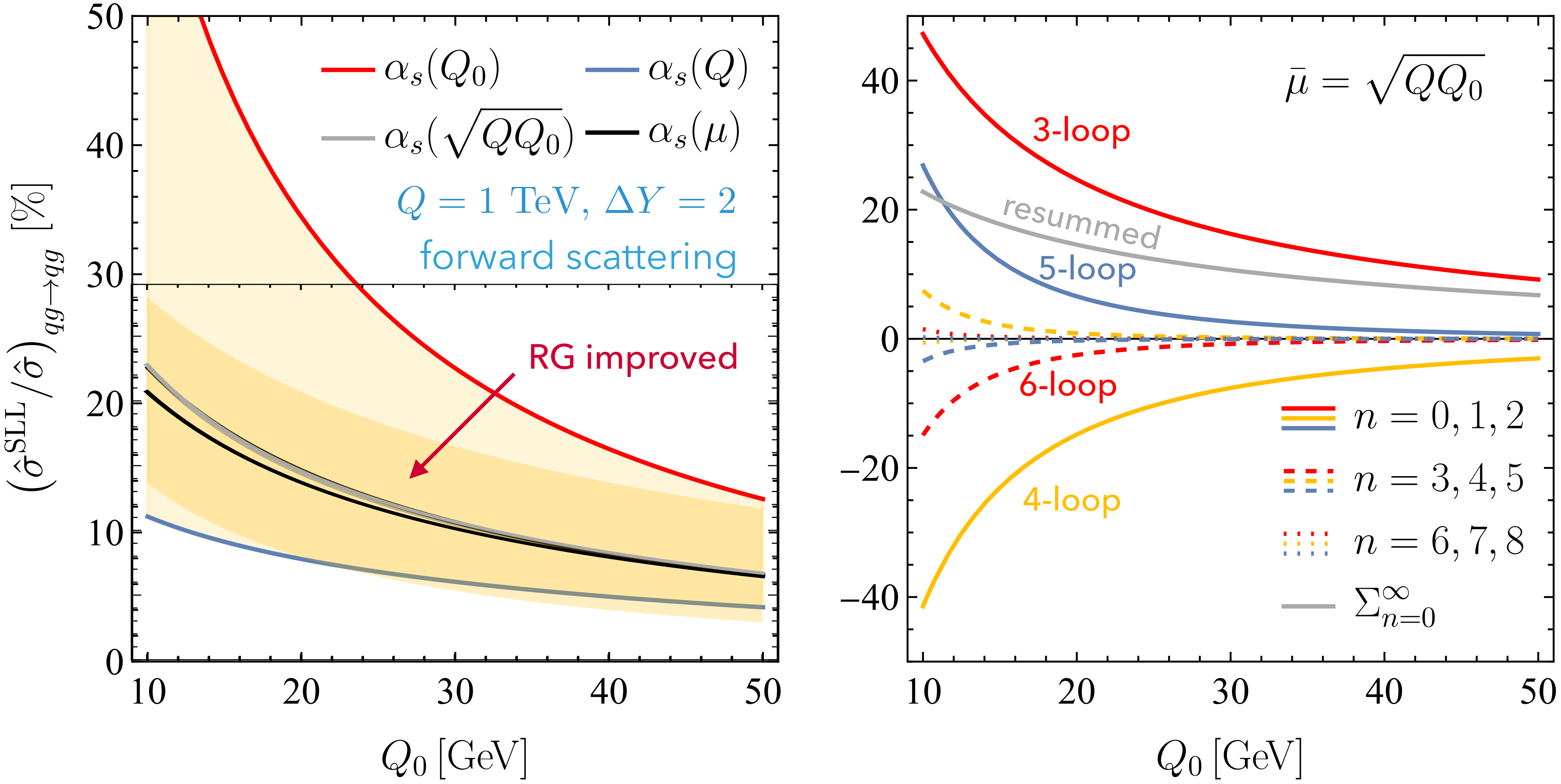
T. Becher, MN, D. Shao, M. Stillger (2023)



PHENOMENOLOGICAL IMPACT (PARTON LEVEL)

Partonic channels contributing to $pp \to$ 2 jets (gap between jets)

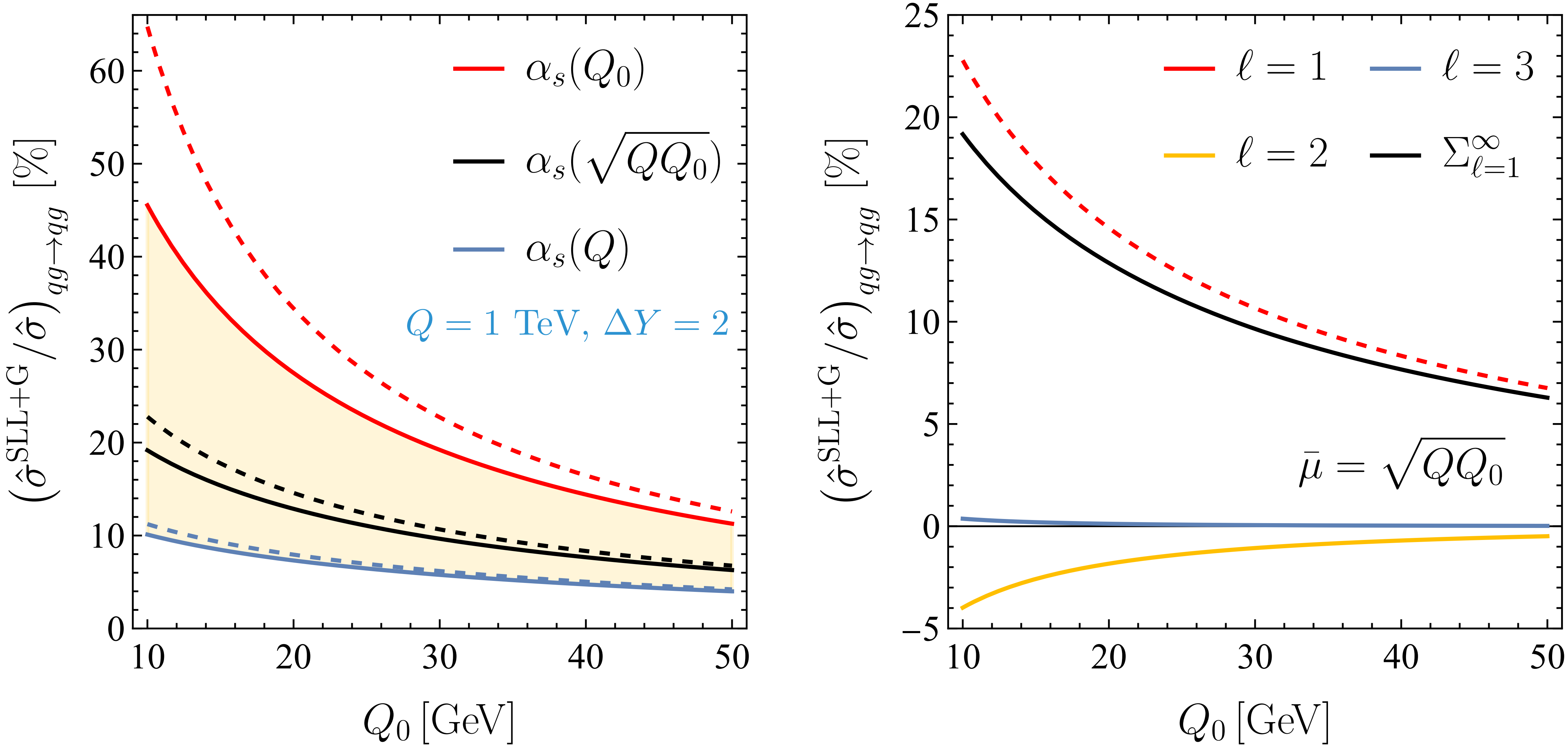
T. Becher, MN, D. Shao, M. Stillger (2023)



PHENOMENOLOGICAL IMPACT (PARTON LEVEL)

Visible effects of higher Glauber contributions

P. Böer, P. Hager, MN, M. Stillger, X. Xu (2024)



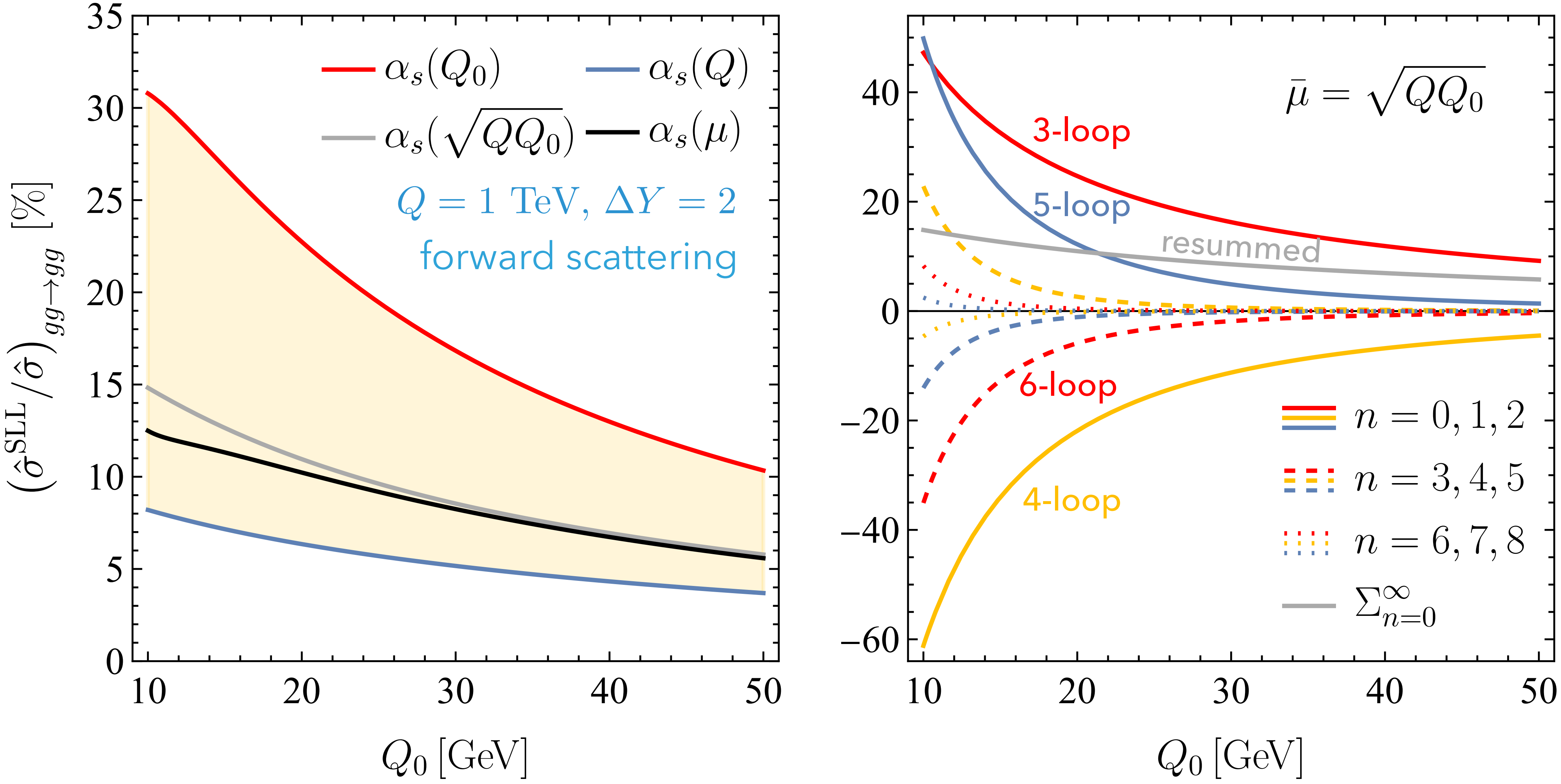
Phenomenology:

$$gg \to gg$$

PHENOMENOLOGICAL IMPACT (PARTON LEVEL)

Partonic channels contributing to $pp \to$ 2 jets (gap between jets)

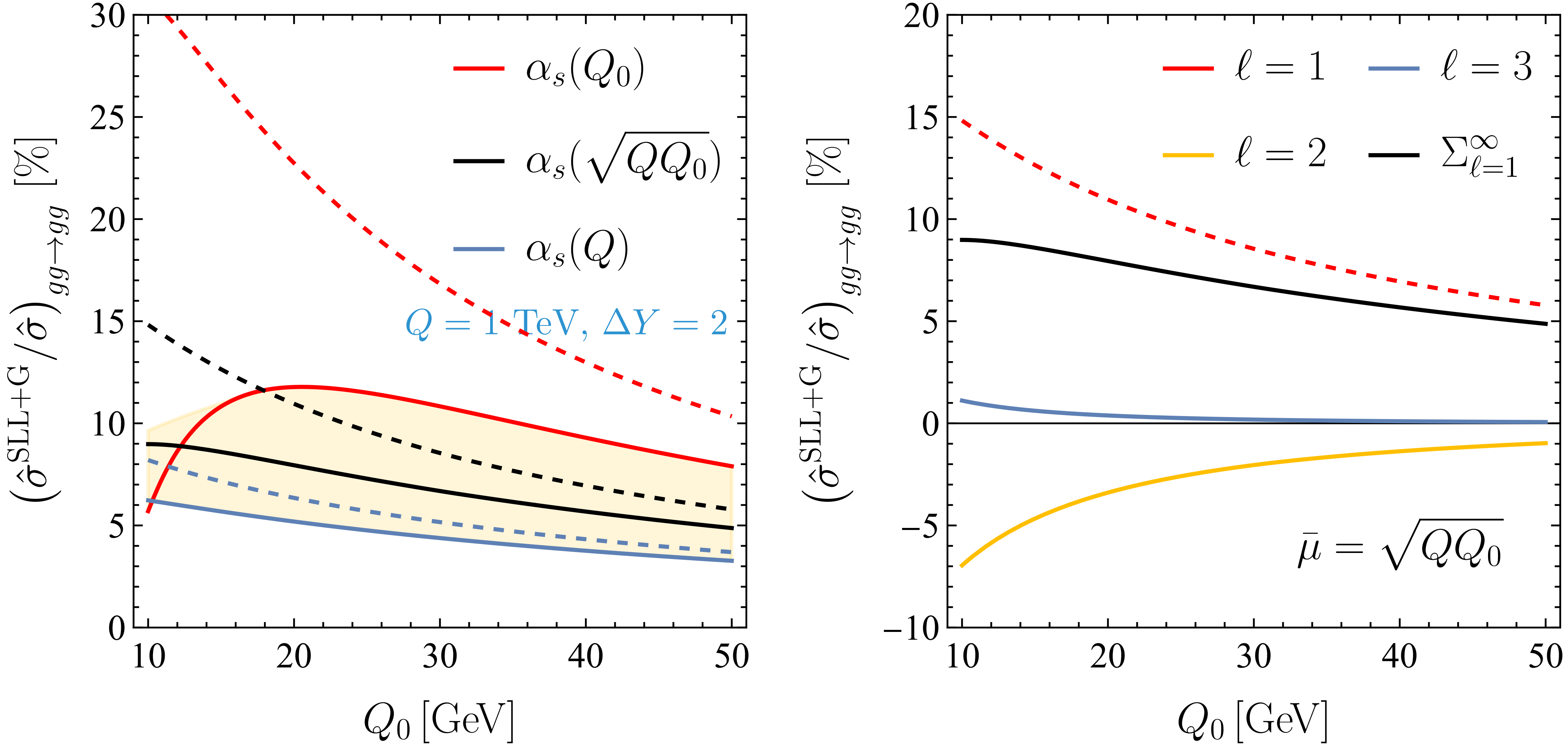
T. Becher, MN, D. Shao, M. Stillger (2023)



PHENOMENOLOGICAL IMPACT (PARTON LEVEL)

Large effects of higher Glauber contributions

P. Böer, P. Hager, MN, M. Stillger, X. Xu (2024)



Asymptotic behavior

@ Matthias Neubert

EXACT ANALYTIC RESULTS WITH FIXED COUPLING

$$L_s = \ln\frac{\mu_h}{\mu_s} \approx \ln\frac{Q}{Q_0}$$

Asymptotics for $\alpha_s L_s \sim 1\,,\ \alpha_s L_s^2 \gg 1$ derived using a fixed coupling

- Analytic expression in terms of Σ-functions:

$$\mathbb{U}_{\text{SLL}}^{(2)}(\mu_h,\mu_s)\,\varsigma = -\frac{2\pi^2}{3}\,N_c\left(\frac{\alpha_s}{\pi}\,L_s\right)^3 \begin{pmatrix} 0 \\ -\frac{1}{2}\,\Sigma(1,1;w) \\ \Sigma(\frac{1}{2},1;w) \\ 2\left[\Sigma(\frac{1}{2},1;w)-\Sigma(1,1;w)\right] \\ \frac{2C_F}{N_c}\left[\Sigma(0,1;w)-\Sigma(\frac{1}{2},1;w)\right] \end{pmatrix}$$

Kampé de Fériet functions

$$w = \frac{N_c\,\alpha_s(\bar{\mu})}{\pi}\,L_s^2$$

$$\mathbb{U}_{\text{SLL}}^{(4)}(\mu_h,\mu_s)\,\varsigma = \frac{\pi^4}{30}\,N_c^3\left(\frac{\alpha_s}{\pi}\,L_s\right)^5 \begin{pmatrix} 0 \\ -\frac{1}{2}\left[K_{12}\,\Sigma(1,1,1,1;w)+\frac{4}{N_c^2}\,\Sigma(1,1,\frac{1}{2},1;w)\right] \\ K_{12}\,\Sigma(\frac{1}{2},1,1,1;w)+\frac{4}{N_c^2}\,\Sigma(\frac{1}{2},1,\frac{1}{2},1;w) \\ 2\left[K_{12}\,\Sigma(\frac{1}{2},1,1,1;w)+\frac{4}{N_c^2}\,\Sigma(\frac{1}{2},1,\frac{1}{2},1;w)\right] -2\left[K_{12}\,\Sigma(1,1,1,1;w)+\frac{4}{N_c^2}\,\Sigma(1,1,\frac{1}{2},1;w)\right] \\ \frac{2C_F}{N_c}\left[K_{12}\,\Sigma(0,1,1,1;w)+\frac{4}{N_c^2}\,\Sigma(0,1,\frac{1}{2},1;w)\right] -\frac{2C_F}{N_c}\left[K_{12}\,\Sigma(\frac{1}{2},1,1,1;w)+\frac{4}{N_c^2}\,\Sigma(\frac{1}{2},1,\frac{1}{2},1;w)\right] \end{pmatrix}$$

integrals of
Kampé de Fériet functions

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$$\mathbb{U}^{(4)}_{\rm SLL}(\mu_h,\mu_s)\,\varsigma = \frac{\pi^4}{30}\,N_c^3\left(\frac{\alpha_s}{\pi}\,L_s\right)^5$$

$$\Sigma(1,1;w) = \frac{3}{w} - \frac{3\sqrt{\pi}}{2\,w^{3/2}} + \mathcal{O}(e^{-w})\,,$$

$$\Sigma(\tfrac12,1;w) = \frac{3\sqrt{2}\,\ln(1+\sqrt{2})}{w} - \frac{3\sqrt{\pi}}{\sqrt{2}\,w^{3/2}} + \mathcal{O}(w^{-2})\,,$$

$$\Sigma(0,1;w) = \frac{3}{2}\,\frac{\ln(4w)+\gamma_E-2}{w} + \frac{3}{4w^2} + \mathcal{O}(w^{-3})\,,$$

$$\Sigma(1,1,1,1;w) = \frac{10}{w^2} - \frac{15\sqrt{\pi}}{2w^{5/2}} + \mathcal{O}(e^{-w})\,,$$

$$\Sigma(\tfrac12,1,1,1;w) = \frac{15\left[2-\sqrt{2}\ln(1+\sqrt{2})\right]}{w^2} - \frac{15\sqrt{\pi}\,(2-\sqrt{2})}{w^{5/2}} + \mathcal{O}(e^{-\frac{w}{2}})\,,$$

$$\Sigma(0,1,1,1;w) = \frac{15}{w^2} - \frac{15\sqrt{\pi}}{w^{5/2}} + \mathcal{O}(w^{-3})\,,$$

$$\Sigma(1,1,\tfrac12,1;w) = \frac{60}{w^2}\left[\sqrt{2}\ln(1+\sqrt{2})-1\right] - \frac{30\sqrt{\pi}\,(\sqrt{2}-1)}{w^{5/2}} + \mathcal{O}(e^{-\frac{w}{2}})\,,$$

$$\Sigma(\tfrac12,1,\tfrac12,1;w) = \frac{15\sqrt{2}}{w^2}\left[\frac{5\pi^2}{4} - \frac{3}{2}\ln^2 2 - 12\,\mathrm{Li}_2\!\left(\frac{1}{\sqrt{2}}\right)\right] - \frac{15\sqrt{2\pi}\,\ln 2}{w^{5/2}} + \mathcal{O}(e^{-\frac{w}{2}})\,,$$

$$\Sigma(0,1,\tfrac12,1;w) = \frac{30\ln^2(1+\sqrt{2})}{w^2} - \frac{30\sqrt{\pi}\,\ln(1+\sqrt{2})}{w^{5/2}} + \mathcal{O}(w^{-3})\,.$$

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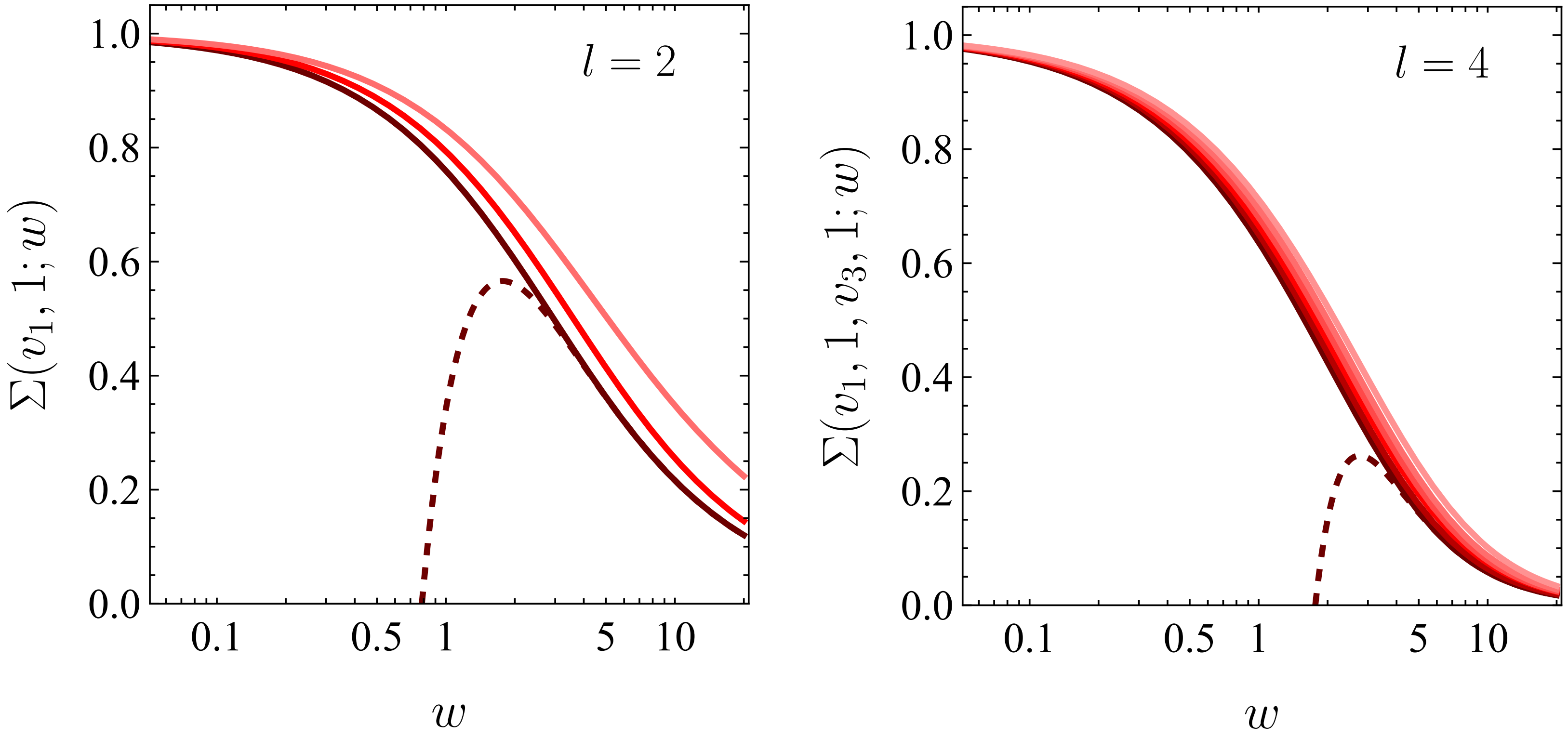

- Parametric suppression:

$$\mathbb{U}_{\rm SLL}^{(l)}(\mu_h,\mu_s) \sim \frac{(i\pi)^l}{(l+1)!}\,N_c^{l-1}\left(\frac{\alpha_s L_s}{\pi}\right)^{l+1}\frac{1}{w^{l/2}} = \frac{i^l}{(l+1)!}\,\frac{\alpha_s L_s}{\pi N_c}\,w_\pi^{l/2}$$

FUTURE CHALLENGES

Important open questions

- How to include multiple soft emissions (single-log effects), and how large is their effect? Does large N_c help?

FUTURE CHALLENGES

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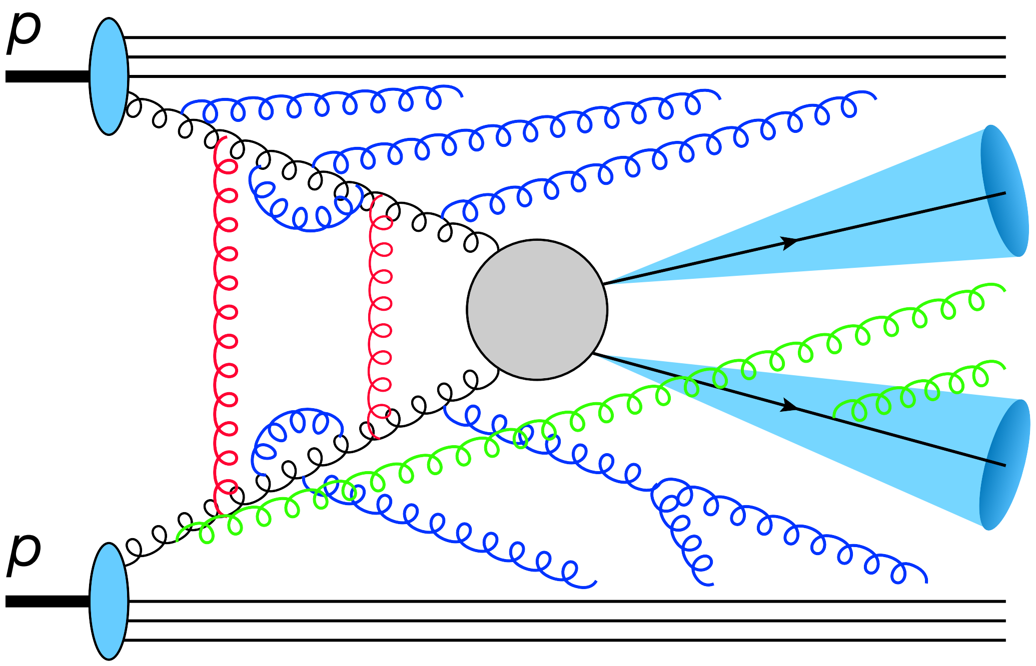
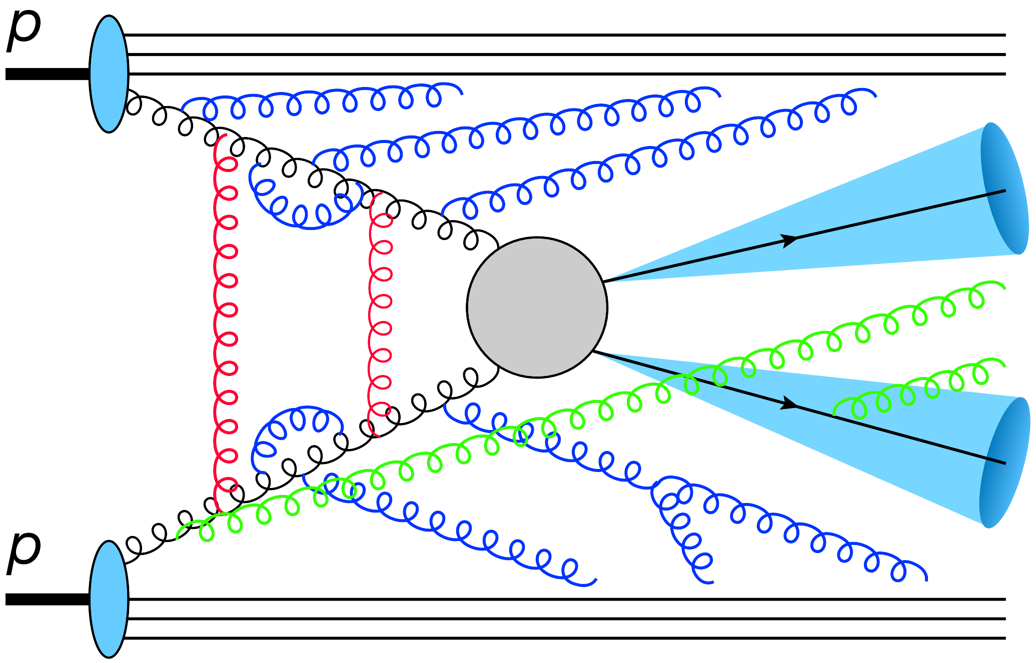


- Can collinear factorization violations be understood in a quantitative way, and at which scale (Q_0 or Λ_{QCD}) do they occur?

FUTURE CHALLENGES

Important open questions

- How to include multiple soft emissions (single-log effects), and how large is their effect? Does large N_c help?
- Can collinear factorization violations be understood in a quantitative way, and at which scale (Q_0 or $\Lambda_{\rm QCD}$) do they occur?
- Implications for LHC phenomenology?



FUTURE CHALLENGES

Important open questions

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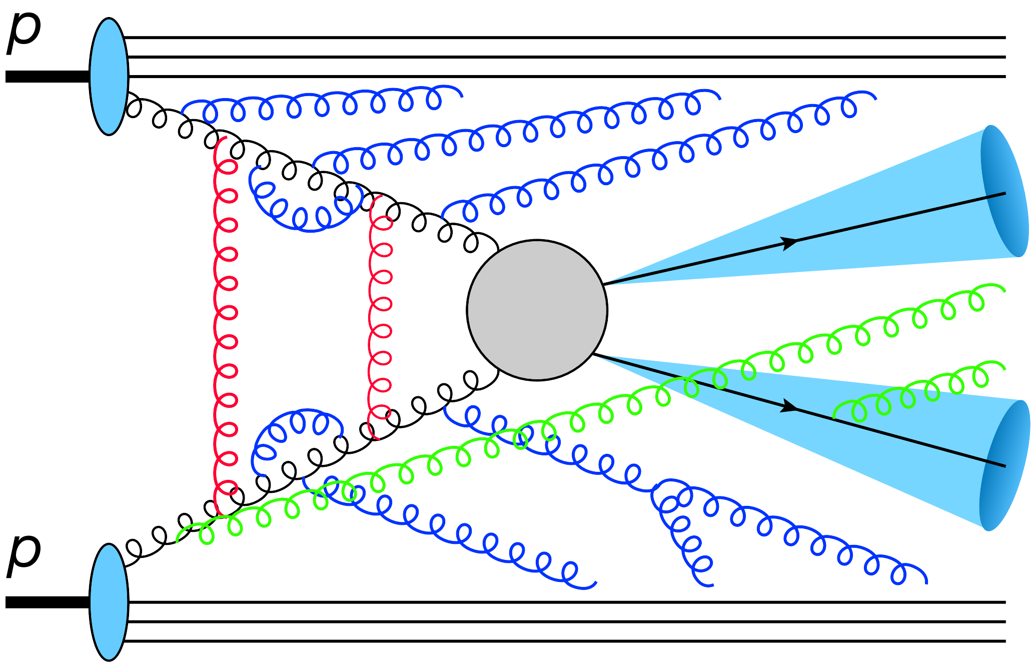


- Can collinear factorization violations be understood in a quantitative way, and at which scale (Q_0 or $\Lambda_{\rm QCD}$) do they occur?
- Implications for LHC phenomenology?
- Our analytical results will be relevant for validations of parton showers with quantum interference

Z. Nagy, D.E. Soper (2007, 2008, 2012, …)
R.A. Martínez, M. De Angelis, J.R. Forshaw, S. Plätzer, M.H. Seymour (2018); J.R. Forshaw, J. Holguin, S. Plätzer (2019–2022)
M. Dasgupta, F.A. Dreyer, K. Hamilton, P.F. Monni, G.P. Salam (2020); M. van Beekveld, S.F. Ravasio, G.P. Salam, A. Soto-Ontoso, G. Soyez *et al.* (2022, 2023)

Thank you

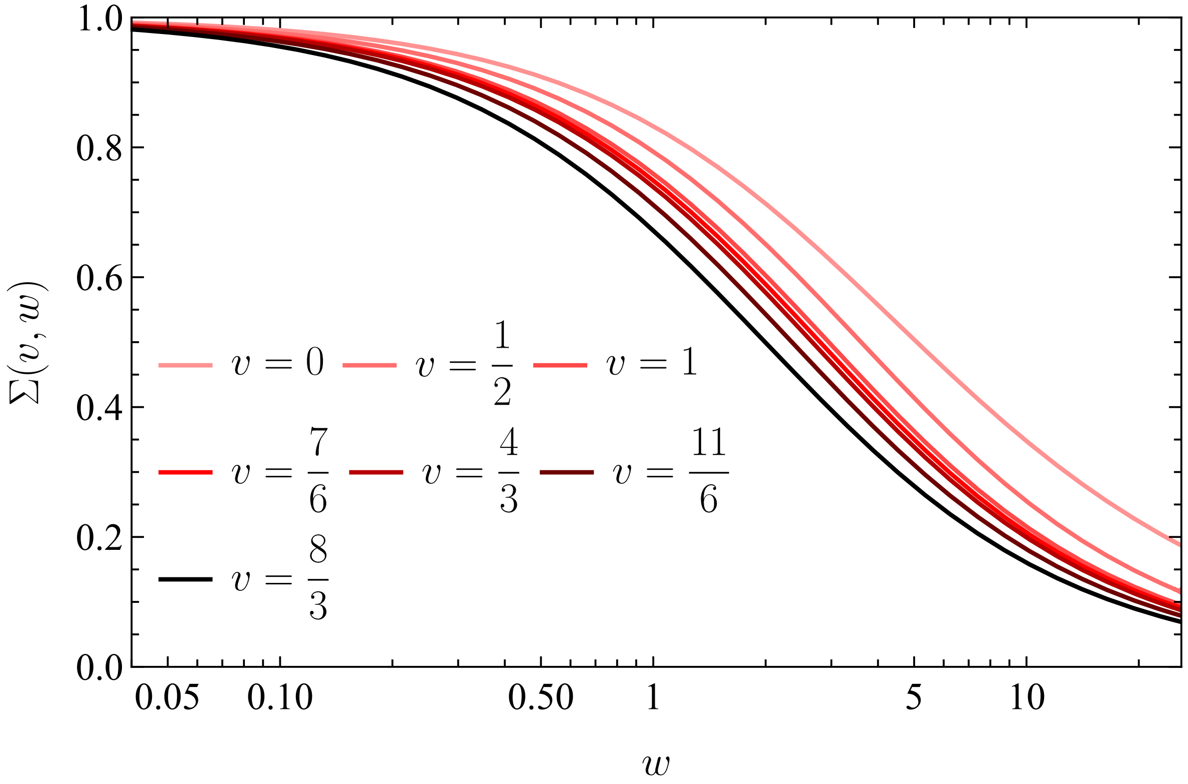
Backup slides

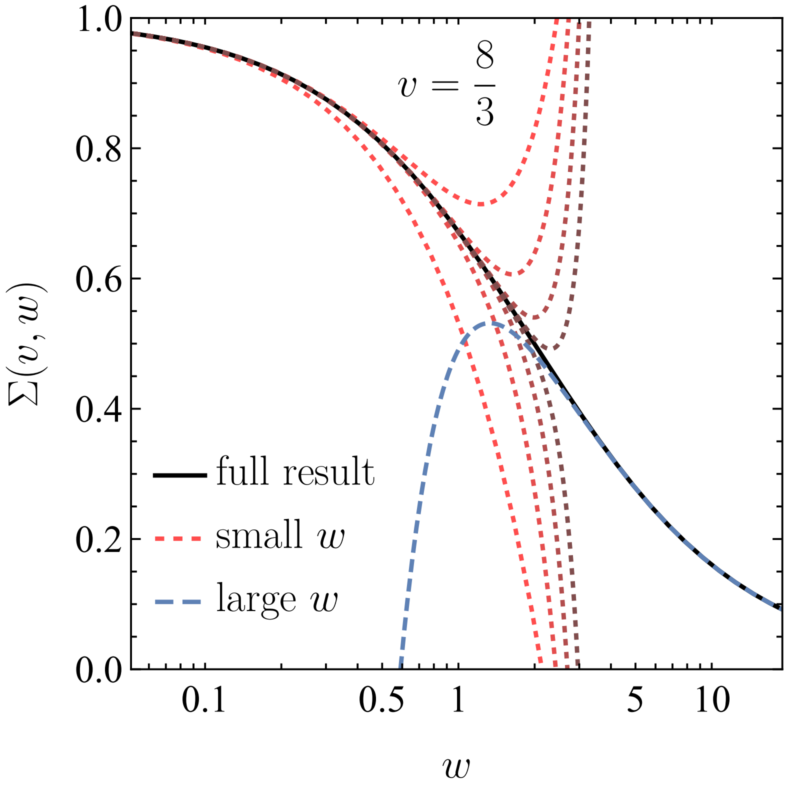
ANALYTIC RESUMMATION AT FIXED COUPLING

Contribution to partonic cross sections (fixed coupling approximation)

- Infinite series can be expressed in closed form in terms of a prefactor times Kampé de Fériet functions $\Sigma(v_i, w)$ with $w = \frac{N_c \alpha_s}{\pi} L^2$ and

$$v_0 = 0\,, \qquad v_1 = \frac{1}{2}\,, \qquad v_2 = 1\,, \qquad v_{3,4} = \frac{3N_c \pm 2}{2N_c}\,, \qquad v_{5,6} = \frac{2\,(N_c \pm 1)}{N_c}$$



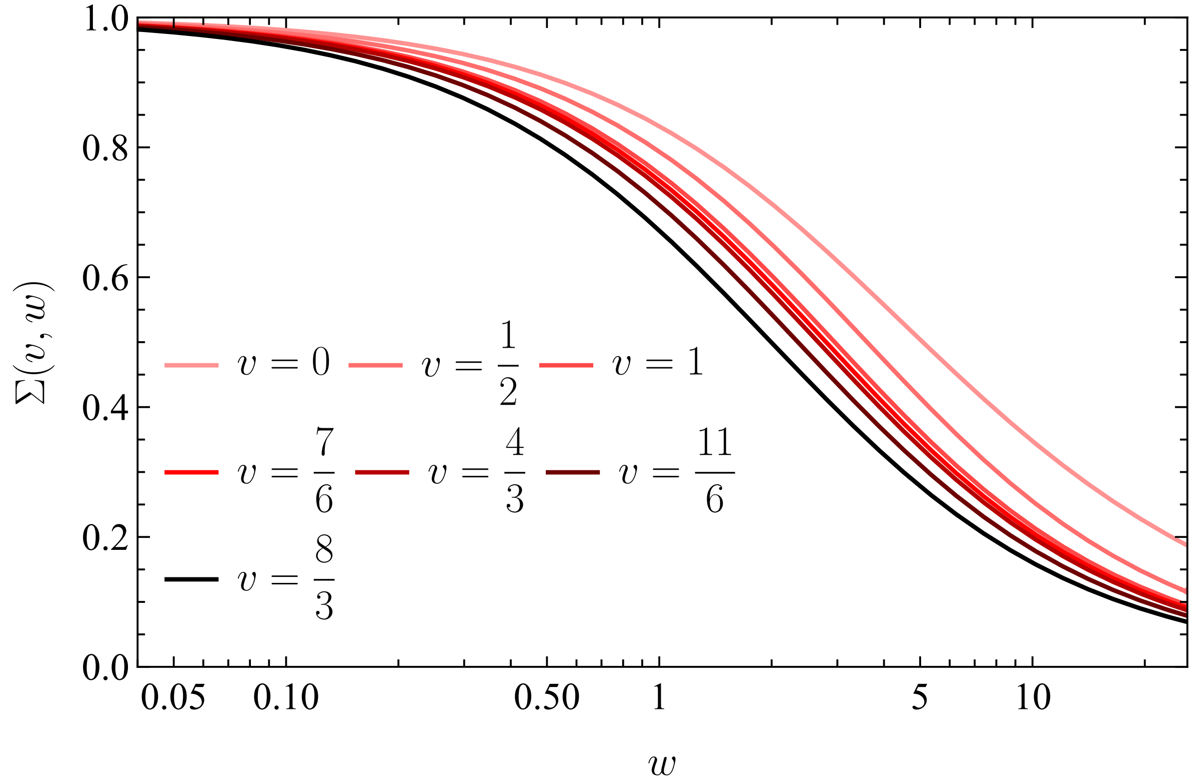


ANALYTIC RESUMMATION AT FIXED COUPLING

Contribution to partonic cross sections (fixed coupling approximation)

- Infinite series can be expressed in closed form in terms of a prefactor times Kampé de Fériet functions $\Sigma(v_i, w)$ with $w = \frac{N_c\alpha_s}{\pi} L^2$ and

$$v_0 = 0\,, \qquad v_1 = \frac{1}{2}\,, \qquad v_2 = 1\,, \qquad v_{3,4} = \frac{3N_c \pm 2}{2N_c}\,, \qquad v_{5,6} = \frac{2\,(N_c \pm 1)}{N_c}$$



Asymptotic behavior for $w \gg 1$:

$$\Sigma_0(w) = \frac{3}{2w}\Big(\ln(4w) + \gamma_E - 2\Big) + \frac{3}{4w^2} + \mathcal{O}(w^{-3})$$

$$\Sigma(v,w) = \frac{3\arctan\left(\sqrt{v-1}\right)}{\sqrt{v-1}\,w} - \frac{3\sqrt{\pi}}{2\sqrt{v}\,w^{3/2}} + \mathcal{O}(w^{-2})$$

$\Rightarrow$ much slower fall-off than Sudakov form factors ~ e^{-cw}

CONSTRUCTION OF THE COLOR BASIS OPERATORS

Comments on the construction of the color basis

- Recall that $\boldsymbol{\Gamma}^c$ and $\boldsymbol{V}^G$ only depend on generators of partons 1 and 2, whereas $\overline{\boldsymbol{\Gamma}}$ brings in the generator of one additional parton *j*

- Hence, there are two types of structures:

$$\zeta\,\boldsymbol{\mathcal{C}}_1\,\widetilde{\boldsymbol{\mathcal{C}}}_2\,\boldsymbol{T}_j \qquad \text{or} \qquad \zeta\,\boldsymbol{\mathcal{C}}_1\,\widetilde{\boldsymbol{\mathcal{C}}}_2$$

- Color structures $\boldsymbol{\mathcal{C}}_i$ contain products of color generators of parton *i*; they carry two *matrix indices* (fundamental or adjoint) as well as an *open adjoint index* for each generator

- Such structures can be built from symmetric products:

$$\boldsymbol{\mathcal{C}}_i^{(k)a_1\dots a_k} = \frac{1}{k!}\sum_{\sigma\in S_k} \boldsymbol{T}_i^{a_{\sigma(1)}}\dots\boldsymbol{T}_i^{a_{\sigma(k)}}$$

CONSTRUCTION OF THE COLOR BASIS OPERATORS

Comments on the construction of the color basis

- Open adjoint indices are contracted with ζ, which can be built from Kronecker δ, f- and d-symbols (higher d-symbols defined recursively):

$$\zeta^{(0)} = 1\,, \qquad \zeta^{(2)a_1a_2} = \delta^{a_1a_2}\,, \qquad \zeta^{(3)a_1a_2a_3} \in \{if^{a_1a_2a_3}\,,\, d^{a_1a_2a_3}\}$$

- For identical initial-state particles, the structures (including angular integrals J_j) need to be symmetric under $1 \leftrightarrow 2$

- For initial-state quarks or anti-quarks, symmetric products of generators can be reduced to linear form

- For initial-state gluons, all indices are adjoint ones

for more details, see **Section 2** in: P. Böer, P. Hager, MN, M. Stillger, X. Xu (arXiv:2311.18811)