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PSR 2024 – Graz

Resummation of non-global observables at subleading accuracy

Nicolas Schalch

based on *Phys. Rev. Lett.* **132** (2024) 081602 with Thomas Becher & Xiaofeng Xu

3RD JULY 2024

Jet processes involve large logarithms

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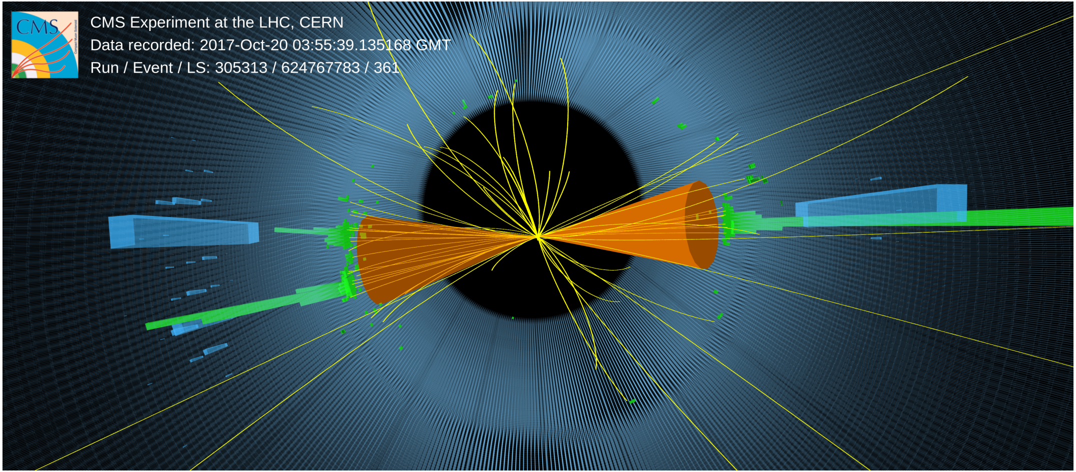

What are non-global logarithms?

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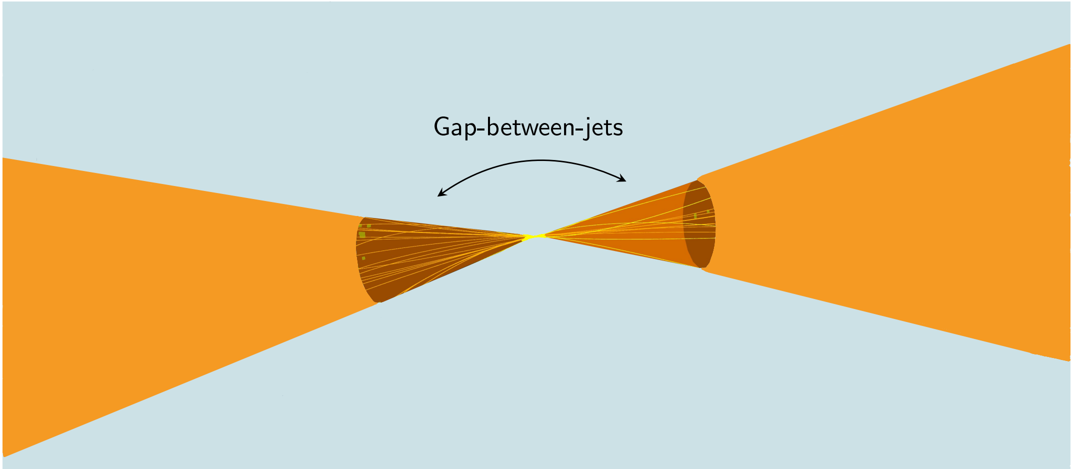

Non-global logarithms are ubiquitous

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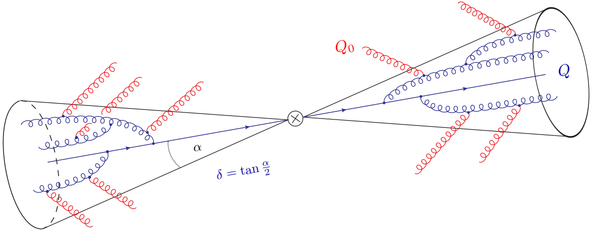
- Jet cross sections involve angular cuts which constrain radiation within a corner of the phase space. As a consequence, logarithmically enhanced higher-order corrections known as Non-Global Logarithms (NGLs) arise. [Dasgupta and Salam, hep-ph/0104277]



Non-global logarithms are ubiquitous

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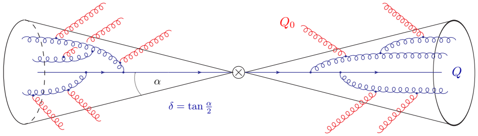
- Jet cross sections involve angular cuts which constrain radiation within a corner of the phase space. As a consequence, logarithmically enhanced higher-order corrections known as Non-Global Logarithms (NGLs) arise.
  [Dasgupta and Salam, hep-ph/0104277]

- Canonical example for NGLs interjet energy flow, one-loop (global) logarithm is

$$\sigma \sim 1 + \frac{g_s^2}{3\pi^2}\left(3\log\delta + 4\log\delta\,\log\frac{Q_0}{Q} + \text{const.}\right)$$

  [Sterman and Weinberg, 1977]

  Large logarithms $\alpha_s^n L^m$ with $L = \log\frac{Q_0}{Q}$ arise, starting at two-loop order



Non-global logarithms

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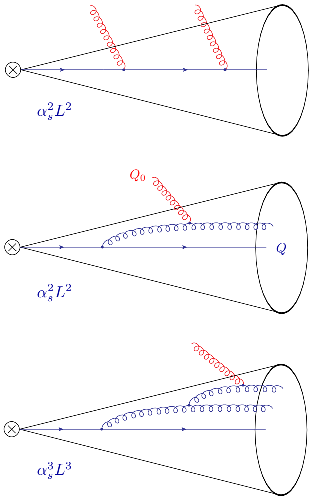
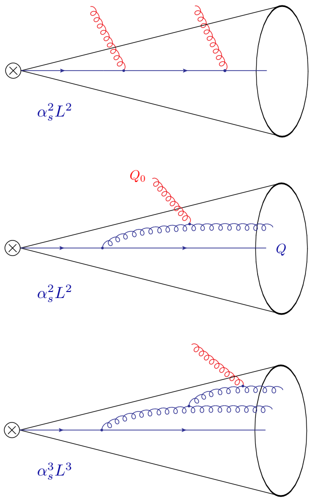


- NGLs arise due to secondary soft gluon emissions inside jets
- Not captured by standard resummation methods
  $\Rightarrow$ even leading NGLs $(\alpha_s L)^n$ do not simply exponentiate
- At large N_c leading NGLs can be obtained with a parton shower [Dasgupta and Salam, hep-ph/0104277] or by solving the non-linear BMS integral equation [Banfi et. al., hep-ph/0206076]

Non-global logarithms: Recent advances

- LL at large N_c with general-purpose shower
  - PanScales [2002.11114, 2207.09467]
  - ALARIC [2208.06057, 2404.14360]
- Finite-N_c results for leading NGLs [Weigert, hep-ph/0312050], [Hatta, Ueda, Hagiwara; 1304.6930, 1507.07641, 2011.04154], [De Angelis, Forshaw, Plätzer; 2007.09648]
- First NLL numerical results in the large-N_c limit
  - Extension of BMS framework to NLL [Monni et. al., 2104.06416] and numerical implementation in MC code GNOLE [Monni et. al., 2111.02413]
  - Ingredients for resummation of subleading effects of NGLs using modern EFT techniques [Becher et. al., 1605.02737, 1901.09038, 2112.02108]
  - Double-soft effects implemented in the PanGlobal family of showers, numerical results available [PanScales, 2307.11142]



Plan

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Parton shower framework

- RG methods
- $\Gamma^{(2)}$ anomalous dimension

Comparison against

- GNOLE
- PanScales

Gap-between-jets at hadron collider

Factorisation theorem

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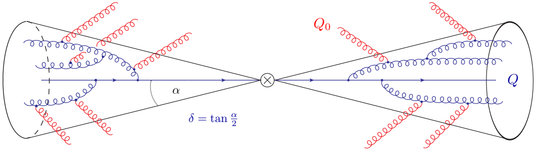
- Cross section for jet production in e^+e^- collisions with veto on radiation factorises into hard $\mathcal{H}_m$ and soft $\mathcal{S}_m$ functions [Becher et. al., 1508.06645]

$$\sigma(Q,Q_0) = \sum_{m=m_0}^{\infty} \langle \mathcal{H}_m(\{\underline{n}\},Q,\mu) \otimes \mathcal{S}_m(\{\underline{n}\},Q_0,\mu)\rangle$$



- Factorisation separates contributions from scales Q and Q_0
  $\Rightarrow$ natural way to perform resummation via RGEs

- Hard functions fulfill RG equations

$$\frac{\mathrm{d}}{\mathrm{d}\log\mu}\mathcal{H}_m(Q,\mu) = -\sum_{l=2}^{m}\mathcal{H}_l(Q,\mu)\,\Gamma_{lm}(Q,\mu)$$

Factorisation theorem

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- $\mathcal{H}_m(\{\underline{n}\},Q,\mu) \sim |\mathcal{M}_m\rangle\langle\mathcal{M}_m|$ describes m hard partons along fixed directions $\{n_1,\cdots,n_m\}$

$$\mathcal{H}_m(\{\underline{n}\},Q,\epsilon) = \frac{1}{2Q^2}\sum_{\text{spins}}\prod_{i=1}^{m}\int\frac{dE_i\,E_i^{d-3}}{\tilde{c}^{\epsilon}\,(2\pi)^2}\,|\mathcal{M}_m(\{\underline{p}\})\rangle\langle\mathcal{M}_m(\{\underline{p}\})|$$
$$\times(2\pi)^d\,\delta\Big(Q-\sum_{i=1}^{m}E_i\Big)\,\delta^{(d-1)}(\vec{p}_{\text{tot}})\,\Theta_{\text{in}}(\{\underline{n}\})$$

- $\mathcal{S}_m(\{\underline{n}\},Q_0,\mu)$ is the squared amplitude with m Wilson lines along fixed directions $\{n_1,\cdots,n_m\}$

$$\mathcal{S}_m(\{\underline{n}\},Q_0,\epsilon) = \sum\!\!\!\!\!\!\int_{X_s}\langle 0|\,\boldsymbol{S}_1^{\dagger}(n_1)\,\ldots\,\boldsymbol{S}_m^{\dagger}(n_m)\,|X_s\rangle\langle X_s|\,\boldsymbol{S}_1(n_1)\,\ldots$$
$$\times\ldots\,\boldsymbol{S}_m(n_m)\,|0\rangle\,\theta(Q_0-E_{\text{out}})$$

Resummation by RG evolution

- Cross section for jet production in e^+e^- collisions with veto on radiation factorises into hard $\mathcal{H}_m$ and soft $\mathcal{S}_m$ functions [Becher et. al., 1508.06645]

$$\sigma(Q, Q_0) = \sum_{m=m_0}^{\infty} \langle \mathcal{H}_m(\{\underline{n}\}, Q, \mu) \otimes \mathcal{S}_m(\{\underline{n}\}, Q_0, \mu) \rangle$$

- Factorisation separates contributions from scales Q and Q_0

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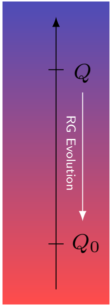
- Hard functions fulfill RG equations

$$\frac{\mathrm{d}}{\mathrm{d}\log\mu} \mathcal{H}_m(Q, \mu) = -\sum_{l=2}^{m} \mathcal{H}_l(Q, \mu)\, \Gamma_{lm}(Q, \mu)$$

Procedure to solve the RGEs

1. Compute $\mathcal{H}_m$ at hard scale $\mu_h = Q$
2. Evolve $\mathcal{H}_m$ to soft scale $\mu_s = Q_0$
3. Evaluate $\mathcal{S}_m$ at soft scale $\mu_s = Q_0$

$\Rightarrow$ Resums large logarithms $\log \frac{Q_0}{Q}$



Resummation by RG evolution

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Clear prescription how to perform resummation at any given accuracy

- LL $\mathcal{H}_2(\mu_h) = \sigma_0 \mathbb{1}$
  $\mathcal{H}_m(\mu_h) = 0$ for $m > 2$
  $\mathcal{S}_m(\mu_s) = \mathbb{1}$
  $\Gamma^{(1)}_{lm}$ one-loop anomalous dimension

- NLL $\mathcal{H}_2(\mu_h) = \sigma_0 |C_V|^2 \, \mathbb{1}$ one-loop virtual
  $\mathcal{H}_3(\mu_h)$ hard real emission corrections
  $\mathcal{S}^{(1)}_m(\mu_s)$ one-loop soft corrections
  $\Gamma^{(2)}_{lm}$ two-loop anomalous dimension

$\mathbf{\Gamma}^{(1)}$ [Becher et. al., 1508.06645]

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- One-loop anomalous dimension $\mathbf{\Gamma}^{(1)}$

$$\mathbf{\Gamma}^{(1)} = \begin{pmatrix} \boldsymbol{V}_2 & \boldsymbol{R}_2 & 0 & 0 & \dots \\ 0 & \boldsymbol{V}_3 & \boldsymbol{R}_3 & 0 & \dots \\ 0 & 0 & \boldsymbol{V}_4 & \boldsymbol{R}_4 & \dots \\ 0 & 0 & 0 & \boldsymbol{V}_5 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$\boldsymbol{R}_m = -4 \sum_{(ij)} \boldsymbol{T}^a_{i,L} \boldsymbol{T}^{\tilde{a}}_{j,R} \, W^q_{ij} \, \theta_{\text{in}}(n_q)$$

$$\boldsymbol{V}_m = 2 \sum_{(ij)} (\boldsymbol{T}_{i,L} \cdot \boldsymbol{T}_{j,L} + \boldsymbol{T}_{i,R} \cdot \boldsymbol{T}_{j,R}) \int [\mathrm{d}\Omega_q] \, W^q_{ij}$$

$$- 2 \sum_{(ij)} \left[\boldsymbol{T}_{i,L} \cdot \boldsymbol{T}_{j,L} - \boldsymbol{T}_{i,R} \cdot \boldsymbol{T}_{j,R}\right] \times i\pi \, \Pi_{ij}$$

Glauber/Coulomb contribution

! $\mathbf{\Gamma}^{(1)}$ infinite dimensional matrix

- No Glauber contribution in e^+e^-

- Singular when soft emission is collinear to hard partons
  $\Rightarrow$ Collinear finite by combining real and virtual

- Two-loop anomalous dimension $\boldsymbol{\Gamma}^{(2)}$ has been calculated by considering (double) soft limits of hard functions

$$\boldsymbol{\Gamma}^{(2)} = \begin{pmatrix} \boldsymbol{v}_2 & \boldsymbol{r}_2 & \boldsymbol{d}_2 & 0 & \dots \\ 0 & \boldsymbol{v}_3 & \boldsymbol{r}_3 & \boldsymbol{d}_3 & \dots \\ 0 & 0 & \boldsymbol{v}_4 & \boldsymbol{r}_4 & \dots \\ 0 & 0 & 0 & \boldsymbol{v}_5 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$\boldsymbol{d}_m$: double real emission

$\boldsymbol{r}_m$: real-virtual correction

$\boldsymbol{v}_m$: double-virtual correction

$$\boldsymbol{d}_m = \sum_{(ij)} \sum_{k} i f^{abc} \left(\boldsymbol{T}^a_{i,L} \boldsymbol{T}^b_{j,L} \boldsymbol{T}^c_{k,R} - \boldsymbol{T}^a_{i,R} \boldsymbol{T}^b_{j,R} \boldsymbol{T}^c_{k,L} \right) K_{ijk;qr} \theta_{\mathsf{in}}(n_q) \theta_{\mathsf{in}}(n_r)$$

$$-2 \sum_{(ij)} \boldsymbol{T}^c_{i,L} \boldsymbol{T}^c_{j,R} K_{ij;qr} \theta_{\mathsf{in}}(n_q) \theta_{\mathsf{in}}(n_r)$$

$$\boldsymbol{r}_m = -2 \sum_{i} \sum_{(jk)} i f^{abc} (\boldsymbol{T}^a_{i,L} \boldsymbol{T}^b_{j,R} \boldsymbol{T}^c_{k,R} - \boldsymbol{T}^a_{i,R} \boldsymbol{T}^b_{j,L} \boldsymbol{T}^c_{k,L}) \int \left[d^2 \Omega_r \right] K_{ijk;qr} \theta_{\mathsf{in}}(n_q)$$

$$- \sum_{(ij)} \boldsymbol{T}^a_{i,L} \boldsymbol{T}^a_{j,R} \left\{ W^q_{ij} \left[4\beta_0 \ln(2W^q_{ij}) + \gamma_1^{\text{cusp}} \right] - 2 \int \left[d^2 \Omega_r \right] K_{ij;qr} \right\} \theta_{\mathsf{in}}(n_q)$$

$$+8 i \pi \sum_{i} \sum_{(jk)} i f^{abc} \left(\boldsymbol{T}^a_{i,L} \boldsymbol{T}^b_{j,R} \boldsymbol{T}^c_{k,R} + \boldsymbol{T}^a_{i,R} \boldsymbol{T}^b_{j,L} \boldsymbol{T}^c_{k,L} \right) W^q_{ij} \ln W^q_{jk} \, \theta_{\mathsf{in}}(n_q)$$

$$\boldsymbol{v}_m = \sum_{(ijk)} i f^{abc} \left(\boldsymbol{T}^a_{i,L} \boldsymbol{T}^b_{j,L} \boldsymbol{T}^c_{k,L} - \boldsymbol{T}^a_{i,R} \boldsymbol{T}^b_{j,R} \boldsymbol{T}^c_{k,R} \right) \int \left[d^2 \Omega_q \right] \int \left[d^2 \Omega_r \right] K_{ijk;qr}$$

$$+ \sum_{(ij)} \frac{1}{2} \left(\boldsymbol{T}^a_{i,L} \boldsymbol{T}^a_{j,L} + \boldsymbol{T}^a_{i,R} \boldsymbol{T}^a_{j,R} \right) \int \left[d^2 \Omega_q \right] W^q_{ij} \left[4\beta_0 \ln(2W^q_{ij}) + \gamma_1^{\text{cusp}} \right]$$

$$- i \pi \sum_{(ij)} \frac{1}{2} \left(\boldsymbol{T}^a_{i,L} \boldsymbol{T}^a_{j,L} - \boldsymbol{T}^a_{i,R} \boldsymbol{T}^a_{j,R} \right) \Pi_{ij} \, \gamma_1^{\mathsf{cusp}}$$

+ additional terms from converting to angular integrals in $d = 4$

Towards a solution of the RGEs

- Started with a factorisation theorem for jet production with veto on radiation which provides natural way to perform resummation via RG evolution of hard functions $\mathcal{H}_m$

$$\sigma(Q, Q_0) = \sum_{m=m_0}^{\infty} \langle \mathcal{H}_m(\{\underline{n}\}, Q, \mu) \otimes \mathcal{S}_m(\{\underline{n}\}, Q_0, \mu) \rangle$$

$$\frac{\mathrm{d}}{\mathrm{d}\log\mu} \mathcal{H}_m(Q, \mu) = -\sum_{l=2}^{m} \mathcal{H}_l(Q, \mu)\, \Gamma_{lm}(Q, \mu)$$

The RG evolution is governed by the anomalous dimension which has been extracted up to two-loops

$$\mathbf{\Gamma} = \frac{\alpha_s}{4\pi}\mathbf{\Gamma}^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 \mathbf{\Gamma}^{(2)} + \cdots \qquad \Rightarrow \quad \text{NLL accuracy}$$

? How to solve complicated RGEs ?

$\Rightarrow$ Monte-Carlo Methods

Parton Shower at LL accuracy

- In practice coupled RGEs for hard functions $\mathcal{H}_m$, however these simplify at LL due to the form of $\mathbf{\Gamma}^{(1)}$

$$\mathbf{\Gamma}^{(1)} = \begin{pmatrix} \boldsymbol{V}_2 & \boldsymbol{R}_2 & 0 & 0 & \dots \\ 0 & \boldsymbol{V}_3 & \boldsymbol{R}_3 & 0 & \dots \\ 0 & 0 & \boldsymbol{V}_4 & \boldsymbol{R}_4 & \dots \\ 0 & 0 & 0 & \boldsymbol{V}_5 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

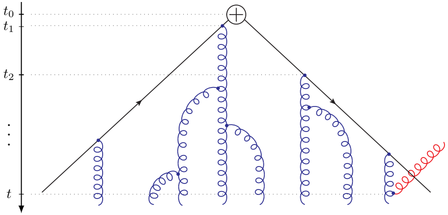
$$\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{H}_m(t) = \mathcal{H}_m(t)\boldsymbol{V}_m + \mathcal{H}_{m-1}(t)\boldsymbol{R}_{m-1}$$

$$\mathcal{H}_m(t) = \mathcal{H}_m(t_0)e^{(t-t_0)\boldsymbol{V}_m} + \int_{t_0}^{t}\mathrm{d}t'\,\mathcal{H}_{m-1}(t')\boldsymbol{R}_{m-1}e^{(t-t')\boldsymbol{V}_m}$$

- Introduce shower-time t

$$\mu \to t = \frac{\alpha_s}{4\pi}\log\frac{\mu_h}{\mu_s} + \mathcal{O}(\alpha_s^2)$$

[Balsiger et. al., 1803.07045]

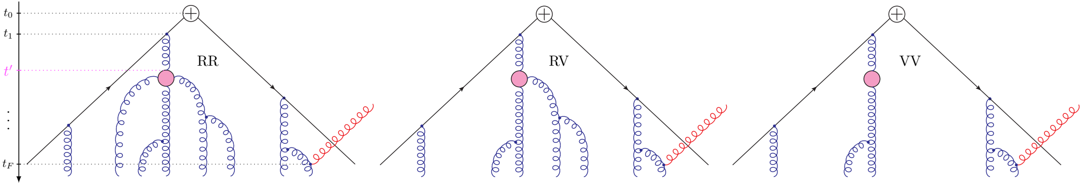


Parton Shower: NLL

- Include corrections of $\mathcal{H}_m$ due to $\Gamma^{(2)}$ $\Rightarrow$ NLL resummation

$$\Delta\mathcal{H}_m(t) = \mathcal{H}_k(t_0)\Delta\boldsymbol{U}_{km}(t,t_0)$$
$$= \mathcal{H}_k(t_0)\int_{t_0}^{t}\mathrm{d}t'\, \boldsymbol{U}_{kl}(t'-t_0)\cdot \frac{\alpha(t')}{4\pi}\left(\Gamma^{(2)}_{ll'} - \frac{\beta_1}{\beta_0}\Gamma^{(1)}_{ll'}\right)\cdot \boldsymbol{U}_{l'm}(t-t')$$

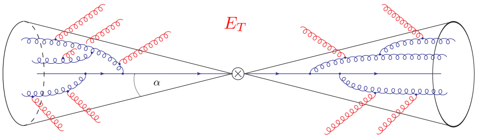
LL evolution | Insertion of $\Gamma^{(2)}$ | LL evolution



Validation: NLL resummation for the interjet energy flow

- Gap fraction: fraction of events with transverse energy E_T in gap below Q_0

$$R(Q_0) \equiv \frac{1}{\sigma_{\rm tot}} \int_0^{Q_0} dE_T \, \frac{d\sigma}{dE_T}$$



- Ongoing work with Pier Monni on detailed numerical comparison with
  - GNOLE [2111.02413]
  - PanScales [2307.11142]
- Compare both
  - 't Hooft limit: $N_c \to \infty, n_F$ fixed
  - Veneziano limit: $N_c \to \infty, \frac{n_F}{N_c}$ fixed
- Delicate to isolate pure NLL correction
  - GNOLE and PanScales: extrapolation $\alpha_s \to 0$
  - Very small collinear cutoffs & high statistics

Subleading NGLs at Hadron Colliders

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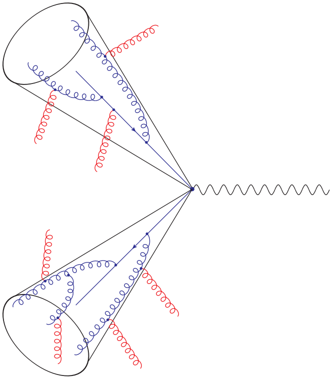
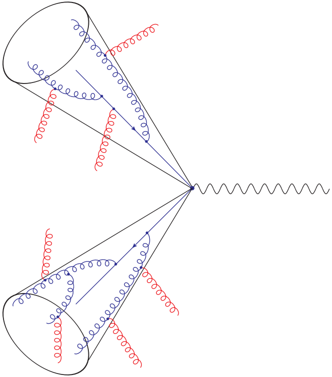


Simplest observable: gap fraction in Z – production

- Gap is defined by vetoing hadronic radiation at central rapidities in interval ΔY around the beam
- Cross section factorises in the large-N_c limit exactly in the same way as for e^+e^-
- Same observables used by PanScales collaboration
- First relevant process in a long list

Subleading NGLs at Hadron Colliders



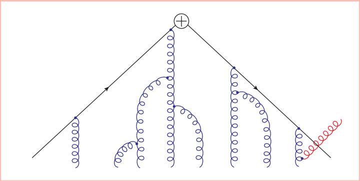
- Required ingredients to resume subleading NGLs
  - One-loop virtual $\mathcal{H}_2(\mu_h) = \sigma_0 |C_V|^2 \, \mathbb{1}$ (✓)
  - Hard real emission corrections $\mathcal{H}_3^{(1)}(\mu_h)$ ✗
  - One-loop soft corrections $\mathcal{S}_m^{(1)}(\mu_s)$ (✓)
  - Two-loop anomalous dimension $\Gamma_{lm}^{(2)}$ (✓)

Extraction of $\mathcal{H}_2^{(1)}$ and $\mathcal{H}_3^{(1)}$

- The virtual corrections due to $\mathcal{H}_2^{(1)}$ factorise such that we obtain these by multiplying the standard dijet hard function H_2 to the LL result $\mathcal{S}_2$

$$\frac{\alpha_s(\mu_h)}{4\pi}\sum_{m=2}^{\infty}\langle \mathcal{H}_2^{(1)}(Q,\mu_h)\otimes \boldsymbol{U}_{2m}(\mu_s,\mu_h)\,\hat{\otimes}\,1\rangle = \sigma_0\, H_2(Q,\mu_h)\, \langle \mathcal{S}_2(\{\bar{n},n\},Q_0,\mu_h)\rangle$$

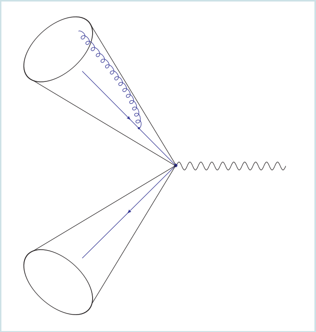
$$C_F\left[-8\ln^2\frac{\mu}{Q}-12\ln\frac{\mu}{Q}-16+\frac{7}{3}\pi^2\right]$$

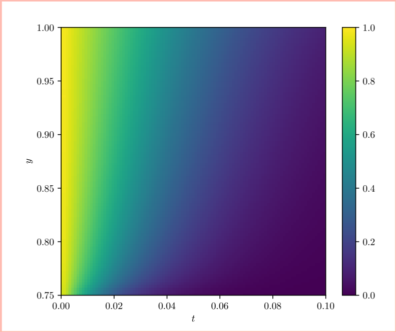


Extraction of $\mathcal{H}_2^{(1)}$ and $\mathcal{H}_3^{(1)}$

- To calculate the contribution due to $\mathcal{H}_3^{(1)}$, which depends on $\theta_{qg}(y)$, we convolute the real corrections of $q\,\bar{q} \to Z$ with the three particle soft function which is obtained via a LL RG evolution

$$\frac{\alpha_s(\mu_h)}{4\pi} \sum_{m=3}^{\infty} \langle\, \mathcal{H}_3^{(1)}(\{n_1, n_2, n_3\}, Q, \mu_h) \otimes \boldsymbol{U}_{3m}(\{\underline{n}\}, \mu_s, \mu_h)\, \hat{\otimes}\, 1 \,\rangle$$

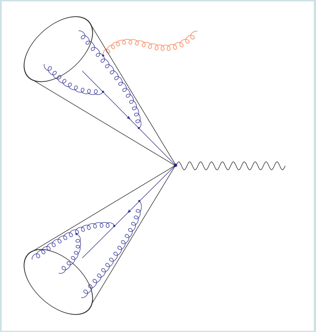




Calculation of $\mathcal{S}_m^{(1)}$ and ΔU_{2m}

- The one-loop corrections to the soft function $\mathcal{S}_m^{(1)}$, which represents the emission of a soft particle into the gap, is directly obtained from our shower

$$\frac{\alpha_s(\mu_h)}{4\pi}\sum_{m=2}^{\infty}\left\langle \mathcal{H}_2^{(0)}(Q,\mu_h)\otimes \boldsymbol{U}_{2m}(\mu_s,\mu_h)\,\hat{\otimes}\,\mathcal{S}_m^{(1)}(Q_0,\mu_s)\right\rangle$$

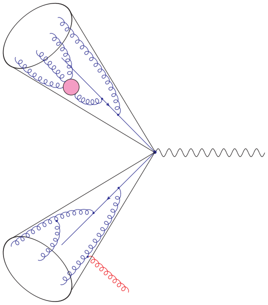


$$\int \frac{d^dk}{(2\pi)^{d-1}}\frac{W_{ij}^k}{E_k^2}\,\delta(k^2)\,\theta(k^0)\,\theta(Q_0-k_T)\,\Theta_{\text{out}}(n_k)$$

Calculation of $\mathcal{S}_m^{(1)}$ and ΔU_{2m}

- The contributions due to the insertion of the two-loop anomalous dimension is obtained within our parton shower framework; in practice, we start a LL shower prior to the insertion and then restart a LL shower

$$\sum_{m=2}^{\infty} \langle \mathcal{H}_2^{(0)}(Q,\mu_h) \otimes \boldsymbol{\Delta U}_{2m}(\mu_s,\mu_h) \,\hat{\otimes}\, 1 \rangle$$



$$\begin{aligned}
\boldsymbol{d}_m &= +N_c \left(K_{ij;qr} + K_{ji;qr}\right) \theta_{\text{in}}(n_q)\, \theta_{\text{in}}(n_r) \\
&\quad - 8\, N_c^2\, M_{ij;qr}\, \theta_{\text{in}}(n_q)\, \theta_{\text{in}}(n_r) \\
\boldsymbol{r}_m &= -N_c \int [d^2\Omega_r] \left(K_{ij;qr} + K_{ji;qr}\right) \theta_{\text{in}}(n_q) \\
&\quad + 8\, N_c^2 \int [d^2\Omega_r]\, M_{ij;qr}\, \theta_{\text{in}}(n_q) \\
&\quad - N_c \left(4\beta_0\, X_{ij}^q - \gamma_1^{\text{cusp}}\, W_{ij}^q\right) \theta_{\text{in}}(n_q) \\
\boldsymbol{v}_m &= +N_c \int [d^2\Omega_q] \left(4\beta_0\, X_{ij}^q - \gamma_1^{\text{cusp}}\, W_{ij}^q\right)
\end{aligned}$$

Resummed gap fraction for $pp \to Z \to \ell^+\ell^- + X_{had}$

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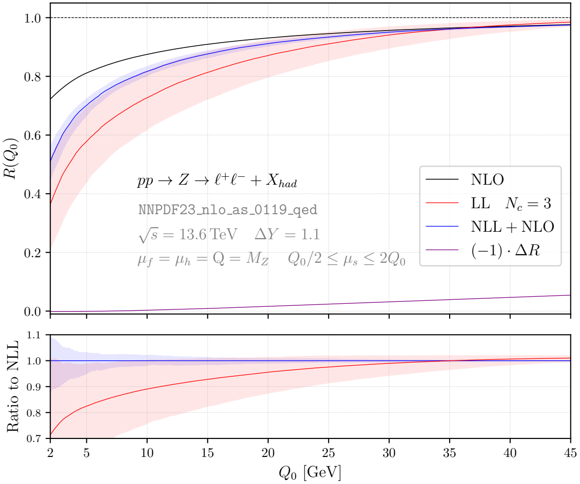

- Many ingredients the same as for e^+e^- case
- $N_c = 3$ LL obtained from [Hatta and Ueda; 1304.6930]

Glauber phases neglected, but superleading logarithms turn out to be small for $q\bar{q} \to Z$

Fixed order versus resummation

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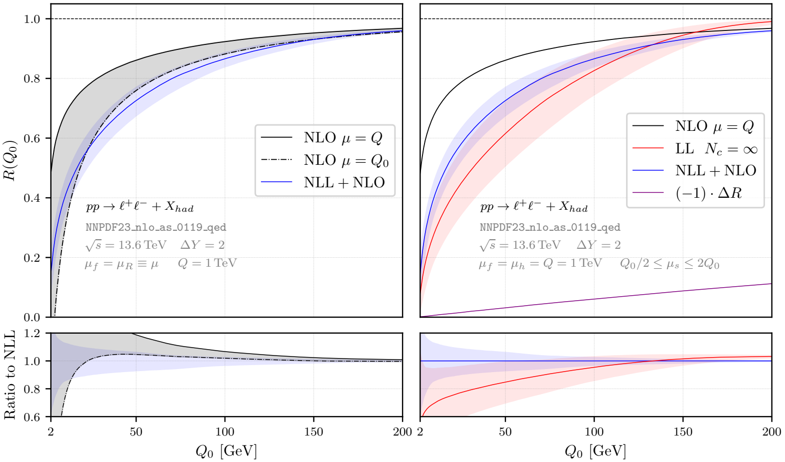
$\mu = Q$ or $\mu = Q_0$?



Conclusion

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Summary

- Fiducial cuts lead to phase-space constraints
  $\Rightarrow$ Intricate pattern of logs: NGLs (& SLL)
- Implemented two-loop anomalous dimension in PS
  $\Rightarrow$ NLL resummation for gap fraction
  $\Rightarrow$ First results for Hadron Collider process

Outlook

- NNLL accuracy for non-global event shapes
  $\Rightarrow$ e.g. Jet mass
- Photon isolation at the LHC

BACKUP

Extraction of anomalous dimension

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- Anomalous dimension $\mathbf{\Gamma}$ arises from soft singularities of hard functions

$$\mathcal{H}_m(Q,\mu) = \sum_{l=2}^{m} \mathcal{H}_l^{\text{bare}}(Q,\mu)\ \left(\mathbf{Z}^{-1}\right)_{lm}(Q,\mu)$$

$$\left(\mathbf{Z}^{-1}\right) = \mathbb{1} + \frac{\alpha_s}{4\pi}\frac{1}{2\epsilon}\mathbf{\Gamma}^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2\left[\frac{1}{8\epsilon^2}\mathbf{\Gamma}^{(1)}\otimes\mathbf{\Gamma}^{(1)} - \frac{\beta_0}{4\epsilon^2}\mathbf{\Gamma}^{(1)} + \frac{1}{4\epsilon}\mathbf{\Gamma}^{(2)}\right]$$

$\mathbf{\Gamma}^{(1)}$ is $(-2)\times$ soft divergence of the one-loop hard function

Renormalisation $Z_\alpha = 1 - \frac{\beta_0}{\epsilon}\frac{\alpha_S}{4\pi}$

$\mathbf{\Gamma}^{(2)}$ is (-4) times single pole of the two-loop hard function

- At $\mathcal{O}(\alpha_s)$ soft singularities arise when either a real or a virtual gluon becomes soft

Angular functions

- Three-leg correlations combine with ϵ-terms

$$K_{ijk;qr} = 8\left(W^q_{ik}W^r_{jk} - W^q_{ik}W^r_{jq} - W^q_{ir}W^r_{jk} + W^q_{ij}W^r_{jq}\right)\ln\left(\frac{n_{kq}}{n_{kr}}\right)$$

$$M_{ij;qr} = \left(W^q_{ik}W^r_{jk} - W^q_{ik}W^r_{jq} - W^q_{ir}W^r_{jk} + W^q_{ij}W^r_{jq}\right)\ \ln\left(\frac{s^2_{\phi_{qr}}}{s^2_{\phi_{qx}}}\right)$$

- Two-leg correlations (diverges for $q \parallel r$)

$$K_{ij;qr} = C_A K^{(a)}_{ij;qr} + [n_F T_F - 2C_A]\, K^{(b)}_{ij;qr} + [C_A - 2n_F T_F + n_S T_S]\, K^{(c)}_{ij;qr}$$

$$K^{(a)}_{ij;qr} = \frac{4n_{ij}}{n_{iq}n_{qr}n_{jr}}\left[1 + \frac{n_{ij}n_{qr}}{n_{iq}n_{jr} - n_{ir}n_{jq}}\right]\ln\frac{n_{iq}n_{jr}}{n_{ir}n_{jq}}$$

$$K^{(b)}_{ij;qr} = \frac{8n_{ij}}{n_{qr}(n_{iq}n_{jr} - n_{ir}n_{jq})}\ln\frac{n_{iq}n_{jr}}{n_{ir}n_{jq}}$$

$$K^{(c)}_{ij;qr} = \frac{4}{n^2_{qr}}\left(\frac{n_{iq}n_{jr} + n_{ir}n_{jq}}{n_{iq}n_{jr} - n_{ir}n_{jq}}\ln\frac{n_{iq}n_{jr}}{n_{ir}n_{jq}} - 2\right)$$

Numerical solution of RGEs

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RGEs not yet in a suitable form for implementation in a MC framework

- Change variables from $\mu \to t = \frac{\alpha_s}{4\pi} \log \frac{\mu_h}{\mu_s} + \mathcal{O}(\alpha_s^2)$

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle \mathcal{H}(t)| = \langle \mathcal{H}(t)|\,\hat{\Gamma}(t) \qquad \to \text{formal solution} \qquad \langle \mathcal{H}(t)| = \langle \mathcal{H}(0)|\,\mathbb{P}\exp\left[\int_0^t \mathrm{d}t'\,\hat{\Gamma}(t')\right]$$

- Expand anomalous dimension perturbatively $\hat{\Gamma}(t) = \Gamma^{(1)}(t) + \frac{\alpha_s}{4\pi}\Delta\Gamma(t) + \mathcal{O}(\alpha_s^2)$ → Interaction picture

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle \mathcal{H}^I(t)| = \langle \mathcal{H}^I(t)|\, e^{t\,\Gamma^{(1)}}\left[\frac{\alpha_s}{4\pi}\Delta\Gamma(t)\right]e^{-t\,\Gamma^{(1)}}$$

- Solve RG evolution iteratively including subleading contributions due to $\Delta\Gamma$

$$\sigma \sim \langle \mathcal{H}(t)|\mathcal{S}(t)\rangle = \langle \mathcal{H}(0)|\left[e^{t\,\Gamma^{(1)}} + \int_0^t \mathrm{d}t'\, e^{t'\,\Gamma^{(1)}}\left[\frac{\alpha_s}{4\pi}\Delta\Gamma(t')\right]e^{(t-t')\,\Gamma^{(1)}}\right]|\mathcal{S}(t)\rangle \quad \to \quad \text{suitable for MC}$$

Perturbative setup $q\,\bar{q} \to Z$

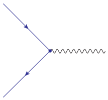
- The differential cross section for Z – Boson production is written as a convolution between the partonic cross section $\mathrm{d}\hat{\sigma}_{ij}$ and the PDFs f_l

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2} = \frac{4\pi^2\alpha}{N_c\, s} \sum_{i,j} \int \mathrm{d}x_1 \mathrm{d}x_2\, f_i(x_1)\, f_j(x_2)\, \frac{\mathrm{d}\hat{\sigma}_{ij}}{\mathrm{d}Q^2}$$

$$\frac{\mathrm{d}\hat{\sigma}_{ij}}{\mathrm{d}Q^2} = \int \mathrm{d}\Pi_f\, |\mathcal{M}_{ij}|^2\, \delta\left(z - \frac{Q^2}{\hat{s}}\right) = \hat{\Sigma}^{(0)}_{ij} + \left(\frac{\alpha_S}{4\pi}\right)\hat{\Sigma}^{(1)}_{ij} + \mathcal{O}(\alpha_s^2)$$

- The LO partonic cross section is expressed in terms of a delta-distribution in $z = \frac{Q^2}{\hat{s}}$

$$\frac{\hat{\Sigma}^{(0)}_{q\bar{q}}}{e_q^2} \sim \int \mathrm{d}\Pi_f \,\Bigg|$$



$$\Bigg|^2 \sim \delta(1-z)$$

NLO corrections to $q\,\overline{q} \to Z$

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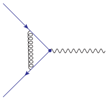
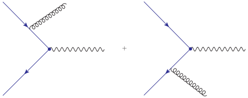
- At NLO we obtain virtual and real contributions $\Rightarrow$ dimensional regularisation in $d = 4 - 2\epsilon$ to make both UV and IR divergences explicit, e.g. the $q\,\overline{q}$ - channel yields

$$\frac{\hat{\Sigma}^{(1)}_{q\overline{q}}}{e_q^2} \sim \int \mathrm{d}\Pi_f \left| \quad \right|^2 + \int \mathrm{d}\Pi_f \left| \quad + \quad \right|^2$$





$$\sim C_F \Big\{ \big(\delta(y) + \delta(1-y)\big) \Big[\delta(1-z)\big(2\zeta_2 - 4\big) + 4\left[\frac{\log(1-z)}{1-z}\right]_+ - 2\,(1+z)\log(1-z) - \frac{(1+z^2)}{1-z}\log(1-z) + 1 - z \Big]$$

$$+ \Big[(1+z^2)\left[\frac{1}{1-z}\right]_+ \left(\left[\frac{1}{y}\right]_+ + \left[\frac{1}{1-y}\right]_+ \right) - 2\,(1-z) \Big] + \mathcal{O}(\epsilon) \Big\}$$

! New channel opens up: $\hat{\Sigma}^{(1)}_{qg}$ also needs to be taken into account

Finite-N_c corrections

- Obtained result for $N_c = 3$ from [Hatta and Ueda; 1304.6930 + improved numerics]