Towards a NLO Parton shower: Gauge choice

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DEDUCTOR

In Decuctor we did everything at LO level that is humanly possible:

- ✓ All order definition of parton shower cross sections
  - ✓ Evolution at density operator level (*aka* amplitude level evolution)
  - ✓ Multi-variable evolution (angular ordering, …)
- ✓ Colour evolution beyond the leading colour approximation
  - ✓ LC+ base approximation
  - ✓ subleading colour treated perturbatively
  - ✓ Fully exponentiated Glauber gluon effect
- ✓ Summation of threshold logarithms
  - ✓ PDF factorisation schemes vs. shower evolution variable
- ✓ Testing log accuracy of parton showers analytically
- ✗ We haven’t implemented spin dependence

It is time to move on and go beyond the first order!

Shower Kernel

At second order level we are not that lucky. The shower kernel is much more complicated:

$$\frac{1}{\mu_S^2} S^{(2)}(\mu^2) = \mathcal{F}(\mu^2)\left(\frac{\partial \mathcal{D}^{(2)}(\mu^2,\mu_S^2)}{\partial \mu_S^2} - \mathcal{D}^{(1)}(\mu^2)\frac{\partial \mathcal{D}^{(1)}(\mu^2,\mu_S^2)}{\partial \mu_S^2}\right)_{\mu_S^2=\mu^2} \mathcal{F}^{-1}(\mu^2)$$
$$-\left(\frac{\partial \mathcal{V}^{(2)}(\mu^2,\mu_S^2)}{\partial \mu_S^2} - \mathcal{V}^{(1)}(\mu^2)\frac{\partial \mathcal{V}^{(1)}(\mu^2,\mu_S^2)}{\partial \mu_S^2}\right)_{\mu_S^2=\mu^2}$$
$$+\left[\mathcal{V}^{(1)}(\mu^2), \frac{1}{\mu^2}\mathcal{S}^{(1)}(\mu^2)\right]$$

This is highly non-trivial operator and cancelation of all the singularities in the first term is rather delicate.

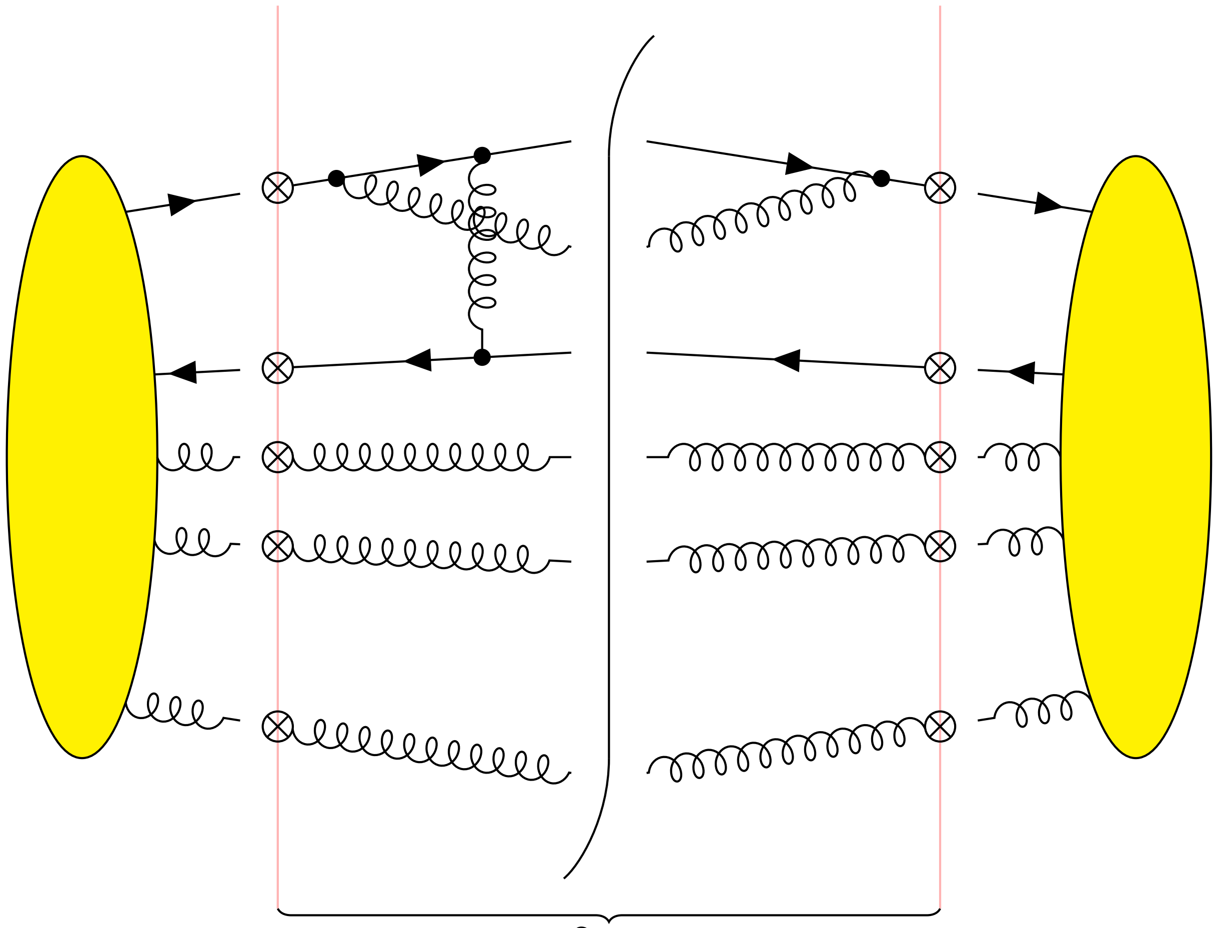
$$\mathcal{D}^{(2)}(\mu^2,\mu_S^2) = \overbrace{\mathcal{D}^{(2,0)}(\mu^2,\mu_S^2)}^{\text{Double real}} + \underbrace{\mathcal{D}^{(1,1)}(\mu^2,\mu_S^2)}_{\text{Real-virtual}} + \overbrace{\mathcal{D}^{(0,2)}(\mu^2,\mu_S^2)}^{\text{Double virtual}}$$

and

$$\mathcal{D}^{(1)}(\mu^2,\mu_S^2) = \overbrace{\mathcal{D}^{(1,0)}(\mu^2,\mu_S^2)}^{\text{Single real}} + \underbrace{\mathcal{D}^{(0,1)}(\mu^2,\mu_S^2)}_{\text{Single virtual}}$$

Infrared sensitive operator

Consider the momenta coming from the hard part as fixed and on shell.



This gives us an operator as

$$\big(\{\hat{p},\hat{f},\hat{s},\hat{s}',\hat{c},\hat{c}'\}_{m+n_\mathrm{R}}\big|\rho(\mu^2)\big)$$

$$\sim \frac{1}{m!}\int[d\{p\}_m]\sum_{\{f\}_m}\sum_{\{s,s',c,c'\}_m}$$

$$\times\big(\{\hat{p},\hat{f},\hat{s},\hat{s}',\hat{c},\hat{c}'\}_{m+n_\mathrm{R}}\big|\mathcal{D}(\mu^2)\big|\{p,f,s,s',c,c'\}_m\big)$$

$$\times\big(\{p,f,s,s',c,c'\}_m\big|\rho_\mathrm{hard}(\mu^2)\big)$$

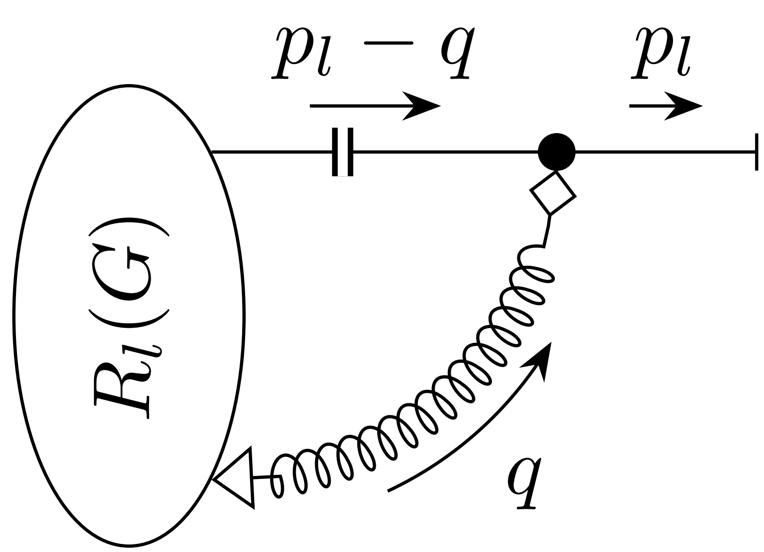
Interpolating Gauge

- I will describe the **interpolating gauge** invented by Doust (1987) and probably reinvented by Baulieu and Zwanziger (1999) and we also reinvented (2022) before we found it in the literature.
  - Doust wanted to fix the ambiguities of the Coulomb gauge Feynman integrals.
  - Baulieu and Zwanziger wanted to understand confinement.
- Our interest in simplifying the soft and collinear singularities of QCD amplitudes.
- This may be useful to **define the infrared sensitive operator** that is need for subtraction terms for fixed order calculation and for splitting operators in parton showers.
- Our particular interest in defining the splitting operators for a **parton shower at order** α_s^2 **level**.
- Interpolating gauge interpolates between **Feynman gauge** (or Lorenz gauge) and **Coulomb gauge**.
- With our different goal we adopt a different notation and emphasise different features of the gauge.
- We also explore technical issues in some detail.

Why not Feynman gauge?

The gluon propagator is very simple in Feynman gauge, but consider the graphs with a virtual gluon that couples to an external leg

$$\sum_l \int \frac{d^d q}{(2\pi)^d} \sum_G$$



- There are **collinear singularities** that give a logarithmic divergence from $q \to x p_l$.
- The collinear divergences appear even when the gluon connects to an off-shell internal line in the graph.
- The unphysical polarisations of the gluon causes the problem and we can get rid of the by using the Ward identity.
- This is rather simple at 1-loop level, but higher order level it gets extremely complicated very quickly.
- It might be better if these **unphysical collinear singularities don't occur**.

Interpolating gauge

- Use a **special reference** frame defined by a light like vector n^μ, with $n^2 = 1$.
- Define a **tensor** $h^{\mu\nu}$ as

$$h^{\mu\nu} = g^{\mu\nu} - \left(1 - \frac{1}{v^2}\right) n^\mu n^\nu$$

In the $\vec{n} = 0$ frame this tensor is

$$h^{\mu\nu} = \begin{pmatrix} 1/v^2 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

- For any vector q^μ we define an **associated vector** $\tilde{q}^\mu$ by

$$\tilde{q}^\mu = h^{\mu\nu} q_\nu \qquad \tilde{q}^\mu = \left(\frac{1}{v^2} E, \vec{q}\right)$$

- Note, in the $v^2 \to 0$ limit $h^{\mu\nu} \to g^{\mu\nu}$
- Use the gauge fixing condition $G[A] = 0$ with

$$G[A]_c(x) = \tilde{\partial}_\mu A^\mu_c(x) - \omega_c(x)$$

Compare this to

$$G[A]_c(x) = \partial_\mu A^\mu_c(x) - \omega_c(x)$$

for covariant gauge

Interpolating gauge

➠ The gauge fixing Langrangian with two gauge parameters (v, ξ) is

$$\mathcal{L}_{\rm GF}(x) = -\frac{v^2}{2\xi}\,(\tilde{\partial}_\mu A_a^\mu(x))(\tilde{\partial}_\nu A_a^\nu(x))$$

➠ With this the **gluon propagator** is

$$D^{\mu\nu}(q) = \frac{1}{q^2+\mathrm{i}0}\left[-\,g^{\mu\nu} + \frac{q^\mu\,\tilde{q}^\nu + \tilde{q}^\mu\,q^\nu}{q\cdot\tilde{q}+\mathrm{i}0} - \left(1+\frac{1}{v^2}\right)\frac{q^\mu\,q^\nu}{q\cdot\tilde{q}+\mathrm{i}0}\right] - \frac{\xi-1}{v^2}\,\frac{q^\mu\,q^\nu}{(q\cdot\tilde{q}+\mathrm{i}0)^2}$$

➠ Usually we choose $\xi = 1$.

➠ The **ghost propagator** is

$$\widetilde{D}(q) = \frac{1}{q\cdot\tilde{q}+\mathrm{i}0}$$

Gluon propagator

Introducing the polarisation vectors in the usual way, the gluon propagator can be decomposed as

Describes the propagation of the **transversely polarised** gluons (T-gluons)

$$D^{\mu\nu}(q) = \overbrace{\frac{1}{q^2 + \mathrm{i}0} \sum_{\lambda=1,2} \varepsilon_\lambda^\mu(q) \varepsilon_\lambda^\nu(q)^*}+ \underbrace{\frac{-\varepsilon_0^\mu(q)\varepsilon_0^\nu(q)^* + \varepsilon_3^\mu(q)\varepsilon_3^\nu(q)^*}{q\cdot\tilde{q} + \mathrm{i}0}}$$

Describes the propagation of the **longitudinally polarised** gluons (L-gluons)

Note, that $q\cdot\tilde{q} = \frac{1}{v^2}E^2 - \vec{q}^{\,2}$, and the condition for on-shell propagation is

$$E = \pm v|\vec{q}|$$

L-gluons propagate with speed v in the $\vec{n} = 0$ frame.

We usually take $v > 1$, this means the **L-gluons are tachyonic states**. This is also true for the ghost fields.

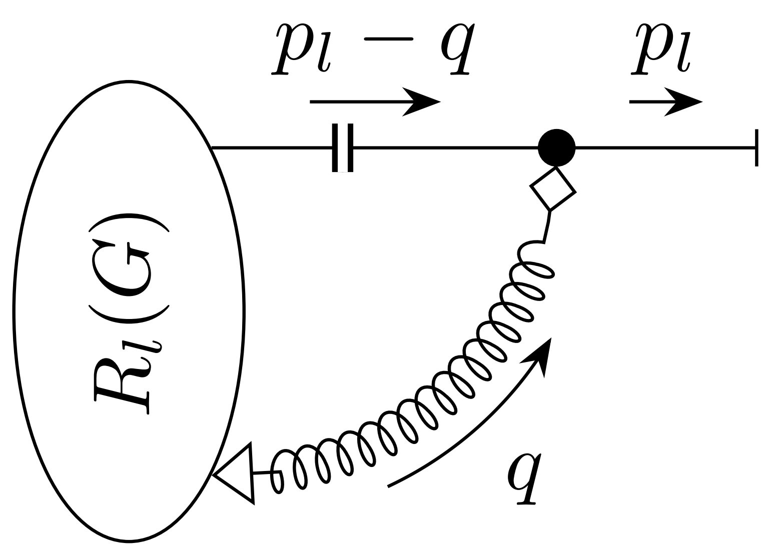
Interpolating gauge

Why is it called interpolating gauge?

- If we take $v = 1$, we get Feynman gauge for $\xi = 1$ and Lorenz gauge for $\xi = 0$.
- If we take $v \to \infty$, we get Coulomb gauge.
- The L-gluons give the **Coulomb force**, which propagates with **infinite speed** in the $\vec{n} = 0$ frame.
- Now the Coulomb gauge is defined as a limit. This way the loop integrals are defined unambiguously.
- We don't need $v \to \infty$, $v = \sqrt{2}$ is **perfectly fine**.

Collinear singularities

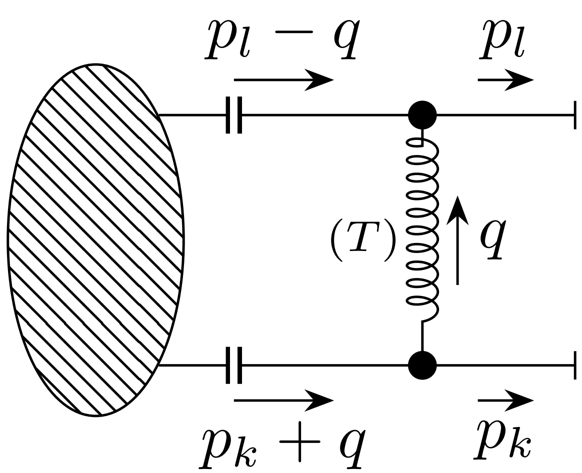
$$\sum_l \int \frac{d^d q}{(2\pi)^d} \sum_G$$



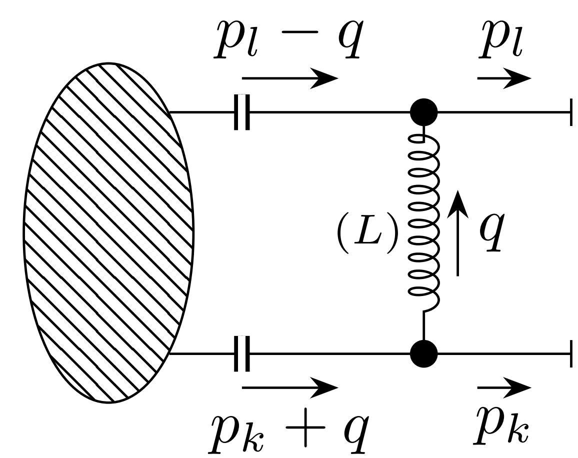
- T-gluons **do not** give collider singularities except for self-energy insertion on external legs, that is because $q \cdot \varepsilon_\lambda(q) = 0$.
- L-gluon **do not** give collinear singularities because if $p_l^2 = 0$ and $q \to x p_l$, then $(p_l - q)^2 \to 0$, but $q \cdot \tilde{q} \neq 0$.
- Thus interpolating gauge is **like physical gauge** respect to collinear divergences.

Soft singularities

$$\sum_{l<k} \int \frac{d^d q}{(2\pi)^d}$$



$$\sum_{l<k} \int \frac{d^d q}{(2\pi)^d}$$



- Both T-gluons and L-gluons create soft divergences ($q^\mu \to 0$) when they couple to two external legs.
- These are pure soft divergences, but without collinear divergences.
- The interpolating gauge looks a powerful tool to disentangle the soft and collinear divergences.

Renormalization

- Renormalization works, we did it at 1-loop level and calculated all the renormalisation constants,

$$Z_\psi, Z_A^{\mu\nu}, Z_\eta, Z_g, Z_v, Z_\xi$$

- The BRST invariance shows that the S-matrix **independent** of the gauge parameters, v, ξ, n^μ.

$$Z_A^{\mu\nu}(\alpha_s, v, \xi) = g^{\mu\nu} + \frac{\alpha_s}{4\pi}\frac{1}{\epsilon_{\overline{\rm MS}}}\left(\left[\frac{22v^3+35v^2+20v-1}{6v(1+v)^2} - \frac{\xi}{2v}\right]C_A - \frac{4}{3}T_R n_f\right)g^{\mu\nu} - \frac{\alpha_s}{4\pi}\frac{1}{\epsilon_{\overline{\rm MS}}}\frac{4v(1+2v)}{3(1+v)^2}C_A\,h^{\mu\nu} + \mathcal{O}(\alpha_s^2)$$

$$Z_g(\alpha_s, v, \xi) = 1 - \frac{\alpha_s}{4\pi}\frac{1}{\epsilon_{\overline{\rm MS}}}\left(\frac{11}{6}C_A - \frac{4}{6}T_R n_f\right) + \mathcal{O}(\alpha_s^2)$$

Conclusion

- Interpolating gauge is not useful to calculate exact matrix element, for that purpose I still recommend Feynman gauge.
- But for defining subtraction terms for fixed order calculation or splitting operators for parton shower or for studying IR structures, it might be a very useful gauge.
- Our goal is to define a NLO level parton shower.

Shower Kernel

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$$+ \left[\mathcal{V}^{(1)}(\mu^2), \frac{1}{\mu^2} \mathcal{S}^{(1)}(\mu^2) \right]$$

This is highly non-trivial operator and cancelation of all the singularities in the first term is rather delicate.

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Just for illustration!

Stay tuned!