Heavy Flavor Jet Substructure for Heavy Ion Collisions

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Based on 2312.15560 and ongoing works

Parton Showers and Resummation 2024
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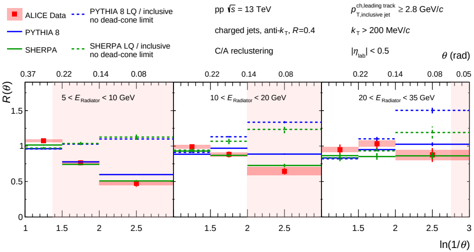
Motivation

- Dead-cone effect: radiation is suppressed within an angular size of m/E
- First direct experimental observation of collinear radiation suppression
  ALICE: ArXiv: 2106.05713



- But only a handful of theoretical studies for heavy flavour jet substructure:
  - L. Cunqueiro, D. Napoletano and A. Soto-Ontoso ArXiv: 2211.11789
  - S. Caletti, A. Ghira and S. Marzani ArXiv: 2312.11623
  - B. Blok, C. Wu ArXiv: 2312.15560
- Our goal: study medium modification effects on the parton splitting functions

Soft drop grooming and z_g distribution

Soft drop (SD) grooming: clean the jets up by removing soft radiation **(More details in Andrea Ghira's talk)**

- identify the "correct" angular scale
- throw away what is soft & large angle
- left a groomed jet

Declustering the jet constituents until the subjets satisfy the SD condition:

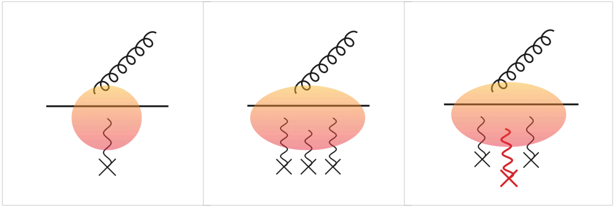
$$z_g = \frac{min\left(p_1, p_2\right)}{p_1 + p_2} > z_{cut}\theta_g^{\beta},\ \theta_g = \frac{\Delta R_{12}}{R}$$

- For $\beta \geq 0$, collinear splittings always pass the SD condition, z_g not IRC safe, need applying Sudakov safe techniques.
- For $\beta = 0$, i.e. modified mass drop, z_g provides a direct measurement of the splitting function.

$$p_i\left(z_g\right) = \frac{\bar{P}_i\left(z_g\right)}{\int_{z_{cut}}^{1/2} \bar{P}_i\left(z_g\right) dz}\Theta\left(z_g - z_{cut}\right)$$

Parton propagation through medium

- Dilute medium: For low medium opacity, only one scattering occurs.
- Dense medium:
  - Bethe-Heitler regime, $\omega < \omega_{BH}$
  - BDMPS-Z regime, $\omega_{BH} < \omega < \omega_c$: Multiple scatterings based on a path-integral formalism
  - Hard GLV regime, $\omega > \omega_c$: Opacity expansion in terms of the number of scattering centers



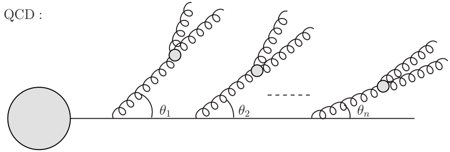
Three regimes of the radiative spectrum in dense media [ArXiv: 2206.02811].

Parton propagation in dense medium

- Vacuum-like emissions (VLE): double differential probability for bremsstrahlung at DLA

$$d^2P = \frac{\alpha_s C_R}{\pi}\frac{d\omega}{\omega}\frac{d\theta^2}{\theta^2}$$

  - Duration: $t_f \sim \omega/k_T^2 = 1/\left(\omega\theta^2\right)$
    Parent parton and the emitted gluon lose their mutual quantum coherence
  - Angular ordering: $\theta_{n+1} \ll \theta_n$ radiation is confined in a cone



  - Heavy flavor VLE: dead-cone approximation

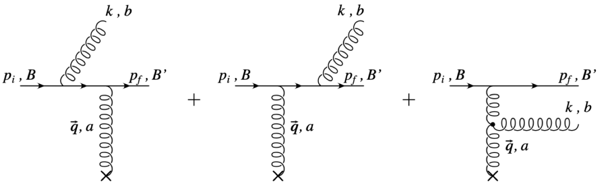
$$d^2P = \frac{\alpha_s C_R}{\pi}\frac{d\omega}{\omega}\frac{d\theta^2}{\theta^2}\cdot\frac{1}{\left(1+\theta_0^2/\theta^2\right)^2}$$

Parton propagation in dense medium

- Medium-induced emissions (MIE): no collinear divergence

$$d^3P \sim \frac{\alpha_s C_R}{\pi} \frac{d\omega}{\omega} \frac{dt}{t_{med}} P_{broad}(\theta)\, d\theta, \text{ with } t_{med} = \sqrt{\omega/q}$$

- Transverse momentum broadening:
  - Gaussian distribution, with a width $\langle k_\perp^2 \rangle \sim q\Delta t$
  - The broadening accumulated momentum over the formation time.

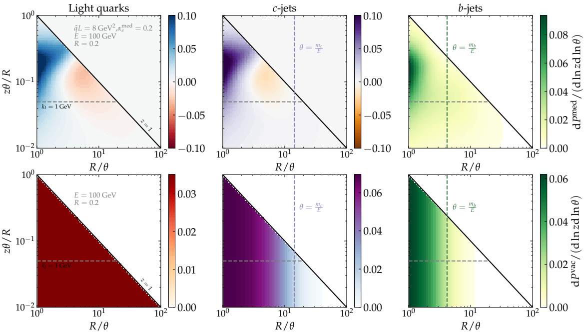


- Heavy flavor MIE: the radiation is also suppressed, but less effective due to the reduction of LPM effect.

Dead-cone and radiation in dense QCD medium

Radiation from an energetic, massive quark is strongly suppressed within the dead-cone

$$\theta_0 = \frac{m_Q}{E}$$



Lund plane density: Medium-induced (top) and vacuum emissions (bottom) [ArXiv: 2211.11789].

Definition (Jet modification factor)

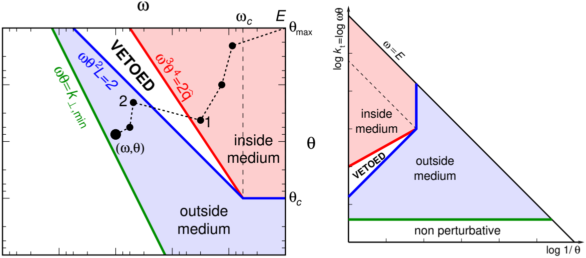
$$R_i\left(z_g\right) \equiv f_{i,med}\left(z_g\right) / f_{i,vac}\left(z_g\right)$$

Physical picture: factorization between VLE and MIE

- Physical picture: factorization between VLE and MIE:

$$t_f\left(\omega,\theta\right) \ll t_{med}\left(\omega\right)$$

  - The medium $k_\perp$ cannot be smaller than $k_f^2 = \hat{q}t_{med}$
  - No VLE allowed: $t_{med} < t_f < L$
- Jet factorizes into three regions:
  - angular ordered vacuum-like shower inside the medium
  - medium-induced emissions triggered by previous sources
  - vacuum-like shower outside the medium

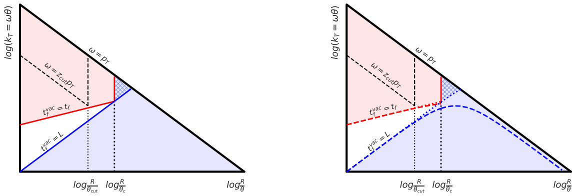


The phase-space for VLE and MIE [P. Caucal, E. Iancu, G, Soyez 1907.04866]

Physical picture: extension to heavy flavor

Factorization between vacuum-like and medium-induced emissions:

$$t_f = \frac{\omega}{k_t^2 + \theta_0^2\omega^2} \ll t_{med} = \sqrt{\frac{\omega}{q}}$$



Lund diagram representation of the phase space for the in-medium radiation for massless case (left) and heavy flavor jets (right) with c-jets (dotted line) and b-jets (dashed line).

- Blue region: $t_f^{vac} > L$, outside of the medium, the blue crossed region is between $t_f^{vac} < L$ and $\theta < \theta_c$, i.e. not resolved by the medium
- Red region: $t_f^{vac} \leq t_{med}$, VLE emissions inside the medium
- White region: $L \gg t_f^{vac} > t_{med}$, the VLEs are vetoed.

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BDMPS-Z

BDMPS formula: The medium-induced gluon spectrum is given by

$$\omega \frac{dI}{d\omega} = \frac{\alpha_s C_R}{\omega^2} 2Re \int_0^\infty dt_2 \int_0^{t_2} dt_1$$
$$\partial_{\vec{x}} \cdot \partial_{\vec{y}} \left[K(\vec{x}, t_2 | \vec{y}, t_1) - K_0(\vec{x}, t_2 | \vec{y}, t_1) \right] |_{\vec{x}=\vec{y}=0}$$

Alternative method: Zakharov approach

$$\omega \frac{dI}{d\omega} = \frac{\alpha_s C_R}{\omega^2} 2\text{Im} \int_0^L d\xi (L - \xi) \frac{d}{d\rho} \frac{\tilde{F}}{\sqrt{\rho}} |_{\rho=0},$$

where $\tilde{H}$ is the solution of radial Schrodinger equation

$$\left(i\partial_\xi + \frac{1}{2\omega} \partial_\rho^2 - V(\rho) - \frac{4m^2 - 1}{8\omega\rho^2} \right) \tilde{F} = 0$$

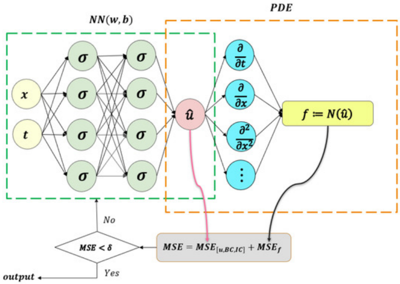
with the initial condition

$$\tilde{F}(0, \rho) = V(\rho) / \sqrt{\rho}.$$

NN-based differential equation solver

Neural network can solve differential equations as an optimization problem. In general, there are three approaches:

- Continuous time approach
- Discrete time approach
- Connection between PDEs and stochastic processes: backward stochastic differential equation



Neural network structures.

$$MSE_f = \frac{1}{N_f}\sum_{n=1}^{N_f} \left| f\left(x_f^n, t_f^n\right)\right|^2, \; MSE_u = \frac{1}{N_u}\sum_{n=1}^{N_u} \left|\hat{u}^n - u\left(x_u^n, t_u^n\right)\right|^2.$$

NN predicted solution for harmonic oscillator approximation

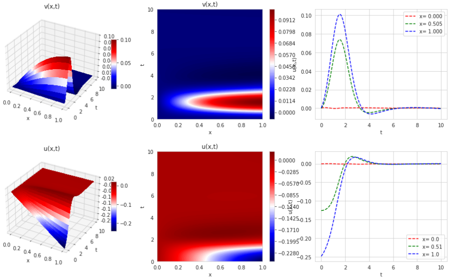
For harmonic potential

$$V(\rho) = \frac{\omega\Omega^2}{2}\rho^2,$$

with imaginary frequency $\Omega = \frac{1-i}{2}\sqrt{\frac{q}{\omega}}$. One can obtain the famous BDMPS spectrum

$$\omega\frac{dI}{d\omega} = \frac{2\alpha_s C_R}{\pi} log|cos(\Omega L)| \overset{\omega \ll \omega_c}{\to} \frac{\alpha_s C_R}{\pi}\sqrt{\frac{2\omega_c}{\omega}}$$

On the other hand, from my NN solver we can solve the TDSE, we have



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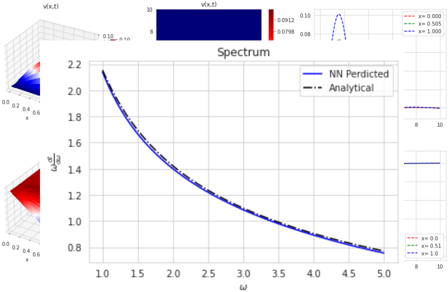
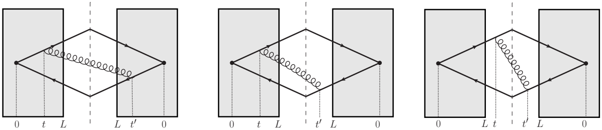
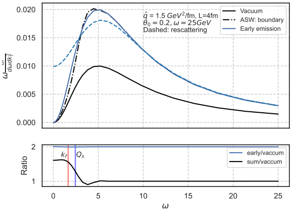


Early/Late emission factorization and broadening



The three parts of the gluon spectrum in the presence of a medium

Early emission, $t < t_{med}$



Late emission, $t > t_{med}$: at massless limit

$$\omega \frac{dI}{d\omega d^2 k_t} = \frac{\alpha_s C_F}{\pi^2 \omega} Re \int_0^L dt \int \frac{d^2 k'}{(2\pi)^2} P\left(\vec{k}_t - \vec{k}', t, L\right) e^{-(1+i)\frac{k'^2}{2k_f^2}}$$

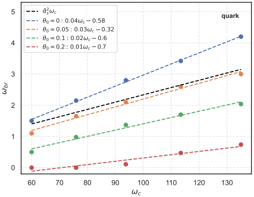
$$\xrightarrow{k_t \gg k} \frac{\alpha_s C_F}{\pi^2} \sqrt{\frac{2\omega_c}{\omega}} \tilde{P}(k_t, q, L)$$

Multiplicity and energy loss

Recall the physical picture: VLEs in the medium act as the source of medium-induced radiation.

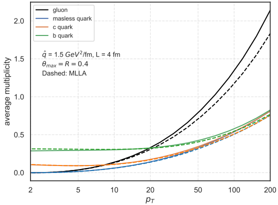
- multiple branching scale ω_{br}: $\omega < \omega_{br}$
  MIEs need to be resumed to all-order

$$\int_{\omega_{br}}^{\omega_c} \frac{dI}{d\omega} d\omega \sim O(1)$$



  for massless case: $\omega_{br}^{(R)} = \frac{\alpha_s^2}{\pi^2} C_A C_R \frac{qL^2}{2}$

- In medium VLE multiplicity: $\nu(z, R) = \int_{\theta_{cut}}^{R} d\theta \int_{zp_T}^{p_T} d\omega \frac{d^2 N_{VLE}}{d\omega d\theta}$



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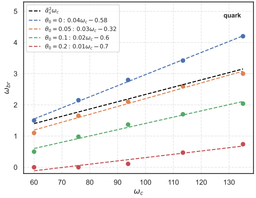
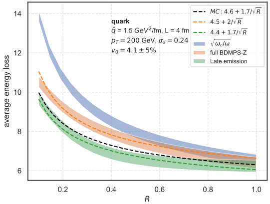
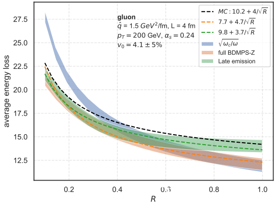


- Energy loss: multiple soft branchings at large angles, and semi-hard gluons with angle $\theta > R$

$$\epsilon_{flow}(E) = E\left(1 - e^{-v_0 \frac{\omega_{br}}{E}}\right), \; \epsilon_{spec}(R) = \int_{\omega_{br}}^{\bar{\omega}} d\omega \omega \frac{dI}{d\omega}, \; \bar{\omega} \sim Q_s/R$$





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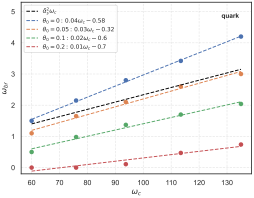
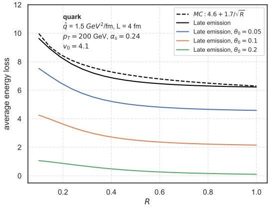


- multiple branching scale ω_{br}: $\omega < \omega_{br}$
  MIEs need to be resumed to all-order

$$\int_{\omega_{br}}^{\omega_c} \frac{dI}{d\omega} d\omega \sim O(1)$$

  for massless case: $\omega_{br}^{(R)} = \frac{\alpha_s^2}{\pi^2} C_A C_R \frac{qL^2}{2}$

- Energy loss for heavy flavor jet: smaller energy loss for heavy quarks than for light quarks, a net effect due to the filling of dead-cone



In medium z_g distribution

- Full shower formula: included VLE multicity due to the fact each VLE act as a source of MIE Sudakov safe:

$$f(z_g) = N \int_{\theta_{cut}}^{R} d\theta_g \Delta^{tot}(R, \theta_g) P^{tot}(z_g, \theta_g) \Theta(z_g - z_{cut})$$

$$P^{tot}(z, \theta_g) = P_{VLE}(z, \theta_g) + \nu(z, \theta_g) P_{MIE}(z, \theta_g)$$

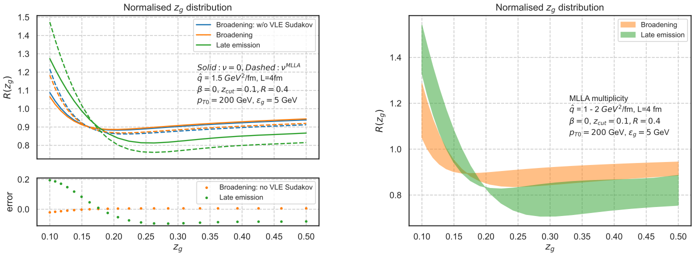
Alternative, due to no collinear singularities for MIE spectrum:

$$f(z_g) = N \int_{\theta_{cut}}^{R} d\theta_g \left[P_{VLE}(z_g, \theta_g) \Delta^{VLE}(R, \theta_g) + \nu(z_g, \theta_g) P_{MIE}(z_g, \theta_g) \right] \Theta(z_g - z_{cut})$$

- Definition of z_g with energy loss:

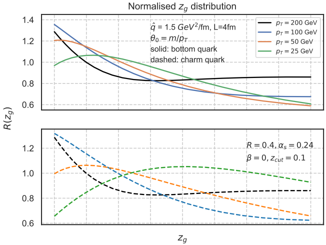
$$z_g \equiv \frac{p_{T1}}{p_{T1} + p_{T2}} = \frac{z p_T - \mathcal{E}_g(z p_T, \theta_g)}{p_T - \mathcal{E}_i(p_T, \theta_g)} \equiv Z_g(z, \theta_g),$$

Phenomenology: z_g in dense medium



Combination of incoherent energy loss affecting vacuum-like splitting and a small z_g peak associated with the SD condition being triggered by MIE.

Phenomenology: heavy flavor z_g in dense medium



R ratio is sensitive to the dead-cone angle and can be used to help probe gluon filling the dead-cone

- vacuum emissions are more suppressed compared with the MIEs.
- in some limited regions of phase space the dead cone is filled

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Towards medium-induced radiation in expanding medium

Expanding QGP medium: non-uniform, time-dependent, $q \equiv q(t)$

- Bjorken expanding medium: $q(t) = q_0 (t_0/t)^{\alpha}$
- Exponential decaying medium: $q(t) = q_0 e^{-t/L}$

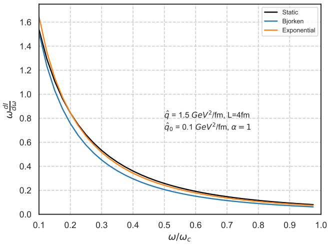
For massless case:

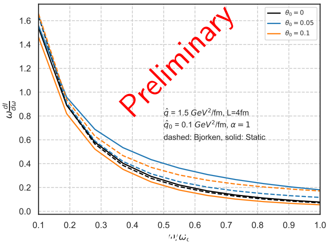
$$\omega \frac{dI}{d\omega} = \frac{\alpha}{\pi} x P_{i\to g}(x) \lim_{t\to\infty} log|C(t;0)|$$

Scaling law: an equivalent static scenario for expanding medium

C. Salgado, U. Wiedemann ArXiv: hep-ph/0302184, 0204221

$$\langle q \rangle = \frac{2}{L^2}\int_{t_0}^{L+t_0} dt\,(t-t_0)\,q(t)$$





Conclusions and outlook

Summary:

- We extended the factorization picture for heavy flavor and further extended it by factorizing early and late emissions
- Heavy flavor jet substructure can help probe dead-cone effect

Ongoing and future works:

- Neural network approach to solve DGLAP-like evolution equation and its application to medium-induced heavy flavor jet evolution
- Heavy flavor jet substructure in expanding medium
- Heavy flavor extension for the Improved Opacity Expansion framework
- Towards precision phenomenology of jet quenching

Thank you for your attention

Extra Slides

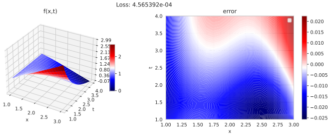
Two examples

Unintegrated gluon distribution in the small x limit

$$u\frac{d}{du}\phi(x,u) = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y} P\left(\frac{x}{y}\right)\phi(y,u)$$

$$\phi(x,1) = x$$

For NN, the integral part is calculated via matrix multiplication, we have

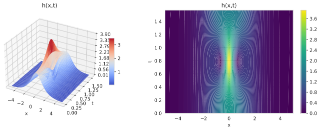


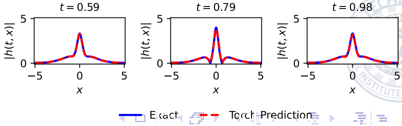
Comparison of the NN predicted and exact solution, fixed-coupling limit

Network parameters:

Parameter	TDSE	DGLAP
Hidden layers	5	5
Internal width	100	100
Activation function	Swish	Tanh
Train samples	2000	200
Batch size	1000	20
Epochs	10	100
Optimizer	L-BFGS	Adam
$(\alpha, \beta_1, \beta_2)$	(0.1, 1, 0.99)	$(1 \times 10^{-2}$, 0.9, 0.999)
Max iteration	200	

Non-linear time-dependent Schrodinger equation [Arxiv 1711.10561]





Full shower result for in-medium z_g