

Heavy flavour jet substructure

Andrea Ghira

Graz, 3rd July 2024

Based on [2312.11623](#) in collaboration with S. Caletti and S. Marzani, and on a work in progress with P. Dhani, O. Fedkevych, S. Marzani and G. Soyez

Jet substructure in a nutshell

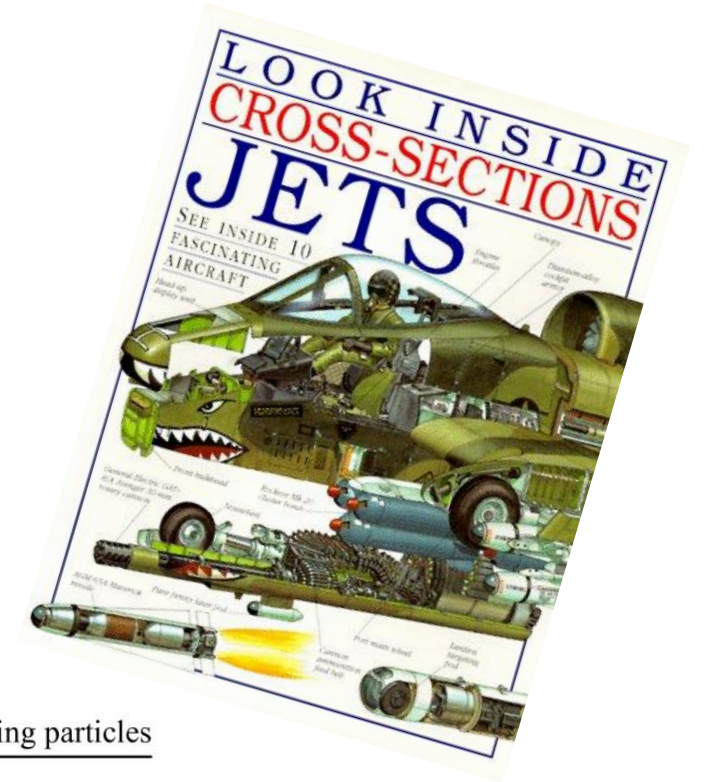
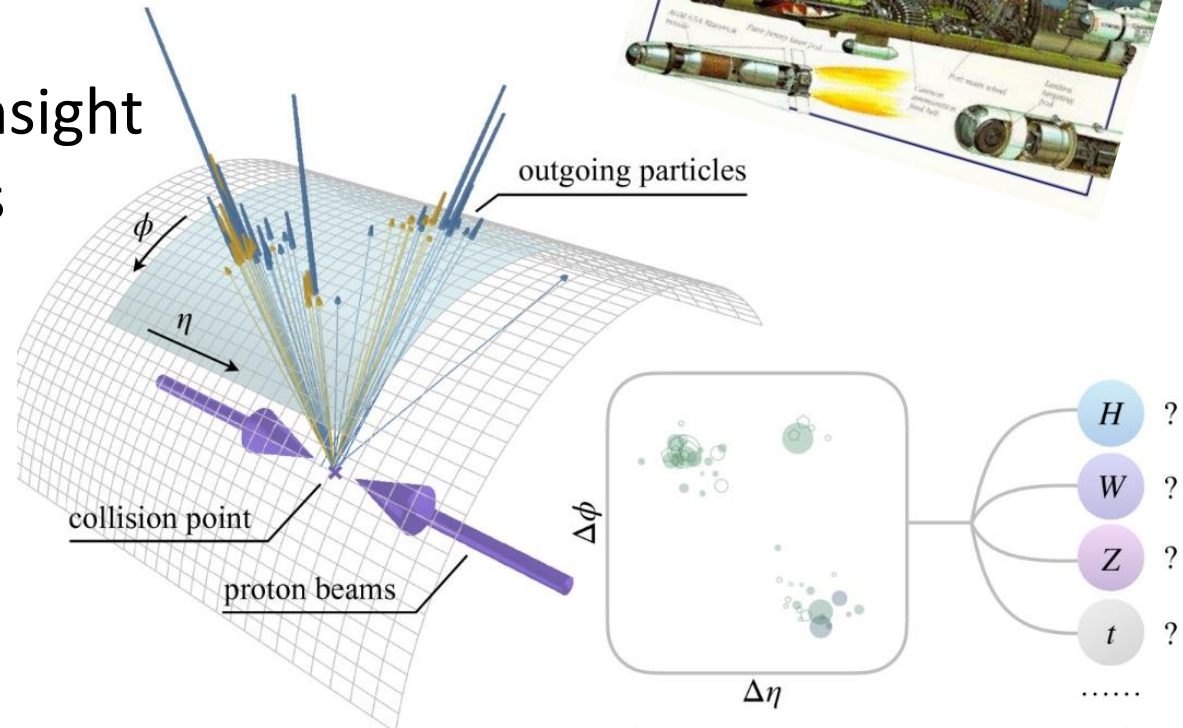
High energy collisions result in collimated sprays of particles



Internal structure of jets gives an insight on the originating splitting process

In a massless theory, the collinear emission is enhanced:

$$\alpha_S \int \frac{d\theta^2}{\theta^2} \gg 1$$



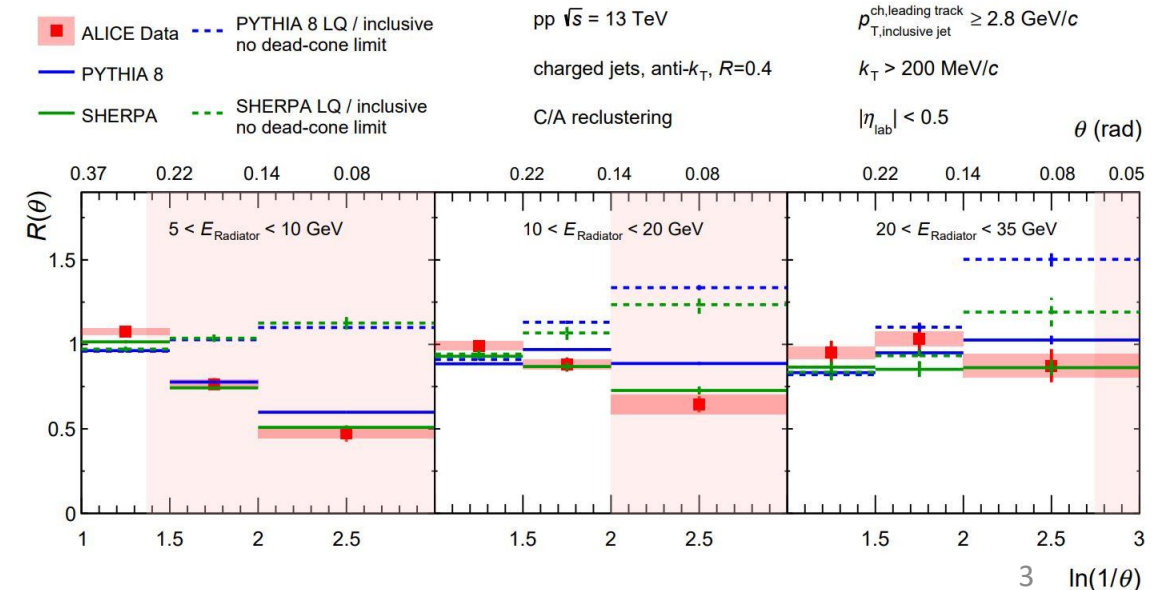
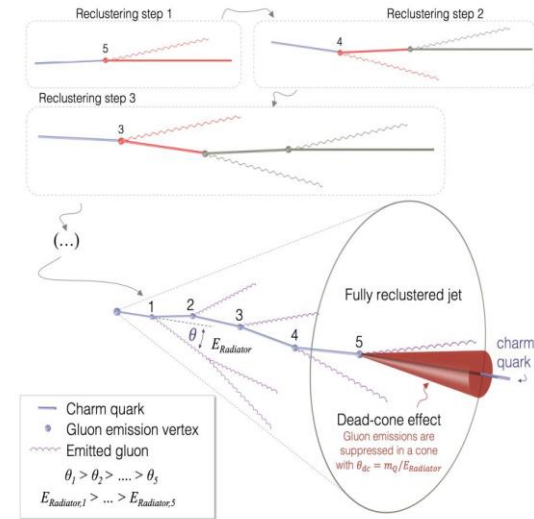
Jets to probe heavy flavours

When jets are initiated by a heavy flavour, the quark mass shields the collinear singularity

$$\alpha_s \int \frac{d\theta^2}{\theta^2 + \frac{m^2}{E^2}} \simeq \alpha_s \log \frac{m^2}{E^2}$$

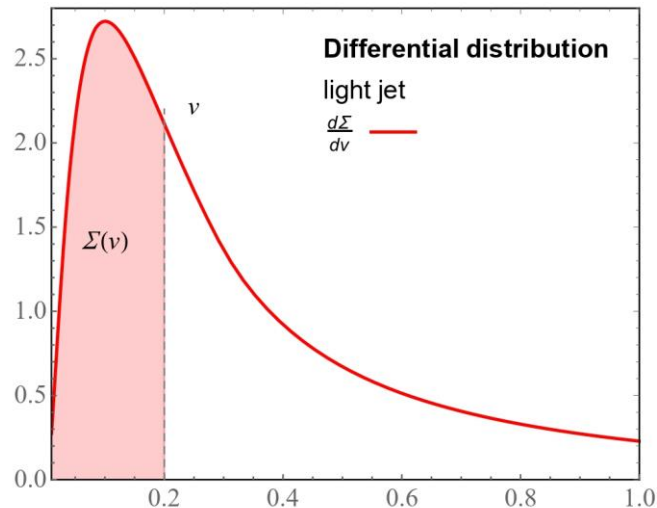
Dead Cone effect

the radiation emitted off a heavy flavour is suppressed inside a cone of opening angle $\theta \sim m/E$ ([ALICE](#))



Theoretical Framework: light jet

Given an observable v , from a theoretical point of view it is natural to compute the resummation of the cumulative distribution



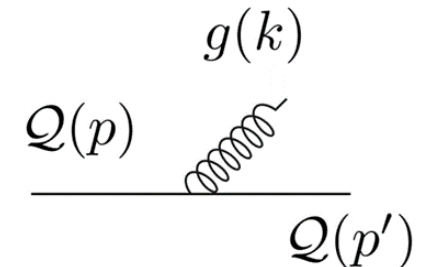
$$\Sigma(v) = \frac{1}{\sigma_0} \int_0^v dv' \frac{d\sigma}{dv'}$$

- v is a function of momenta that vanish when no emissions occur (Born level)
- v must be IRC safe

Collinear factorization

We begin studying the case of the single emission off a quark.

The matrix element factorizes in the collinear limit, thus we can write the cumulative as:

$$\Sigma(v) = 1 - \sum_{\ell} \int_0^{Q^2} \frac{dk_t^2}{k_t^2} \int_0^1 dz P_{Qg}(z) \frac{\alpha_s}{2\pi} \Theta(\mathcal{V}^{\ell}(k_{t\ell}^2, \eta_{\ell}) - v)$$


\mathcal{V} represents the soft and collinear limit of the observable and in general can be written [\(Banfi, Salam, Zanderighi\)](#)

$$\mathcal{V}^{\ell}(k_{t\ell}^2, \eta) = d_{\ell} \left(\frac{k_{t\ell}^2}{Q^2} \right)^{\frac{a_{\ell}}{2}} e^{-b_{\ell} \eta}$$

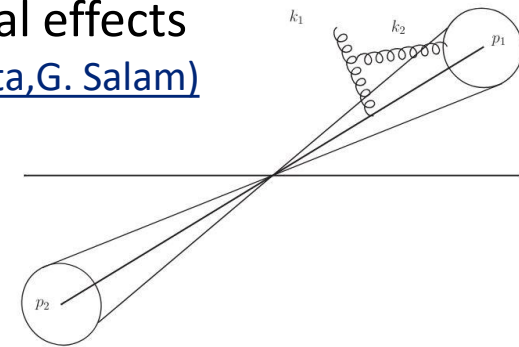
Going to all orders

Taking into account an infinite number of emissions, at NLL accuracy we have:

$$\Sigma(v) = e^{-R(v)} \mathcal{F} S$$

Multiple emission

Non-global effects
[\(M. Dasgupta, G. Salam\)](#)

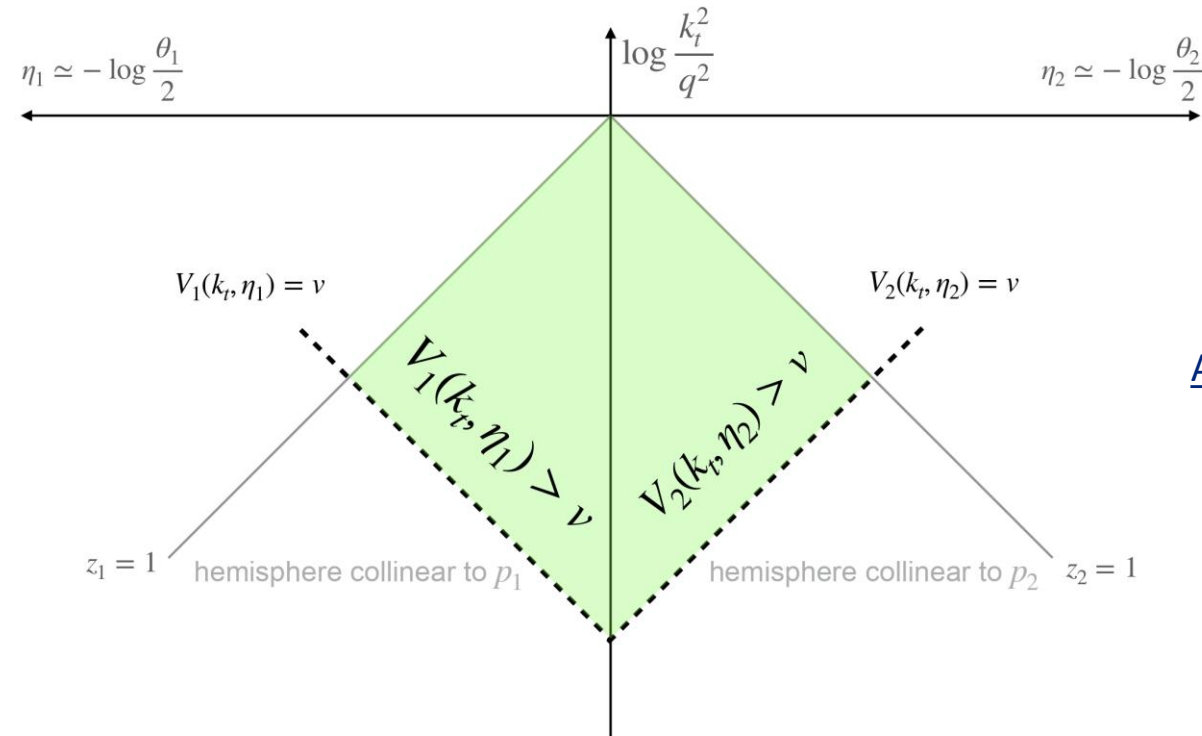


with R the radiator defined as:

$$R(v) = \sum_{\ell} \int_0^{Q^2} \frac{dk_t^2}{k_t^2} \int_0^1 dz P_{Qg}(z) \frac{\alpha_s^{\text{CMW}}(k_{t\ell}^2)}{2\pi} \Theta(\mathcal{V}^{\ell}(k_{t\ell}^2, \eta_{\ell}) - v)$$

Lund Plane geography for light quarks

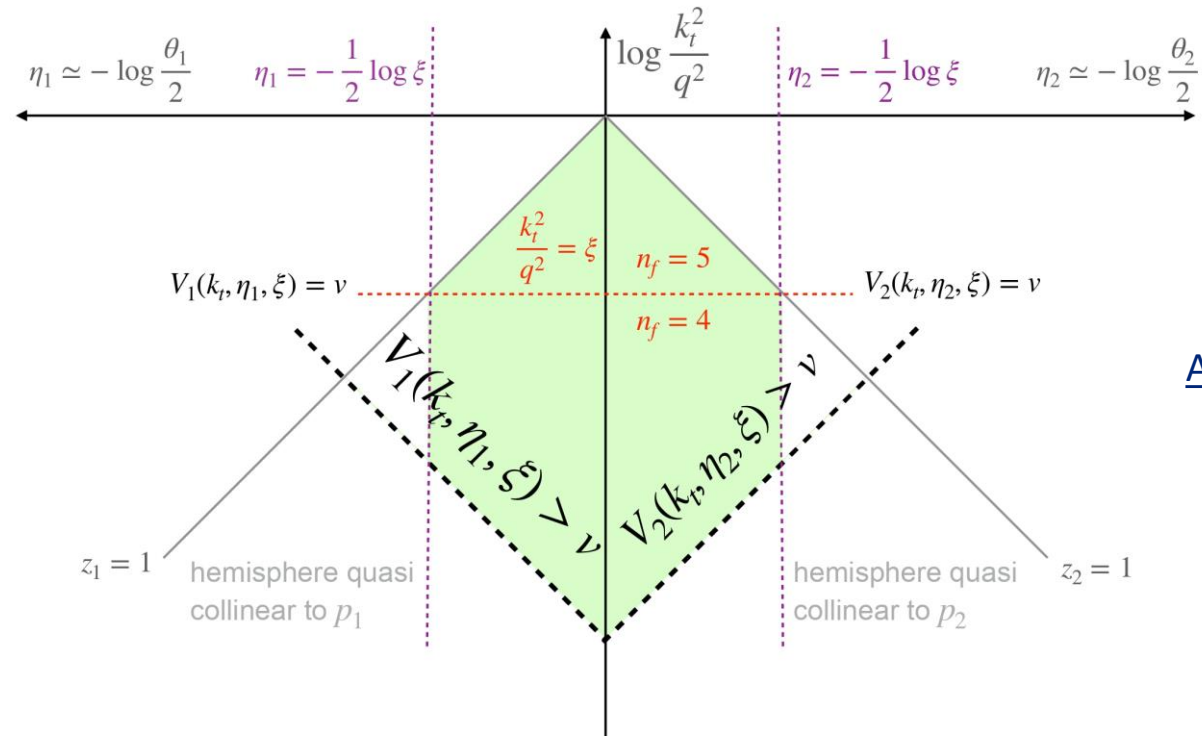
In order to perform the calculation of R , we exploit the Lund diagrams.



A.G., S. Marzani, G. Ridolfi

Differences with the massive case

- Threshold in the running of the coupling
- Dead-cone effect

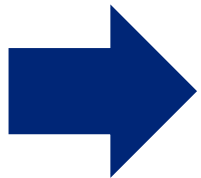


A.G., S. Marzani, G. Ridolfi

Mass effects on θ_g and z_g distribution

We want to examine observables which are sensitive to the dead cone.

To start, we consider heavy-flavour initiated jets groomed with the Soft-Drop procedure ([A. Larkoski, S. Marzani, G. Soyez, J. Thaler](#)).



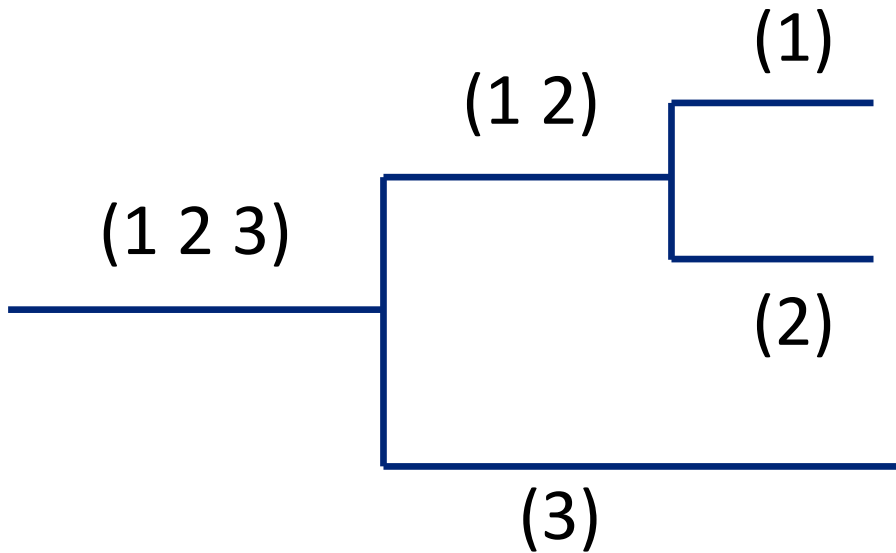
θ_g : angular opening of the groomed jet (access to the dead-cone)

z_g : allows us to probe the heavy quark splitting function

Recent measurement by [ALICE](#) of the SD observables on c-jets.

The Soft Drop Procedure

The SD algorithm removes consistently soft emission at large angle



$$\frac{\min(p_{t(12)}, p_{t(3)})}{p_{t(12)} + p_{t(3)}} > z_{\text{cut}} \left(\frac{\Delta_{(12)(3)}}{R_0} \right)^\beta,$$

$$\Delta_{(12)(3)} = \sqrt{(y_{(12)} - y_{(3)})^2 + (\phi_{(12)} - \phi_{(3)})^2}.$$

The jet constituents of an anti- k_t are re-clustered according to C/A, to form an angular ordered tree. The declustering is then applied.

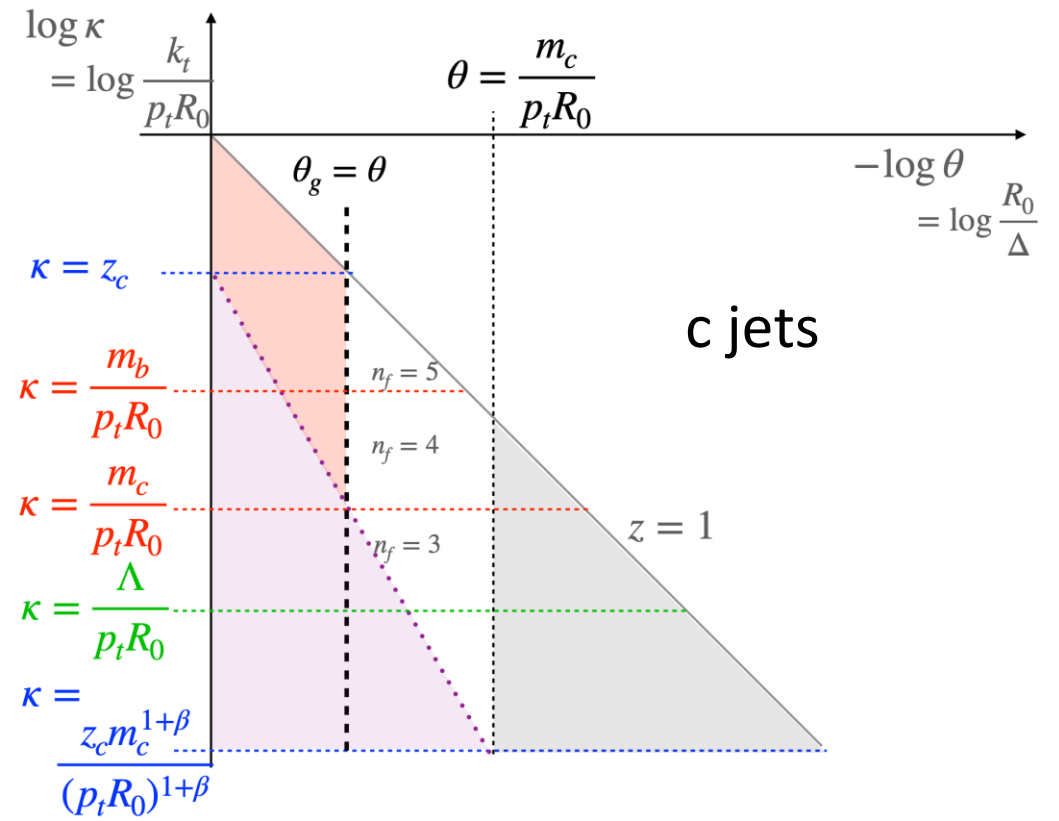
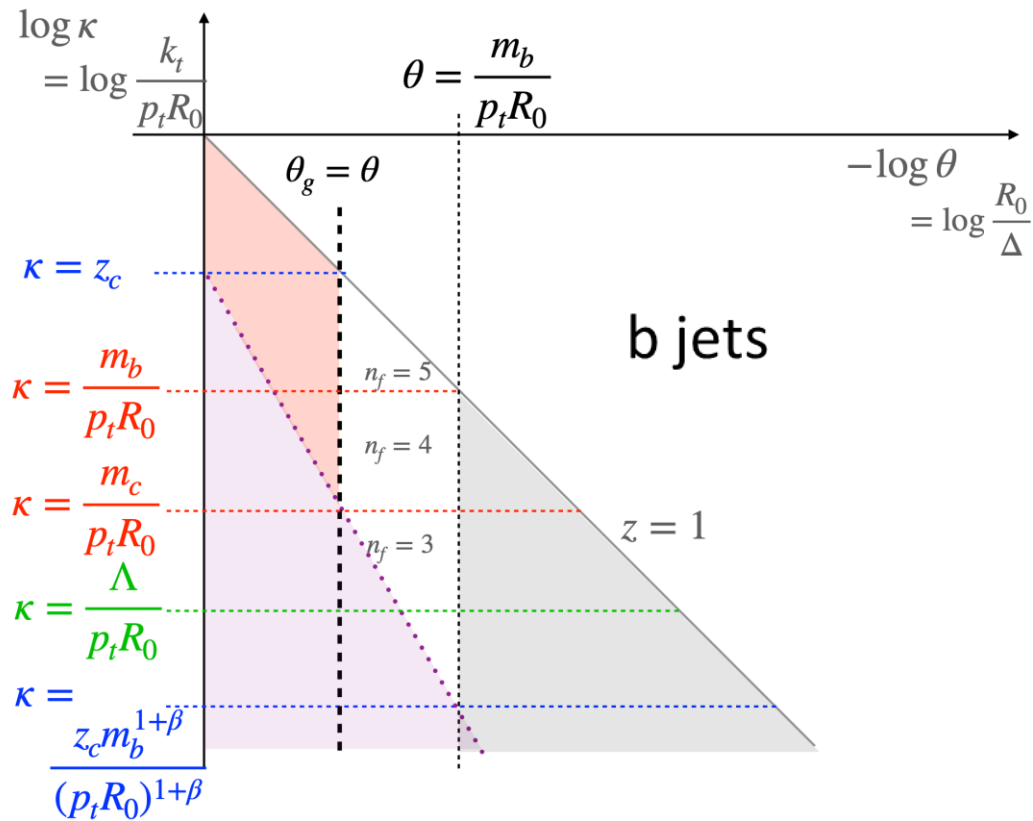
Definition of the observables

The first branching that passes the soft-drop procedure defines the groomed jet radius θ_g and the groomed fraction of momentum z_g

$$\theta_g = \frac{\Delta_{(12)(3)}}{R_0}, \quad z_g = \frac{\min(p_{t(12)}, p_{t(3)})}{p_{t(12)} + p_{t(3)}}.$$

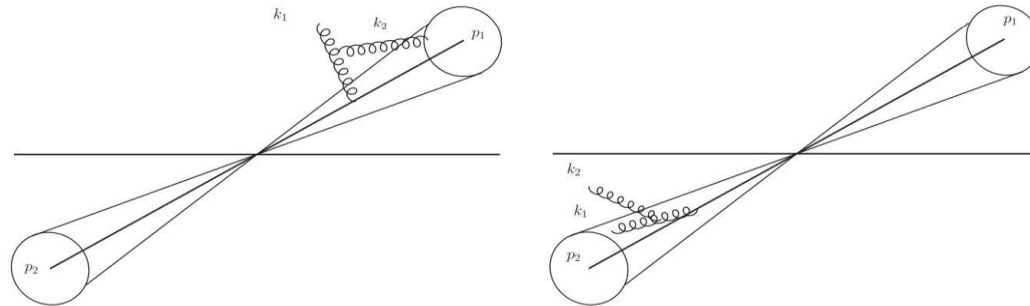
- The θ_g distribution is IRC safe both for massive and massless particles.
- The z_g distribution is IRC safe for massless particles only for $\beta < 0$. Conversely, it is always IRC safe for massive particles.

Analytic understandings of θ_g distribution



Non global Logs

- Non global logs represent correlated gluon emissions ([M. Dasgupta, G. Salam and A. Banfi, M. Dasgupta, K. Khelifa-Kerfa, S. Marzani](#))



- Their contribution is reduced with the inclusion of clustering effects

([Z. Kang, K. Lee, X. Liu, D. Neill, F. Ringer](#))

$$S_0(\theta_g) = 1 + \left(\frac{\alpha_s}{\pi}\right)^2 \frac{\pi^2}{108} (C_F^2 - 4C_F C_A) \log(z_{\text{cut}} \theta_g^\beta)^2 + \mathcal{O}(\alpha_s^3)$$

Mass effects on NGLs

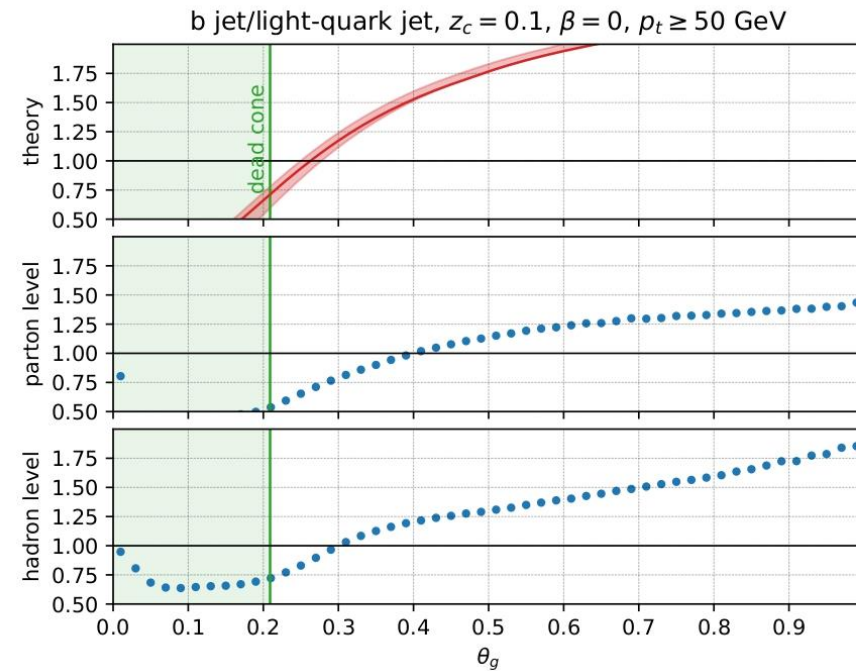
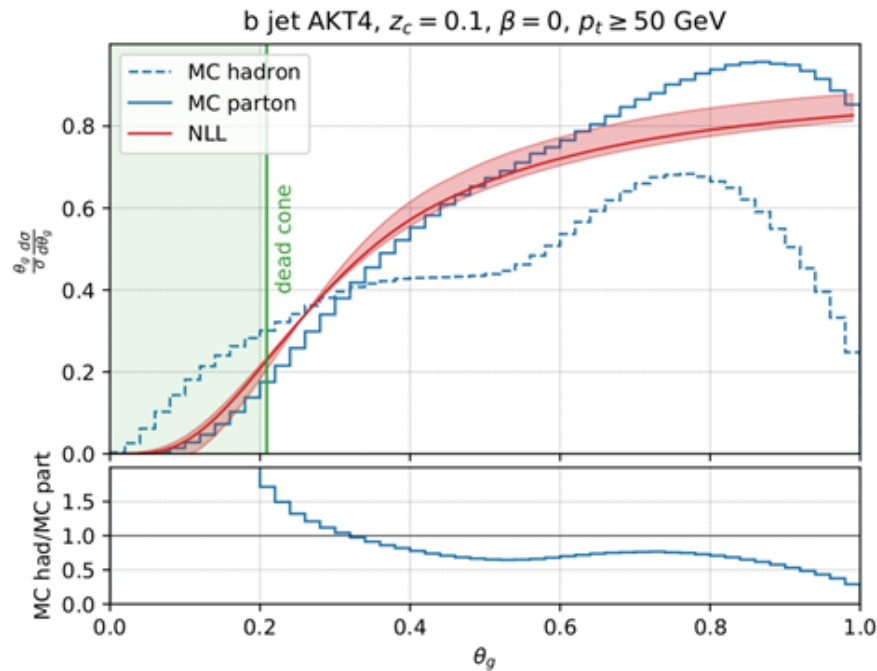
The inclusion of the masses in the computation further reduce the effect of the NGLs.

$$S(\theta_g, \theta_i) = 1 + \left(\frac{\alpha_s}{\pi}\right)^2 (C_F^2 \mathcal{F}_1(\theta_g, \theta_i) + C_F C_A \mathcal{F}_2(\theta_g, \theta_i)) \log(z_{\text{cut}} \theta_g^\beta)^2 + \mathcal{O}(\alpha_s^3)$$

$$\lim_{\theta_g \rightarrow 0} \mathcal{F}_1(\theta_g, \theta_i) = \lim_{\theta_g \rightarrow 0} \mathcal{F}_2(\theta_g, \theta_i) = 0$$

- This result is related to the requirement that one of the two gluons is within the jet radius. Performing the small θ_g limit of such configuration $S \rightarrow 1$.
- The resummed expression is approximated by exponentiating the two-loop result with RC corrections

θ_g distribution: Comparison with Monte Carlo



- Good agreement with Monte Carlo simulations (especially for $\beta = 0$).
- Sensitive to the dead-cone
- The two distributions (heavy and light) are normalized to have area 1 (tagging mode).

z_g distribution: Sudakov Safety

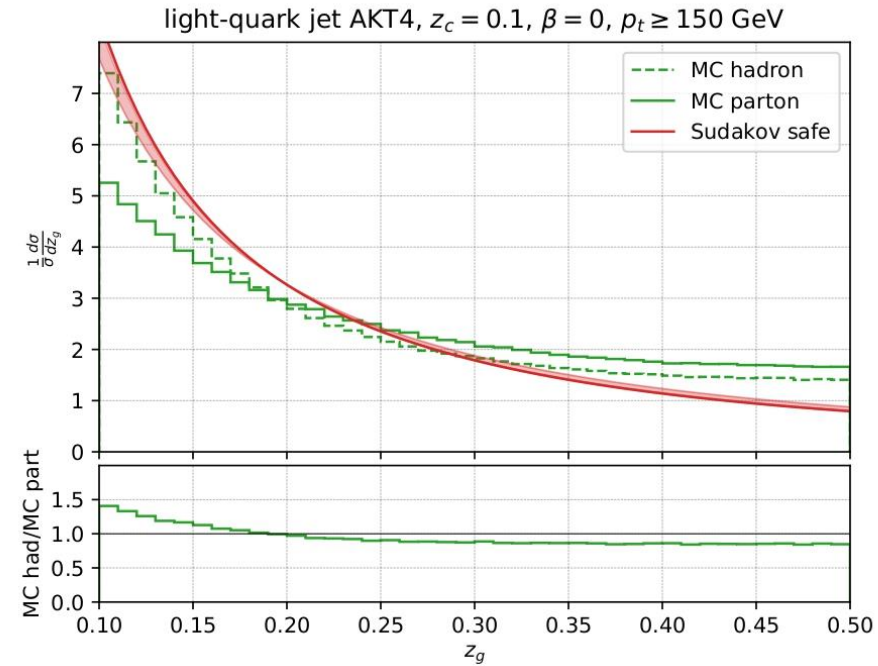
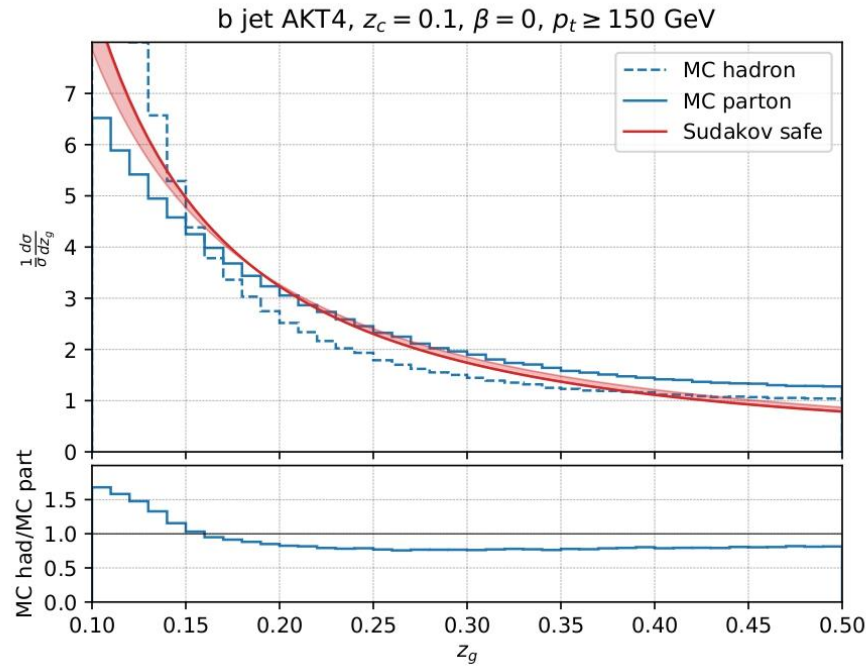
In the massless case Z_g is not an IRC safe observable

➔ Sudakov Safety ([A. Larkoski, S. Marzani, J. Thaler](#)).

$$\frac{1}{\sigma_0} \frac{d\sigma}{dz_g} = \int_0^1 d\theta_g \left[\frac{p(z_g, \theta_g)}{p(\theta_g)} \right]^{\text{F.O}} p(\theta_g)^{\text{RES}}, \quad p(\theta_g) = \frac{1}{\sigma_0} \frac{d\sigma}{d\theta_g}$$

- In the massless case, this mechanism allows to obtain a finite expression of the differential cross section.
- The full resummation of the logs of z_g and z_{cut} is performed in ([P. Cal, K. Lee, F. Ringer, W. Waalewijn](#)), but not considered here

z_g distribution: Comparison with Monte Carlo

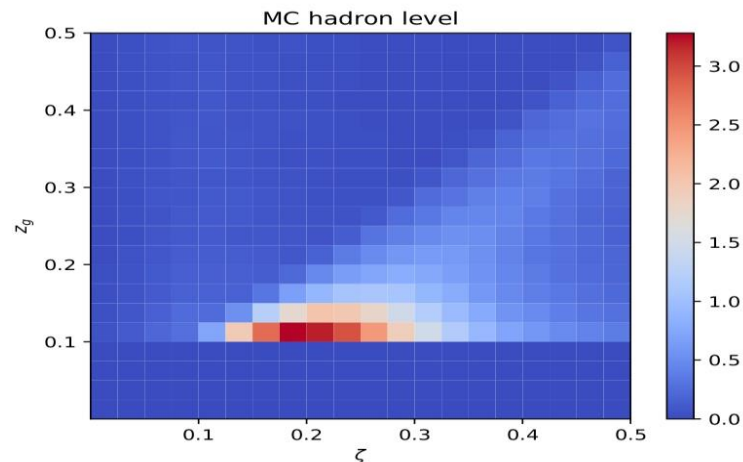


- Similar behaviour of the two differential distributions \rightarrow same leading term in the splitting function $1/z_g$
- Lack of agreement in the large z_g region \rightarrow missing symmetrization of the splitting function

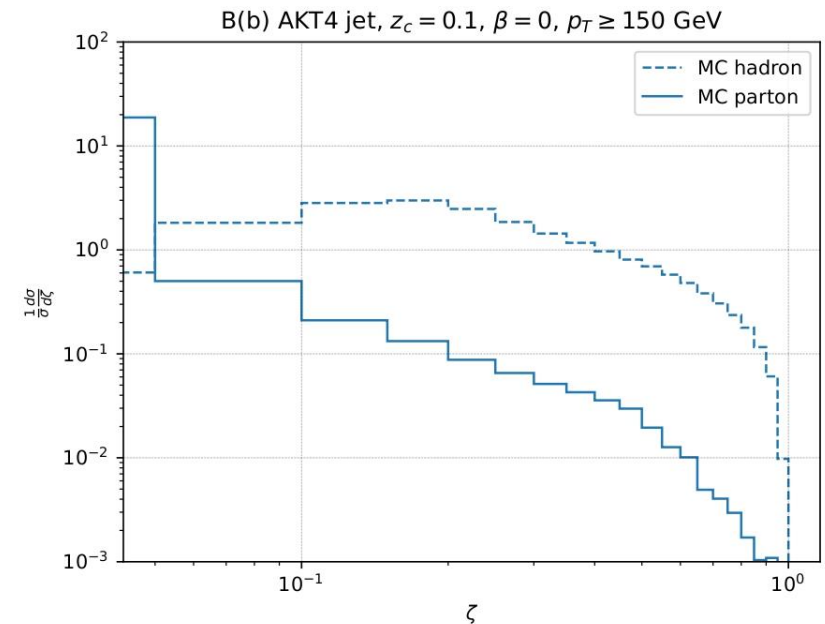
Correlation with fragmentation distribution

We study the correlation between the fragmentation variable $\zeta = 1 - \frac{p_{T,b}}{p_{T,J}}$ and z_g :

$$\text{At order } \alpha_S: \frac{1}{\sigma_0} \frac{d\sigma^{(\text{f.o.})}}{d\zeta} = \frac{\alpha_S C_F}{\pi} \frac{\log\left(\frac{m_i^2}{p_t^2 R_0^2}\right)}{\zeta}$$



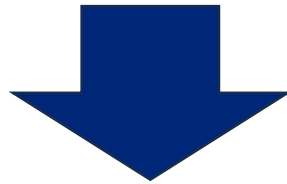
Identical to z_g distribution at this order!



- The spectrum of the b quark is strongly affected by hadronization.
- z_g distribution remains more stable with the inclusion of NP corrections

Mass effects on angularities and ECF

- We now study another class of observables: jet angularities and energy-correlation function (ECF).
- Many possible choices in the definition of observables sensitive to the dead-cone
[\(C. Lee, P. Shrivastava, V. Vaidya\)](#)



Which one is more sensitive to dead-cone effect?

Example: ECF in pp collisions

$$e_\alpha = \sum_{i \neq j \in \text{Jet}} \frac{p_{t_i} p_{t_j}}{p_t^2} \left(\frac{\Delta_{ij}}{R_0} \right)^\alpha, \quad \dot{e}_\alpha = \sum_{i \neq j \in \text{Jet}} \frac{p_{t_i} p_{t_j}}{p_t^2} \left(\frac{2p_i \cdot p_j}{p_{t_i} p_{t_j} R_0^2} \right)^{\frac{\alpha}{2}}$$

Linked to the mass
of the particles

Massless theory

e_α coincides with \dot{e}_α in the collinear limit

Massive theory

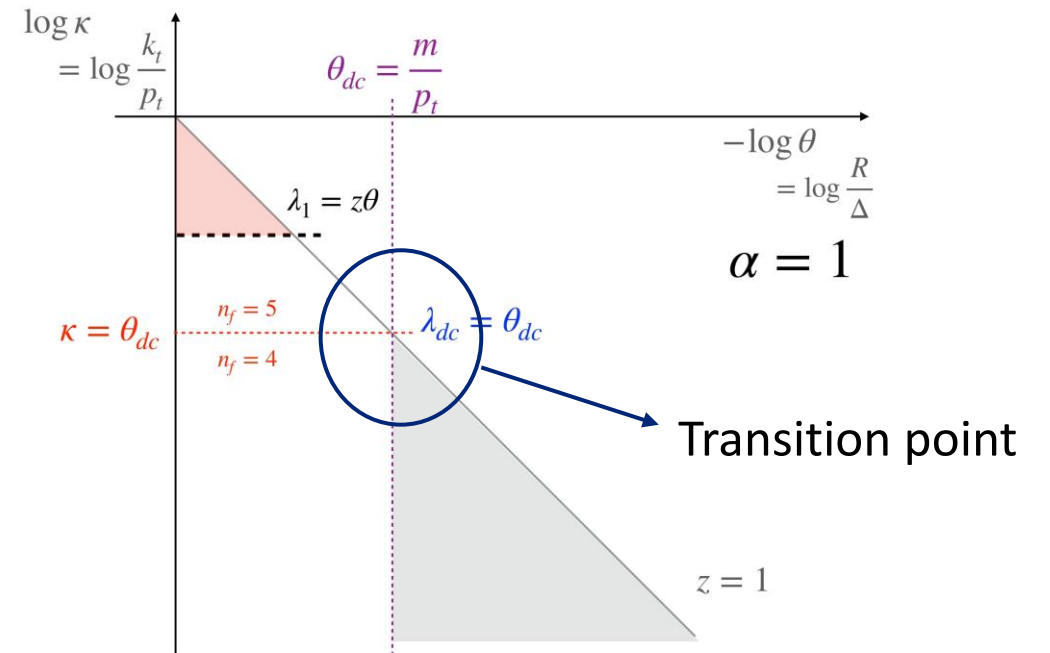
e_α does not coincide with \dot{e}_α in the quasi-collinear limit

Transition from 5 to 4 flavours

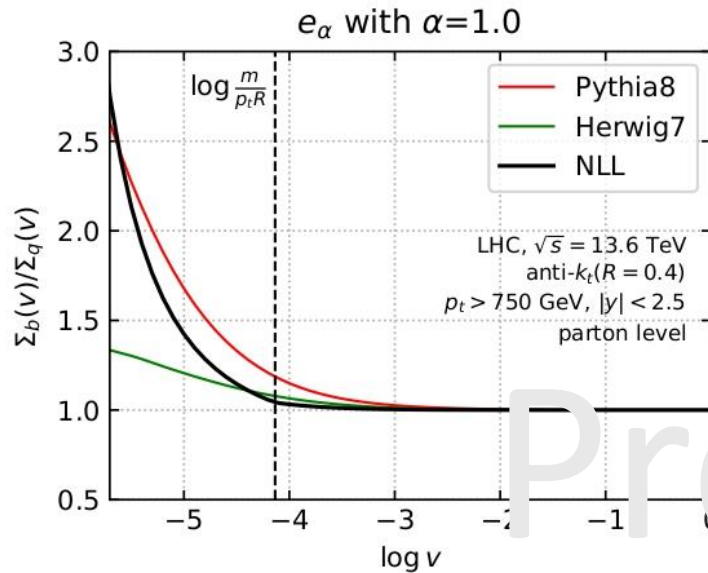
Differential distribution exhibit discontinuity for any value of α in the transition



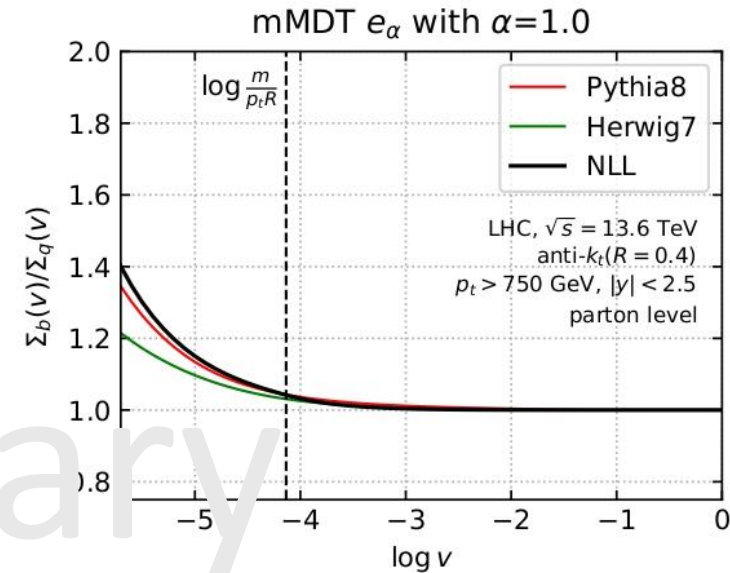
- To smooth the transition we decide to incorporate fixed order calculation
- These are NNLL contributions, which depend on the specific definition of the observable



Comparison Analytics and Monte Carlo



Ungroomed



Groomed ($\beta = 0$)

- Plot of the ratio of the cumulative distribution massive/massless (NGL simplify in the ungroomed case)
- It appears that the dead cone effect manifests earlier than predicted by theoretical calculations ($v \simeq \frac{m^\alpha}{p_T^\alpha R_0^\alpha}$).

Conclusions and Outlook

- We have discussed about mass effects on different types of variables
- Good agreement with Monte Carlo for θ_g and z_g distribution

- The situation for angularities and ECF is more complicated
→ need to access NNLL accuracy for heavy flavoured jets

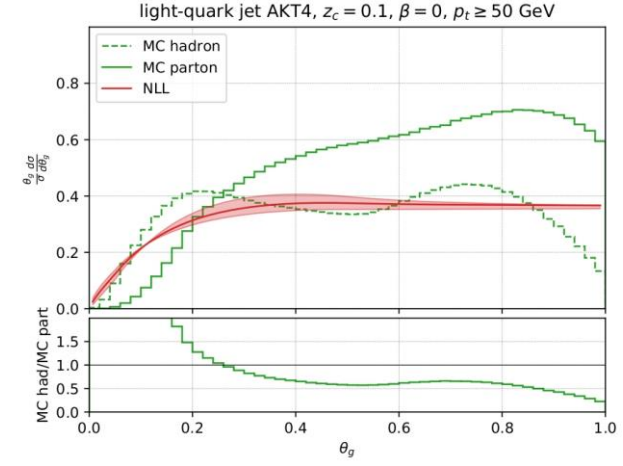
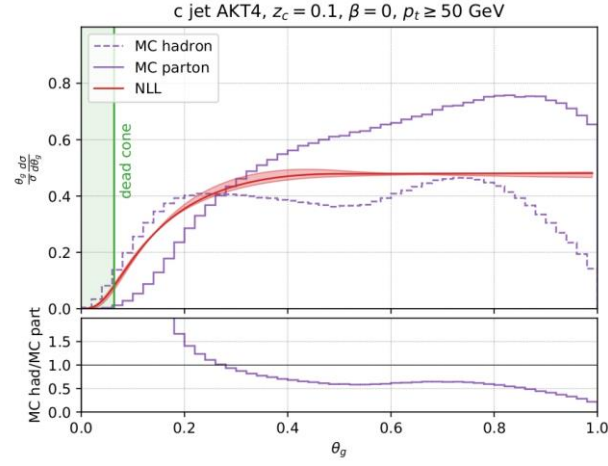
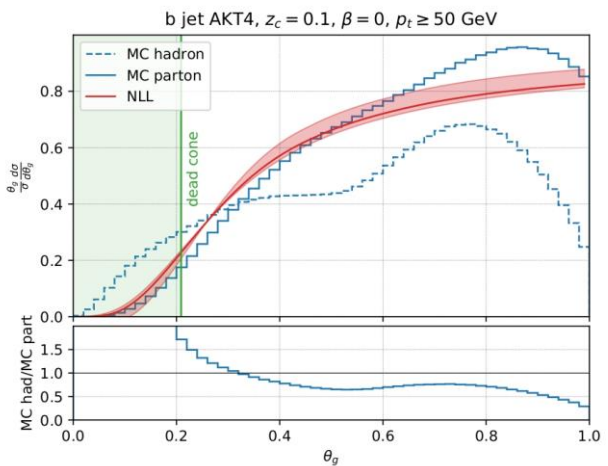
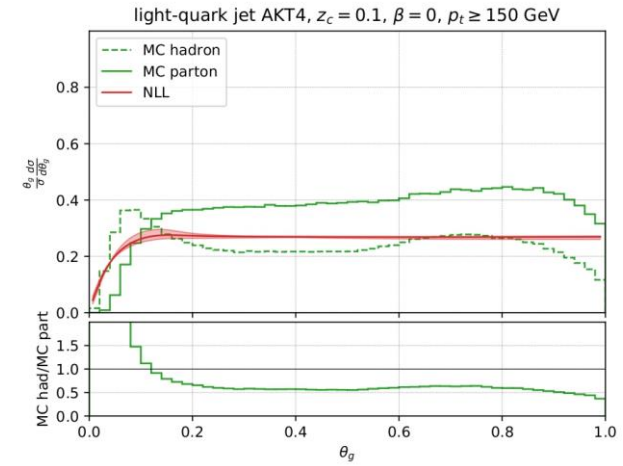
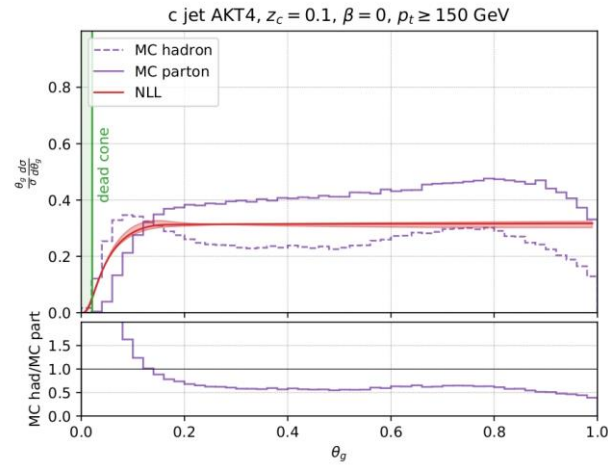
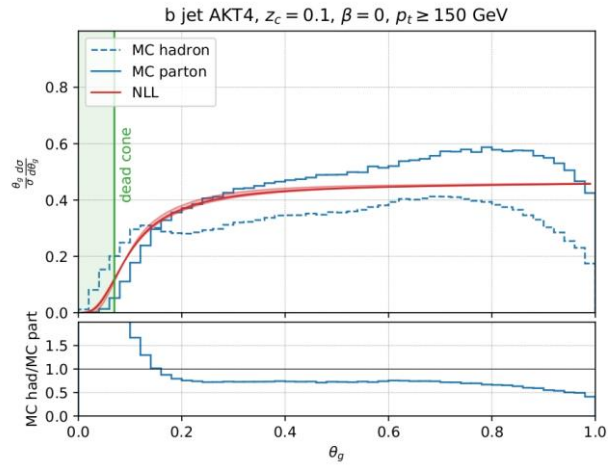
- More observable to study: Lund plane density

**Special acknowledgment to the Cost Action COMETA CA22130
(Comprehensive Multiboson Experiment Theory Action)**

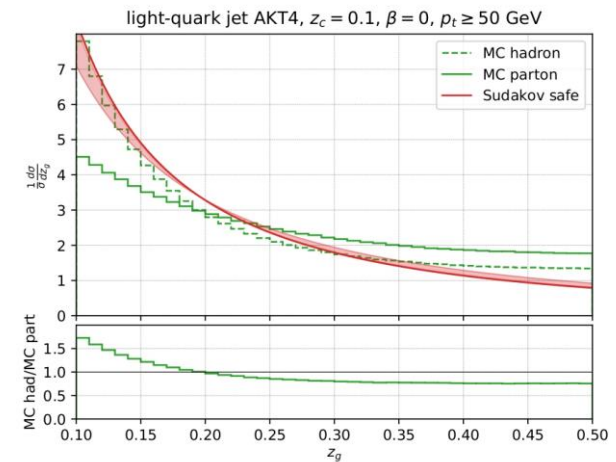
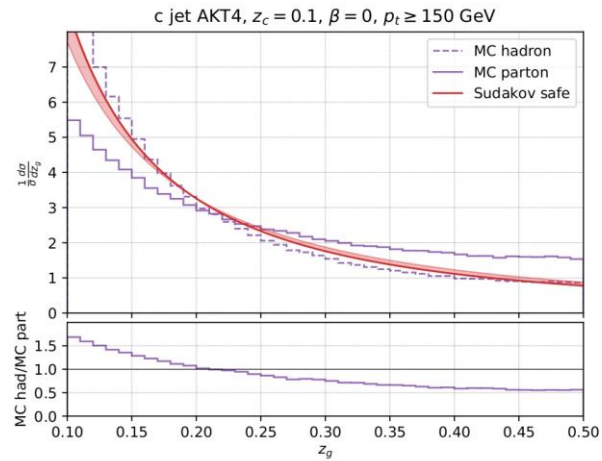
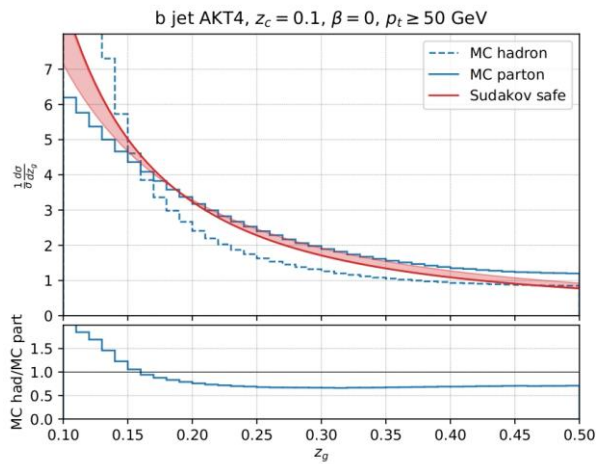
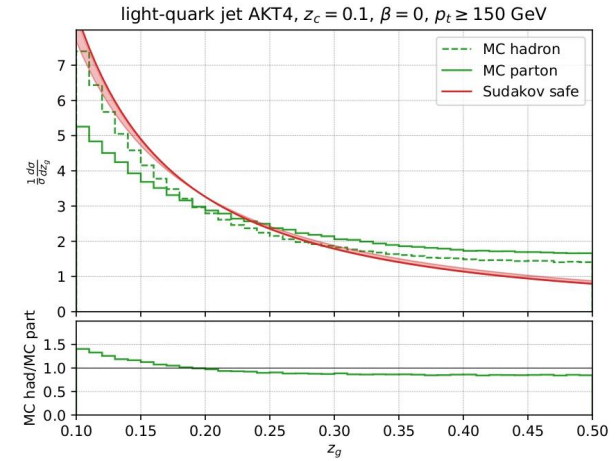
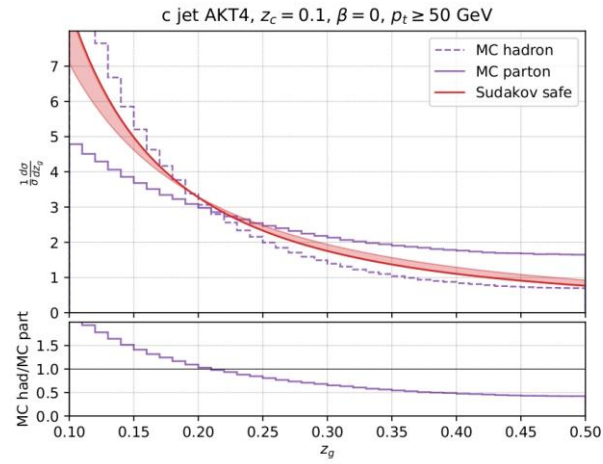
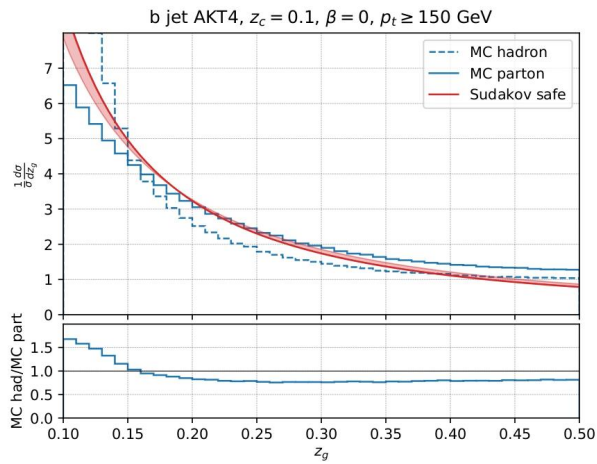


Thanks for your attention !!

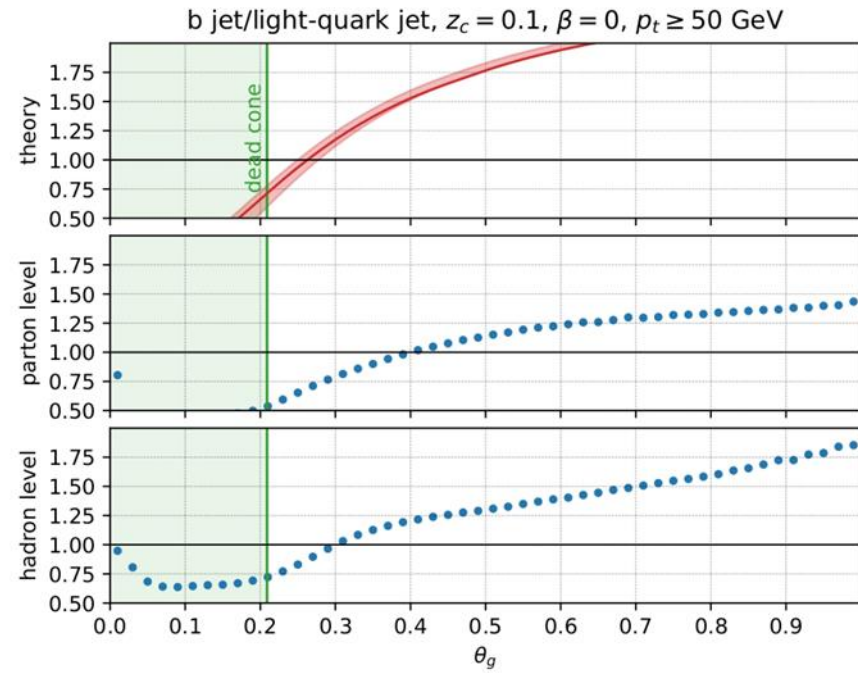
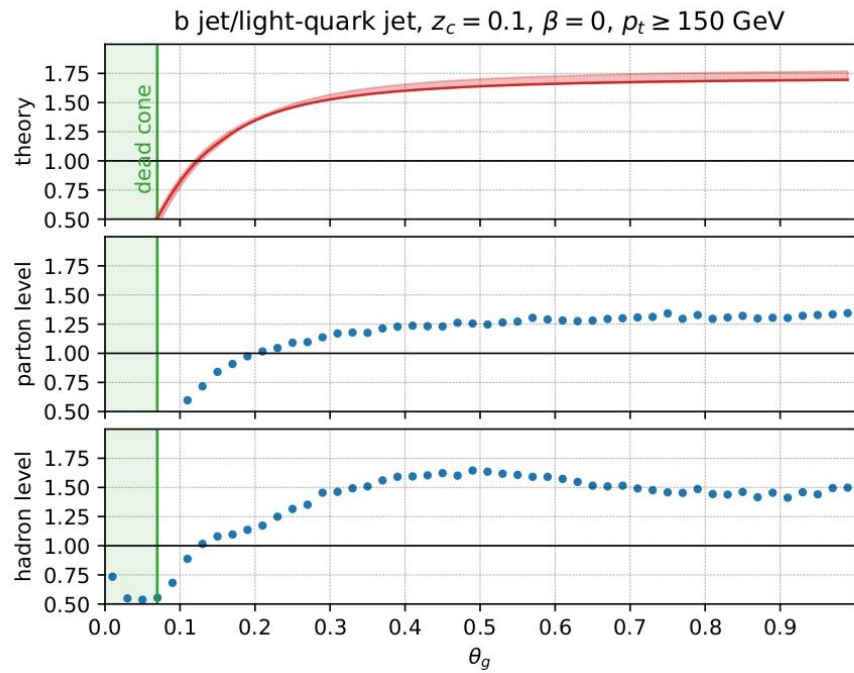
θ_g distribution: Comparison with Monte Carlo



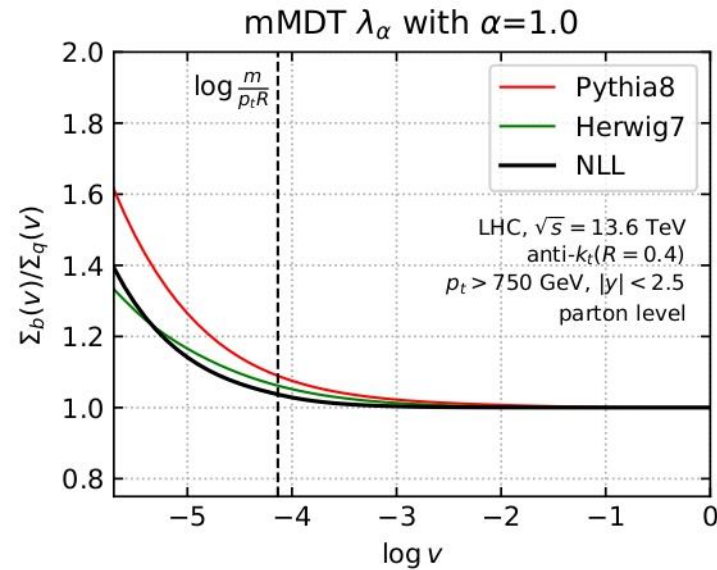
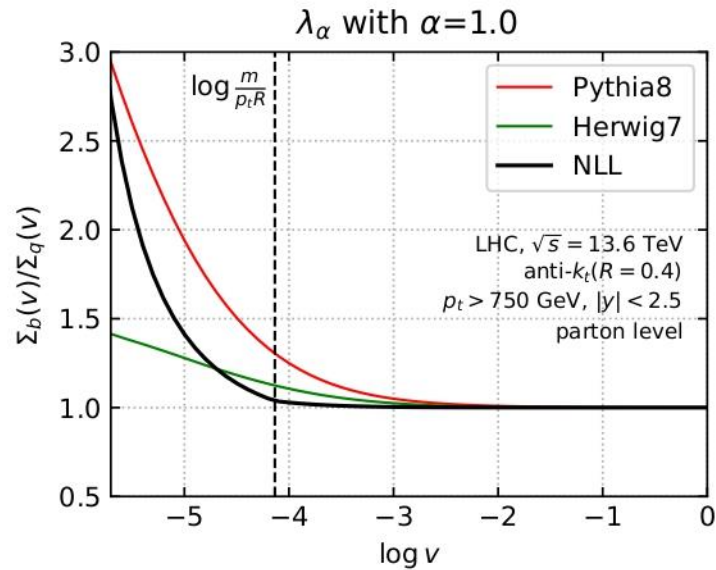
z_g distribution: Comparison with Monte Carlo



Ratio plots



Angularities: λ_α for pp collisions



$$\lambda_\alpha = \sum_{i \in \text{Jet}} \frac{p_{t_i}}{p_t} \left(\frac{\Delta R_i}{R_0} \right)^\alpha$$

Mass effects manifests earlier than predicted