NSL collinear fragmentation and parton showers

Jack Helliwell

Based on:

arxiv:2402.05170 Van Beekveld, Dasgupta, El-Menoufi, JH, Karlberg, Monni
arxiv:2407.xxxxx Van Beekveld, Dasgupta, El-Menoufi, JH, Monni, Salam

Parton Showers and Resummation 2024

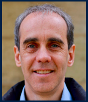
- Parton showers play a crucial role in the interpretation of collider data.

- A natural way to classify the accuracy of a parton shower is its logarithmic accuracy.

- Several NNLL accuracy milestones reached already in PanScales showers.
  2406.02661: van Beekveld, Dasgupta, El-Menoufi, Ferrario Ravasio, Hamilton, JH, Karlberg, Monni, Salam, Scyboz, Soto-Ontoso, Soyez, 2307.11142: Ferrario Ravasio, Hamilton, Karlberg, Salam, Scyboz, Soyez

PanScales People

Oxford



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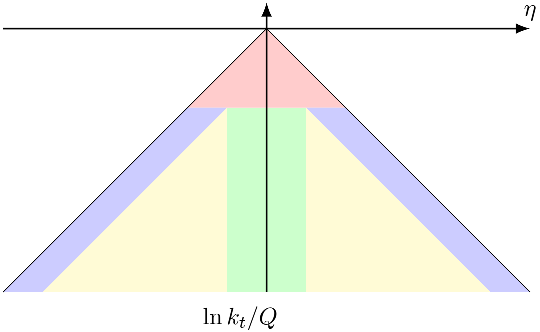
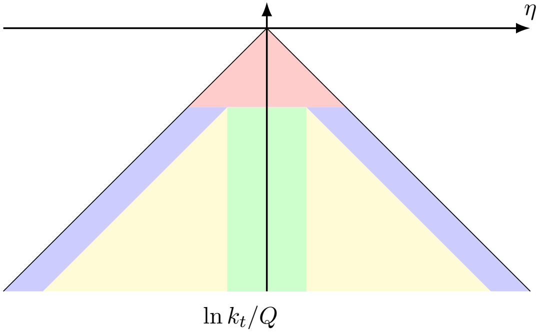
Alba Soto-Ontoso

PanScales current members
A project to bring logarithmic understanding and accuracy to parton showers

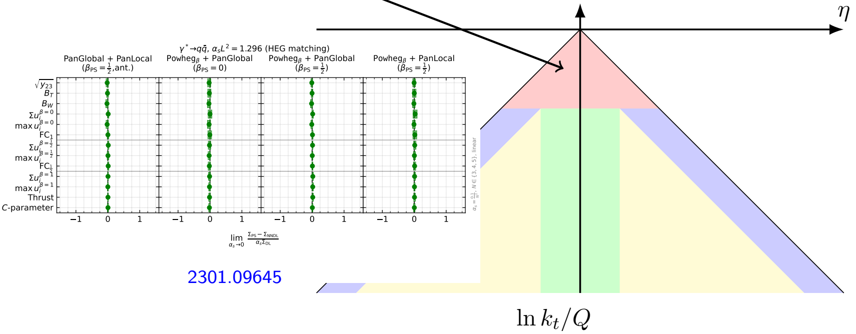
What do we need for NNLL accuracy?



Start with an NLL shower



NLO Matching
NNDL event shape accuracy



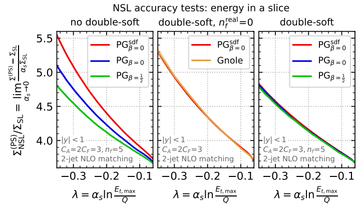
2301.09645

What is needed for NNLL?

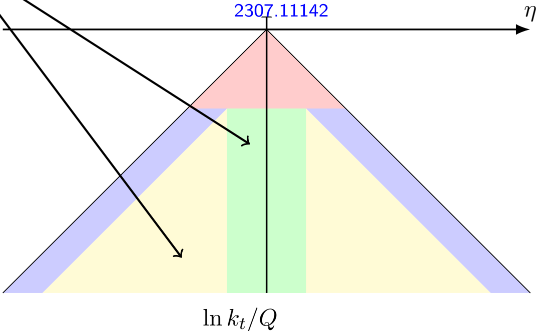
Double Soft corrections

NSL Non-global logarithms

NNDL Multiplicity



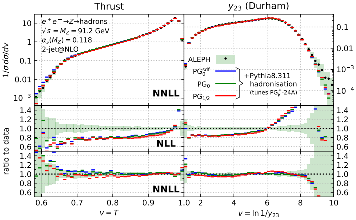
2307.11142



Integrated soft collinear rate to α_s^3

NNLL Event Shapes

*(Also needs integrated hard collinear rate to α_s^2)



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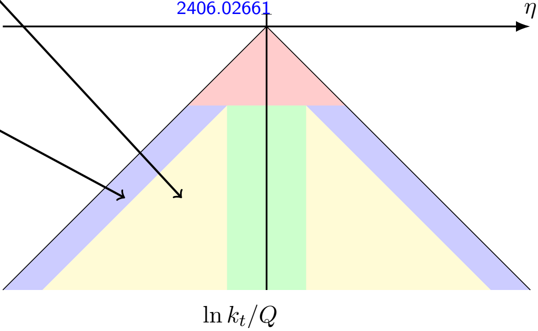
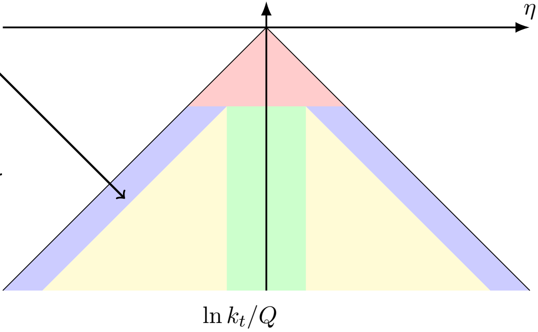


Triple collinear and corresponding virtual corrections

NSL DGLAP
NSL Small R jets



This talk

*Not in a full dipole shower

- Reference observables
- Collinear parton shower algorithm
- Tests of logarithmic accuracy
- Summary

- We need resummations differentially sensitive to the triple collinear region to compare an NSL shower prediction to.

- NSL fragmentation function evolution - well known Curci, Furmanski, Petronzio 1980 .

- NSL Small radius jet evolution

- We are interested in the energy spectrum of small radius jets (inclusive micro-jet spectrum).

$$\frac{1}{\sigma_0}\frac{d\sigma^{\text{jet}}}{dz} \equiv \sum_{i=q,\bar{q},g}\int_z^1 \frac{d\xi}{\xi} C_i^{\text{jet}}(\xi,\mu,Q)\, D_i^{\text{jet}}\left(\frac{z}{\xi},\mu,E\,R\right)$$

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- At LL the micro-jet fragmentation function (D_i^{jet}) evolves according to DGLAP.

$$\frac{dD_k^{\text{jet}}\left(z,\mu,E\,R\right)}{d\ln\mu^2} = \sum_i \int_z^1 \frac{d\xi}{\xi} \hat{P}_{ik}\left(\frac{z}{\xi},\mu\right) D_i^{\text{jet}}\left(\xi,\mu,E\,R\right)\,.$$

- At NSL we need the coefficient function to $\mathcal{O}(\alpha_s)$.
- And the two loop anomalous dimension

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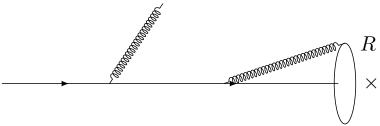
- And the two loop anomalous dimension - Closely related to (but not equal to) the timelike DGLAP kernel.

We can directly calculate the difference between the DGLAP and small R anomalous dimensions at 2 loops e.g

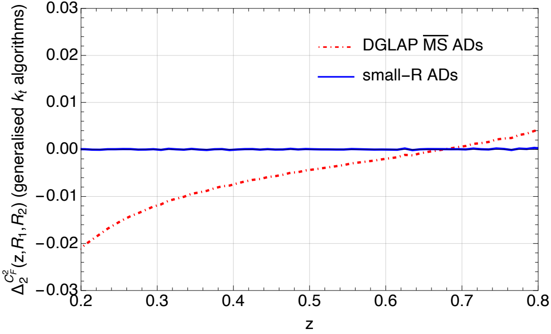
$$\delta\hat{P}^{(1)}_{qq}(z) \sim$$



$$\times\Big(\delta(z - xz_p) - \delta(z - x)\Big)$$

$$\delta\hat{P}^{(1)}_{qq}(z) \equiv \Big(2\,\ln z\,\hat{P}^{(0)}_{qq}\Big) \otimes \hat{P}^{(0)}_{qq}$$

- Similar considerations in other colour channels.
- Can test at α_s^2 with EVENT2 through the difference in the energy spectrum between two small radii.
- Zero shows consistency between the calculation and EVENT2.



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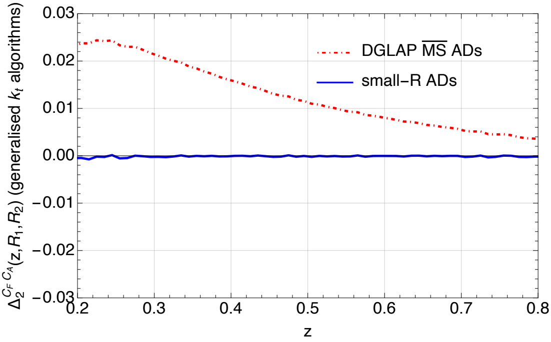


- Now we consider defining jets with a small y_{cut}, instead of a small radius
- This replaces the angular cuttoff of with a k_t-like cuttoff

$$y_{ij} = 2\frac{\min\left(E_i^2, E_j^2\right)}{Q^2}\left(1 - \cos(\theta_{ij})\right) < y_{\mathrm{cut}}$$

- In this case the 2-loop anomalous dimension coincides with that of DGLAP
- This suggests a correspondance between the scale of the $\overline{\mathrm{MS}}$ fragmentation function and a transverse momentum cuttoff

A NSL collinear shower

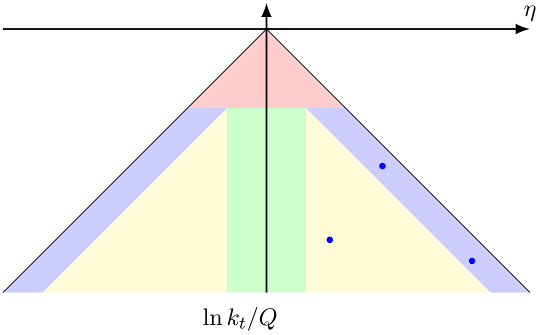
- Construction of an NSL ($\alpha_s^n L^{n-1}$) collinear parton shower algorithm
- Will consider the case of a shower designed specifically for non-singlet collinear fragmentation observables.
- The goal is to implement the ideas and understanding developed and tested here to a full shower.

$$d\mathcal{P}_i = \frac{\alpha_s^{\text{eff}}}{\pi}\frac{dv_i}{v_i}dz_i\frac{d\phi_i}{2\pi}P\exp\left[-\int_{v_i}^{v_{i-1}}\frac{\alpha_s^{\text{eff}}}{\pi}\frac{dv}{v}dz\frac{d\phi}{2\pi}P\right]$$

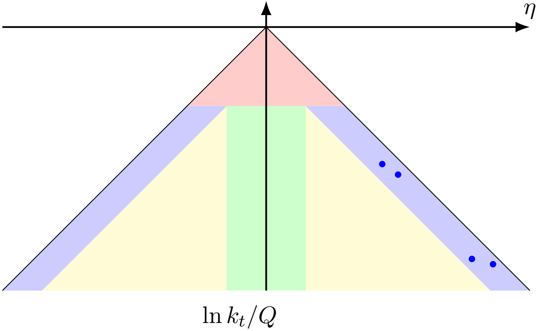
$$\alpha_s^{\text{eff}} = \alpha_s(v_i)\left[1+\frac{\alpha_s(v_i)}{2\pi}K(z_i)\right]$$

- $K(z_i)$ defines an NLO inclusive emission probability ($=K_{\text{CMW}}$ in the soft limit), which accounts for virtual corrections.
- P is the shower matrix element.

- At NLL accuracy the real ME needs to be correct for emissions logarithmically separated in phase space (shown on the Lund plane) 2002.11114, Dasgupta, Dreyer, Hamilton, Monni, Salam, Soyez .



- At NNLL, must additionally reproduce the correct matrix element for pairs of emissions close by in phase space 2002.11114, Dasgupta, Dreyer, Hamilton, Monni, Salam, Soyez .
- Here, this means triple collinear. configurations



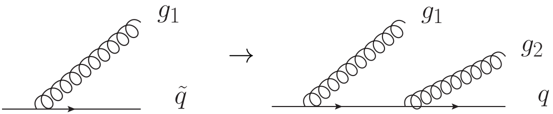
- Having accepted a first emission, we use the following shower matrix element

$$P = J(\Phi_i, \Phi_p)\frac{p_{1\to 3}(\Phi_i, \Phi_p)}{P_{qq}(z_p)}\Theta(v_{g_i q_i} < v_{g_p q_i})$$

So that for two emissions we reproduce the correct triple collinear matrix element and phase space

- Applying this for successive emissions, the shower will correctly reproduces the matrix elements for pairs of emissions, strongly ordered with respect to other pairs

- A typical shower would generate emissions with the ordering $\Theta(v_{g_1,\tilde{q}} > v_{g_2,q})$



- When applying triple collinear matrix element corrections, the ordering variable needs to be symmetric between the 2 emissions to exactly account for the $1/2$ symmetry factor.

- Generate a disordered emission and apply the ordering condition, $\Theta(v_{g_1,q} > v_{g_2,q})$ on the three particle kinematics.

Inclusive emission probability - gluon branching channels

- We consider a non-singlet shower, so can treat gluon branching inclusively ($C_F C_A$ and $C_F n_f$ channels).
- $K(z)$ found by fixing the kinematics of a collinear gluon and integrate inclusively over a subsequent (collinear) branching combining with the corresponding 1-loop correction.
- very closely related to $B_2^q(z)$ from 2109.07496: Dasgupta, El-Menoufi .

$$P_{qq}(z)K(z) = \frac{2C_F}{1-z}K_{\mathrm{CMW}} + B_2^{q,\mathrm{nab}}(z)$$

Inclusive emission probability - C_F^2 channel

- More care is needed in defining the inclusive emission probability as subsequent real emissions are resolved.

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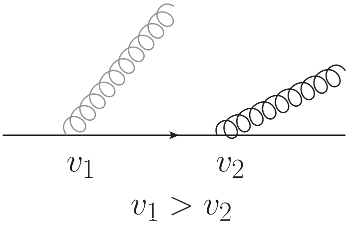
- $K(z)$ encodes the probability of producing an emission, inclusive over the virtual corrections and subsequent branchings that are correlated with the presence of that emission.



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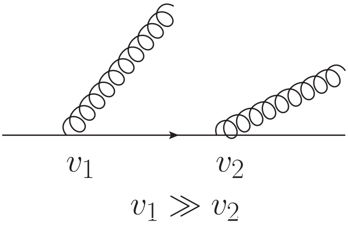

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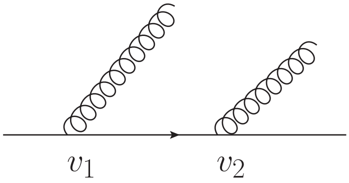


$v_1 \sim v_2$

Inclusive emission probability - C_F^2 channel

- More care is needed in defining the inclusive emission probability as subsequent real emissions are resolved.

- $K(z)$ encodes the probability of producing an emission, inclusive over the virtual corrections and subsequent branchings that are correlated with the presence of that emission.

Start by considering the case of $e^+e^- \to q\bar{q}g$

$$\frac{\alpha_s}{2\pi}K(z_g) = \frac{V_{q\bar{q}g}}{B_{q\bar{q}g}} - \frac{V_{q\bar{q}}}{B_{q\bar{q}}} + \int_0^{\tilde{v}_g} \frac{d\Phi_{q\bar{q}ij}}{d\Phi_{q\bar{q}g}} \frac{B_{q\bar{q}ij}}{B_{q\bar{q}g}} - \int_0^{v_g} \frac{d\Phi_{q\bar{q}g'}}{d\Phi_{q\bar{q}}} \frac{B_{q\bar{q}g'}}{B_{q\bar{q}}}$$

*A similar equation appears in the context of embedding NLO 3-jet with NLO 2-jet in a shower 1303.4974, 1611.00013, 2108.07133; Li, Skands et.al .

Taking the collinear limit:

Factorises into a process dependant virtual multiplied by a splitting kernel plus the one loop correction to the splitting kernel

$$\frac{\alpha_s}{2\pi}K(z_g) = \frac{V_{q\bar{q}g}}{B_{q\bar{q}g}} - \frac{V_{q\bar{q}}}{B_{q\bar{q}}} + \int_0^{\tilde{v}_g} \frac{d\Phi_{q\bar{q}ij}}{d\Phi_{q\bar{q}g}} \frac{B_{q\bar{q}g_1g_2}}{B_{q\bar{q}g}} - \int_0^{v_g} \frac{d\Phi_{q\bar{q}g'}}{d\Phi_{q\bar{q}}} \frac{B_{q\bar{q}g'}}{B_{q\bar{q}}}$$

$$\frac{V_{q\bar{q}g}}{B_{q\bar{q}g}} - \frac{V_{q\bar{q}}}{B_{q\bar{q}}} = \frac{P^{(1)}_{q\to qg}(s_{qg}, z_g, \epsilon, \mu^2)}{P_{qq}(z, \epsilon)}$$

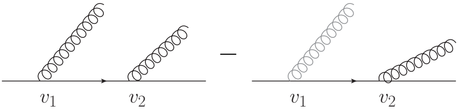
The process dependence cancels between the two terms

Taking the collinear limit:

Factorises into a product of $B_{q\bar{q}}$ with a $1 \to 3$ or $1 \to 2$ splitting function

$$\frac{\alpha_s}{2\pi}K(z_g) = \frac{V_{q\bar{q}g}}{B_{q\bar{q}g}} - \frac{V_{q\bar{q}}}{B_{q\bar{q}}} + \underbrace{\int_0^{\tilde{v}_g} \frac{d\Phi_{q\bar{q}ij}}{d\Phi_{q\bar{q}g}} \frac{B_{q\bar{q}g_1g_2}}{B_{q\bar{q}g}} - \int_0^{v_g} \frac{d\Phi_{q\bar{q}g'}}{d\Phi_{q\bar{q}}} \frac{B_{q\bar{q}g'}}{B_{q\bar{q}}}}_{\text{Evaluated with shower kinematics and ordering}}$$

Evaluated with shower kinematics and ordering



Taking the collinear limit:

Factorises into a product of $B_{q\bar{q}}$ with a $1 \to 3$ or $1 \to 2$ splitting function

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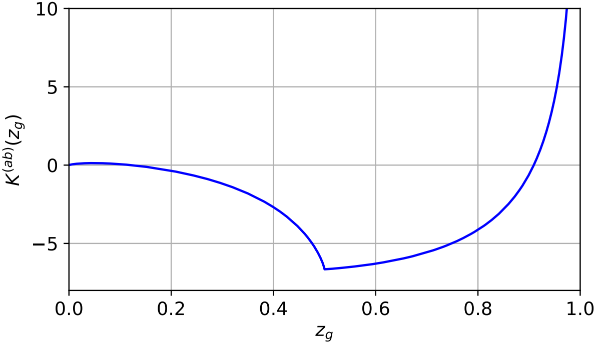
This is crucial so that the shower Sudakov will properly cancel this term down to the cutoff scale.

Evaluated with shower kinematics and ordering

Again, the process dependence cancels between the two terms.

For the the ordering variable $v_{ij} = \min(E_i, E_j)\theta_{ij}$, we have



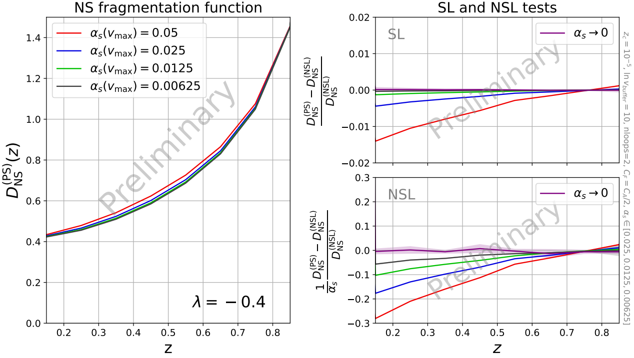
- The kink at $z = 1/2$ is a result of our choice of ordering variable

- $K(z)$ is process independent (and can be computed without reference to process dependent terms).

- This definition is also applicable in the soft limit (consistent with the approaches in 2307.11142, 2406.02661).

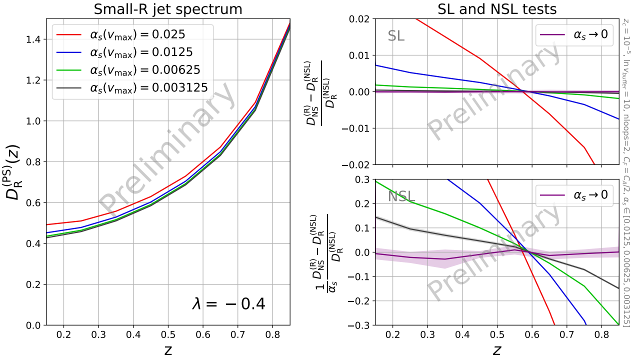
- Can also be applied (now with process dependence) to matching between fixed-order and parton showers 1303.4974, 1611.00013, 2108.07133; Li, Skands et.al .

- Non-singlet partonic fragmentation function evolution
- Comparison between shower and HOPPET DGLAP evolution code
- Zero in the $\alpha_s \to 0$ limit signals NSL agreement



*$C_F = C_A/2 = 3/2$ to avoid the N_c suppressed $q \to q\bar{q}q$ contribution

- Non-singlet small radius jet spectrum
- same as fragmentation function at SL but distinct at NSL
- NSL prediction implemented in HOPPET



*$C_F = C_A/2 = 3/2$ to avoid the N_c suppressed $q \to q\bar{q}q$ contribution

Summary

- Presented 2-loop anomalous dimension for small R and small y_{cut} jets
- Described the construction of a shower with NSL accuracy for non-singlet collinear fragmentation observables.
- Presented a definition of the NLO inclusive emission probability which is applicable beyond the collinear limit and is consistent with 2307.11142, 2406.02661 .
- The goal is to implement the ideas developed here into the PanScales showers to give almost complete NNLL accuracy (Double soft and soft collinear inclusive emission probability are already implemented).

Backup slides

To test the single logarithmic accuracy of the shower, we construct

$$\frac{D_{\mathrm{NS}}^{(\mathrm{PS})}(z, v_{\min}, v_{\max})}{D_{\mathrm{NS}}^{(\mathrm{NSL})}(z, v_{\min}, v_{\max})} - 1\,.$$

and take the $\alpha_s \to 0$ limit.

For NSL accuracy, the relevant quantity is

$$\frac{1}{\alpha_s}\frac{D_{\mathrm{NS}}^{(\mathrm{PS})}(z, v_{\min}, v_{\max})}{D_{\mathrm{NS}}^{(\mathrm{NSL})}(z, v_{\min}, v_{\max})} - 1\,.$$

- We start the shower at scale $v_{\max}$ with a quark with momentum fraction $z = 1$.
- Run the shower down to $v_{\min}$ and measure the energy distribution of quarks.

$$\lambda = \alpha_s(v_{\max}) \ln(v_{\min}/v_{\max})$$

$$D_{\mathrm{NS}}^{(\mathrm{NSL})}(z, v_{\min}, v_{\max}) = C(v_{\min}) \otimes \exp\left[\int_{v_{\min}^2}^{v_{\max}^2} \frac{dv^2}{v^2} \hat{P}(v)\right] \otimes C^{-1}(v_{\max})$$

The coeficient function accounts for the scheme change between $\overline{\mathrm{MS}}$ fragmentation function, using dimensional regularisation, and the shower scheme, using a cuttoff regularisation scheme.

- Start the shower at sufficiently high scale with a quark with momentum fraction $z = 1$, so that, for any $z \in [z_c, 1 - z_c]$, the angular scale R_0 can be generated by the shower.

- Run the shower down to a low scale, so that for any $z \in [z_c, 1 - z_c]$, the angular scale R can be generated by the shower.

- Veto emissions with angle larger than R_0, so as to mimic starting with a jet of radius R_0.

- Cluster jets with radius R, and study the energy spectrum of jets containing the quark.

$$D_{\mathrm{R}}^{(\mathrm{NSL})}(z, ER, ER_0) = C^{(R)}(ER) \otimes \exp\left[2\int_{ER}^{ER_0} \frac{d\mu}{\mu}\hat{P}^{(R)}(\mu, ER)\right] \otimes [C^{(R)}(ER_0)]^{-1},$$

- As we don't implement matching in our shower, the hard matching coefficient is replaced by $[C^{(R)}(ER_0)]^{-1}$ which accounts for starting with a jet of radius R_0.

- We find that the scale associated with $\delta\hat{P}_{ik}^{(1)}$ is not simply μ but ER.

$$\hat{P}_{ik}\left(z,\mu,E\,R\right)=\frac{\alpha_s(\mu^2)}{2\pi}\left(\hat{P}_{ik}^{(0)}(z)+\frac{\alpha_s(\mu^2)}{2\pi}\hat{P}_{ik}^{(1),\,\mathrm{AP}}(z)-\frac{\alpha_s(E^2R^2)}{2\pi}\delta\hat{P}_{ik}^{(1)}\right)$$

so that:

$$\frac{dD_k^{\mathrm{jet}}\left(z,\mu,E\,R\right)}{d\ln\mu^2}=\sum_i\int_z^1\frac{d\xi}{\xi}\hat{P}_{ik}\left(\frac{z}{\xi},\mu,E\,R\right)D_i^{\mathrm{jet}}\left(\xi,\mu,E\,R\right)\,.$$

- This can be derived using an NSL generating functional approach 2307.15734
- Emerges as a consequence of the change in energy of the quark over the course of the evolution

Small R jet evolution - Varying the lower boundary

- This effect can be incorporated into the structure of an evolution equation where now it is the low scale being varied

$$\frac{dD^{\text{jet}}_{\text{NS}}(z,E,\mu)}{d\ln\mu^2} = -\,\hat{P}^{(0)}_{qq}(z)\otimes\left(\frac{\alpha_s(z^2\mu^2)}{(2\pi)}\,D^{\text{jet}}_{\text{NS}}(z,E,\mu)\right) - \hat{P}^{(1),\,\text{NS}}_{qq}(z)\otimes\left(\frac{\alpha_s^2(z^2\mu^2)}{(2\pi)^2}\,D^{\text{jet}}_{\text{NS}}(z,E,\mu)\right)$$